DIS at low and high transverse momentum: matches and mismatches

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Jefferson Lab
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Based on:

AB, Daniel Boer, Markus Diehl, Piet J. Mulders

arXiv:0803.0227 [hep-ph]

Outline

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- Scales in semi-inclusive DIS

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- Calculations at high and low transverse momentum

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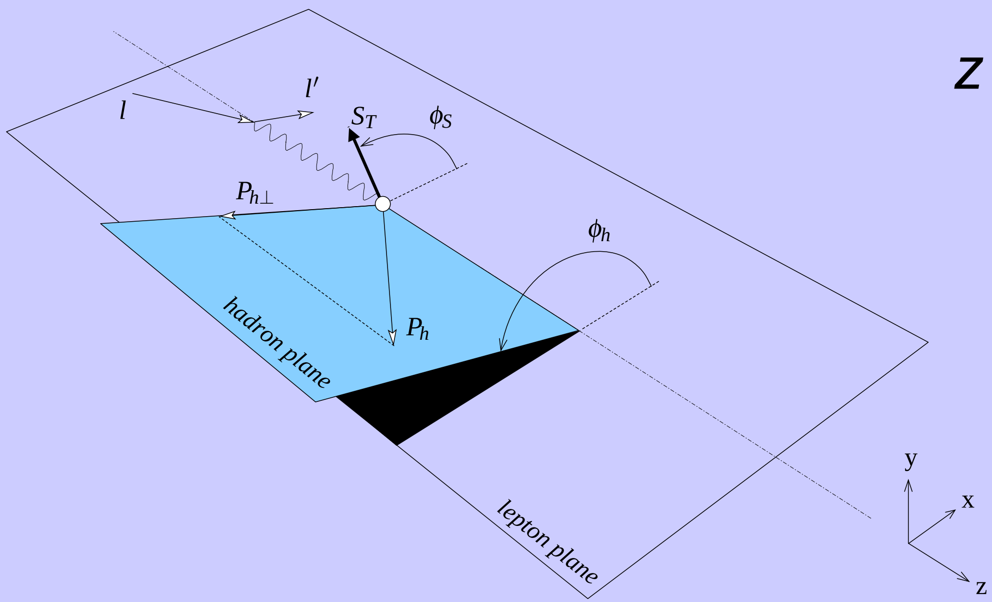
- Scales in semi-inclusive DIS
- Calculations at high and low transverse momentum
- Selected examples of matching and mismatching structure functions
- Summary of all structure functions

Semi-inclusive DIS

$$\ell(l) + p^{\uparrow}(P) \to \ell(l') + h(P_h) + X$$

$$x = \frac{Q^2}{2P \cdot (l - l')}$$

$$z = \frac{P \cdot P_h}{P \cdot (l - l')}$$



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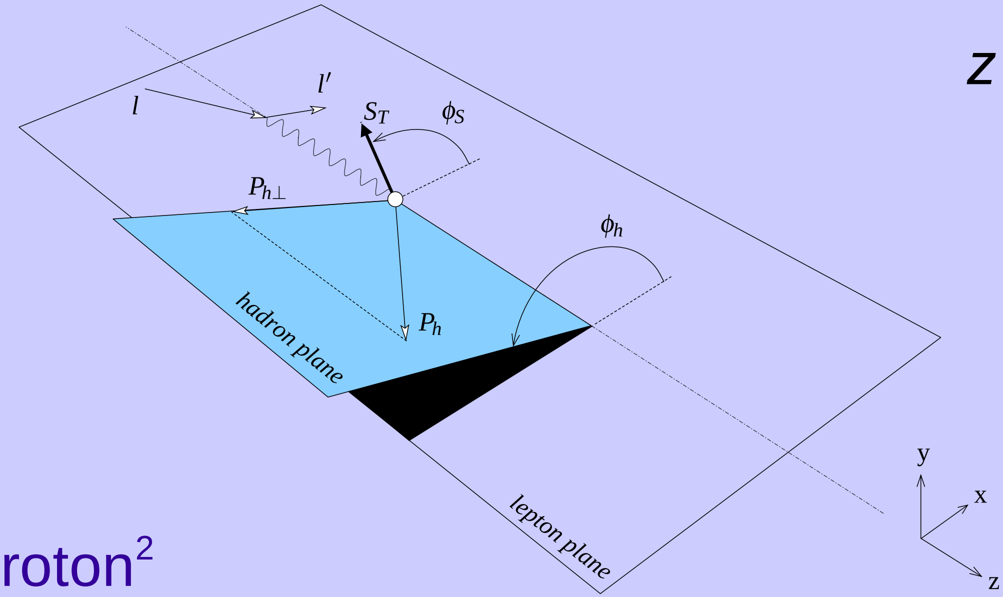

$M^2 =$ mass of proton^2

$Q^2 = -(l-l')^2 =$ virtuality of photon

$P_{h\perp}^2 =$ transverse momentum of pion^2

Relation between $P_{h\perp}$ and q_T

$$\frac{P_{h\perp}^{\mu}}{z} = -q_T^{\mu} - 2x\,\frac{q_T^2}{Q^2}\,P^{\mu}$$

$$q_T^2 \approx P_{h\perp}^2 \big/ z^2$$

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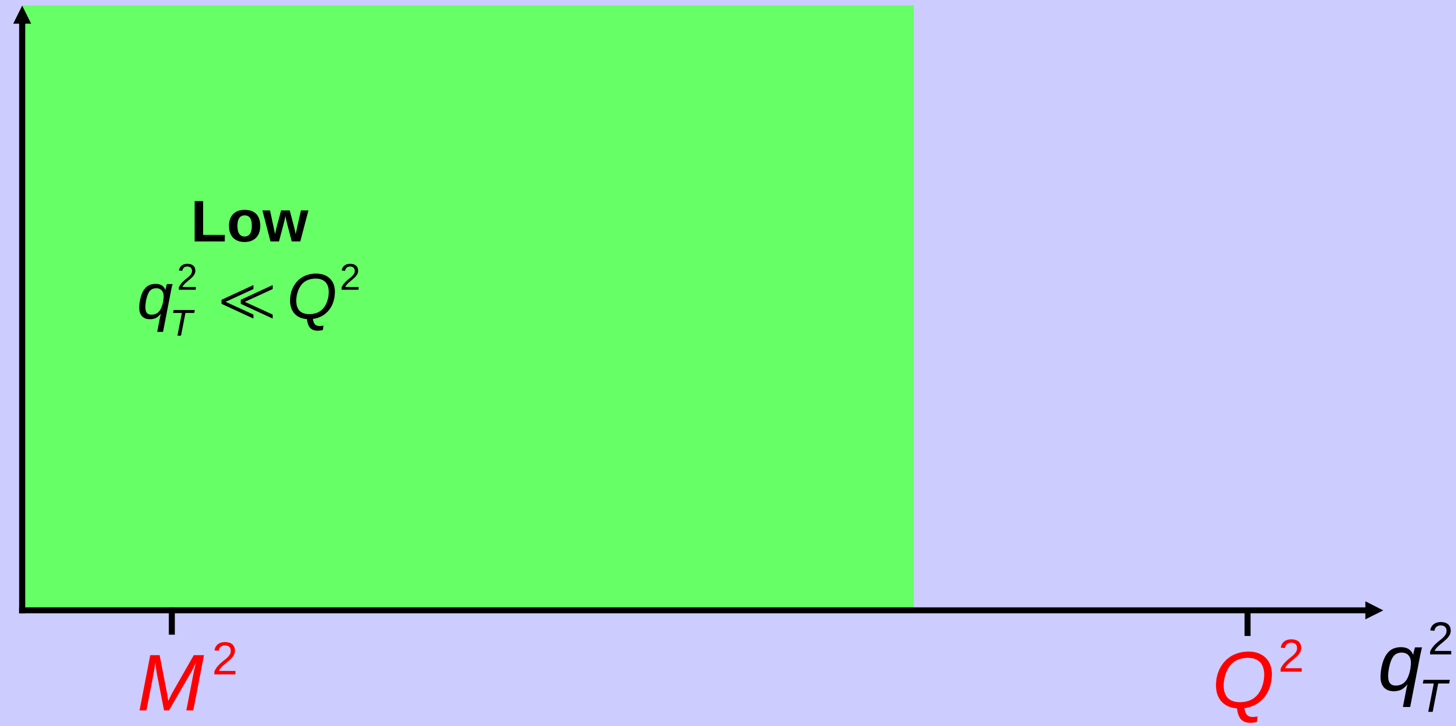
NOTE: q_T taken to be a positive scalar

