JLab Hall C Users' Summer Workshop
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Linking Nuclear Observables through Quark-Hadron Duality

Wally Melnitchouk
Jefferson Lab

Outline

1. Quark-hadron (“Bloom-Gilman”) duality
2. Local duality
   - elastic duality
   - medium modifications
3. Truncated moments

Quark-Hadron
(“Bloom-Gilman”) duality

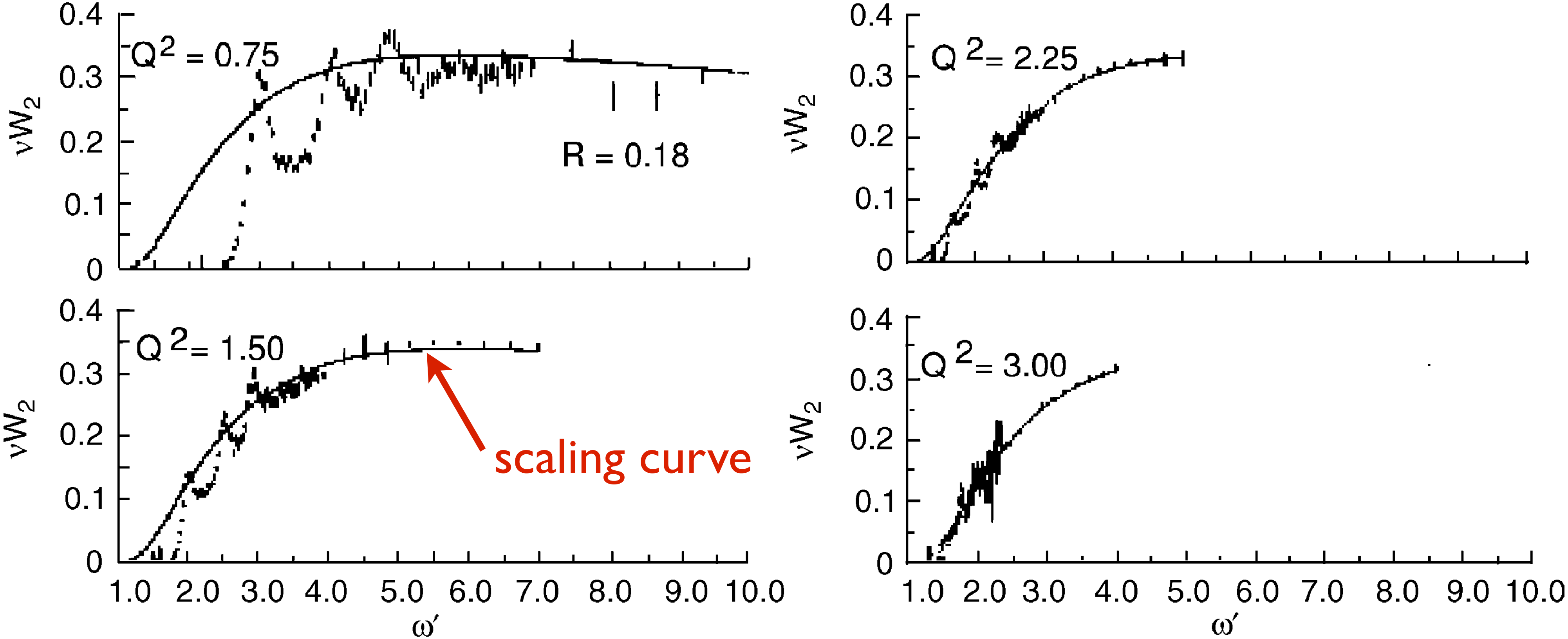
Quark-hadron duality

Complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

Can use either set of complete basis states to describe all physical phenomena

Electron scattering



Bloom, Gilman, Phys. Rev. Lett. 85 (1970) 1185

➡ resonance – scaling duality in proton $\nu W_2 = F_2$ structure function

Bloom-Gilman duality

Average over (strongly Q^2 dependent) resonances
$\approx Q^2$ independent scaling function

Finite energy sum rule for eN scattering

$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

measured structure function
(function of ν and Q^2)

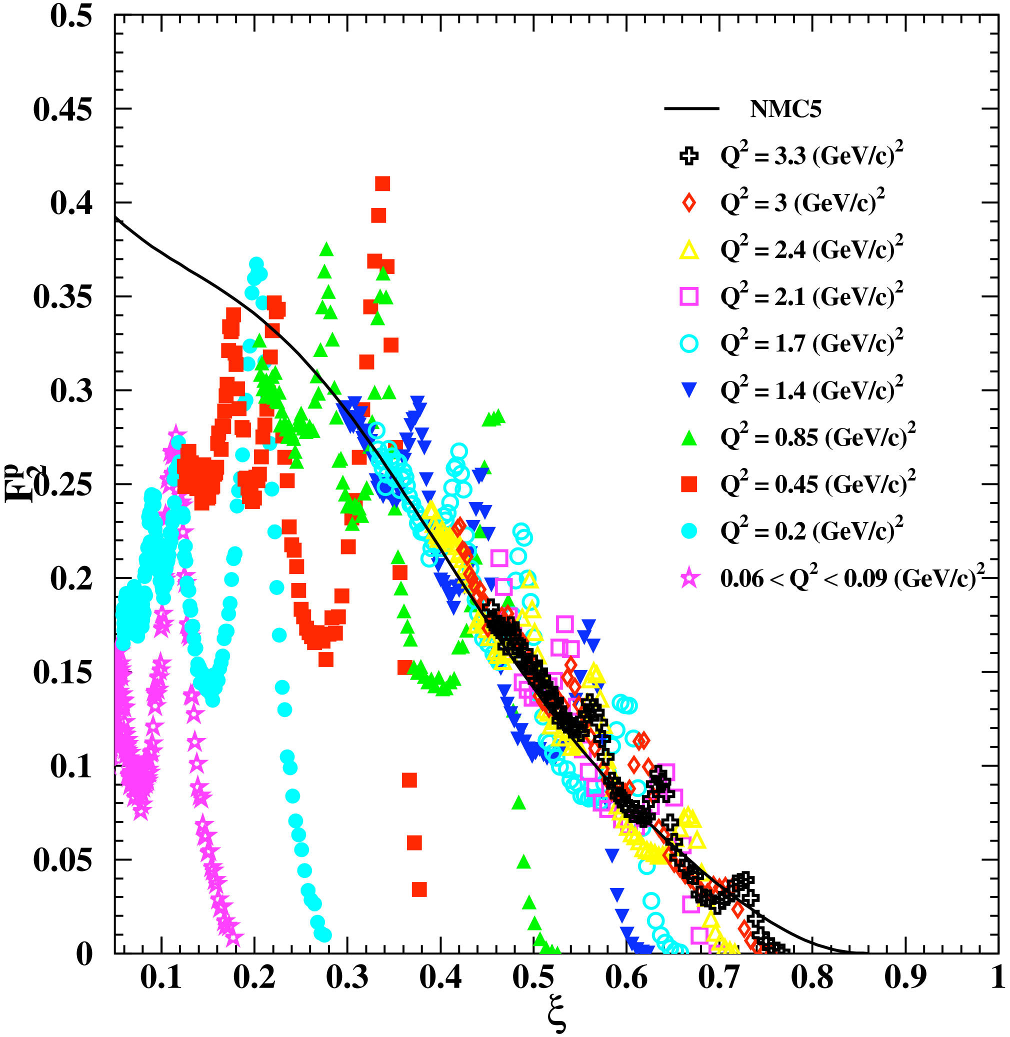
"hadrons"

$$\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$$

scaling function
(function of ω' only)

"quarks"

Bloom-Gilman duality



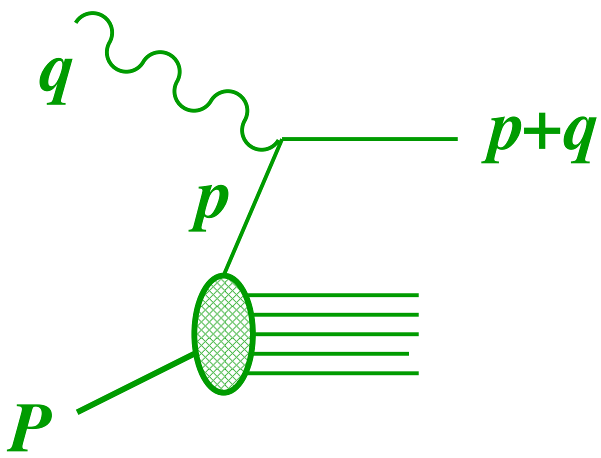
Jefferson Lab (Hall C)

