Los Alamos National Lab.
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Two-Boson Exchange in Electron-Nucleon Scattering

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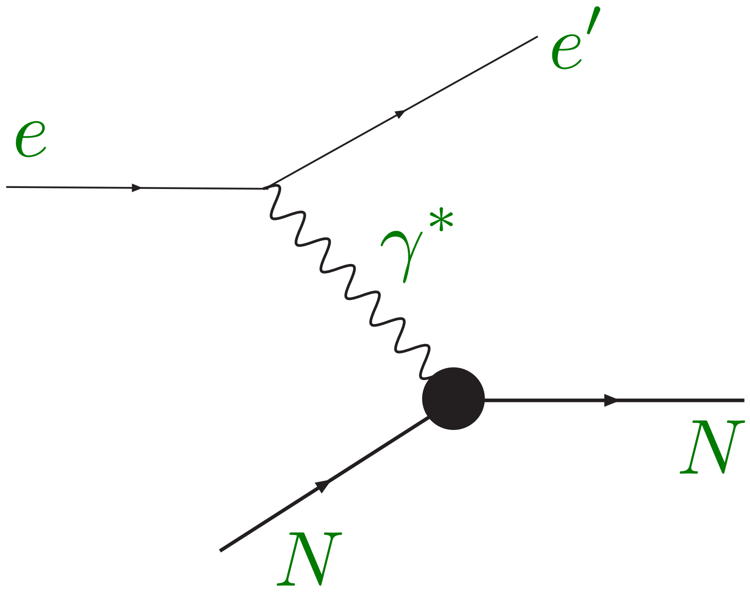
Outline

- Elastic ep scattering
- Two-photon exchange
  - → Rosenbluth separation *vs.* polarization transfer
  - → first global analysis of form factors including TPE
- Parity-violating electron scattering
  - → photon-Z interference & strangeness in the proton
  - → dispersive corrections to proton's weak charge
- Summary

Elastic eN scattering

- Elastic cross section



$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \, \frac{\tau}{\varepsilon\,(1+\tau)} \, \sigma_R$$

$$\tau = Q^2/4M^2$$

$$\varepsilon = \left(1 + 2(1+\tau)\tan^2(\theta/2)\right)^{-1}$$

$$\sigma_{\text{Mott}} = \frac{\alpha^2 E' \cos^2\frac{\theta}{2}}{4E^3 \sin^4\frac{\theta}{2}}$$

cross section for scattering from point particle

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

reduced cross section

$$G_E \ , \ G_M$$

Sachs electric and magnetic form factors

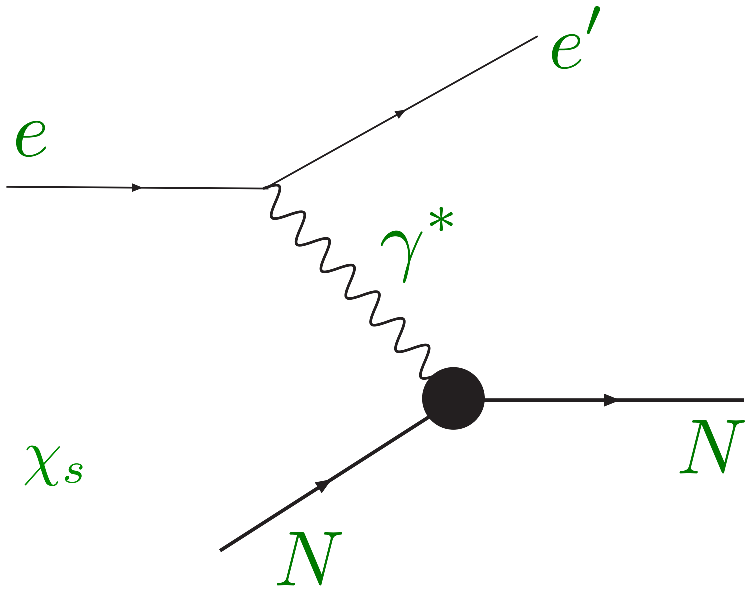
Traditional interpretation

- In Breit frame

$$\nu = 0 \ , \quad Q^2 = \vec{q}^{\,2}$$

electromagnetic current is

$$\bar{u}(p', s')\Gamma^\mu u(p, s) = \chi^\dagger_{s'} \left(G_E + \frac{i\vec{\sigma} \times \vec{q}}{2M} G_M \right) \chi_s$$



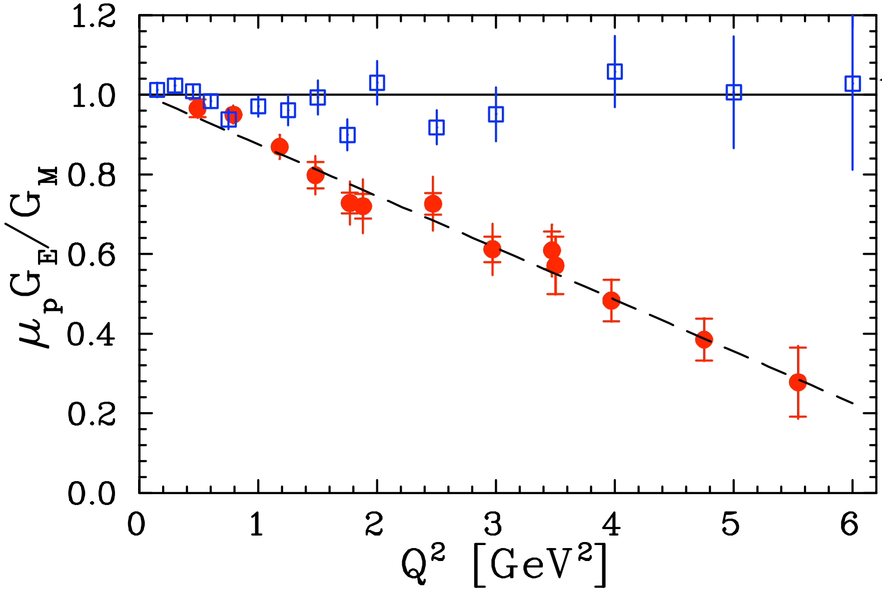
→ *cf.* classical (Non-Relativistic) current density

$$J^{\mathrm{NR}} = \left(e\ \rho_E^{\mathrm{NR}} \ , \ \mu\ \vec{\sigma} \times \vec{\nabla} \rho_M^{\mathrm{NR}} \right)$$

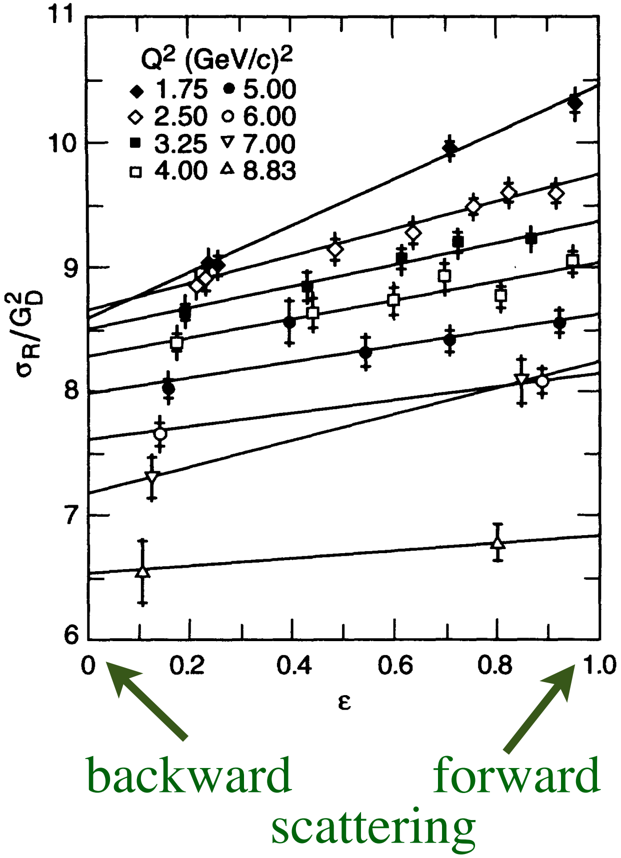
→ $$\rho_E^{\mathrm{NR}}(r) = \frac{2}{\pi} \int_0^\infty dq\ \vec{q}^{\,2}\ j_0(qr)\ G_E(\vec{q}^{\,2})$$ ← charge density

$$\mu\ \rho_M^{\mathrm{NR}}(r) = \frac{2}{\pi} \int_0^\infty dq\ \vec{q}^{\,2}\ j_0(qr)\ G_M(\vec{q}^{\,2})$$ ← magnetization density

Proton G_E/G_M ratio



Rosenbluth (Longitudinal-Transverse) Separation



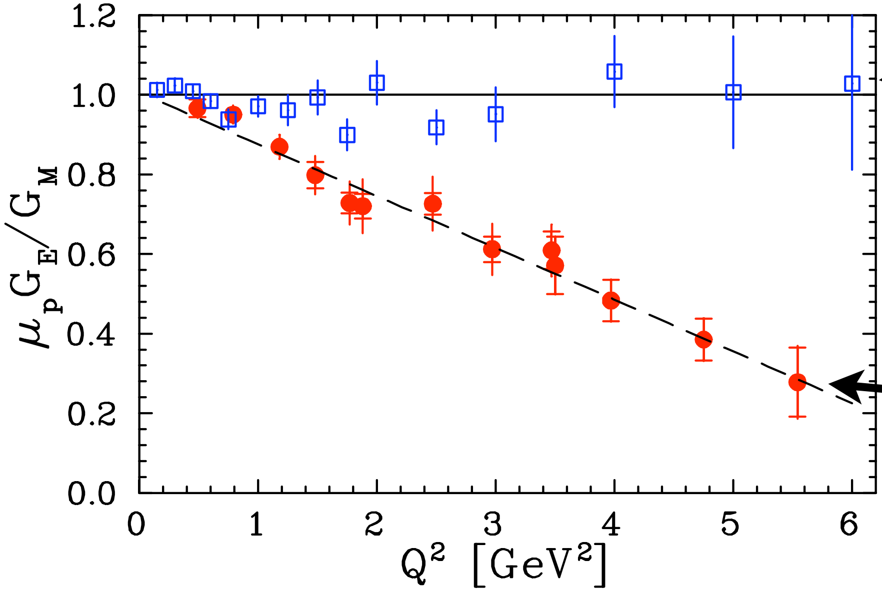
LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

→ suppressed at large Q^2

Proton G_E/G_M ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

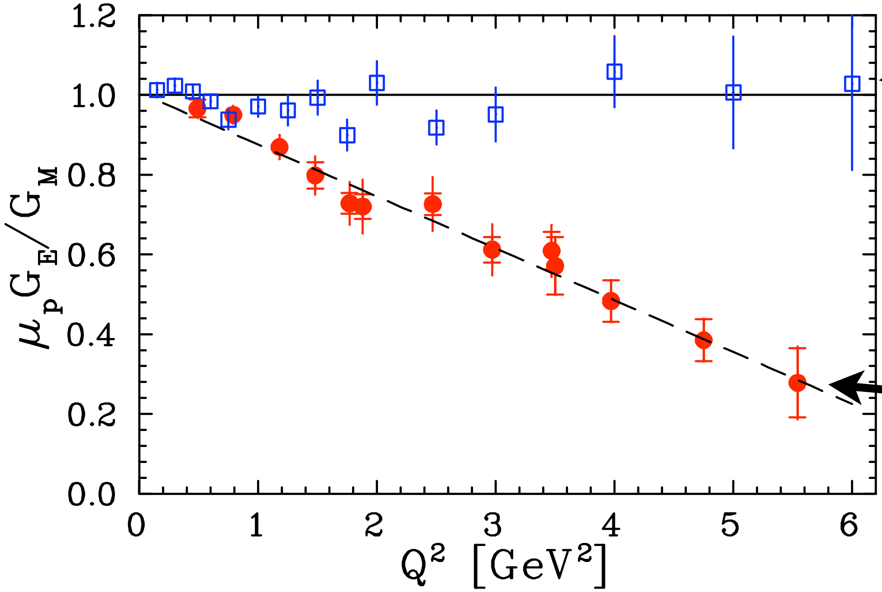
→ suppressed at large Q^2

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→ $P_{T,L}$ recoil proton polarization in $\vec{e}\, p \to e\, \vec{p}$