High and low transverse momentum



High and low transverse momentum

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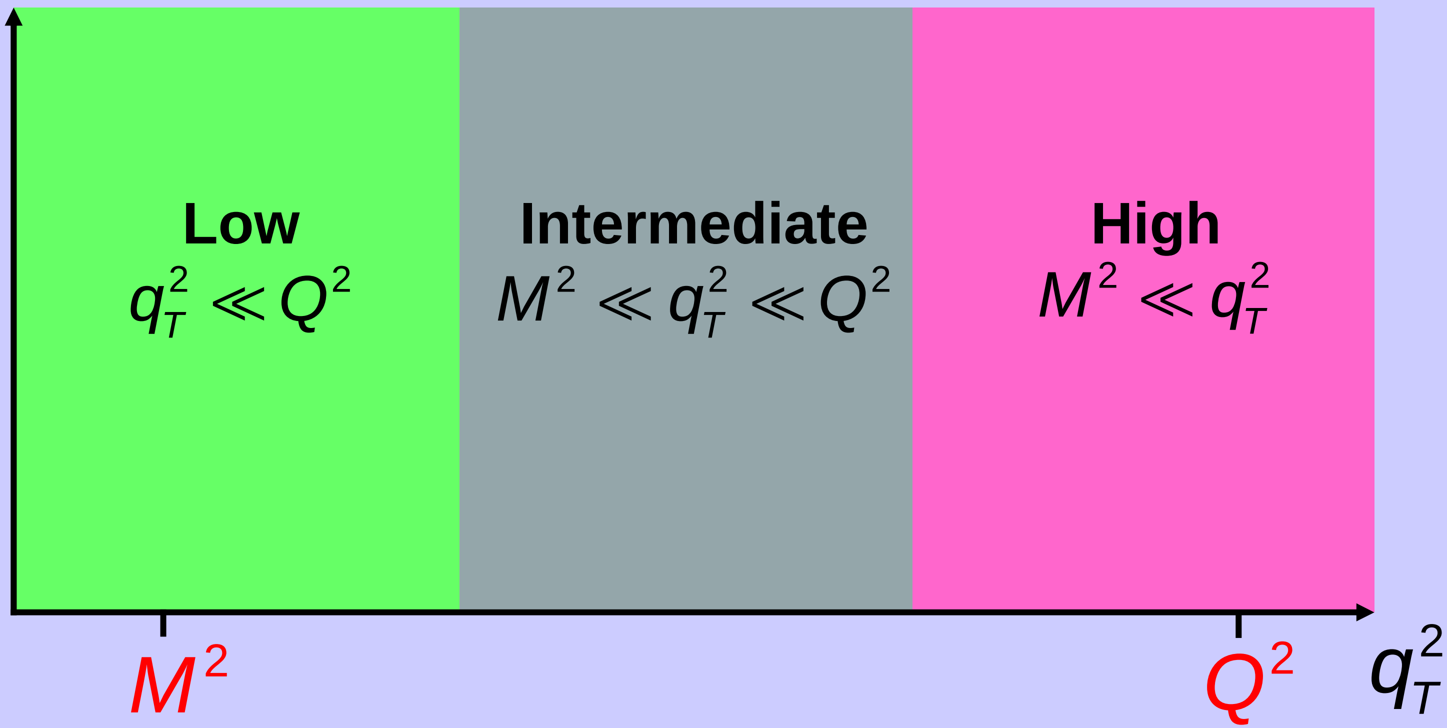
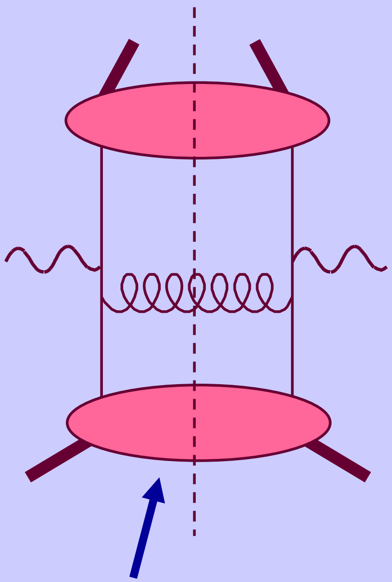


Calculation at high q_T



Twist-2 integrated PDFs

e.g. $f_1(x)$

see e.g. Koike, Nagashima, Vogelsang, NPB744 (06)

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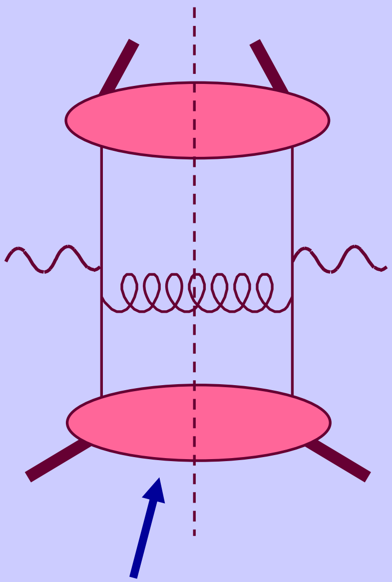

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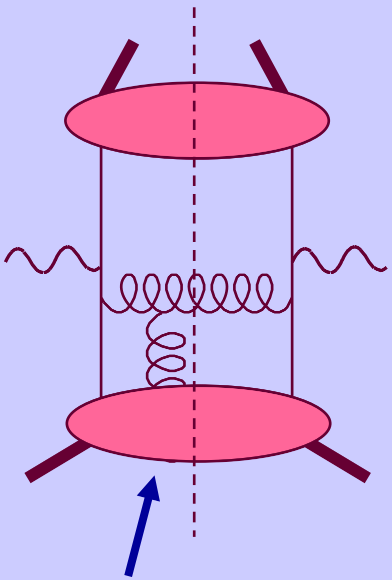


Twist-3 integrated PDFs

e.g. $G_F(x_1, x_2)$

Eguchi, Koike, Tanaka, NPB752 (06) & NPB763 (07)

From high to intermediate

Take the limit $\dfrac{q_T^2}{Q^2} \to 0$

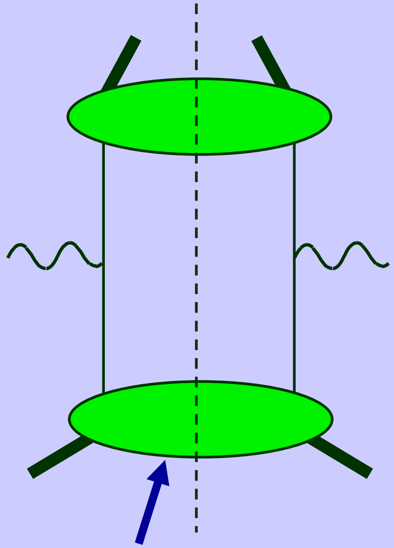
From high to intermediate

Take the limit $\frac{q_T^2}{Q^2} \to 0$

Easier to say than to do…

see e.g. Meng, Olness, Soper, PRD54 (96)

Calculation at low q_T

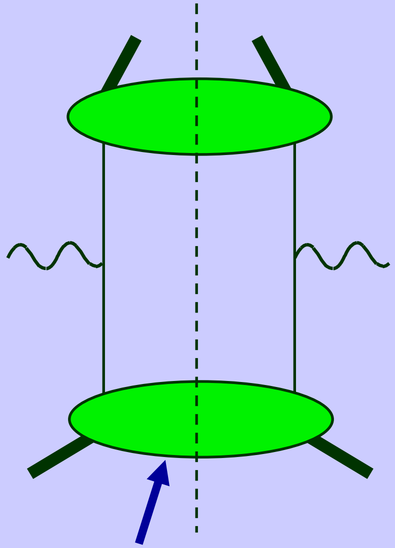


Twist-2 TMD PDFs
(a.k.a. unintegrated PDFs)

e.g. $f_1(x, p_T^2)$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Calculation at low q_T



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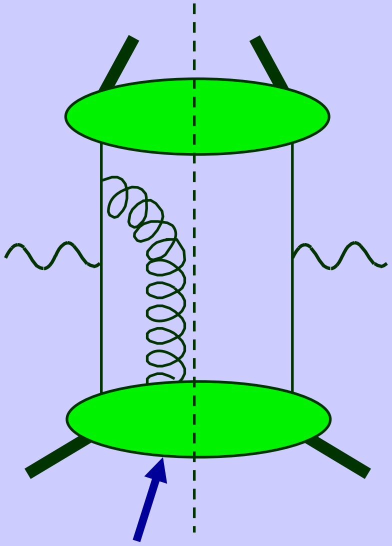

Twist-3 TMD PDFs
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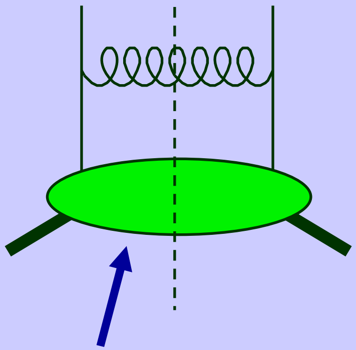
e.g. $f^{\perp}(x, p_T^2)$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

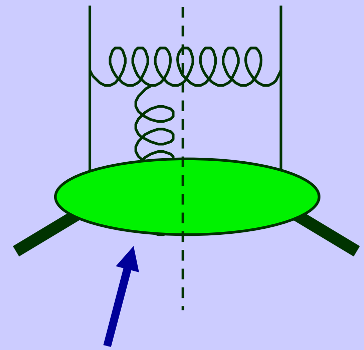
From low to intermediate

see e.g. Ji, Qiu, Vogelsang, Yuan, PLB638 (06)

- Compute the high-transverse-momentum behavior of the TMD PDFs by considering diagrams such as



Twist-2 integrated PDFs



Twist-3 integrated PDFs

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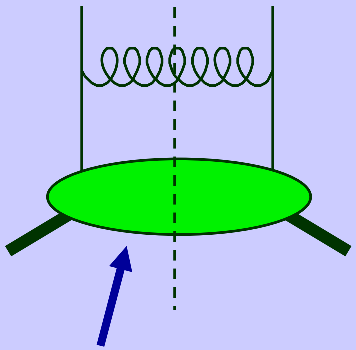

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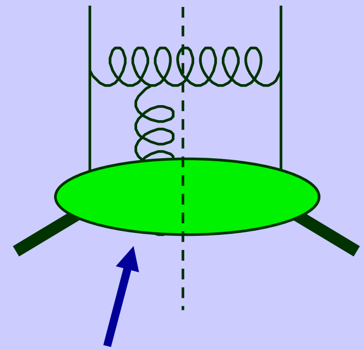

