

Recent results in chiral EFT for the two-nucleon system

Dear reader,

These are the slides from my presentation.
They do not represent the entire content of
the talk. What was actually said is quite
important.

Daniel Phillips

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RESEARCH SUPPORTED BY THE US DEPARTMENT OF ENERGY

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- Consequences of QCD's spontaneously and explicitly broken chiral symmetry for $A \geq 2$
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- Clean living \rightarrow error estimates, model-independent results
- χ PT: low scales: m_π , p ; high scales: m_ρ , M_N , $M_\Delta - M_N \equiv \Lambda_{\chi\text{SB}}$

Outline

- The proposal: Weinberg's counting for the NN potential, aka naive dimensional analysis for V
- One-pion exchange and renormalization: how strong interactions taught us to be not-quite-as-naïve
- A “new leading order” and its discontents
- Higher orders in χ EFT: what comes where?
- Selected applications to EM reactions
- Conclusion

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- χ PT \Rightarrow pion interactions are weak at low energy.

Weinberg (1990), apply χ PT to V , i.e. expand it in $P=(p/\Lambda_{\chi\text{SB}}, m_{\pi}/\Lambda_{\chi\text{SB}})$

$$(E - H_0)|\psi\rangle = V|\psi\rangle$$

$$V = V^{(0)} + V^{(2)} + V^{(3)} + \dots$$

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- Leading-order V :

$$V^{(0)} = \text{[diagram: contact vertex]} + \text{[diagram: pion exchange]} ;$$

$$\langle \mathbf{p}' | V | \mathbf{p} \rangle = C^{3S1} P_{3S1} + C^{1S0} P_{1S0} + V_{1\pi}(\mathbf{p}' - \mathbf{p})$$

Higher orders in V

	Two-nucleon force	Three-nucleon force	Four-nucleon force
P^0		—	—
P^2		—	—
P^3			—
P^4			

—
CONSISTENT 3NFS, 4NFS,
SEE TALK OF H. KREBS
 —

work in progress...

Courtesy
 E. Epelbaum

2 nucleon force \gg 3 nucleon force \gg 4 nucleon force ...

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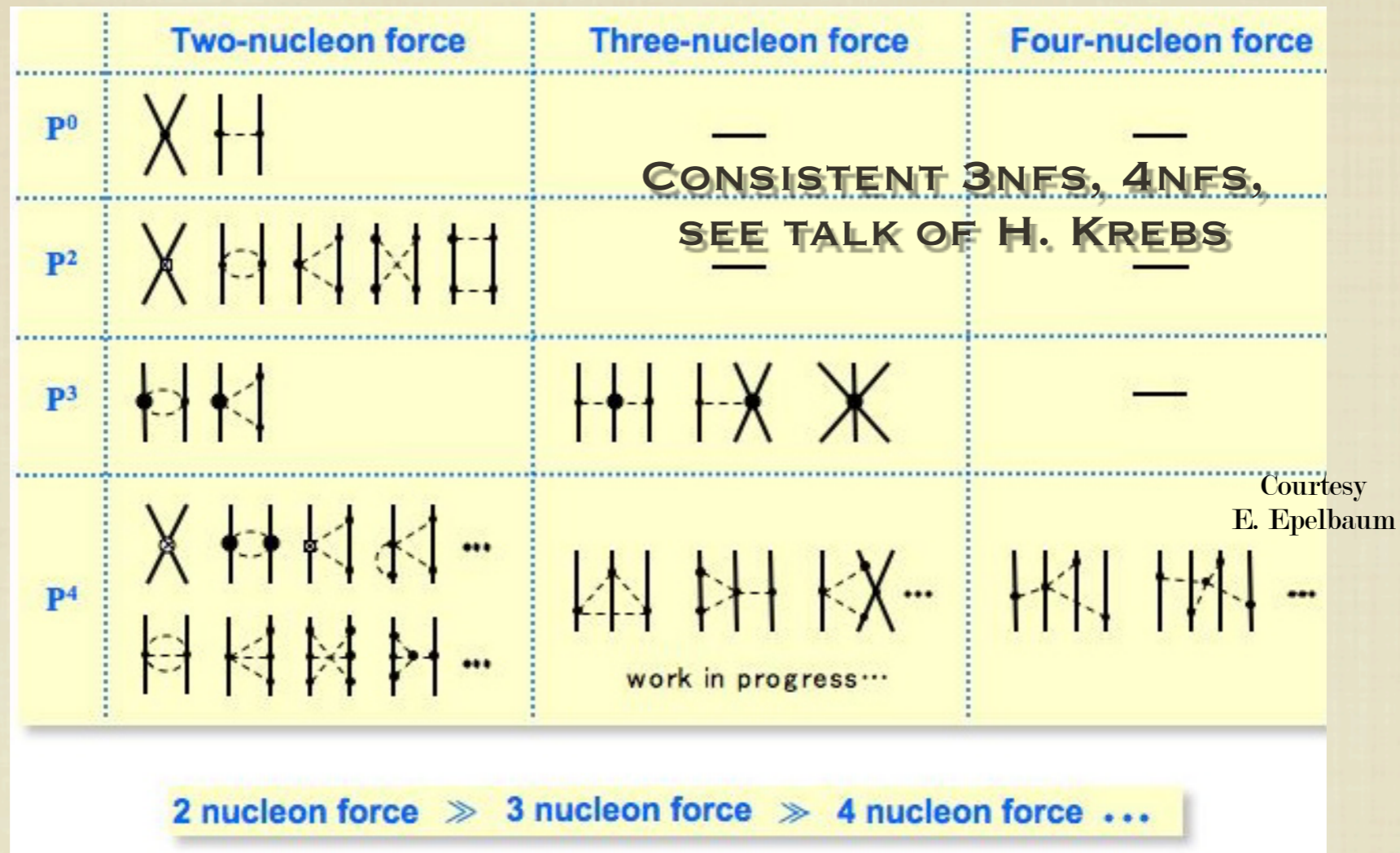
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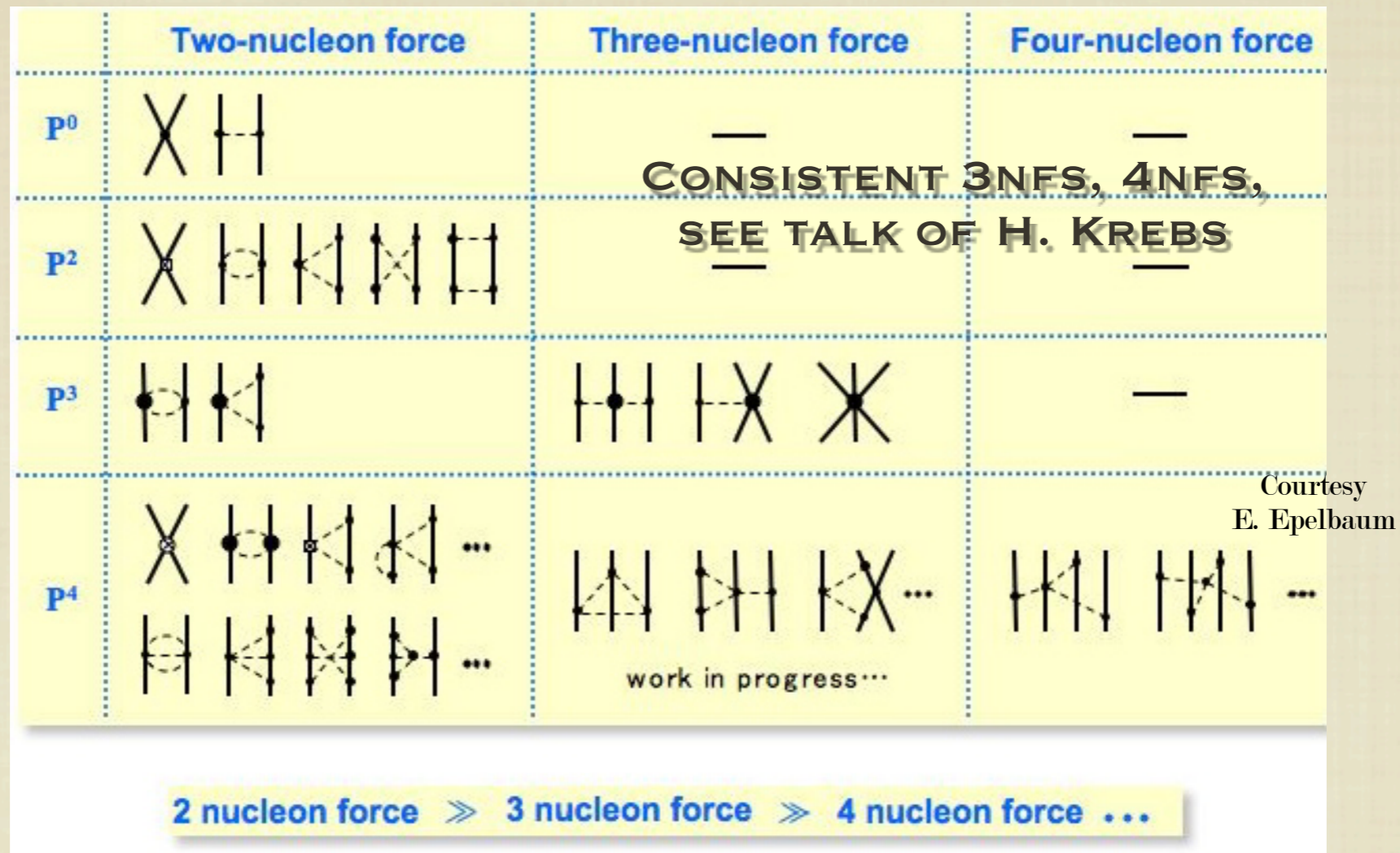
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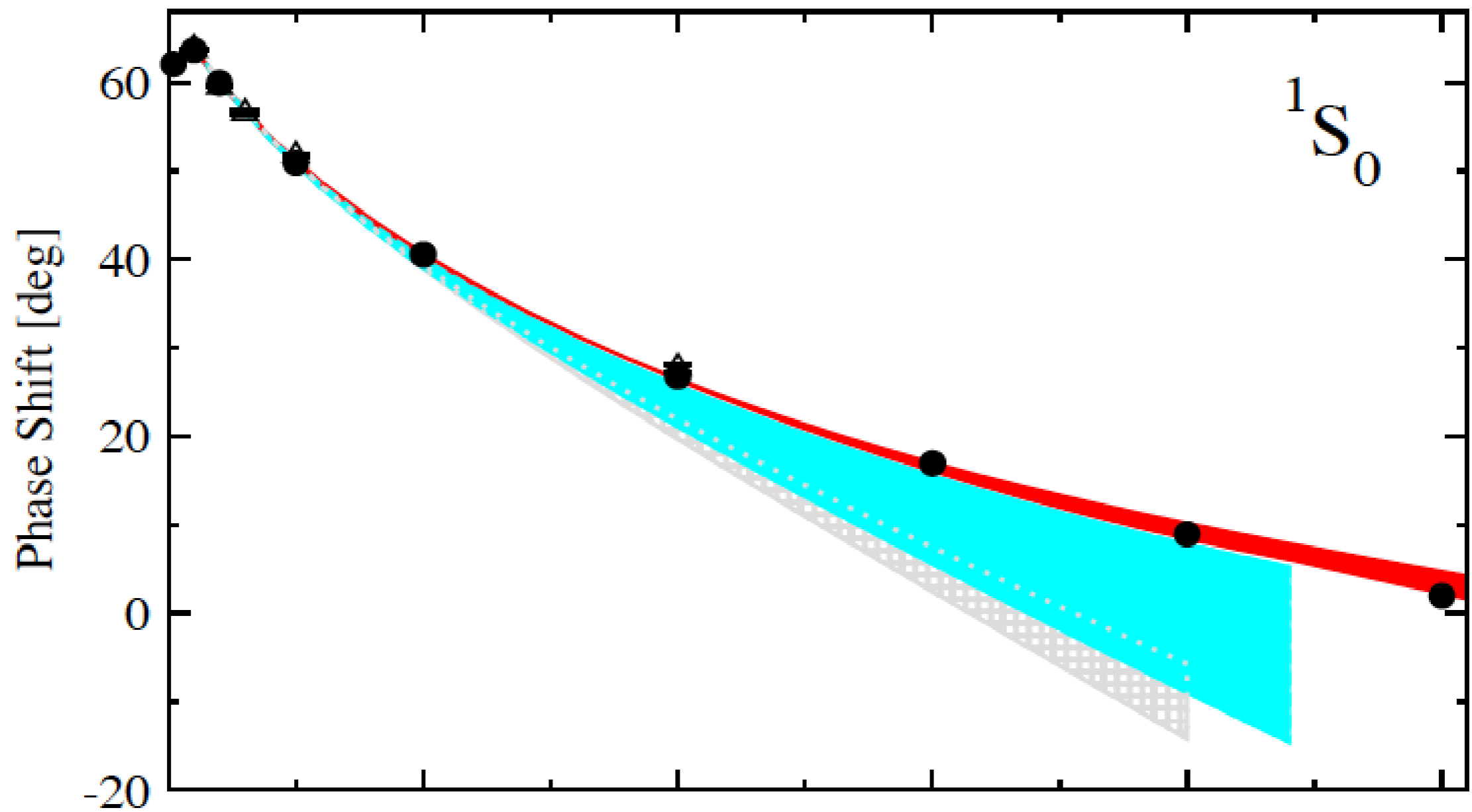


- No difficulties with counting for long-distance pieces
- Here I present discussion of “Delta-less” potential

“Weinberg” counting to $O(P^4)$ [N^3LO]

