meson spectra from lattice QCD

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& Jefferson Lab

this talk

not a review !

... rather the ongoing story of the hadron spectrum collaboration's travails on the road to the excited hadron spectrum ...

"Therefore, my dear friend and companion,... if I should sometimes put on a fool's cap with a bell to it, for a moment or two as we pass along, don't fly off, but rather courteously give me credit for a little more wisdom than appears upon my outside; and as we jog on, either laugh with me, or at me, or in short do any thing, only keep your temper."

Laurence Sterne: The Life and Opinions of Tristram Shandy, Gentleman

hadron spectrum collaboration

Dudek, Edwards, Joo, L. Liu, Mathur, Moir, Peardon, Richards, Ryan, C. Thomas, Vilaseca, Wallace

Jefferson Lab, Trinity College, Dublin, Old Dominion University, Tata, Mumbai, University of Maryland

meson spectrum

- "Excited and exotic charmonium spectroscopy from lattice QCD" - JHEP 07 (2012) 126
- "The lightest hybrid meson supermultiplet in QCD" - PRD.84.074023 (2011)
- "Isoscalar meson spectroscopy from lattice QCD" - PRD.83.071504 (2011)
- "Toward the excited meson spectrum of dynamical QCD" - PRD.82.034508 (2010)
- "Highly excited and exotic meson spectrum from dynamical lattice QCD" - PRL.103.262001 (2009)

baryon spectrum

- "Hybrid Baryons in QCD" - PRD.85.054016 (2012)
- "Excited state baryon spectroscopy from lattice QCD" - PRD.84.074508 (2011)

hadron scattering

- "S and D-wave phase shifts in isospin-2 ππ scattering from lattice QCD" - arXiv:1203.6041 (PRD in press)
- "The phase-shift of isospin-2 ππ scattering from lattice QCD" - PRD.83.071504 (2011)

lattice tech.

- "Helicity operators for mesons in flight on the lattice" - PRD.85.014507 (2012)
- "A novel quark-field creation operator construction for hadronic physics in lattice QCD" - PRD.80.054506 (2009)

hadron spectrum collaboration

'our' lattices (generated to make spectroscopy as simple as possible)

Clover improved Wilson quarks $\left[\mathcal{O}(a^2) \text{ discretisation errors ?} \right]$

anisotropy (finer in time)

$$a_s \sim 0.12\,\mathrm{fm}$$

$$a_t \sim 0.035\,\mathrm{fm} \sim \frac{1}{5.8\,\mathrm{GeV}}$$

two light dynamical flavours, plus dynamical strange quarks

$$m_\pi \sim \mathit{230},\ 400,\ 450,\ 525,\ 700\ \mathrm{MeV}$$

230 MeV: $24^3 \times 128$, $32^3 \times 256$, $40^3 \times 256$

400 MeV: $16^3 \times 128$, $20^3 \times 128$, $24^3 \times 128$, $32^3 \times 256$

450, 525, 700 MeV: $16^3 \times 128$, $20^3 \times 128$

& ≈ physical pion mass
in preparation ...
($48^3 \times 512$?)

excited meson states

in a finite-volume, expect a discrete spectrum of states

extract from meson two-point correlators

$$C_{ij}(t) = \left\langle 0 \middle| \mathcal{O}_i(t)\mathcal{O}_j(0) \middle| 0 \right\rangle$$

$\mathcal{O}_i$ combination of quark and gluon fields with meson quantum number

$$C_{ij}(t) = \sum_{\mathfrak{n}} Z_i^{(\mathfrak{n})} Z_j^{(\mathfrak{n})} e^{-E_{\mathfrak{n}} t}$$

finite-volume eigenstates of H_{QCD}

first practical question:

“can we extract an excited-state spectrum ?”

➟ which correlators should we compute ?
➟ how to extract the spectrum ?

excited meson states - spectrum extraction

variational analysis of a matrix of correlators

'optimal' operator for state $\mathfrak{n}$

$$\Omega_{\mathfrak{n}} = \sum_i v_i^{(\mathfrak{n})} \mathcal{O}_i$$

linear combination of basis ops

variational solution (c.f. Rayleigh-Ritz)

$$C(t) v^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t)\, C(t_0) v^{(\mathfrak{n})}$$

eigenvalues (principal correlators)

$$\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$$

takes advantage of an enforced orthogonality of eigenvectors to distinguish near-degenerate states

$$v^{(\mathfrak{m})} C(t_0) v^{(\mathfrak{n})} = \delta_{\mathfrak{n},\mathfrak{m}}$$

excited meson states - ‘simple’ operator basis $\{\mathcal{O}_i\}$

a simple basis of meson operators - fermion bilinears

$$\bar{\psi}\Gamma\overleftrightarrow{D}\ldots\overleftrightarrow{D}\psi$$

smeared quark fields
up to three covariant derivatives

can form definite J^{PC} operators

$$\mathcal{O}_M^{J^{PC}}$$

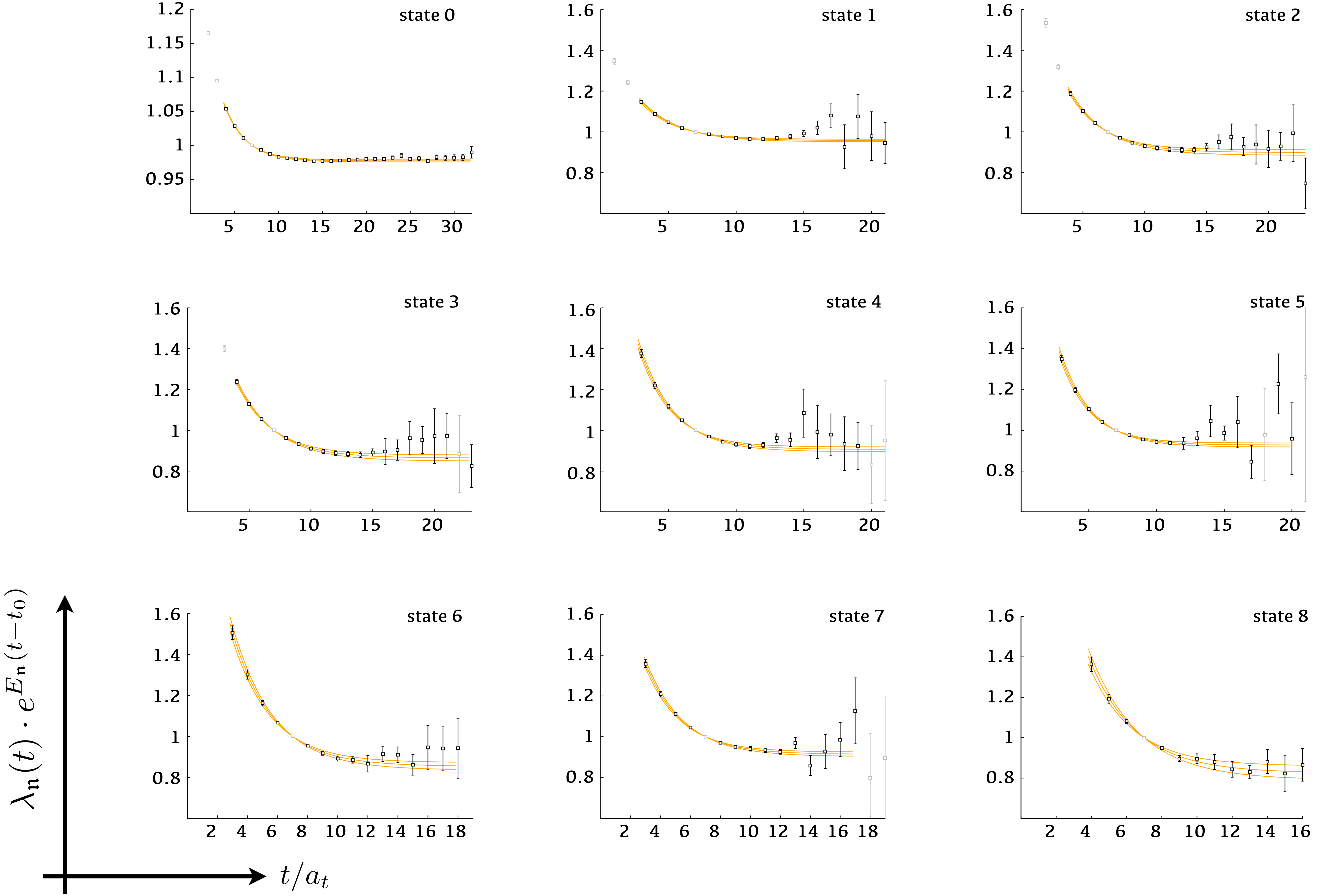
but the lattice symmetry is cubic ⇒ ‘subduce’ into irreducible representations

$$\mathcal{O}_\lambda^{\Lambda^{PC}} = \sum_M \mathcal{S}_{\Lambda,\lambda}^{J,M}\,\mathcal{O}_M^{J^{PC}}$$

Λ	J
A_1	$0, 4\ldots$
T_1	$1, 3, 4\ldots$
T_2	$2, 3, 4\ldots$
E	$2, 4\ldots$
A_2	$3\ldots$

principal correlators (T_1^{--} - 26 ops)

lowest nine states



principal correlators (T_1^{--} - 26 ops)

