Excited light meson spectroscopy from lattice QCD

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With Jo Dudek, Robert Edwards, Mike Peardon, Bálint Joó, David Richards and the *Hadron Spectrum Collaboration*

Outline

- Introduction and motivation
- Excited isoscalar mesons
- Energy-dependent $\pi\pi$ I=2 phase shift
- Summary and outlook

PR D83, 071504 (2011)
PR D83, 111502 (2011)

Motivation

Upcoming experimental efforts in light meson sector (and charmonium)

GlueX and CLAS12 (JLab), BESIII, COMPASS, PANDA, ...

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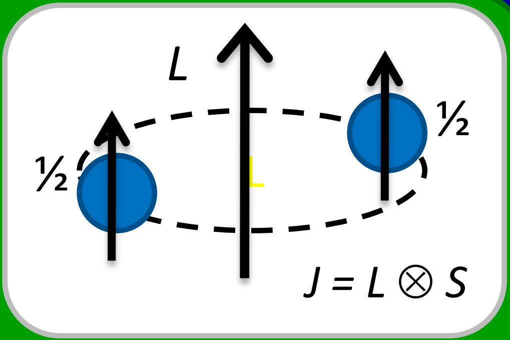
Parity: $P = (-1)^{(L+1)}$

Charge Conj Sym: $C = (-1)^{(L+S)}$



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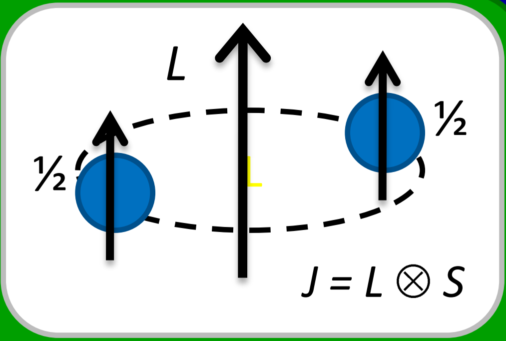

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Probe low energy d.o.f. of QCD

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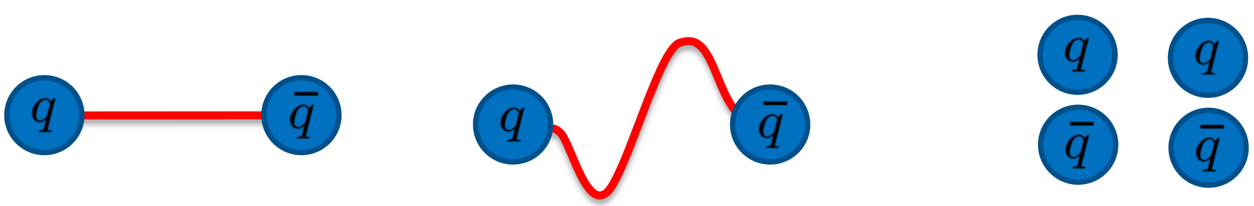

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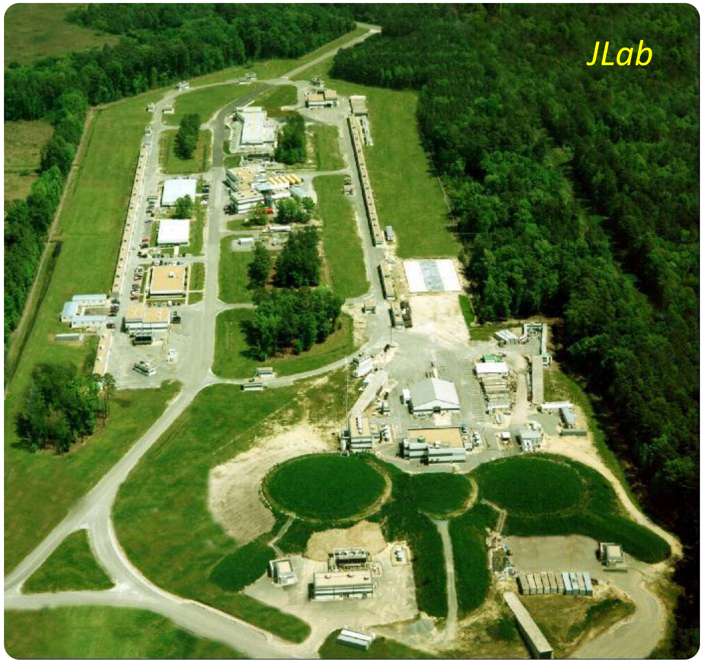
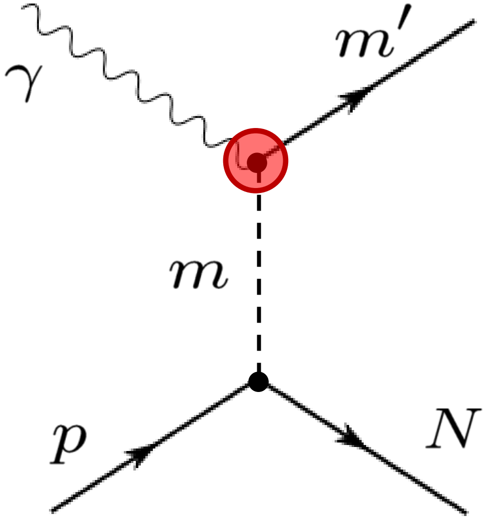
- systematic study of light mesons, particular interest in exotics





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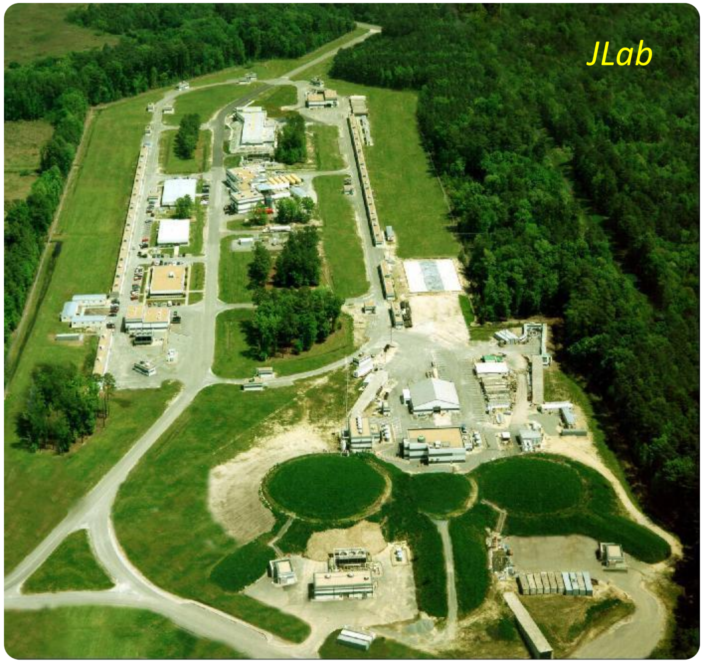

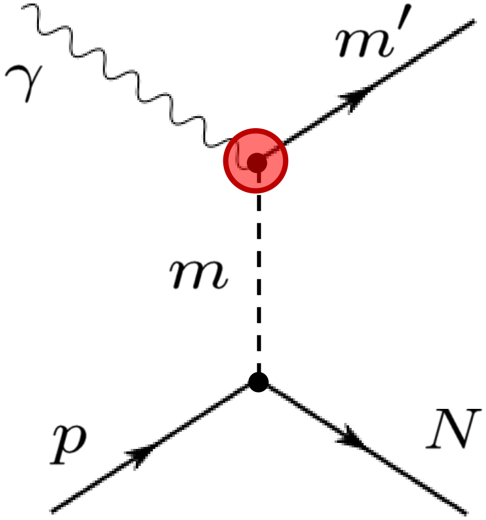

Use Lattice QCD to extract excited spectrum and photocouplings

Isoscalars

Isoscalars (I = 0) e.g. η, η', ω, ϕ

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$m_s = m_u = m_d$ [SU(3) sym]
– eigenstates are octet, singlet

$$\frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

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In general

$$\begin{aligned} |a\rangle &= \cos\alpha \left|\ell\bar{\ell}\right\rangle - \sin\alpha \left|s\bar{s}\right\rangle \\ |b\rangle &= \sin\alpha \left|\ell\bar{\ell}\right\rangle + \cos\alpha \left|s\bar{s}\right\rangle \end{aligned}$$

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Experimentally

ω, ϕ (1^{--}) and $f_2(1270)$, $f_2'(1525)$ (2^{++}) – close to 'ideal'

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Can also mix with glueballs

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Spectroscopy on the lattice

Calculate energies and matrix elements ("overlaps", *Z*'s) from correlation functions of meson interpolating fields

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Construct operators which only overlap with one spin in the continuum limit

$$\mathcal{O}(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\ \bar{\psi}(x)\Gamma_i \overleftrightarrow{D}_j \overleftrightarrow{D}_k \ldots \psi(x)$$

definite J^{PC}

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$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2 E_n} < 0|\mathcal{O}_i(0)|n >< n|\mathcal{O}_j^\dagger(0)|0 >$$

Variational Method

Large basis of operators → matrix of correlators

$$C_{ij}(t) =< 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0 >$$

Generalised eigenvector problem:

$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$$

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Also → optimal linear combination of operators to overlap on to a state

$$\Omega^{(n)} \sim \sum_i v_i^{(n)}\mathcal{O}_i$$

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Var. method uses orthog of eigenvectors; don't just rely on separating energies

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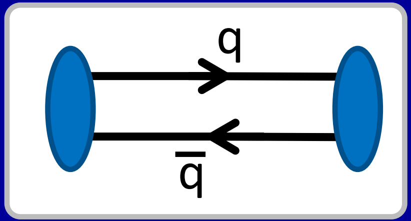

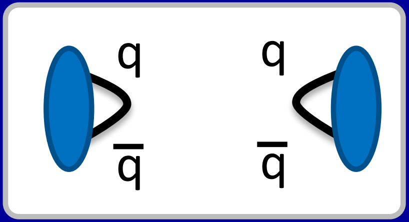

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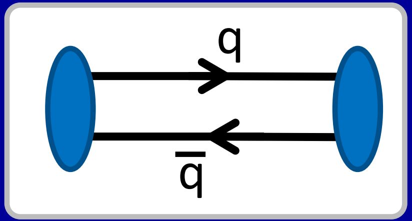

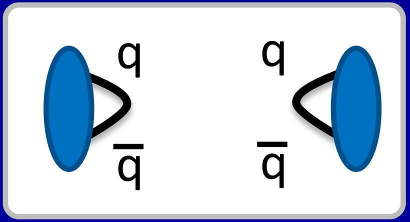

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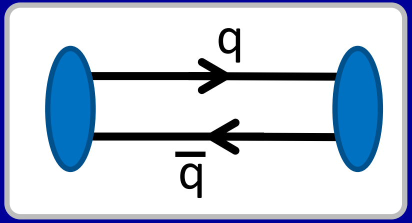