Proton G_E/G_M ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

PT method

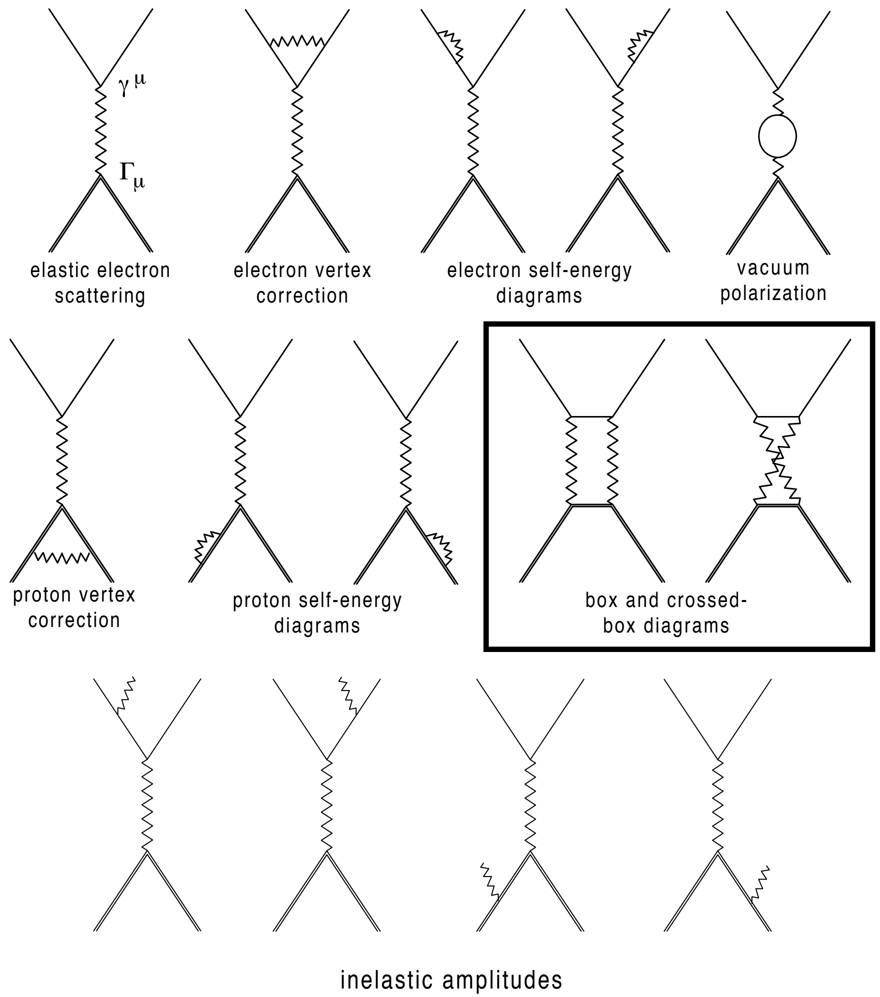
$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→ are the G_E^p/G_M^p data consistent?

Two-photon exchange

QED radiative corrections

- cross section modified by 1γ loop effects



Born

TPE

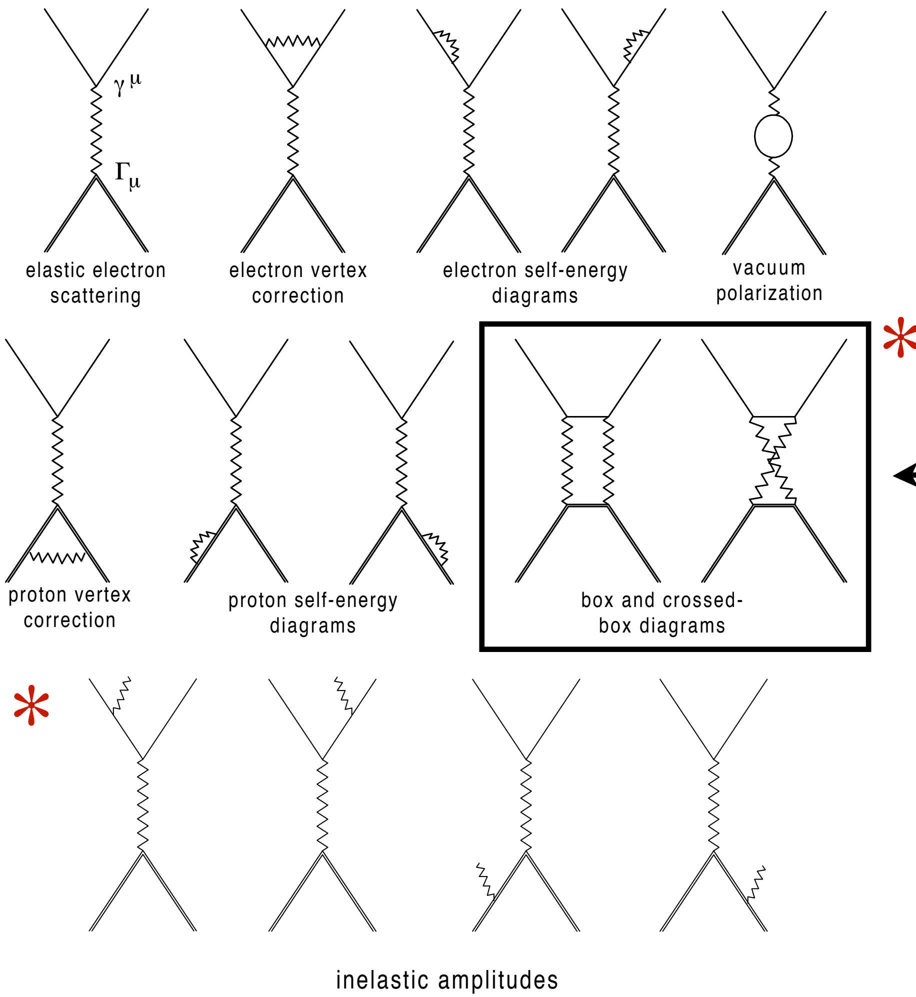
$$d\sigma = d\sigma_0\,(1+\delta)$$

δ contains additional ε dependence, mostly from box diagrams

(most difficult to calculate)

QED radiative corrections

- cross section modified by 1γ loop effects



$$d\sigma = \overset{\text{Born}}{d\sigma_0}\,(1+\overset{\text{TPE}}{\delta})$$

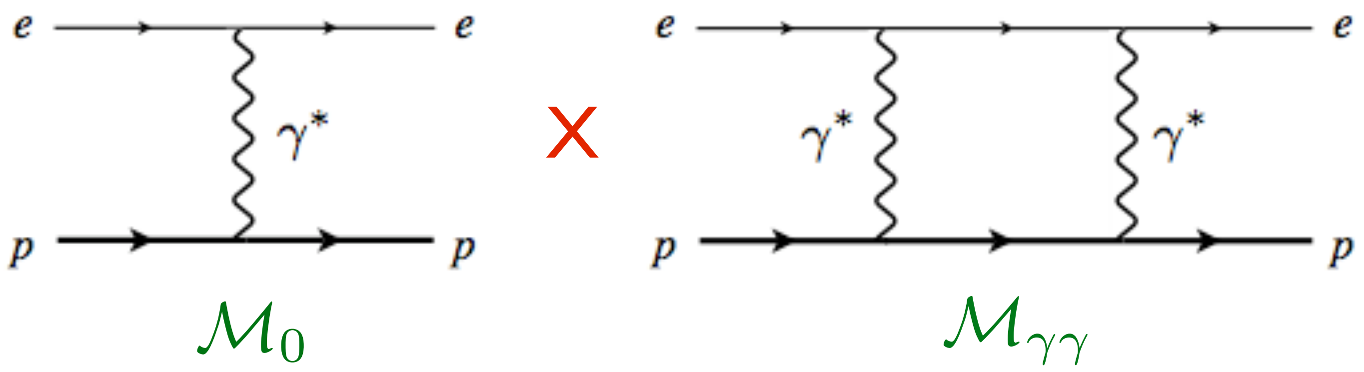
δ contains additional ε dependence, mostly from box diagrams

(most difficult to calculate)

* IR divergences cancel

Two-photon exchange

- interference between Born and TPE amplitudes



- contribution to cross section:

$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_0^\dagger\,\mathcal{M}_{\gamma\gamma}\right\}}{\left|\mathcal{M}_0\right|^2}$$

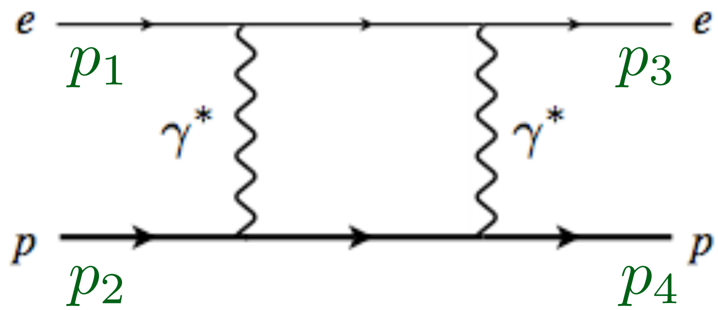
- "soft photon approximation" (used in most data analyses)

  $\longrightarrow$ approximate integrand in $\mathcal{M}_{\gamma\gamma}$ by values at γ^* poles

  $\longrightarrow$ neglect nucleon structure (no form factors)

Mo, Tsai (1969)

Two-photon exchange



$$\mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{D(k)}$$

where

$$N(k) = \bar{u}(p_3)\, \gamma_\mu (\not{p}_1 - \not{k} + m_e)\gamma_\nu \, u(p_1) \times \bar{u}(p_4)\, \Gamma^\mu(q-k)\, (\not{p}_2 + \not{k} + M)\, \Gamma^\nu(k)\, u(p_2)$$

and

$$D(k) = (k^2 - \lambda^2)\, ((k-q)^2 - \lambda^2) \times ((p_1 - k)^2 - m^2)\, ((p_2 + k)^2 - M^2)$$

with λ an IR regulator, and e.m. current is

$$\Gamma^\mu(q) = \gamma^\mu \, F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M}\, F_2(q^2)$$

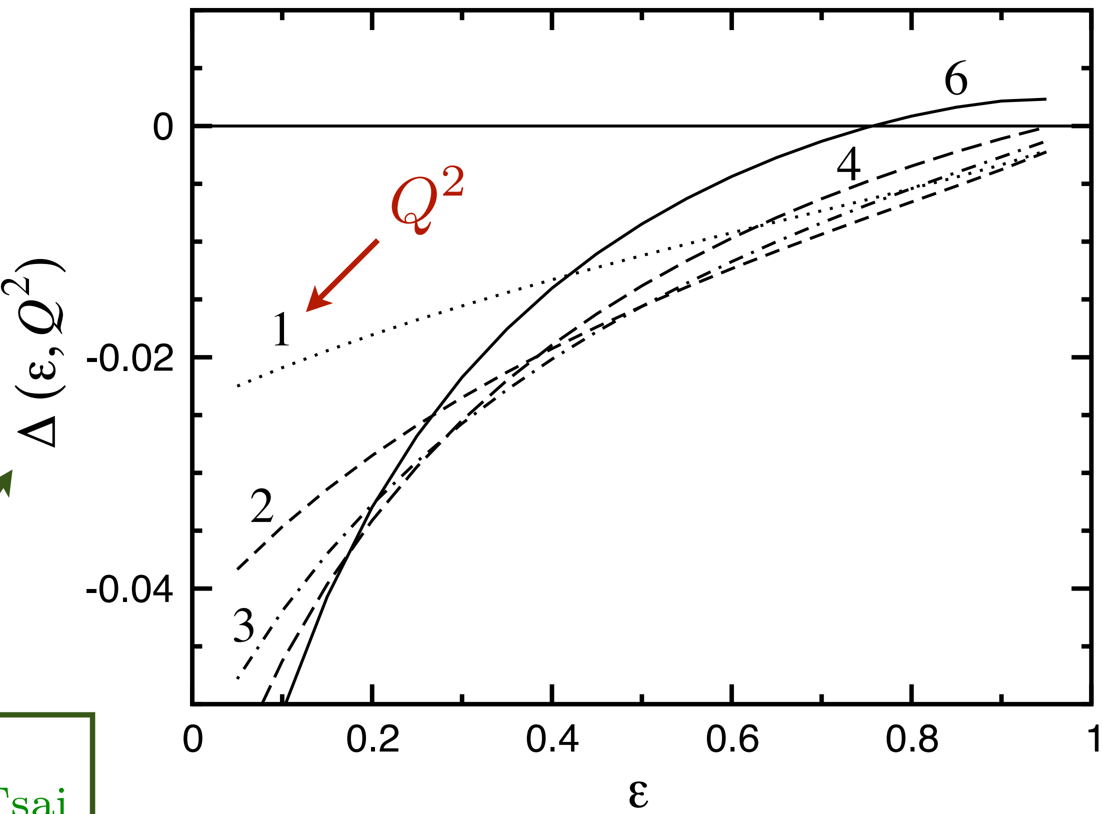
- Mo-Tsai: soft γ approximation
  $\longrightarrow$ integrand most singular when $k = 0$ and $k = q$
  $\longrightarrow$ replace γ propagator which is not at pole by $1/q^2$
  $\longrightarrow$ approximate numerator $N(k) \approx N(0)$
  $\longrightarrow$ neglect all structure effects