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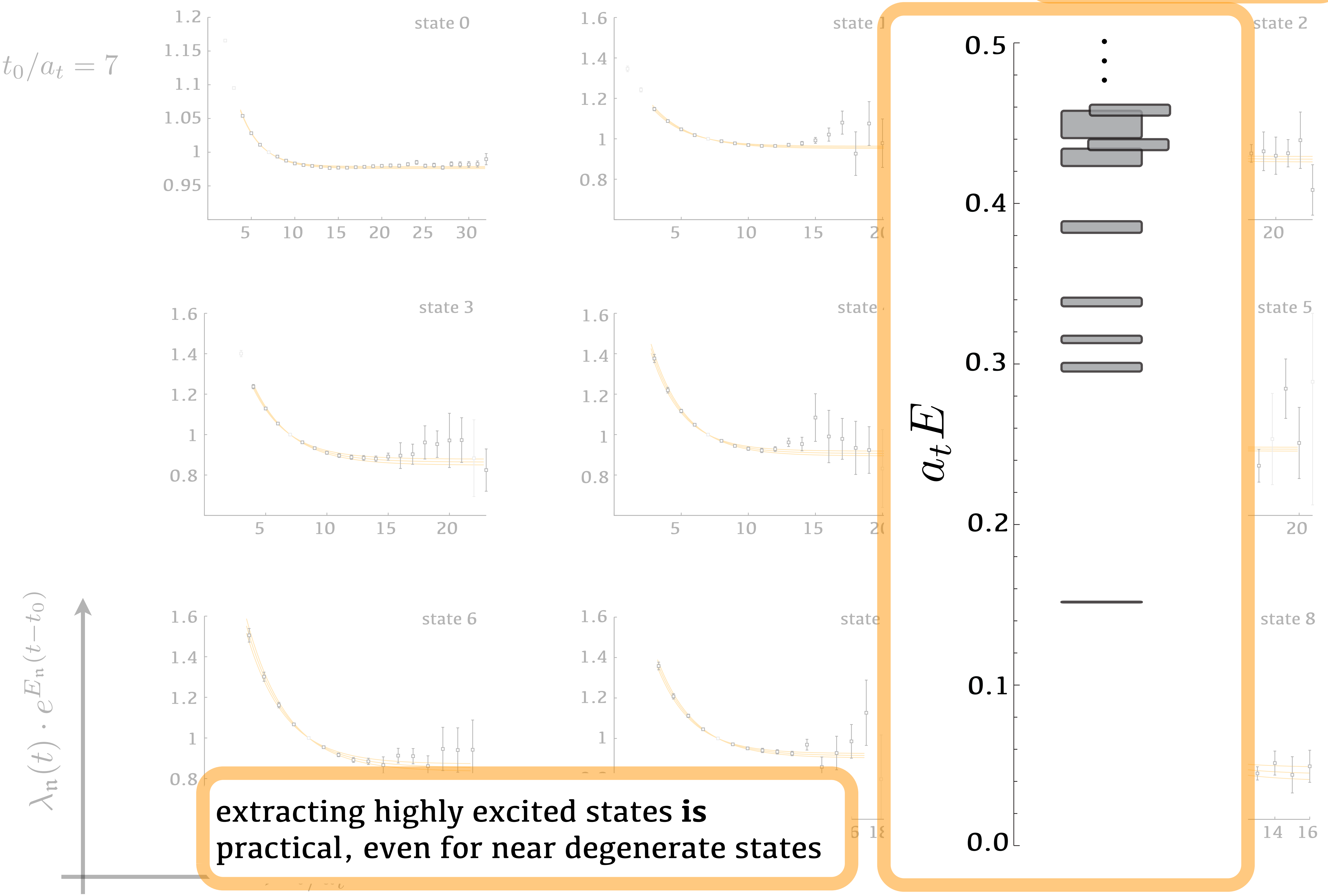
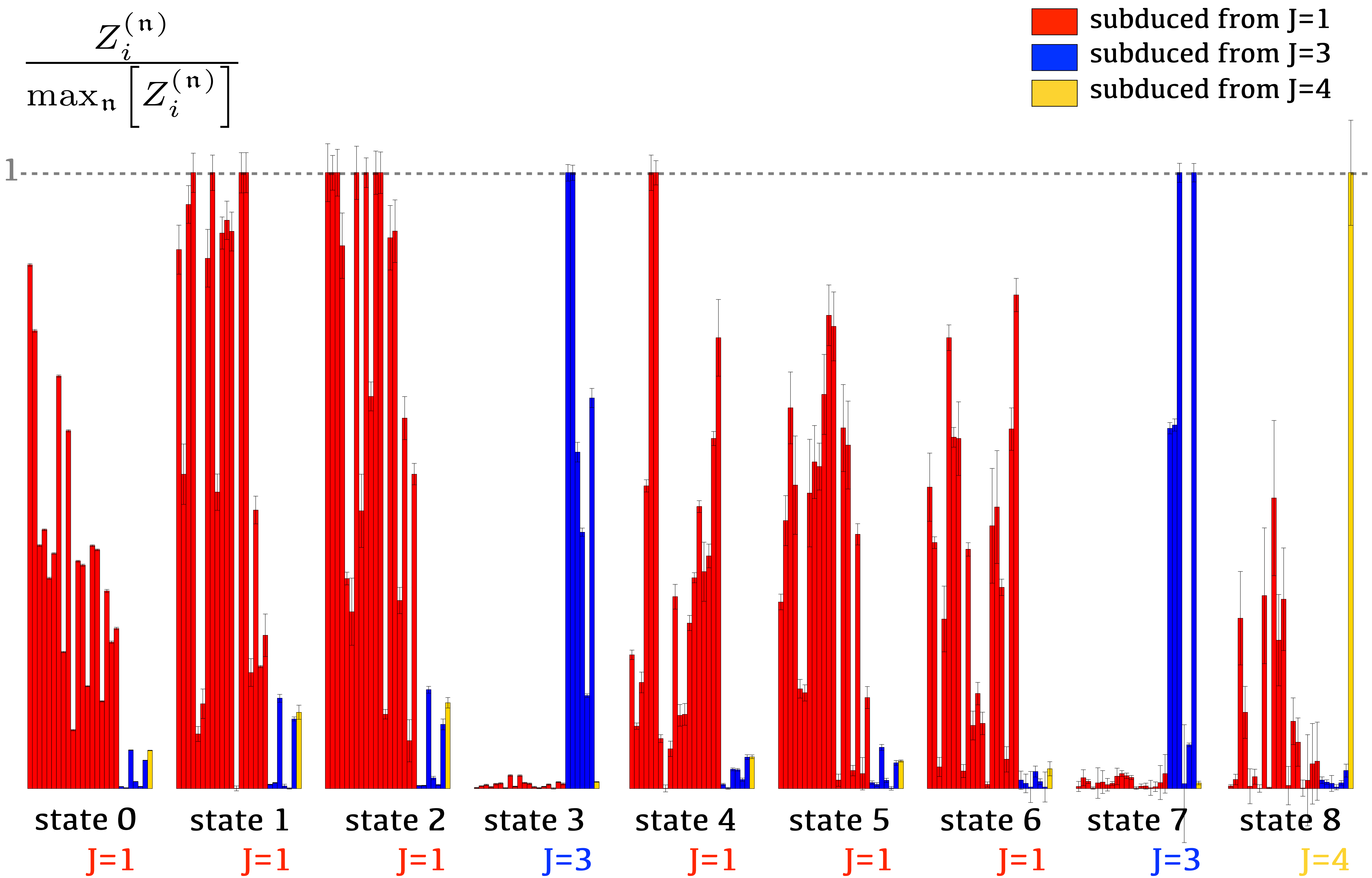


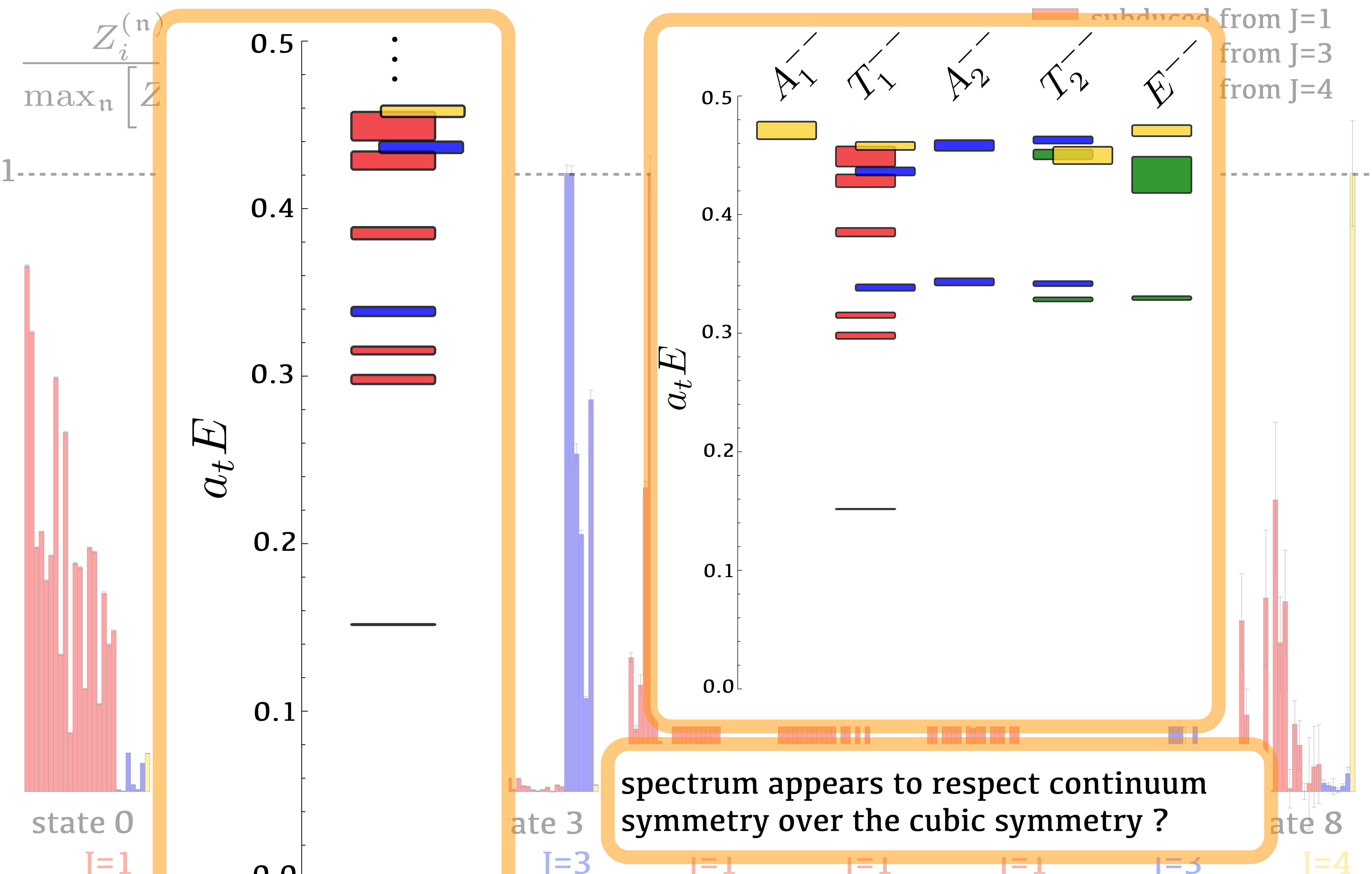
extracting highly excited states **is** practical, even for near degenerate states

overlap matrix elements - T_1^{--} $\quad Z_i^{(\mathfrak{n})} = \langle \mathfrak{n} | \mathcal{O}_i | 0 \rangle$



overlap matrix elements - T_1^{--}

$$Z_i^{(\mathfrak{n})} = \langle \mathfrak{n} | \mathcal{O}_i | 0 \rangle$$



spectrum appears to respect continuum symmetry over the cubic symmetry ?

