

# Dual parameterization of nucleon and nuclear GPDs

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## References:

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## Outline

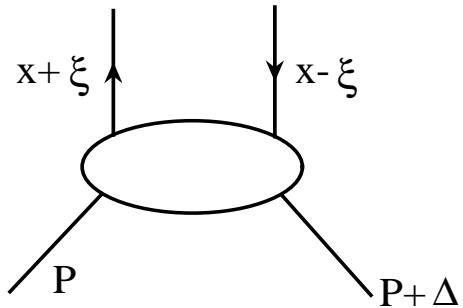
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## Introduction

- Generalized Parton Distributions (GPDs) of a hadron (nucleon, pion, nucleus) describe correlations of partons in the target. For the nucleon

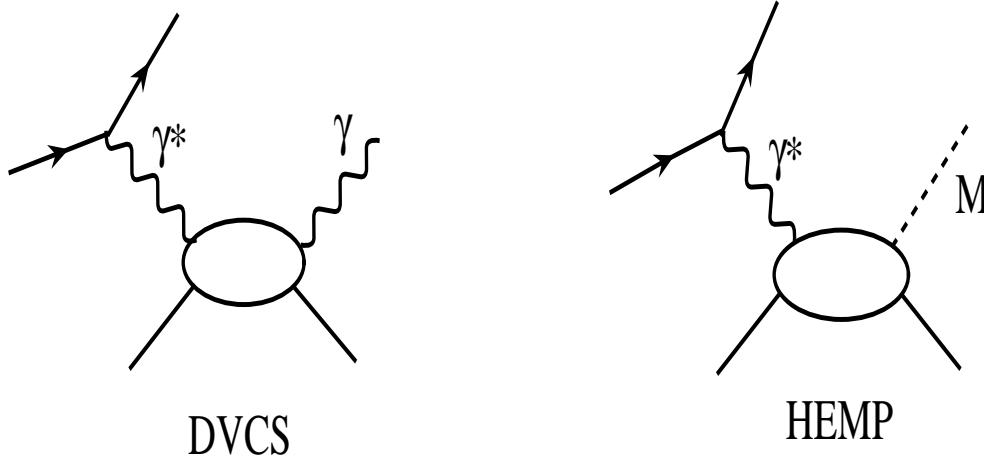
$$\bar{P}^+ \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \left\langle P' | \bar{\psi} \left( -\frac{z^-}{2} \right) \gamma_+ \psi \left( \frac{z^-}{2} \right) | P \right\rangle = H(x, \xi, t) \bar{N}(P') \gamma^+ N(P) + E(x, \xi, t) \bar{N}(P') \frac{i\sigma^{+\mu} \Delta^\mu}{2m_N} N(P)$$

- Schematically



- GPDs contain more microscopic information about the parton structure of the target than usual parton distributions and form factors.

- GPDs can be accessed in hard exclusive reactions such as Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP)



- QCD factorization theorems for DVCS and HEMP allow to express the corresponding scattering amplitudes as convolution of coefficient functions with the GPDs

$$T_{\text{DVCS}}^{\mu\nu}(\xi, t, Q^2) = -\frac{1}{2} g_{\perp}^{\mu\nu} \int_{-1}^1 dx \, C^+(x, \xi) [H(x, \xi, t, Q^2) \bar{N}(p') \hat{n} N(p) + E(x, \xi, t, Q^2) \bar{N}(p') i \sigma^{k\lambda} \frac{n_k \Delta_\lambda}{2m_N} N(p)] + \dots$$

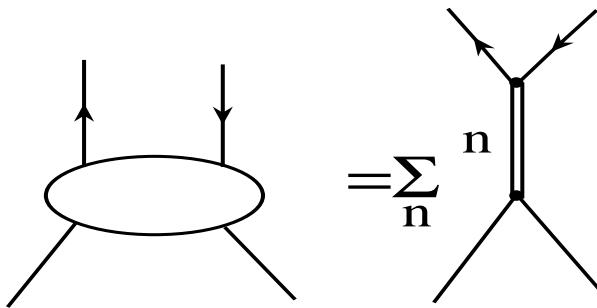
- Corollaries of factorization
  - GPDs are universal (process-independent)
  - GPDs have a well-defined dependence on the factorization scale (virtuality  $Q^2$ )
- DVCS experimental observables (cross section, asymmetries) expressed in terms of the Compton Form Factors

$$\mathcal{H}(\xi, t, Q^2) = \sum e_q^2 \int_0^1 dx \, H^q(x, \xi, t, Q^2) \left( \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right)$$

- Since GPDs enter via convolution and depend on three variables, extraction from the data is difficult/impossible
- At the present stage, models of GPDs are necessary

## Dual parameterization of nucleon GPDs

- The essence of the dual parameterization of GPDs is the assumption of **duality** between the  $s$  and  $t$ -channel description of the quark-hadron scattering amplitude



- Duality in hadronic physics – the Veneziano model
- The dual Veneziano amplitude is given as an **infinite series** of infinitely narrow resonances in either  $s$  or  $t$  channel.

The series is **formally divergent** to provide singularities of the amplitude  $\Rightarrow$  same in the dual parameterization of GPDs.

- The dual representation of quark GPDs of the **pion** is a formal solution reproducing Mellin moments of the pion GPDs, **M. Polyakov, 1998**.

- **Derivation:**

- Two-pion distribution amplitude  $\Phi^I(z, \xi, w^2)$  is expanded in terms of eigenfunctions of QCD evolution and in partial waves of produced pions

$$\Phi^I(z, \zeta, w^2, \mu^2) = 6z(1-z) \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}^I(w^2, \mu^2) C_n^{3/2}(2z-1) P_l(2\zeta-1)$$

- \*  $I = 0, 1$  isospin
- \*  $p_1$  and  $p_2$  momenta of final pions,  $P = p_1 + p_2$
- \*  $z = k^+/P^+$  quark light-cone fraction
- \*  $\zeta = p_1^+/P^+$  distribution of light-cone momenta between pions
- \*  $w^2 = (p_1 + p_2)^2$

- Consider Mellin moments of  $\Phi^I$

$$\int_0^1 dz (2z - 1)^{N-1} \Phi^I(z, \zeta, w^2) = \frac{1}{[p_1^+ + p_2^+]^N} \langle p_1 p_2 | \bar{\psi} \gamma^+ (\overleftrightarrow{\nabla}^+)^{N-1} \psi | 0 \rangle$$

- As matrix elements of a local operator, the Mellin moments can be continued to the crossed, GPD channel

$$\langle p_1 p_2 | \bar{\psi} \gamma^+ (\overleftrightarrow{\nabla}^+)^{N-1} \psi | 0 \rangle = \langle p_2 | \bar{\psi} \gamma^+ (\overleftrightarrow{\nabla}^+)^{N-1} \psi | -p_1 \rangle$$

- Changing appropriately the kinematic variables, we have

$$\xi^N \sum_{n=0}^{N-1} \sum_{l=0}^{n+1} B_{nl}^I(t) P_l \left( \frac{1}{\xi} \right) \int_0^1 dx \frac{3}{4} (1 - x^2) x^{N-1} C_n^{3/2}(x) = \int_0^1 dx x^{N-1} H^I(x, \xi, t)$$

- The quark GPDs of the pion are reconstructed as a formal divergent series

$$H^I(x, \xi, t, \mu^2) = \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}^I(t, \mu^2) \theta(\xi - |x|) \left( 1 - \frac{x^2}{\xi^2} \right) C_n^{3/2} \left( \frac{x}{\xi} \right) P_l \left( \frac{1}{\xi} \right)$$

- Shuvaev and Polyakov (2002) postulated similar dual parameterization for nucleon GPDs,

$$H^i(x, \xi, t, \mu^2) = \sum_{n=1}^{\infty} \sum_{\substack{l=0 \\ \text{odd}}}^{n+1} B_{nl}^i(t, \mu^2) \theta(\xi - |x|) \left(1 - \frac{x^2}{\xi^2}\right) C_n^{3/2} \left(\frac{x}{\xi}\right) P_l \left(\frac{1}{\xi}\right)$$

$$E^i(x, \xi, t, \mu^2) = \sum_{n=1}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} C_{nl}^i(t, \mu^2) \theta(\xi - |x|) \left(1 - \frac{x^2}{\xi^2}\right) C_n^{3/2} \left(\frac{x}{\xi}\right) P_l \left(\frac{1}{\xi}\right)$$

- $i$  the quark flavor
- $B_{nl}^i$  and  $C_{nl}^i$  unknown form factors
- Formula is written for singlet combinations of the GPDs,  $H^i(x, \xi, t) \equiv H^i(x, \xi, t) - H^i(-x, \xi, t)$  and  $E^i(x, \xi, t) \equiv E^i(x, \xi, t) - E^i(-x, \xi, t)$
- The important property of polynomiality is by construction

## Main features of dual parameterization

- Easy QCD evolution to leading order accuracy

$$B_{nl}^i(\mu^2) = B_{nl}^i(\mu_0^2) \left( \frac{\ln(\mu_0^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)} \right)^{\gamma_n/B}$$

- $\gamma_n$  anomalous dimension
- $B = 11 - (2/3)n_{\text{flav}}$
- Simple expression for the Compton Form Factors to the LO accuracy (see later)  
→ use the dual parameterization of the GPDs as a **LO parameterization**.
- The formal series diverge → cannot be used in this form to study GPDs themselves.  
However, the series can be decomposed over other orthogonal polynomials on  $x \in [-1, 1]$  ([Belitsky et al., 1997](#)) or it can actually be summed using the trick of [Polyakov and Shuvaev](#).