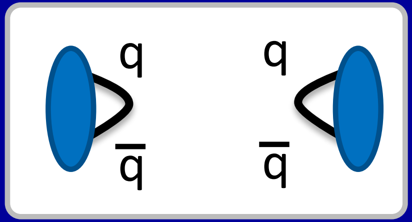

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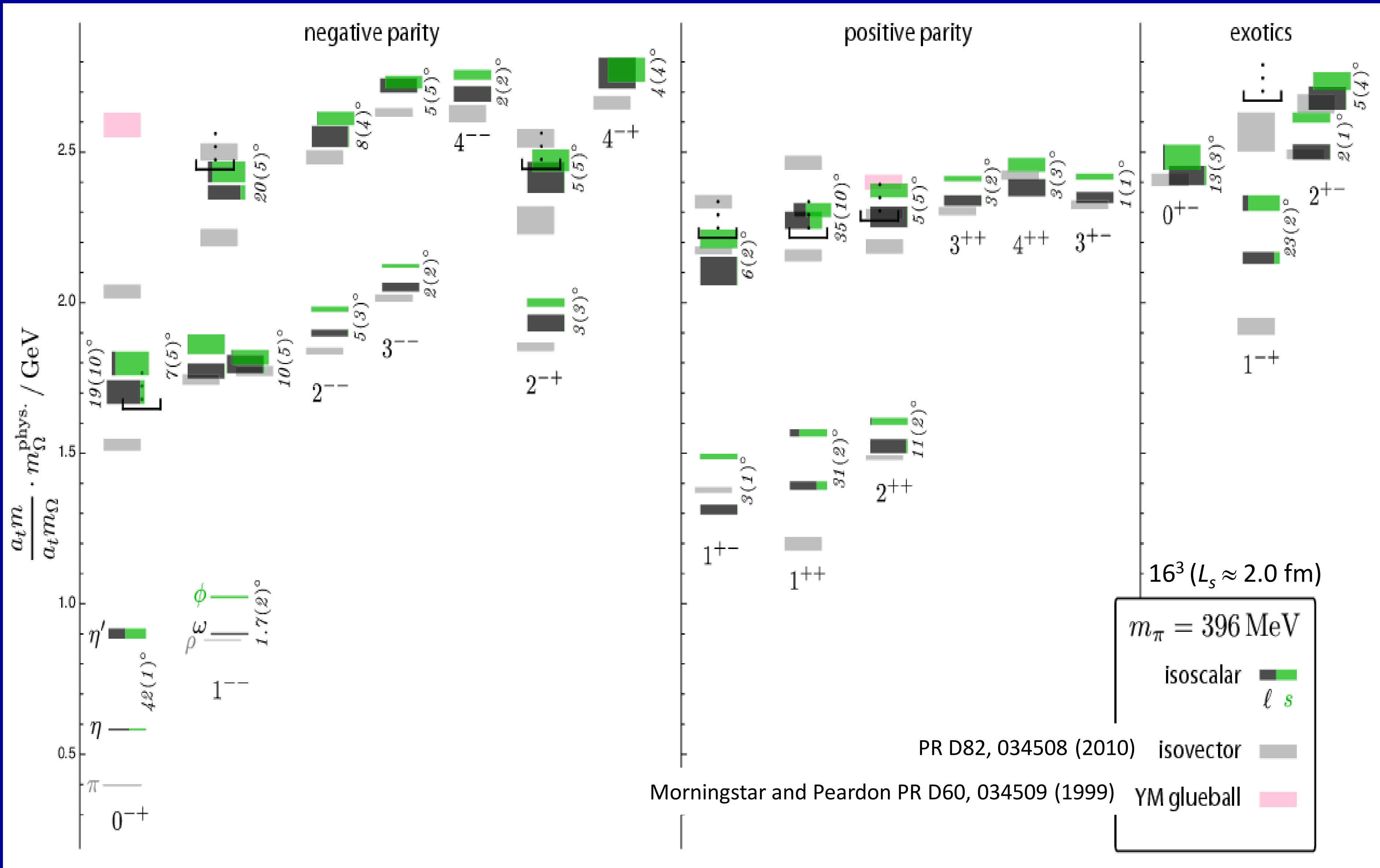
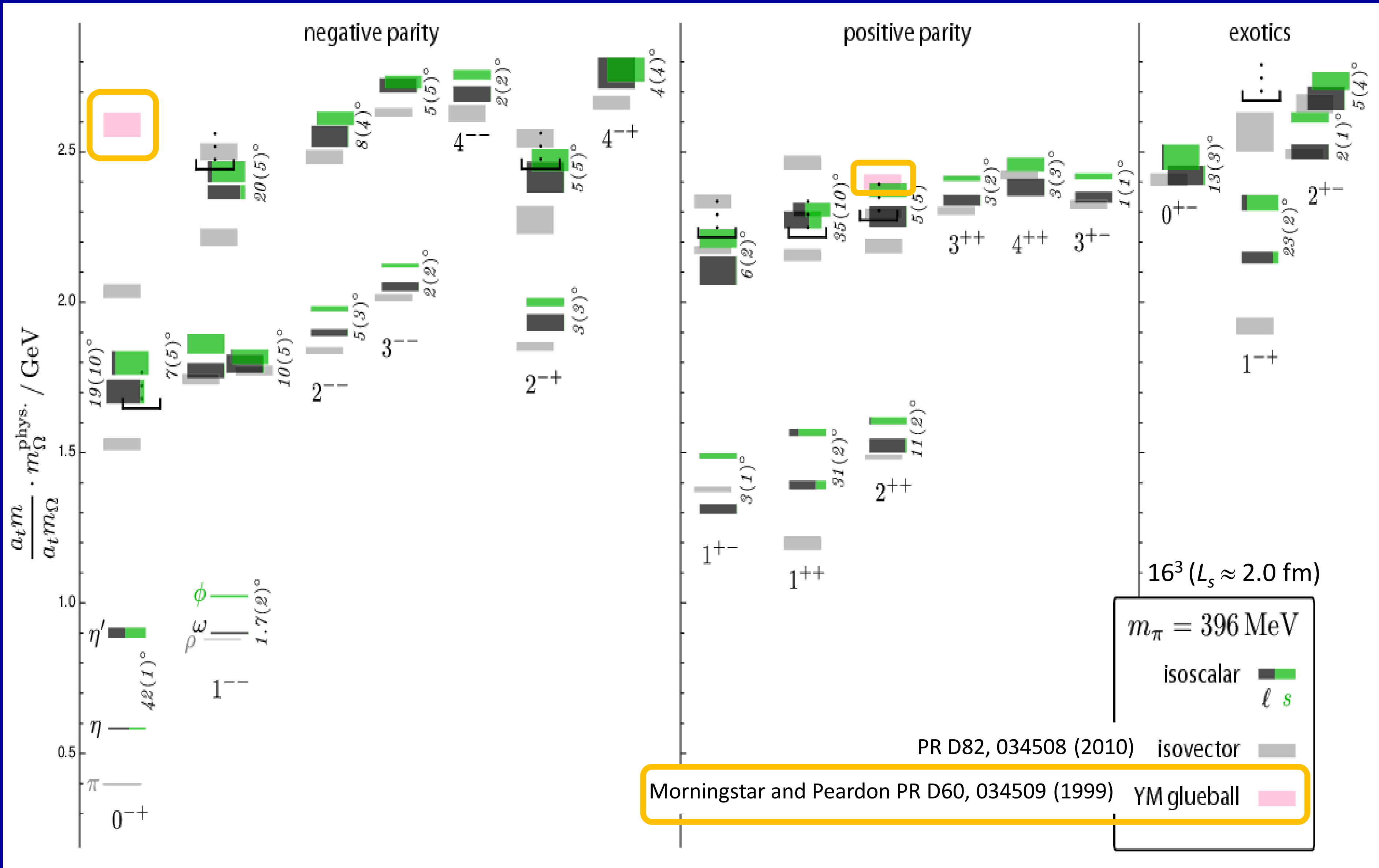
Anisotropic lattices (a_s/a_t = 3.5), a_s ~ 0.12 fm; 16^3 (2.0 fm)

Dynamical, N_f = 2+1, $M_\pi \approx$ 400 MeV

Lattice details in:
PR D78, 054501;
PR D79, 034502



PR D83, 111502 (2011)



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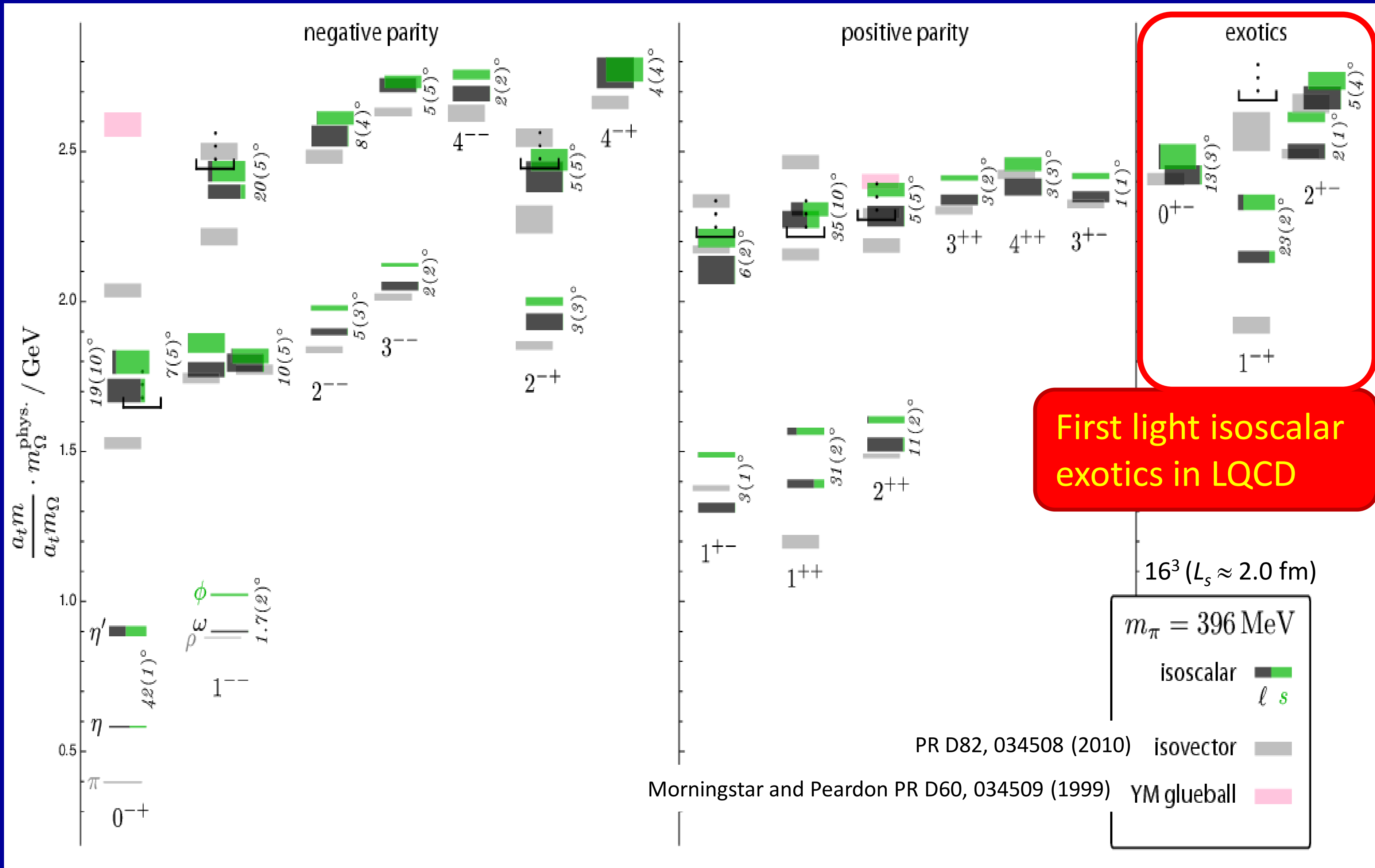
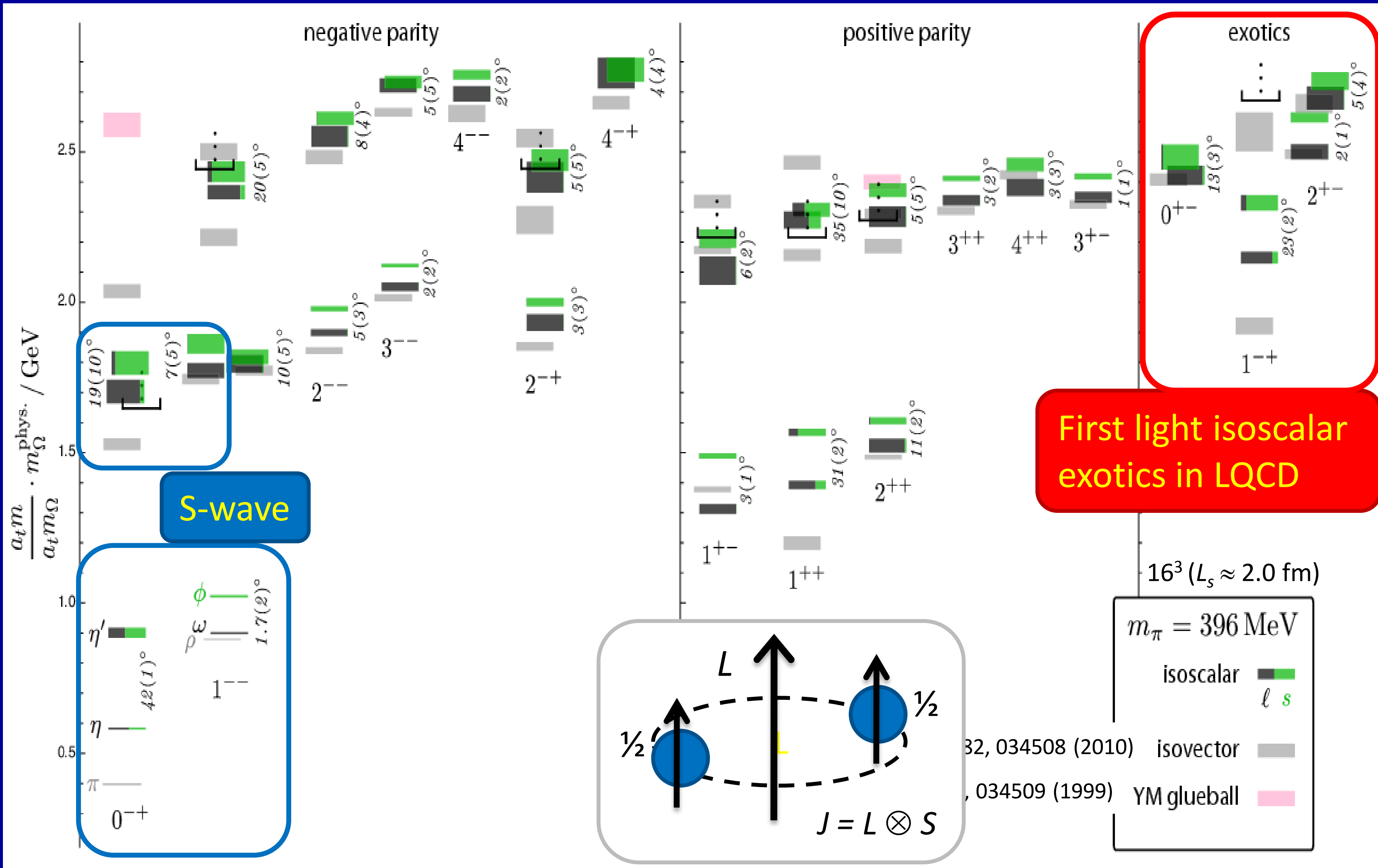