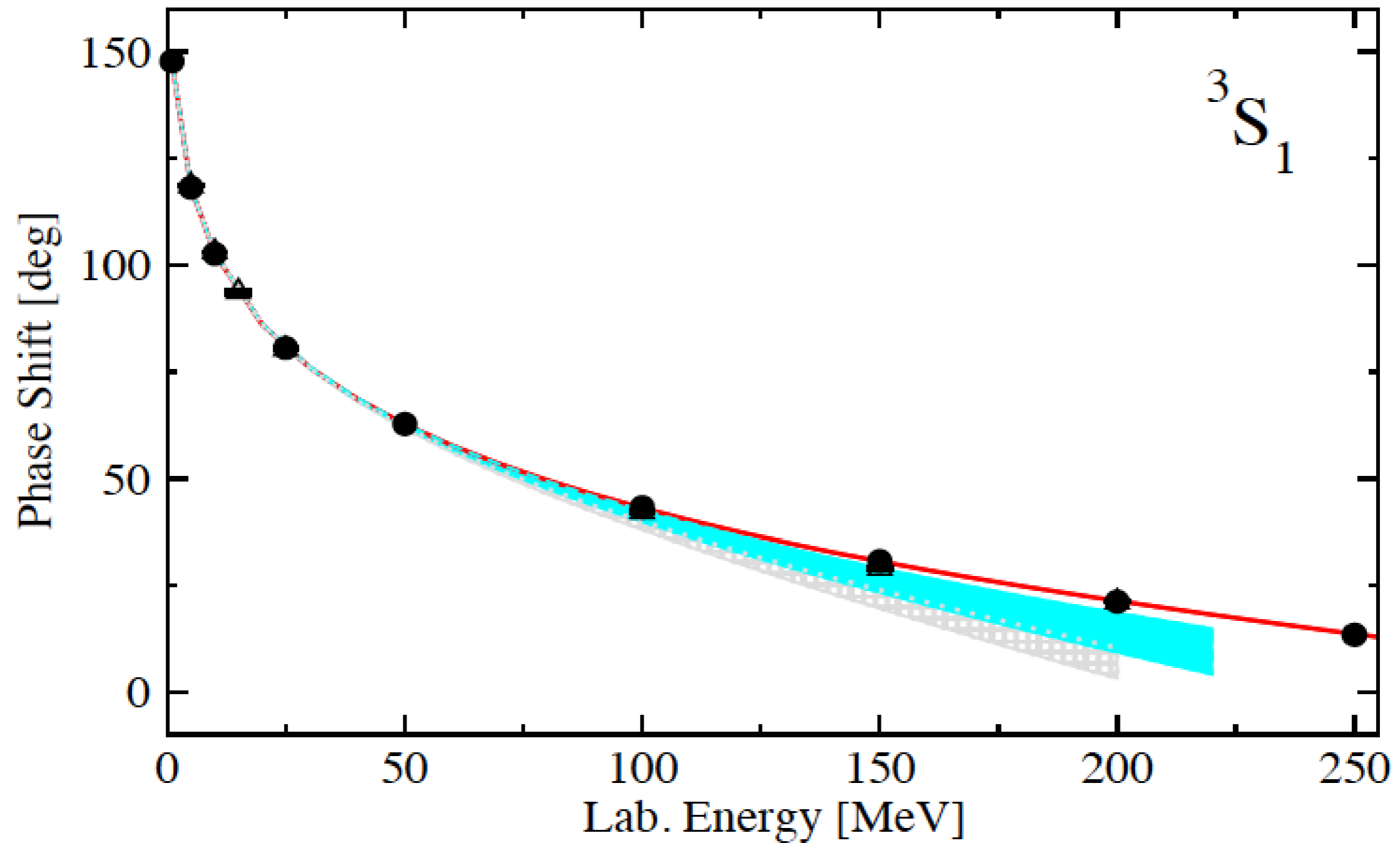
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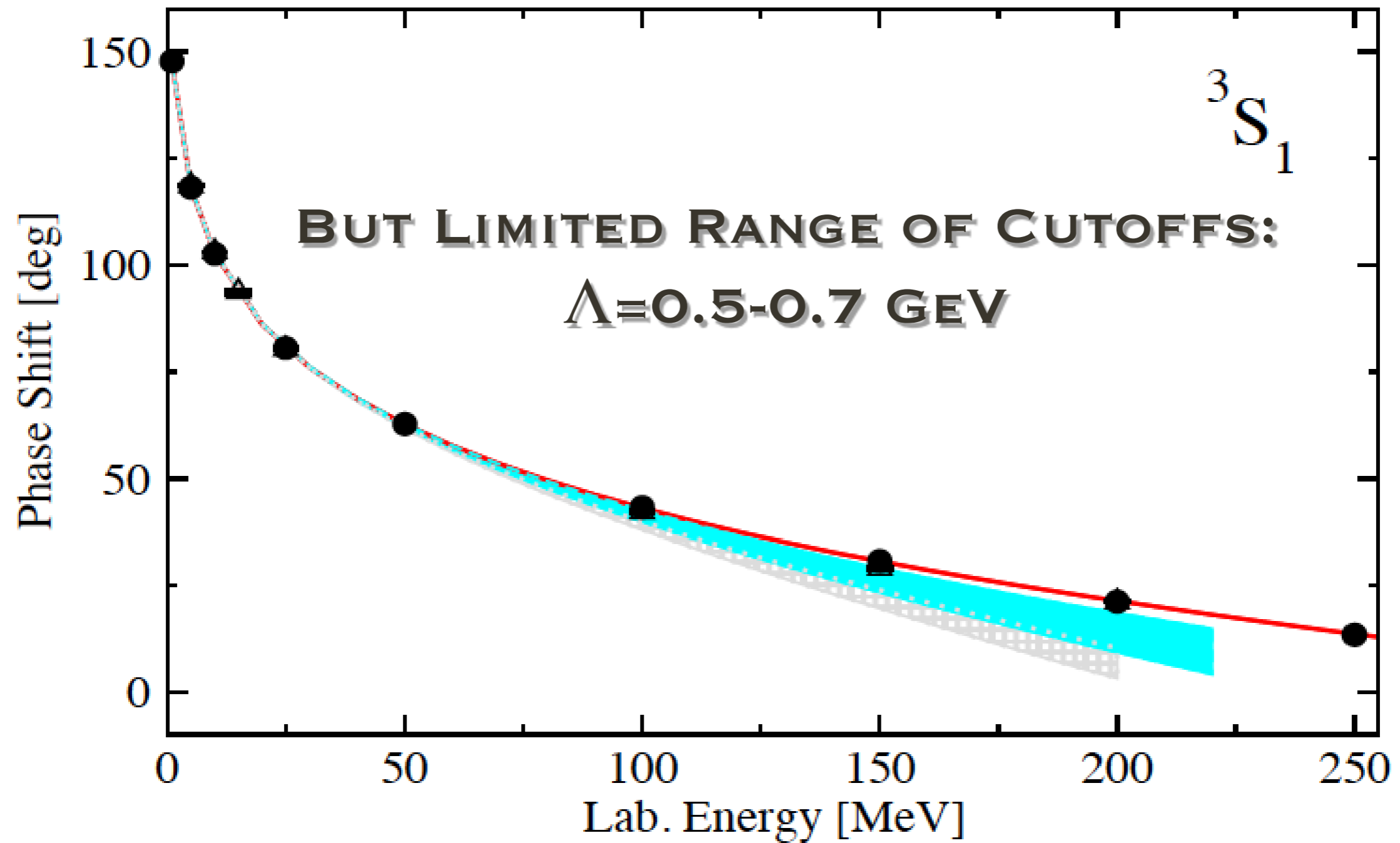
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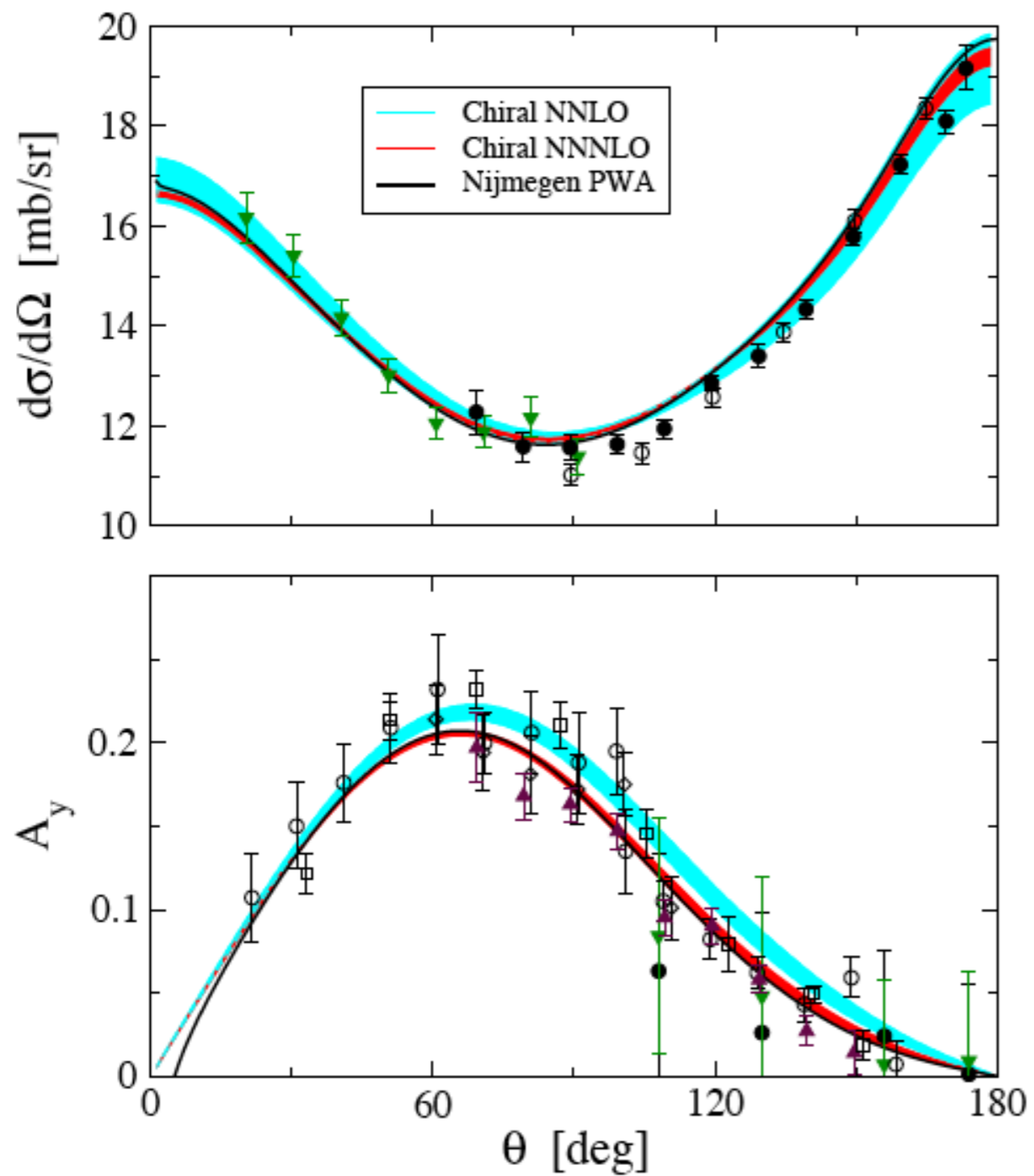
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Successes in $A=2-4$

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$T_{\text{lab}}=50 \text{ MeV}$

- $N^3\text{LO}$ potential, χ^2/dof comparable to AV18.

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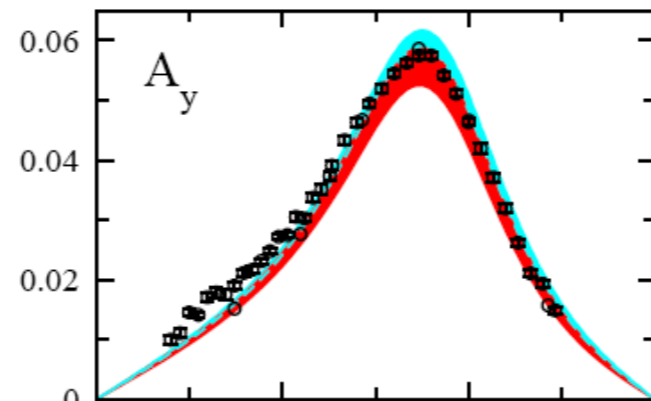
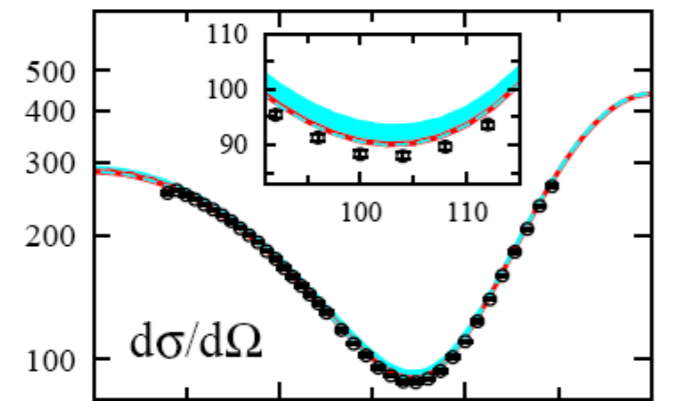
- Reproduce $A=3$ and 4 observables

Epelbaum, Nogga, et al.(2002)

	NLO	NNLO	"Exp."
${}^3\text{H}$	-7.53..-8.54	-8.68	-8.68
${}^4\text{He}$	-23.87..-29.57	-29.51..-29.98	-29.6