Twist-3 integrated PDFs

- Consider also the high-transverse-momentum contribution of the *soft factor*

Collins, Soper, NPB193 (81)

SIDIS structure functions

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\bigg[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\bigg] \\
&\quad + S_L\,\lambda_e\bigg[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\bigg] + S_T\bigg[\sin(\phi_h-\phi_S)\,\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) \\
&\quad + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\bigg] + S_T\lambda_e\bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\bigg]\bigg\}
\end{aligned}
$$

SIDIS structure functions

$$
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&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big] \\
&\quad + S_L\,\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big] + S_T\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) \\
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\end{aligned}
$$

18 structure functions

SIDIS structure functions

$$\frac{d\sigma}{dx\, dy\, d\phi_S\, dz\, d\phi_h\, dP_{h\perp}^2}$$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Big\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\,\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\,\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big]$$

$$+ S_L\,\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big] + S_T\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big)$$

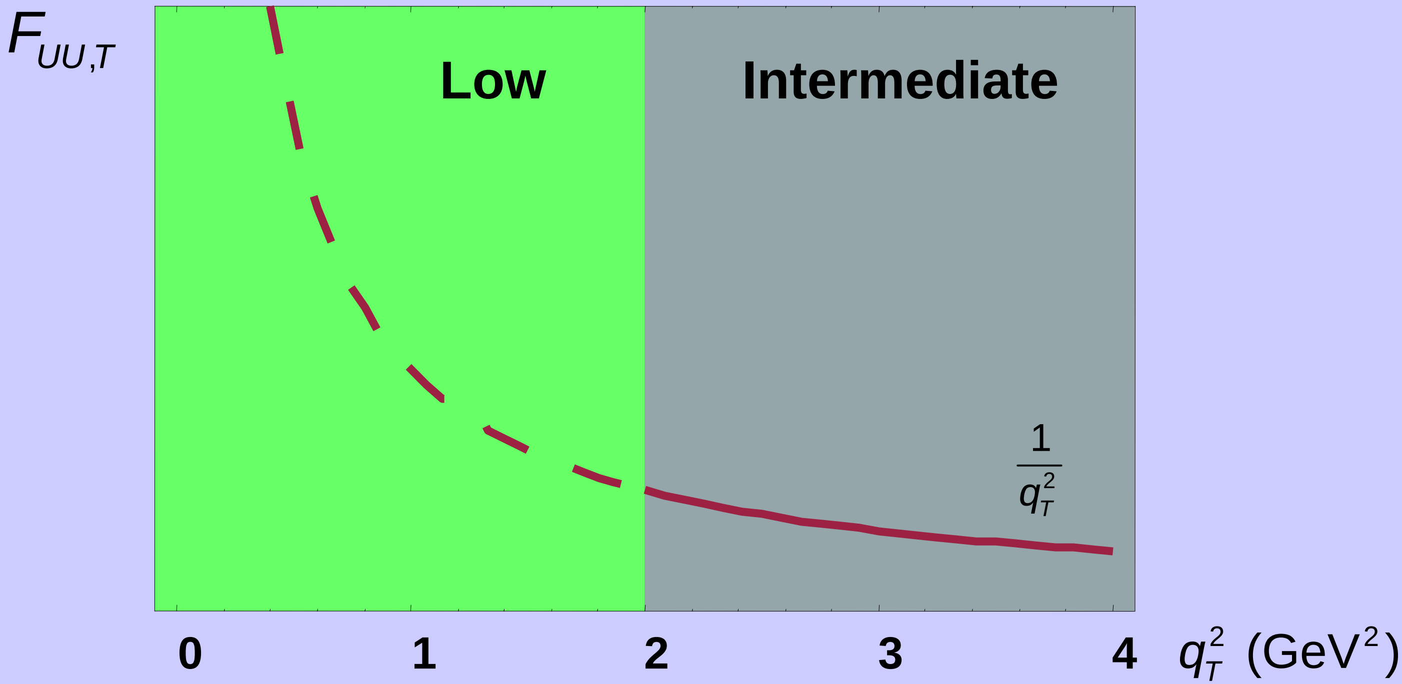
$$+ \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$

$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] + S_T\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

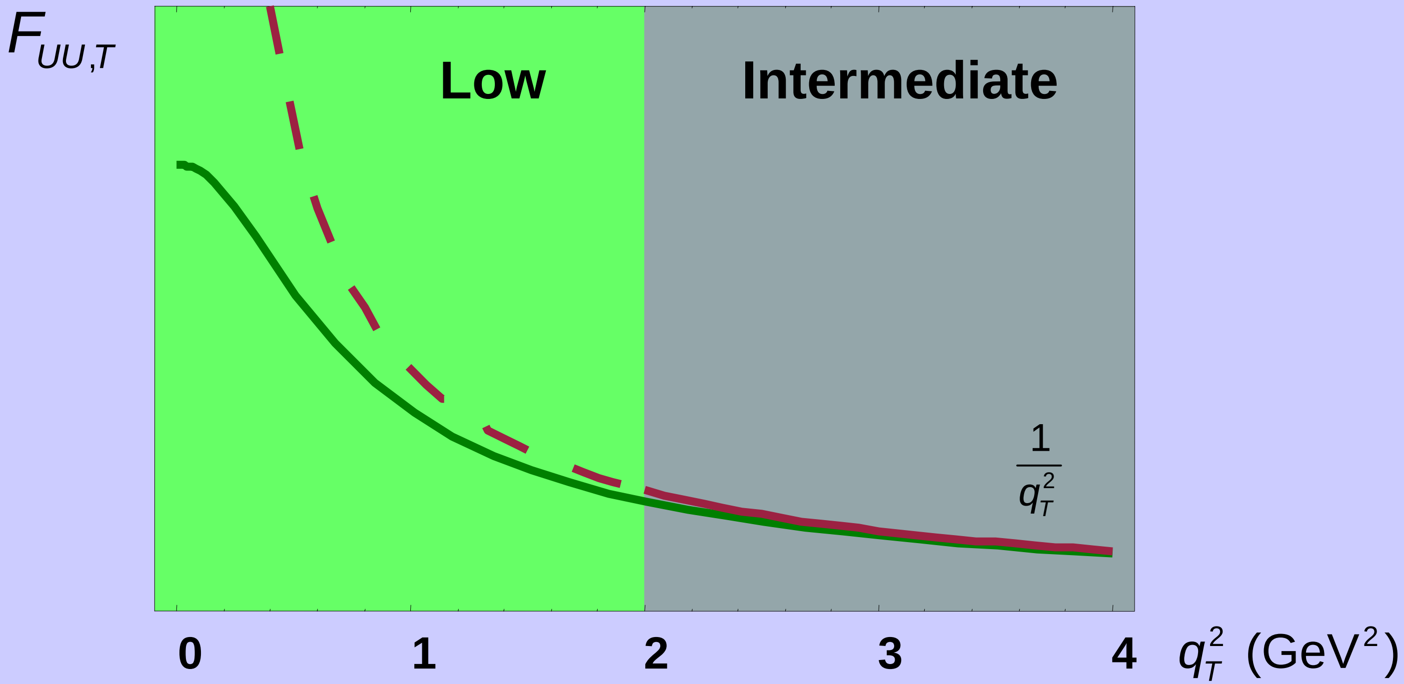
18 structure functions

$F_{UU,T}$ structure function



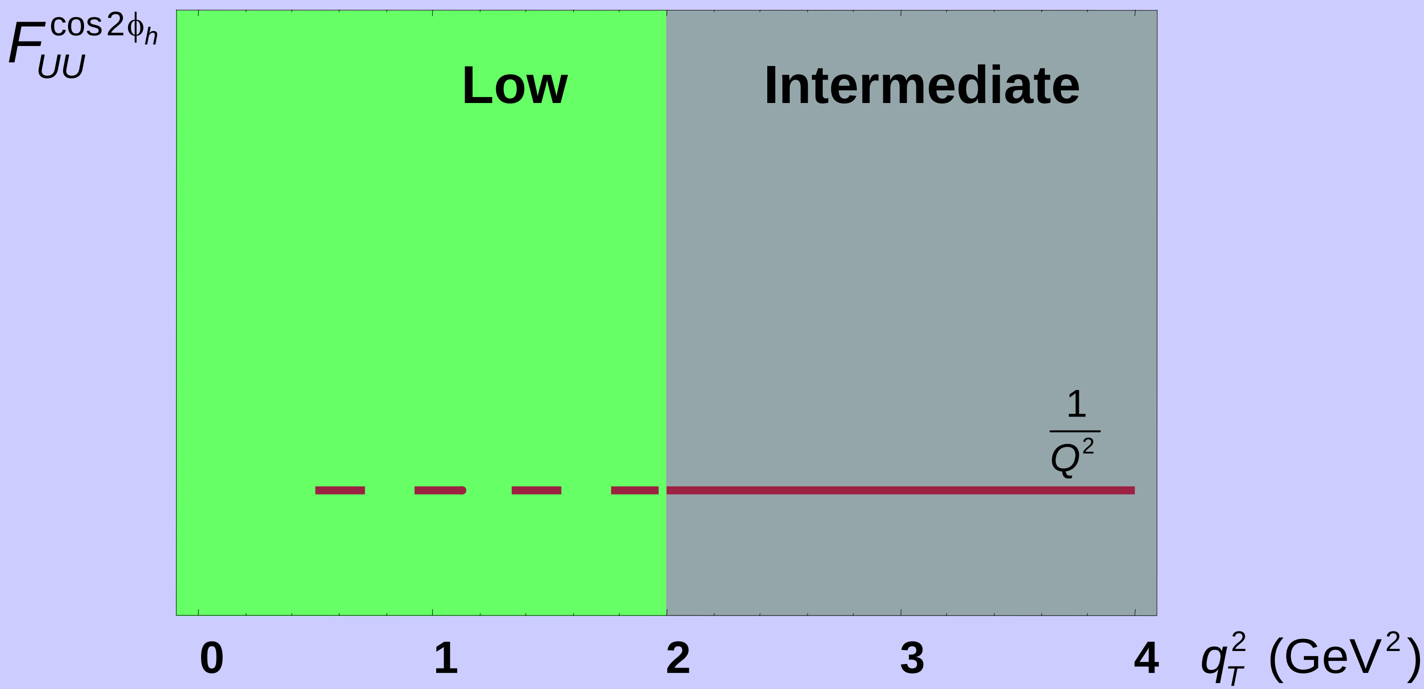
Collins, Soper, Sterman, NPB250 (85)