an isovector meson spectrum

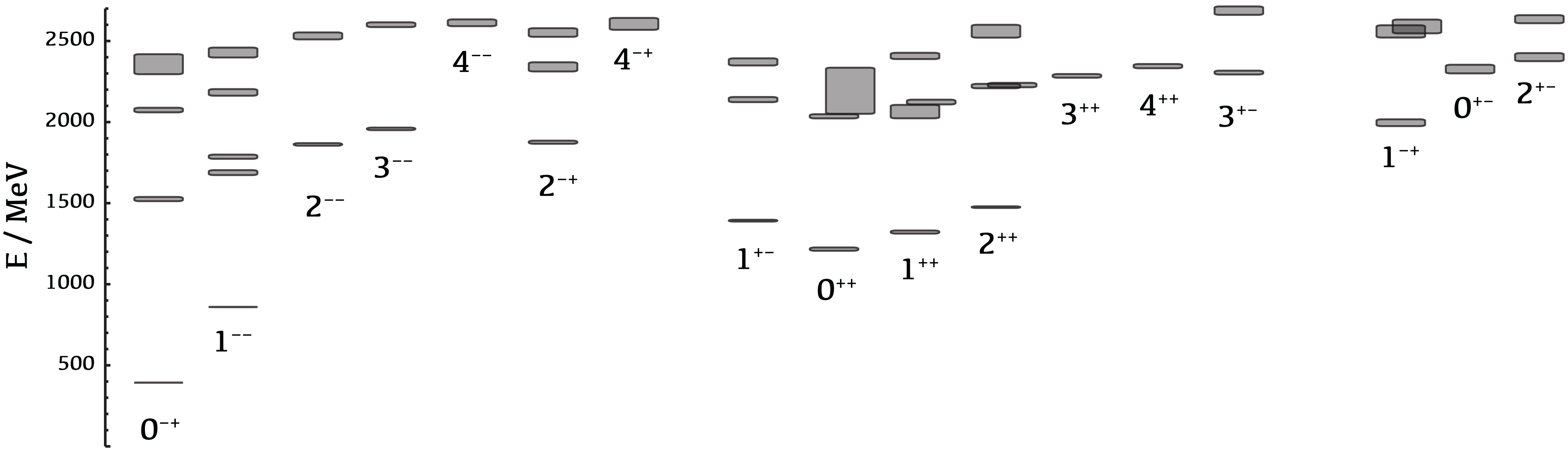
$m_\pi = 396\,\mathrm{MeV}$

$24^3 \times 128$

$L \sim 3\,\mathrm{fm}$

$m_\pi L \sim 5.7$

taking the continuum spin assignments seriously :



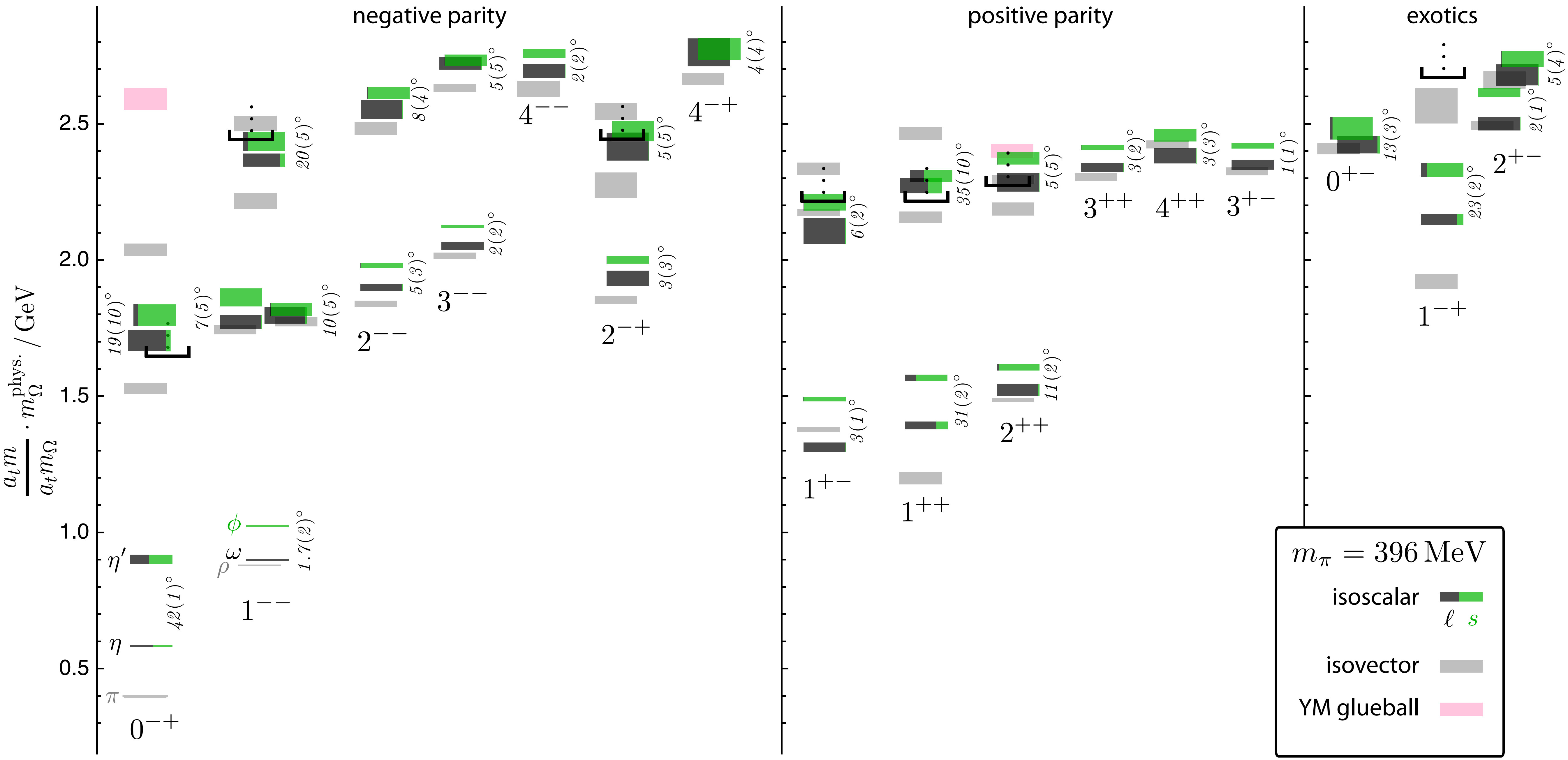
scale setting

$$m = \frac{am}{am_\Omega} m_\Omega^{\mathrm{phys.}}$$

smaller volumes in
Phys.Rev.D82 034508 (2010)

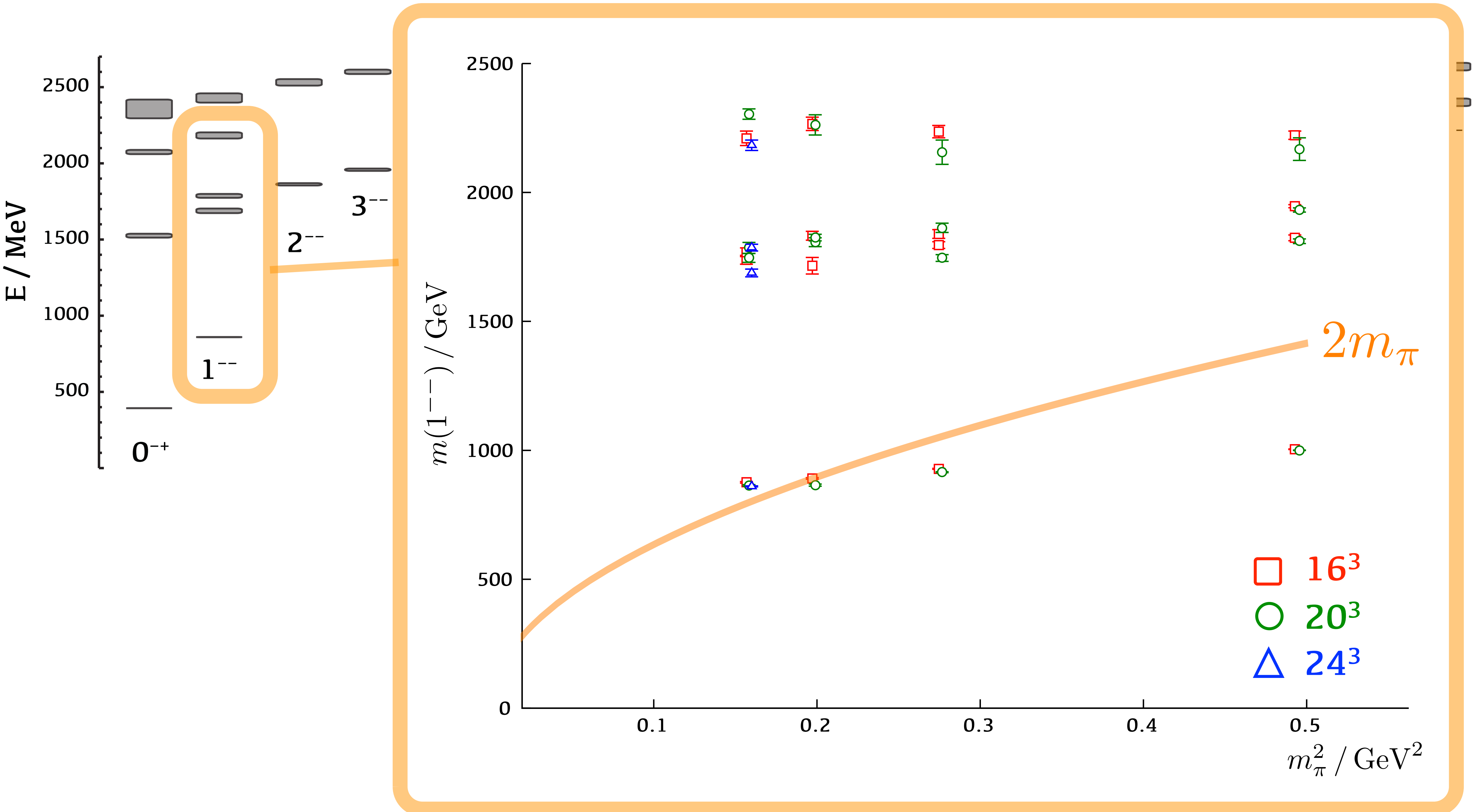
$16^3 \times 128$

same methods in the isoscalar sector:

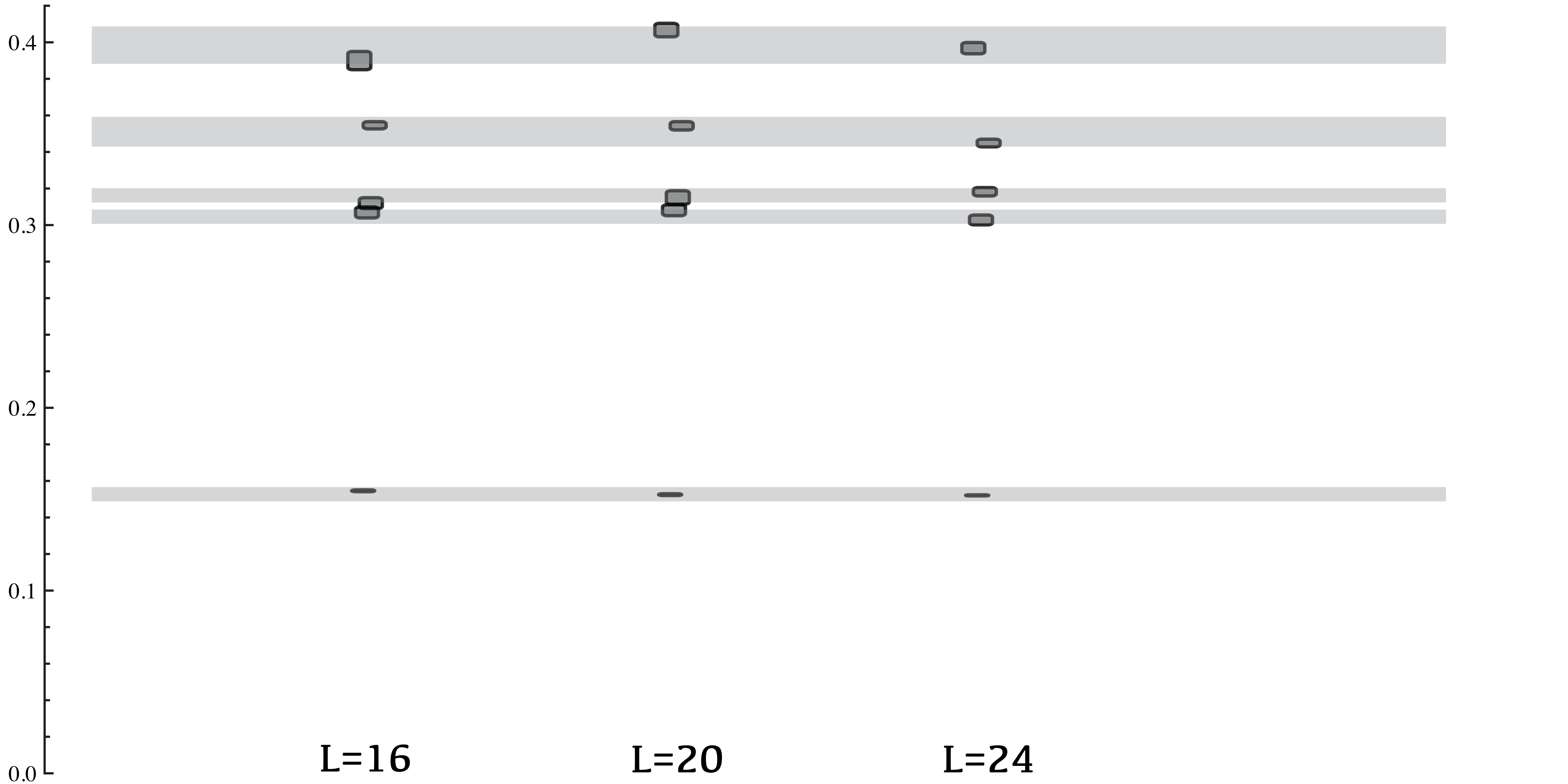


Hadron Spectrum Collab.
Phys.Rev. D83 (2011) 111502

isovector meson spectrum - quark mass dependence



volume dependence - T_1^{--}



no significant volume dependence ...?

success . . . ?

so is this successful ?

➟ a spectrum of excited meson states

➟ J^{PC} assignment possible (irrelevance of cubic lattice at small distances?)

➟ no observed dependence on the size of the box (box much bigger than the states?)

obviously not completely!

➟ meson resonances shouldn't have a unique energy

➟ enhancements in meson-meson continuum

➟ no meson-meson continuum in finite-volume
➟ but should be 'extra' discrete states
➟ strong volume dependence

periodic b.c.

$\psi(x) = \psi(x+L)$ $\psi(x) = e^{ikx}$ $k = \frac{2\pi}{L}n$

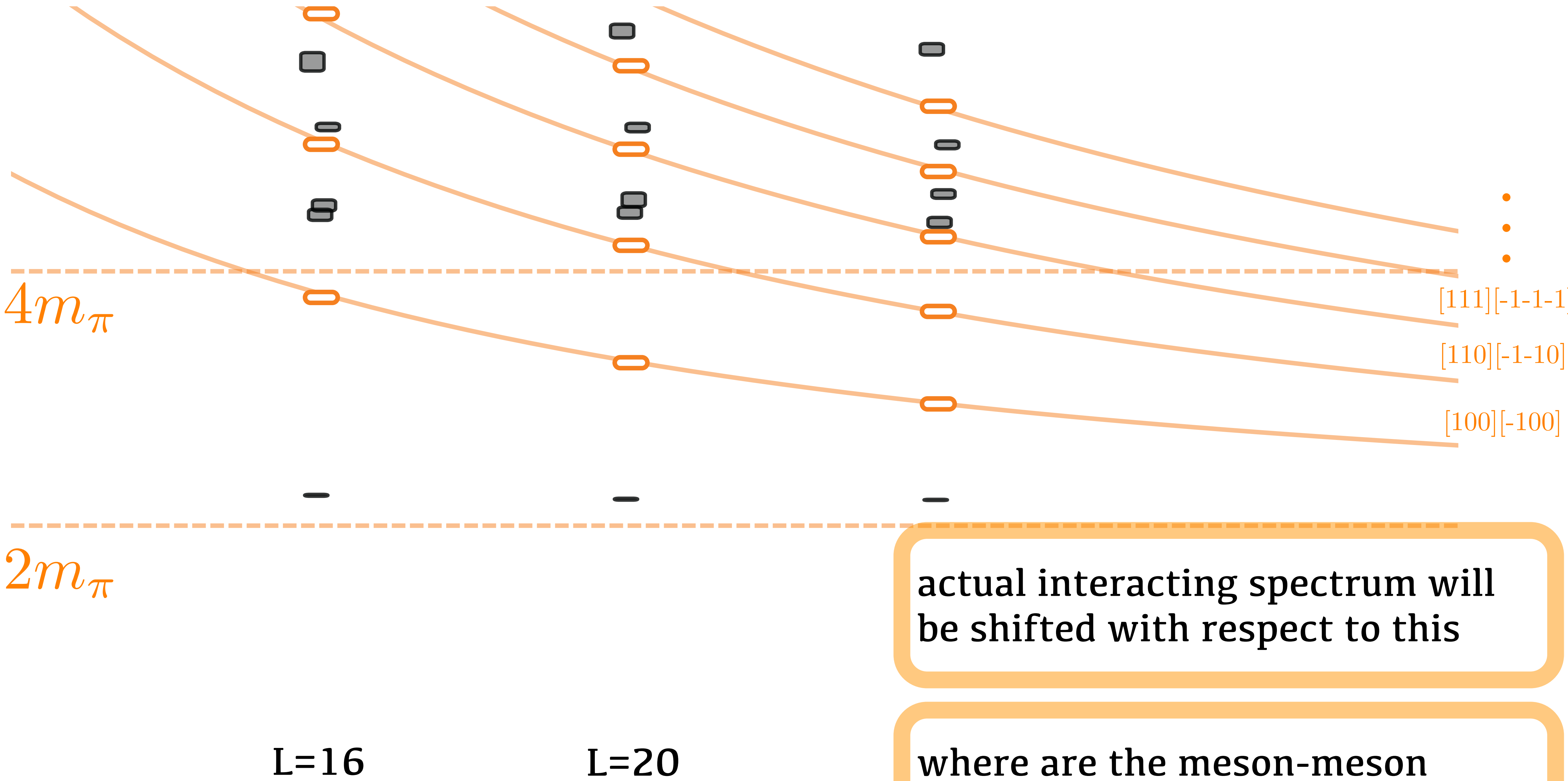
➟ should respect cubic (boundary) symmetry

volume dependence & multi-meson states

$$E^{(0)}_{\pi\pi} = 2\sqrt{m_\pi^2 + \left(\frac{2\pi}{L}\right)^2 n^2}$$

$$k = \frac{2\pi}{L}\left[n_x, n_y, n_z\right]$$

non-interacting ππ energies



actual interacting spectrum will be shifted with respect to this

where are the meson-meson states ... ?

where are the rest of the levels ?

it would seem that 'local' operators aren't overlapping strongly onto the meson-meson components of the finite-volume eigenstates

➟ overlap likely suppressed by the volume $\langle MM|\overline{\psi}\ldots\psi|0\rangle \sim \frac{1}{V}$

the lack of cubic symmetry restriction in the spectrum
- reflects an effectively fine lattice spacing
- and not sampling states that 'feel' the cubic boundary

solution - include operators that resemble meson-meson (which sample the whole volume of the lattice) ...

meson-meson operators

$$\mathcal{O}^{\Lambda\lambda} = \sum_{\hat{\vec{k}}_1, \hat{\vec{k}}_2} C^{\Lambda\lambda}(\hat{\vec{k}}_1, \hat{\vec{k}}_2) \mathcal{O}_\pi(\vec{k}_1) \mathcal{O}_\pi(\vec{k}_2)$$

'pions' of definite momentum

variational basis is increasing values of $|\mathbf{k}|$

$$\mathcal{O}^{\Lambda\lambda} = \sum_{\hat{\vec{k}}_1, \hat{\vec{k}}_2} C^{\Lambda\lambda}(\hat{\vec{k}}_1, \hat{\vec{k}}_2) \sum_{\vec{x}} \mathcal{O}_\pi(\vec{x}) e^{i\vec{k}_1\cdot\vec{x}} \sum_{\vec{y}} \mathcal{O}_\pi(\vec{y}) e^{i\vec{k}_2\cdot\vec{y}}$$

all relative positions are summed over

(technical challenge to implement this in lattice QCD ... distillation)