- Maximon-Tjon: improved loop calculation
  $\longrightarrow$ exact treatment of propagators
  $\longrightarrow$ still evaluate $N(k)$ at $k = 0$
  $\longrightarrow$ first study of form factor effects
  $\longrightarrow$ additional ε dependence

- Blunden-WM-Tjon: exact loop calculation
  $\longrightarrow$ no approximation in $N(k)$ or $D(k)$
  $\longrightarrow$ include form factors

Two-photon exchange

- "exact" calculation of loop diagram (including hadron structure)



Blunden, Melnitchouk, Tjon
*PRL **91** (2003) 142304;*
*PRC **72** (2005) 034612*

$$\delta^{(2\gamma)}_{\rm full} - \delta^{(2\gamma)}_{\rm Mo-Tsai}$$

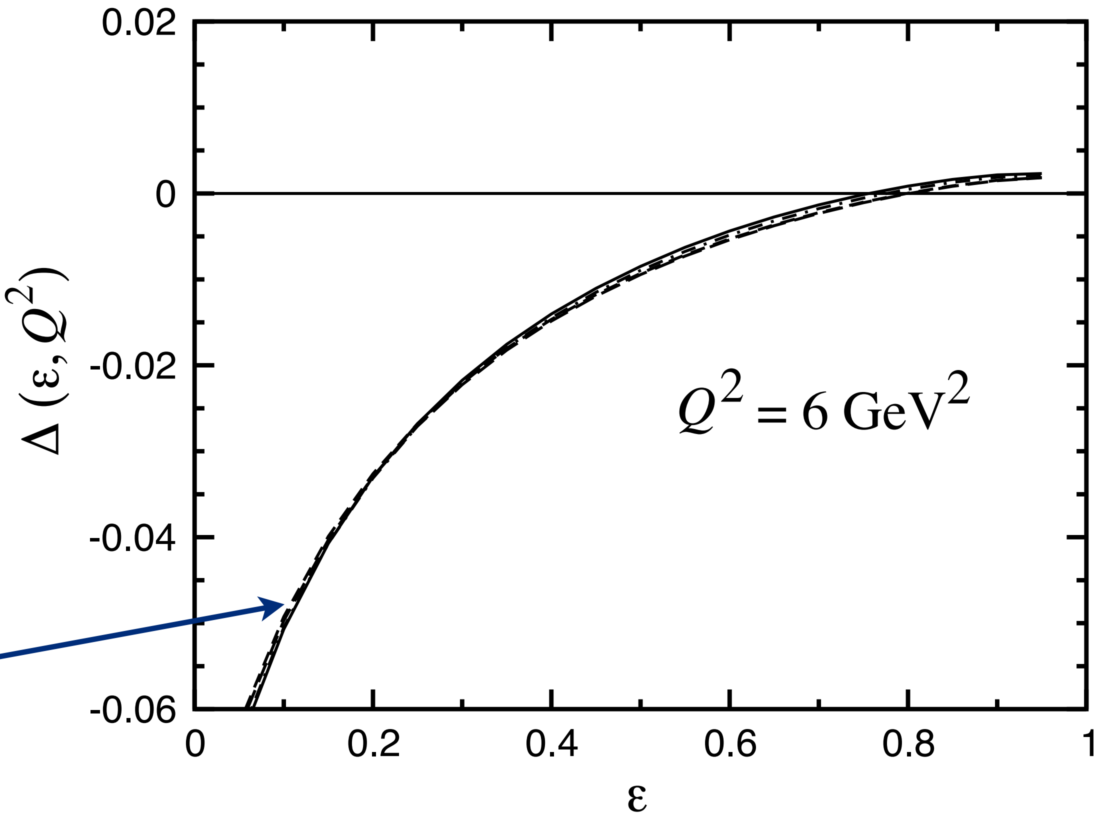
→ few % magnitude, non-linear in ε

→ *positive* slope

(will <u>*reduce*</u> Rosenbluth ratio)

Two-photon exchange

- "exact" calculation of loop diagram (including hadron structure)

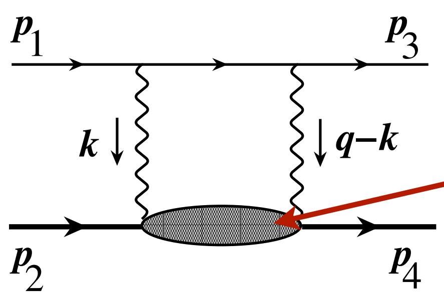


form factors:
Mergell et al. (1996)
Brash et al. (2002)
Arrington LT (2004)
Arrington PT (2004)

Blunden, Melnitchouk, Tjon
PRL ***91*** *(2003) 142304;*
PRC ***72*** *(2005) 034612*

→ results do not depend strongly on input form factors

Higher-mass intermediate states



$N,\ \Delta,\ P_{11},\ S_{11},\ S_{31}, \ldots$

- lowest mass excitation is $P_{33}\ \Delta(1232)$ resonance

→ relativistic $\gamma^* N\Delta$ vertex

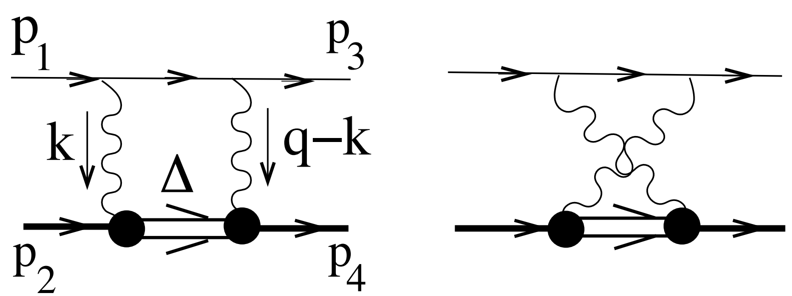
form factor $\dfrac{\Lambda_\Delta^4}{(\Lambda_\Delta^2 - q^2)^2}$

$$\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q) \equiv iV^{\nu\alpha}_{\Delta in}(p,q) = i\frac{eF_\Delta(q^2)}{2M_\Delta^2}\Big\{ g_1\left[g^{\nu\alpha}\not{p}\not{q} - p^\nu\gamma^\alpha\not{q} - \gamma^\nu\gamma^\alpha p\cdot q + \gamma^\nu\not{p}q^\alpha \right]$$
$$+ g_2\left[p^\nu q^\alpha - g^{\nu\alpha}p\cdot q \right] + (g_3/M_\Delta)\left[q^2(p^\nu\gamma^\alpha - g^{\nu\alpha}\not{p}) + q^\nu(q^\alpha\not{p} - \gamma^\alpha p\cdot q) \right]\Big\}\gamma_5\, T_3$$

→ coupling constants

g_1	magnetic	⇨	7
$g_2 - g_1$	electric	⇨	9
g_3	Coulomb	⇨	-2 ... 0

TPE amplitude with Δ intermediate state



$$\mathcal{M}_{\Delta}^{\gamma\gamma} = -e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N_{box}^{\Delta}(k)}{D_{box}^{\Delta}(k)} - e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N_{x-box}^{\Delta}(k)}{D_{x-box}^{\Delta}(k)}$$

with numerators

$$\begin{aligned} N_{box}^{\Delta}(k) &= \overline{U}(p_4) V_{\Delta in}^{\mu\alpha}(p_2+k, q-k) \left[\not{p}_2 + \not{k} + M_{\Delta}\right] \mathcal{P}_{\alpha\beta}^{3/2}(p_2+k) V_{\Delta out}^{\beta\nu}(p_2+k, k) U(p_2) \\ &\times \; \overline{u}(p_3)\gamma_{\mu} \left[\not{p}_1 - \not{k} + m_e\right] \gamma_{\nu} u(p_1) \end{aligned}$$

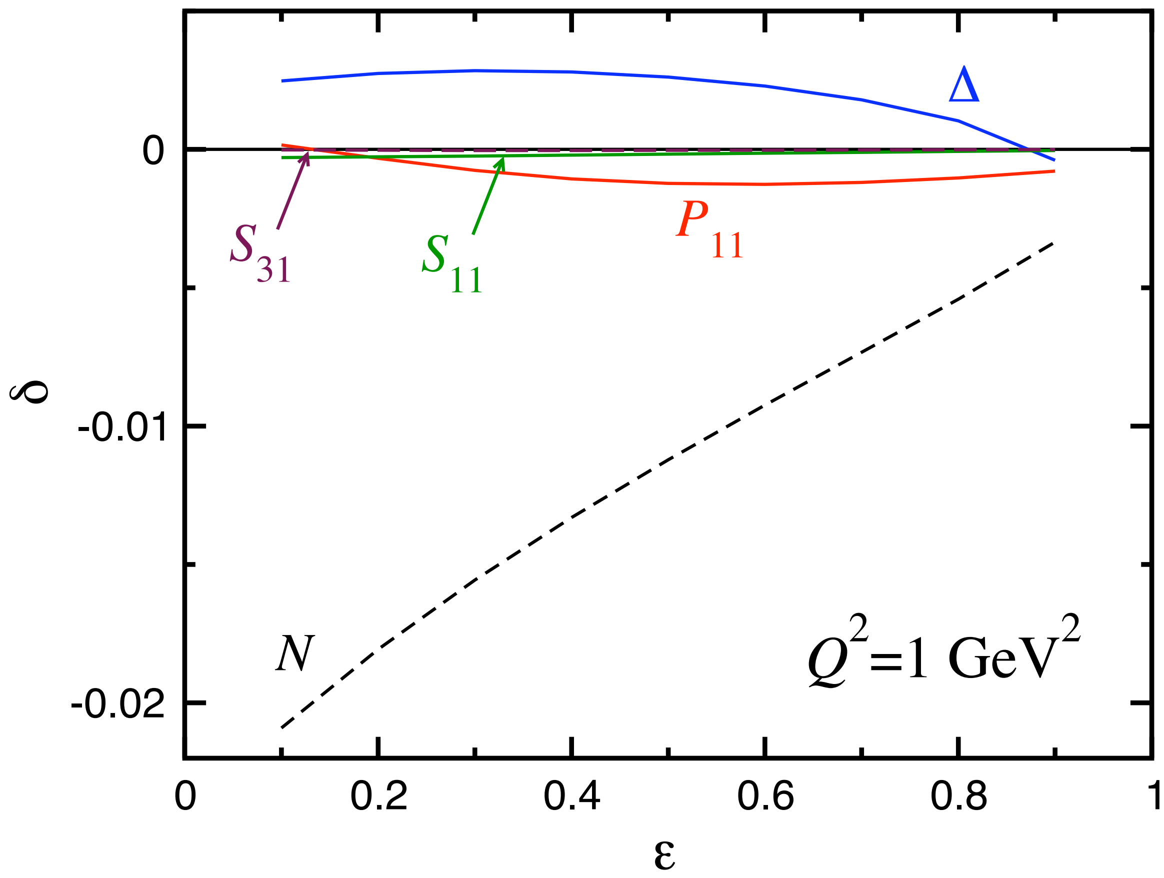
$$\begin{aligned} N_{x-box}^{\Delta}(k) &= \overline{U}(p_4) V_{\Delta in}^{\mu\alpha}(p_2+k, q-k) \left[\not{p}_2 + \not{k} + M_{\Delta}\right] \mathcal{P}_{\alpha\beta}^{3/2}(p_2+k) V_{\Delta out}^{\beta\nu}(p_2+k, k) U(p_2) \\ &\times \; \overline{u}(p_3)\gamma_{\nu} \left[\not{p}_3 + \not{k} + m_e\right] \gamma_{\mu} u(p_1) \end{aligned}$$

spin-3/2 projection operator

$$\mathcal{P}_{\alpha\beta}^{3/2}(p) = g_{\alpha\beta} - \frac{1}{3}\gamma_{\alpha}\gamma_{\beta} - \frac{1}{3p^2}\left(\not{p}\gamma_{\alpha}p_{\beta} + p_{\alpha}\gamma_{\beta}\not{p}\right)$$