Successes in $A=2-4$

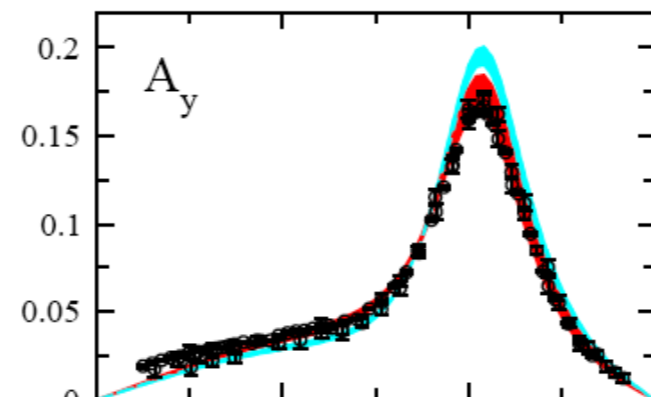
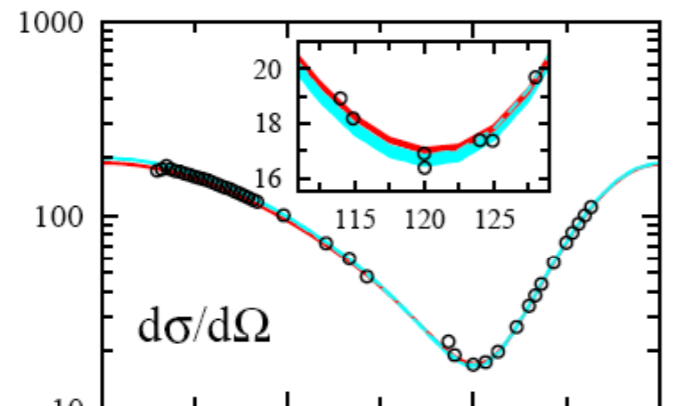
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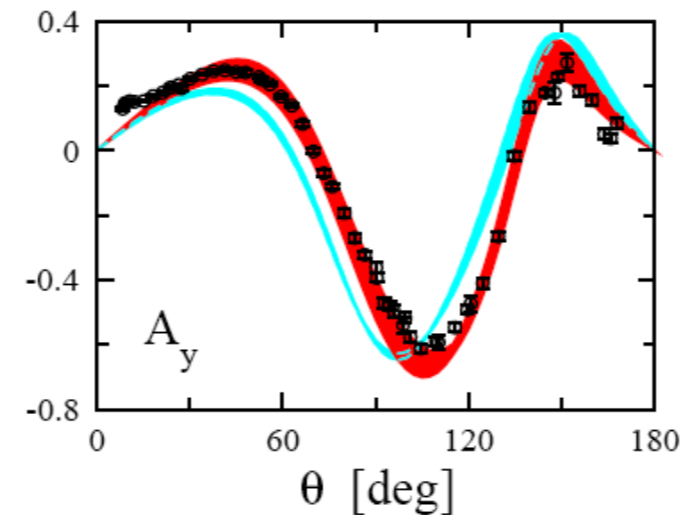
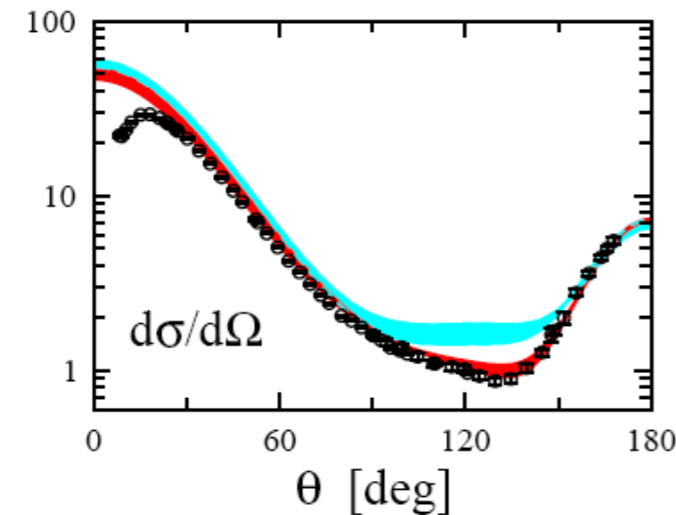
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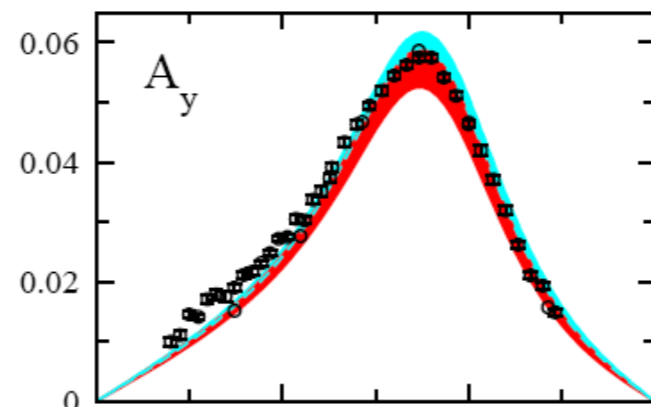
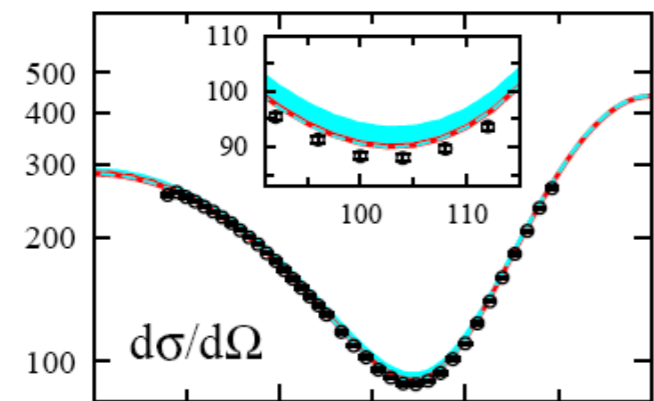
$E=65$ MeV



Courtesy E. Epelbaum

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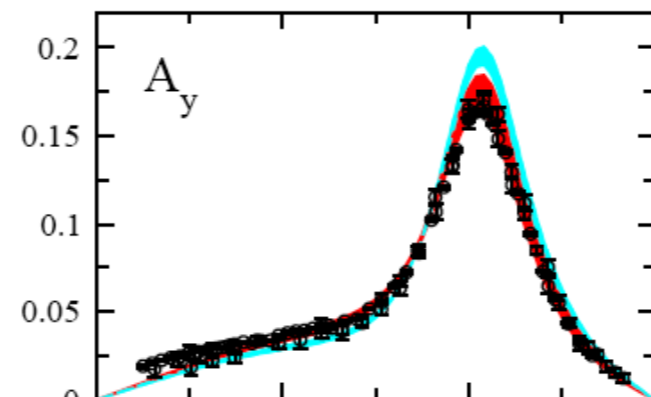
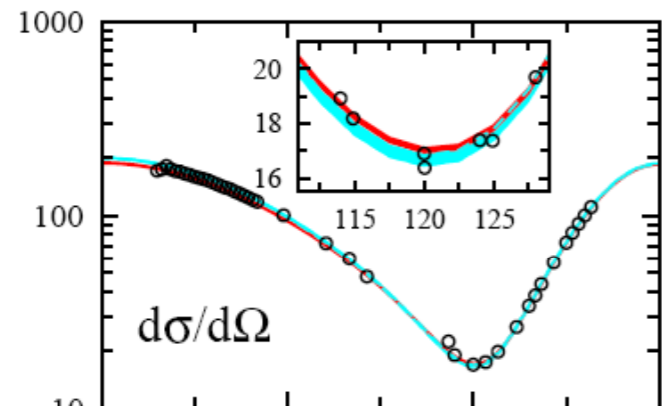
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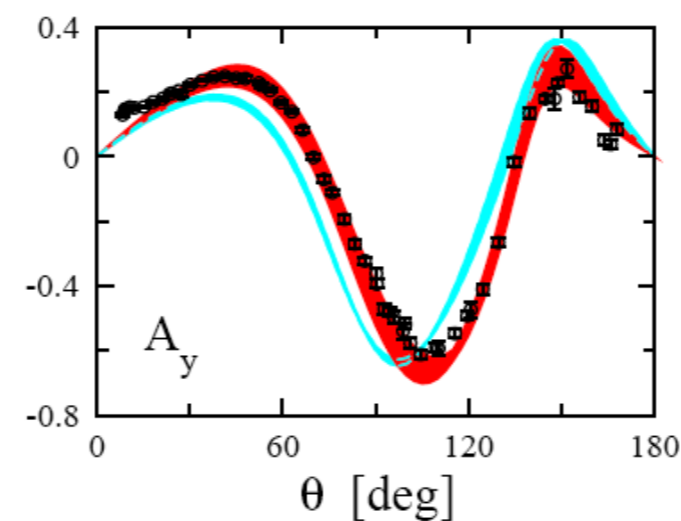
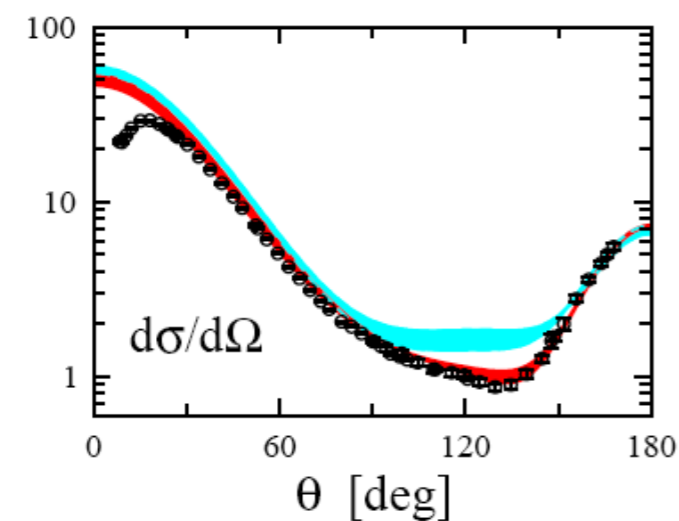
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- Applications to many-body systems: see talk of R. Roth

Courtesy E. Epelbaum

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Goal: once we understand what terms are present in χEFT up to some order, we can include them in a potential, and use it with a low cutoff in order to do nuclear physics calculations

Fun facts about one-pion exchange

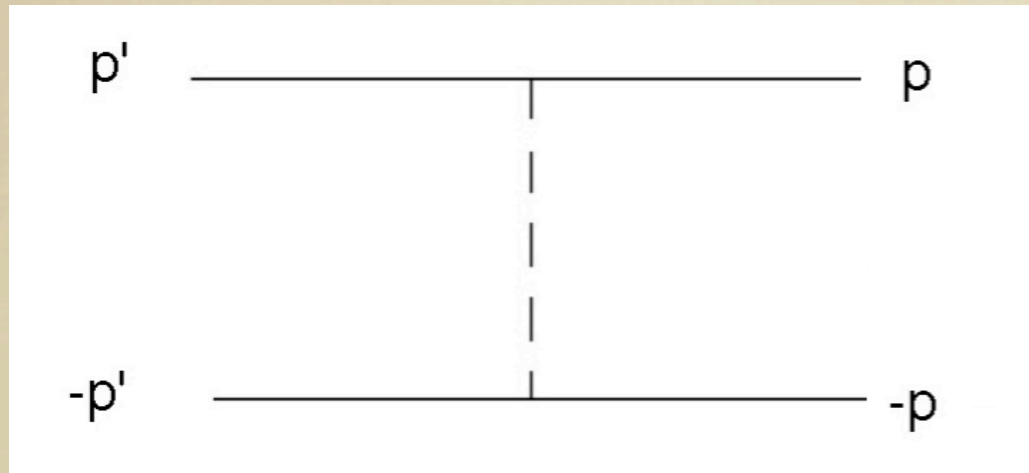
$$V(\mathbf{r}) = \tau_1^a \tau_2^a [\sigma_1 \cdot \sigma_2 Y(r) + S_{12}(\hat{r}) T(r)]$$

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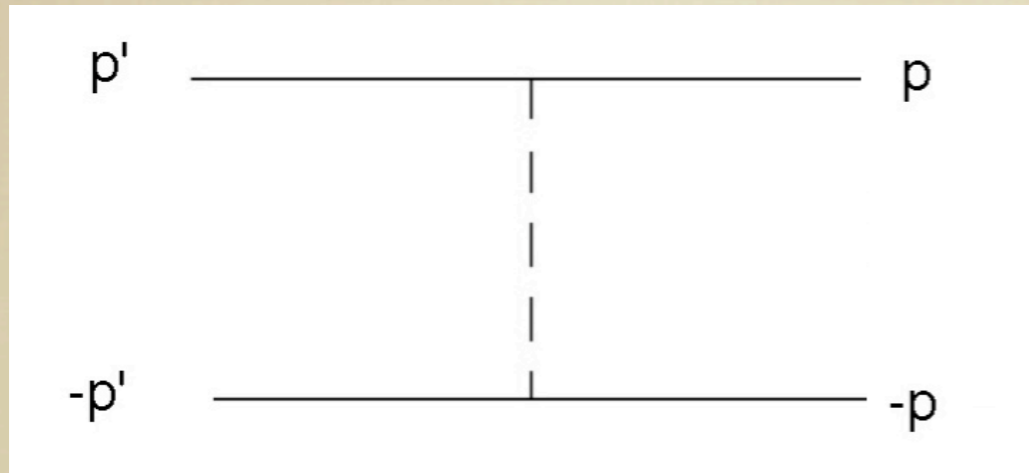
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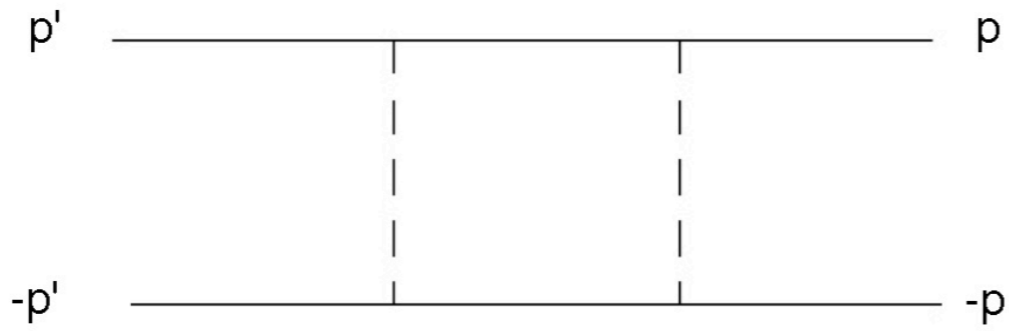
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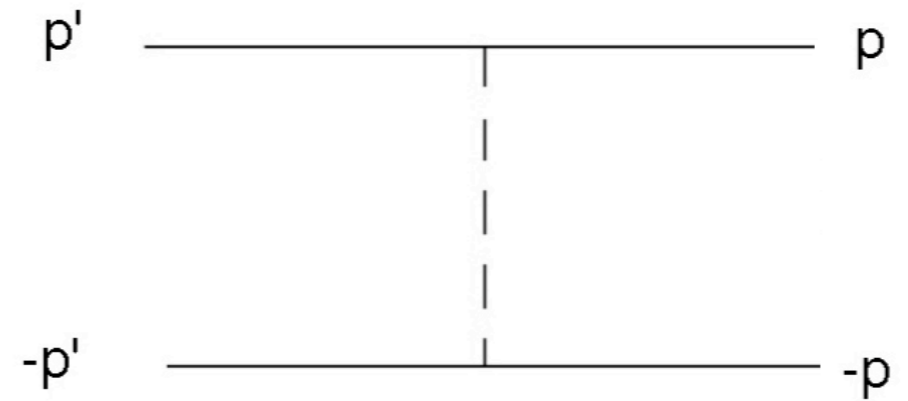
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- Momentum scales present: m_π and $\Lambda_{NN} = \frac{16\pi f_\pi^2}{g_A^2 M} \approx 300 \text{ MeV}$
- χ SB predicts $1/r^3$ potential that couples waves with $\Delta L=2$
- Tensor part of 1π exchange does not appear for $S=0$
- $1/r^3$ part of 1π exchange “screened” by centrifugal barrier for large L