ππ I=2

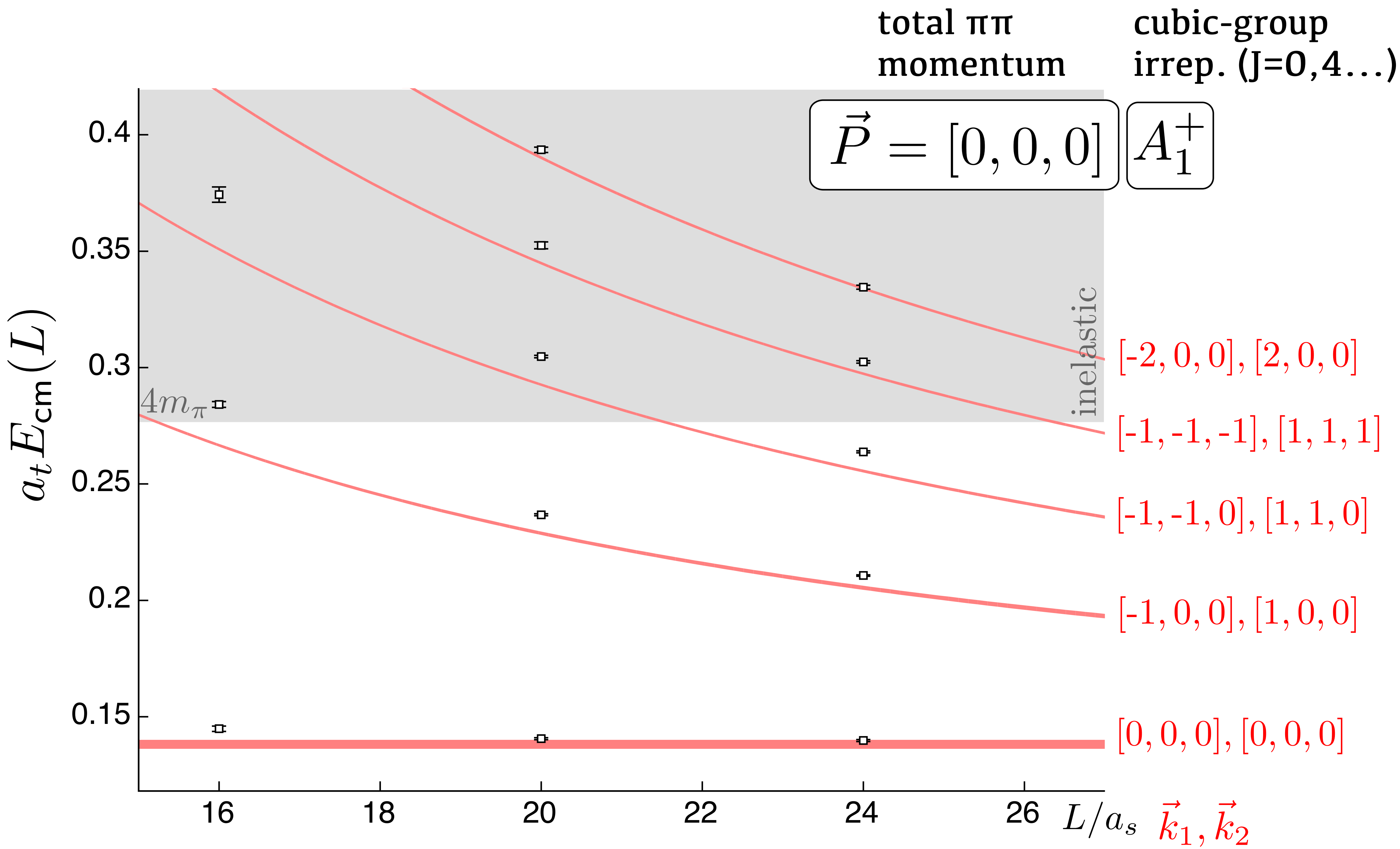
a relatively simple channel

empirically weak and repulsive - no resonances

no quark-antiquark operator constructions required

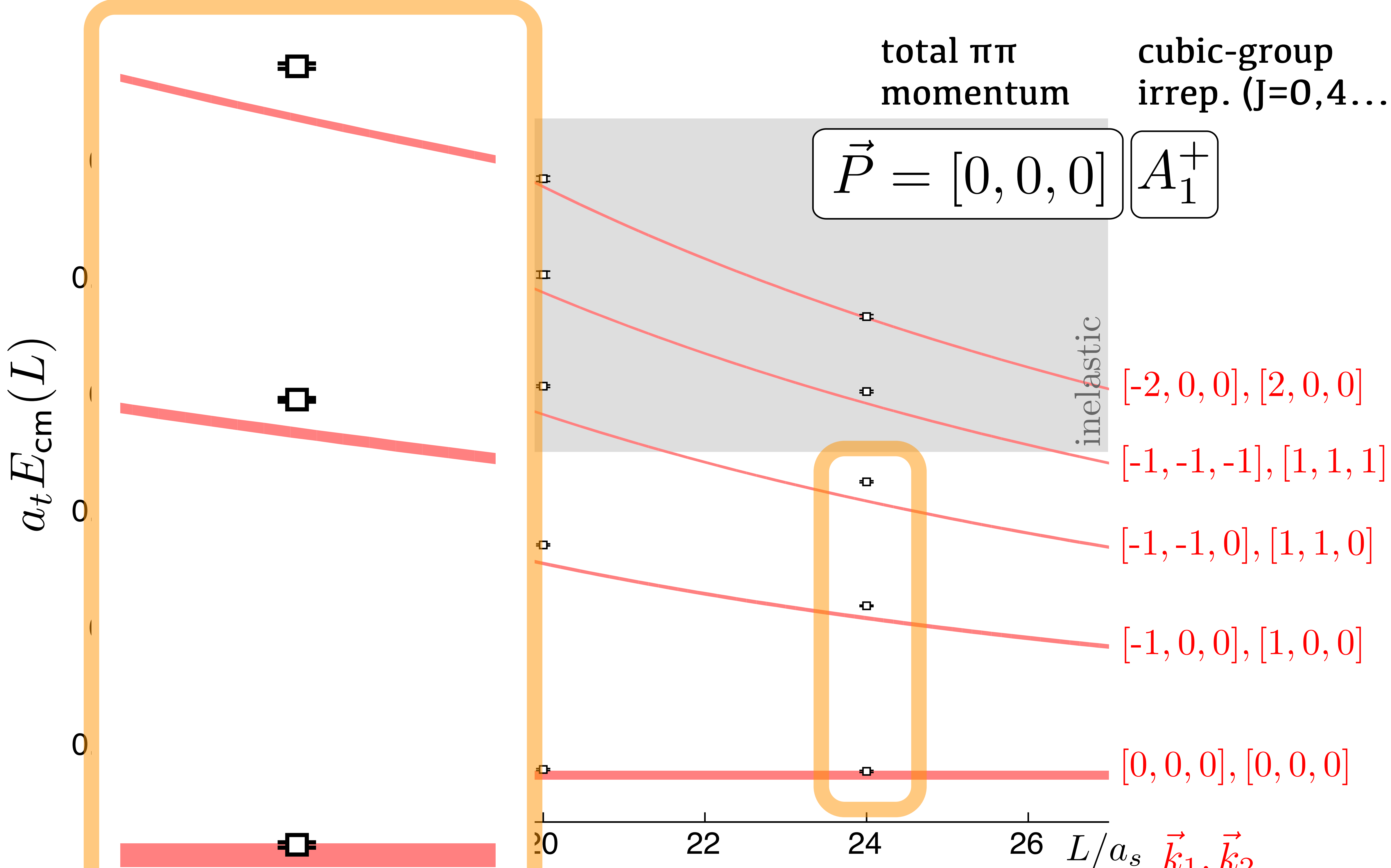
no quark-line annihilation diagrams in $qq\bar{q}\bar{q} \rightarrow qq\bar{q}\bar{q}$

ππ I=2



total ππ momentum
cubic-group irrep. (J=0,4...)
$\vec{P} = [0,0,0]$
A_1^+
$a_t E_{\mathrm{cm}}(L)$
0.4
0.35
0.3
0.25
0.2
0.15
$4m_\pi$
inelastic
[-2, 0, 0], [2, 0, 0]
[-1, -1, -1], [1, 1, 1]
[-1, -1, 0], [1, 1, 0]
[-1, 0, 0], [1, 0, 0]
[0, 0, 0], [0, 0, 0]
16
18
20
22
24
26
L/a_s
$\vec{k}_1, \vec{k}_2$

ππ I=2



statistical uncertainties < 0.2 %

spectrum → phase-shift

very roughly speaking

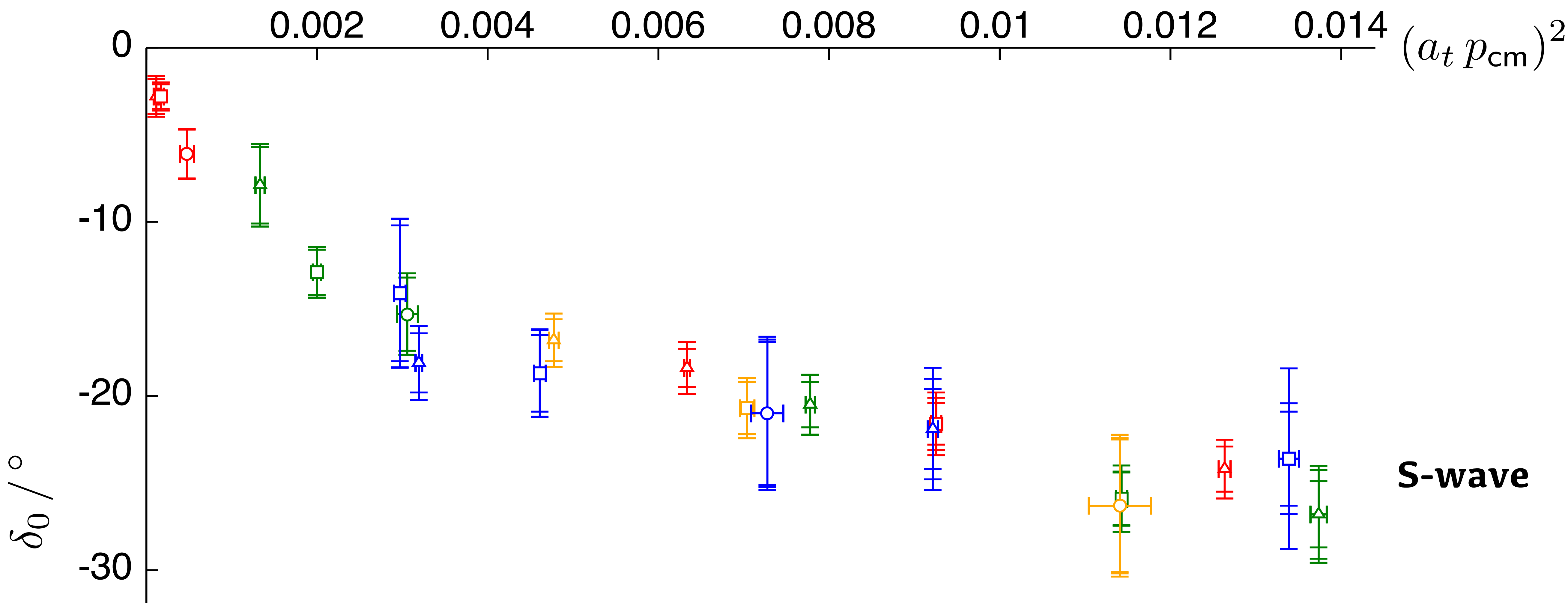
discrete energies
in finite volume

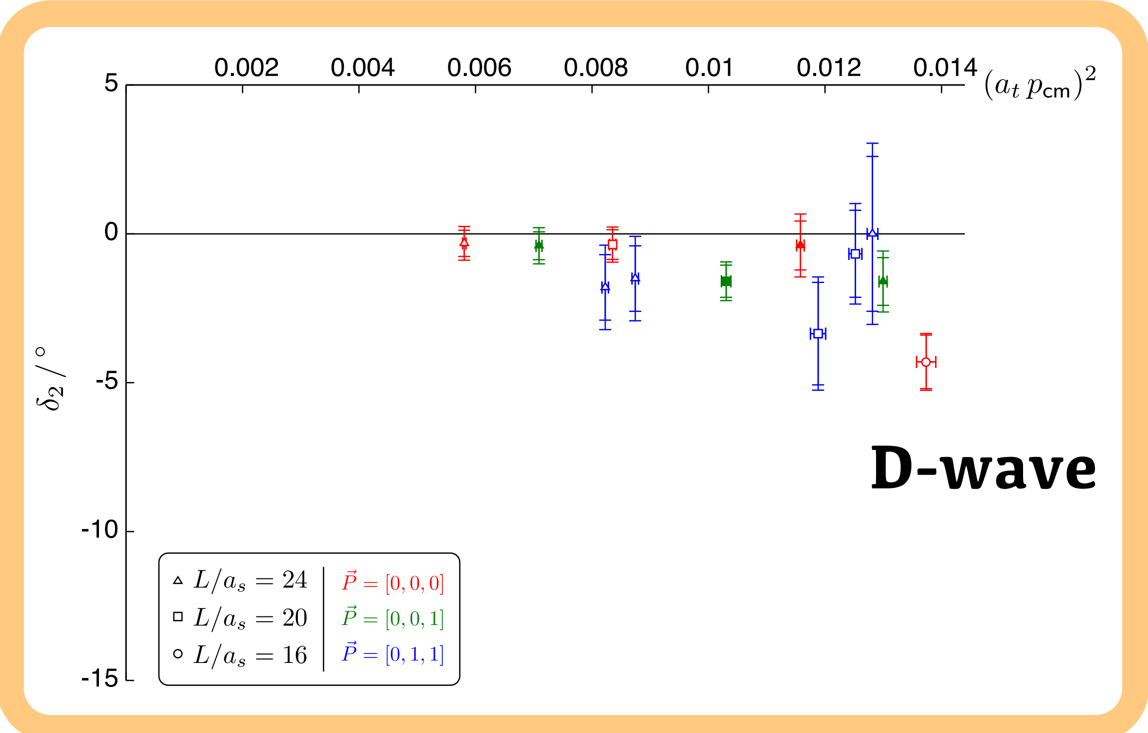
elastic scattering
phase shift at
discrete energies

actually complications from the cubic symmetry
mixes up different angular momenta