## Polyakov-Shuvaev trick

Let us introduce of a set of generating functions  $Q_k^i$  and  $R_k^i$

$$B_{n n+1-k}^i(t, \mu^2) = \int_0^1 dx x^n Q_k^i(x, t, \mu^2)$$

$$C_{n n+1-k}^i(t, \mu^2) = \int_0^1 dx x^n R_k^i(x, t, \mu^2) \rightarrow$$

$$\begin{aligned} H^i(x, \xi, t, \mu^2) &= \sum_{\substack{k=0 \\ \text{even}}}^{\infty} \left[ \frac{\xi^k}{2} \left( H^{i(k)}(x, \xi, t, \mu^2) - H^{i(k)}(-x, \xi, t, \mu^2) \right) \right. \\ &\quad \left. + \left( 1 - \frac{x^2}{\xi^2} \right) \theta(\xi - |x|) \sum_{\substack{l=1 \\ \text{odd}}}^{k-3} C_{k-l-2}^{3/2} \left( \frac{x}{\xi} \right) P_l \left( \frac{1}{\xi} \right) \int_0^1 dy y^{k-l-2} Q_k^i(y, t, \mu^2) \right] \\ H^{i(k)}(x, \xi, t, \mu^2) &= \frac{1}{\pi} \int_0^1 \frac{dy}{y} \left[ \left( 1 - y \frac{\partial}{\partial y} \right) Q_k^i(y, t, \mu^2) \right] \int_{-1}^1 ds \frac{x_s^{1-k}}{\sqrt{2x_s - x_s^2 - \xi^2}} \theta(2x_s - x_s^2 - \xi^2) \\ - \lim_{y \rightarrow 0} Q_k^i(y, t, \mu^2) &\int_{-1}^1 ds \frac{x_s^{1-k}}{\sqrt{2x_s - x_s^2 - \xi^2}} \theta(2x_s - x_s^2 - \xi^2) \end{aligned}$$

## Minimal model of the dual parameterization

**Essence of the minimal model:** GPDs  $H^i$  and  $E^i$  are expressed in terms of the forward parton distributions, unknown forward limit of  $E^i$  and Gegenbauer moments of the  $D$ -term.

- Keep only  $Q_0^i$  and  $Q_2^i$  for  $H^i$  and  $R_0^i$  and  $R_2^i$  for  $E^i$ .  
In the HERA kinematics ( $\xi < 0.005$ ), the contribution of  $Q_k^i$  and  $R_k^i$  with  $k \geq 2$  is kinematically suppressed by  $\xi^k$ .  
In HERMES kinematics ( $\xi < 0.1$ ), we keep  $Q_2^i$  and  $R_2^i$  as a first correction.
- Relation between Mellin moments of  $H^i$  and form factors  $B_{nl}^i$  in the  $\xi \rightarrow 0$  limit

$$B_{nn+1}^i(t, \mu^2) = \frac{2n+3}{2n+4} \int_{-1}^1 dx x^n H^i(x, 0, t, \mu^2) \equiv \frac{2n+3}{2n+4} \int_0^1 dx x^n (q^i(x, t, \mu^2) + \bar{q}^i)$$

$$C_{nn+1}^i(t, \mu^2) = \frac{2n+2}{2n+4} \int_{-1}^1 dx x^n E^i(x, 0, t, \mu^2) \equiv \frac{2n+3}{2n+4} \int_0^1 dx x^n (e^i(x, t, \mu^2) + \bar{e}^i)$$

- Since all  $B_{nn+1}^i$  and  $C_{nn+1}^i$  are fixed, the generating functions  $Q_0^i$  and  $R_0^i$  can be restored

$$Q_0^i(x, t, \mu^2) = q^i(x, t, \mu^2) + \bar{q}^i(x, t, \mu^2) - \frac{x}{2} \int_x^1 \frac{dz}{z^2} (q^i(z, t, \mu^2) + \bar{q}^i(z, t, \mu^2))$$

$$R_0^i(x, t, \mu^2) = e^i(x, t, \mu^2) + \bar{e}^i(x, t, \mu^2) - \frac{x}{2} \int_x^1 \frac{dz}{z^2} (e^i(z, t, \mu^2) + \bar{e}^i(z, t, \mu^2))$$

In  $t \rightarrow 0$  limit,  $q^i(x, t, \mu^2) + \bar{q}^i(x, t, \mu^2)$  become the singlet singlet combination of **forward quark distribution** and  $e^i(x, t, \mu^2) + \bar{e}^i(x, t, \mu^2)$  become the unknown forward limit of the singlet combination GPDs  $E^i$

Therefore, up to the  $t$ -dependence, the leading functions  $Q_0^i$  and  $R_0^i$  are completely constrained by the **forward parton distributions** and the **forward limit of the GPDs  $E^i$** .

## The $t \rightarrow 0$ limit

- Forward quark PDFs are taken from CTEQ5L at  $\mu_0 = 1 \text{ GeV}$ .
- Since the GPDs  $E^i$  decouple in the forward limit, the functions  $e^i + \bar{e}^i$  are unconstrained. We followed the simple model of [Goeke et al., 2001](#)

$$e^i(x, \mu^2) = A_i(\mu^2) q_{\text{val}}^i(x, \mu^2) + \frac{B_i(\mu^2)}{2} \delta(x)$$

$$\bar{e}^i(x) = \frac{B_i(\mu^2)}{2} \delta(x)$$

where

$$A_i(\mu^2) = \frac{\mathbf{2J}^i(\mu^2) - M_2^i(\mu^2)}{M_2^{i,\text{val}}}$$

$$B_u(\mu^2) = k_u - 2 A_u(\mu^2), \quad B_d(\mu^2) = k_d - A_d(\mu^2)$$

- Functions  $Q_2^i$  and  $R_2^i$  are not so well-constrained, only their Mellin moments are known. From

$$B_{nn-1}^i(t, \mu^2) = \frac{n}{n+1} B_{nn+1}^i(t, \mu^2) + \frac{d_n^i(t, \mu^2)}{P_{n-1}(0)},$$

where  $d_n$  are Gegenbauer moments of the  $D$ -term, we find

$$Q_2^i(x, t, \mu^2) = Q_0^i(x, t, \mu^2) - \int_x^1 \frac{dz}{z} Q_0^i(z, t, \mu^2) + \tilde{Q}_2^i(x, t, \mu^2)$$

where

$$\int_0^1 dx x^n \tilde{Q}_2^i(x, t, \mu^2) = \frac{d_n^i(t, \mu^2)}{P_{n-1}(0)}$$

The Gegenbauer moments  $d_n^i$  are taken from the chiral quark soliton model.

- Since the  $D$ -term contribution to the GPDs  $E^i$  and  $H^i$  are equal and opposite in sign,

$$R_2^i(x, t, \mu^2) = R_0^i(x, t, \mu^2) - \int_x^1 \frac{dz}{z} R_0^i(z, t, \mu^2) - \tilde{Q}_2^i(x, t, \mu^2)$$

## Two models of $t$ -dependence

- Factorized exponential  $t$ -dependence

$$H^i(x, \xi, t, \mu^2) = \exp\left(\frac{B(\mu^2) t}{2}\right) H^i(x, \xi, t = 0, \mu^2)$$

$$E^i(x, \xi, t, \mu^2) = \exp\left(\frac{B(\mu^2) t}{2}\right) E^i(x, \xi, t = 0, \mu^2)$$

with  $Q^2$ -dependent slope

$$B(\mu^2) = 7.6 \left(1 - 0.15 \ln(\mu^2/2)\right) \text{ GeV}^2$$

- The value of the slope is chosen to reproduce the only measurement of differential DVCS cross section by H1 at HERA fitted to the exponential form:  
 $B(\mu^2 = 8 \text{ GeV}^2) = 6.02 \pm 0.35 \pm 0.39 \text{ GeV}^{-2}$ , *Aktas et al., 2005*.
- The slight decrease of the slope is expected on general grounds.

- Non-factorizable Regge-motivated  $t$ -dependence

$$q^i(x, t, \mu_0^2) - \bar{q}^i(x, t, \mu_0^2) = q_{\text{val}}^i(x, t, \mu_0^2) = \left( \frac{1}{x^{\alpha'_{\text{val}} t}} \right) q_{\text{val}}^i(x, \mu_0^2)$$

$$q^i(x, t, \mu_0^2) + \bar{q}^i(x, t, \mu_0^2) = \left( \frac{1}{x^{\alpha' t}} \right) [q^i(x, \mu_0^2) + \bar{q}^i(x, \mu_0^2)]$$

$$g(x, t, \mu_0^2) = \left( \frac{1}{x^{\alpha'_g t}} \right) g(x, \mu_0^2)$$

with

$$\alpha'_{\text{val}} = 1.1(1-x) \text{ GeV}^{-2}, \quad \alpha' = 0.9 \text{ GeV}^{-2}, \quad \alpha'_g = 0.5 \text{ GeV}^{-2}$$

Note that the data on  $\sigma_{\text{DVCS}}$  forces us to take  $\alpha', \alpha'_g > \alpha_P = 0.25 \text{ GeV}^{-2}$ .

