33-228 Electronics I Spring 2009

Carnegie Mellon University **Department of Physics**

Homework 2 SOLUTIONS

Problems 1.20, 1.21, 2.1, 2.3, 2.5, 2.8, and 2.12.

1.20 FOr a voltage divider, the output voltage is given as $V_{out} = V_{in}\frac{R_2}{R_1+R_2}$. For $12\,V$ in and $4\,V$ out, we find that

$$\begin{aligned} \frac{1}{3} &= \frac{R_2}{R_1+R_2} \\ R_1 &= 2R_2 \end{aligned}$$

For a voltage divider, $R_{th} = R_1 \mid\mid R_2$, or

$$\begin{aligned} R_{th} &= \frac{R_1R_2}{R_1+R_2} \\ R_{th} &= \frac{2R_2R_2}{2R_2+R_2} \\ R_{th} &= \frac{2R_2^2}{3R_2} \\ R_{th} &= \frac{2}{3}R_2 \end{aligned}$$

For the $8\,k\Omega$ resistor not to load down the divider, we must have that $8\,k\Omega$ is much larger than $\frac{2}{3}R_2$. So we could choose R_2 to be $100\,\Omega$, which then gives that $R_1 = 200\,\Omega$.

1.21 We now have hard number for how much the voltage can sag. We also want to minimize the power dissipated in the divider. The power is simply $P = V^2/R$, which is minimized by making R_1 and R_2 as large as possible. The Thevènin equivalent of the voltage divider has $V_{th} = 4\,V$ and $R_{th} = \frac{2}{3}R_2$. This now builds a voltage divider with the $8\,k\Omega$ resistor whose output voltage is $3.8\,V$. This gives that

$$\frac{3.8}{4.0} = \frac{8\,k\Omega}{\frac{2}{3}R_2 + 8\,k\Omega}$$

We can solve this for R_2 to yield $R_2 = 632\,\Omega$ and thus $R_1 = 1263\,\Omega$.

2.1 The RMS voltage is given as

$$V_{RMS} = \sqrt{\left[\frac{1}{T}\int_0^T v^2(t)dt\right]}.$$

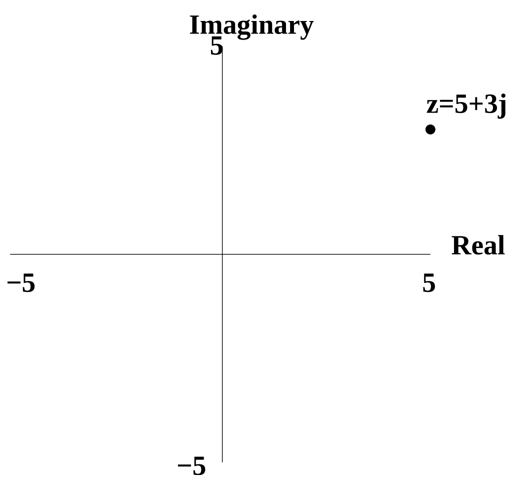
For the voltage form that we have, $v^2 = v_0^2$ except for the point where $t = \frac{\pi}{2}$. So we have

$$\begin{aligned} V_{RMS} &= \sqrt{\left[\frac{1}{\pi}\int_0^{\pi} v_0^2 dt\right]} \\ V_{RMS} &= \frac{1}{\sqrt{\pi}} v_0 \sqrt{t \mid_0^{\pi}} \\ V_{RMS} &= v_0 \,. \end{aligned}$$

2.3 If $z = 5 + 3j$, then $z^* = 5 - 3j$. $z^2 = z * z$ and is given as

$$\begin{aligned} z^2 &= z * z \\ z^2 &= (x + jy) * (x + jy) \\ z^2 &= (x^2 - y^2) + 2jxy \end{aligned}$$

putting in numerical values, we get that $z^2 = 16 + 30j$. The quantity $zz^* = x^2 + y^2$, or with numerical values, 34. The quantity $z + z^*$ is $2x$, or 10 and the quantity $z - z^*$ is $2jy$ or $6j$.