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

Average over (strongly Q^2 dependent) resonances

$\approx$ Q^2 independent scaling function

Scaling variables



$$(p+q)^2 = m_q^2 \quad \begin{cases} m_q = 0 \\ p_T = 0 \end{cases}$$

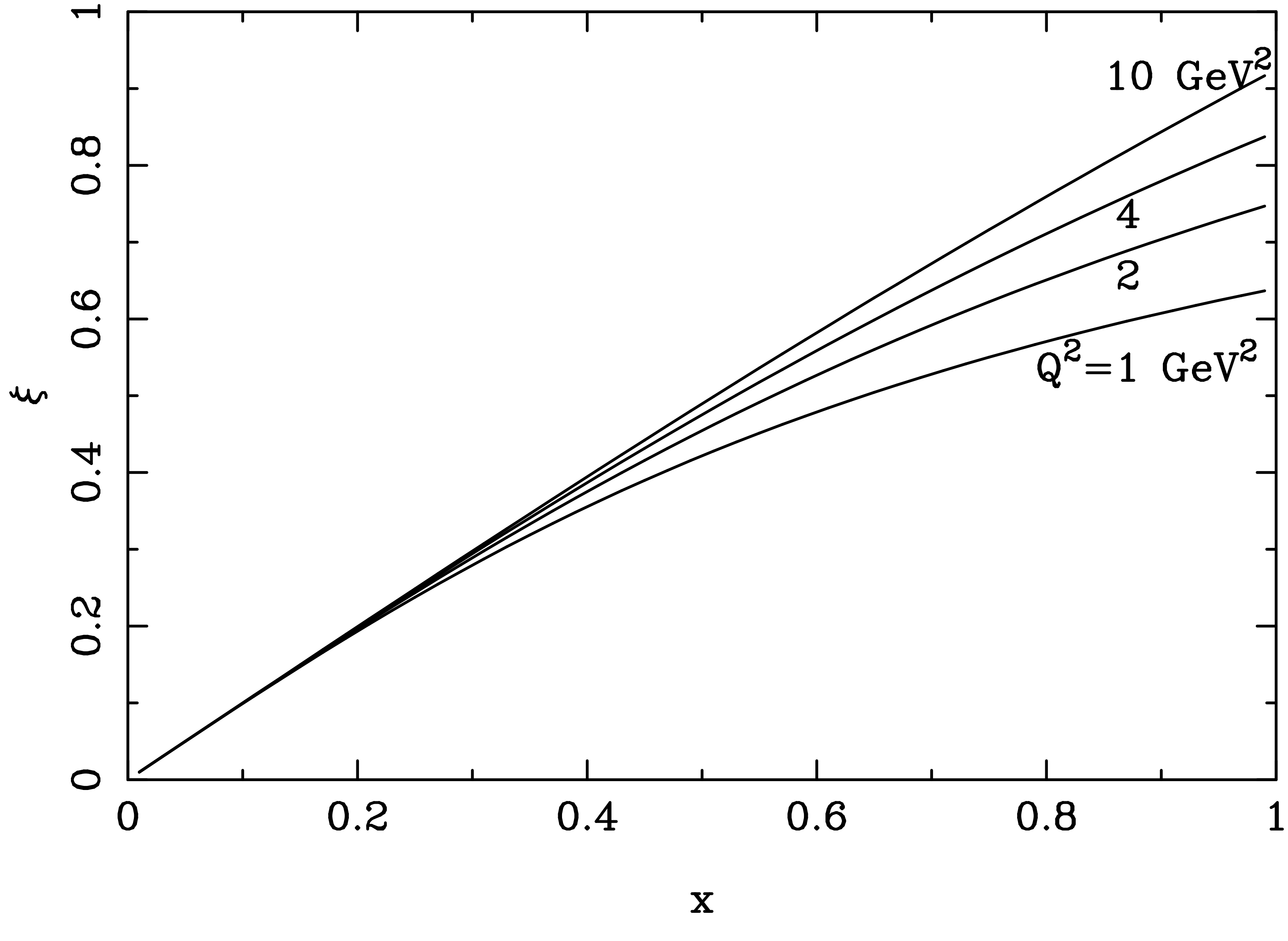
light-cone fraction of target's momentum carried by parton

$$\xi = \frac{p^+}{P^+} = \frac{p^0 + p^z}{M}$$

$$\Longrightarrow \quad \xi = \frac{2x}{1 + \sqrt{1 + 4x^2 M^2 / Q^2}} \quad \rightarrow \quad x \quad \text{as} \quad Q^2 \rightarrow \infty$$

Nachtmann scaling variable

Scaling variables



Duality in QCD

Operator product expansion

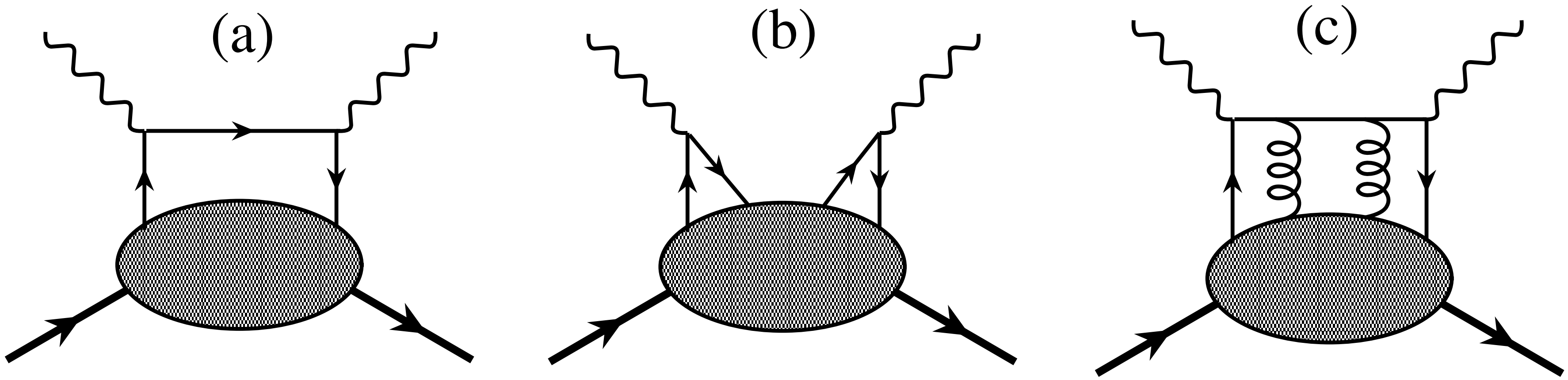
$\Longrightarrow$ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x,Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \mathrm{dimension} - \mathrm{spin}$$

Higher twists



$\tau = 2$

single quark scattering

e.g. $\bar{\psi}\ \gamma_\mu\ \psi$

$\tau > 2$

qq and qg correlations

e.g. $\bar{\psi}\ \gamma_\mu\ \psi\ \bar{\psi}\ \gamma_\nu\ \psi$

or $\bar{\psi}\ \tilde{G}_{\mu\nu}\gamma^\nu\ \psi$

Duality in QCD

Operator product expansion

⟹ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

If moment $\approx$ independent of Q^2

⟹ higher twist terms $A_n^{(\tau>2)}$ small

Duality in QCD

Operator product expansion

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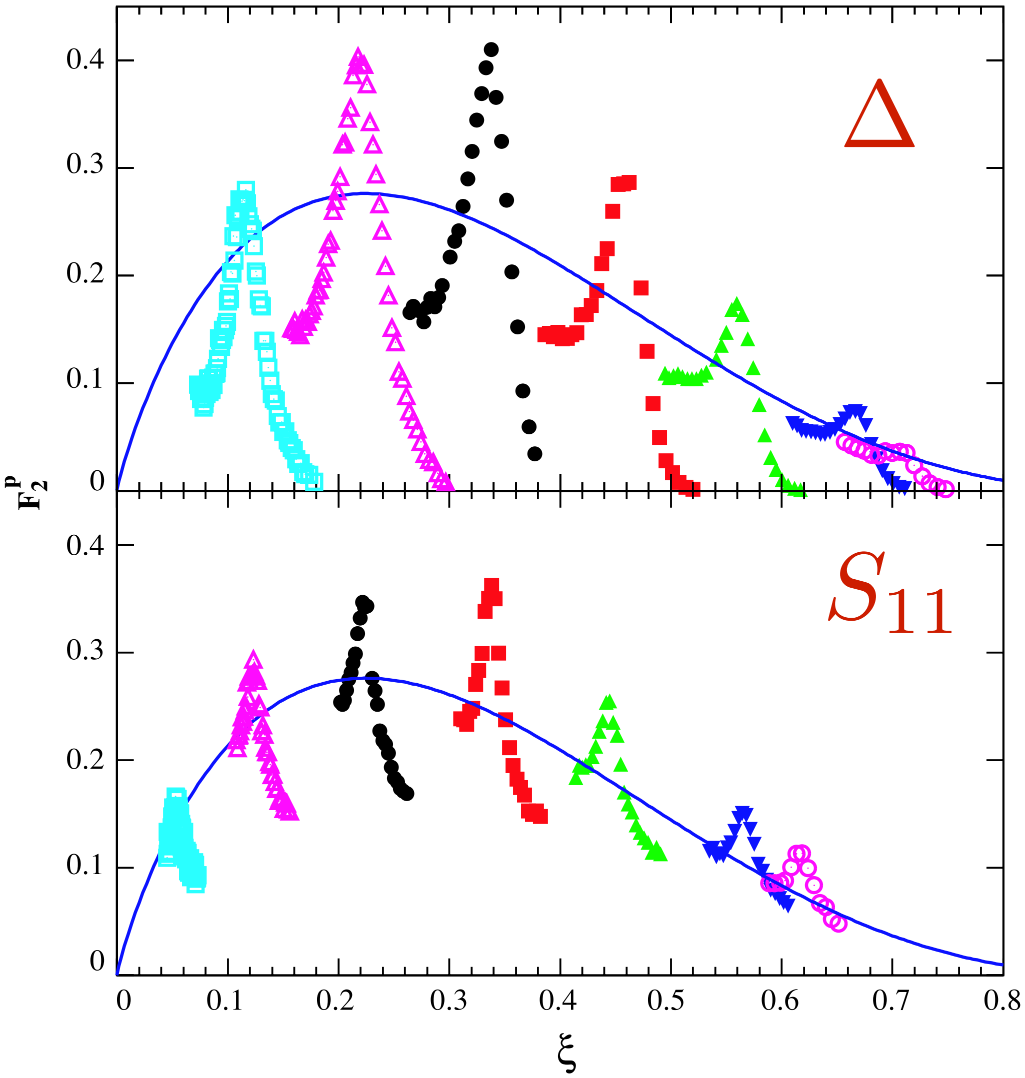
Duality $\Longleftrightarrow$ suppression of higher twists