- higher-mass intermediate states

→ more model dependent, since couplings & form factors not as well known (especially at high Q^2)



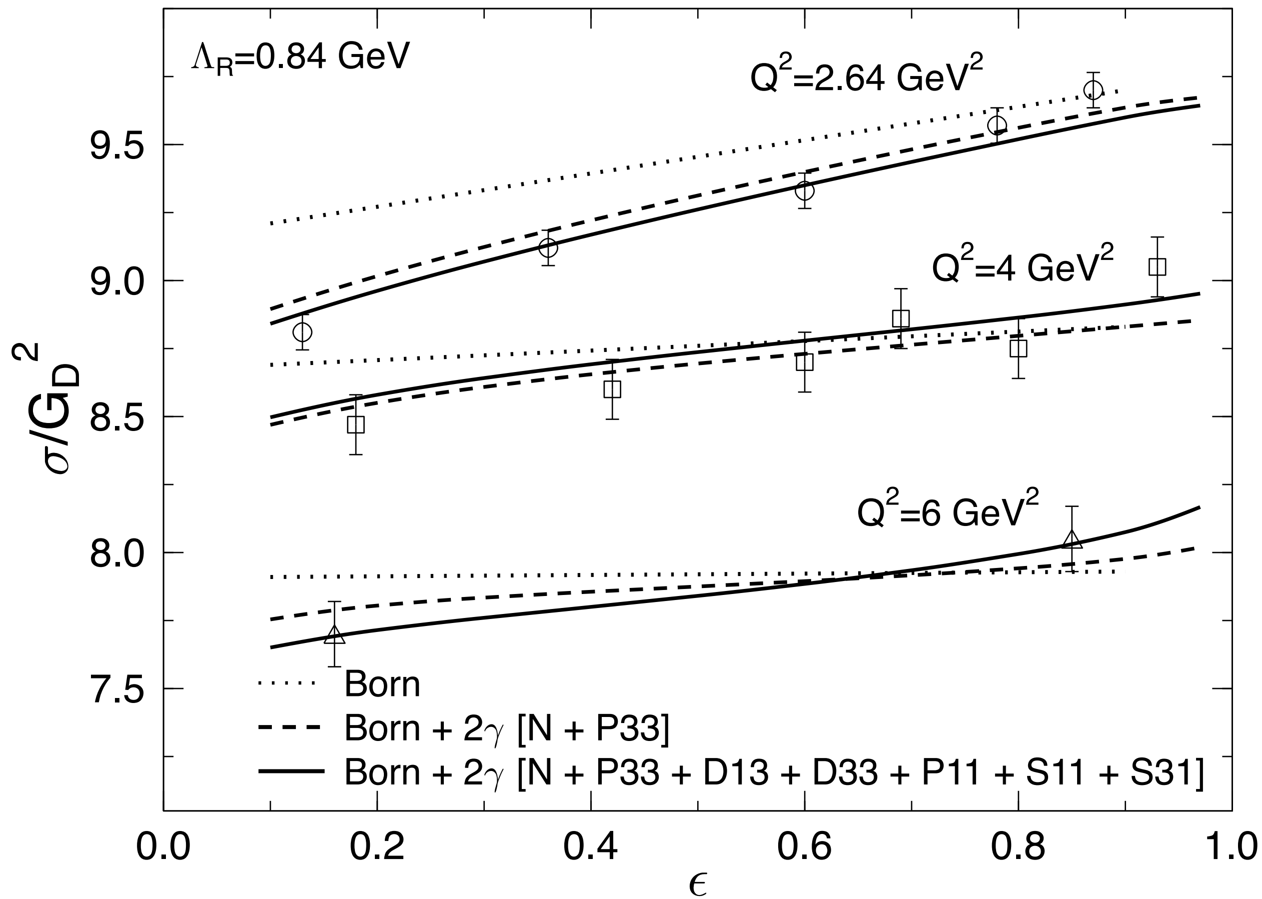
Kondratyuk, Blunden, Melnitchouk, Tjon
PRL ***95*** *(2005) 172503*

Kondratyuk, Blunden
PRC ***75*** *(2007) 038201*

→ dominant contribution from N

→ Δ partially cancels N contribution

higher-mass intermediate states



Kondratyuk, Blunden
*PRC **75** (2007) 038201*

→ higher mass resonance contributions small

→ much better fit to data including TPE

Global analysis

Global analysis

- reanalyze *all* elastic *ep* data (Rosenbluth, PT), including TPE corrections consistently from the beginning

- use explicit calculation of N elastic contribution

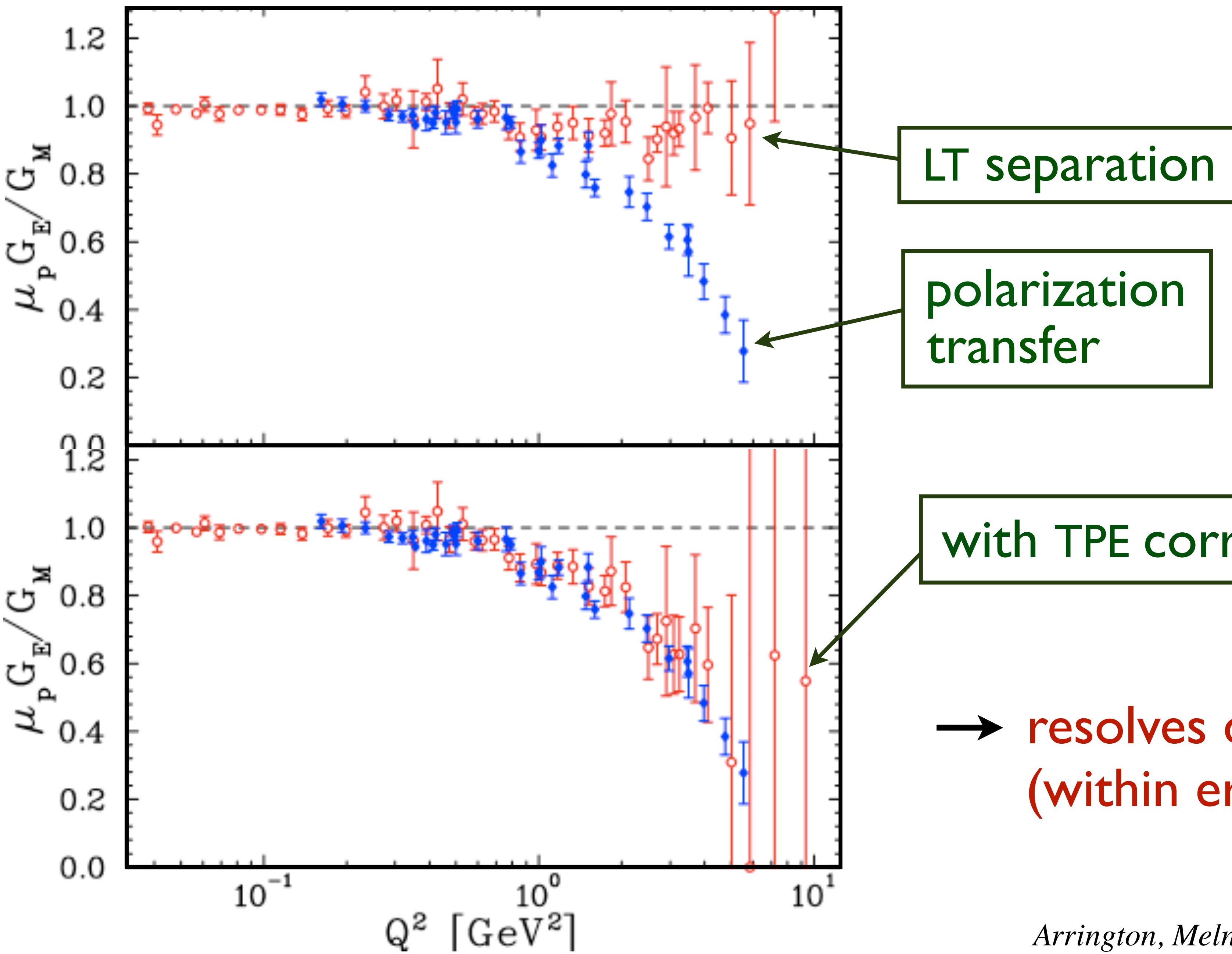
- approximate higher mass contributions by phenomenological form, based on N^* calculations:

$$\delta^{(2\gamma)}_{\text{high mass}} = -0.01\ (1-\varepsilon)\ \log Q^2 / \log 2.2$$

for $Q^2 > 1\ \text{GeV}^2$, with $\pm 100\%$ uncertainty

$\longrightarrow$ decreases $\varepsilon = 0$ cross section by 1% (2%) at $Q^2 = 2.2\ (4.8)\ \text{GeV}^2$

Global analysis



→ resolves discrepancy (within errors)

Arrington, Melnitchouk, Tjon
Phys. Rev. C ***76*** *(2007) 035205*

Non-linearity in ε

- unique feature of TPE correction to cross section
- observation of non-linearity in ε would provide direct evidence of TPE in elastic scattering
- fit reduced cross section as:

$$\sigma_R = P_0 \left[1 + P_1 \left(\varepsilon - \frac{1}{2}\right) + P_2 \left(\varepsilon - \frac{1}{2}\right)^2 \right]$$