First light isoscalar exotics in LQCD

PR D83, 111502 (2011)



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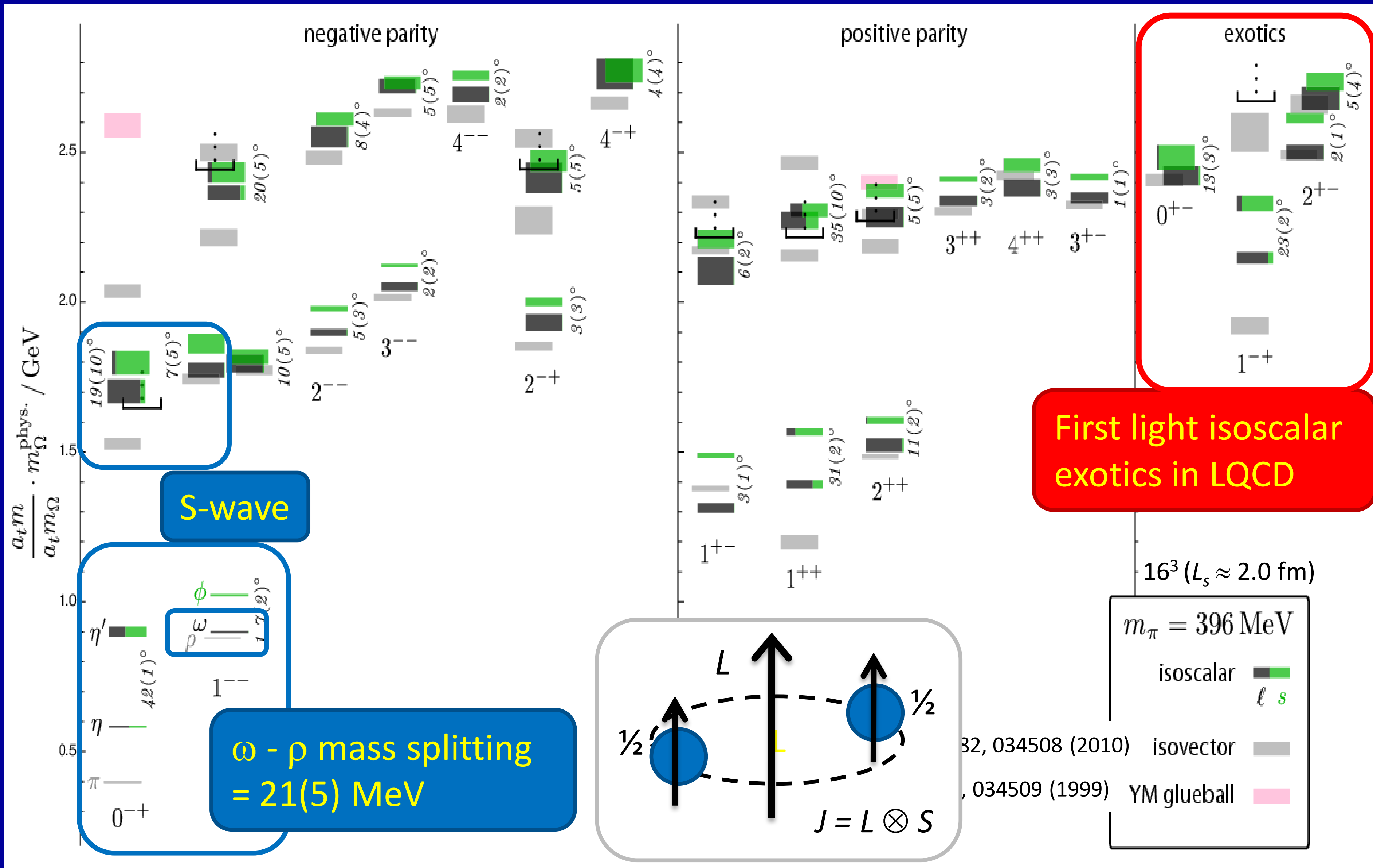

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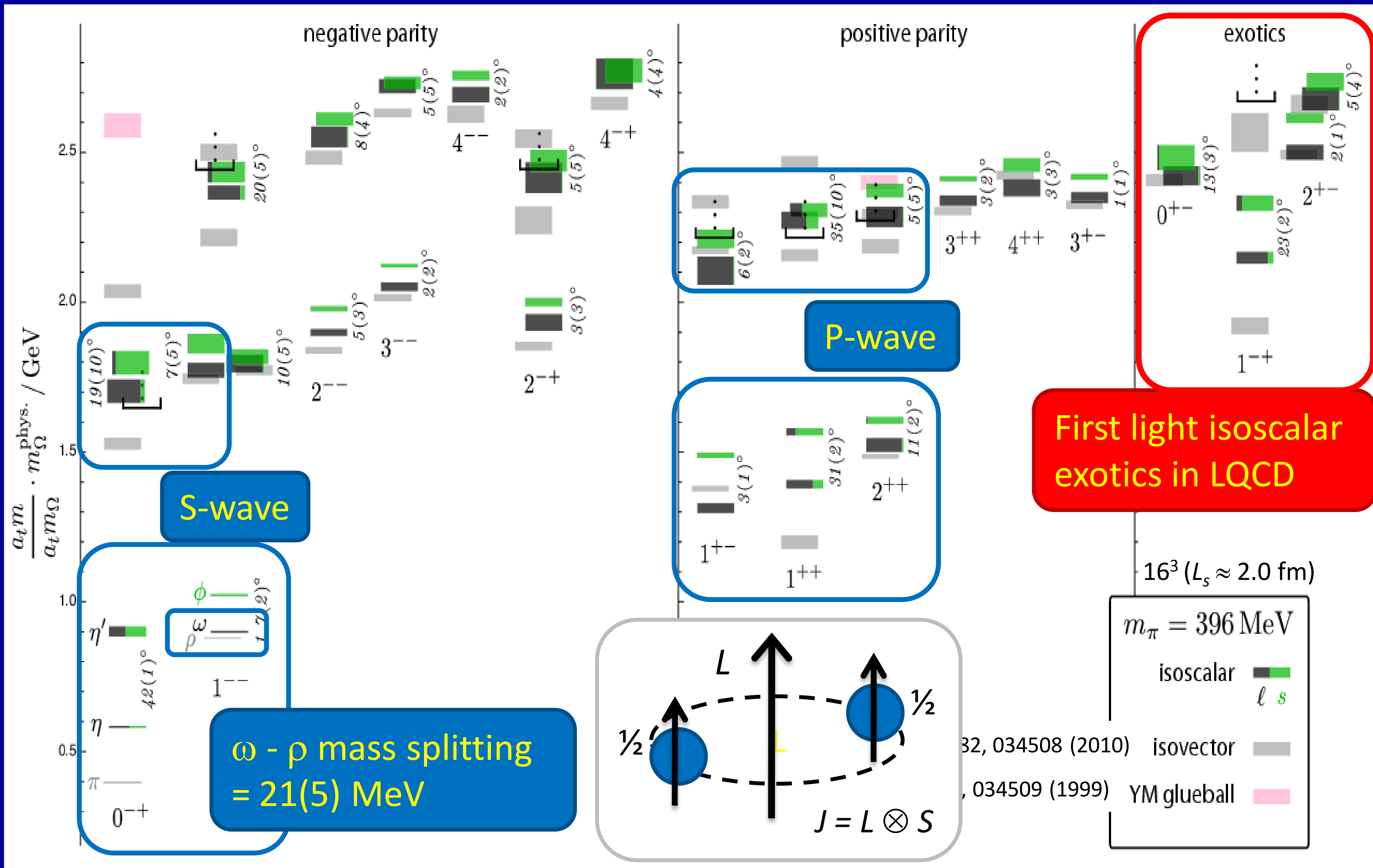

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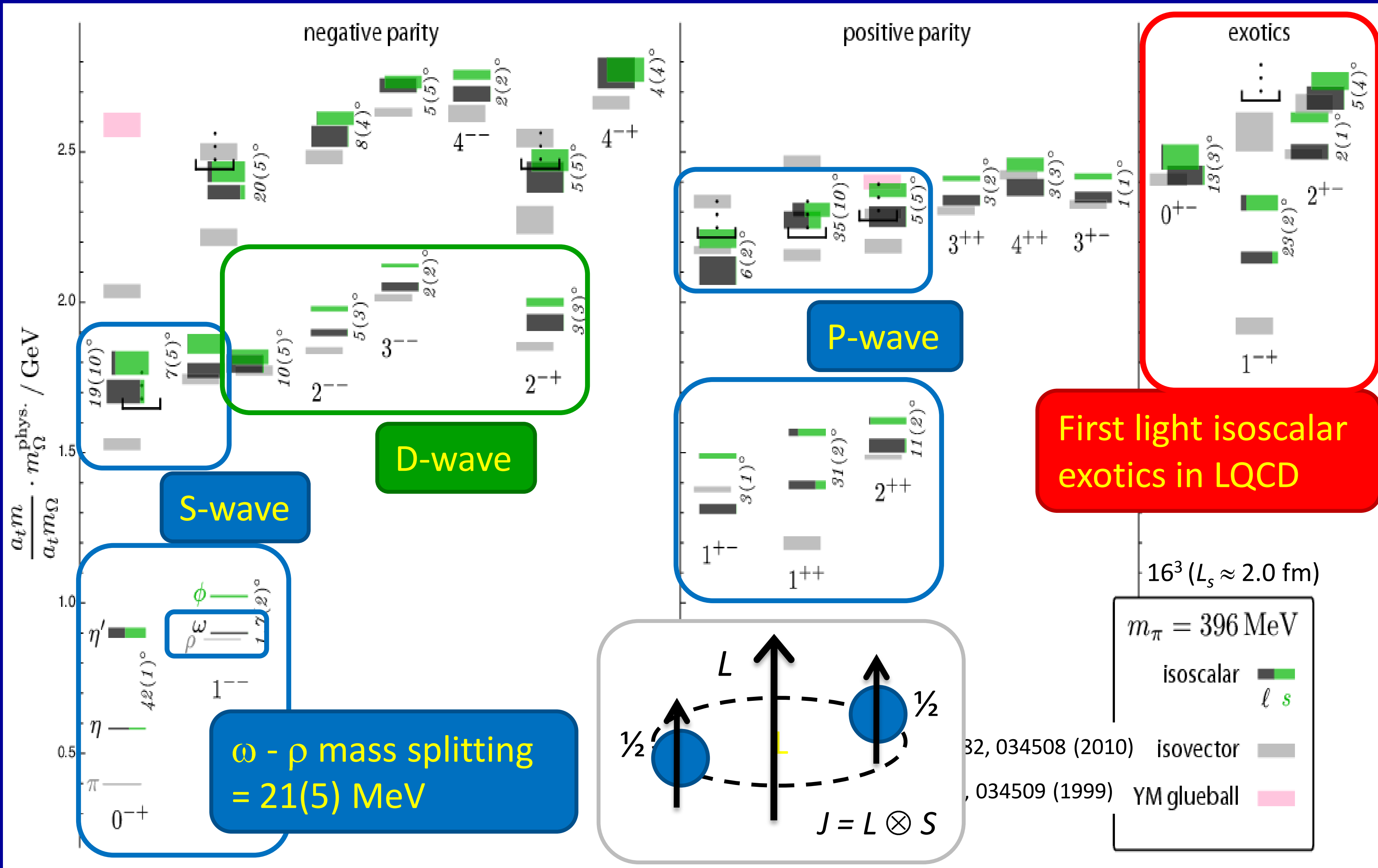