de Rujula, Georgi, Politzer,
Ann. Phys. 103 (1975) 315

Local Duality

Local Bloom-Gilman duality



Local Bloom-Gilman duality

- contribution of (narrow) resonance R to structure function

$$F_2^{(R)} \approx 2M\nu \left(G_R(Q^2)\right)^2 \delta(W^2 - M_R^2)$$

if $G_R(Q^2) \sim (1/Q^2)^n$ then for $Q^2 \gg M_R^2$

$$F_2^{(R)} \approx (1 - x_R)^{2n-1}$$

"Drell-Yan-West relation"

with

$$x_R = \frac{Q^2}{Q^2 + M_R^2 - M^2}$$

$\longrightarrow$ as $Q^2 \to \infty$, $x_R \to 1$

resonances move to larger x

Local elastic duality

- extreme case of local duality for elastic peak

  $\longrightarrow$ elastic contribution to structure function

$$F_2^{(el)} = \frac{2M\tau}{1+\tau}\left(G_E^2 + \tau G_M^2\right)^2 \; \delta(\nu - Q^2/2M) \qquad \tau = \frac{Q^2}{4M^2}$$

- hypothesis: area under elastic peak same as integral of scaling structure function below threshold

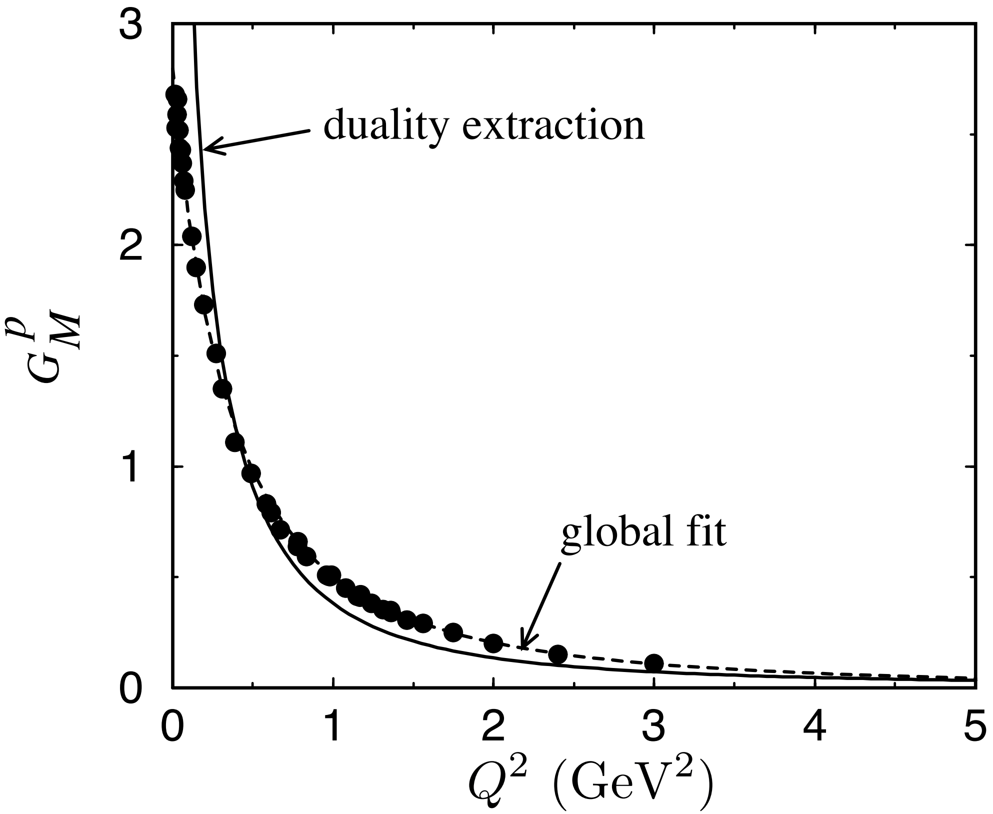
$$\int_1^{\delta\omega'} d\omega' \; F_2^{\mathrm{LT}}(\omega') = \frac{2M}{Q^2} \int d\nu \; F_2^{(el)}(\nu, Q^2) \qquad \omega' = \frac{2M\nu + M^2}{Q^2}$$

$$= \frac{G_E^2 + \tau G_M^2}{1+\tau}$$

Bloom-Gilman scaling variable

Local elastic duality

- extract magnetic form factor from integral of F_2



→ good to ~ 30% for Q^2 ~ few GeV^2

Local elastic duality

- conversely, differentiate local duality relation w.r.t. Q^2 to obtain structure function at threshold

$$F_2(x = x_{\rm th}) \; = \; \beta \left[\frac{G_M^2 - G_E^2}{2M^2(1+\tau)^2} + \frac{2}{1+\tau} \left(\frac{dG_E^2}{dQ^2} + \tau \frac{dG_M^2}{dQ^2} \right) \right]$$

where

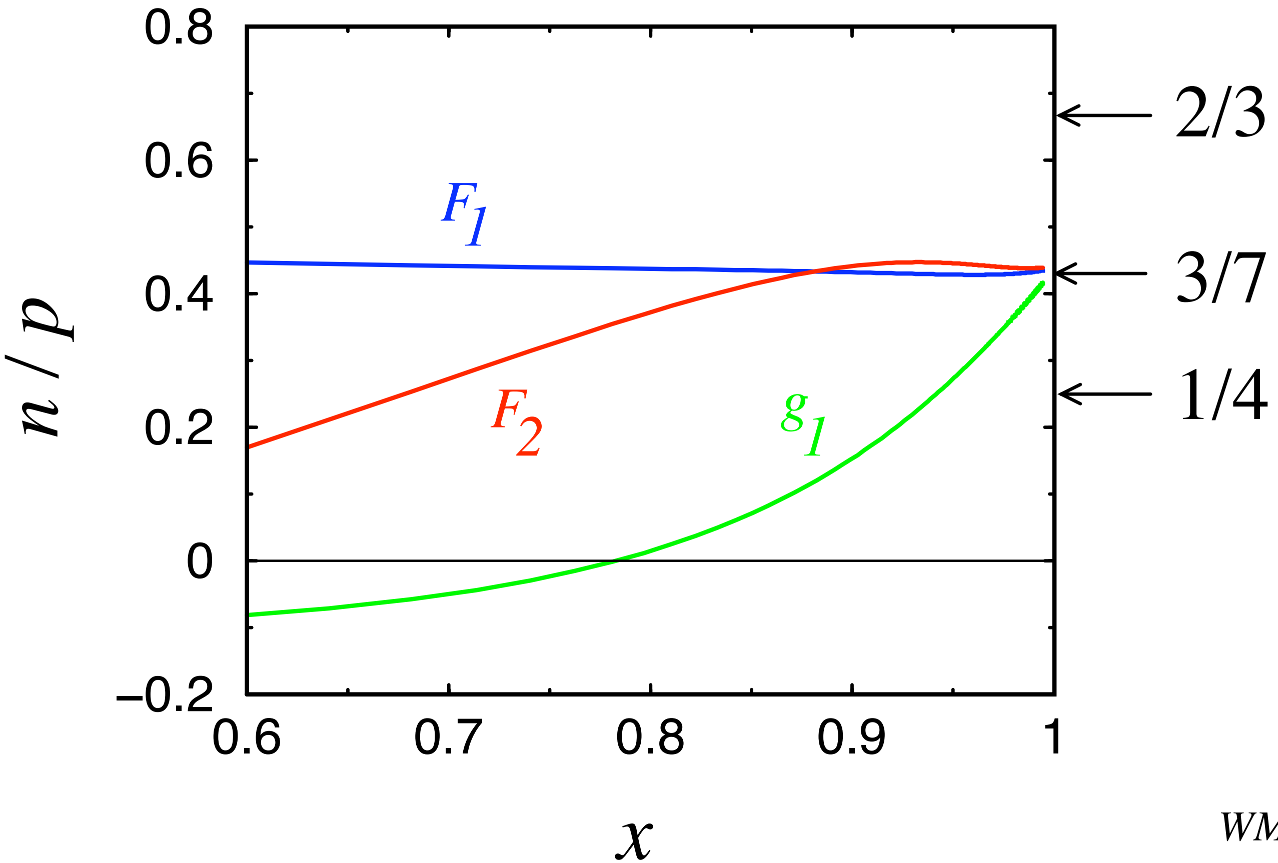
$$\beta = \frac{(Q^4/M^2)(\xi_0^2/\xi^3)(2 - \xi/x)}{2\xi_0 - 4}$$

$$\xi_0 = \xi(x = 1)$$

$\longrightarrow$ structure functions at large x from form factors !