- current data give average non-linearity parameter:

$$\langle P_2 \rangle = 4.3 \pm 2.8\%$$

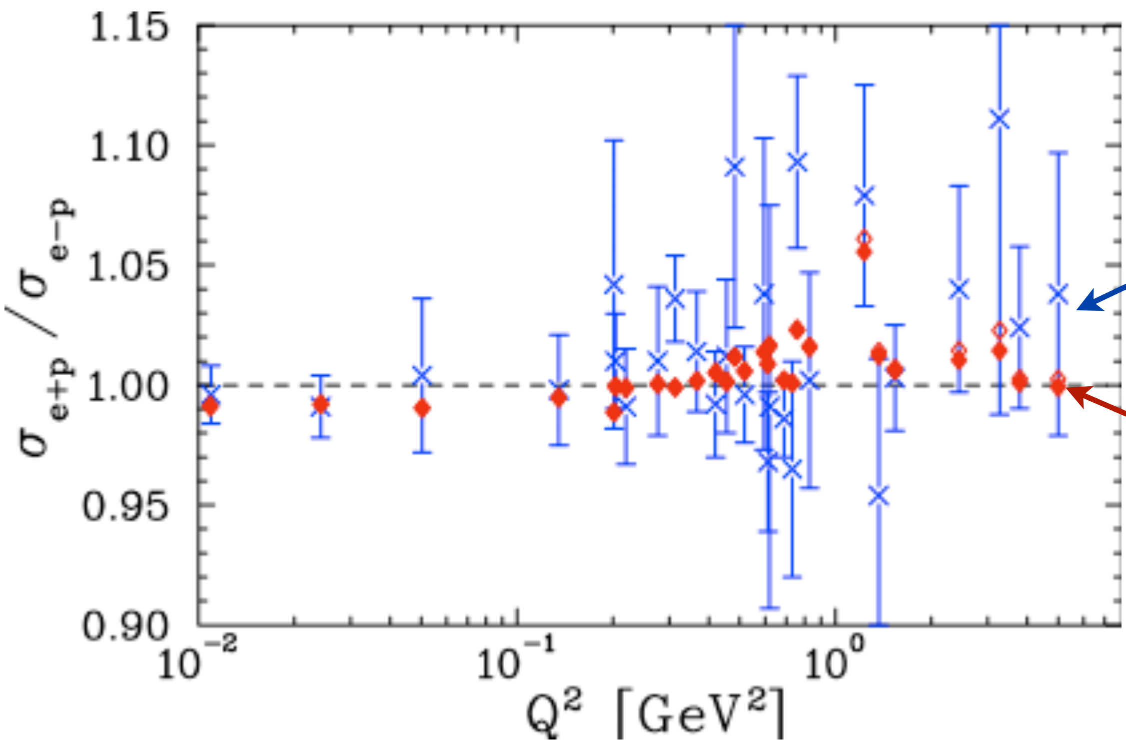
- Hall C experiment E-05-017 will provide accurate measurement of ε dependence

e^+/e^- comparison

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$

  → ratio of $e^+p \, / \, e^-p$ cross sections sensitive to $\Delta(\varepsilon, Q^2)$

$$\sigma_{e^+p}/\sigma_{e^-p} \approx 1 - 2\Delta$$



data at various ε

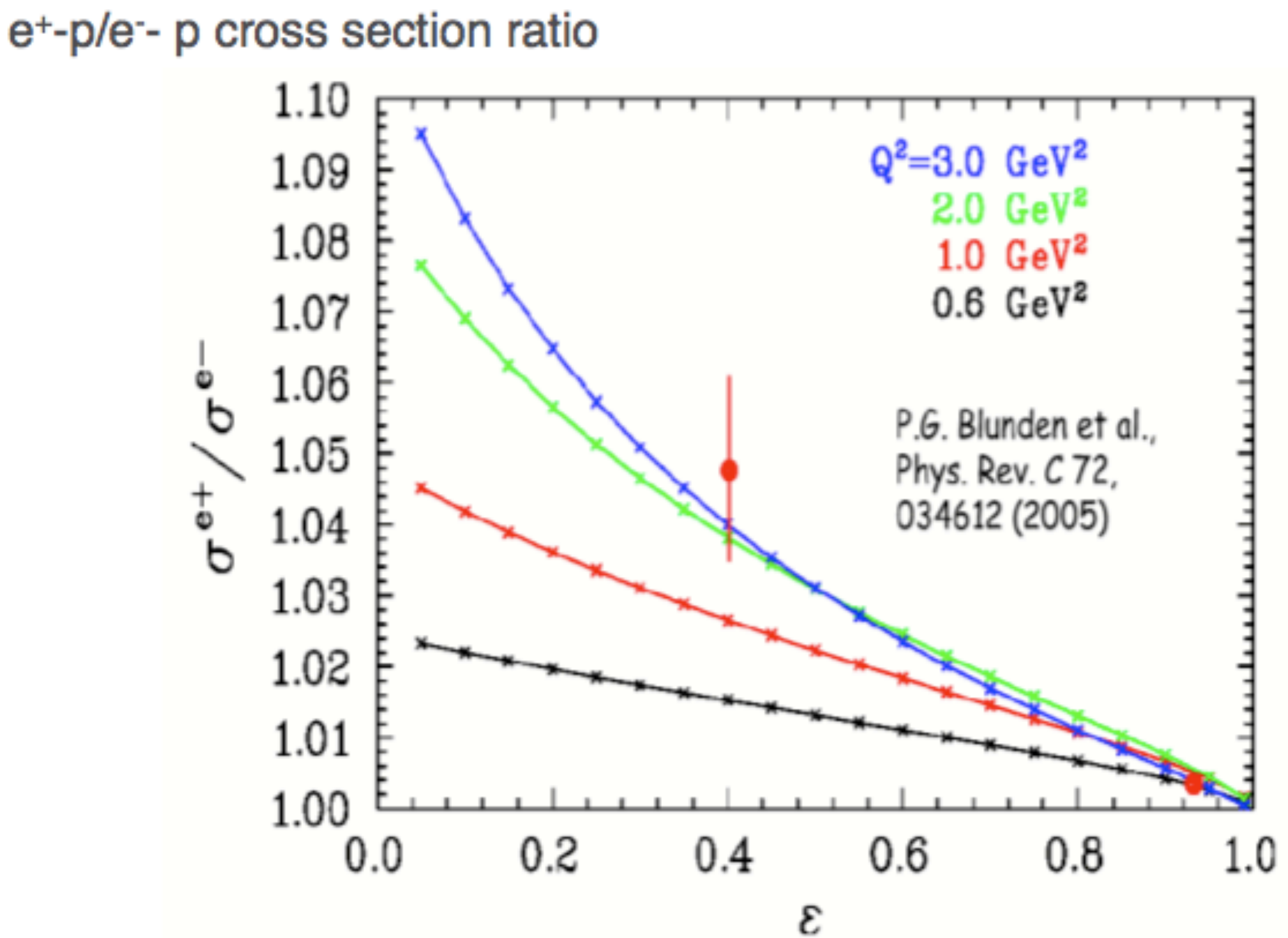
TPE calculation

→ simultaneous e^-p/e^+p measurement using tertiary e^+/e^- beam to $Q^2 \sim$ 1-2 GeV^2 (Hall B experiment E-04-116)

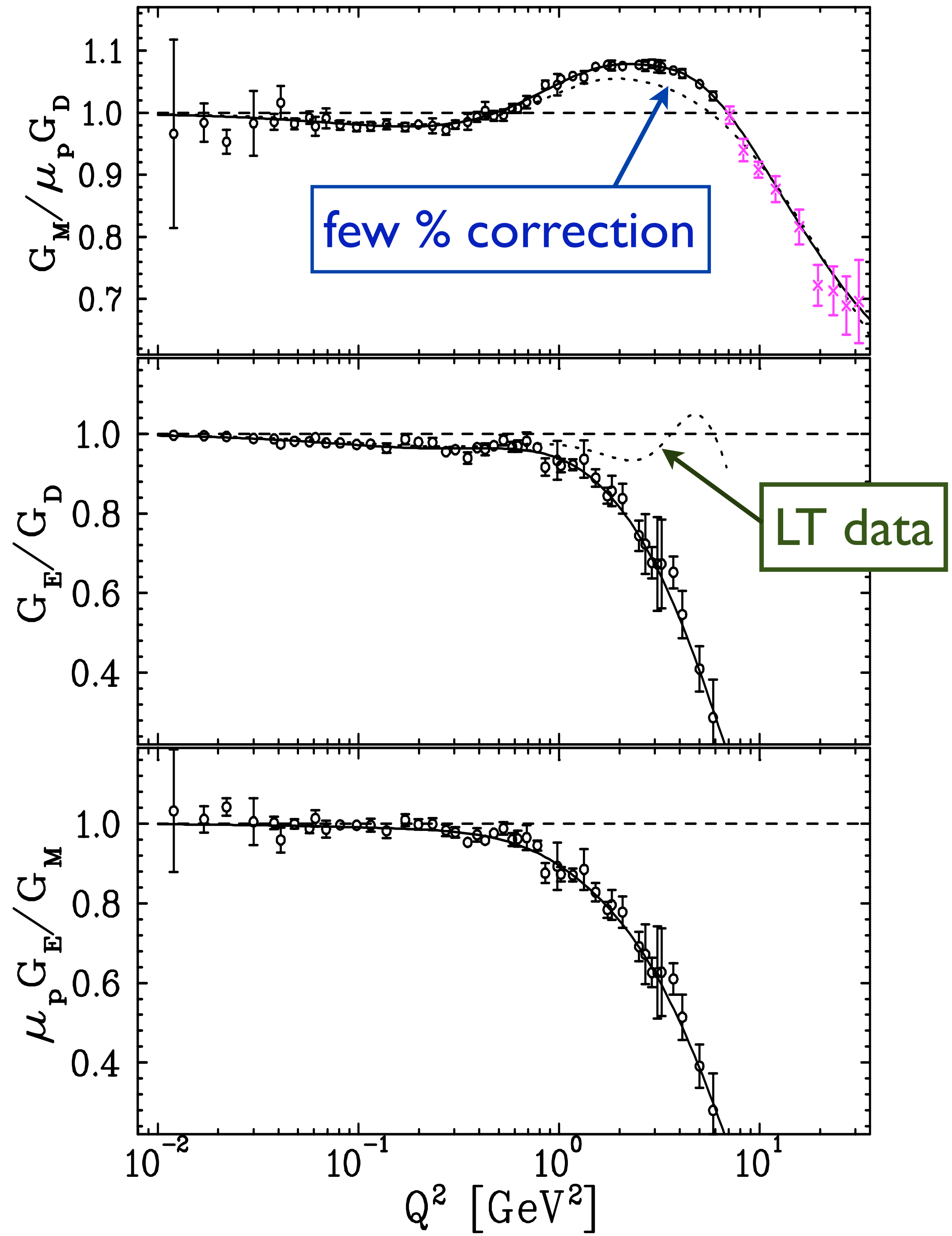
e^+/e^- comparison

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$

Very preliminary Novosibirsk data



Arrington, Holt *et al*. (2010)

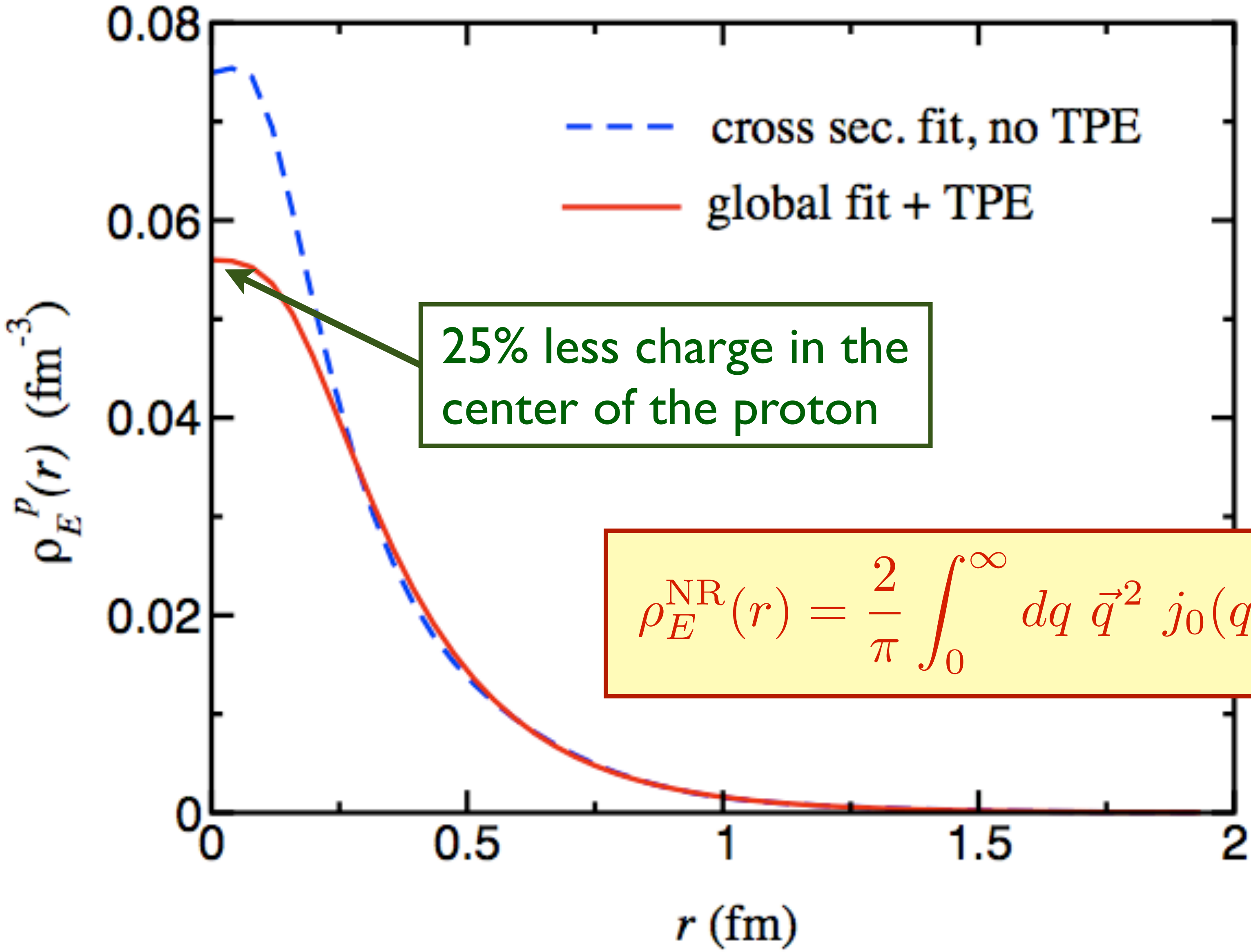


final form factor results from global analysis *including TPE corrections*

$$\left\{G_E,\ \frac{G_M}{\mu_p}\right\} = \frac{1+\sum_{i=1}^{n} a_i \tau^i}{1+\sum_{i=1}^{n+2} b_i \tau^i}$$