$F_{UU,T}$ structure function



Collins, Soper, Sterman, NPB250 (85)

$F_{UU}^{\cos 2\phi_h}$ structure function



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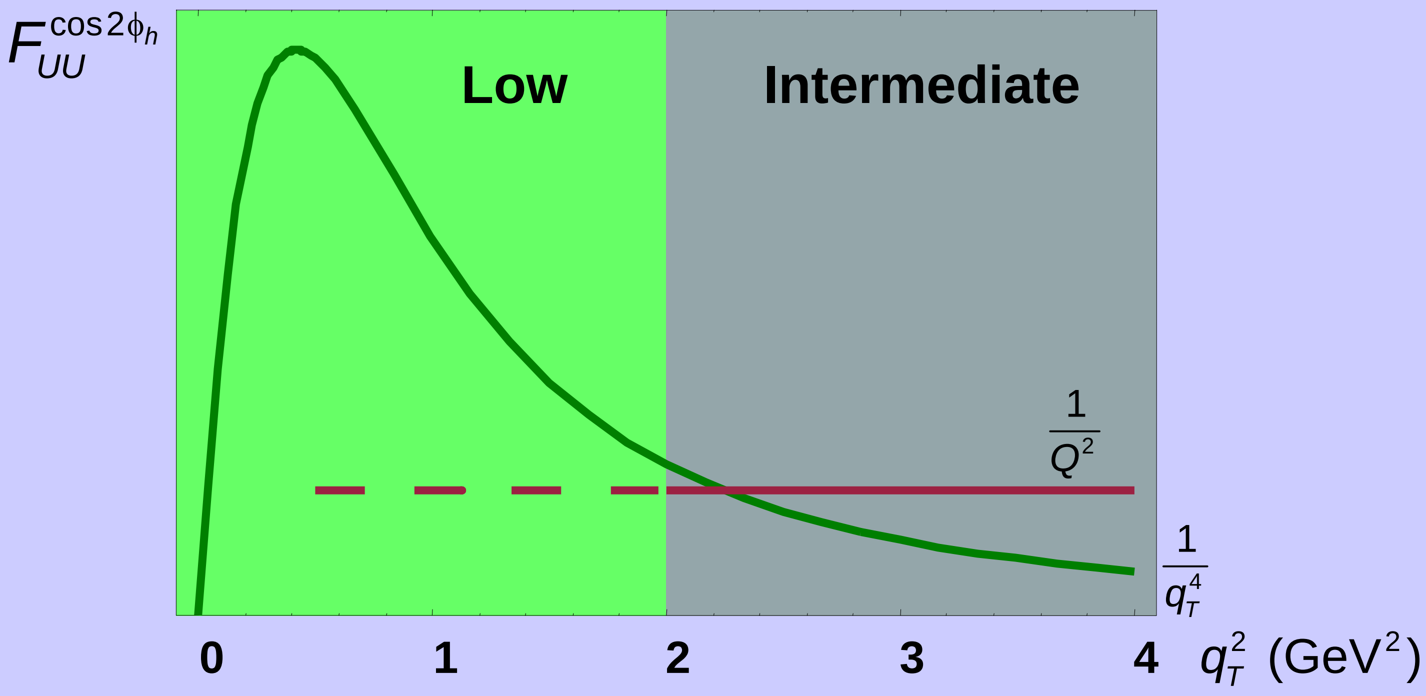

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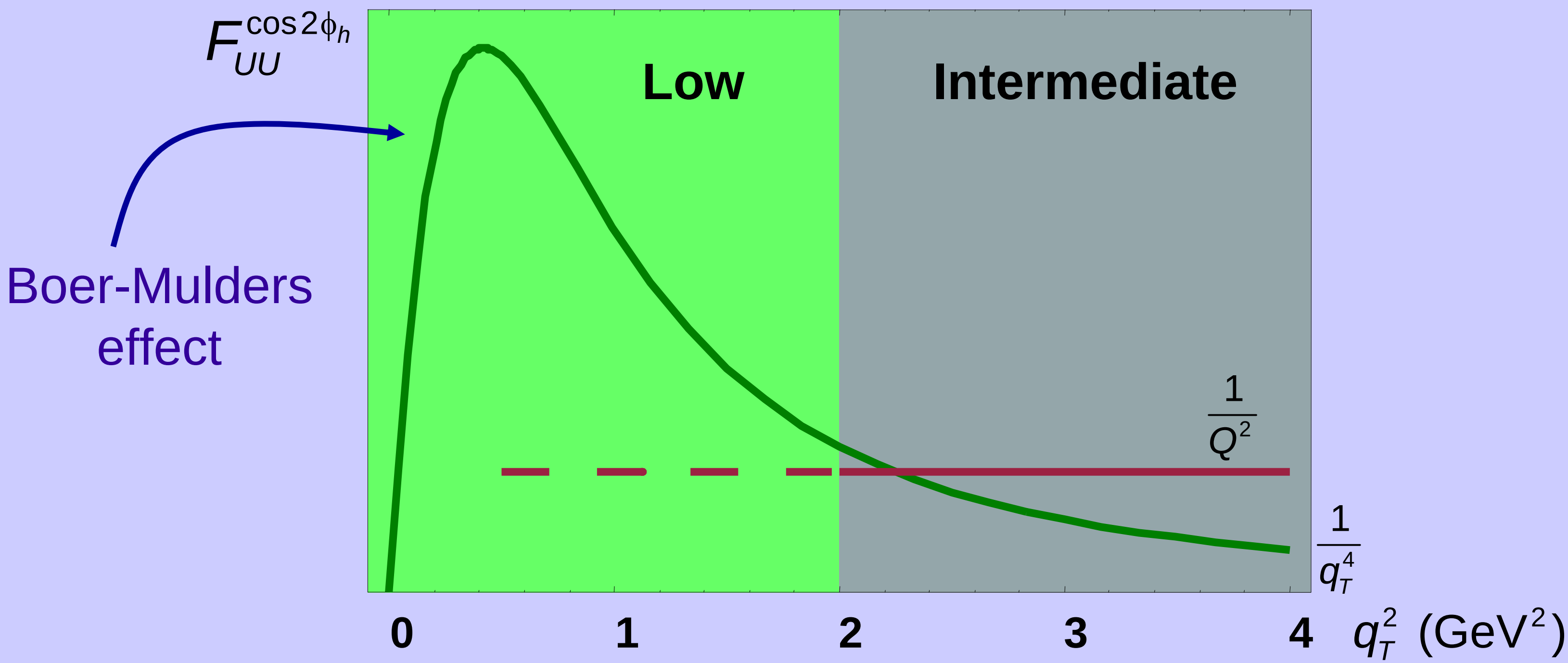