Local elastic duality

- neutron to proton structure function ratios



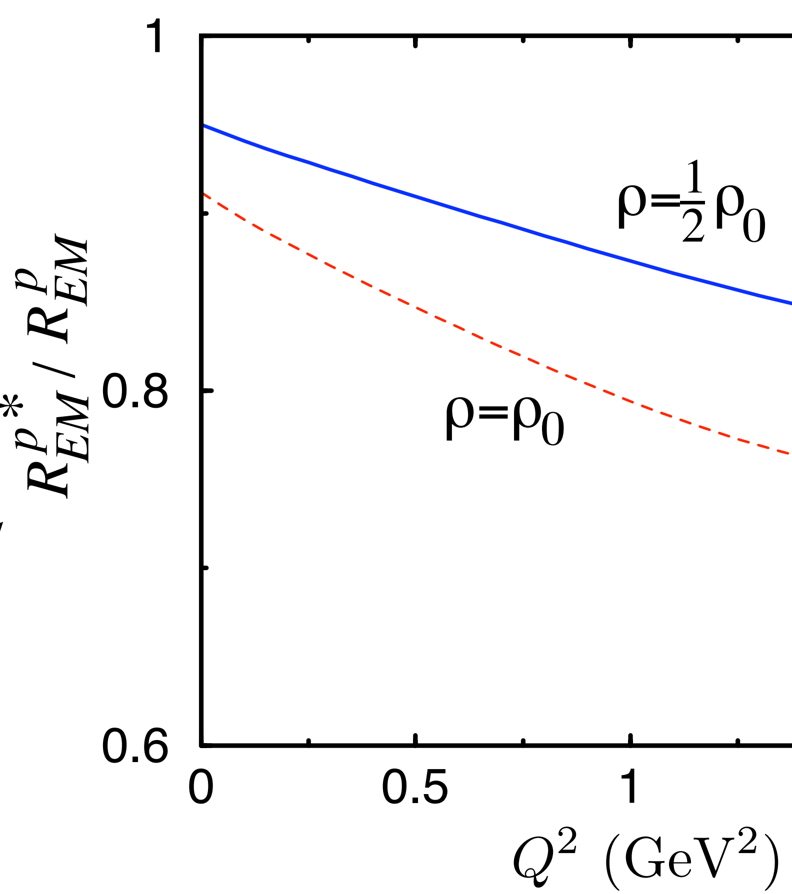
WM, PRL 86 (2001) 35

⟶ testable predictions for $x \to 1$ behavior

Local Duality & Nuclear Modifications

WM, Tsushima, Thomas

- can recent ^{4}He $(e,e'p)$ data be interpreted in terms of medium modified form factors ?
- use local duality to relate medium modified *form factors* to medium modified *structure functions* (EMC effect)



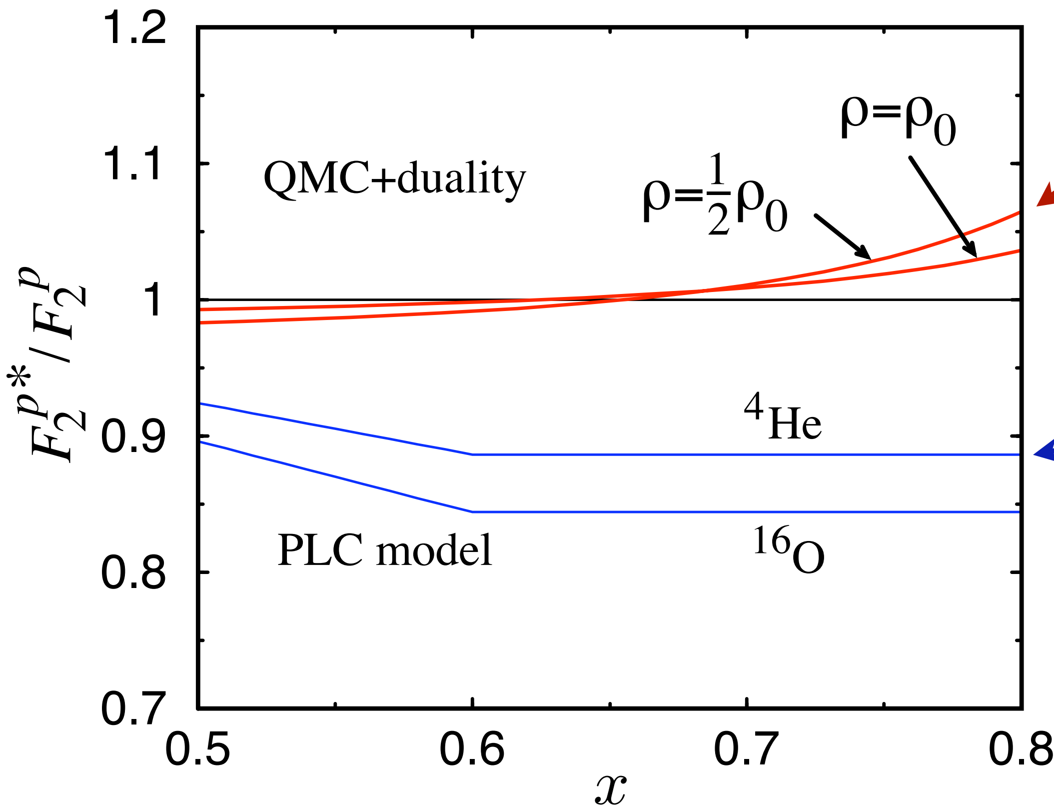
$\mu_p \dfrac{G_E^p}{G_M^p}$

quark-meson coupling (QMC) model

medium modified structure functions

$$\frac{F_2^{p*}}{F_2^p} \approx \frac{dG_M^{p*2}/dQ^2}{dG_M^{p2}/dQ^2} \qquad \text{large } Q^2$$

note: threshold for bound nucleon at $x_{\rm th}^* = \left(\frac{m_\pi(2M + m_\pi^2) + Q^2}{m_\pi(2(M^* + V) + m_\pi) + Q^2} \right) x_{\rm th}$



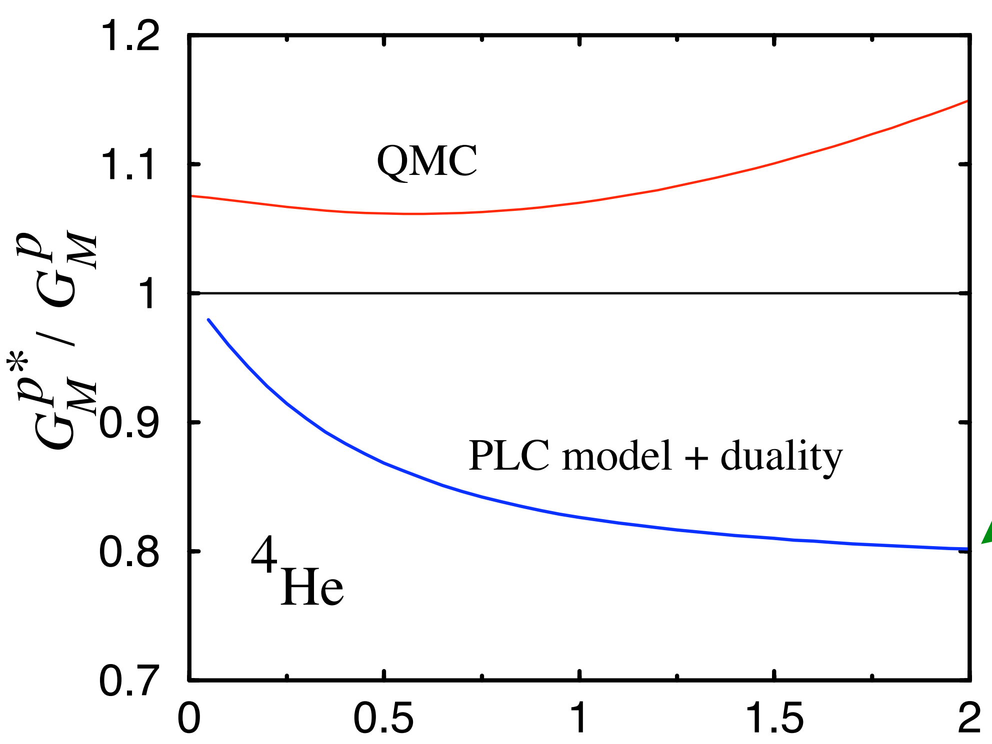
< 5% effect in QMC

point-like configuration model attributes EMC effect to deformation of bound nucleon structure

$$\frac{F_2^*(k,x)}{F_2(x)} = 1 - 2(k^2/2M + \epsilon_A)/\Delta E_a$$

- conversely, change in form factor of bound nucleon implied by change in structure function in medium

$$\left[G_M^p(Q^2)\right]^2 \approx \frac{2-\xi_0}{\xi_0^2}\,\frac{(1+\tau)}{(1/\mu_p^2+\tau)}\int_{\xi_{\rm th}}^{1} d\xi\ F_2^p(\xi)$$



predicts 20% suppression in magnetic form factor

→ enhancement of PT ratio

→ contrary to ^{4}He data

Truncated Moments

Psaker, Christy, Keppel, WM (2007)

Truncated moments

- complete moments can be studied in QCD via twist expansion

  $\longrightarrow$ Bloom-Gilman duality has a precise meaning

  (*i.e.*, duality violation = higher twists)

- for "local" duality, difficult to make rigorous connection with QCD

  $\longrightarrow$ *e.g.* need prescription for how to average over resonances

- *truncated* moments allow study of restricted regions in x (or W) within QCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

Truncated moments

- truncated moments obey DGLAP-like evolution equations, similar to PDFs

$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left(P'_{(n)} \otimes \overline{M}_n\right)(\Delta x, Q^2)$$

where modified splitting function is

$$P'_{(n)}(z, \alpha_s) = z^n \; P_{NS,S}(z, \alpha_s)$$

$\longrightarrow$ can follow evolution of <u>specific resonance (region)</u> with Q^2 in pQCD framework!