Parameter	G_M/μ_p	G_E
a_1	−1.465	3.439
a_2	1.260	−1.602
a_3	0.262	0.068
b_1	9.627	15.055
b_2	0.000	48.061
b_3	0.000	99.304
b_4	11.179	0.012
b_5	13.245	8.650

Charge density



$$\rho_E^{\rm NR}(r) = \frac{2}{\pi}\int_0^\infty dq\ \vec{q}^{\,2}\ j_0(qr)\ G_E(\vec{q}^{\,2})$$

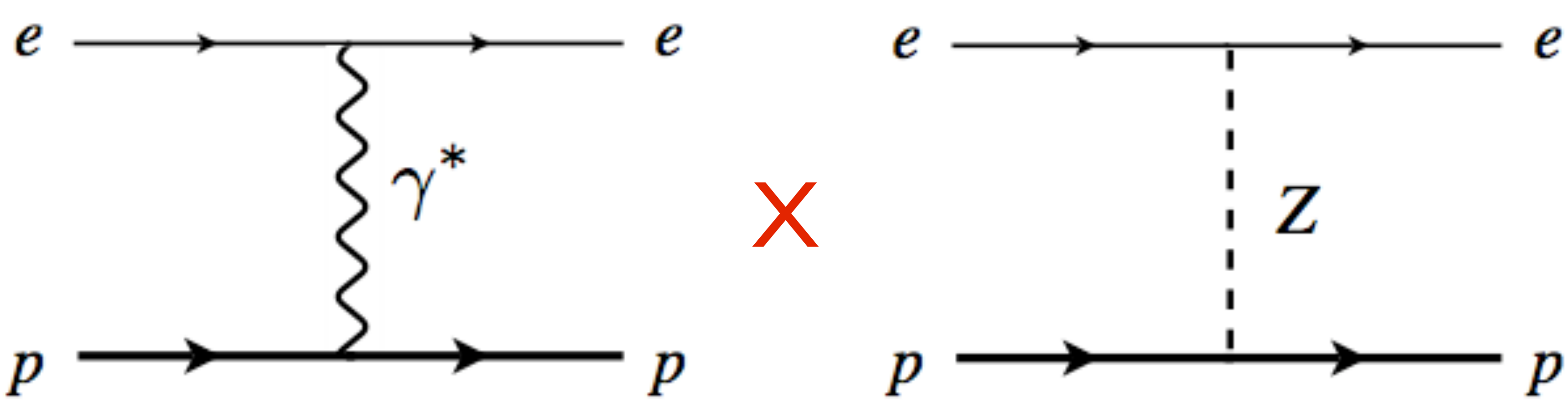
Parity-violating electron scattering

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\,p \to e\,p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents



Born (tree) level

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

$$A_V = g_A^e \rho \left[(1 - 4\kappa \sin^2\theta_W) - (\varepsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n})/\sigma^{\gamma p}\right]$$

radiative corrections, including TBE

→ using relations between weak and e.m. form factors

$$G_{E,M}^{Zp} = (1 - 4\sin^2\theta_W) G_{E,M}^{\gamma p} - G_{E,M}^{\gamma n} - G_{E,M}^{s}$$

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents

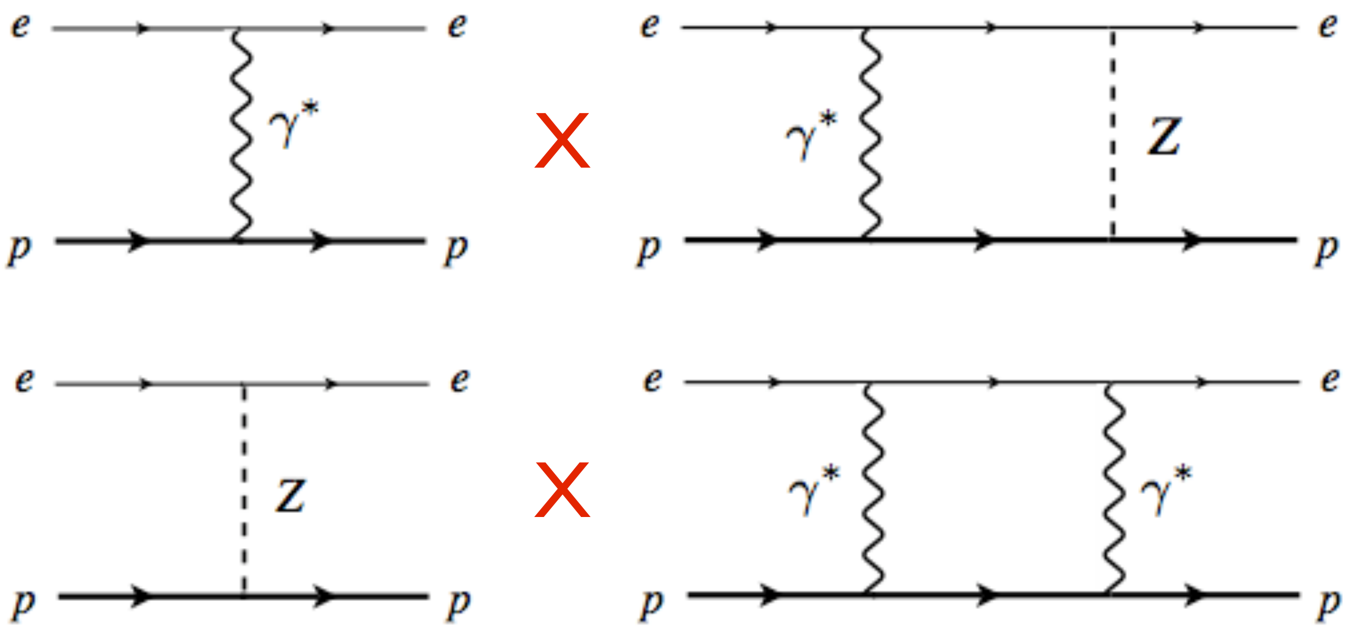
$$A_A = g_V^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)}\; \widetilde{G}_A^{Zp} G_M^{\gamma p} / \sigma^{\gamma p}$$

includes axial RCs + anapole term

$$A_s = -g_A^e \rho \left(\varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s\right) / \sigma^{\gamma p}$$

strange electric &
magnetic form factors

Two-boson exchange corrections



"$\gamma(Z\gamma)$"

"$Z(\gamma\gamma)$"

- current PDG estimates computed at $Q^2 = 0$

Marciano, Sirlin (1980)
Erler, Ramsey-Musolf (2003)

- do not include hadron structure effects

Two-boson exchange corrections

- parameterize corrections to asymmetry as

$$A_{\rm PV} = (1+\delta) A^0_{\rm PV} \equiv \left(\frac{1 + \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)}}{1 + \delta_{\gamma(\gamma\gamma)}} \right) A^0_{\rm PV}$$

Born asymmetry

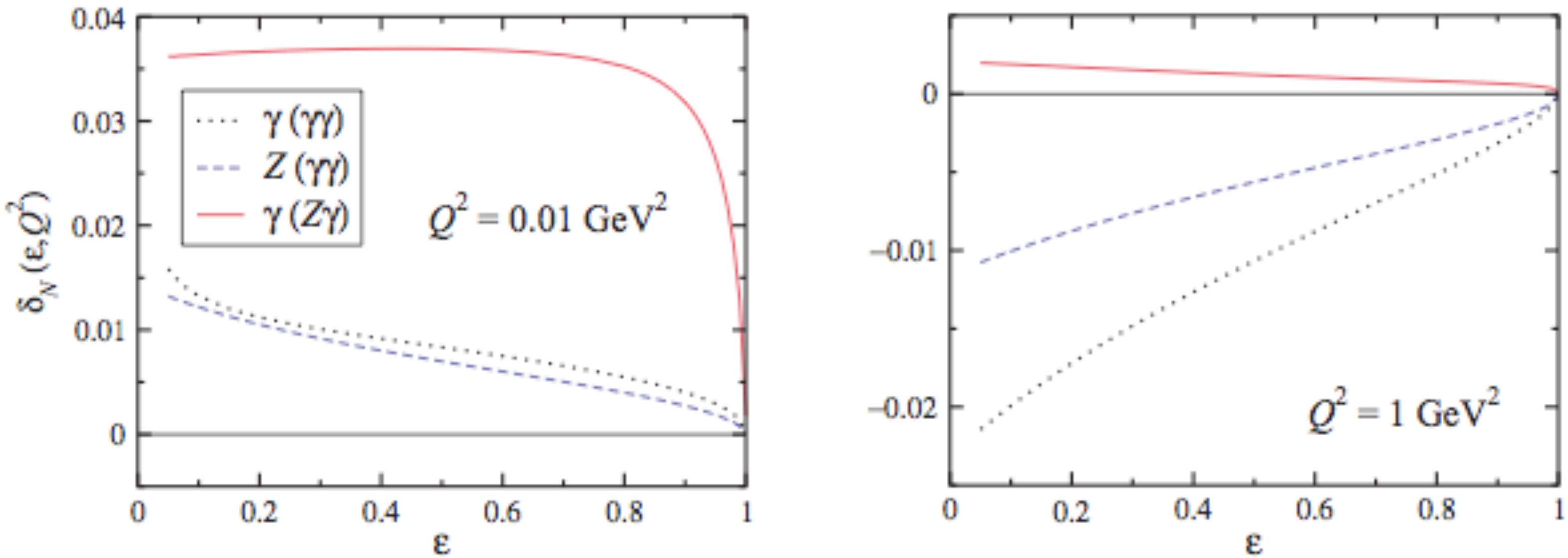
$$\begin{aligned}
\delta_{Z(\gamma\gamma)} &= \frac{2\Re e(\mathcal{M}_Z^* \mathcal{M}_{\gamma\gamma})}{2\Re e(\mathcal{M}_Z^* \mathcal{M}_\gamma)} \\
\delta_{\gamma(Z\gamma)} &= \frac{2\Re e(\mathcal{M}_\gamma^* \mathcal{M}_{\gamma Z} + \mathcal{M}_\gamma^* \mathcal{M}_{Z\gamma})}{2\Re e(\mathcal{M}_Z^* \mathcal{M}_\gamma)} \\
\delta_{\gamma(\gamma\gamma)} &= \frac{2\Re e(\mathcal{M}_\gamma^* \mathcal{M}_{\gamma\gamma})}{|\mathcal{M}_\gamma|^2}
\end{aligned}$$