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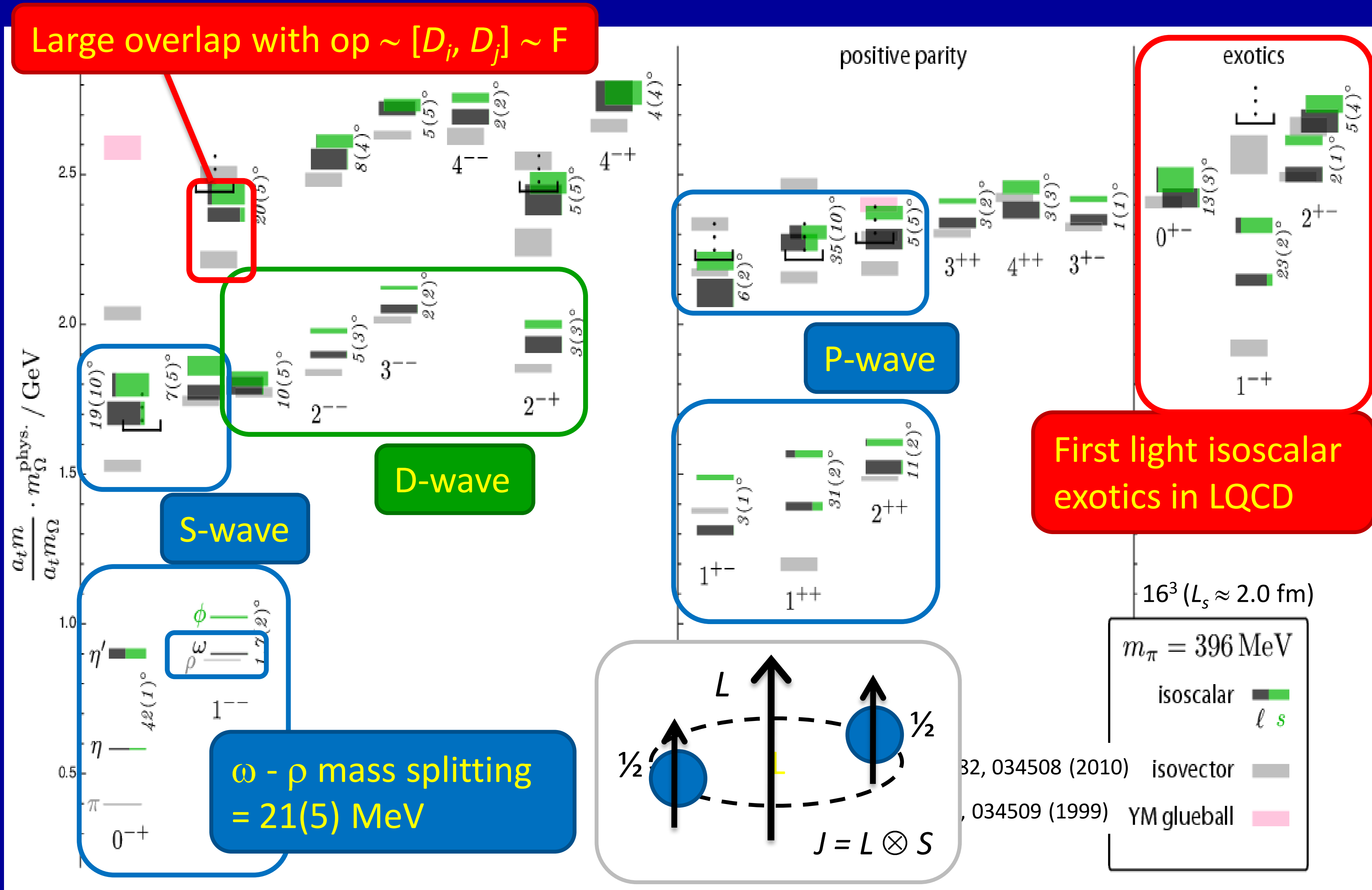

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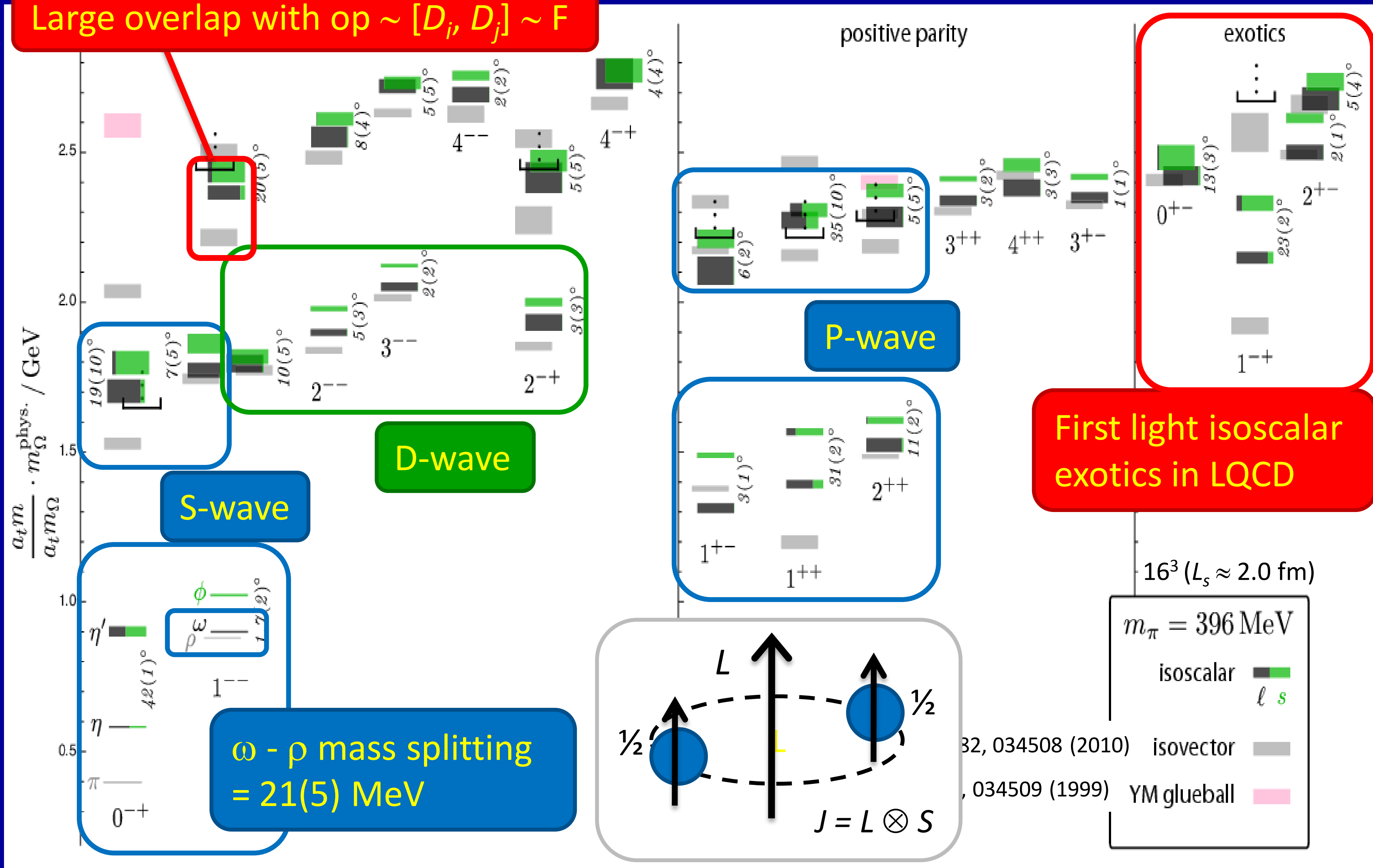

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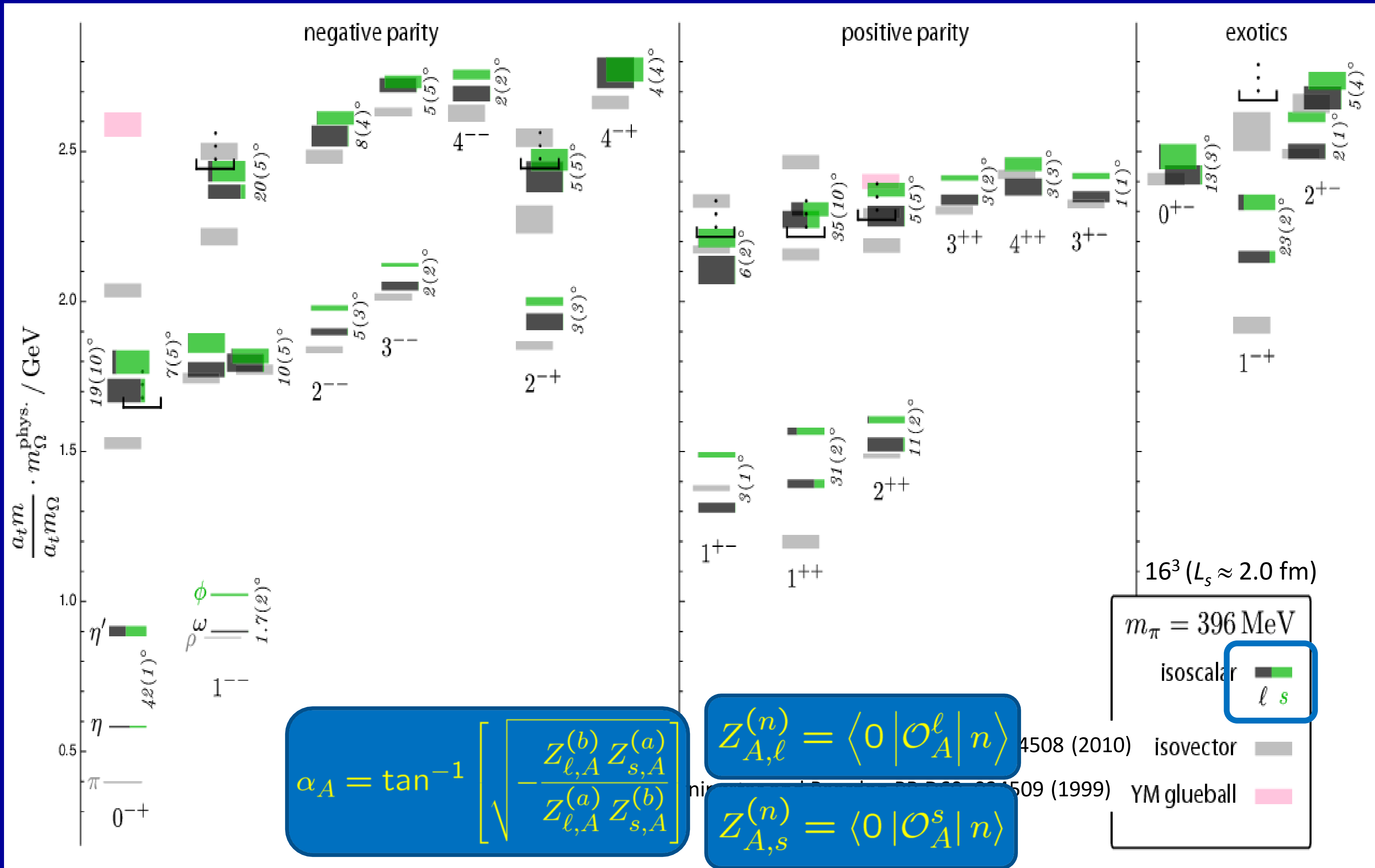
More interpretation of isovector and isoscalar results:
“lowest hybrid meson supermultiplet” arXiv:1106.5515



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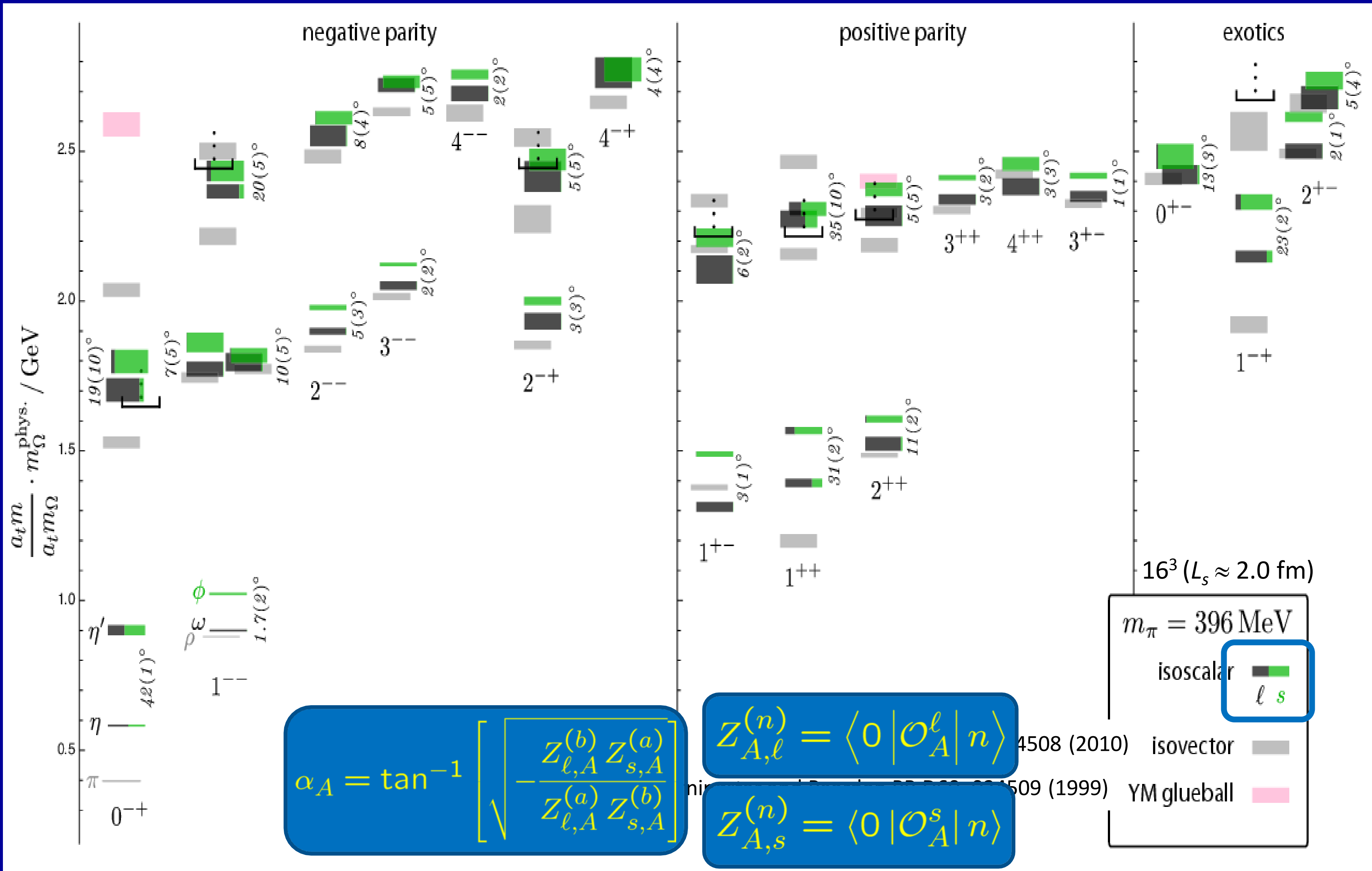