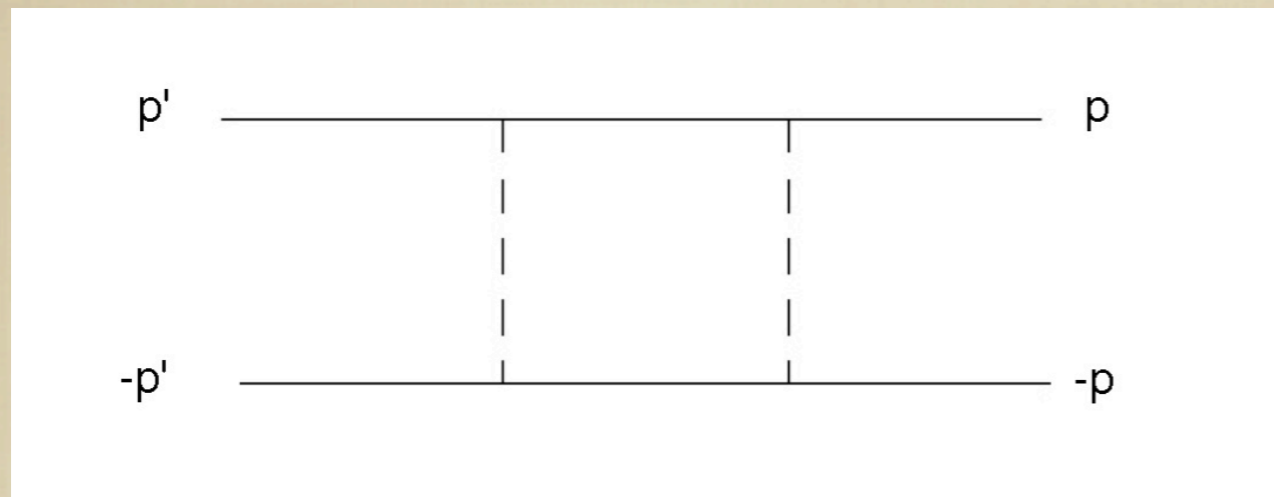
The quest for leading order I



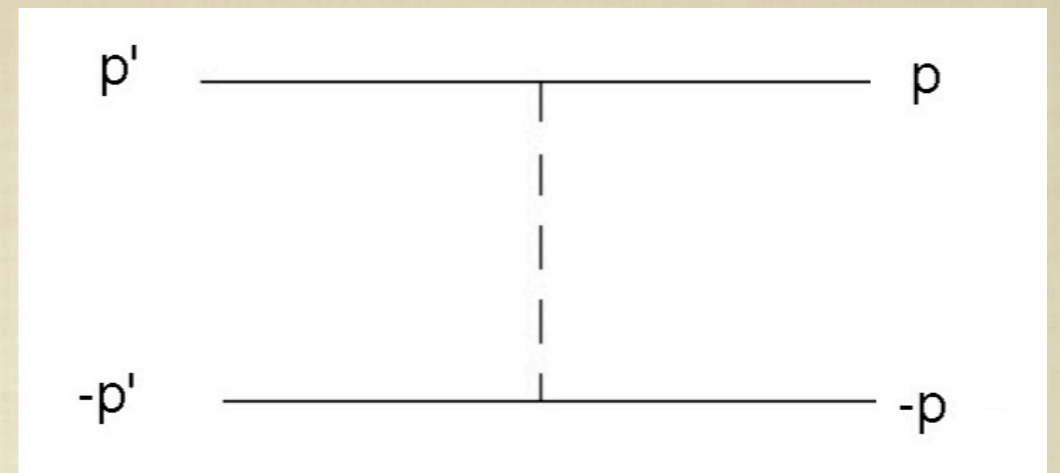
VS



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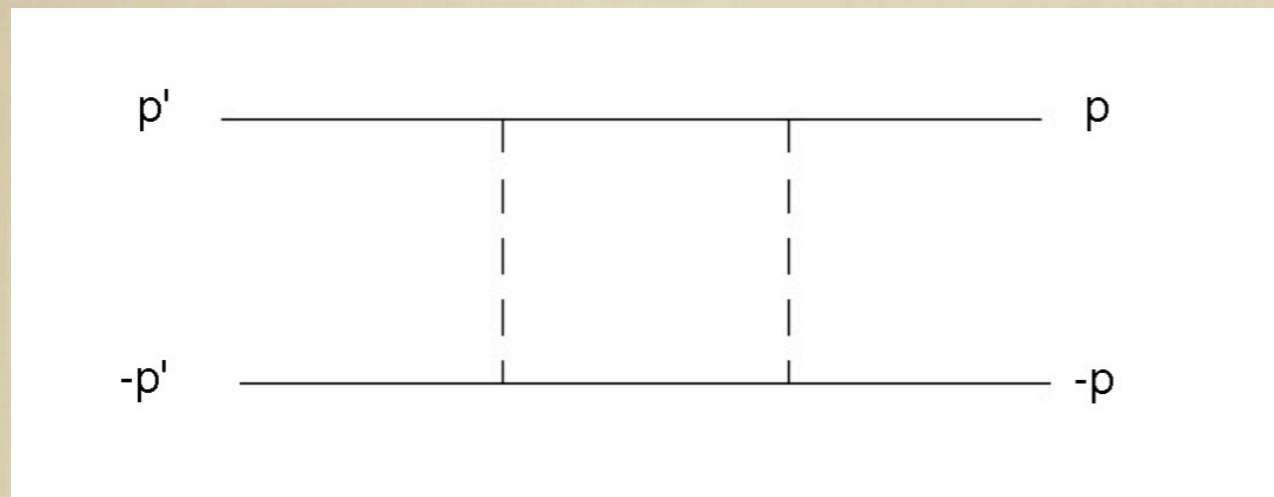
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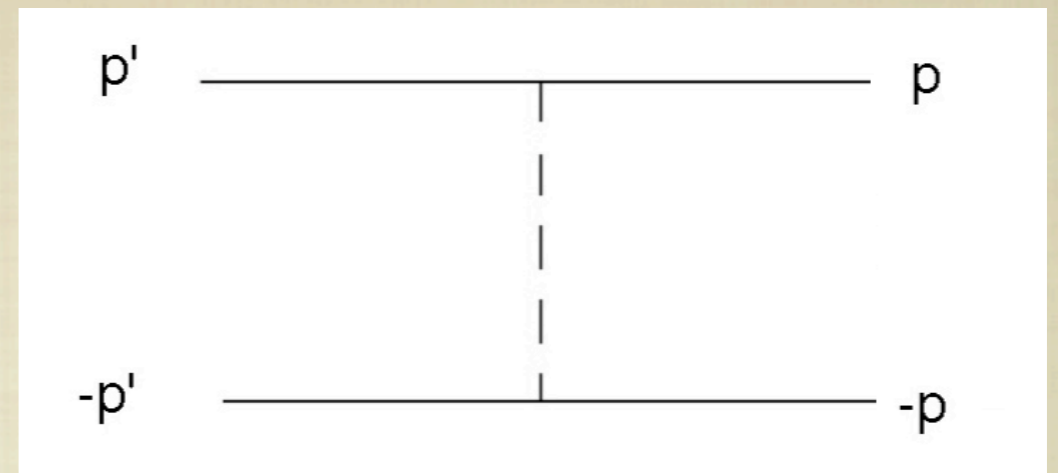
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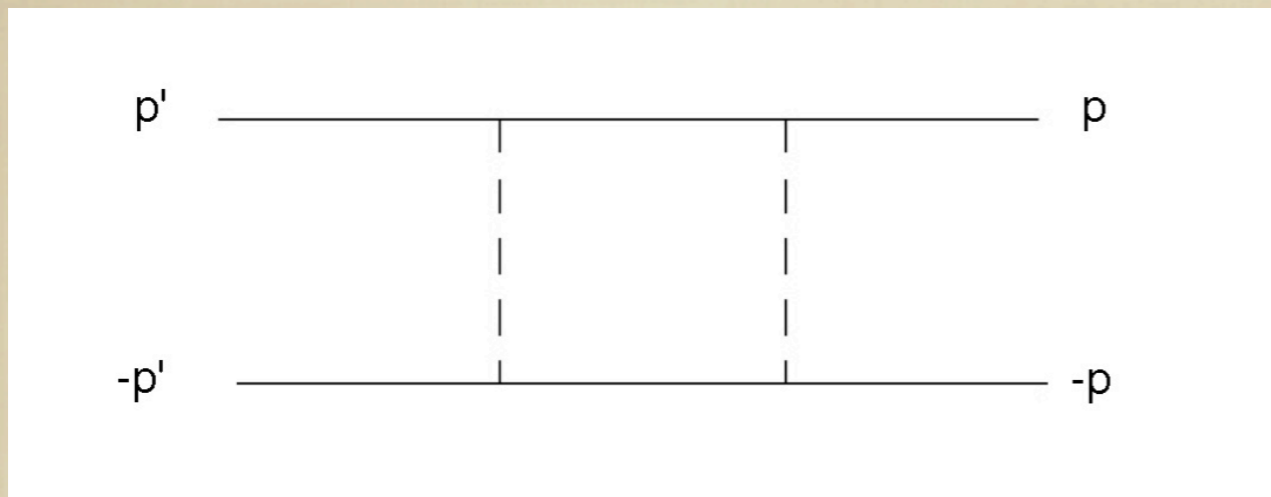


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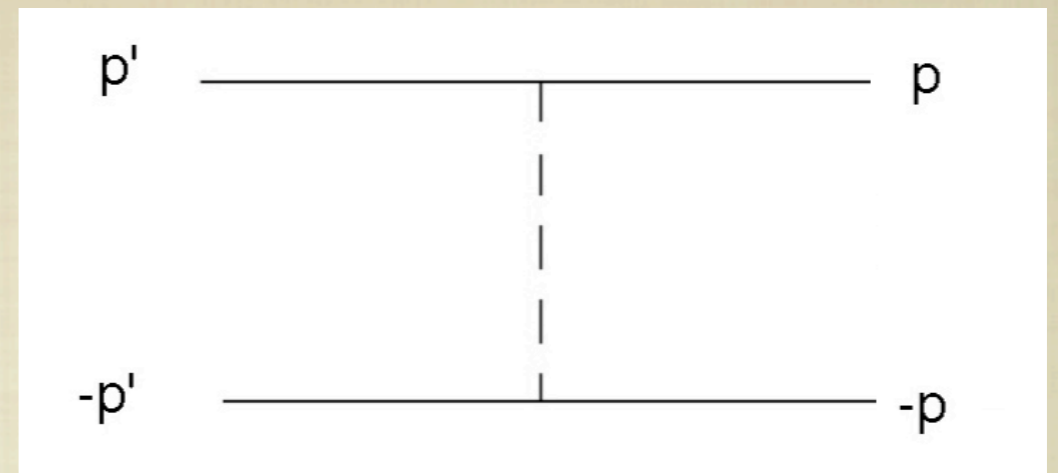


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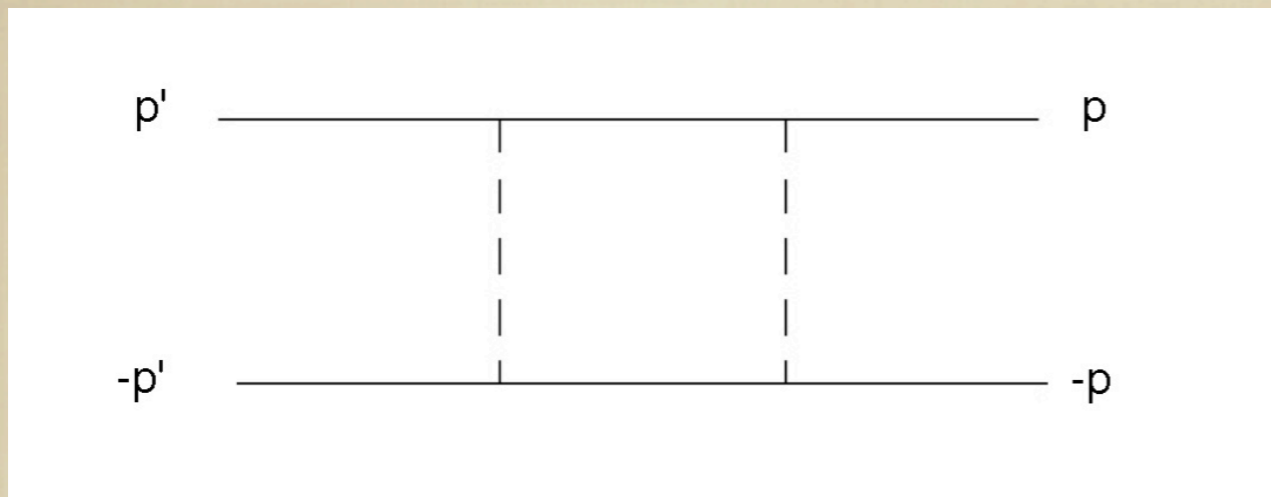


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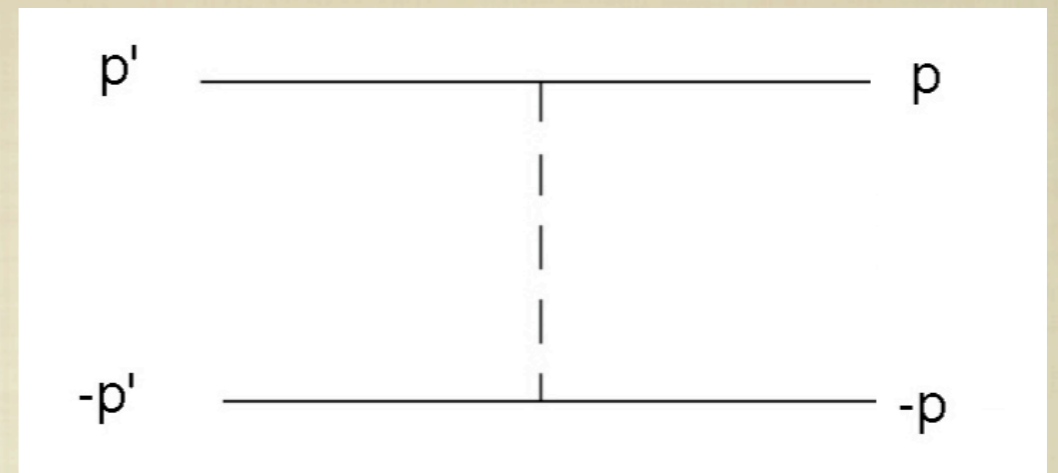