ππ I=2 phase-shifts

$m_\pi \approx 396\,\mathrm{MeV}$

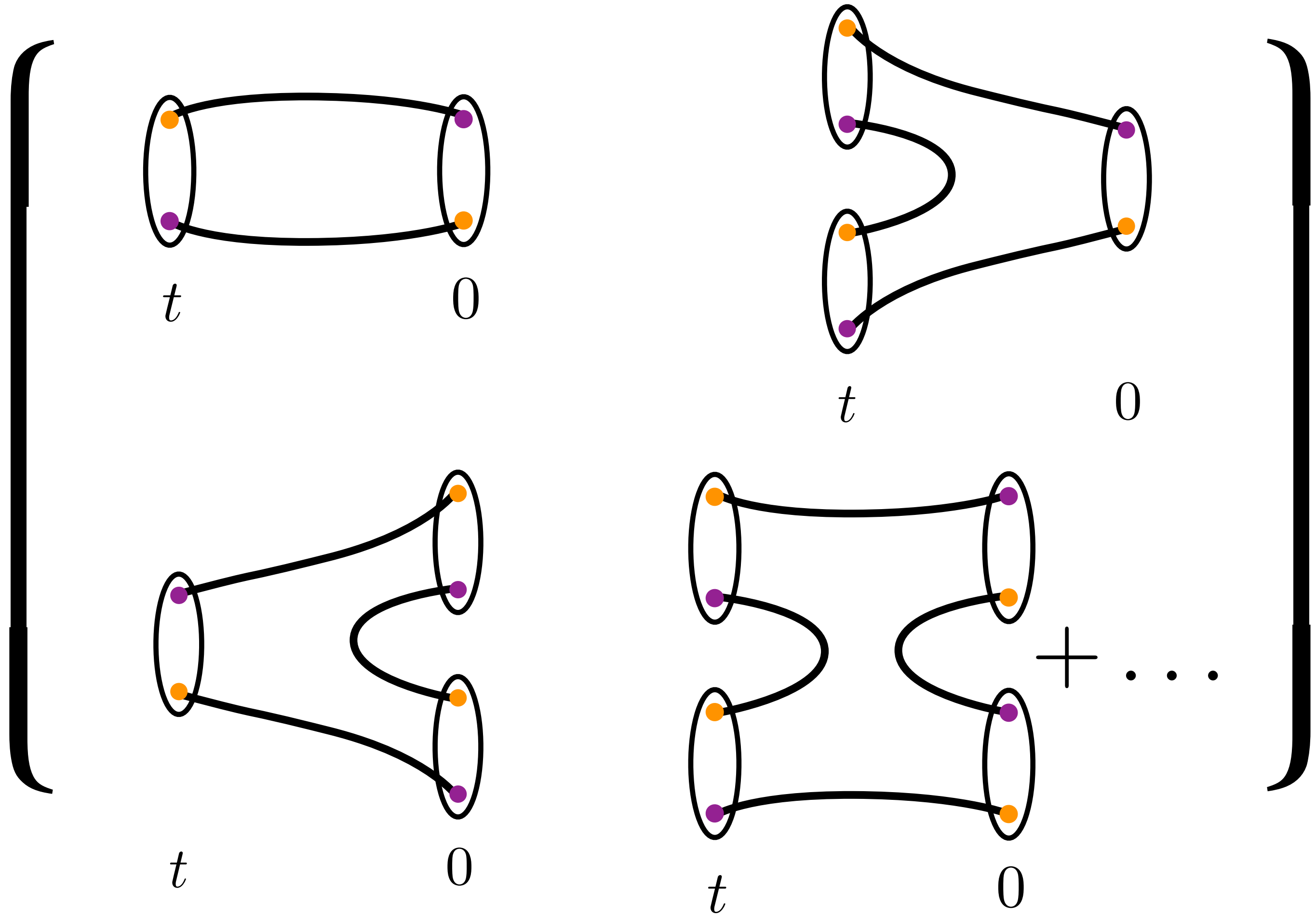




ππ I=1 & the ρ resonance

large basis of ‘local’ quark bilinears
and ππ constructions

$$C(t) =$$

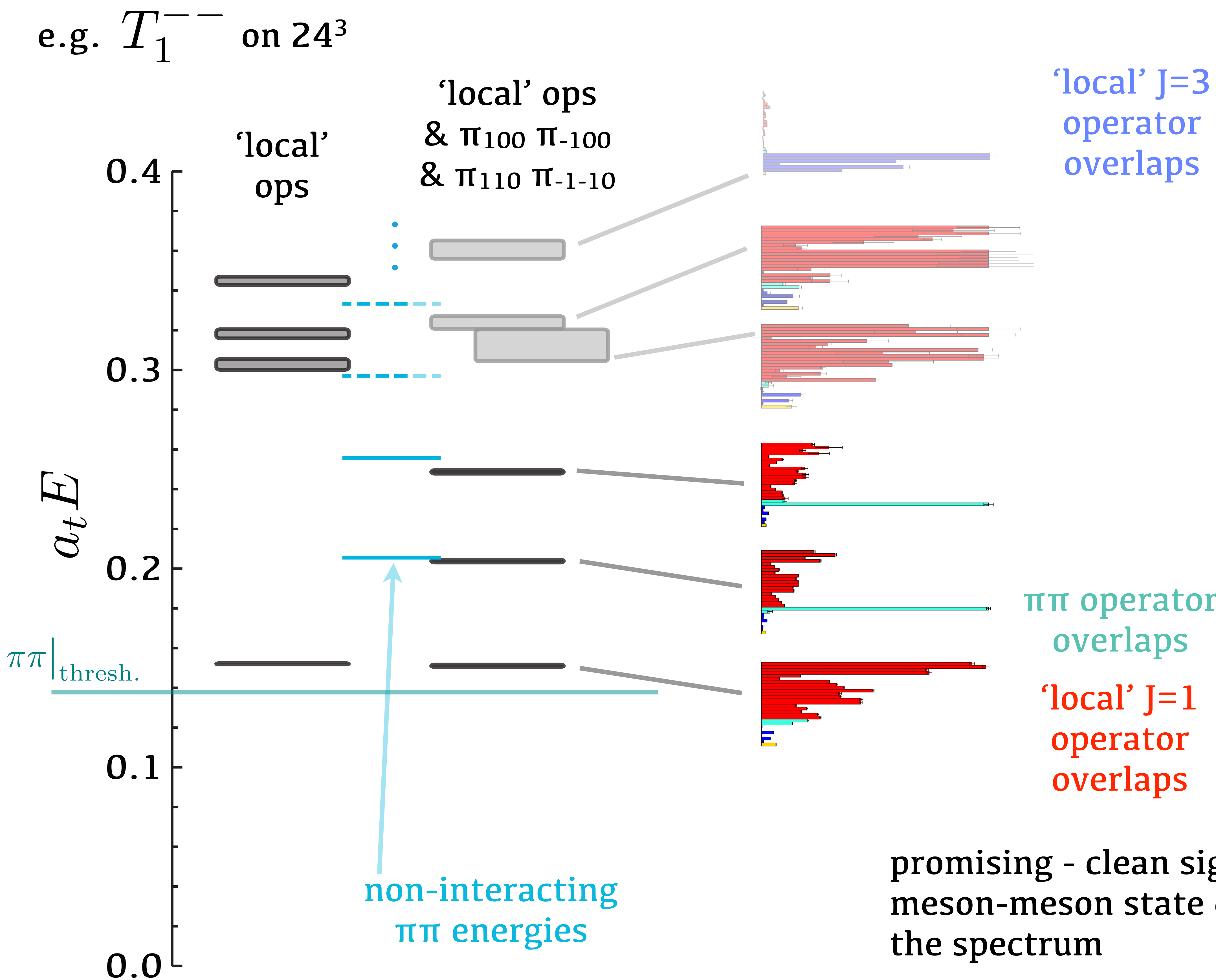


solve variational problem in this extended basis ...

the resulting spectrum changes w.r.t
using just ‘local’ quark bilinears !

including multi-meson operators

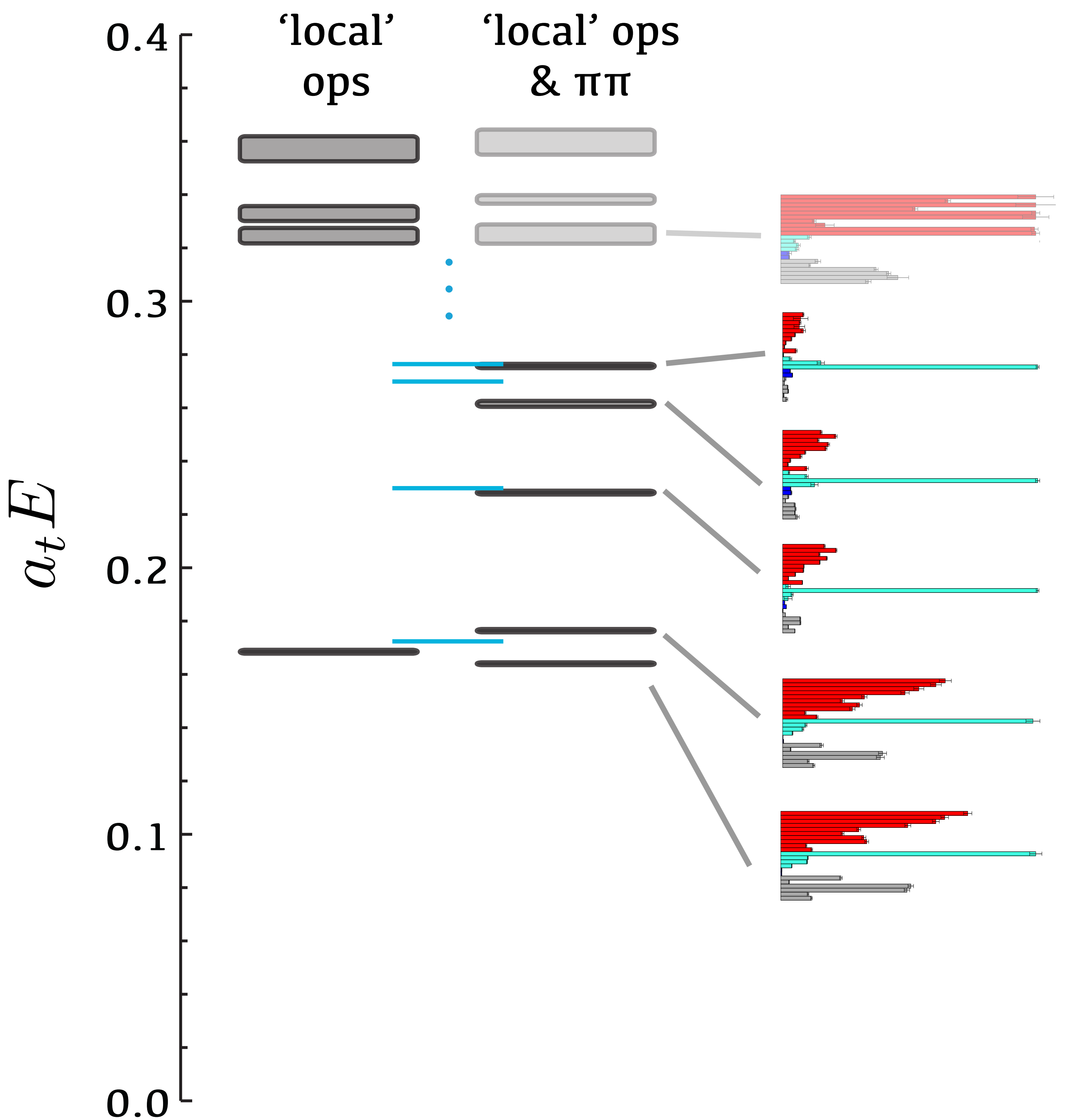
$m_\pi \approx 396\,\mathrm{MeV}$



promising - clean signals for excited meson-meson state contributions in the spectrum