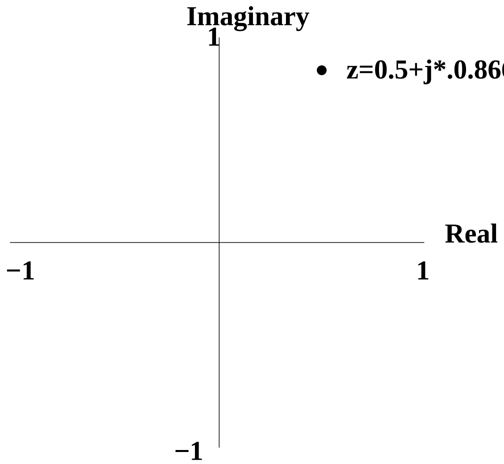
2.5 The number $e^{j\pi/3}$ can be expressed as

$$\begin{aligned} e^{j\pi/3} &= \cos(\pi/3) + j\sin(\pi/3) \\ e^{j\pi/3} &= \frac{1}{2} + j\frac{\sqrt{3}}{2} \end{aligned}$$

The real part is $\frac{1}{2}$ and the imaginary part is $\frac{\sqrt{3}}{2}$.

2.8 For $z = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$, which we can write as $z = \frac{1}{\sqrt{2}}(1 - j)$, we can compute (**a**) z^2 as

$$z^2 = \frac{1}{2}(1 - j)(1 - j)$$



$$z^2 = \frac{1}{2}[(1-1) - 2j]$$
$$z^2 = -j$$

(**b**) For z^3, we have $z * z^2$, which is

$$z^3 = z * z^2$$
$$z^3 = \frac{1}{\sqrt{2}}(1-j) * -j$$
$$z^3 = -\frac{1}{\sqrt{2}}(1+j)$$

(**c**)For z^4, we have:

$$z^4 = z^2 * z^2$$
$$z^4 = -j * -j$$
$$z^4 = -1$$

(**d**) To do these in polar form, we note that the magnitude of z is $\mid z \mid= 1$ anf the phase of z is $\tan\phi = -1/1$, which gives that $\phi = -\frac{\pi}{4}$, so $z = e^{-j\pi/4}$. This gives:

$$z = e^{-j\pi/4}$$
$$z^2 = e^{-j\pi/2} = -j$$
$$z^3 = e^{-j3\pi/2}$$
$$z^4 = e^{-j4\pi/4} = e^{-j\pi} = -1$$

2.12 We are given $Z = 1000(1+j)\Omega$ and a $60\,Hz$ voltage source of amplitude $10\,V$. For $f = 60$, we have $\omega = 2\pi f = 120\pi$. If the voltage is maxiumim at $t = 0$, then we have

$$v(t) = V_0 \cos\omega t$$
$$v(t) = V_0 e^{j\omega t}$$

We can express the impedance in polar form as

$$Z = \sqrt{2000}e^{j\pi/4}\Omega$$

(**a**) The current, $i(t)$ is given as $i = v/Z$, or

$$\begin{aligned} i(t) &= \frac{v(t)}{Z} \\ i(t) &= \frac{V_0 e^{j\omega t}}{\sqrt{2000}e^{j\pi/4}} \\ i(t) &= \frac{10V}{\sqrt{2000}\Omega} e^{j(\omega t - \pi/4)} \\ i(t) &= 0.224\, A e^{j(\omega t - \pi/4)} \end{aligned}$$

(**b**) To get the power dissipated, we need the real part of the voltage and the real part of the current. The voltage is just the cos as given above. For the current, we have

$$i(t) = 0.224\, A\, [\cos(\omega t - \pi/4) + j \sin(\omega t - \pi/4)]$$

or the real part of i is just the cos part. Also, we need that $\cos(a+b) = \cos a \cos b - \sin a \sin b$, so we have

$$Re(i(t)) = 0.224\, A\, [\cos(\pi/4)\cos(\omega t) - \sin(\pi/4)\sin(\omega t)]$$

Power is now the average over one cycle of $Re(v)$ time $Re(i)$. The average of $\cos^2$ over a cycle is $\frac{1}{2}$, while the average of cos times sin over a cycle is 0. This gives

$$\begin{aligned} < P > &= (10\, V)(0.224\, A)\frac{1}{sqrt2}\frac{1}{2} \\ < P > &= 0.792\, W \end{aligned}$$