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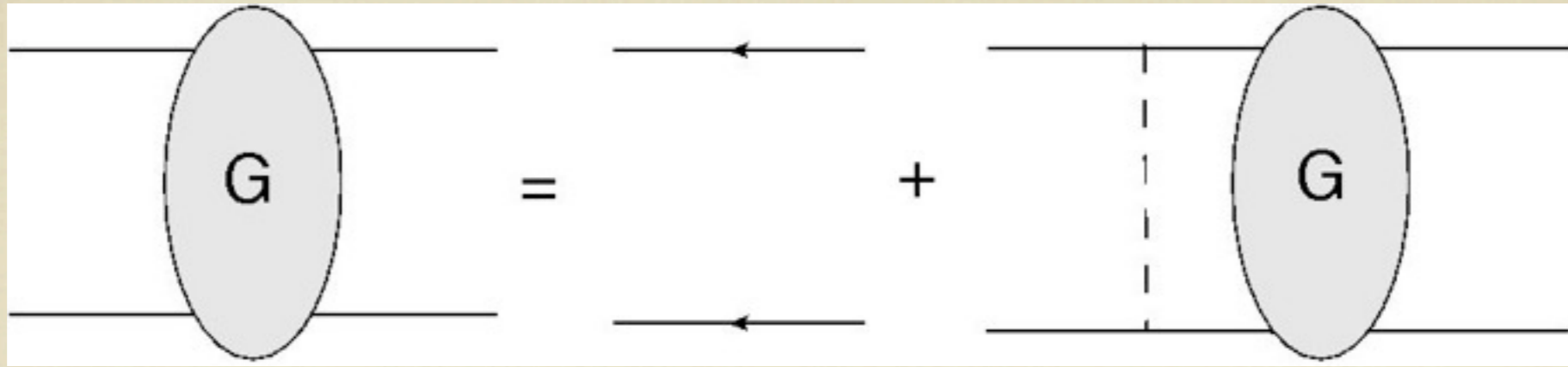


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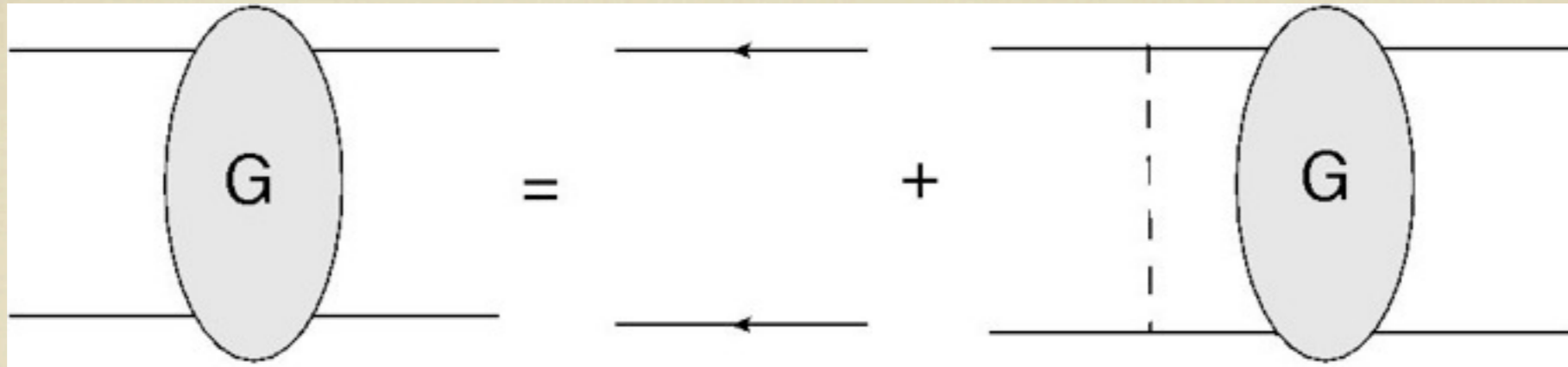
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- Λ_{NN} is a new low-energy scale, thus this is not χ PT. But, higher-order pieces of chiral potential suppressed by $\Lambda_{NN}/\Lambda_{\chi SB}$.
- Perturbation theory should also be OK for: (a) higher partial waves, (b) 1π exchange in singlet waves, (c) $p \ll \Lambda_{NN}$

The quest II: to iterate or not to iterate



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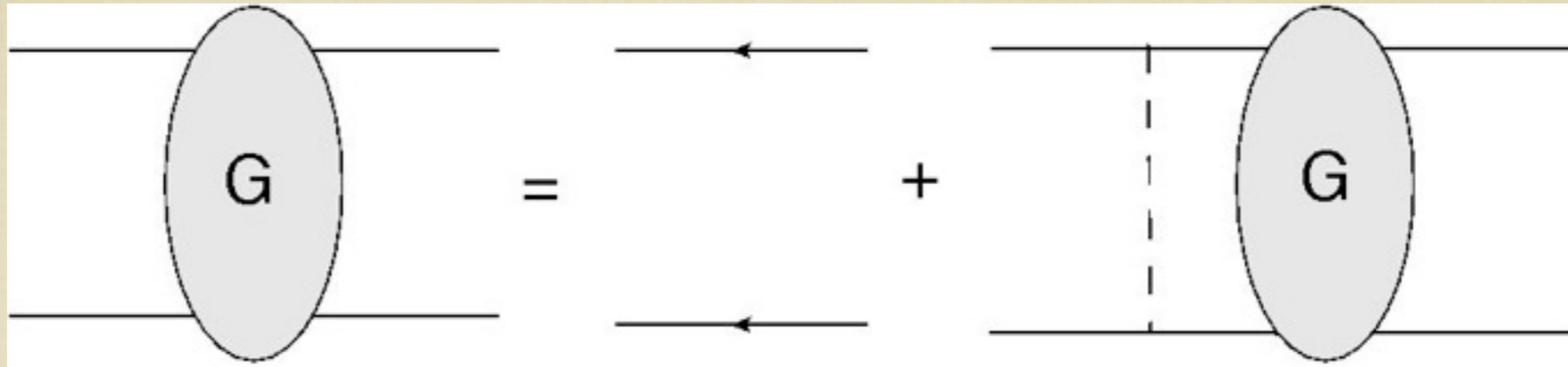
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- Do this in 3S_1 , 3P_0 , 3P_1 , 3P_2 , and possibly D waves

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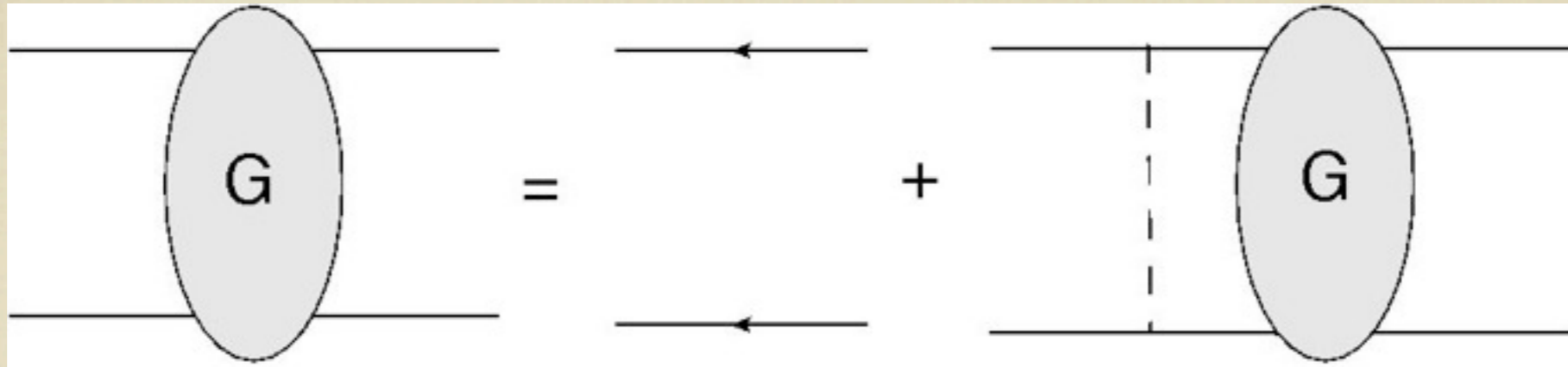
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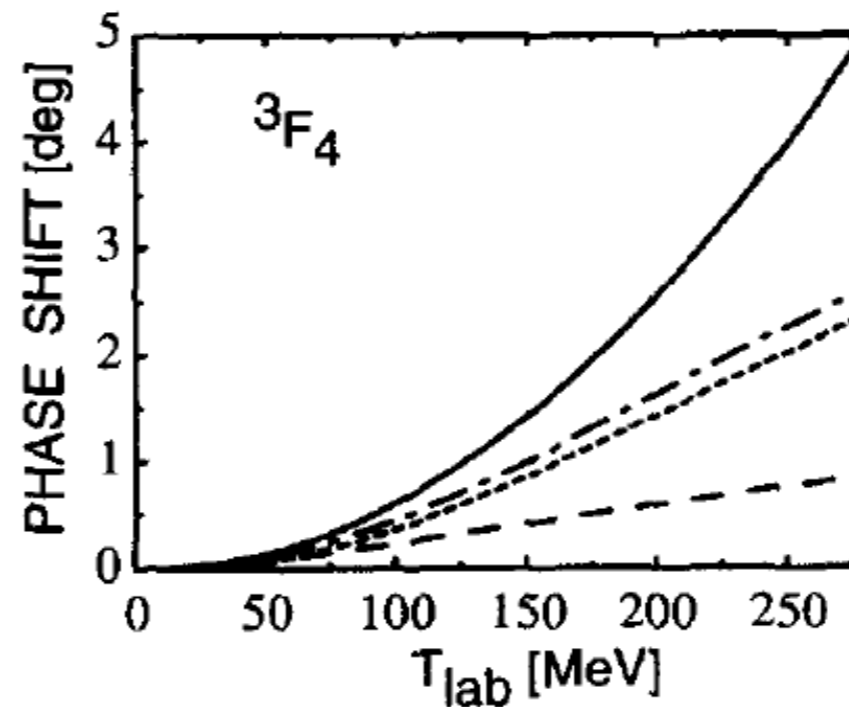
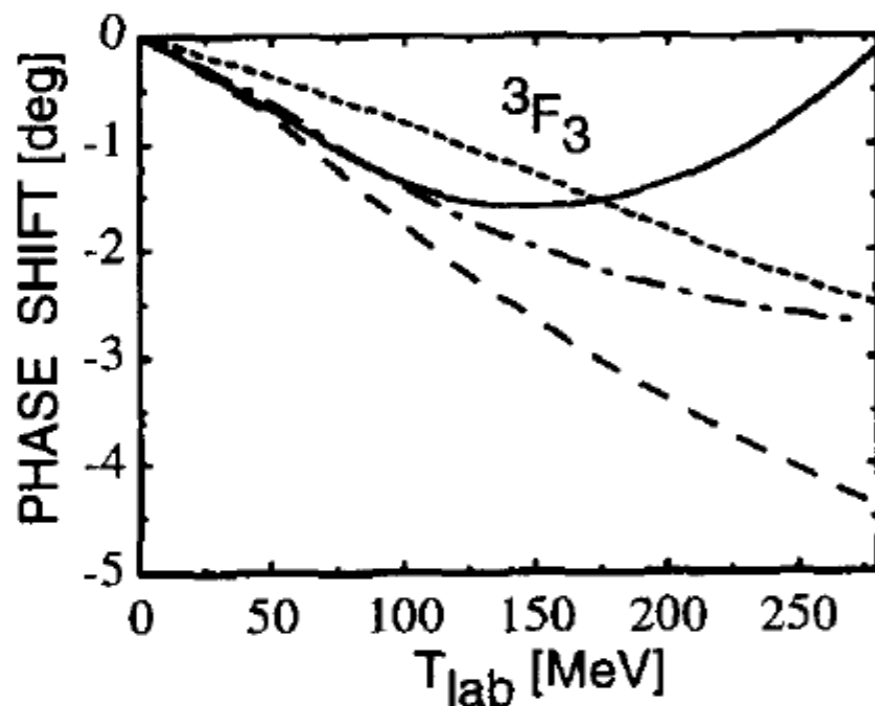
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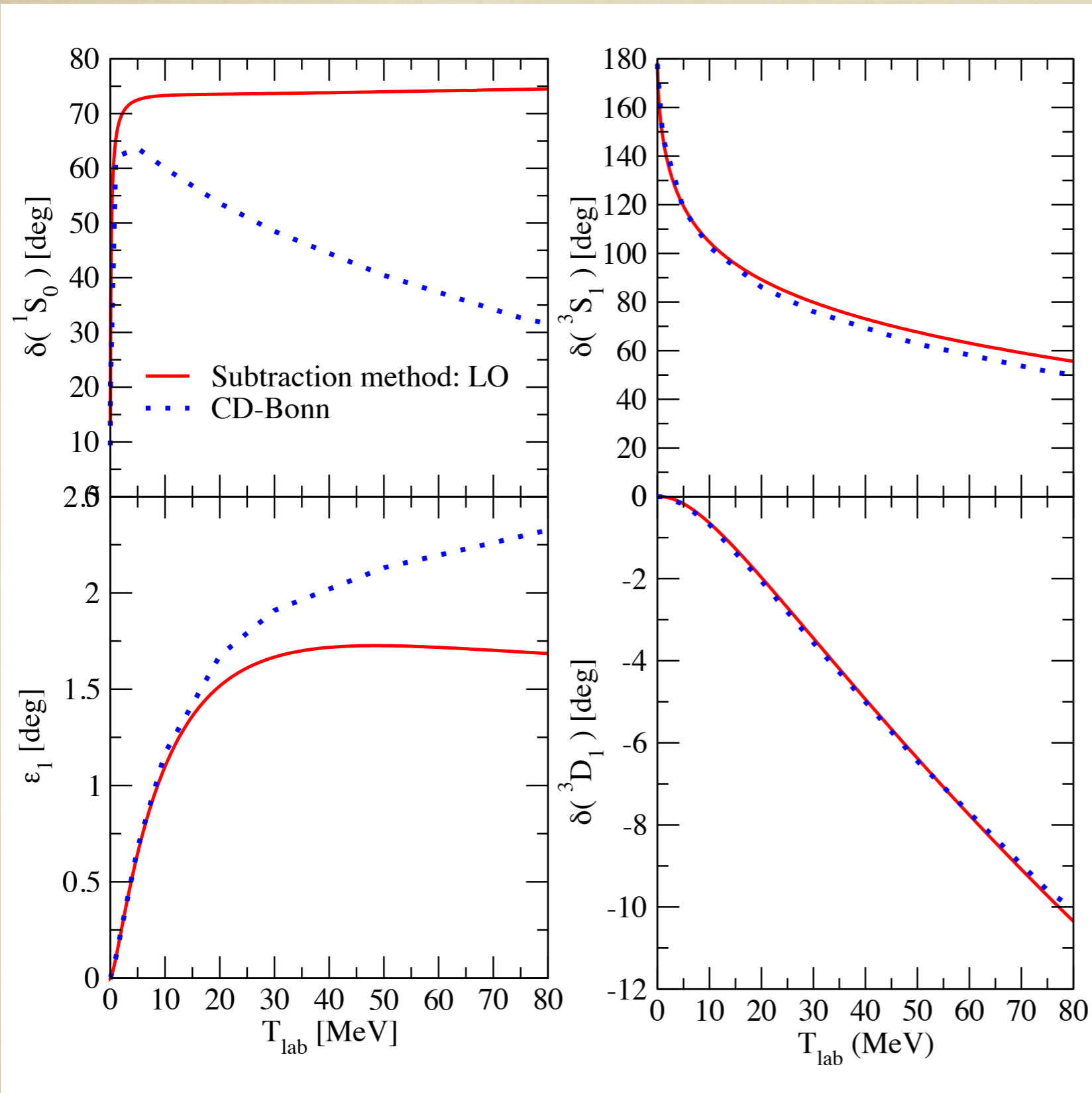
Kaiser, Brockmann, Weise (1997)