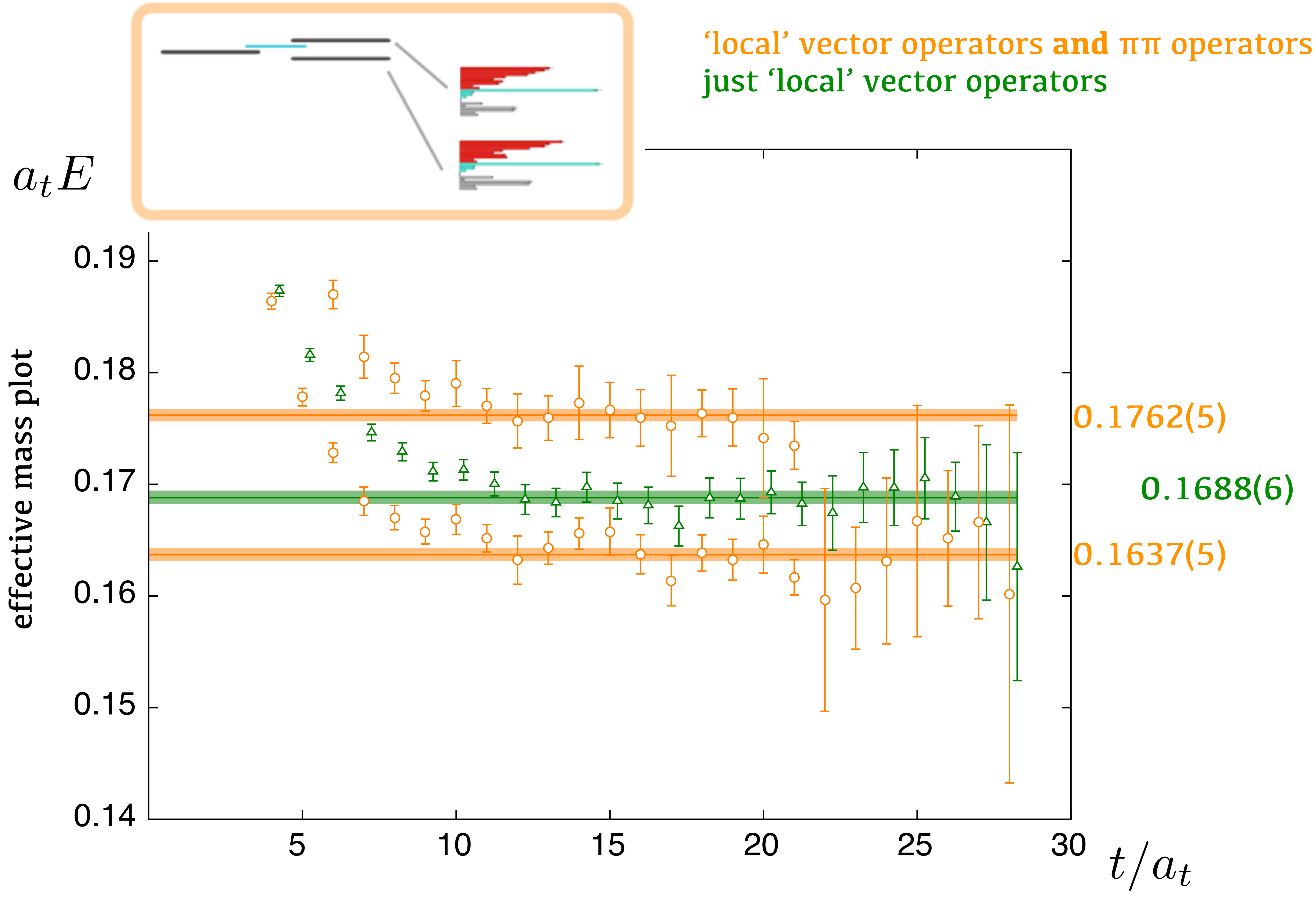
including multi-meson operators - ππ in-flight

e.g. $\mathrm{Dic}_4\ A_1^-$ P=[100] on 24^3 ("in-flight helicity zero")