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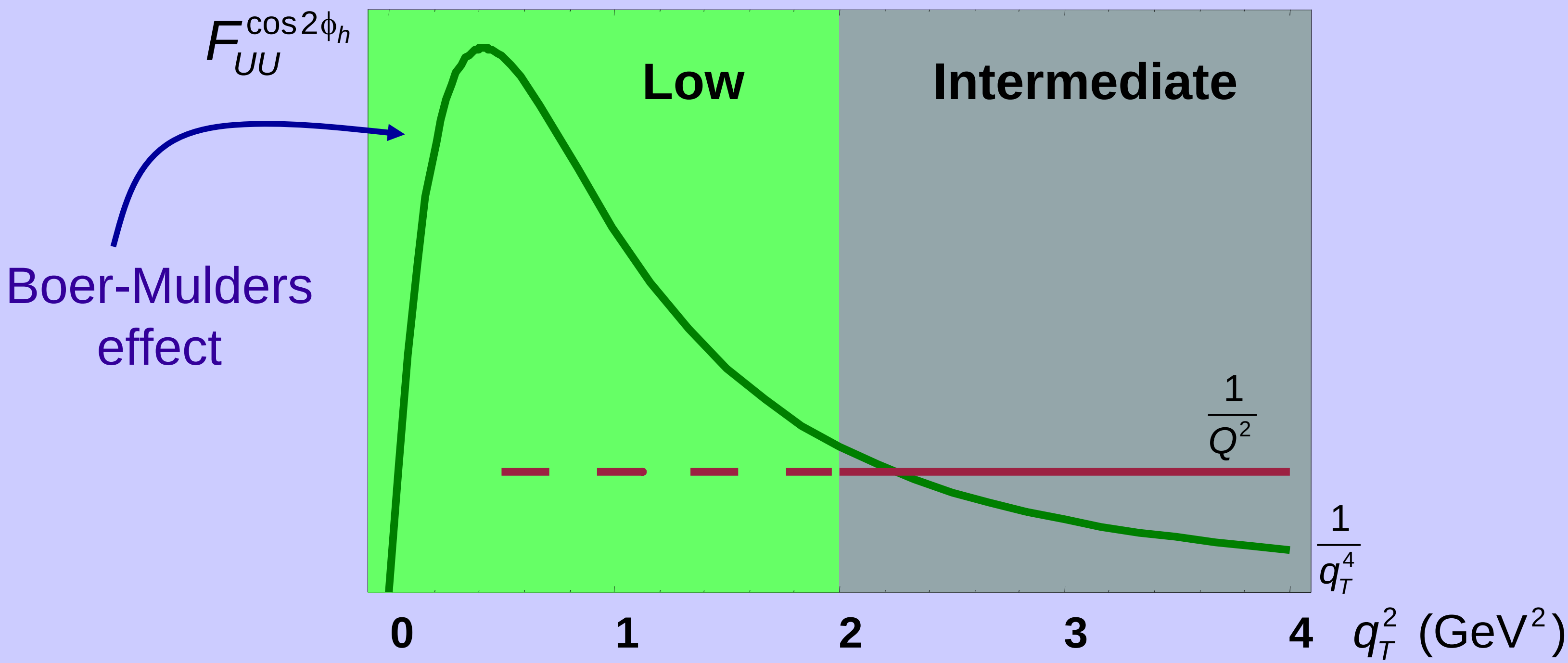
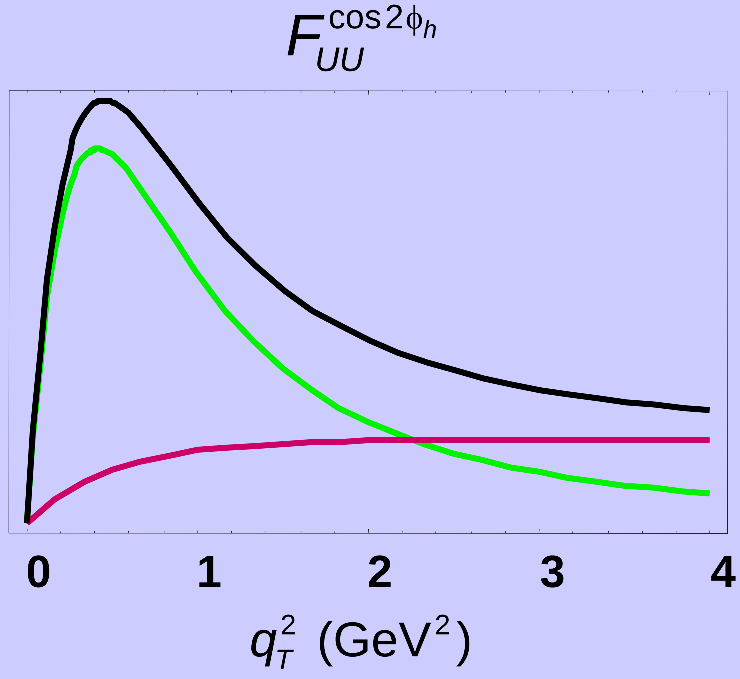


High and low calculations represent two distinct mechanisms

NOTE: it's a twist-2 calculation

$F_{UU}^{\cos 2\phi_h}$ and weighting

unweighted



$F_{UU}^{\cos 2\phi_h}$ and weighting

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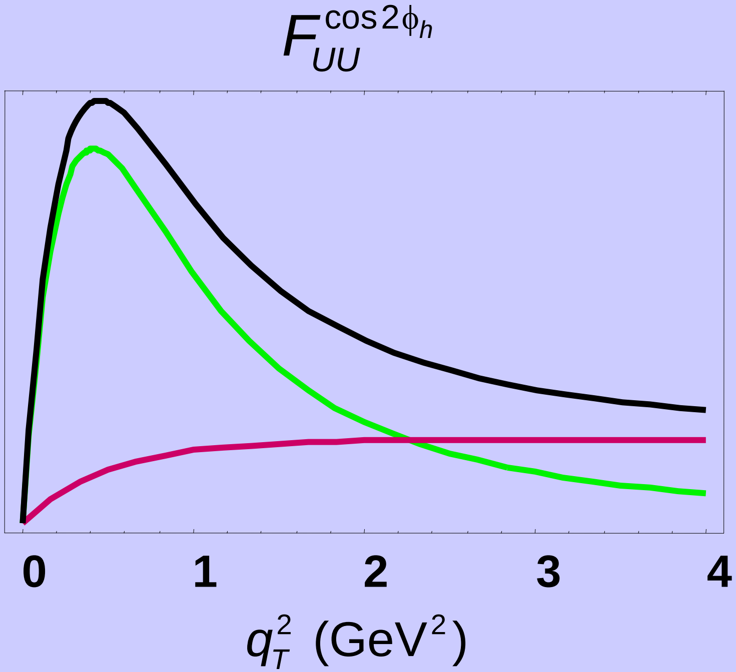
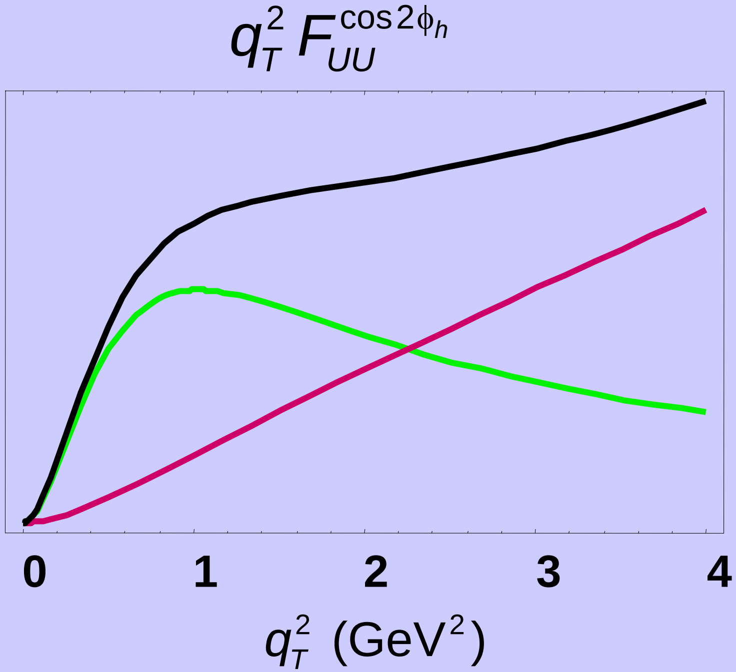
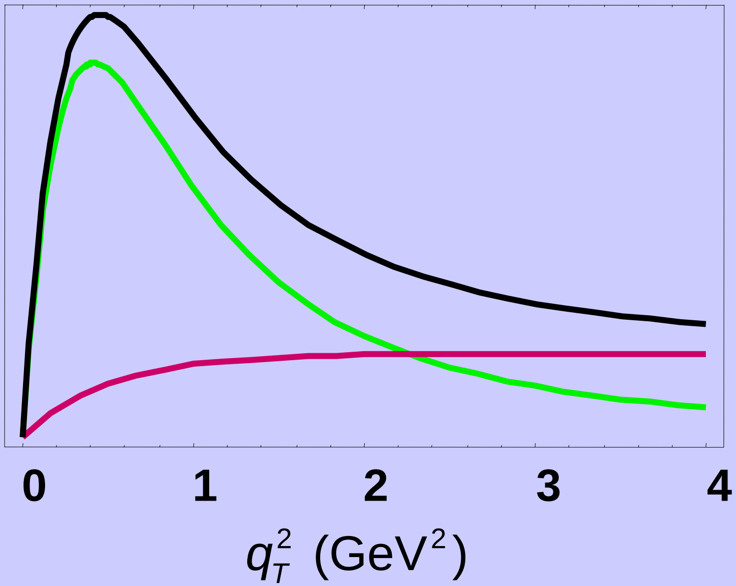