- Since the  $D$ -term does not have a partonic interpretation, we cannot use the Regge model.  
Instead, we use the results of the lattice calculations, **Gockeler *et al.*, 2003**

$$d_i^{u,d}(t) = d_i^{u,d}(t=0) \frac{1}{(1-t/M_D^2)^2}$$

where  $M_D = 1.11 \pm 0.20$  GeV in the continuum limit.

## DVCS cross section in HERA kinematics

- The DVCS cross section on the photon level

$$\sigma_{\text{DVCS}}(x_B, Q^2) = \frac{\pi \alpha^2 x_B^2}{Q^4 \sqrt{1 + 4m_N^2 x^2/Q^2}} \int_{t_{\min}}^{t_{\max}} dt |\mathcal{A}_{\text{DVCS}}(\xi, t, Q^2)|^2$$

– In the small- $\xi$  limit,  $|\mathcal{A}_{\text{DVCS}}(\xi, t, Q^2)|^2 \approx |\mathcal{H}|^2(1 - \xi^2)$

–

$$\mathcal{H}(\xi, t, Q^2) = \sum_i e_i^2 \int_0^1 dx H^i(x, \xi, t, Q^2) \left( \frac{1}{x - \xi + i0} + \frac{1}{x + \xi - i0} \right)$$

- One appealing feature of the dual parameterization is that the convolution integral can be easily taken

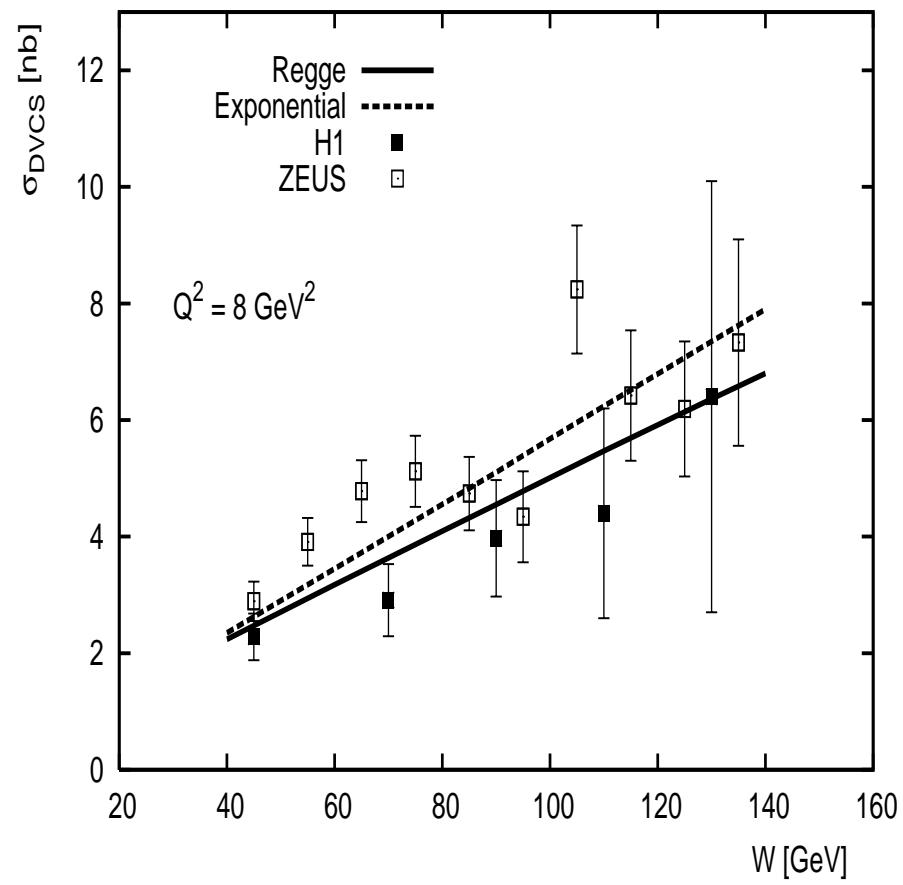
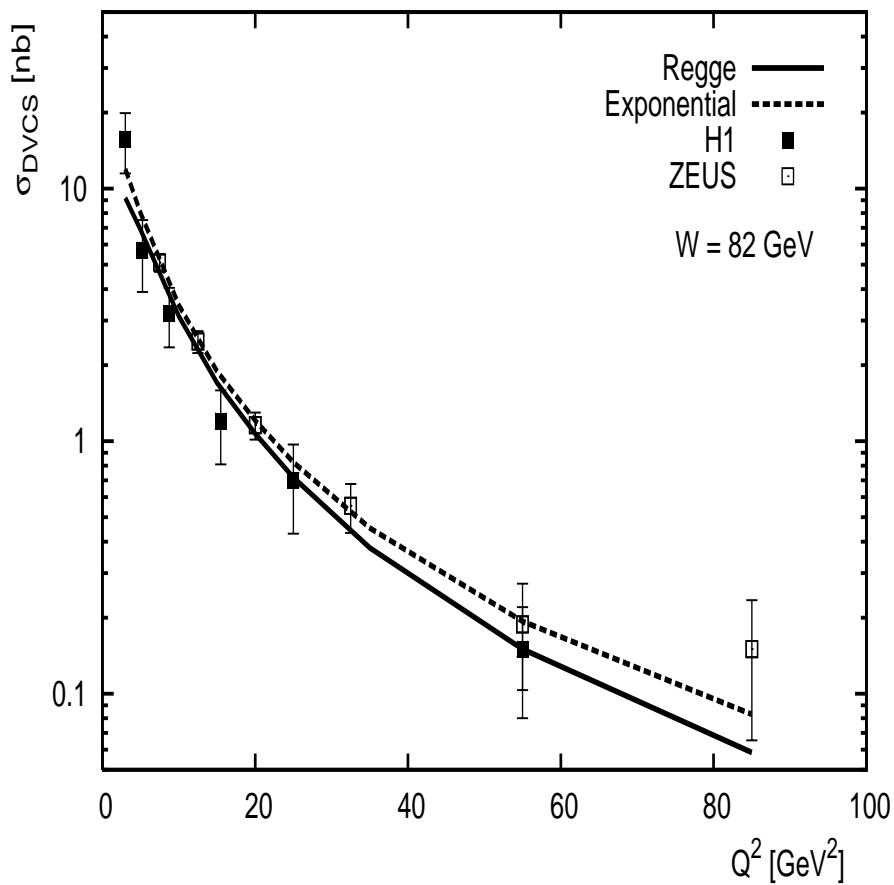
$$\mathcal{H}(\xi, t, Q^2) = - \sum_i e_i^2 \int_0^1 \frac{dx}{x} \sum_{k=0}^{\infty} x^k \mathcal{Q}_k^i(x, t, Q^2) \left( \frac{1}{\sqrt{1 - \frac{2x}{\xi} + x^2}} + \frac{1}{\sqrt{1 + \frac{2x}{\xi} + x^2}} - 2\delta_{k0} \right)$$

Contains both real and imaginary parts

$$\begin{aligned}
 \textcolor{red}{Im} \mathcal{H}(\xi, t) &= - \sum_i e_i^2 \int_a^1 \frac{dx}{x} \frac{1}{\sqrt{2x/\xi - x^2 - 1}} \sum_{\text{even } k} x^k Q_k^i(x, t), \\
 \textcolor{red}{Re} \mathcal{A}^i(\xi, t) &= - \sum_i \int_a^1 \frac{dx}{x} \sum_{\text{even } k} x^k Q_k^i(x, t) \left( \frac{1}{\sqrt{1 + 2x/\xi + x^2}} - 2\delta_{k0} \right) \\
 &\quad - \sum_i e_i^2 \int_0^a \frac{dx}{x} \sum_{\text{even } k} x^k Q_k(x, t) \left( \frac{1}{\sqrt{1 - 2x/\xi + x^2}} + \frac{1}{\sqrt{1 + 2x/\xi + x^2}} - 2\delta_{k0} \right)
 \end{aligned}$$

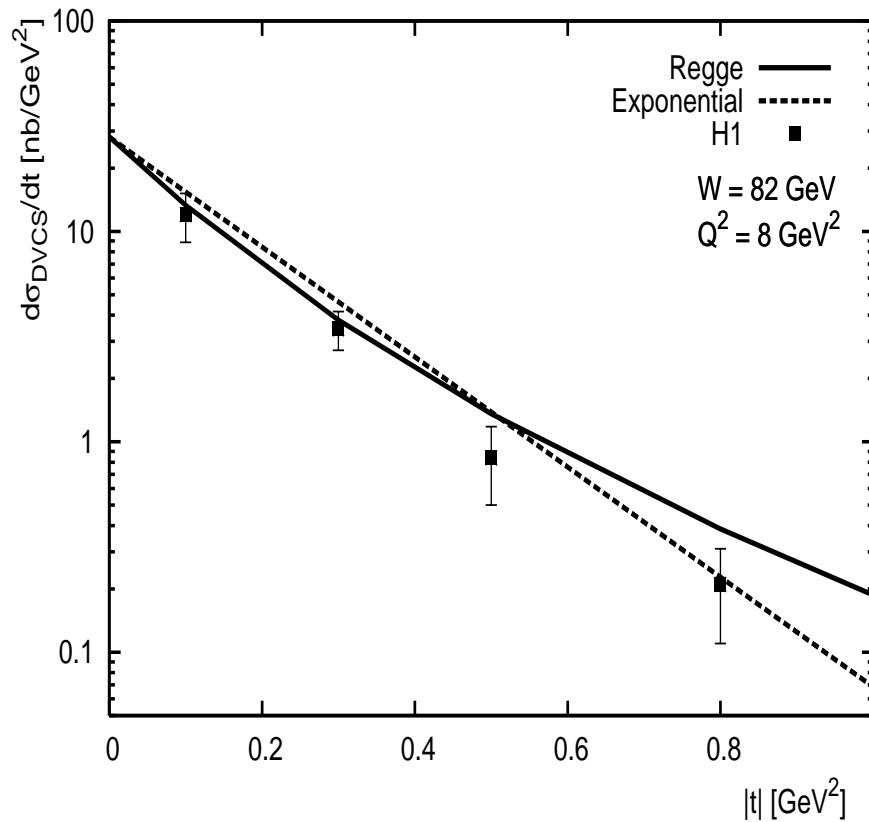
where  $a = (1 - \sqrt{1 - \xi^2})/\xi \approx \xi/2$  at small  $\xi$ .