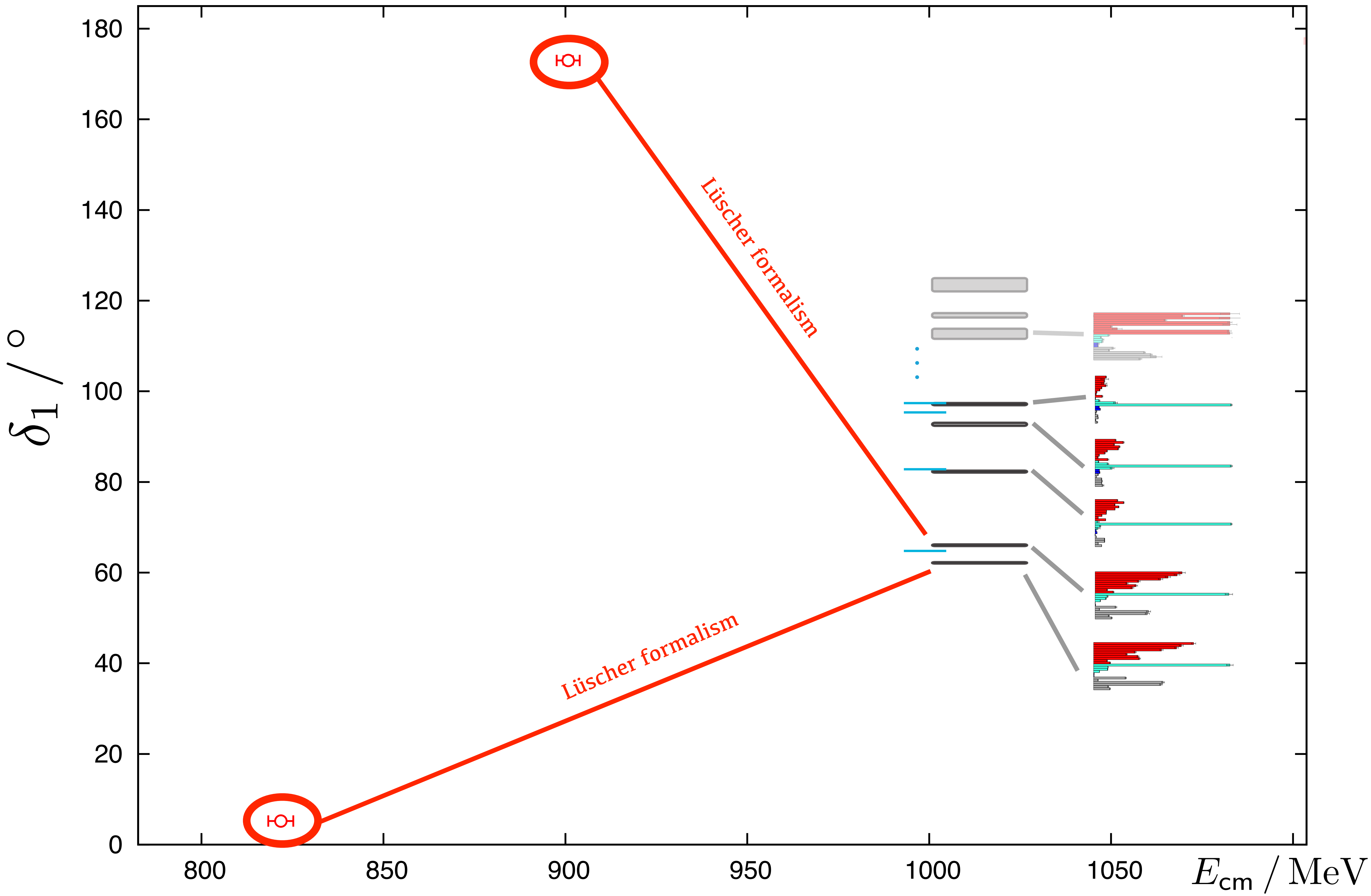


in the elastic scattering region
⇒ can use Lüscher formalism

including multi-meson operators

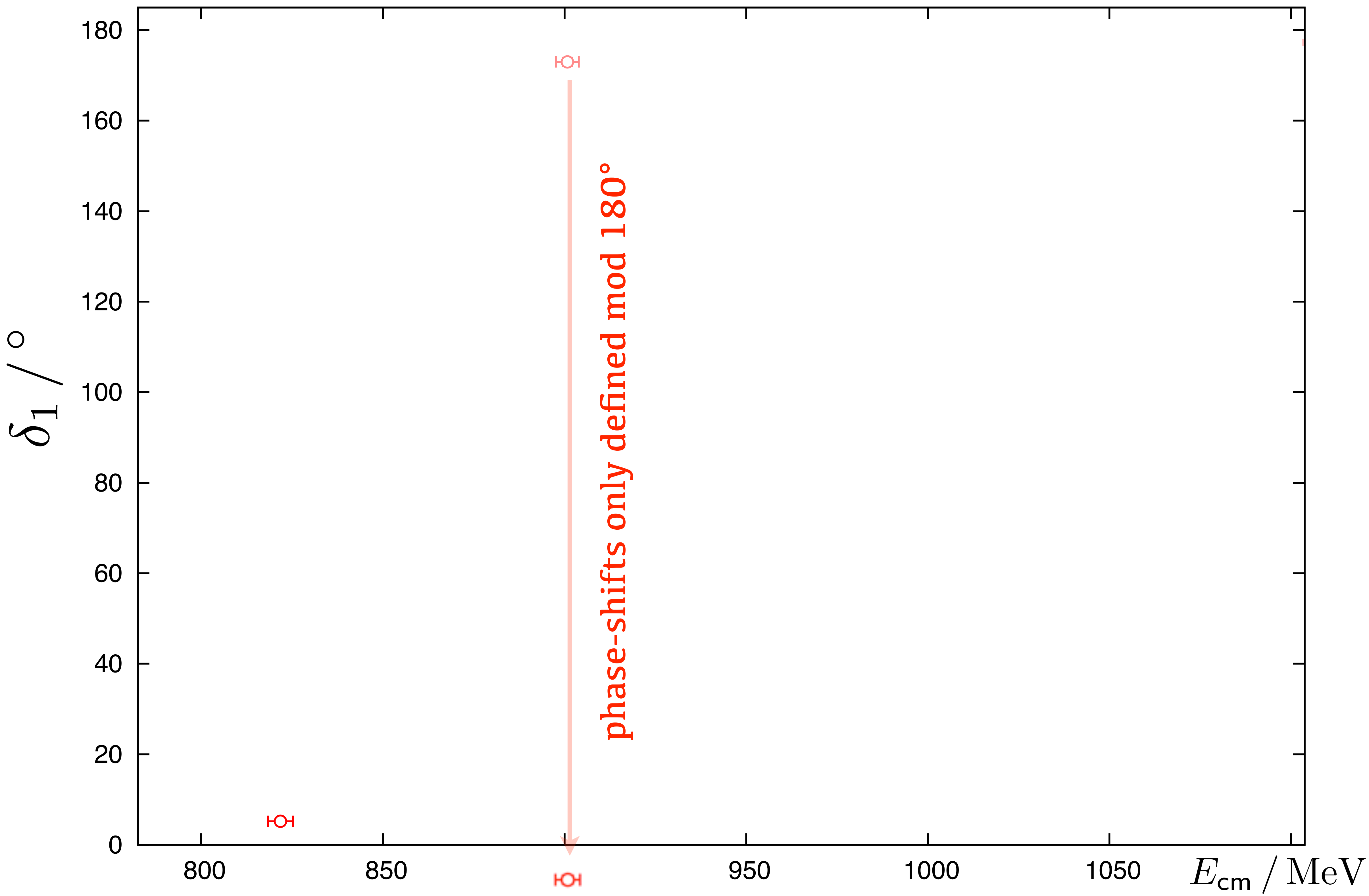


a resonance ?



180
160
140
120
100
80
60
40
20
0
$\delta_1 / ^\circ$
800
850
900
950
1000
1050
E_{cm} / MeV
Lüscher formalism
Lüscher formalism

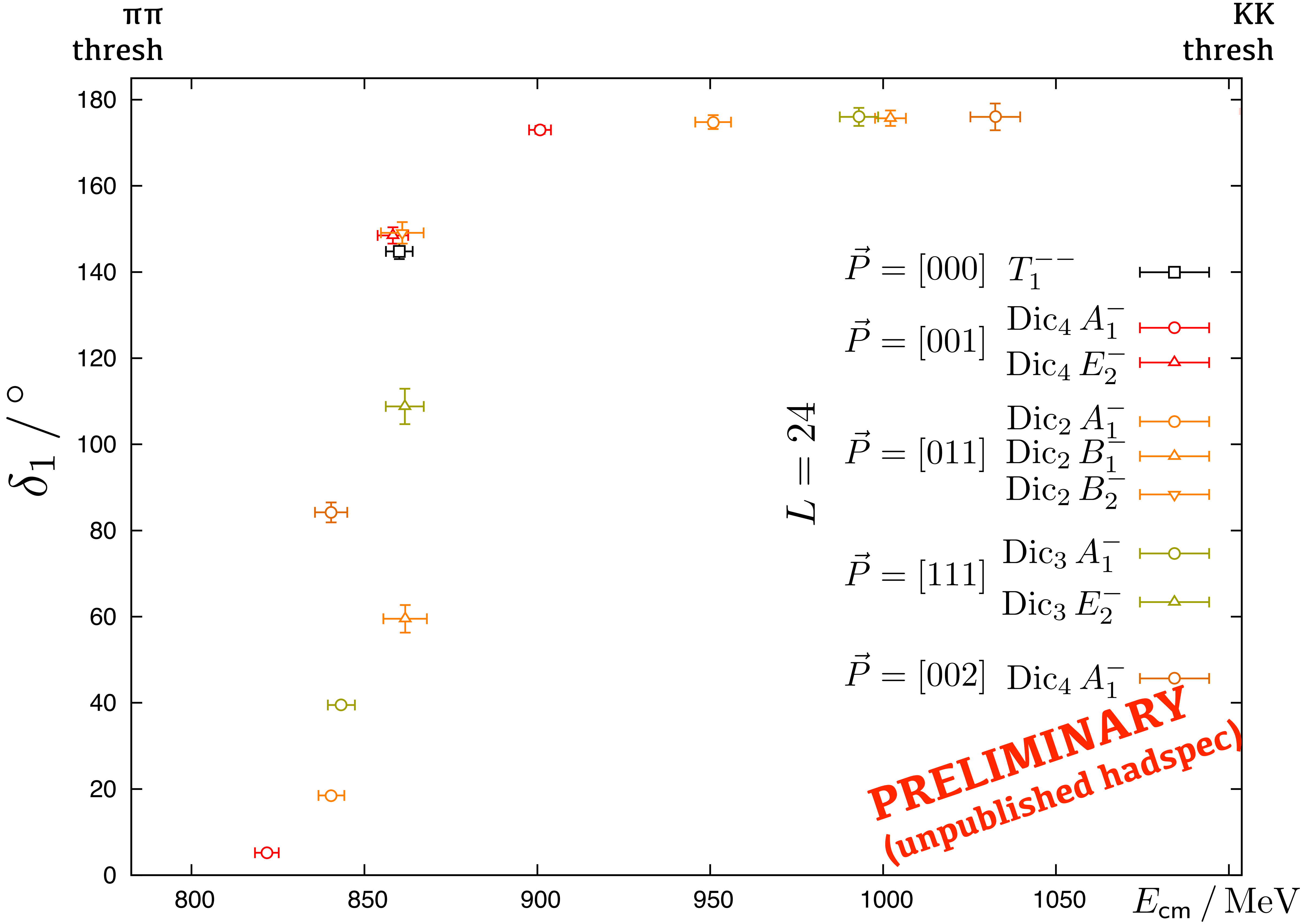
a resonance ?



180
160
140
120
100
80
60
40
20
0
$\delta_1 / ^\circ$
800
850
950
1000
1050
E_{cm} / MeV
phase-shifts only defined mod 180°

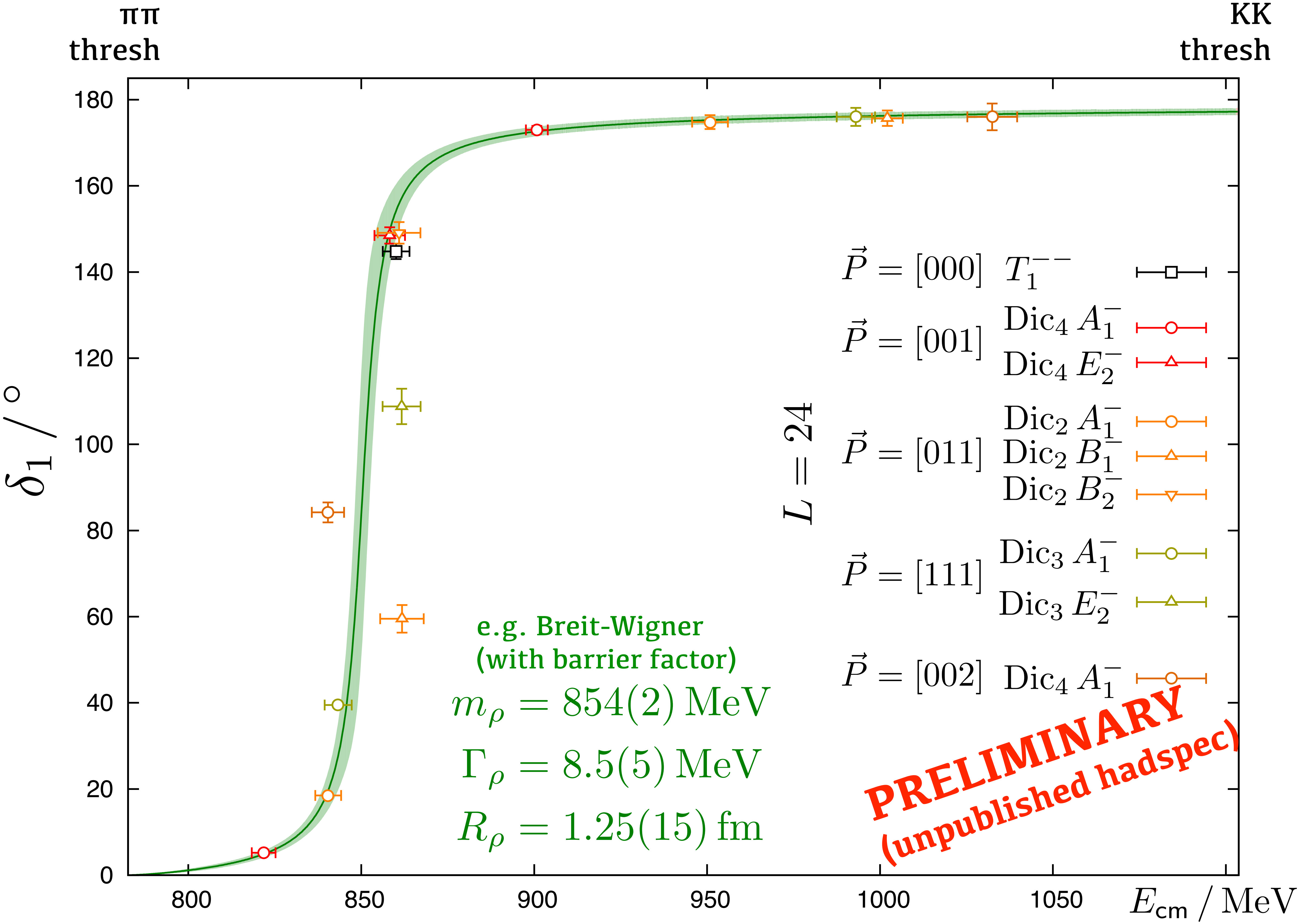
the "ρ" resonance in ππ

$m_\pi \approx 396\,\mathrm{MeV}$



the "ρ" resonance in ππ

$m_\pi \approx 396\,\text{MeV}$



excited meson spectroscopy

hadron spectrum collaboration is focussed on computing excited hadron properties

‘reliable’ extraction of excited states through

- ➟ new correlator construction methods
- ➟ new, large, interpolating operator bases
- ➟ constrained analysis techniques

scattering and resonance properties in finite-volume

- ➟ careful analysis of excited state spectra in finite-volume
  - ➟ $\pi\pi$ I=2 elastic phase-shifts in S-wave and D-wave
  - ➟ $\pi\pi$ I=1 elastic phase-shift in P-wave - shows a ρ resonance

& more to come

- ➟ move toward physical parameters (230 MeV pions soon)
- ➟ more physics quantities: inelastic scattering, three-particle final states …