$F_{UU}^{\cos 2\phi_h}$ and weighting

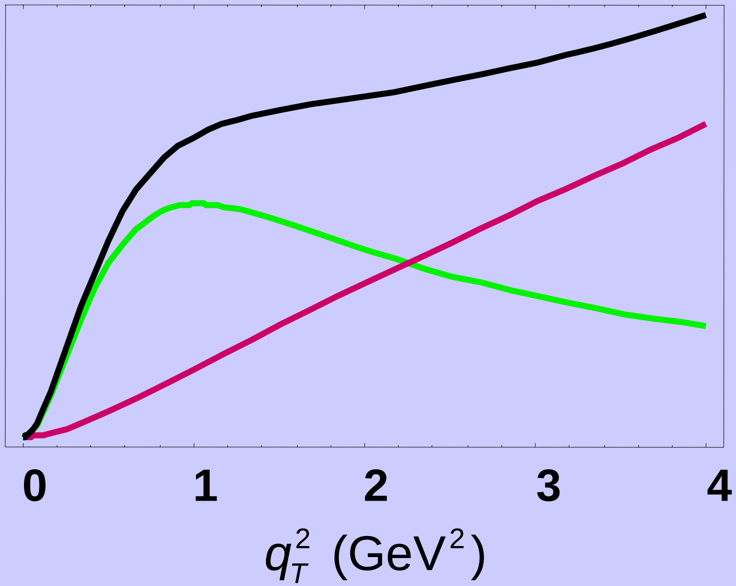
unweighted

$F_{UU}^{\cos 2\phi_h}$



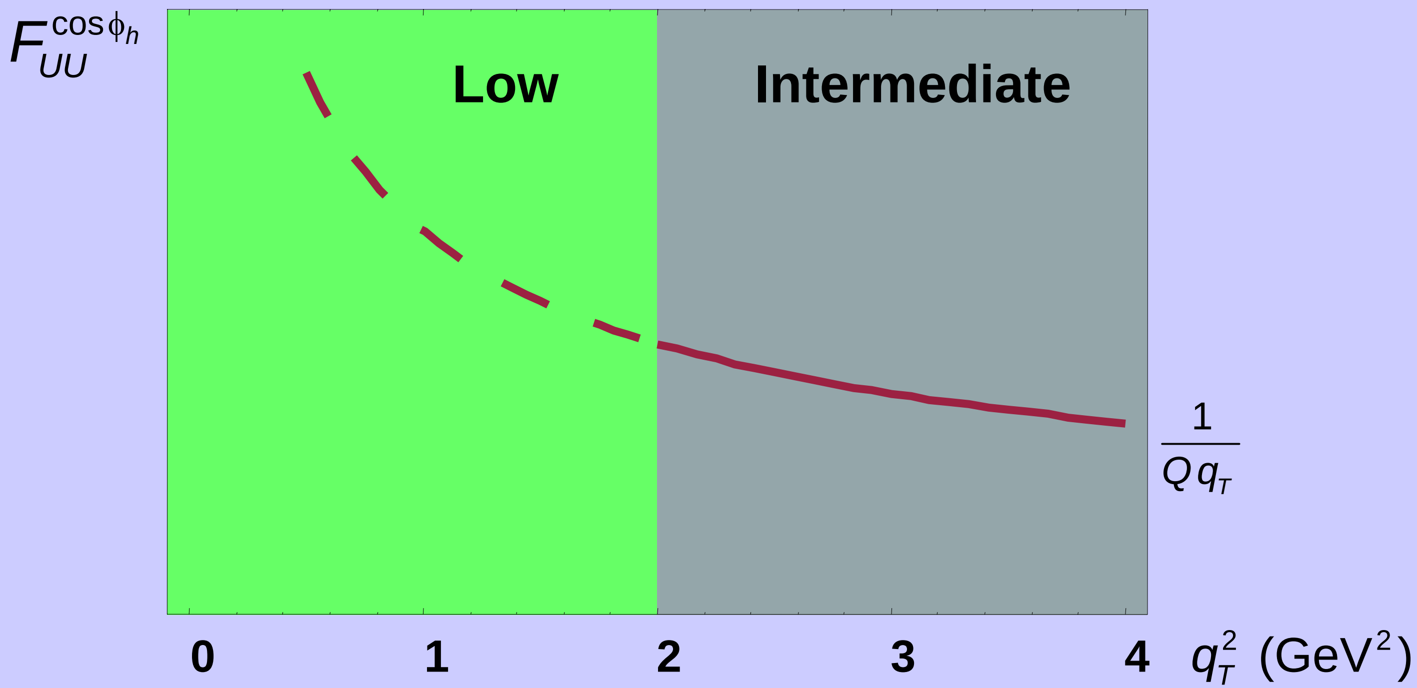
weighted

$q_T^2 \, F_{UU}^{\cos 2\phi_h}$



Weighting is not a good idea to access Boer-Mulders function

$F_{UU}^{\cos\phi_h}$ structure function



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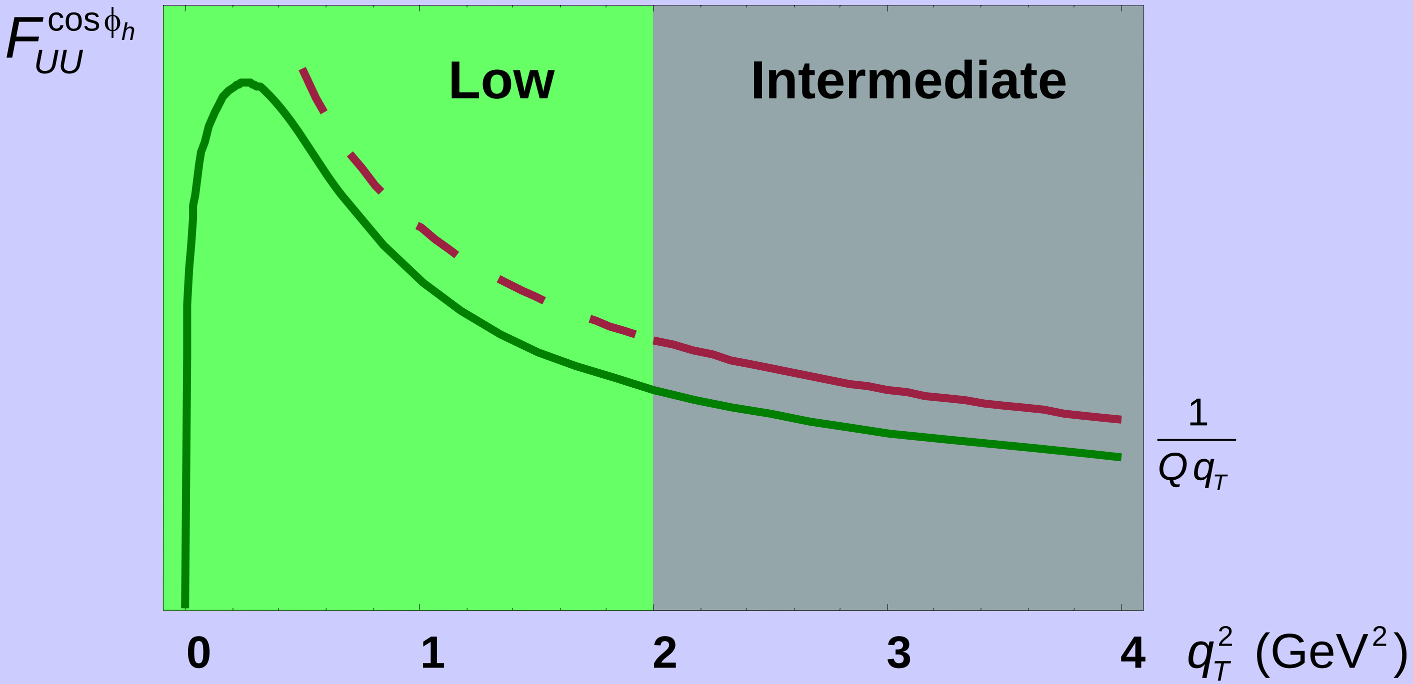
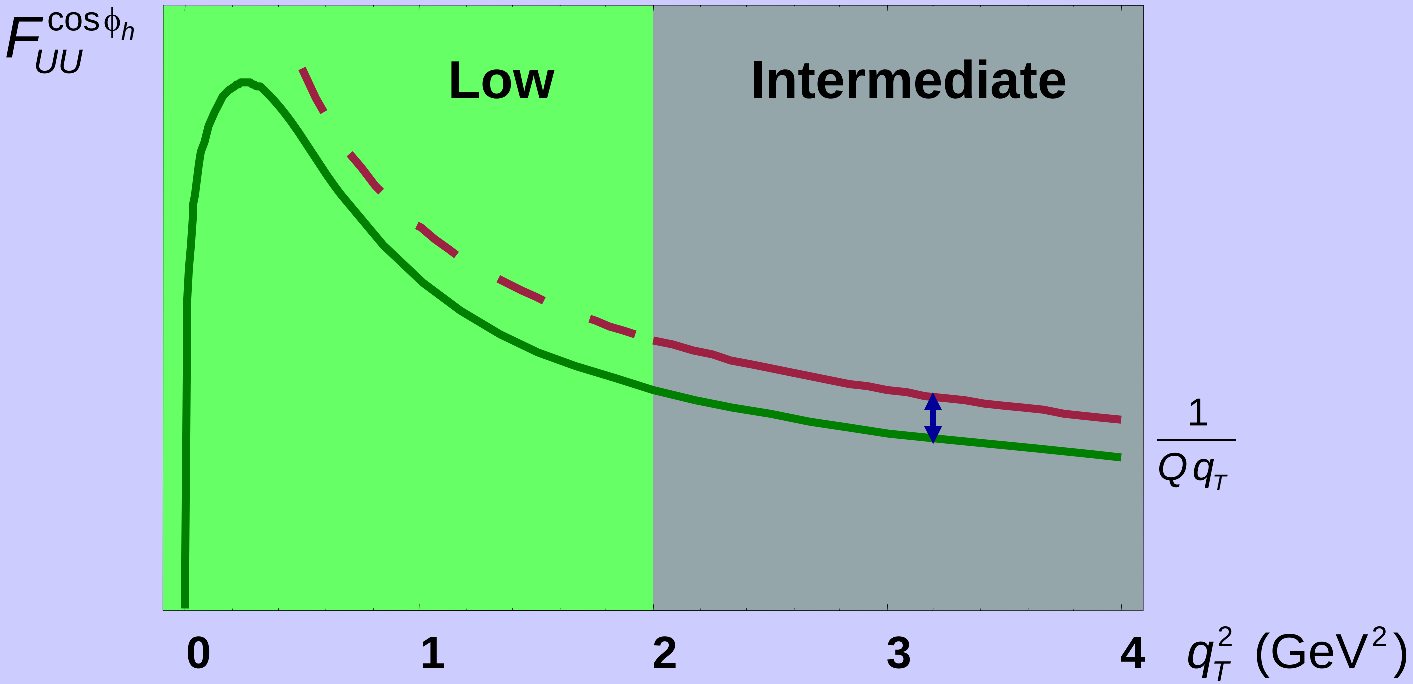