- Moreover, in the HERA kinematics, only  $Q_0^i$  which is given by forward PDFs, is important → **parameter-free\*** predictions for the DVCS cross section.



- The differential DVCS cross section

$$\frac{d\sigma_{\text{DVCS}}(x_B, t, Q^2)}{dt} = \frac{\pi \alpha^2 x_B^2}{Q^4 \sqrt{1 + 4m_N^2 x^2/Q^2}} |\mathcal{A}_{\text{DVCS}}(\xi, t, Q^2)|^2$$



## Beam-spin asymmetry in HERMES kinematics

- The approximate expression for the  $\sin \phi$ -moment of the beam-spin asymmetry,  
*Belitsky et al., 2001*

$$A_{LU}^{\sin \phi} \approx \left( \frac{x_B}{y} \right) 8 K y (2 - y) (1 + \epsilon^2)^2 \frac{\left[ F_1(t) \text{Im } \mathcal{H}(\xi, t) + \frac{|t|}{4m_N^2} F_2(t) \text{Im } \mathcal{E}(\xi, t) \right]}{c_{0,\text{unp}}^{\text{BH}}}$$

- The dual parameterization predictions compare very well to the HERMES measurement at  $\langle x_B \rangle = 0.11$ ,  $\langle Q^2 \rangle = 2.6 \text{ GeV}^2$  and  $\langle t \rangle = -0.27 \text{ GeV}^2$

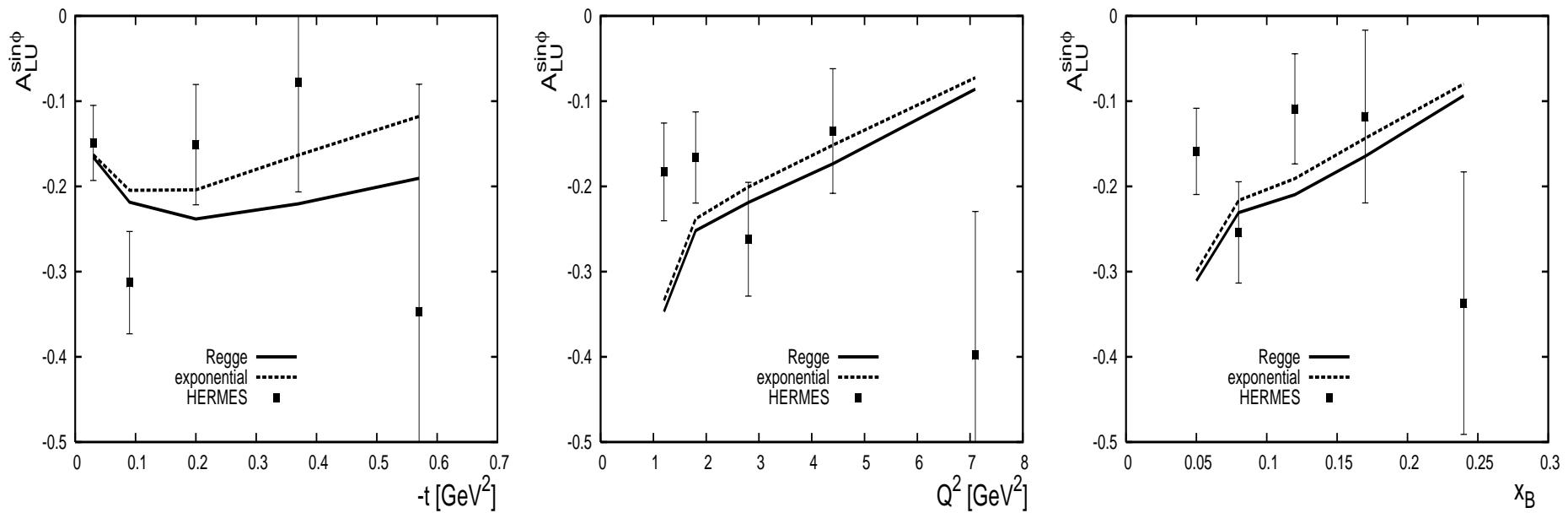
$$A_{LU}^{\sin \phi} = -0.22 \dots - 0.24, \quad \text{exponential } t - \text{dependence}$$

$$A_{LU}^{\sin \phi} = -0.27 \dots - 0.29, \quad \text{Regge } t - \text{dependence}$$

$$A_{LU}^{\sin \phi} = -0.23 \pm 0.04 \pm 0.03, \quad \text{HERMES (Airapetian, 2001)}$$

The range of theoretical prediction comes from varying  $0 \leq J_u \leq 0.4$ .

- Comparison of the dual parameterization predictions for the  $A_{LU}^{\sin\phi}$  dependence on  $t$ ,  $Q^2$  and  $x_B$  in the HERMES kinematics, F. Ellinghaus, Ph.D. thesis, 2004.



- The calculation is done with  $J_u = J_d = 0$ , but the sensitivity to the model for the GPD  $E$  is weak.
- Apart from the last point, the data is described by both models fairly well.

## Beam-spin asymmetry in CLAS kinematics

The 2001 average kinematic point of the CLAS kinematics:  $E = 4.25 \text{ GeV}$ ,  $\langle Q^2 \rangle = 1.25 \text{ GeV}^2$ ,  $\langle x_B \rangle = 0.19$  and  $\langle t \rangle = -0.19 \text{ GeV}^2$ , experimental value,

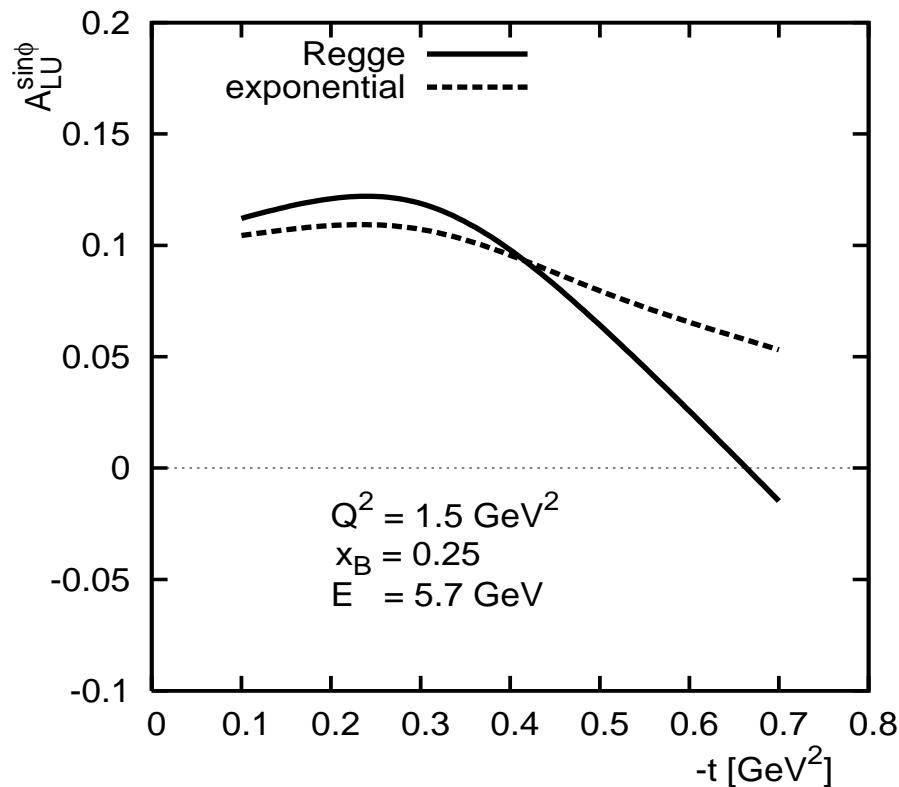
$$A_{LU}^{\sin \phi} = 0.15 \dots 0.17, \quad \text{exponential } t - \text{dependence}$$

$$A_{LU}^{\sin \phi} = 0.18 \dots 0.20, \quad \text{Regge } t - \text{dependence}$$

$$A_{LU}^{\sin \phi} = 0.202 \pm 0.028, \quad \text{CLAS (Stepanyan, 2001)}$$

The range of theoretical prediction comes from varying  $0 \leq J_u \leq 0.4$ .