Standard
 χ^{PT}

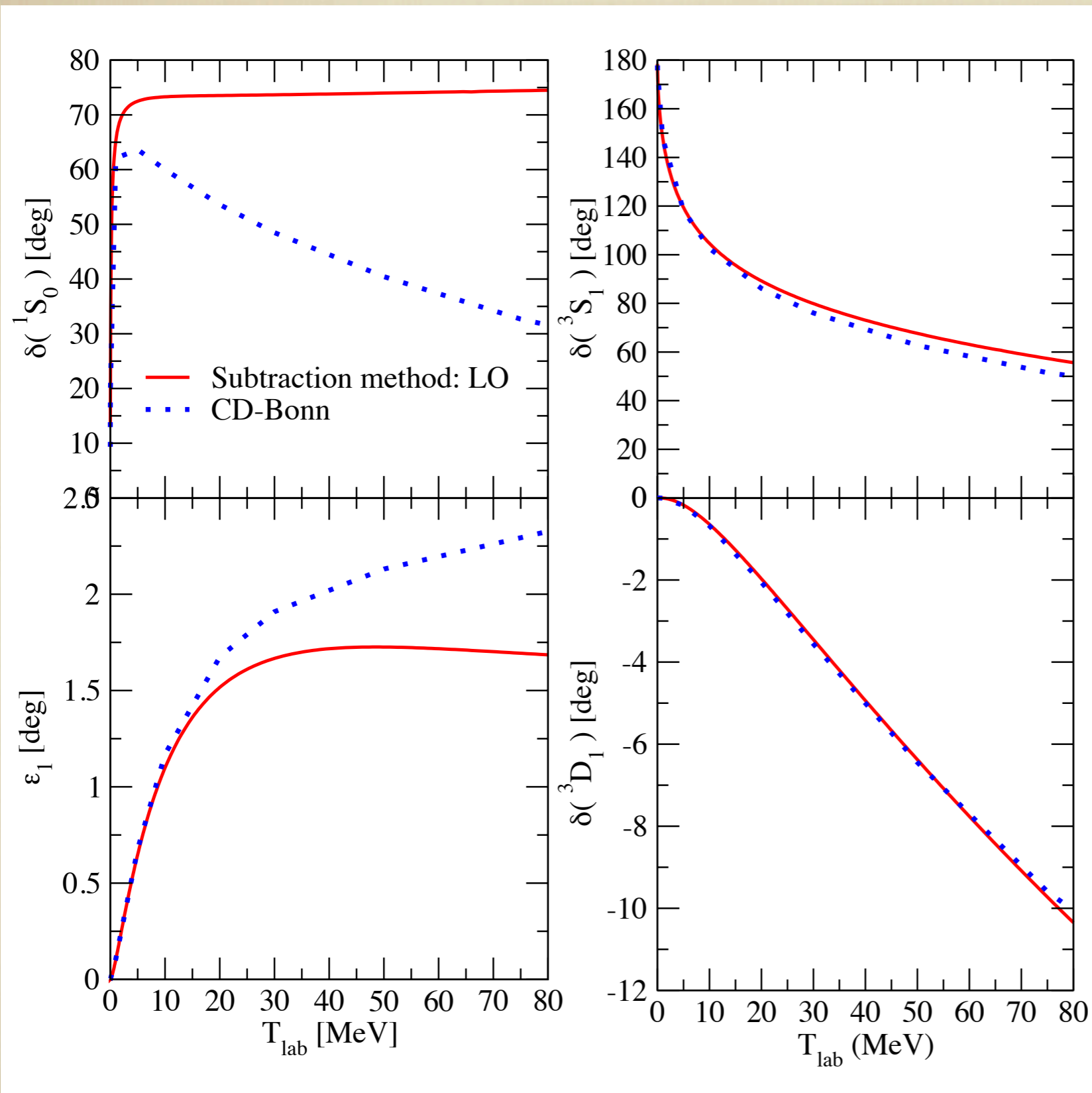
The quest III: S waves

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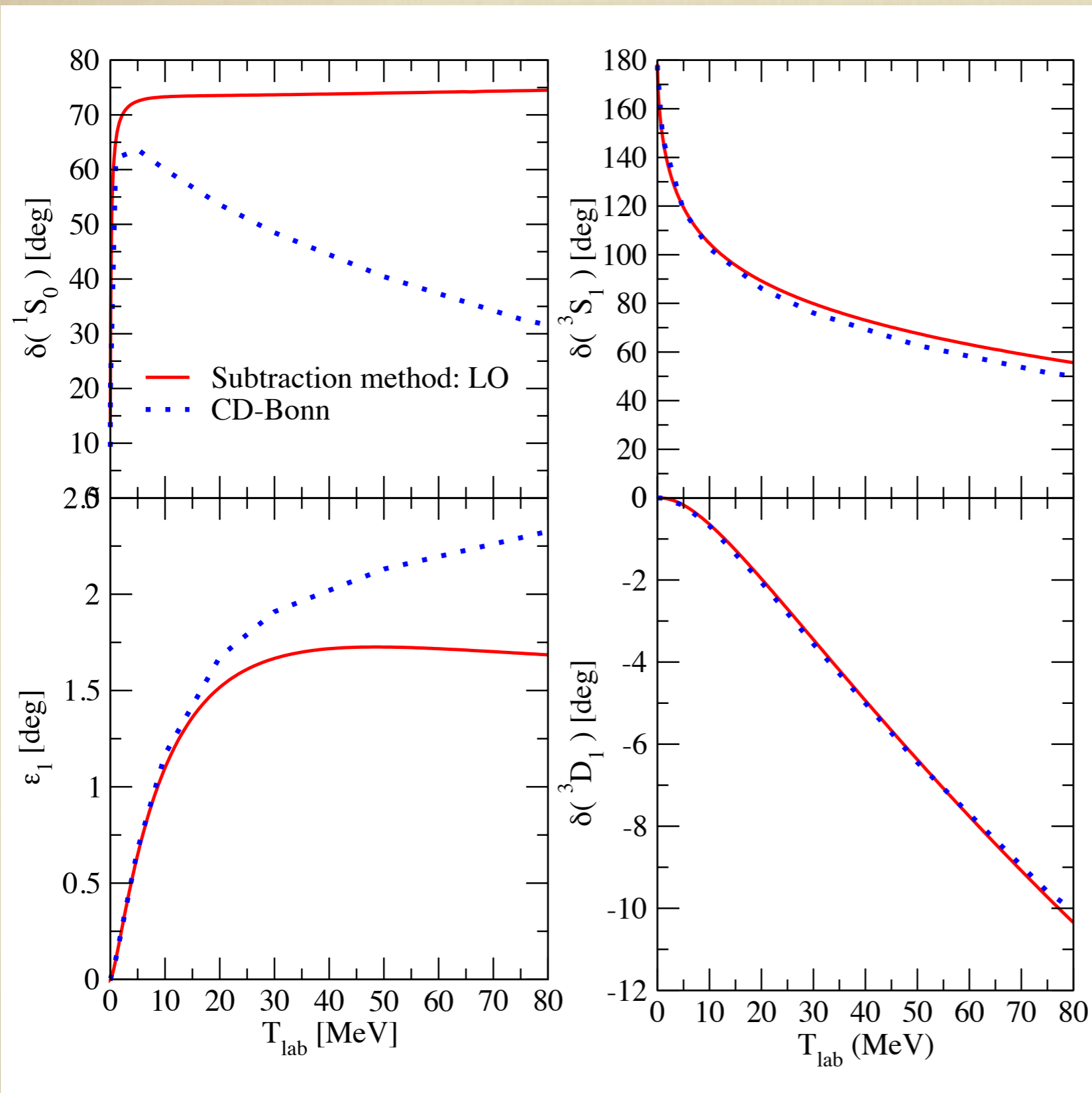
■ Stable for wide range of cutoffs

■ Subtractive renormalization numerically efficient

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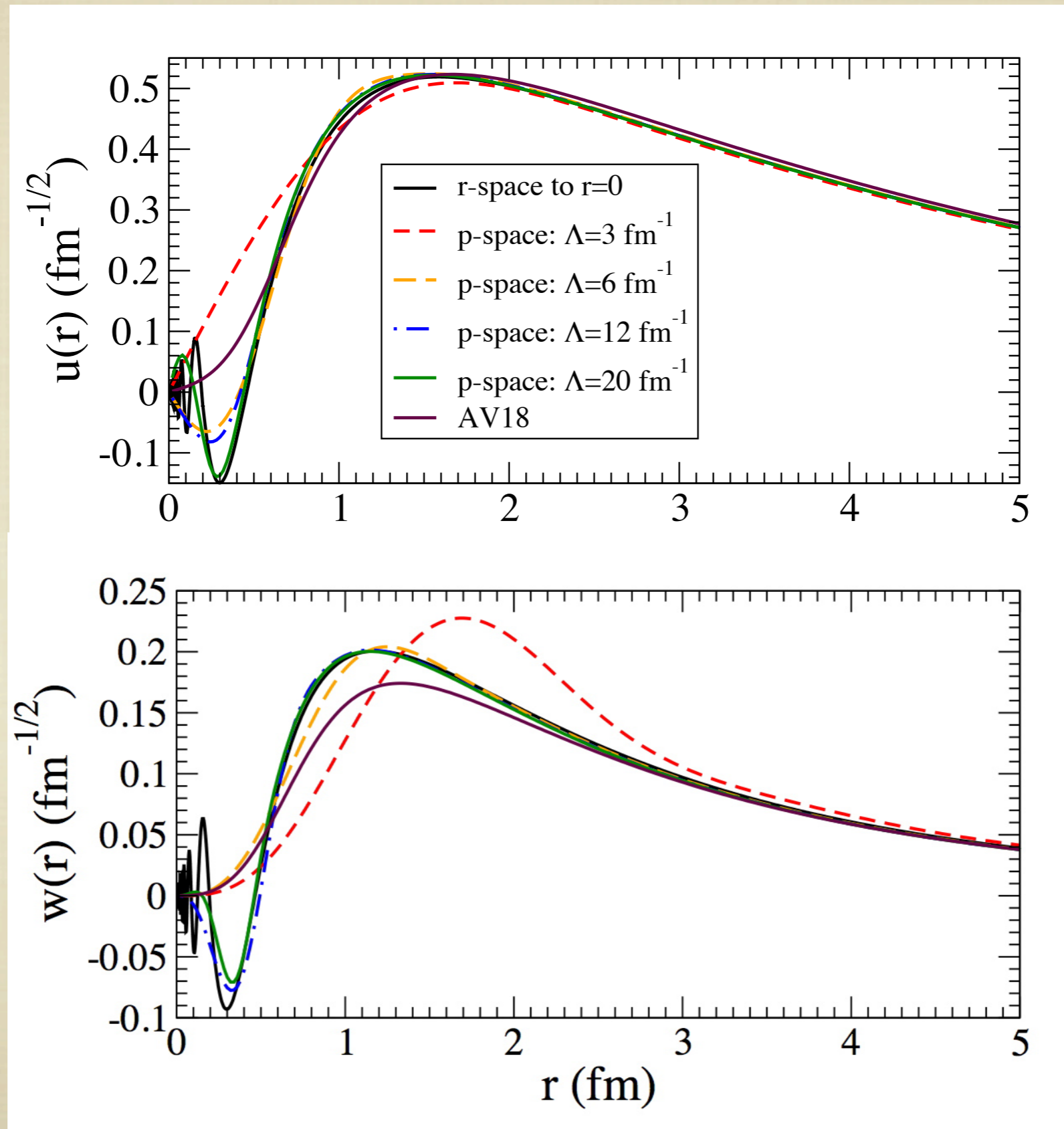
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- One-pion exchange weak in 1S_0

χ EFT deuteron wave functions at leading order

Pavon Valderrama, Nogga, Ruiz Arriola, DP, EPJA 36, 315 (2008)



The quest IV: solving the $1/r^3$ potential

Case (1950), Sprung et al. (1994),

Beane et al. (2001),

Pavon Valderrama, Ruiz Arriola (2004-6)

- Attractive case, for $r \ll 1/\Lambda_{NN}$

$$u_1(r) = (\Lambda_{NN}r)^{3/4} \cos\left(4\sqrt{\frac{1}{\Lambda_{NN}r}}\right); u_2(r) = (\Lambda_{NN}r)^{3/4} \sin\left(4\sqrt{\frac{1}{\Lambda_{NN}r}}\right)$$