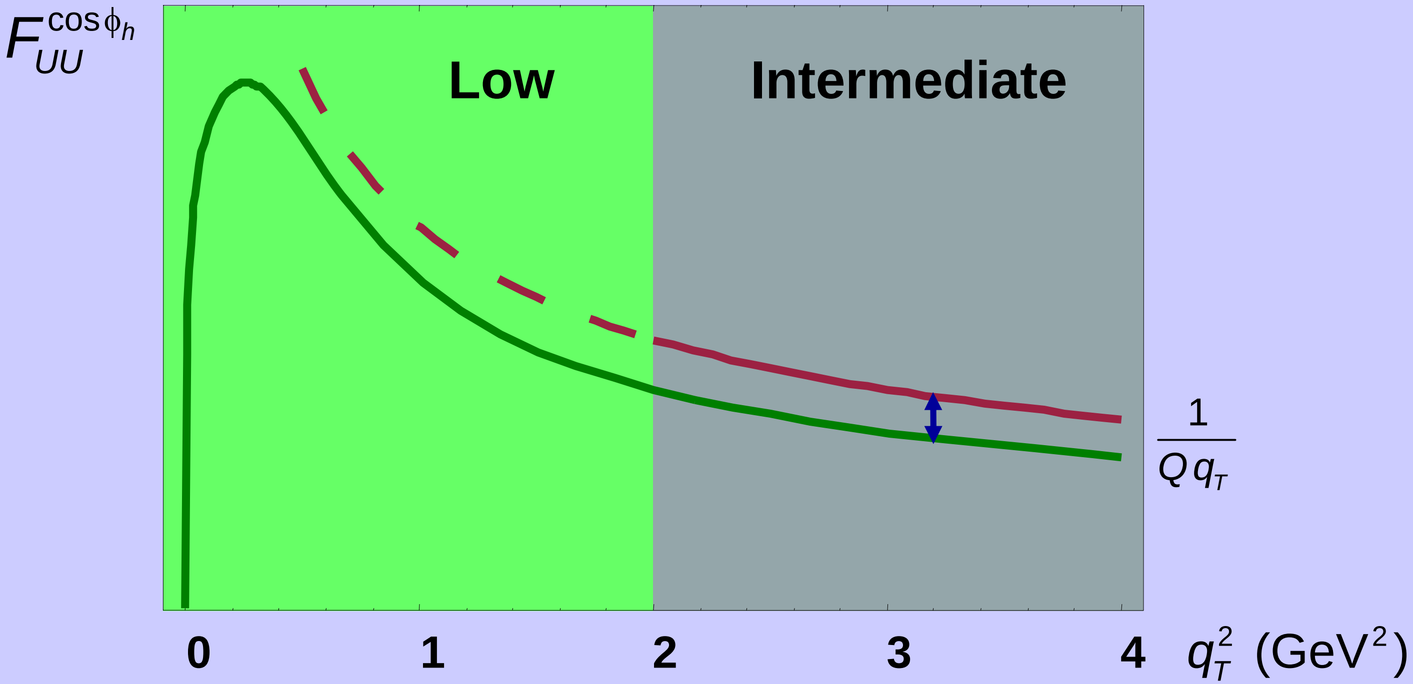
$F_{UU}^{\cos\phi_h}$ structure function



They have the same power behavior, but they don't match

Problems with the formalism at low transverse momentum!

$F_{UU}^{\cos\phi_h}$ structure function



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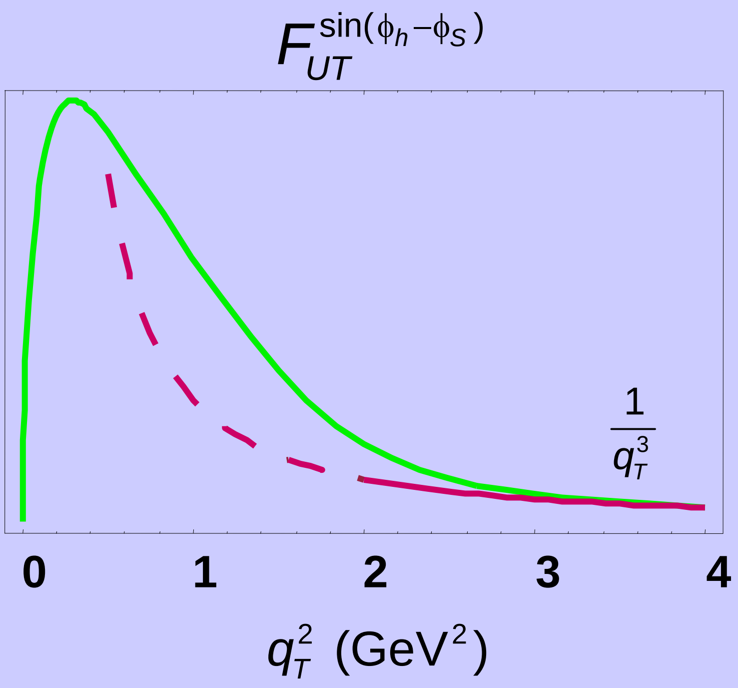
Problems with the formalism at low transverse momentum!

Cf. "Cahn effect calculations" of Anselmino, Boglione, Prokudin, Turk, EPJA31 (07)

$F_{UT}^{\sin(\phi_h-\phi_S)}$ (Sivers) structure funct.

unweighted



Ji, Qiu, Vogelsang, Yuan, PLB638 (06)
Koike, Vogelsang, Yuan, arXiv:0711.0636

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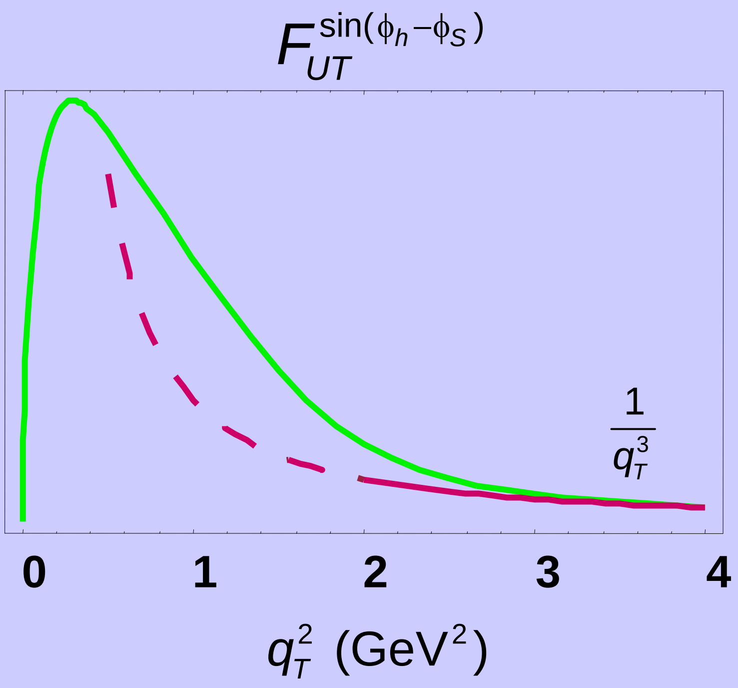