$\longrightarrow$ suitable when complete moments not available

Truncated moments

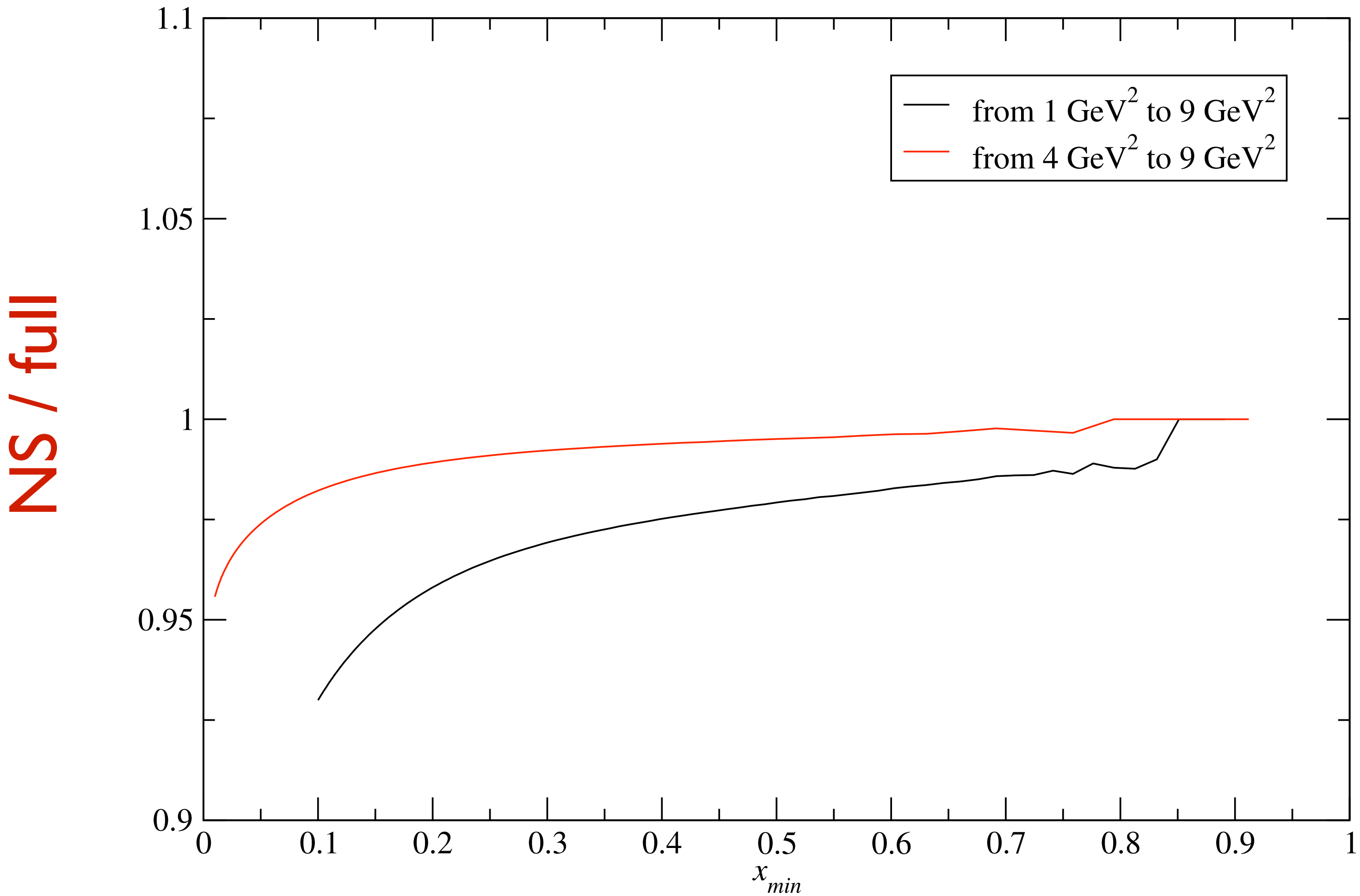
- truncated moment evolution equations exist for singlet (S) and nonsinglet (NS) separately

- for analysis of data, do not know much of experimental structure function is NS and how much is S

  → for lowest ($n=2$) truncated moment, assumption that total $\approx$ NS is good to few % for $x_{\rm min} > 0.2$

  → for higher moments, small-x region is further suppressed, so that NS is a very good approximation to total

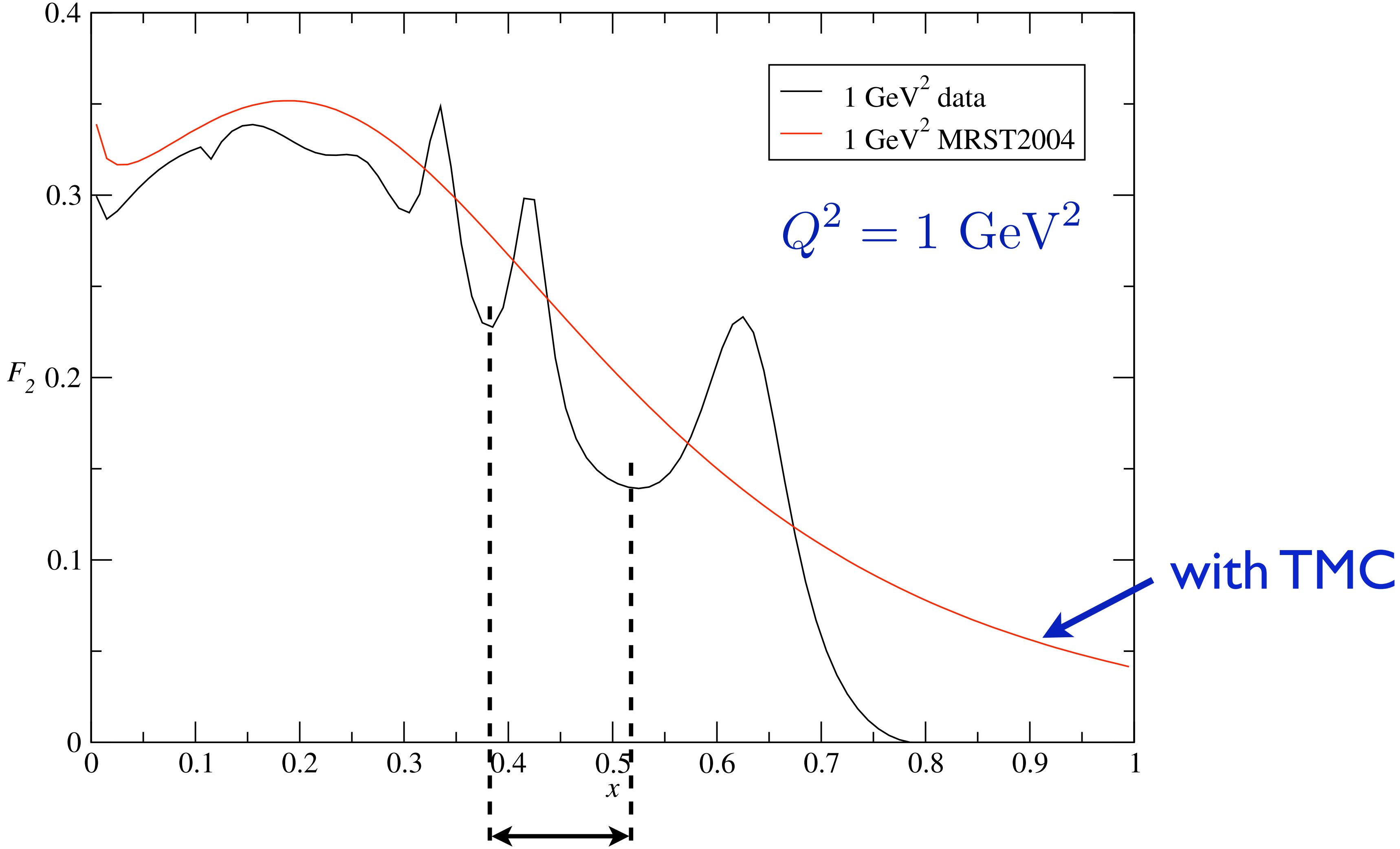
$n = 2$ truncated moment of F_2^p



$$\Delta x = [x_{min}, 1]$$

Psaker et al. (2007)