- Equally regular solutions, need boundary condition to fix phase

- c.f. $j_l(kr)$ and $n_l(kr)$ for plane waves as $r \rightarrow 0$

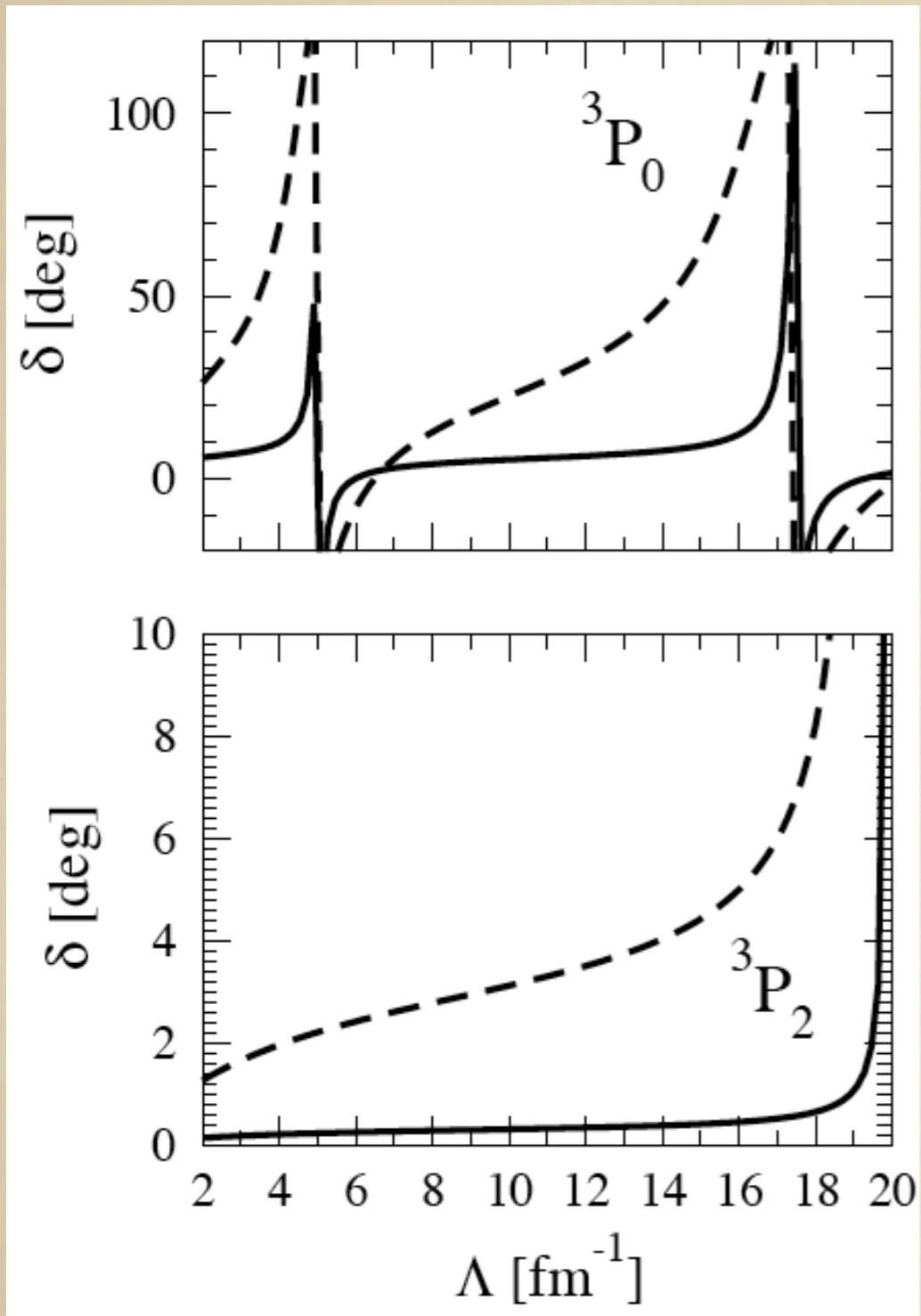
- Repulsive, for $r \ll 1/\Lambda_{NN}$

$$u_1(r) = (\Lambda_{NN}r)^{3/4} \exp\left(4\sqrt{\frac{1}{\Lambda_{NN}r}}\right); u_2(r) = (\Lambda_{NN}r)^{3/4} \exp\left(-4\sqrt{\frac{1}{\Lambda_{NN}r}}\right)$$

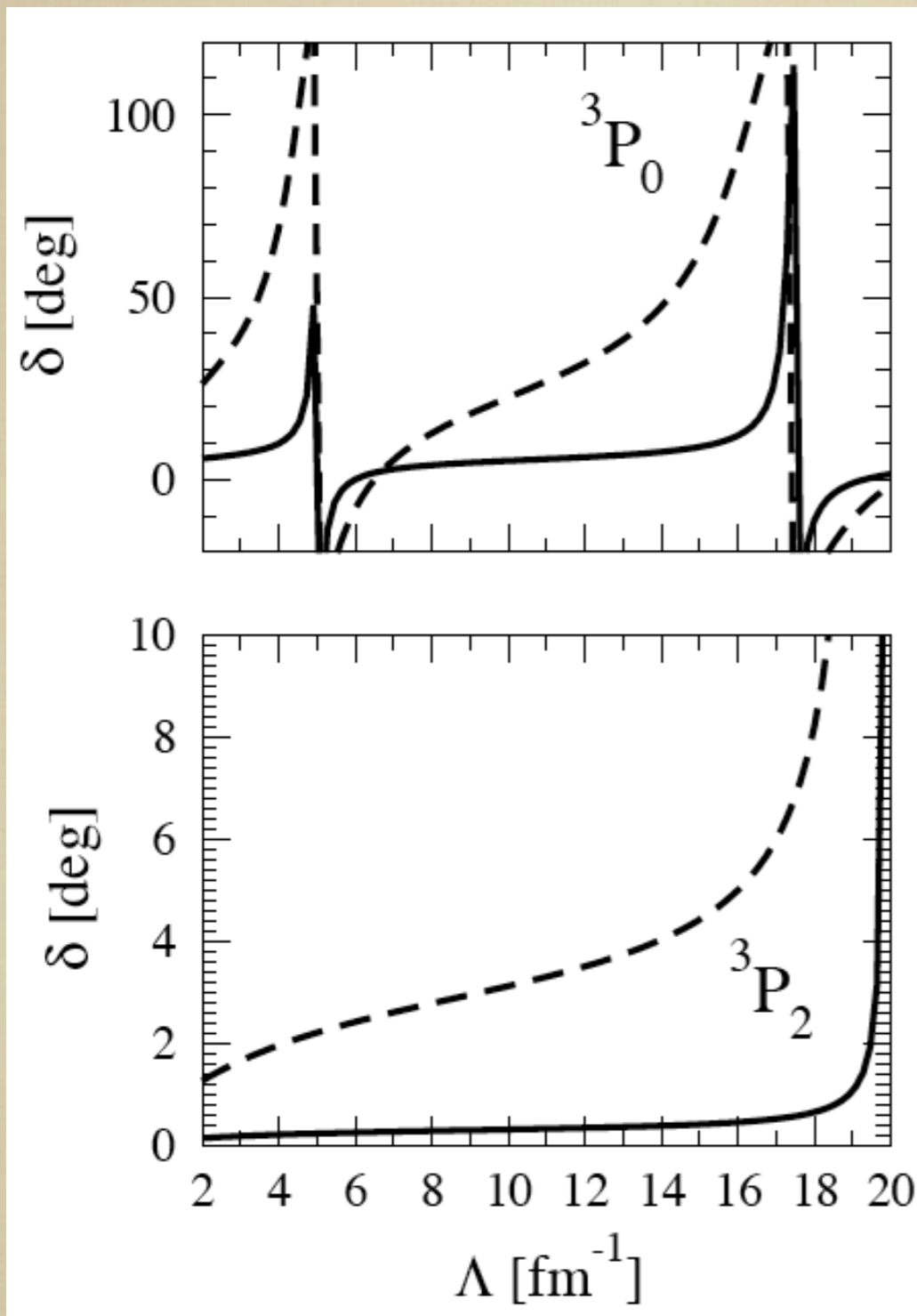
- Still need boundary condition to fix “phase”, but results insensitive to choice

The quest V: power counting

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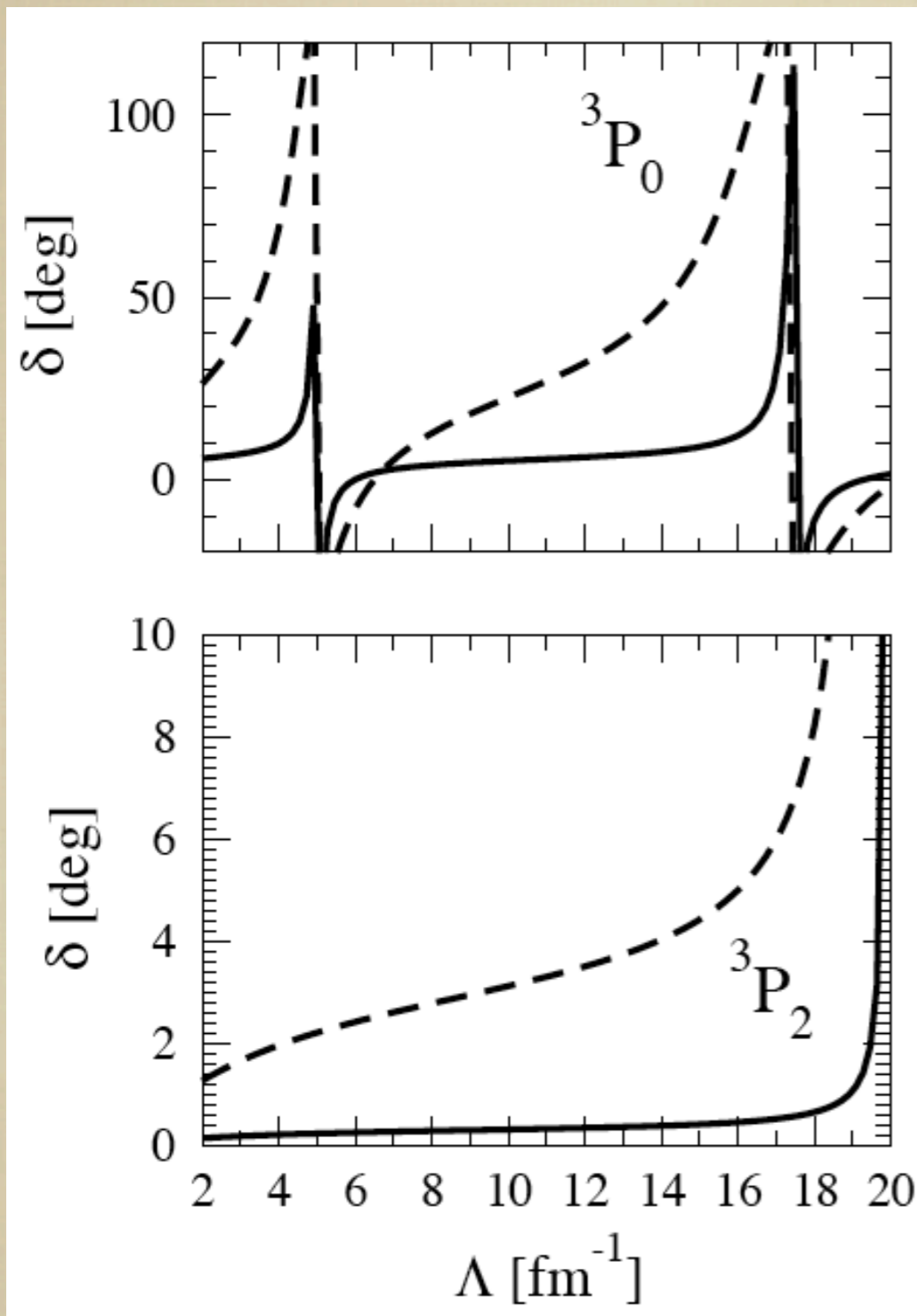
The quest V: power counting



- Need contact terms in certain P waves already at LO, in order to specify short-distance b.c.