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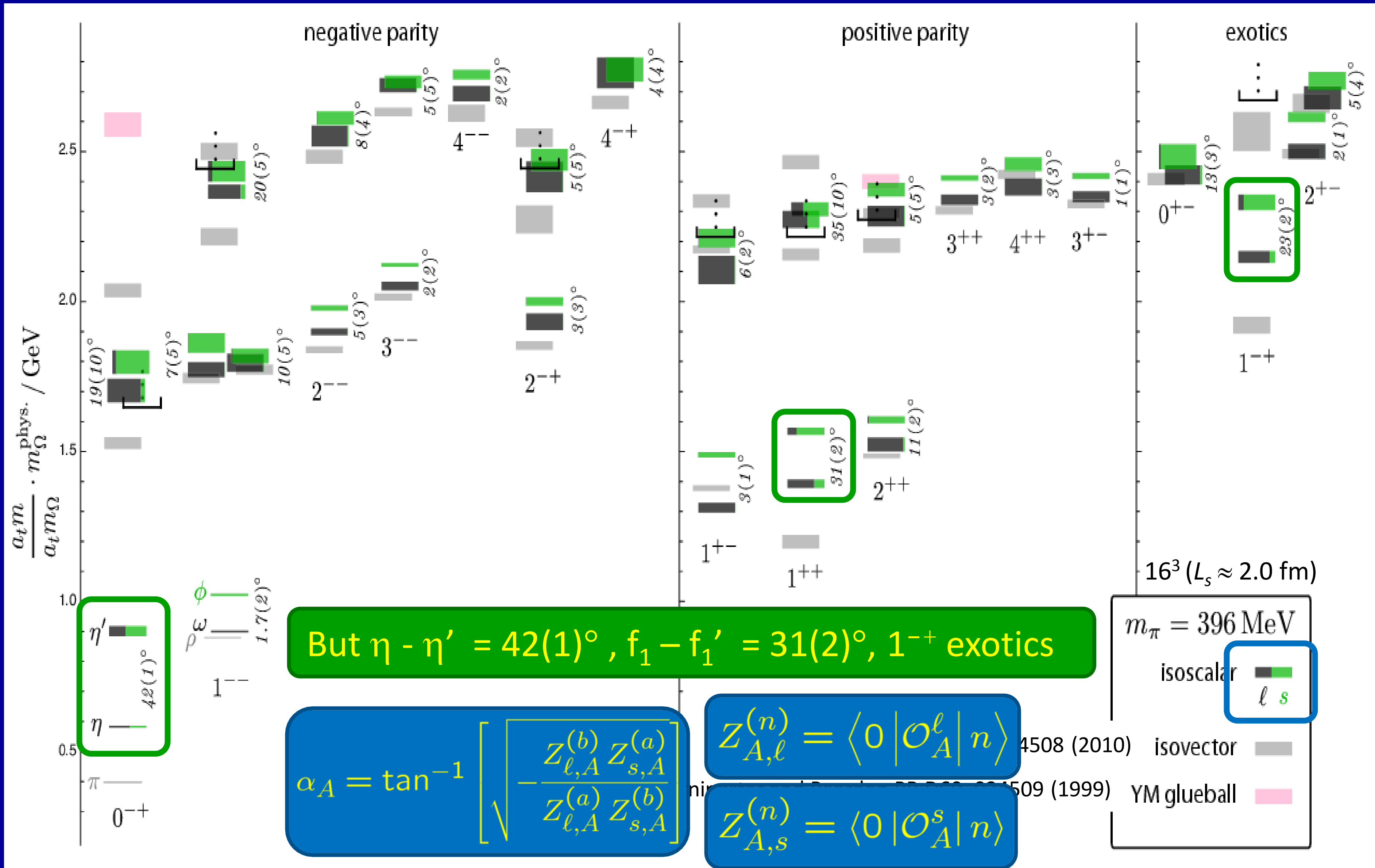
PR D83, 111502 (2011)

Most close to ideally mixed



But η - η' = 42(1)°, $f_1 - f_1'$ = 31(2)°, 1^{-+} exotics

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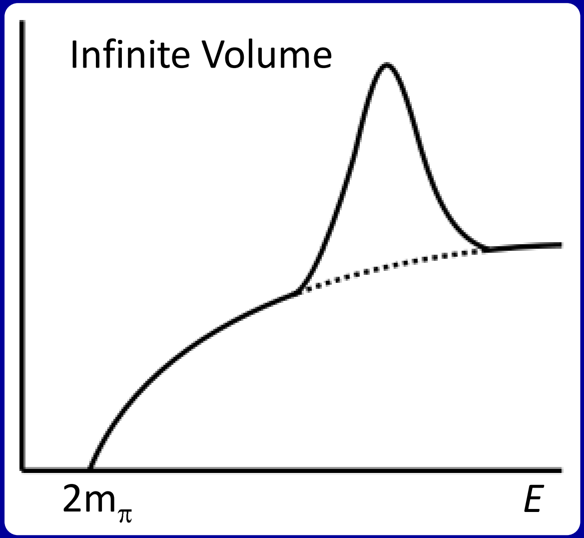

Infinite Volume

Continuous spectrum

$$E_{\pi\pi}(p) = 2\sqrt{m_\pi^2 + \vec{p}^2}$$

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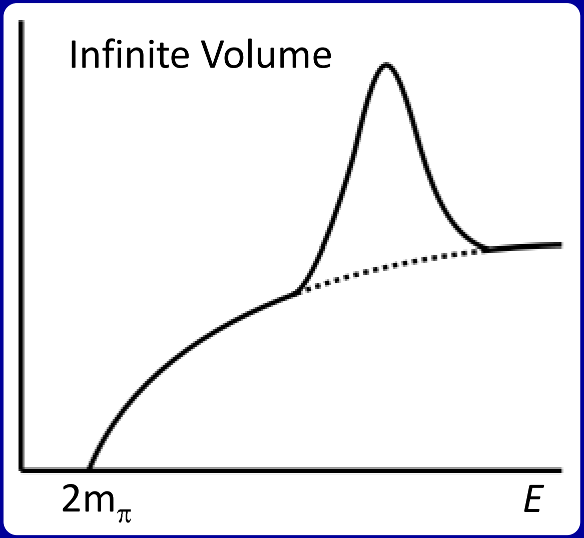

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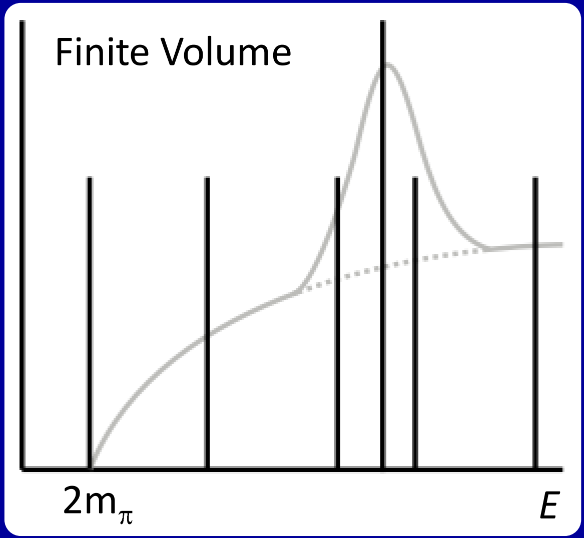

Finite Volume

Cubic box with periodic boundary conditions

Quantised momenta

$$\vec{p} = \frac{2\pi}{L_s}(n_x, n_y, n_z)$$

→ Discrete spectrum

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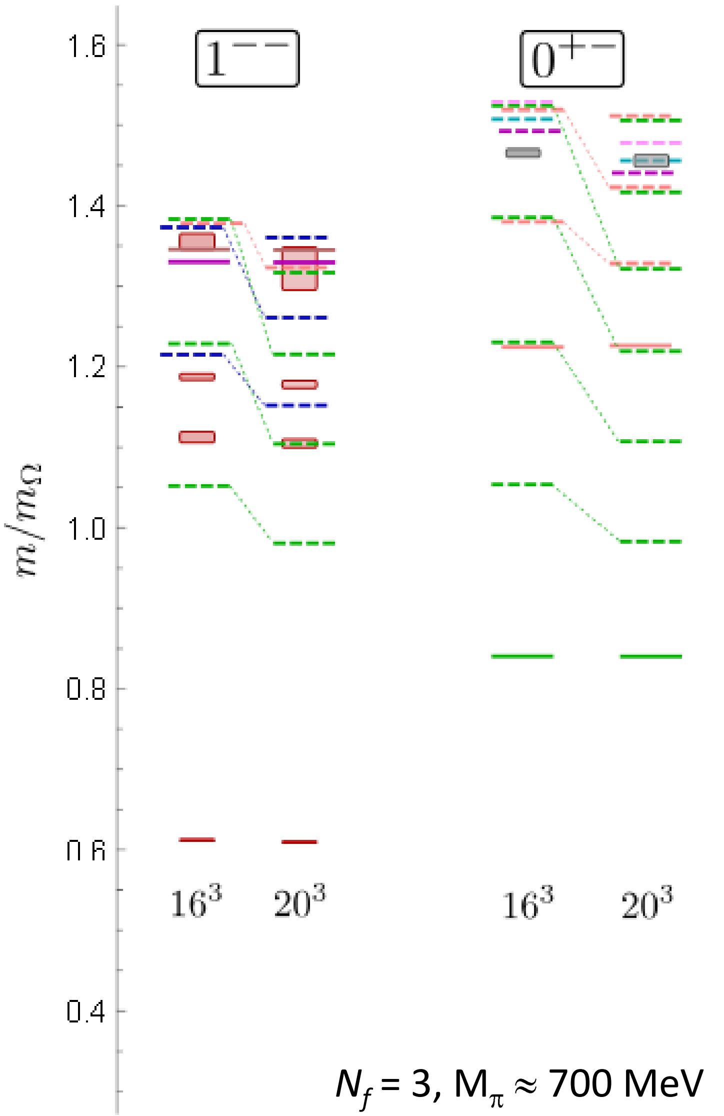

$N_f = 3$, $M_\pi \approx 700$ MeV

Boxes – extracted meson levels (I=1)

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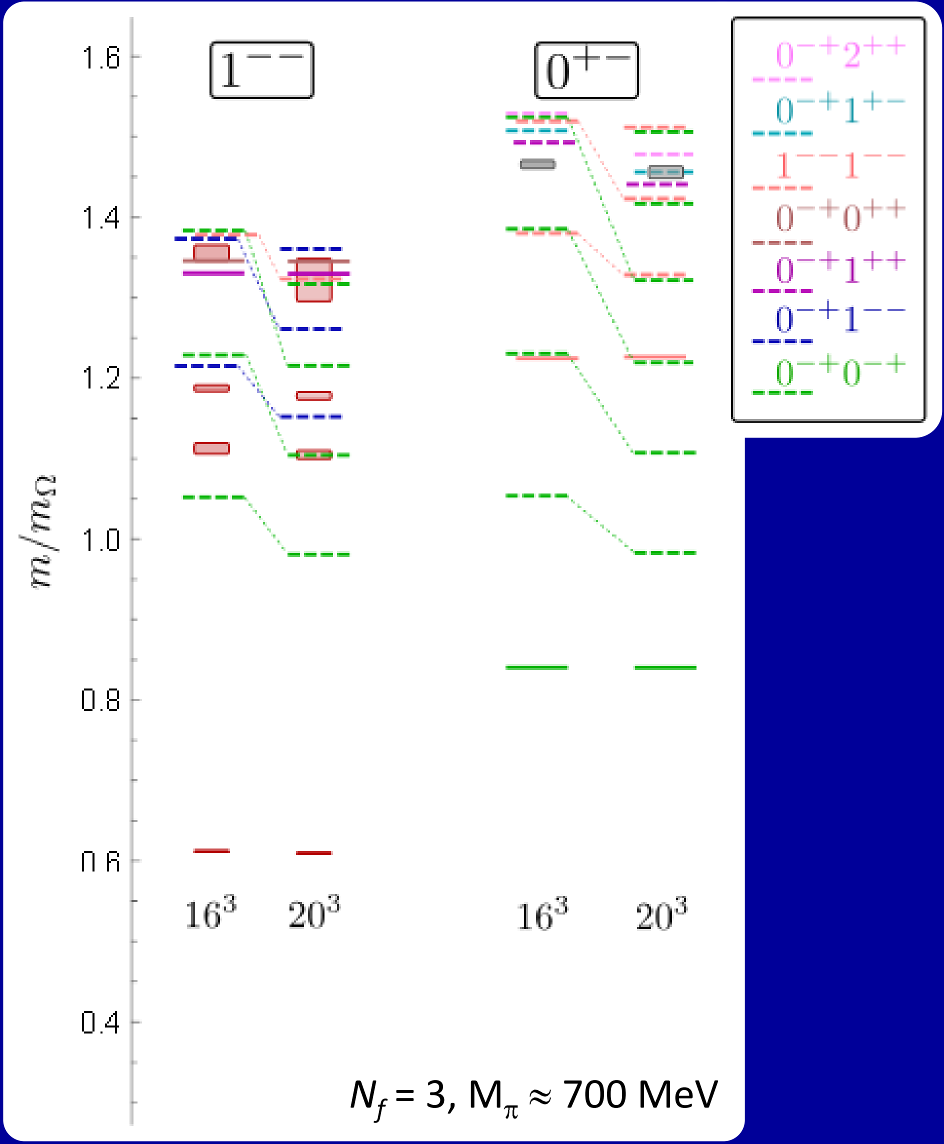