Calculations of  $A_{LU}^{\sin\phi}$  in the present CLAS kinematics:  $E = 5.7$  GeV,  $Q^2 = 1.5$  GeV $^2$  and  $x_B = 0.25$ .



**Note** that our model becomes unstable starting from  $x_B = 0.2 - 0.3$ .

## Beam-charge asymmetry in HERMES kinematics

- The approximate expression for the  $\cos \phi$ -moment of the beam-charge asymmetry,  
*Belitsky et al., 2001*

$$A_C^{\cos \phi} \approx \left( \frac{x_B}{y} \right) 8 K (2 - 2y + y^2) (1 + \epsilon^2)^2 \frac{\left[ F_1(t) \operatorname{Re} \mathcal{H}(\xi, t) + \frac{|t|}{4m_N^2} F_2(t) \operatorname{Re} \mathcal{E}(\xi, t) \right]}{c_{0,\text{unp}}^{\text{BH}}}$$

- The dual parameterization predictions in the average HERMES kinematics,  
 $\langle x_B \rangle = 0.12$ ,  $\langle Q^2 \rangle = 2.8 \text{ GeV}^2$  and  $\langle t \rangle = -0.27 \text{ GeV}^2$

$$A_C^{\cos \phi} = 0.010 \dots 0.030, \quad \text{exponential } t - \text{dependence}$$

$$A_C^{\cos \phi} = 0.19 \dots 0.23, \quad \text{Regge } t - \text{dependence}$$

$$A_C^{\cos \phi} = 0.11 \pm 0.04 \pm 0.03, \quad \text{HERMES (2002, unpub.)}$$

The range of theoretical prediction comes from varying  $0 \leq J_u \leq 0.4$ .

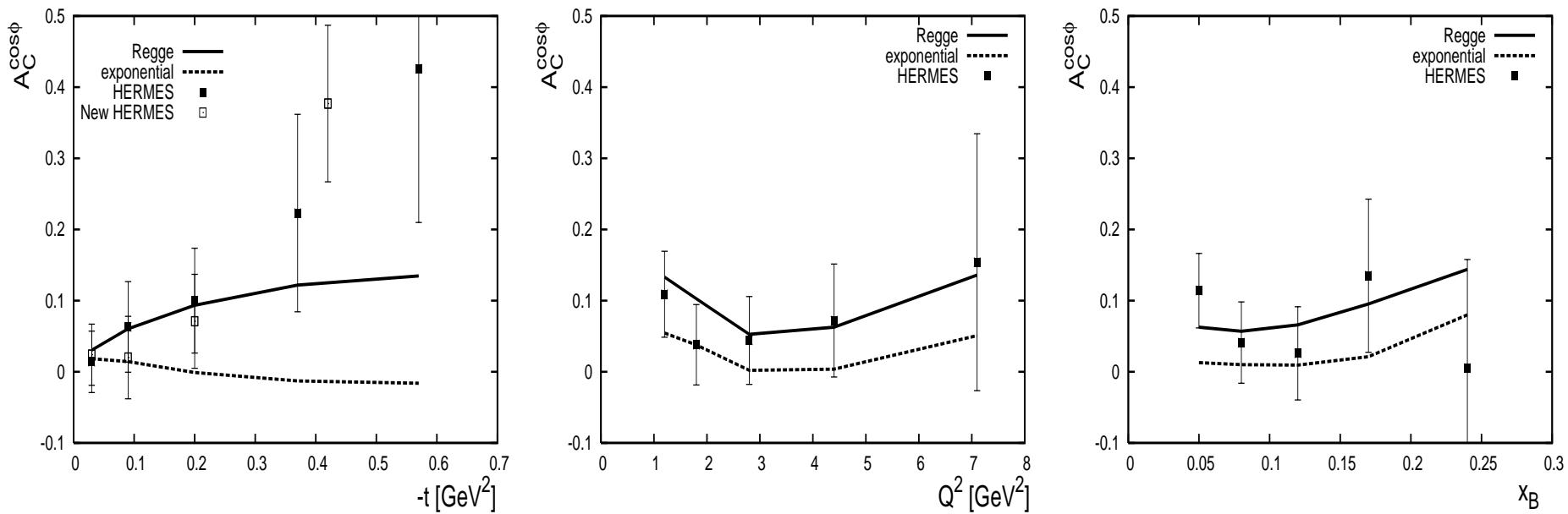
- Also for the 2006 HERMES kinematics:  $\langle x_B \rangle = 0.10$ ,  $\langle Q^2 \rangle = 2.5 \text{ GeV}^2$  and  $\langle t \rangle = -0.12 \text{ GeV}^2$

$A_C^{\cos \phi} = 0.013 \dots 0.022$ , exponential  $t$  – dependence,

$A_C^{\cos \phi} = 0.080 \dots 0.092$ , Regge  $t$  – dependence,

$A_C^{\cos \phi} = 0.063 \pm 0.029 \pm 0.026$ , (HERMES, 2006)

- Comparison of the dual parameterization predictions for the  $A_C^{\cos\phi}$  dependence on  $t$ ,  $Q^2$  and  $x_B$  to the analysis ([F. Ellinghaus, Ph.D. thesis, 2004](#)) and to new HERMES data ([Airapetian, 2006](#)).



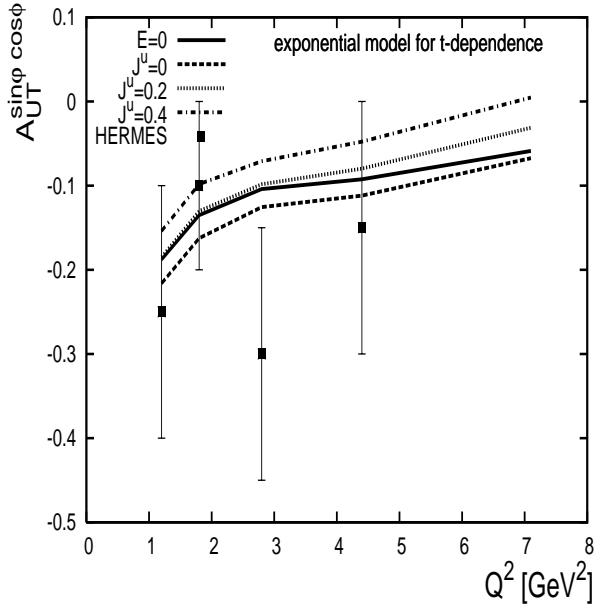
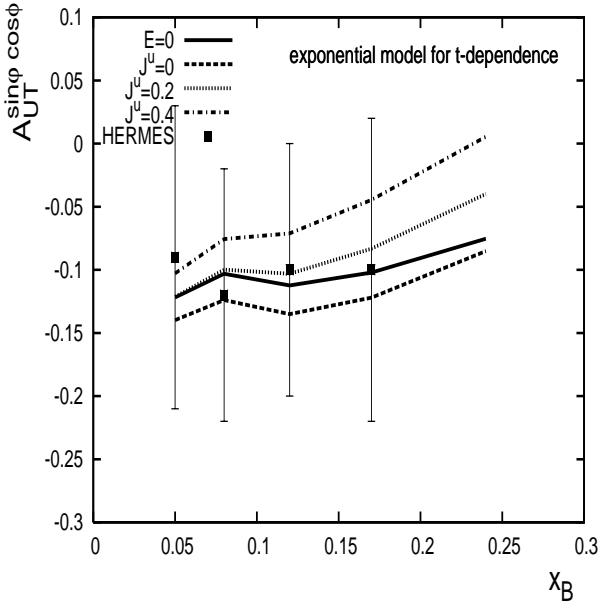
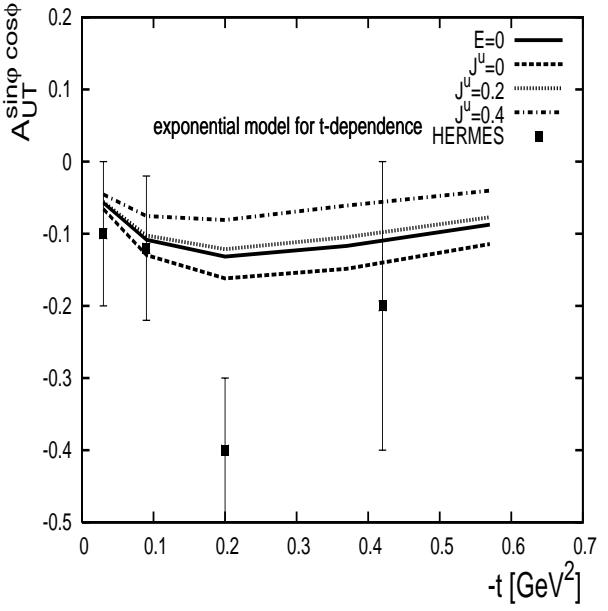
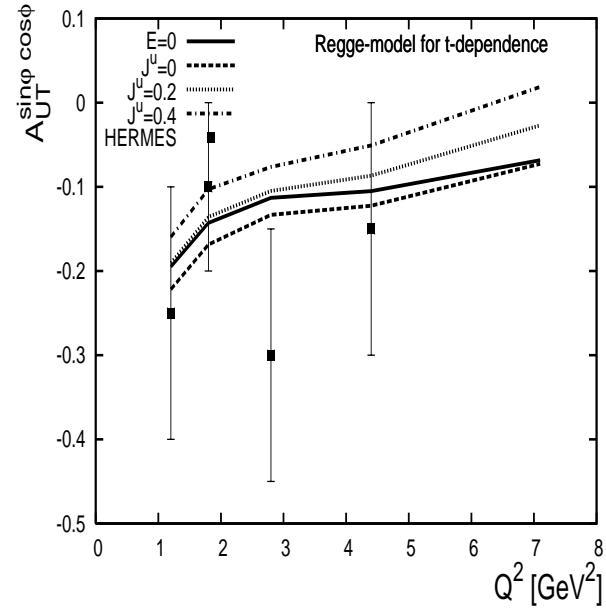
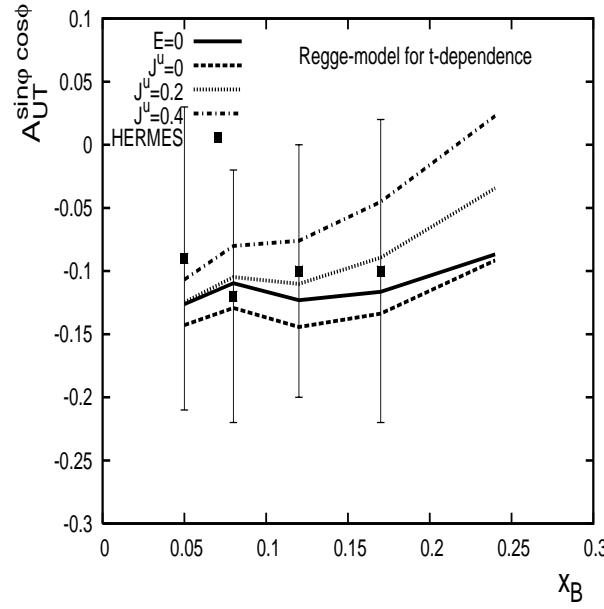
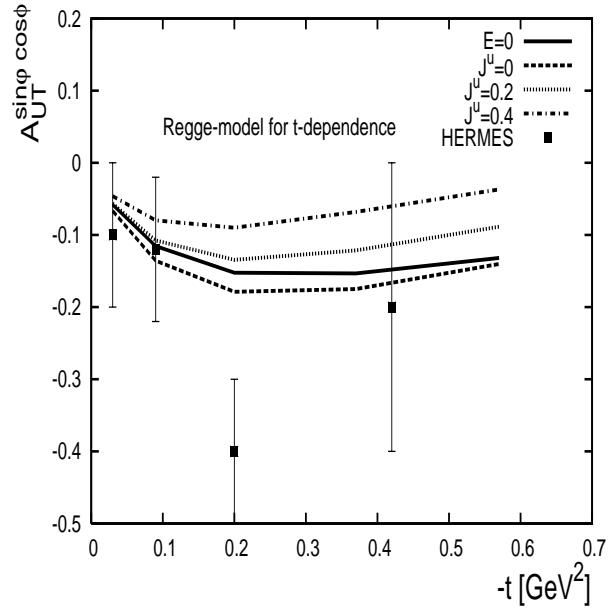
- The calculation is done with  $J_u = J_d = 0$ .
- The Regge model of the  $t$ -dependence gives a much better description of the data.