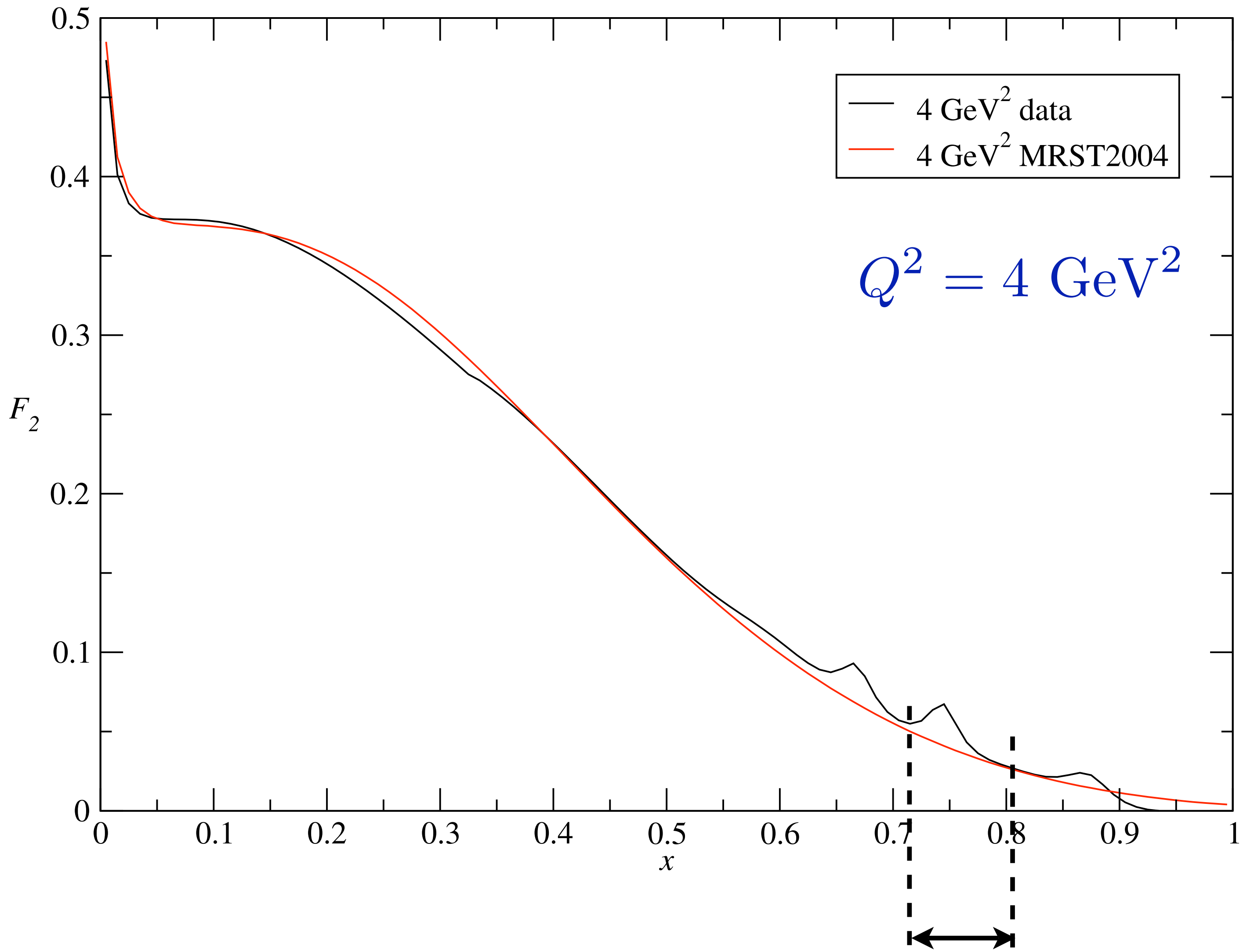
Parameterization of F_2^p data



how much of this region
is <u>leading twist</u> ?

Psaker et al. (2007)

Parameterization of F_2^p data



how much of this region
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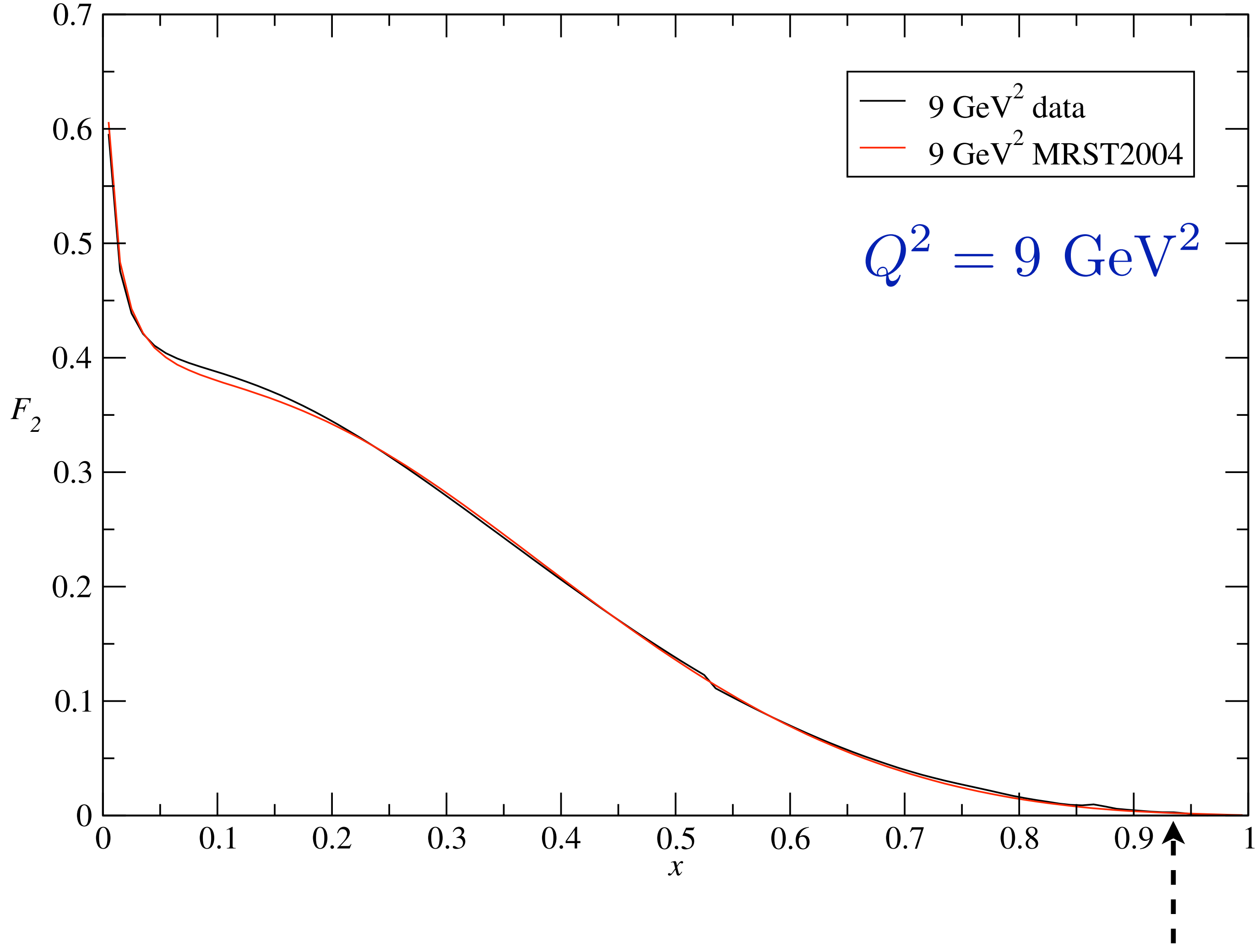
Psaker et al. (2007)

Parameterization of F_2^p data



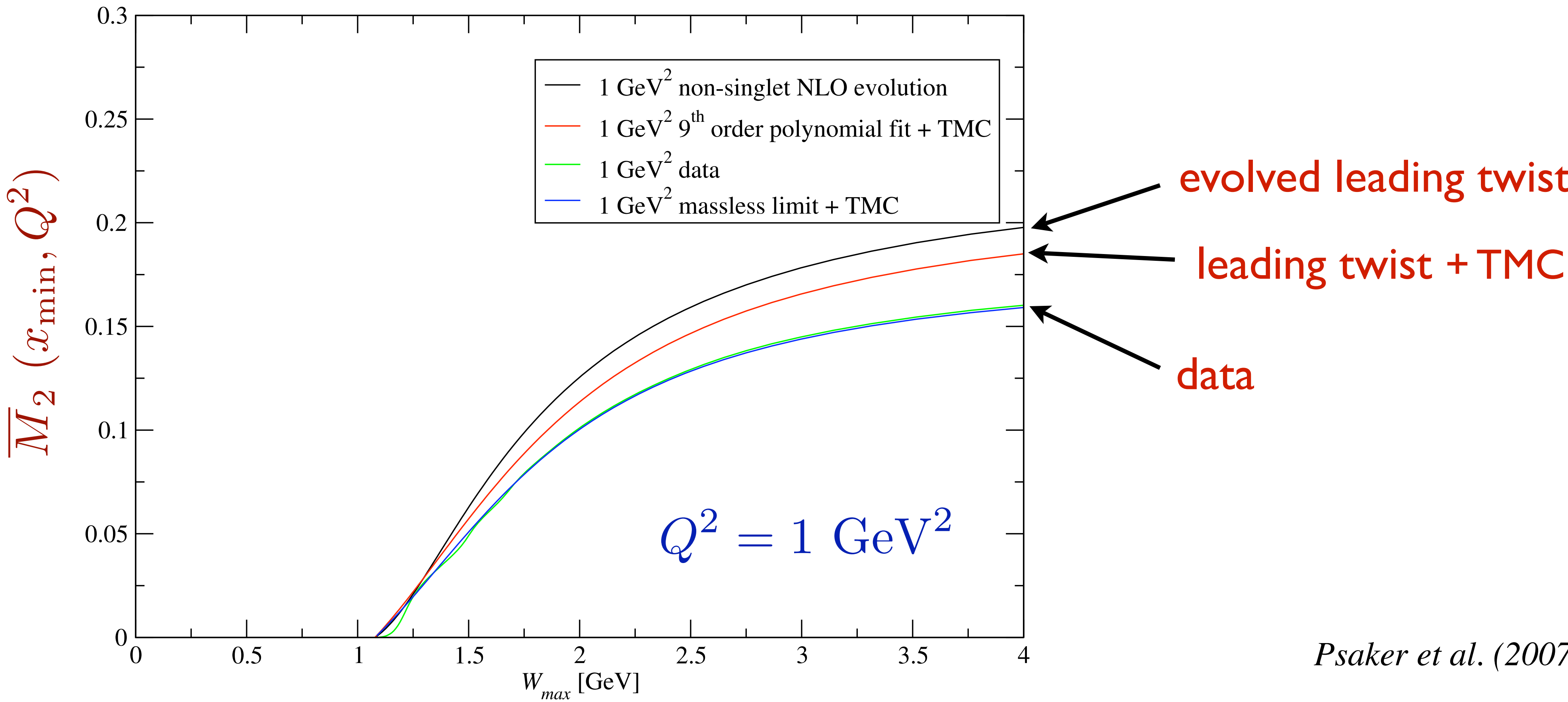
how much of this region
is <u>leading twist</u> ?