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Dashed lines – non-interacting two-meson levels

$$\vec{p} = \frac{2\pi}{L_s}(n_x, n_y, n_z)$$

$$E_{AB}(p) = \sqrt{m_A^2 + \vec{p}^2} + \sqrt{m_B^2 + \vec{p}^2}$$

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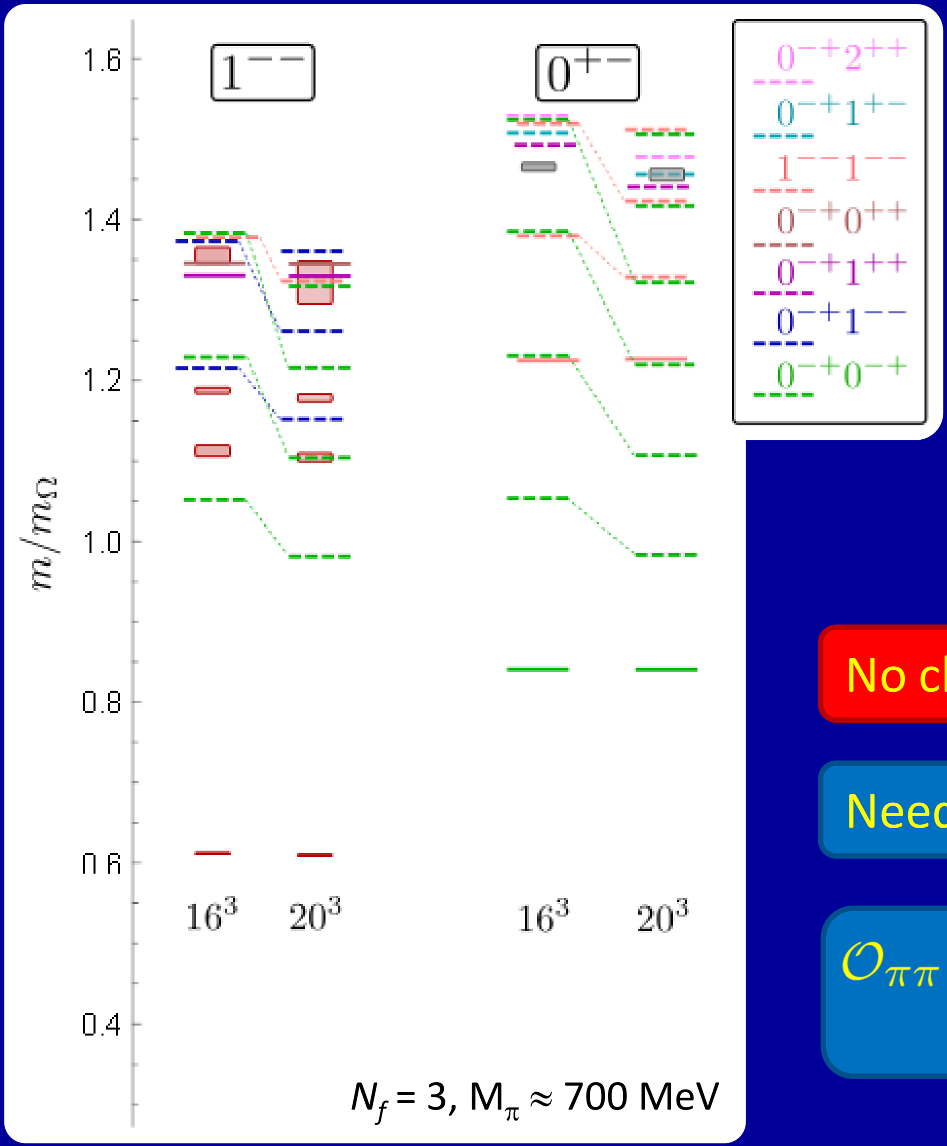

Boxes – extracted meson levels (I=1)

Dashed lines – non-interacting two-meson levels

$$\vec{p} = \frac{2\pi}{L_s}(n_x, n_y, n_z)$$

$$E_{AB}(p) = \sqrt{m_A^2 + \vec{p}^2} + \sqrt{m_B^2 + \vec{p}^2}$$

No clear evidence for two-meson states

Need ops that 'look like' two-mesons

$$\mathcal{O}_{\pi\pi} = \sum_{\Omega_{\vec{p}}} Y_L^M(\Omega_{\vec{p}})\ \mathcal{O}_\pi(\vec{p})\ \mathcal{O}_\pi(-\vec{p})$$

Scattering in a box

Euclidean time: can’t directly study dynamical properties like widths

Lüscher: (elastic) energy shifts in **finite volume** → phase shift

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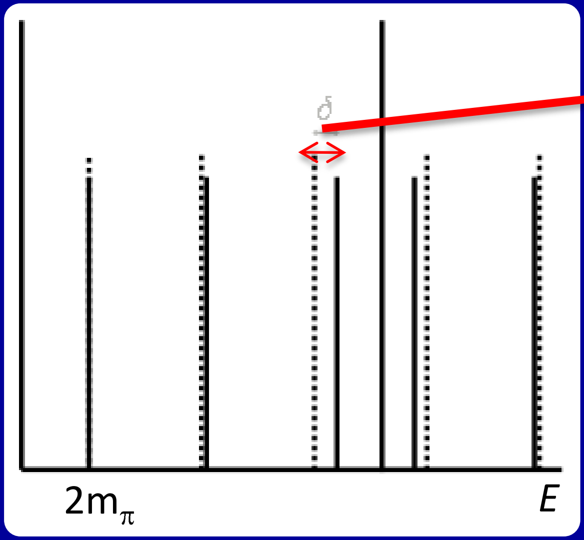

$$\Delta E(L_s) \to \delta(E, L_s)$$

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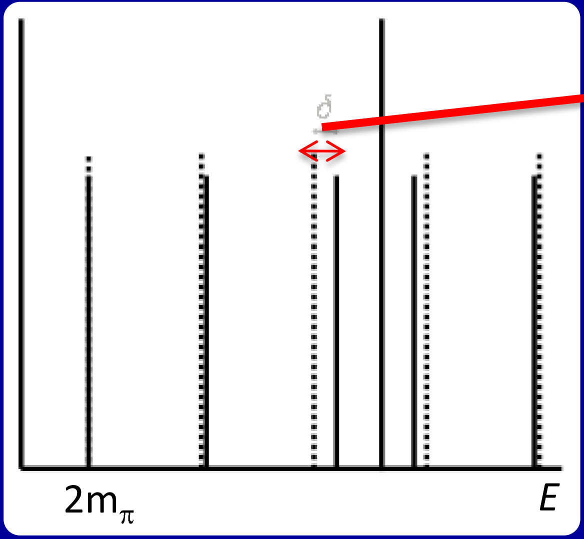


$\Delta E(L_s) \rightarrow \delta(E, L_s)$

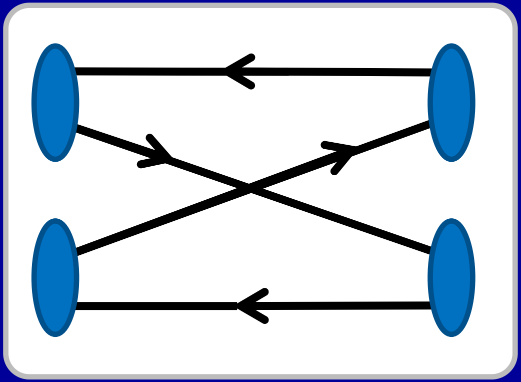
Extract phase shift at discrete *E*

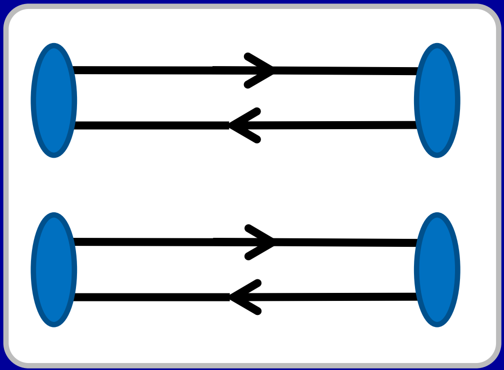
Map out phase shift →
resonance parameters (mass, width)

$\pi\pi$ I=2 spectrum

PR D83, 071504 (2011)

$M_\pi \approx 400$ MeV





+ similar diagrams

'Distillation' for efficient computation

PR D80 054506 (2009)

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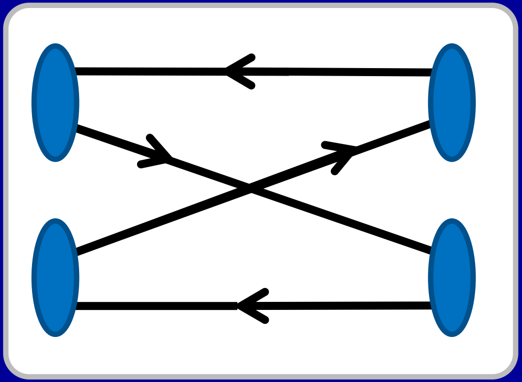

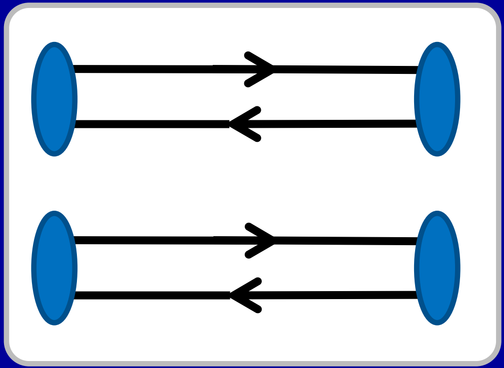

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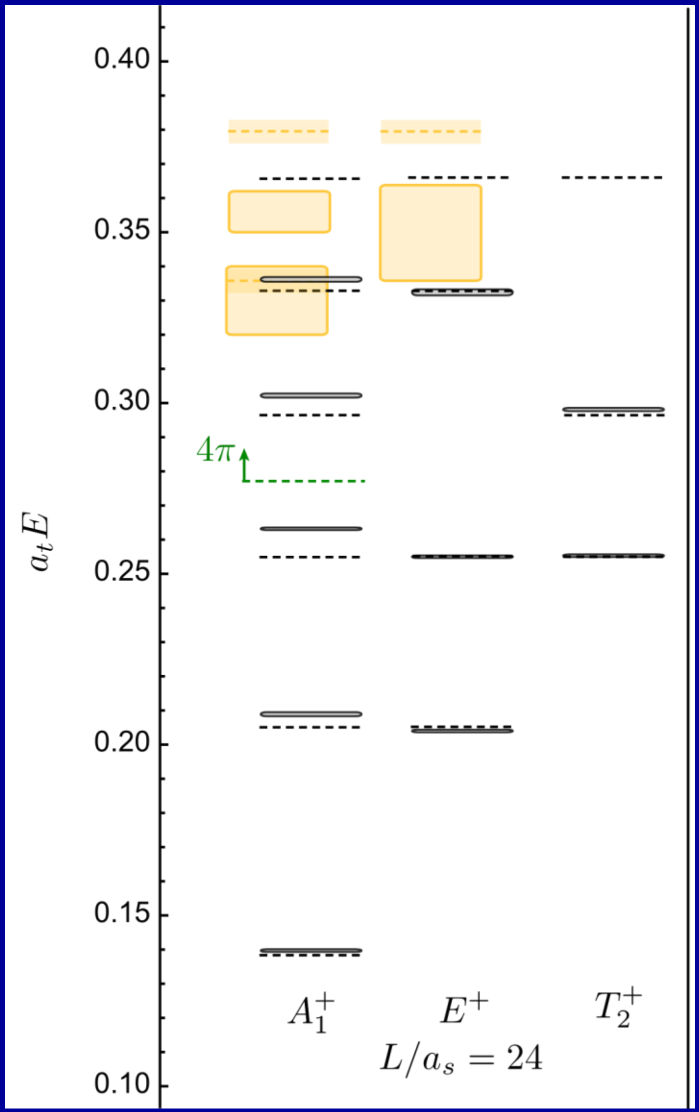