## Transversely-polarized target asymmetry in HERMES kinematics

- The  $\sin \phi \cos \varphi$ -moment of the transversely-polarized target (unpolarized beam) asymmetry is sensitive to the GPD  $E$ , Belitsky *et al.*, 2001

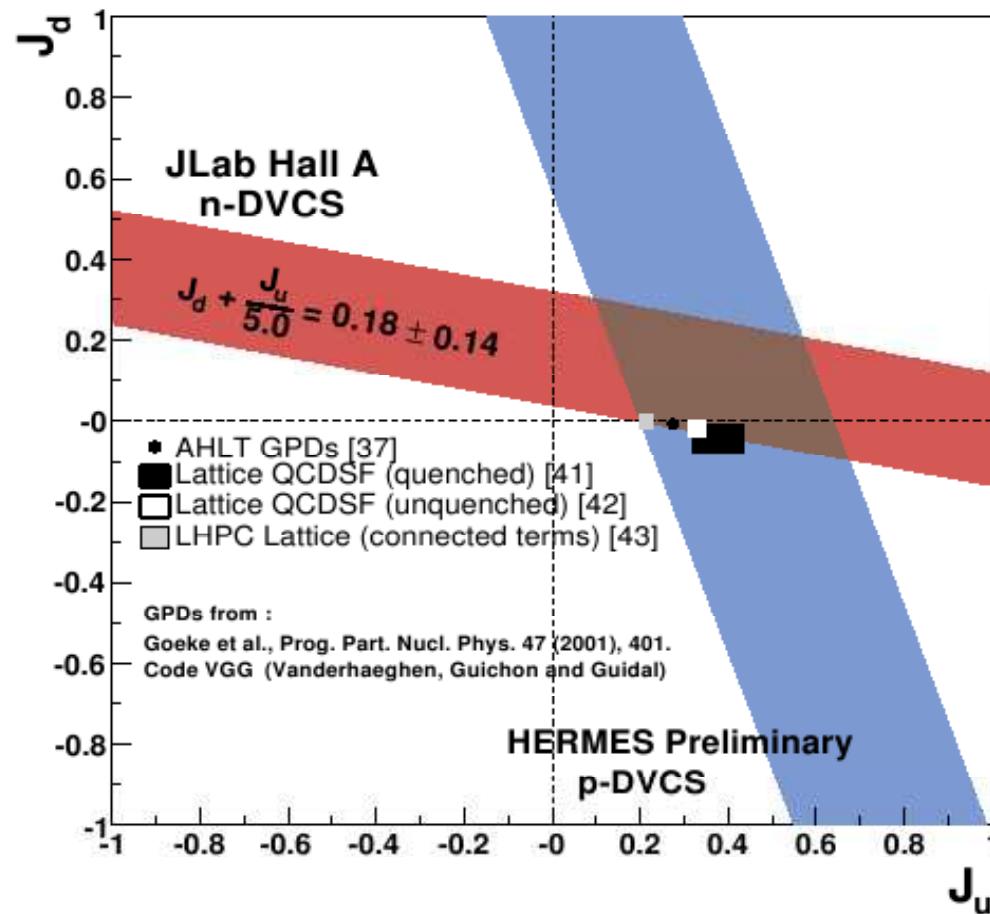
$$A_{UT}^{\sin \phi \cos \varphi} = A_{UT}^{\sin(\phi - \phi_S) \cos \phi} \propto F_2(t) \operatorname{Im} \mathcal{H}(\xi, t) - F_1(t) \operatorname{Im} \mathcal{E}(\xi, t)$$

- Can be used to discriminate between different models of the GPD  $E$
- Can be used to determine the total angular momentum carried by quarks, Ellinghaus, Nowak, Vinnikov, Ye, 2005.
- The dual parameterization predictions for  $A_{UT}^{\sin(\phi - \phi_S) \cos \phi}$  can be compared to the preliminary HERMES data, Ye, 2005. However, because of large experimental errors, no quantitative conclusion from the comparison can be made.