→ total TBE correction

$$\delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$

Two-boson exchange corrections

- nucleon intermediate states



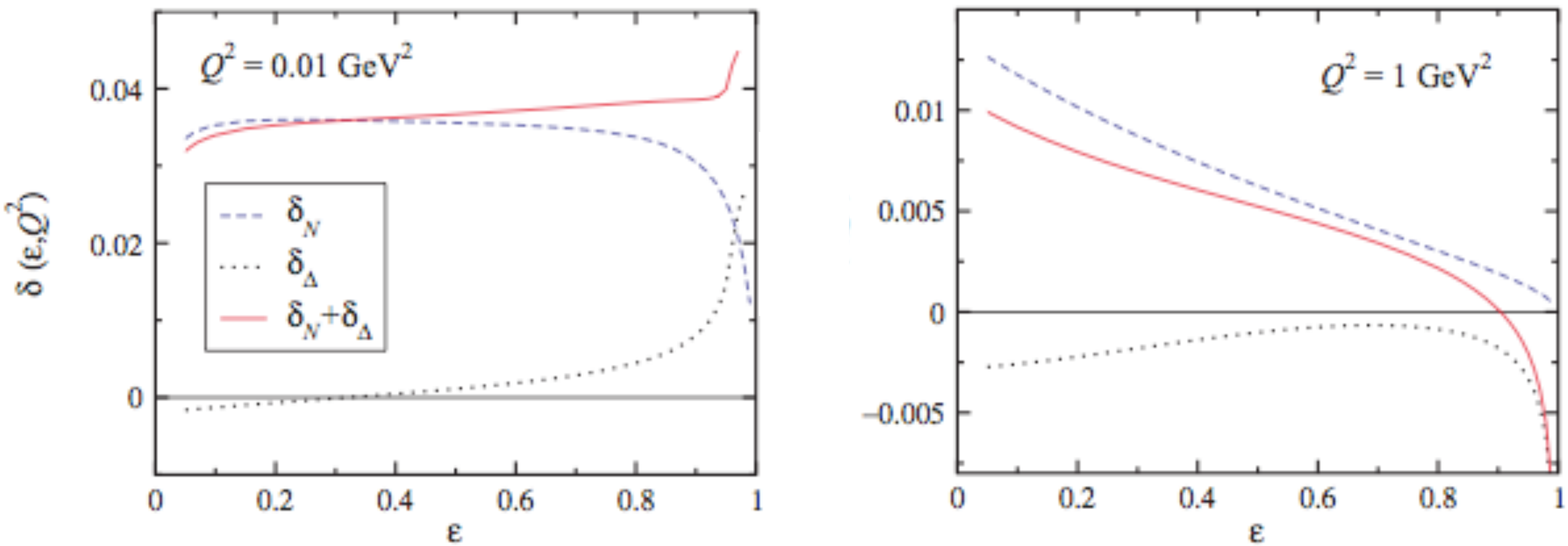
*Tjon, WM, PRL **100**, 082003 (2008)*
*Tjon, Blunden, WM, PRC **79**, 055201 (2009)*

→ cancellation between $Z(\gamma\gamma)$ and $\gamma(\gamma\gamma)$ corrections, especially at low Q^2

→ dominated by $\gamma(Z\gamma)$ contribution

Two-boson exchange corrections

- Δ intermediate states



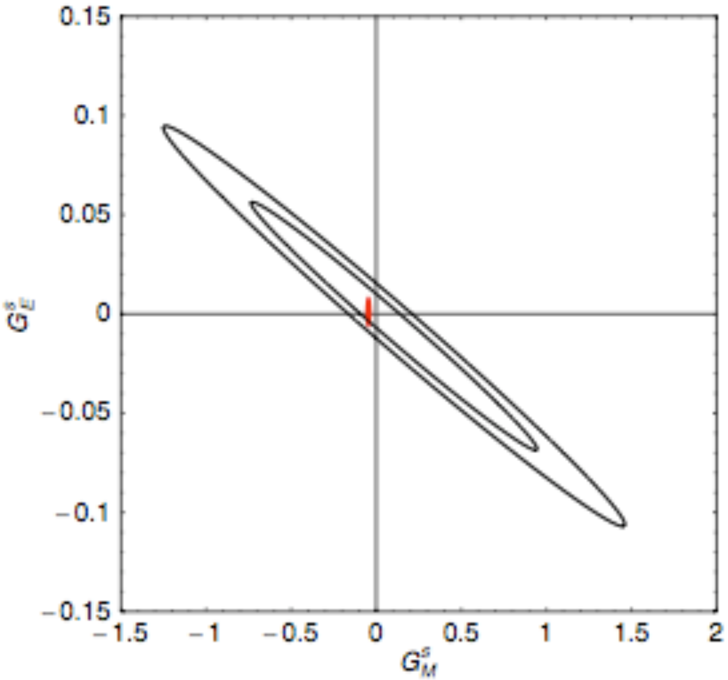
Tjon, WM, PRL ***100****, 082003 (2008)*
Tjon, Blunden, WM, PRC ***79****, 055201 (2009)*

→ Δ contribution small, except at very forward angles
(numerators have higher powers of loop momenta)

→ Δ calculation less reliable for $\varepsilon \to 1$
(grows faster with s than nucleon)

Effects on strange form factors

- global analysis of all PVES data at $Q^2 < 0.3\ \mathrm{GeV}^2$



$$G_E^s = 0.0025 \pm 0.0182$$

$$G_M^s = -0.011 \pm 0.254$$

at $Q^2 = 0.1\ \mathrm{GeV}^2$

Young et al., PRL ***97****, 102002 (2006)*

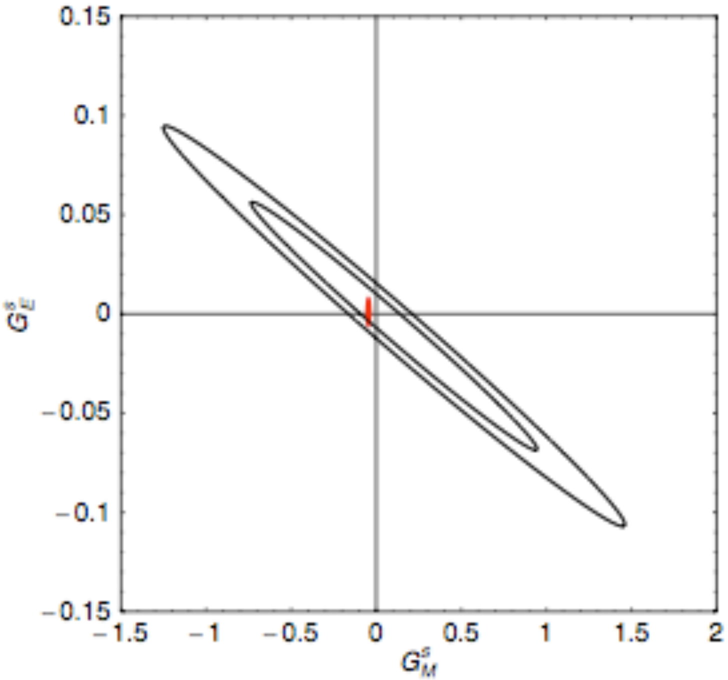
- including TBE corrections:

$$\boxed{\begin{aligned} G_E^s &= 0.0023 \pm 0.0182 \\ G_M^s &= -0.020 \pm 0.254 \end{aligned}}$$

at $Q^2 = 0.1\ \mathrm{GeV}^2$

Effects on strange form factors

- global analysis of all PVES data at $Q^2 < 0.3\ \mathrm{GeV}^2$



$$G_E^s = 0.0025 \pm 0.0182$$

$$G_M^s = -0.011 \pm 0.254$$

at $Q^2 = 0.1\ \mathrm{GeV}^2$

Young et al., PRL ***97****, 102002 (2006)*

- including TBE corrections:

$$\boxed{\begin{aligned} G_E^s &= 0.0023 \pm 0.0182 \\ G_M^s &= -0.020 \pm 0.254 \end{aligned}}$$

at $Q^2 = 0.1\ \mathrm{GeV}^2$

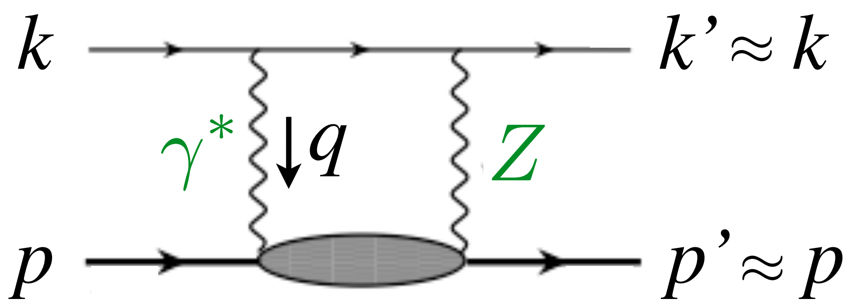
fixed mainly by ^{4}He data ...

... TBE for ^{4}He not yet included

Correction to proton weak charge

- in forward limit $A_{\rm PV}$ measures weak charge of proton Q_W^p

$$A_{\rm PV} \;\rightarrow\; \frac{G_F\, Q_W^p}{4\sqrt{2}\pi\alpha}\, t$$



forward limit

$$t = (k - k')^2 \rightarrow 0$$

$$s = (k + p)^2$$
$$= M(M + 2E)$$

- at tree level Q_W^p gives weak mixing angle

$$Q_W^p = 1 - 4\sin^2\theta_W$$

Correction to proton weak charge

- including higher order radiative corrections

$$Q_W^p \quad = \quad (1 + \Delta\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta_e')$$

"standard" electroweak vertex & other corrections

Correction to proton weak charge

- including higher order radiative corrections

$$Q_W^p = (1 + \Delta\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta'_e) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

box diagrams

→ *WW* and *ZZ* box diagrams dominated by short distances, evaluated perturbatively

→ γZ box diagram sensitive to long distance physics, has two contributions $\Box_{\gamma Z} = \Box^A_{\gamma Z} + \Box^V_{\gamma Z}$

vector e – axial h axial e – vector h

Correction to proton weak charge

- axial h correction $\Box^A_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low energies

$\longrightarrow$ computed by Marciano & Sirlin as sum of two parts:

- low-energy part approximated by *Born* contribution (elastic intermediate state)
- high-energy part (above scale $\Lambda \sim 1\ \mathrm{GeV}$) computed in terms of scattering from *free quarks*

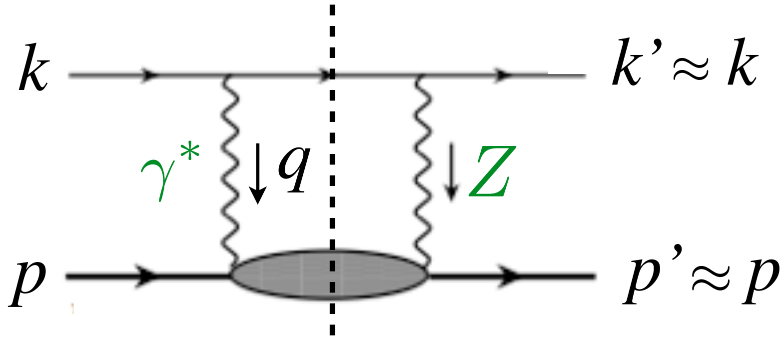
$$\Box^A_{\gamma Z} = \frac{5\alpha}{2\pi}(1 - 4\sin^2\theta_W)\left[\underbrace{\ln\frac{M_Z^2}{\Lambda^2}}_{\text{high-energy}} + \underbrace{C_{\gamma Z}(\Lambda)}_{\text{low-energy}}\right]$$

Marciano, Sirlin, PRD ***29****, 75 (1984)*
Erler et al., PRD ***68****, 016006 (2003)*

Correction to proton weak charge

- vector h correction $\Box^{V}_{\gamma Z}$ negligible for $E \sim m_e$, but what about at $\mathcal{O}(1\ \mathrm{GeV})$?