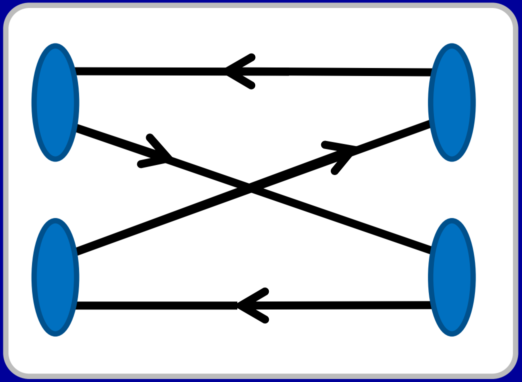

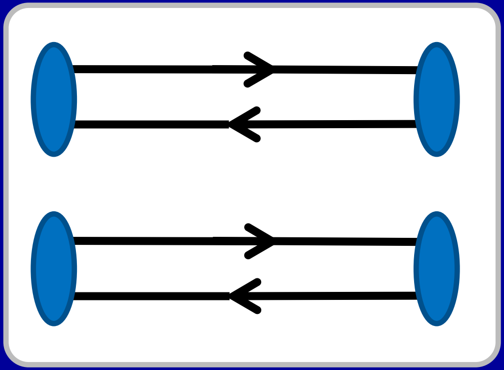

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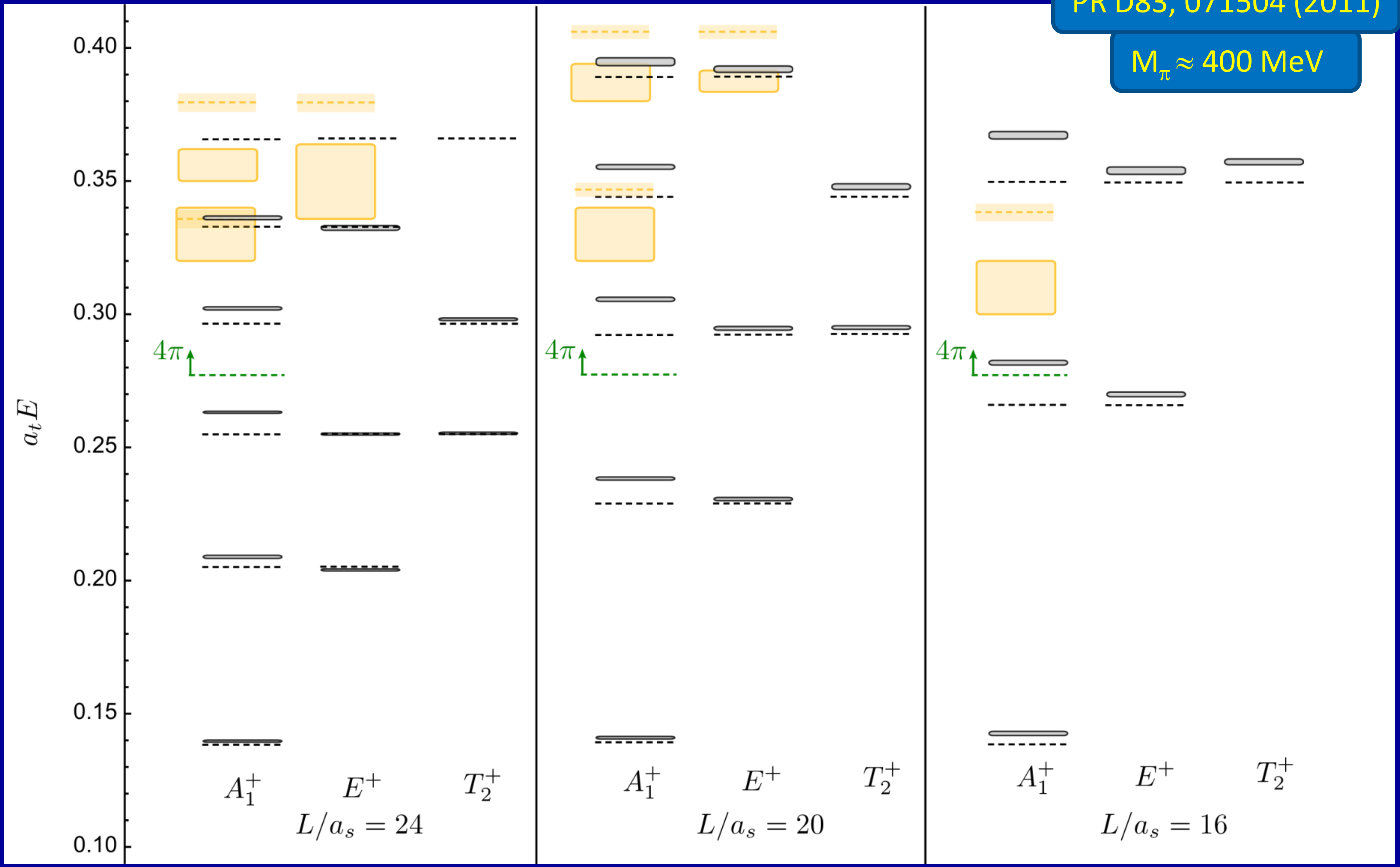
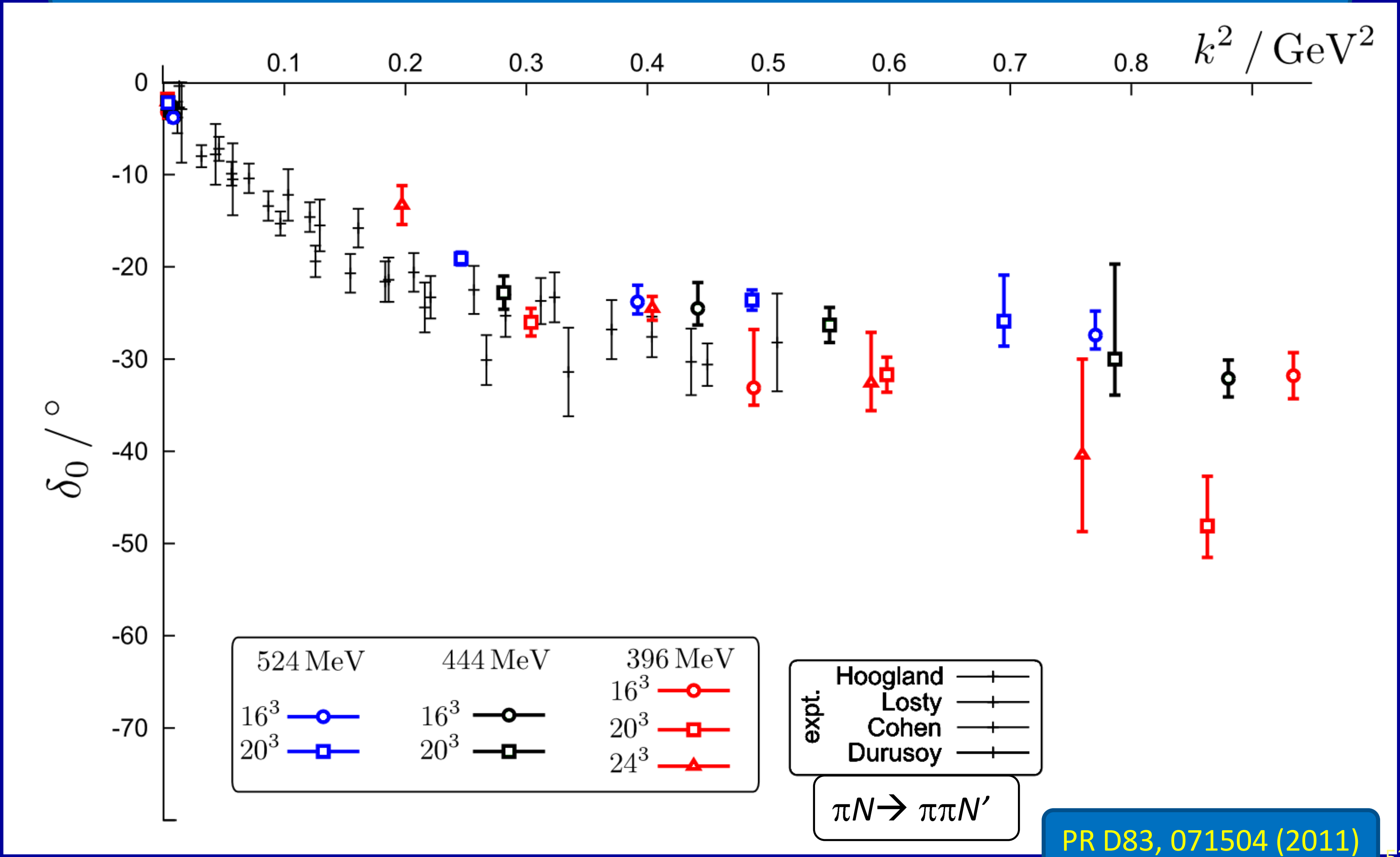