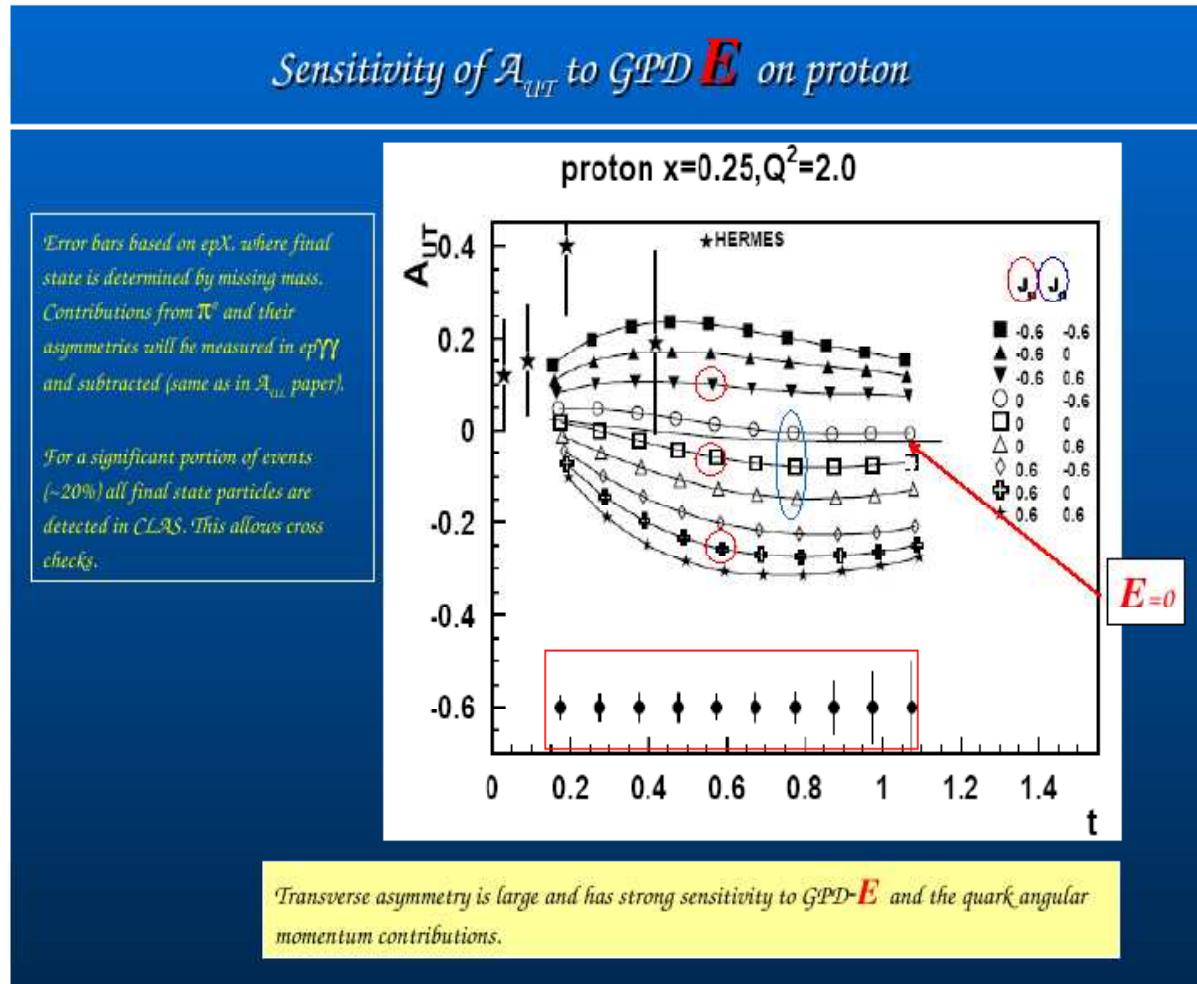
## GPDs and the proton spin crisis

$$\frac{1}{2} = J^q + J^g = \frac{1}{2} \Delta \Sigma + L_q + \Delta G + L_g$$



# Future GPD measurements with transverse target at JLab

## H. Avakian *et al.*, JLab proposal PR-08-021 (2008)



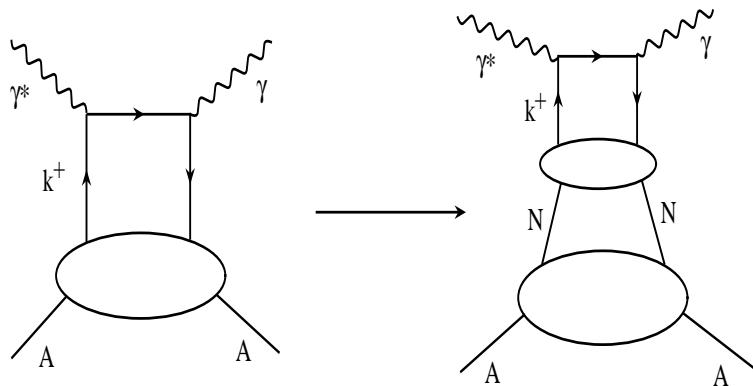
## Dual parameterization of nuclear GPDs

Three roles of nuclear GPDs and nuclear DVCS:

- To give information of the nucleon GPDs (neutron) complimentary to experiments on the protons
- To access novel nuclear effects not present in DIS and in elastic scattering on nuclei
  - Non-nucleonic degrees of freedom
  - Off-forward EMC effect
  - Nuclear shadowing and antishadowing in the real and imaginary parts of the nuclear DVCS amplitude
- To provide constraints on theoretical models of nuclear structure
  - Relativistic description is important for polynomiality

## A simple constituent model for nuclear GPDs

- Assume that the nuclear GPDs is a sum of the free nucleon GPDs (spin-0 nucleus)  
A. Kirchner and D. Mueller, Eur. Phys. J. C **32** (2003) 347 [arXiv:hep-ph/0302007].



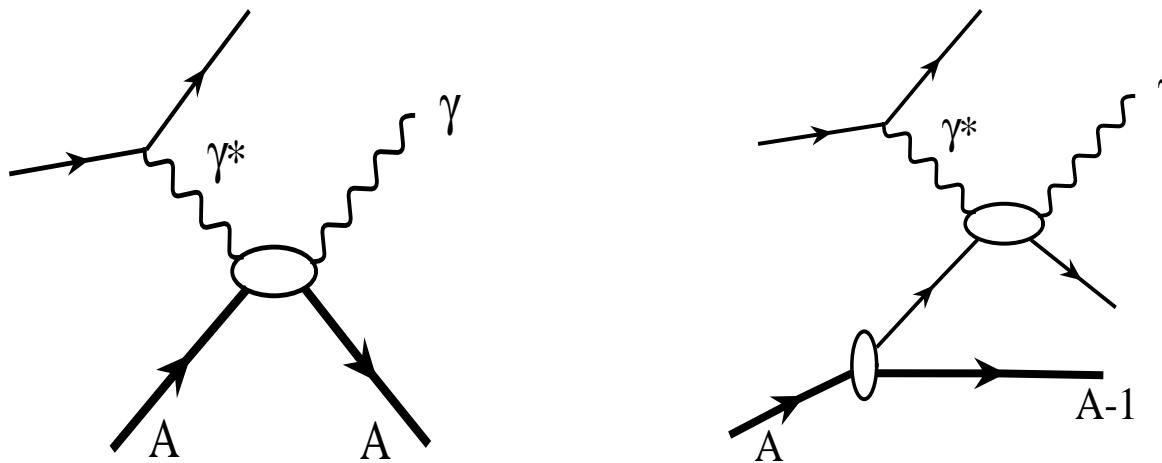
$$\begin{aligned}
H_A^q(x, \xi_A, Q^2, t) &= \left| \frac{dx_N}{dx} \right| \left[ Z \left( H^{q/p}(x_N, \xi_N, Q^2, t) + \frac{t}{4m_N^2} E^{q/p}(x_N, \xi_N, Q^2, t) \right) \right. \\
&\quad \left. + N \left( H^{q/n}(x_N, \xi_N, Q^2, t) + \frac{t}{4m_N^2} E^{q/n}(x_N, \xi_N, Q^2, t) \right) \right] F_A(t)
\end{aligned}$$

## The simple model of nuclear GPDs

- Ignores nuclear modifications and uses free nucleon GPDs (dual parameterization)
- Has the correct forward limit and the nuclear form factor
- Does not satisfy polynomiality

## Coherent and incoherent contributions

- When the recoiled nucleus **not detected**, DVCS observables receive coherent and **incoherent** contributions.

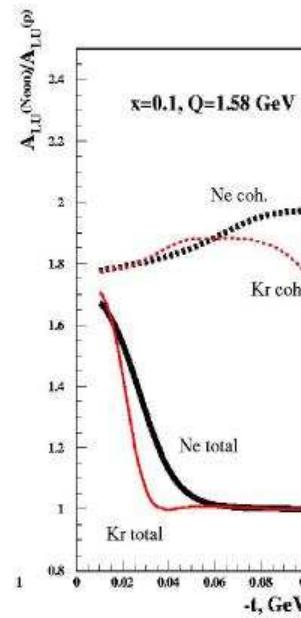
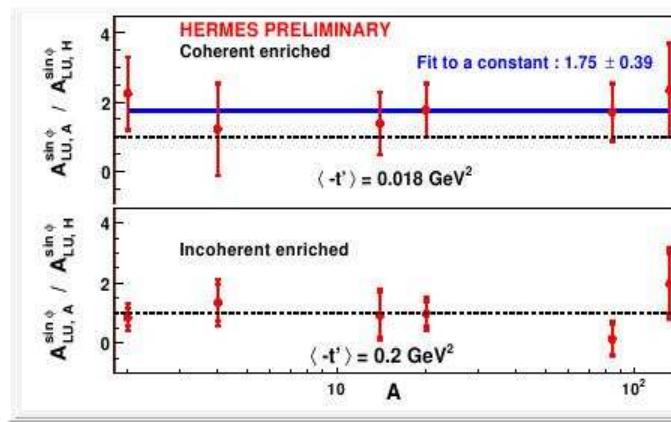


- Coherent dominates at small  $t$ ; Incoherent dominates at large  $t$
- One can write an interpolation formula between the two regimes  
[V. Guzey and M. Strikman, Phys. Rev. C68 \(2003\) 015204](#)

## Nuclear beam-spin asymmetry $A_{LU}$

$$A_{LU}(\phi) = \frac{Z(A-1)F_A^2(t)\{\mathcal{I}_A\} + F_N(t)(Z\{\mathcal{I}_p\} + N\{\mathcal{I}_n\})}{Z(Z-1)\{\text{BH}_A\} + Z(A-1)\{\mathcal{I}_A\} + A(A-1)\{\text{DVCS}_A\} + Z\{\text{BH}_N\} + Z\{\mathcal{I}_N\} + A\{\text{DVCS}_N\}}$$

RATIO  $A_{LU}^A/A_{LU}^p$  (METHOD 1)



- COHERENT ENRICHED: MEAN RATIO DEVIATES FROM UNITY BY  $2\sigma$ . CONSISTENT WITH PREDICTIONS BETWEEN 1.8 AND 1.95: GUZEY/STRIKMAN PHYS.REV.C 68 (2003)
- INCOHERENT ENRICHED: CONSISTENT WITH UNITY AS NAIVELY EXPECTED