Psaker et al. (2007)

Analysis of Hall C data

- assume data at highest Q^2 ($Q^2 = 9\ \mathrm{GeV}^2$) is entirely leading twist
- evolve (as NS) fit to data at $Q^2 = 9\ \mathrm{GeV}^2$ down to lower Q^2

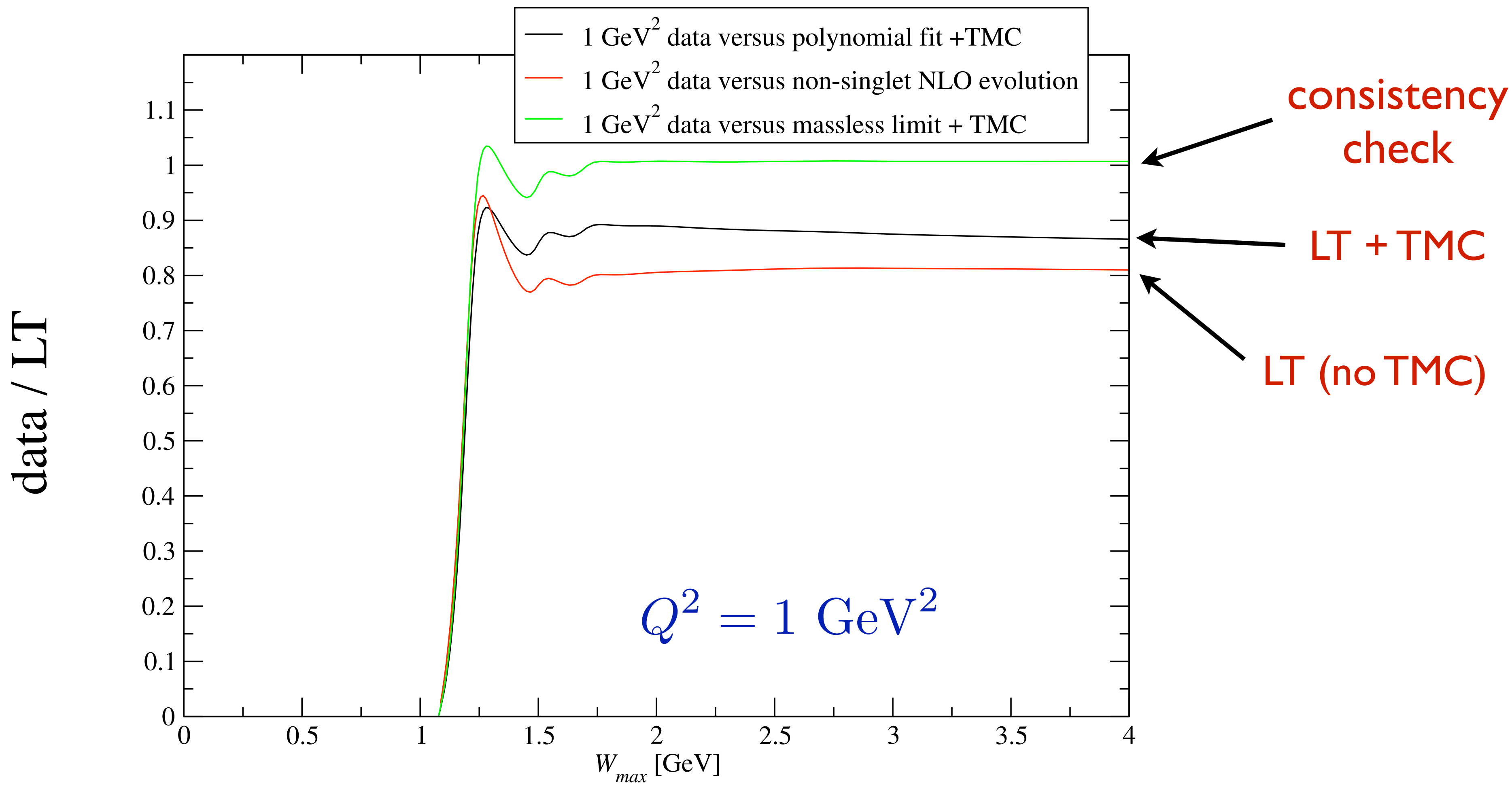
$\longrightarrow$ apply TMC, and compare with data at lower Q^2



Psaker et al. (2007)

Analysis of Hall C data

- ratio of data to leading twist



Analysis of Hall C data

- consider individual resonance regions:

$$W_{\rm thr}^2 < W^2 < 1.9 \text{ GeV}^2 \qquad \text{``}\Delta(1232)\text{''}$$

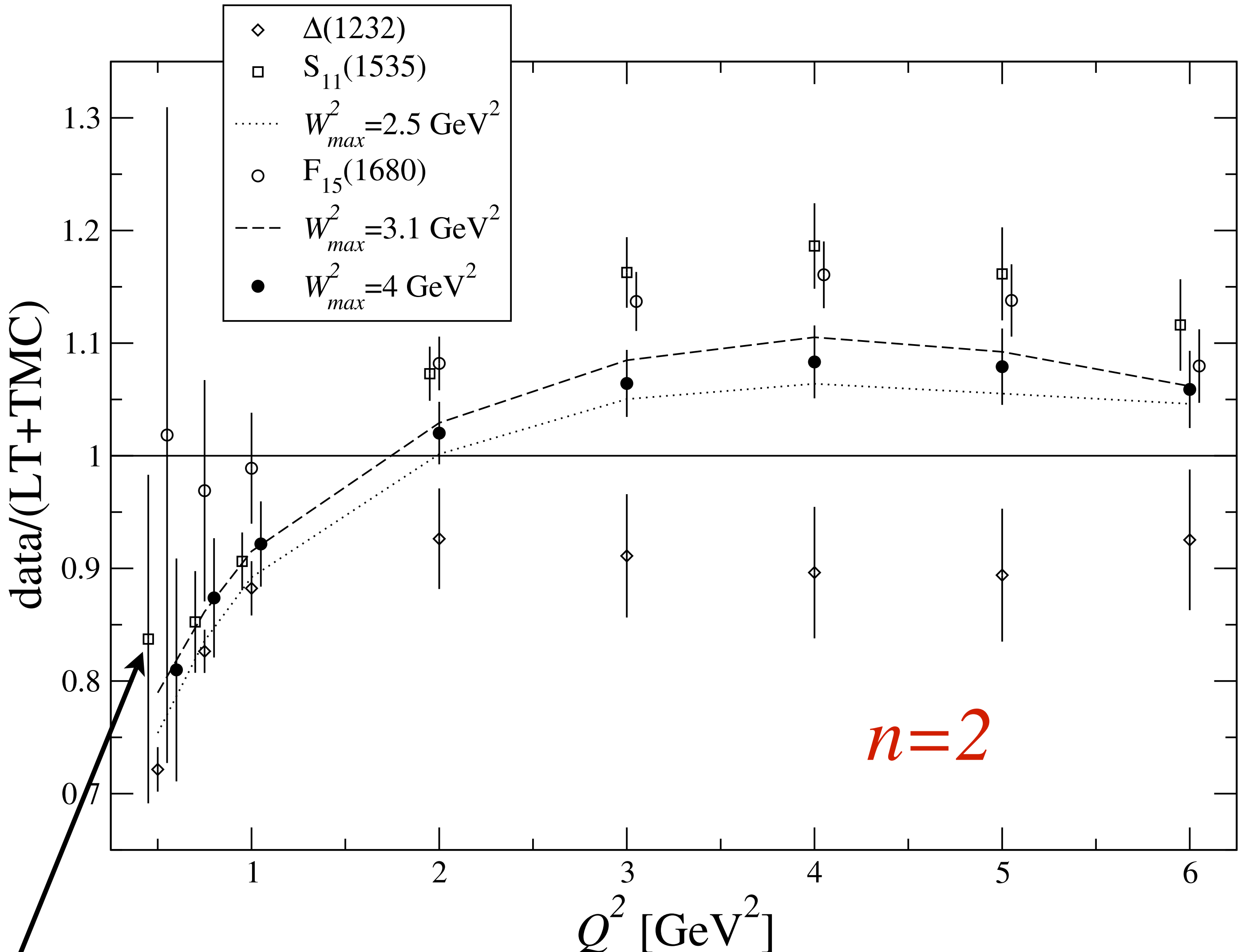
$$1.9 < W^2 < 2.5 \text{ GeV}^2 \qquad \text{``}S_{11}(1535)\text{''}$$