$\pi\pi$ I=2 phase shift



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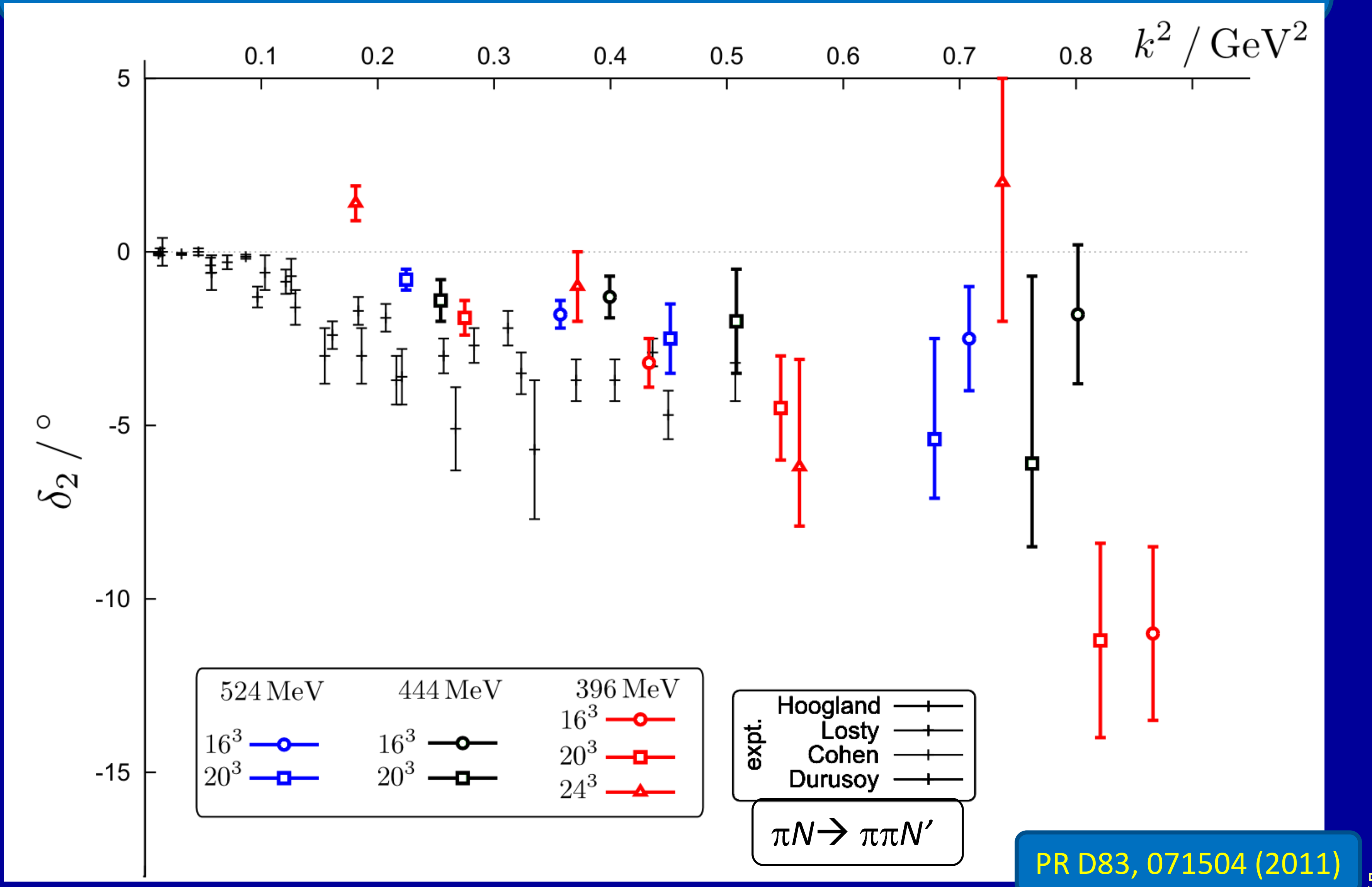
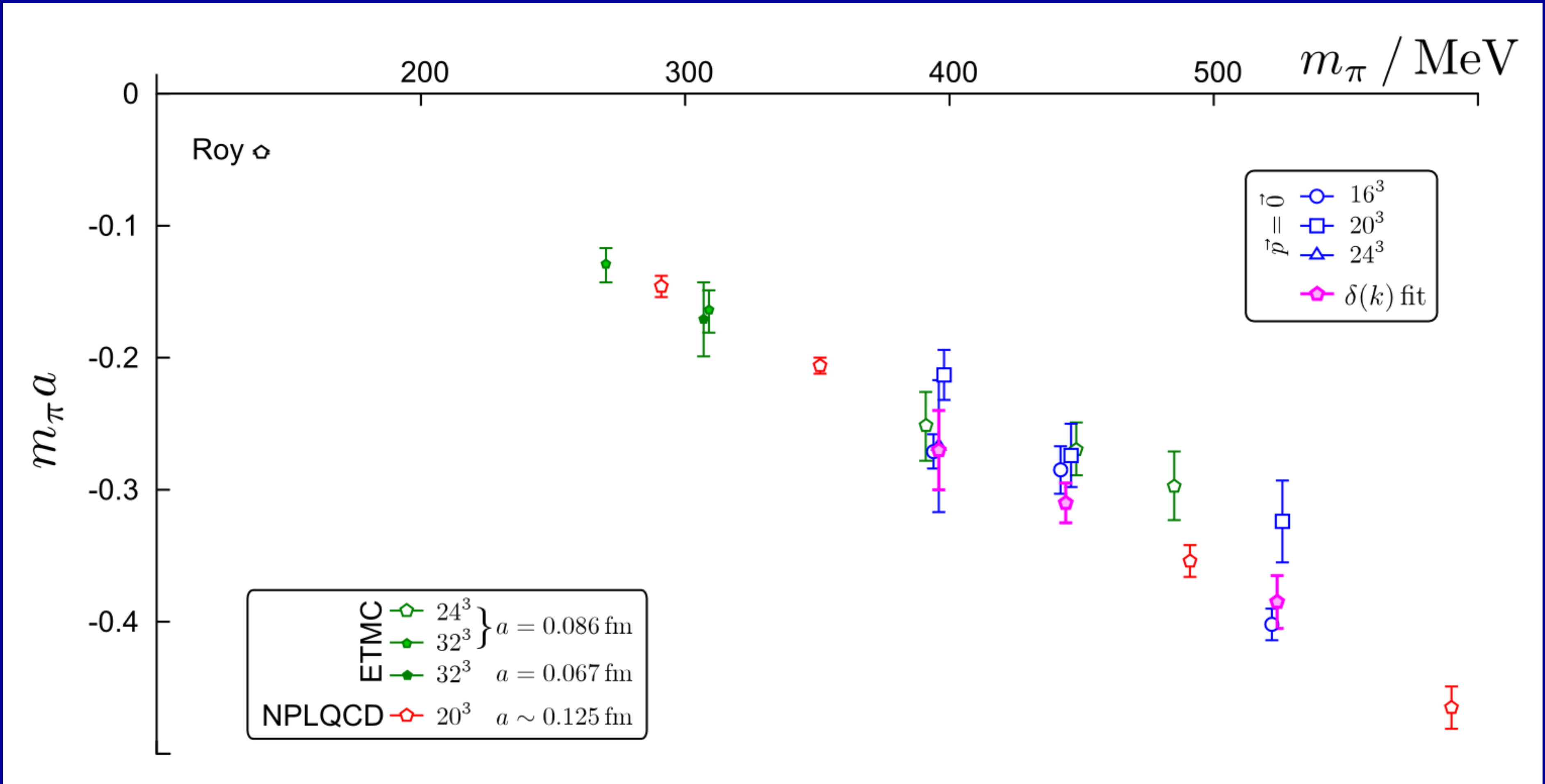


ππ I=2 scattering length



PR D83, 071504 (2011)

Summary and Outlook

Summary

- **Extensive** isoscalar meson spectrum
- **Exotics** and non-exotic **hybrids**, high spin, excited states
- Flavour mixing angles
- Multi-meson operators – $\pi\pi$ I=2 **phase shift mapped out**

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Outlook

- Include **glueball** operators; A_1^{++} (0^{++}) channel
- Other scattering channels – map out **resonances**
- Lighter pion masses, larger volumes, ...

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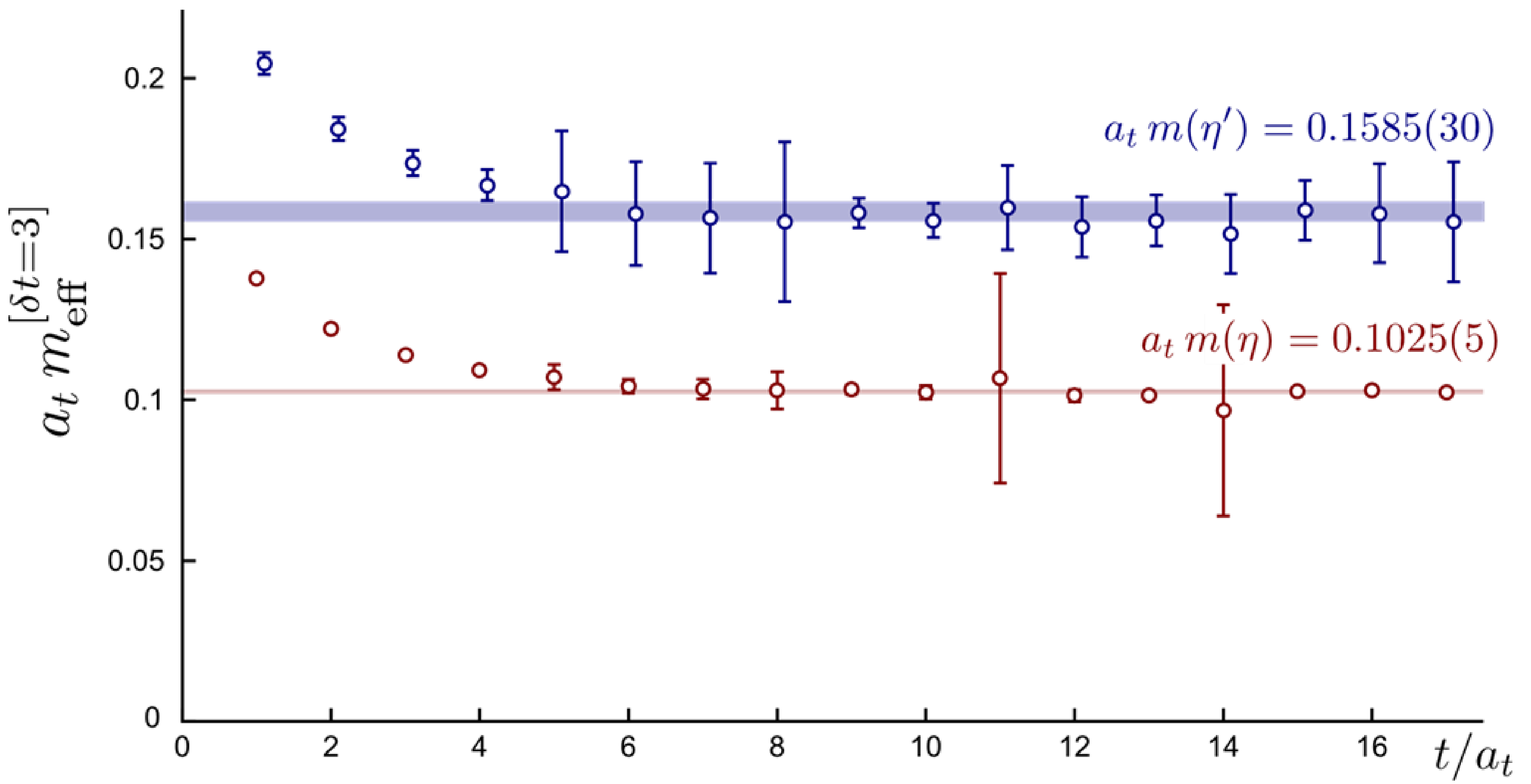
Extra Slides

Prin. Corr. M_{eff}

PR D83, 111502 (2011)



$M_{eff}(t) = -\ln\left[\lambda(t+\delta t)/\lambda(t)\right]/dt$

$\delta t = 3$

Flavour Mixings

Determine mixing of physical states by looking at overlaps

$$\mathcal{O}^{\ell} \sim \frac{1}{\sqrt{2}}\left(\bar{u}\Gamma u + \bar{d}\Gamma d\right)$$

$$\mathcal{O}^{s} \sim \bar{s}\Gamma s$$

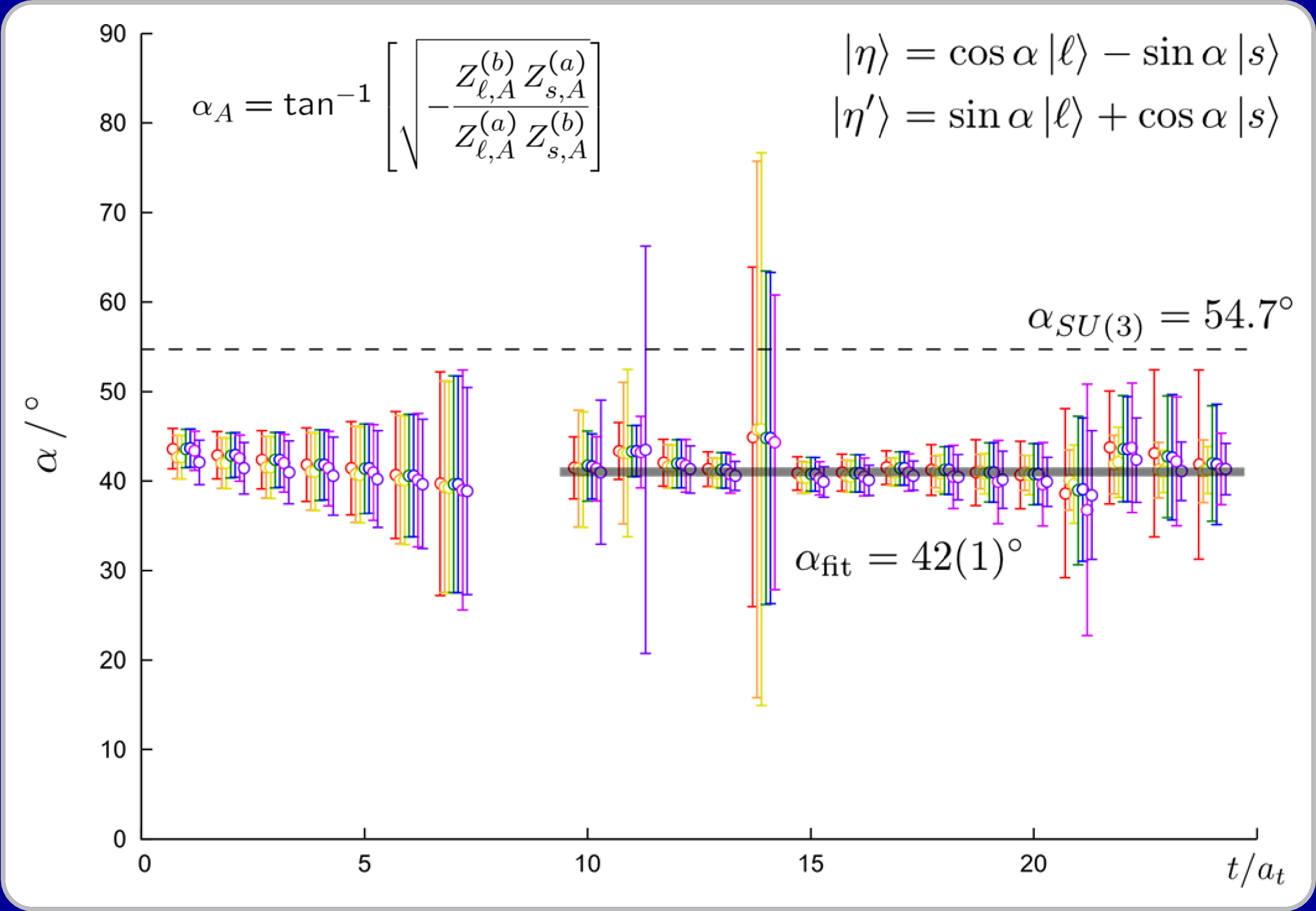
$$Z_{A,\ell}^{(n)} = \left\langle 0 \left| \mathcal{O}_A^{\ell} \right| n \right\rangle$$

$$Z_{A,s}^{(n)} = \langle 0 | \mathcal{O}_A^{s} | n \rangle$$

$$\alpha_A = \tan^{-1}\left[\sqrt{-\frac{Z_{\ell,A}^{(b)}\, Z_{s,A}^{(a)}}{Z_{\ell,A}^{(a)}\, Z_{s,A}^{(b)}}}\right]$$

Mixing Angle

PR D83, 111502 (2011)



Correlators

PR D83, 111502 (2011)