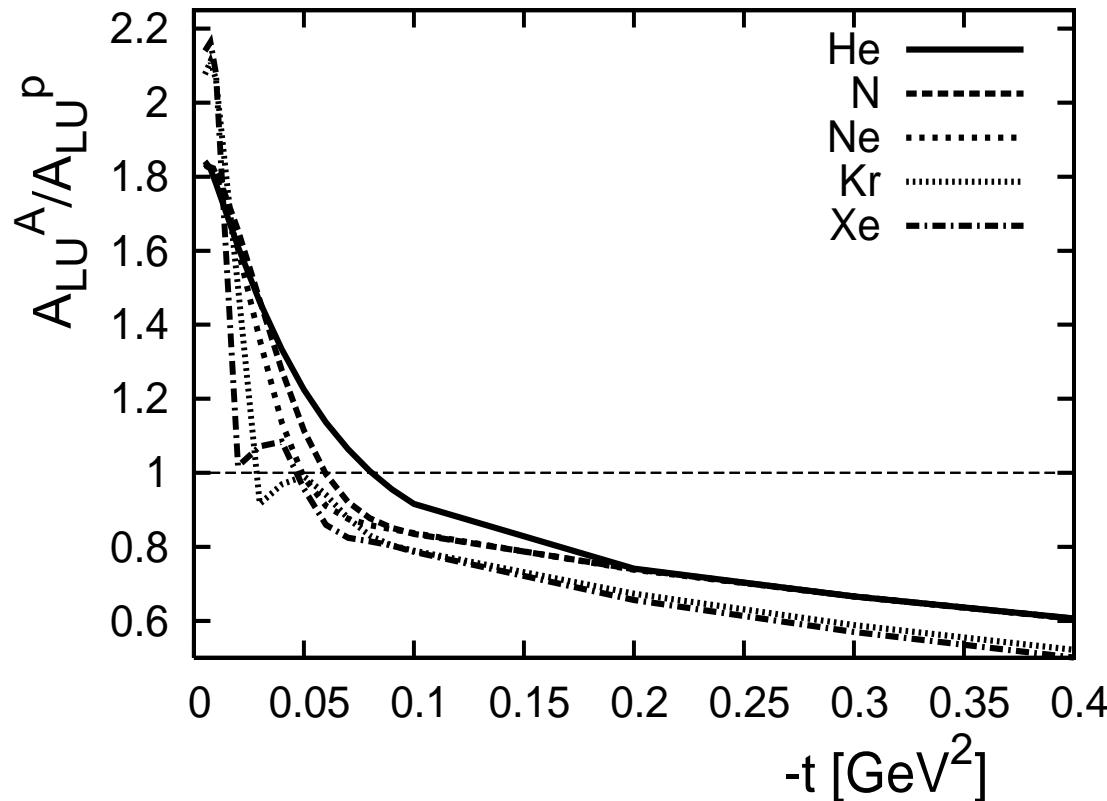


Frank Ellinghaus, University of Maryland, October 2006



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## Nuclear beam-spin asymmetry $A_{LU}$ (keep the neutron)



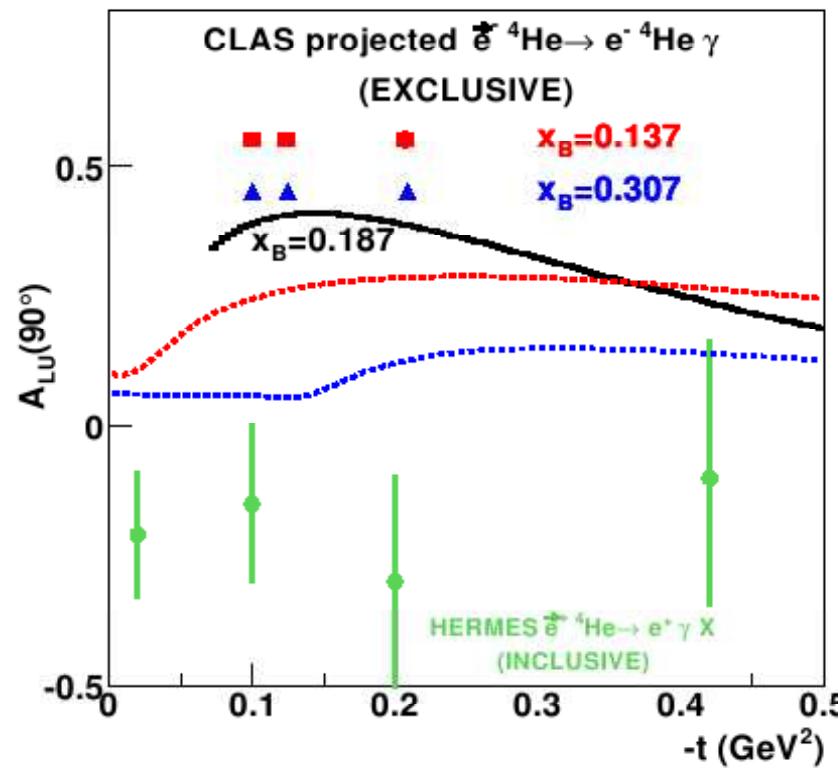
V. Guzey, ArXiv:0801.3235 [nucl-th] (2008)

- Use incoherent DVCS to measure the neutron GPDs

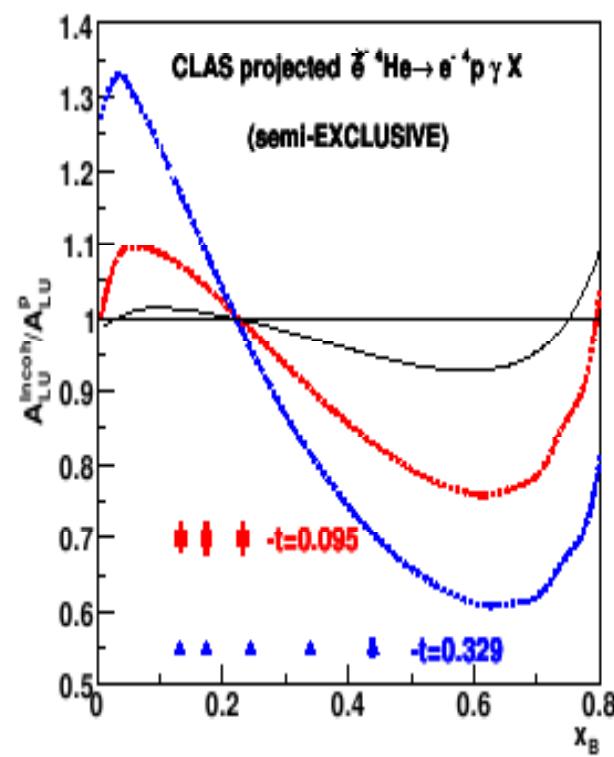
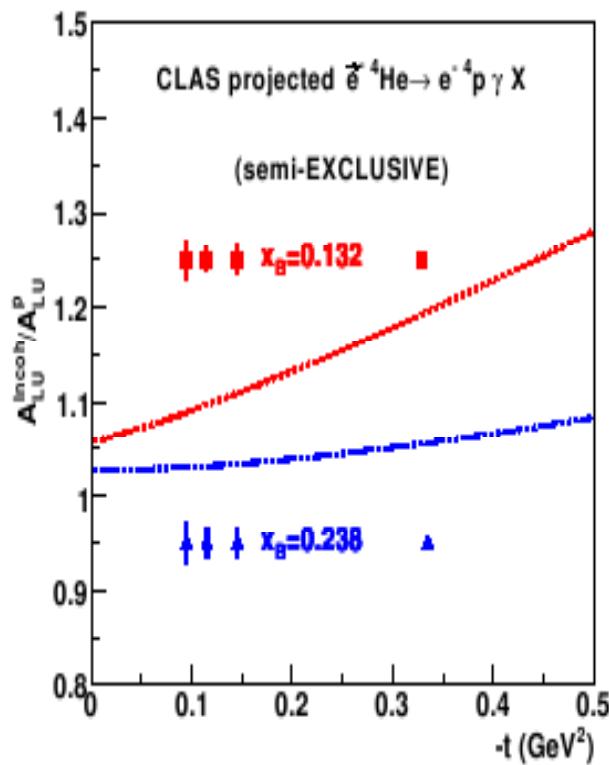
## Nuclear DVCS on $^4\text{He}$ at JLab

K. Hafidi *et al.*, JLab proposal PR-08-024 (2008)

- Measure coherent DVCS on  $^4\text{He}$  using the BoNuS Detector; for the first time and with large accuracy



- In addition, the incoherent measurement  $\vec{e}^4\text{He} \rightarrow e^- p \gamma X$  will probe nuclear modifications of the proton GPDs.



S. Liuti and S. K. Taneja (2005)

## Conclusions and discussion

- Dual parameterization of nucleon GPDs is a new LO parameterization of GPDs  $H$  and  $E$  which
  - Has a simple QCD evolution
  - Allows for an economical and good description of all available data on DVCS
  - Works best for  $x_B < 0.1 – 0.2$
  - Can be systematically improved, [M. Polyakov \(2007\)](#)
- Nuclear DVCS is a new tool to study microscopic structure of nucleons and nuclei.
  - A wide-open field for theorists (relativistic description of nuclear structure, small- $x$  nuclear GPDs, FSI)
  - The future high-precision JLab data on DVCS on  ${}^4\text{He}$  will be an important step