$\longrightarrow$ computed in forward limit using dispersion relations



- $\Re e\, \Box^{V}_{\gamma Z} = \dfrac{2E}{\pi} \displaystyle\int_0^{\infty} dE' \, \dfrac{\Im m\, \Box^{V}_{\gamma Z}(E')}{E'^2 - E^2}$

- integration over $E' < 0$ corresponds to crossed-box, vector h contribution symmetric under $E' \leftrightarrow -E'$

- vanishes as $E \rightarrow 0$

*Gorchtein, Horowitz, PRL **102**, 091806 (2009)*

Correction to proton weak charge

→ imaginary part given by γZ interference structure functions

$$\Im m\, \Box^{V}_{\gamma Z}(E) = \frac{\alpha}{(s-M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1+Q^2/M_Z^2} \times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\max}-Q^2)}{Q^2(W^2-M^2+Q^2)} \right)$$

- little direct data on interference structure functions
  (neutral currents at HERA at very small x)

- in parton model $F_2^{\gamma Z} = 2x \sum_q e_q\, g_V^q\, (q+\bar{q}) = 2x F_1^{\gamma Z}$

  → $F_2^{\gamma Z} \approx F_2^{\gamma}$ good approximation at *low* x

  → provides upper limit at *large* x $(F_2^{\gamma Z} \lesssim F_2^{\gamma})$

Correction to proton weak charge

- in resonance region use phenomenological input for F_2, empirical SLAC fit for $R = \sigma_L/\sigma_T = (1+4M^2x^2/Q^2)F_2/(2xF_1) - 1$

→ for transitions to $I = 3/2$ states ($e.g.$ Δ), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q_W^p)\, F_i^\gamma$

→ for transitions to $I = 1/2$ states, SU(6) wave functions predict Z & γ transition couplings equal to few percent

→ include contributions from four prominent resonances:

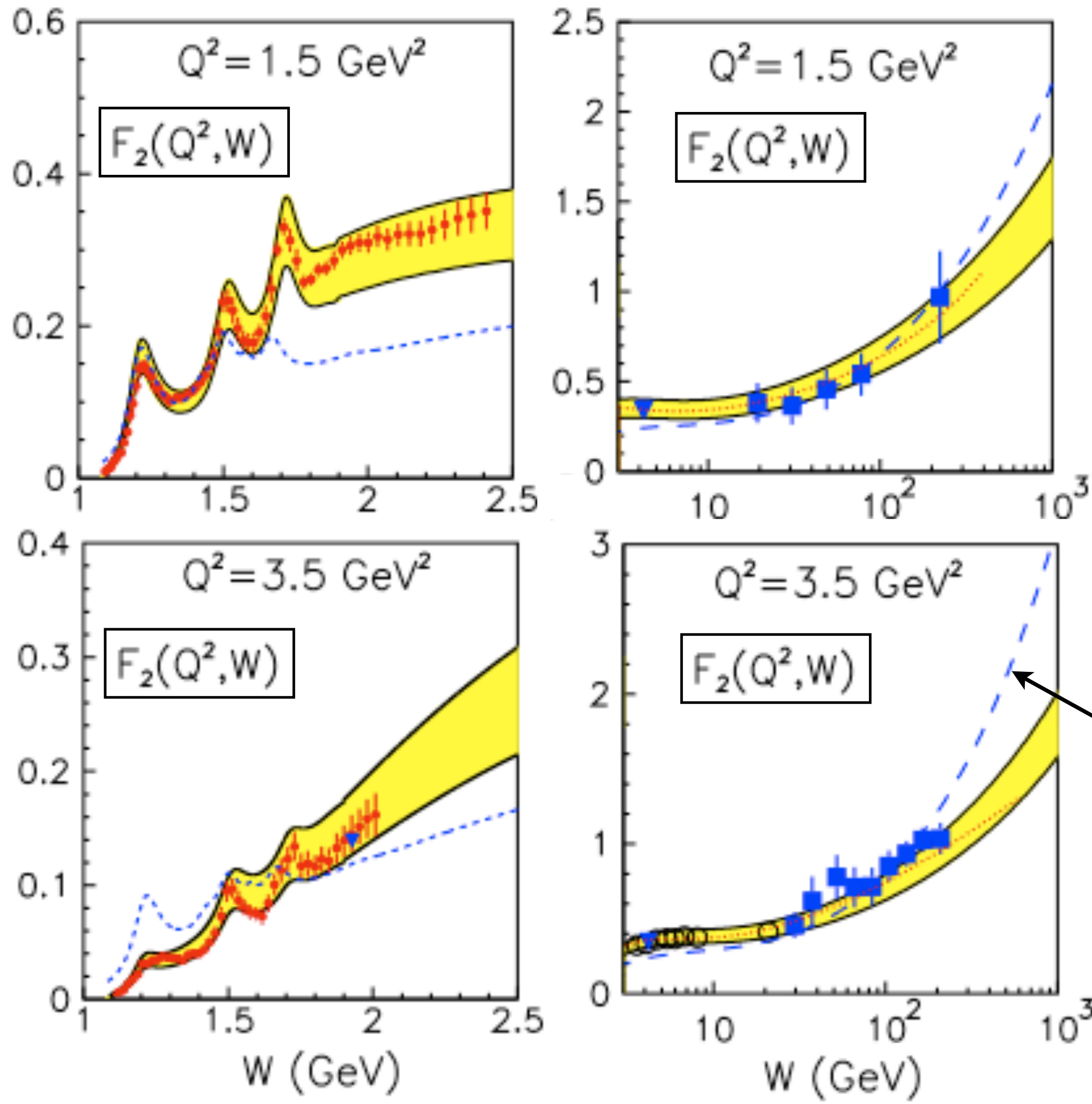
$P_{33}(1232),\ D_{13}(1520),\ F_{15}(1680),\ F_{37}(1950)$

Correction to proton weak charge

- in resonance region use phenomenological input for F_2, empirical SLAC fit for $R = \sigma_L/\sigma_T = (1 + 4M^2x^2/Q^2)F_2/(2xF_1) - 1$

low W

high W



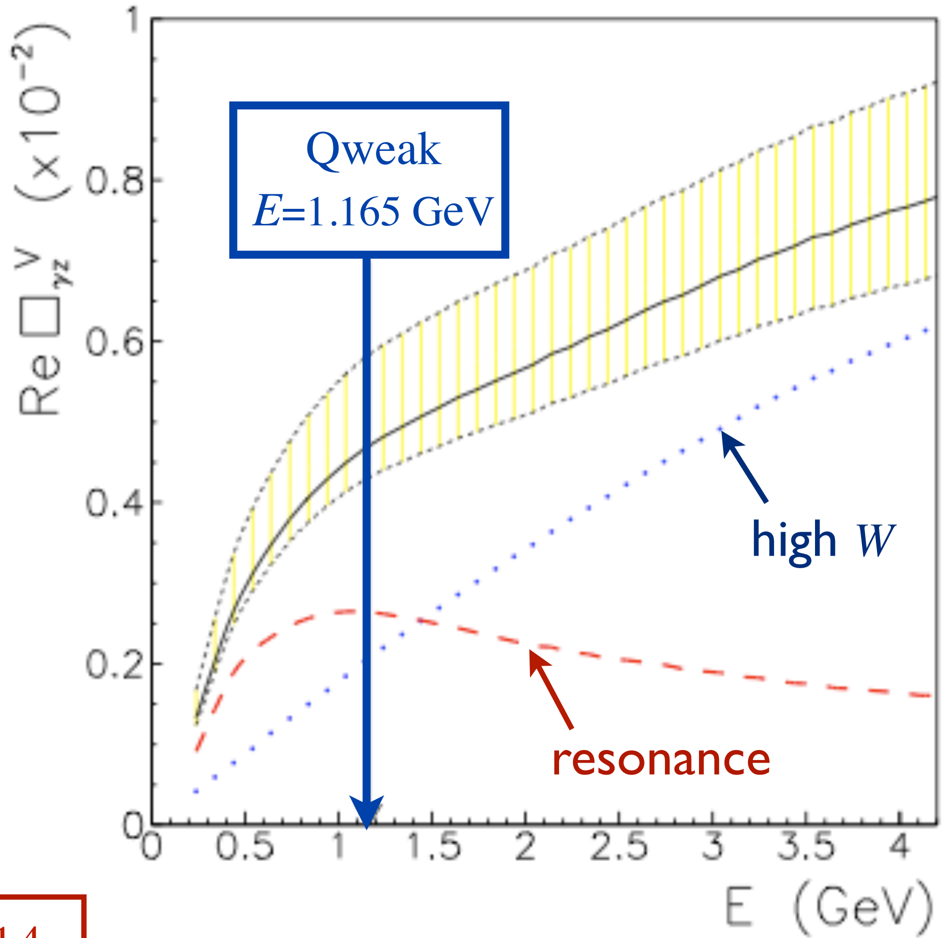
GVMD model
(used as input by
Gorchtein & Horowitz)

Correction to proton weak charge

- final $\Box^V_{\gamma Z}$ correction:

$$\Re e\, \Box^V_{\gamma Z} = 0.0047^{+0.0011}_{-0.0004}$$

or $6.6^{+1.5}_{-0.6}\,\%$ of uncorrected Q^p_W



$$\boxed{Q^p_W = 0.0713(8) \rightarrow 0.0760^{+0.0014}_{-0.0009}}$$

→ significant shift in central value, errors within projected experimental uncertainty $\Delta Q^p_W = \pm 0.003$

Summary

- Two-photon exchange corrections resolve most of Rosenbluth / polarization transfer G_E^p/G_M^p discrepancy

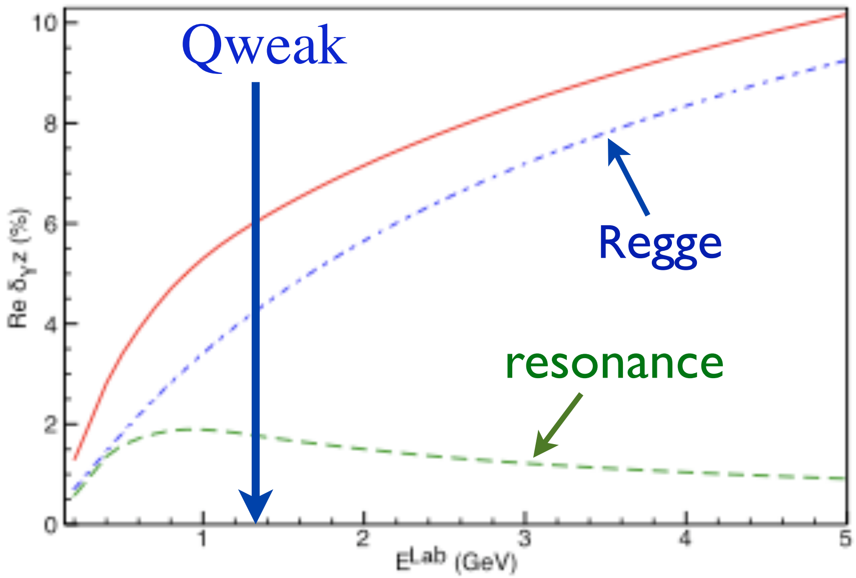
- Reanalysis of global data, including TPE from the outset
  $\longrightarrow$ first consistent form factor fit at order α^3

- $\gamma(Z\gamma)$ and $Z(\gamma\gamma)$ contributions give (hitherto unaccounted) $\sim 2\%$ corrections to PVES at small Q^2
  $\longrightarrow$ affect on extraction of strange form factors small

- More dramatic effect of $\gamma(Z\gamma)$ corrections at forward angles on proton weak charge, $\Delta Q_W^p \sim 6\text{–}7\%$
  $\longrightarrow$ can be better constrained by direct measurement of $F_{1,2}^{\gamma Z}$ $(e.g.$ in PVDIS at JLab)

The End

Gorchtein, Horowitz, PRL ***102****, 091806 (2009)*



$$\Re e\, \delta_{\gamma Z} = \Re e\, \Box^{V}_{\gamma Z} / Q^{p}_{W} \approx 6\%$$

mostly from high-W ("Regge") contribution

→ our formula for $\Im m\, \Box^{V}_{\gamma Z}$ factor 2 larger
(incorrect definition of parton model structure functions: "nuclear physics" *vs.* "particle physics" weak charges!)

→ GH omit factor $(1-x)$ in definition of $F_{1,2}$
(spurious ~30% enhancement)

→ GH use $Q^{p}_{W} \sim 0.05$ *cf.* ~ 0.07
(spurious ~40% enhancement)

→ numerical agreement purely coincidental!