$$2.5 < W^2 < 3.1 \text{ GeV}^2 \qquad \text{``}F_{15}(1680)\text{''}$$

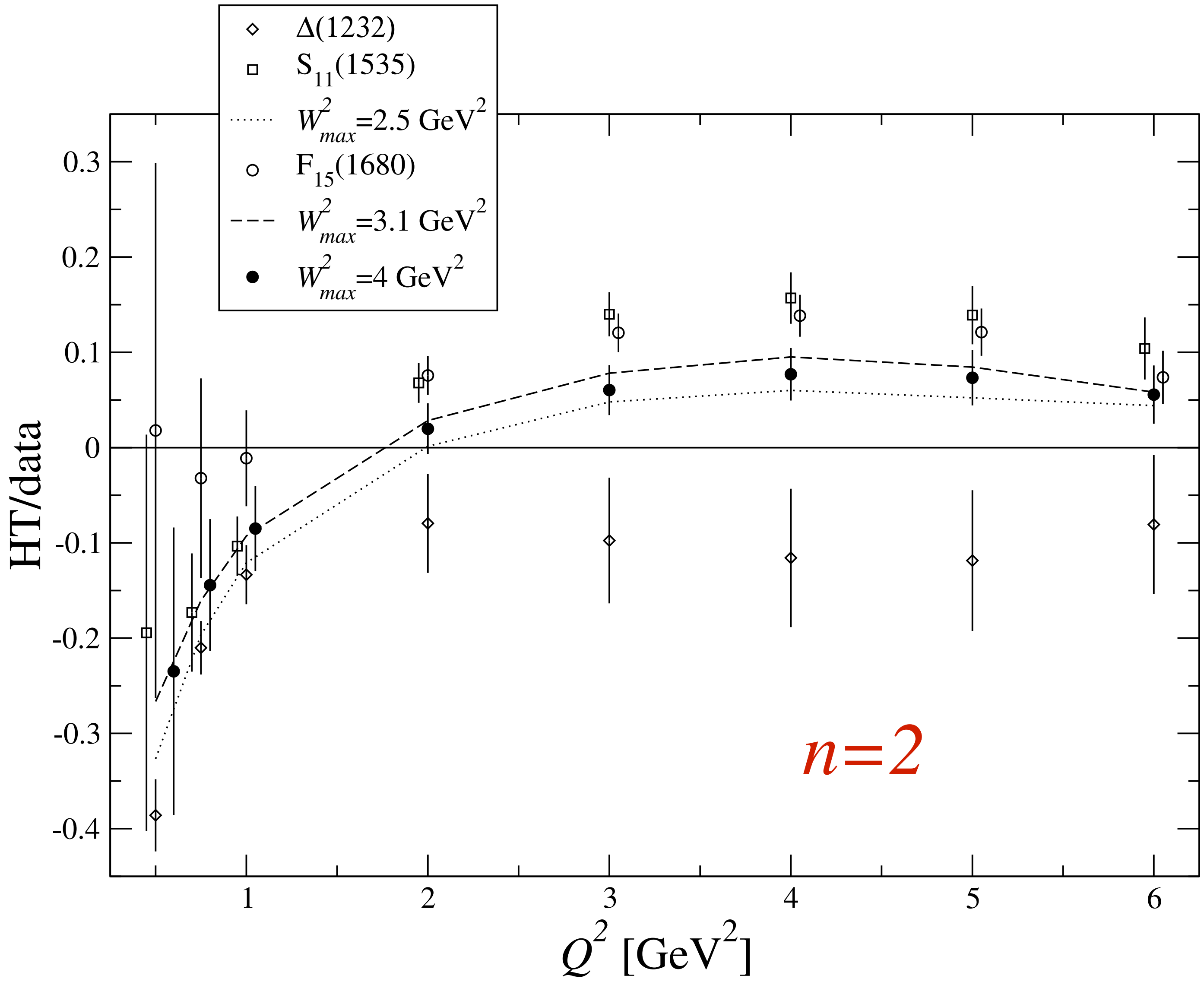
as well as total resonance region:

$$W^2 < 4 \text{ GeV}^2$$

Analysis of Hall C data

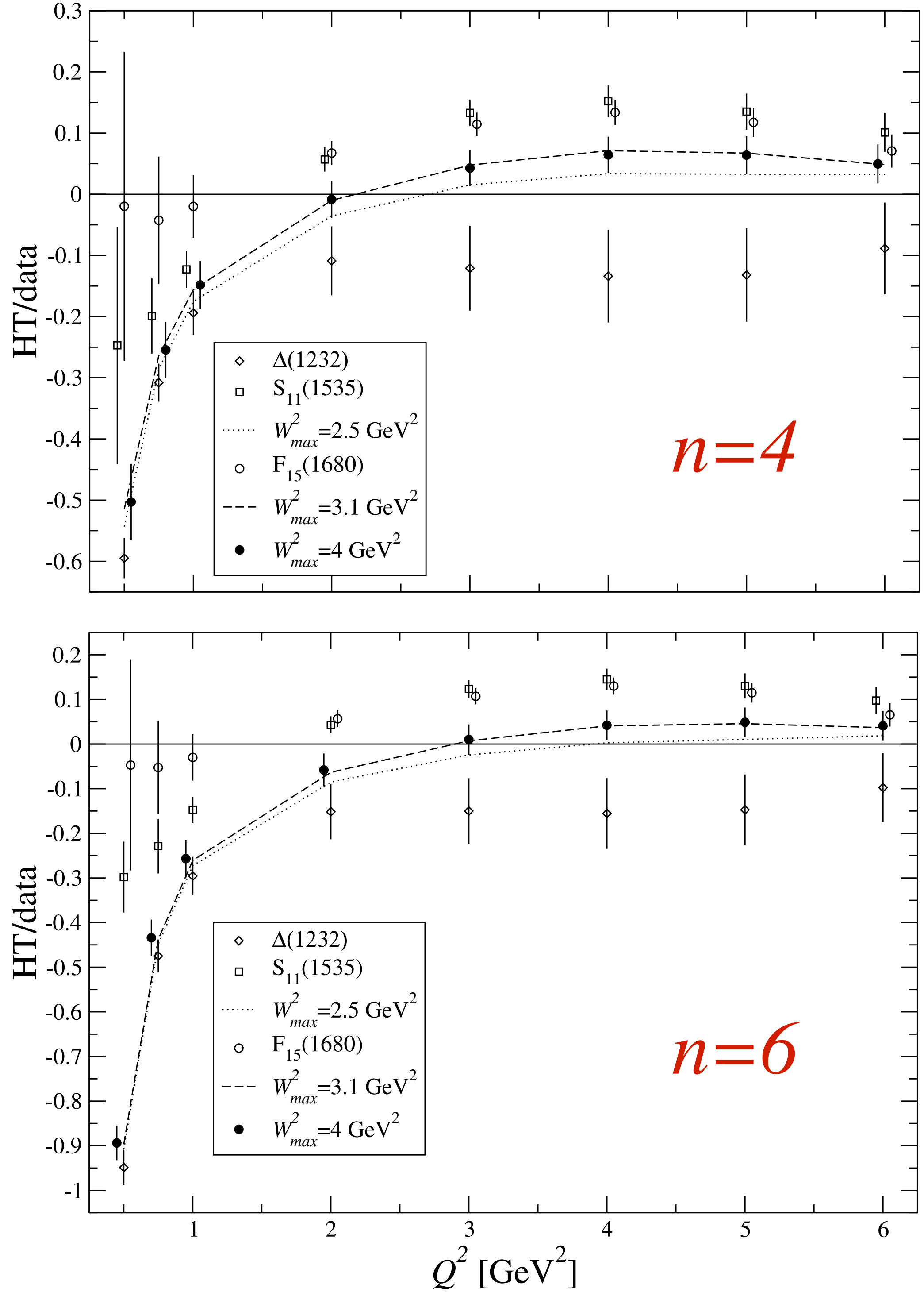


method breaks down for
low x (high W) at low Q^2



→ higher twists < 10% for $Q^2 > 1\ \mathrm{GeV}^2$

- higher moments



Summary

- Remarkable confirmation of quark-hadron duality in structure functions

  → higher twists "small" down to low Q^2 (~ 1 GeV^2)

- Local (elastic) duality

  → constraints on nuclear EMC effect and medium modified form factors

- Truncated moments

  → firm foundation for study of local duality in QCD

  → method can be applied to nuclear cross sections, relating nuclear structure functions to transition form factors