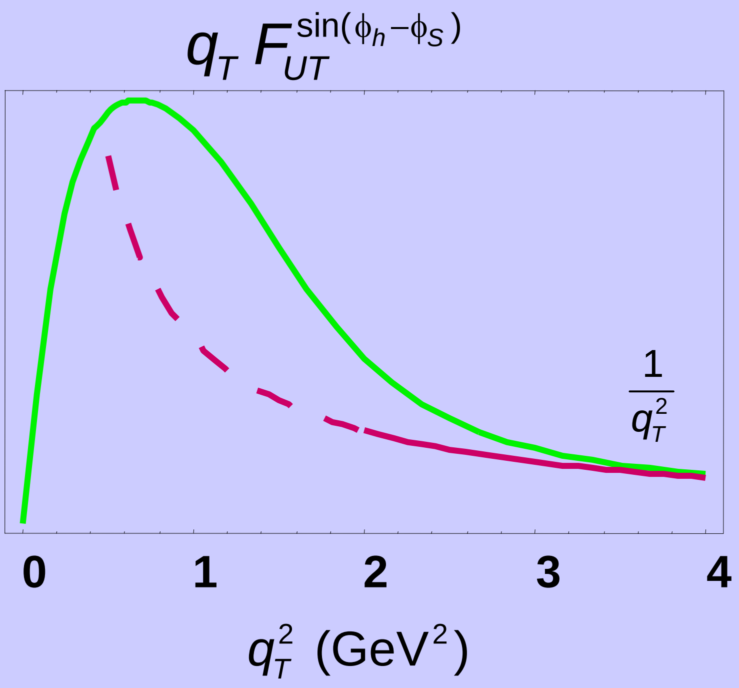

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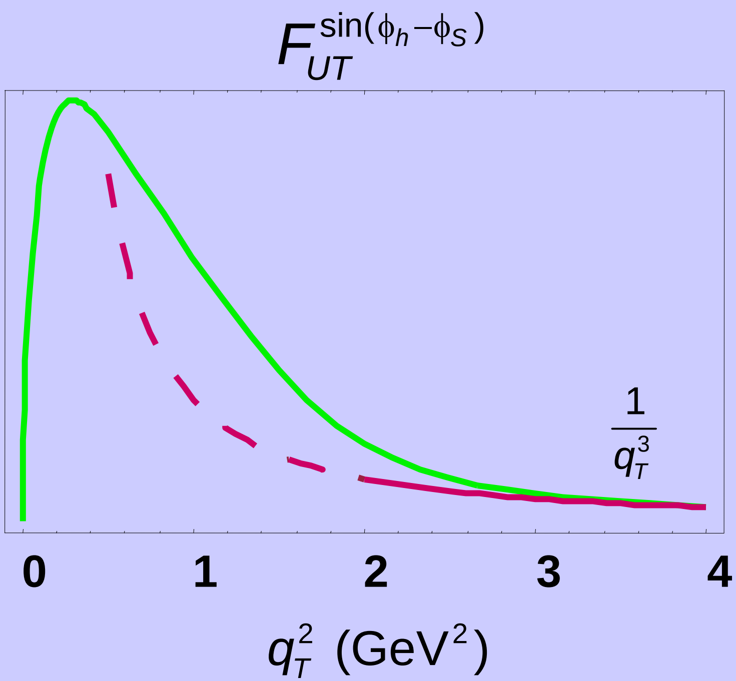
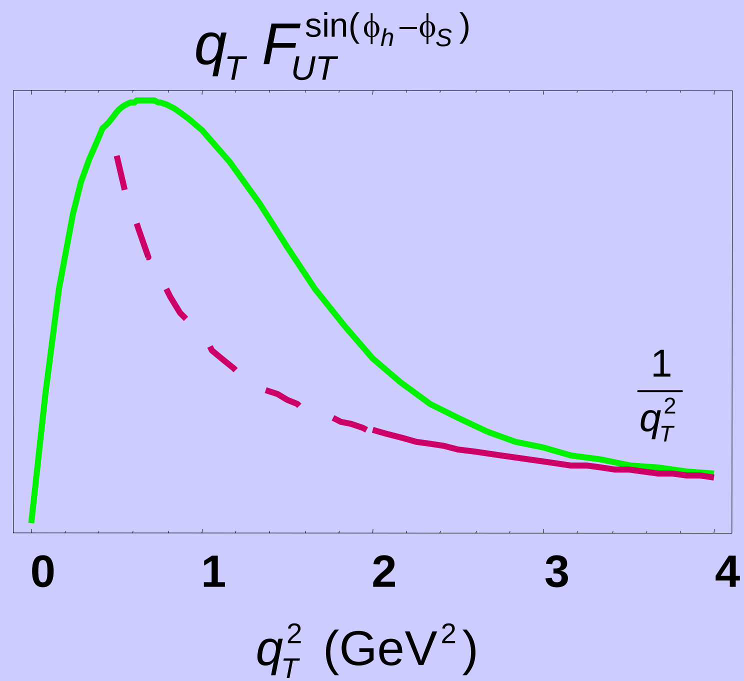


weighted



Ji, Qiu, Vogelsang, Yuan, PLB638 (06)
Koike, Vogelsang, Yuan, arXiv:0711.0636

Weighting is good!

Table of power behaviors

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observable	low-q_T calculation twist	order	power	high-q_T calculation twist	order	power	powers match
$F_{UU,T}$	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{UU,L}$	4			2	α_s	$1/Q^2$	
$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$				
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$				
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$	
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$				
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$				
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$				

Table of power behaviors

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observable	low-q_T calculation twist	order	power	high-q_T calculation twist	order	power	powers match
$F_{UU,T}$	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{UU,L}$	4			2	α_s	$1/Q^2$	?
$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$				?
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$				?
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$	?
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$				?
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$				?
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$				?

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$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$				
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$				
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$	?
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$				
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$				
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conjectures!

Table of exact matching

observable	low-q_T calculation twist	order	power	high-q_T calculation twist	order	power	powers match	exact match
$F_{UU,T}$	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes	✓
$F_{UU,L}$	4			2	α_s	$1/Q^2$		
$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes	✗
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no	
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes	?
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$					
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$					
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes	✓
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes	✗
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes	✓
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$		
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes	?
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no	
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes	?
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes	?
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$					
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$					
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$F_{UU}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes	✗
$F_{UU}^{\cos 2\phi_h}$	2	α_s	$1/q_T^4$	2	α_s	$1/Q^2$	no	
$F_{LU}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$	2	α_s^2	$1/(Q\,q_T)$	yes	✗
$F_{UL}^{\sin\phi_h}$	3	α_s^2	$1/(Q\,q_T)$					
$F_{UL}^{\sin 2\phi_h}$	2	α_s	$1/q_T^4$					
F_{LL}	2	α_s	$1/q_T^2$	2	α_s	$1/q_T^2$	yes	✓
$F_{LL}^{\cos\phi_h}$	3	α_s	$1/(Q\,q_T)$	2	α_s	$1/(Q\,q_T)$	yes	✗
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes	✓
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4			3	α_s	$1/(Q^2\,q_T)$		
$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	α_s	$1/q_T^3$	3	α_s	$1/q_T^3$	yes	✓
$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	α_s^2	$1/q_T^3$	3	α_s	$1/(Q^2\,q_T)$	no	
$F_{UT}^{\sin\phi_S}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes	✗
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$	3	α_s	$1/(Q\,q_T^2)$	yes	✗
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	α_s	$1/q_T^3$					
$F_{LT}^{\cos\phi_S}$	3	α_s	$1/(Q\,q_T^2)$					
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	α_s	$1/(Q\,q_T^2)$					

conjectures!

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- The last case indicates a violation of factorization with twist-3 TMD PDFs
- The study has several phenomenological consequences

$F_{UU,T}$ high to intermediate

$$F_{UU,T} = \frac{1}{Q^2}\,\frac{\alpha_s}{(2\pi z)^2}\sum_a x\,e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$$

$$\times\left[f_1^a\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\left(\frac{x}{\hat{x}}\right) D_1^g\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$

$$\Downarrow$$

$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\,e_a^2\left[f_1^a(x)\,D_1^a(z)\,L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,\big(D_1^a\otimes P_{qq} + D_1^g\otimes P_{gq}\big)(z)\right.$$

$$\left. + \big(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\big)(x)\,D_1^a(z)\right],$$

$F_{UU,T}$ low to intermediate

$$F_{UU,T} = \sum_a x\, e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T\big)\, f^a(x, p_T^2)\, D^a(z, k_T^2)\, U(l_T^2)$$

$\Downarrow$

$$F_{UU,T} = \sum_a x\, e_a^2 \left[f_1^a(x, q_T^2)\, \frac{D_1^a(z)}{z^2} + f_1^a(x)\, D_1^a(z, q_T^2) + f_1^a(x)\, \frac{D_1^a(z)}{z^2}\, U(q_T^2) \right]$$

$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{\boldsymbol{p}_T^2} \left[\frac{L(\eta^{-1})}{2}\, f_1^q(x) - C_F\, f_1^q(x) + \big(P_{qq} \otimes f_1^q + P_{qg} \otimes f_1^g\big)(x) \right]$$

$$D_1^q(z, k_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{z^2\, \boldsymbol{k}_T^2} \left[\frac{L(\eta_h^{-1})}{2}\, D_1^q(z) - C_F\, D_1^q(z) + \big(D_1^q \otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z) \right]$$

$\Downarrow$

$$U(q_T^2) = \frac{\alpha_s C_F}{\pi^2} \frac{1}{q_T^2}$$

$$F_{UU,T} = \frac{1}{q_T^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_a x\, e_a^2 \left[f_1^a(x)\, D_1^a(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\, \big(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z) \right.$$
$$\left. + \big(P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g\big)(x)\, D_1^a(z) \right],$$

Cahn effect

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

$$F_{UU}^{\cos\phi_h} = -\frac{2q_T}{Q}\sum_a x\,e_a^2\left[x\,f^{\perp a}(x,q_T^2)\,\frac{D_1^a(z)}{z^2} - f_1^a(x)\,\frac{\tilde{D}^{\perp a}(z,q_T^2)}{z}\right]$$

$$x\,f^{\perp q}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{2\boldsymbol{p}_T^2}\left[\frac{L(\eta^{-1})}{2}\,f_1^q(x) + \big(P'_{qq}\otimes f_1^q + P'_{qg}\otimes f_1^g\big)(x)\right]$$

$$\frac{\tilde{D}^{\perp q}(z,k_T^2)}{z} = -\frac{\alpha_s}{2\pi^2}\,\frac{1}{2z^2\,\boldsymbol{k}_T^2}\left[\frac{L(\eta_h^{-1})}{2}\,D_1^q(z) - 2C_F\,D_1^q(z) + \big(D_1^q\otimes P'_{qq} + D_1^g\otimes P'_{gq}\big)(z)\right]$$

$$F_{UU}^{\cos\phi_h} = -\frac{1}{Q\,q_T}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\,e_a^2\bigg[f_1^a(x)\,D_1^a(z)\,L\Big(\frac{Q^2}{q_T^2}\Big) + f_1^a(x)\,\big(D_1^a\otimes P'_{qq} + D_1^a\otimes P'_{gq}\big)(z)$$
$$+ \big(P'_{qq}\otimes f_1^a + P'_{qg}\otimes f_1^g\big)(x)\,D_1^a(z) - 2C_F\,f_1^a(x)\,D_1^a(z)\bigg],$$