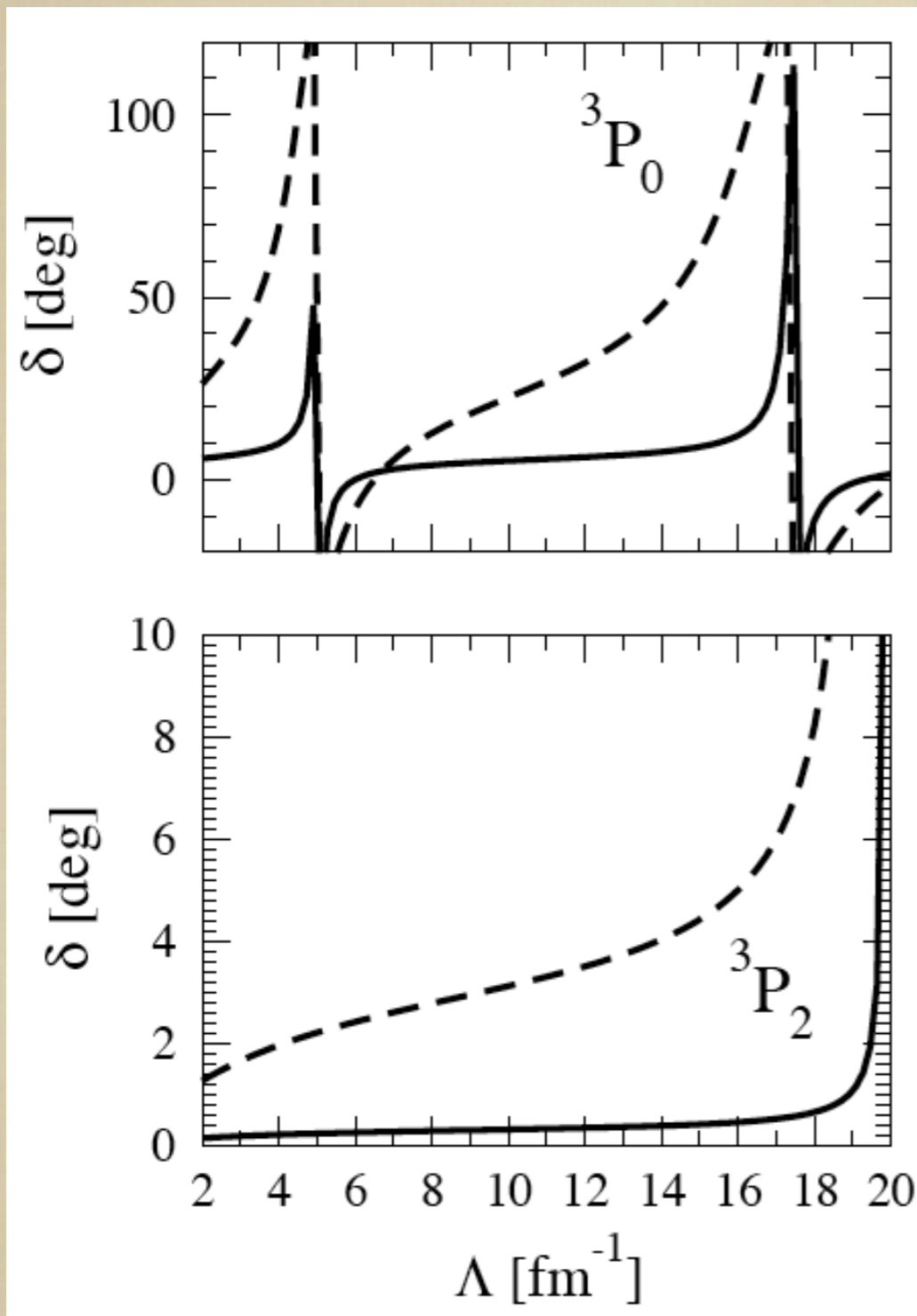
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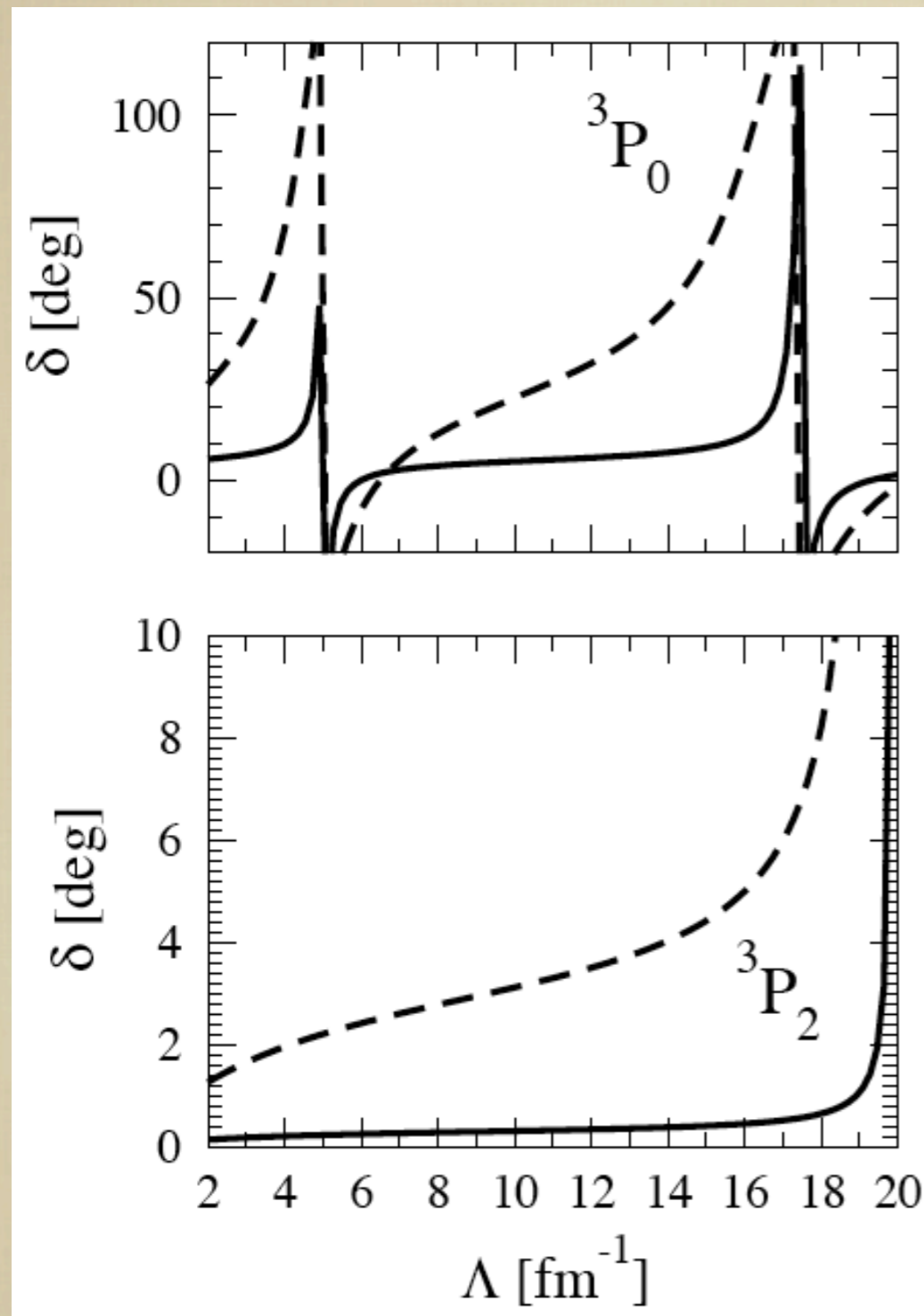
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Birse

Moral: NDA doesn't predict scaling of short-distance operators needed for renormalization if LO wave functions are not plane waves

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- Argue that cutoff should never get above m_ρ Epelbaum, Meissner (2006)

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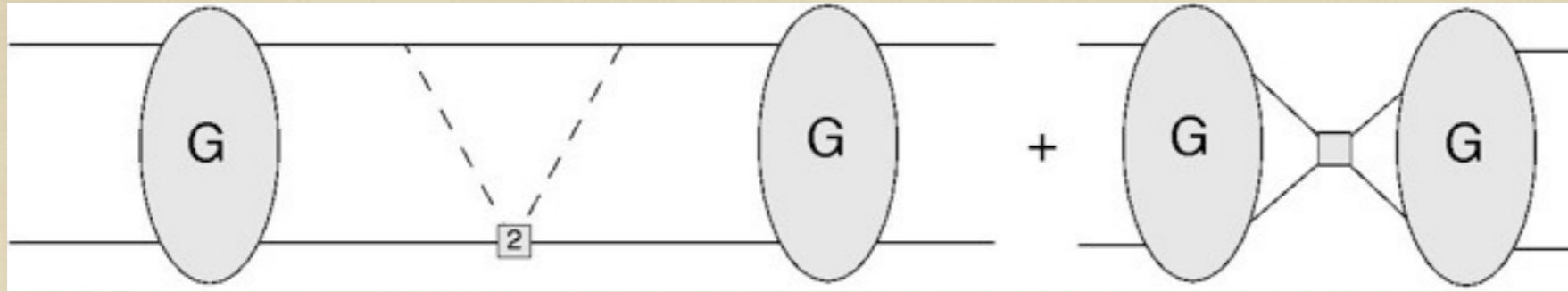
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- Analysis tool: co-ordinate space matrix elements of $V^{(3)}$ (say) between $|\psi^{(0)}\rangle$
- Equivalent momentum-space formulation

An example: sub-leading TPE in 3S1

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- Need two counterterms, same as in NDA, although scaling of matrix element with r_c modified
- Real difference in P waves, where $\sim r^2$ gets replaced by $\sim r^{3/4}$
Birse (2006)
- Two NN contact interactions needed to renormalize $V^{(3)}$ in attractive triplet P waves

Shallow poles: why the 1S_0 is special

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- Since deuteron is also fine-tuned there is a similar (but not the same!) enhancement of contact interactions in the 3S_1 channel

Birse (2009)

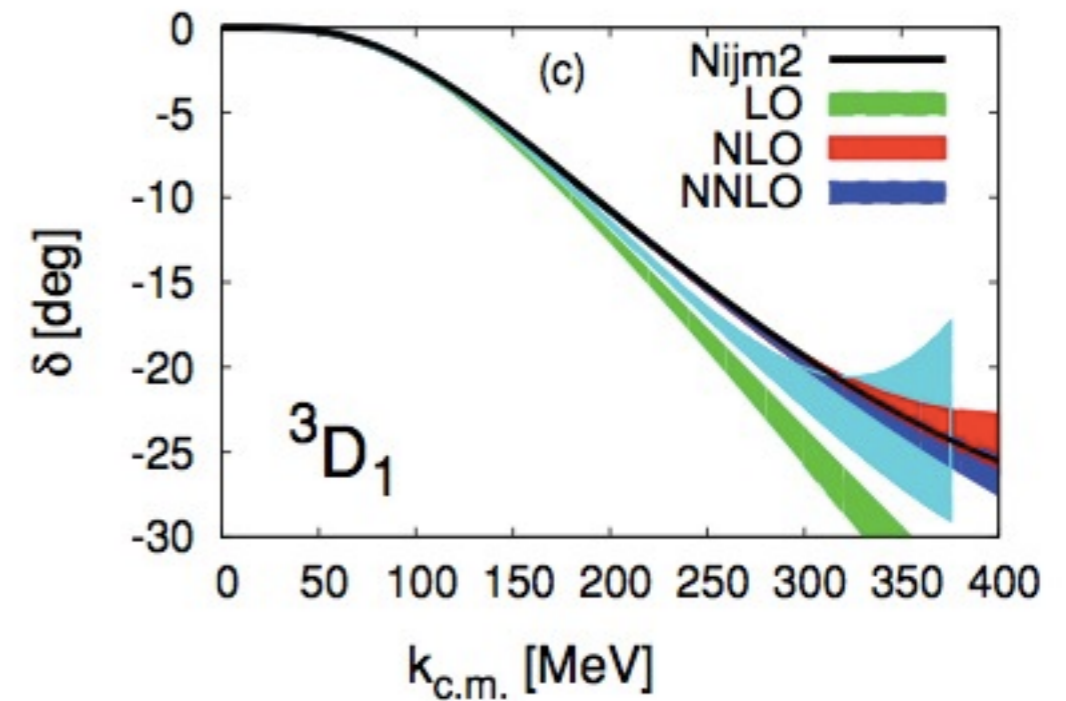
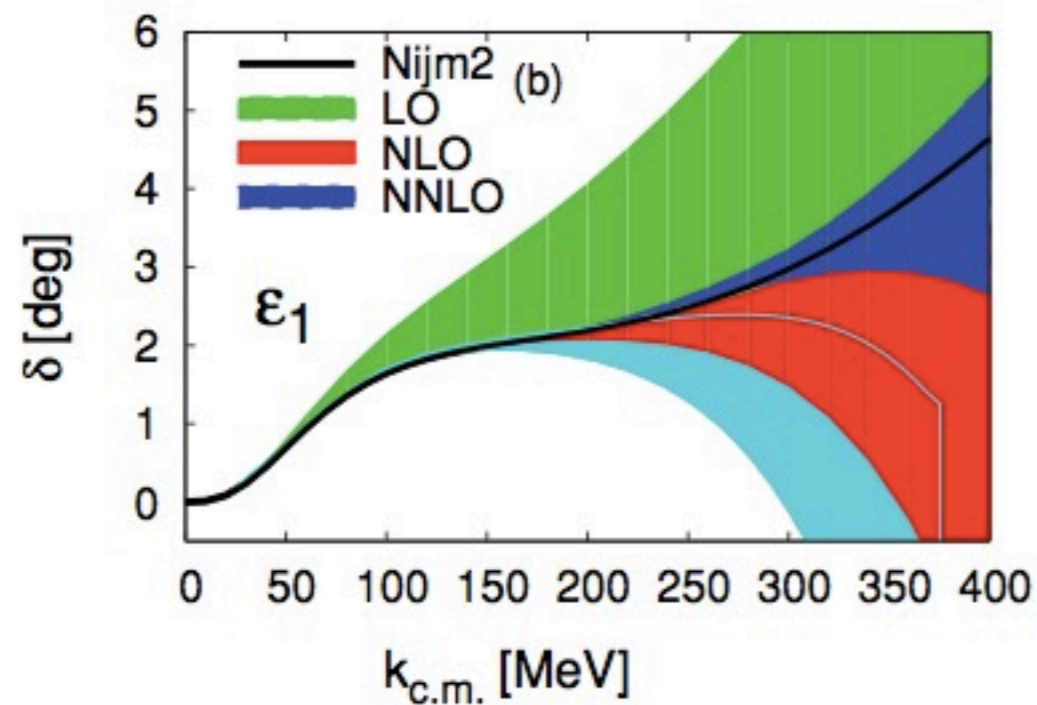
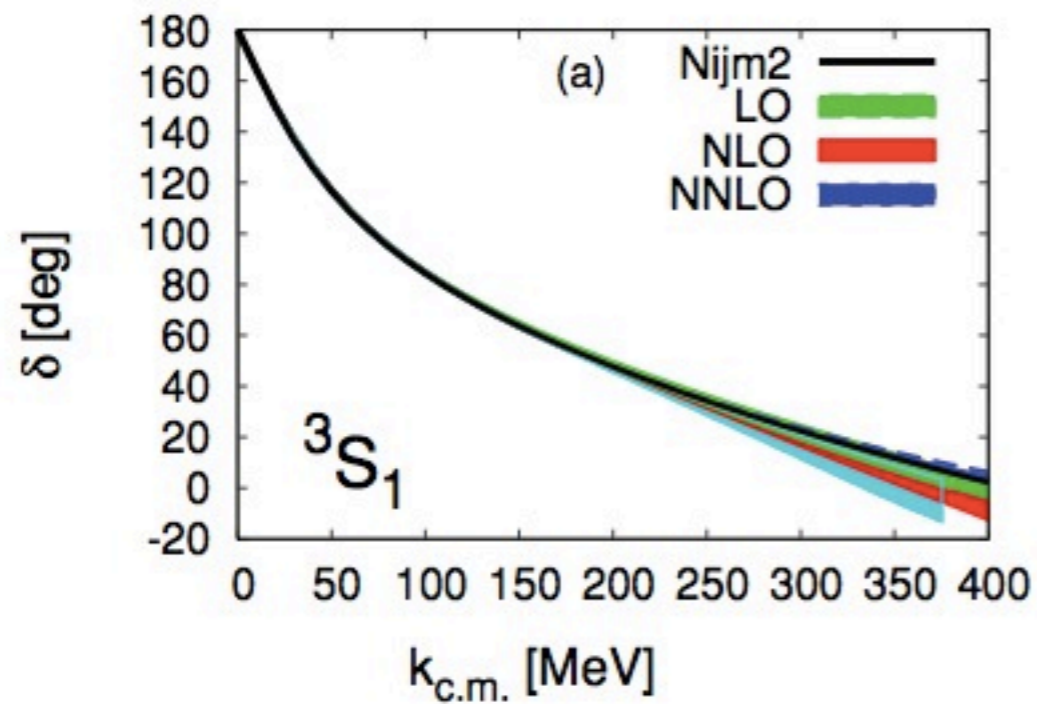
Summary of results I

Birse (2009)

ORDER	INCLUDED
P^{-1}	C^{1S0} , C^{3S1} , 1π exchange
$P^{-1/2}$	C^{3P0} , C^{3P2}
P^0	C_2^{1S0}
$P^{1/2}$	C_2^{3S1}
$P^{3/2}$	C_2^{3P0} , C_2^{3P2}
P^2	Renormalized leading 2π exchange, C^{1P1} , C^{3P1} , C_4^{1S0} , $C^{\epsilon 1}$
$P^{5/2}$	C_4^{3S1}
P^3	Renormalized sub-leading 2π exchange

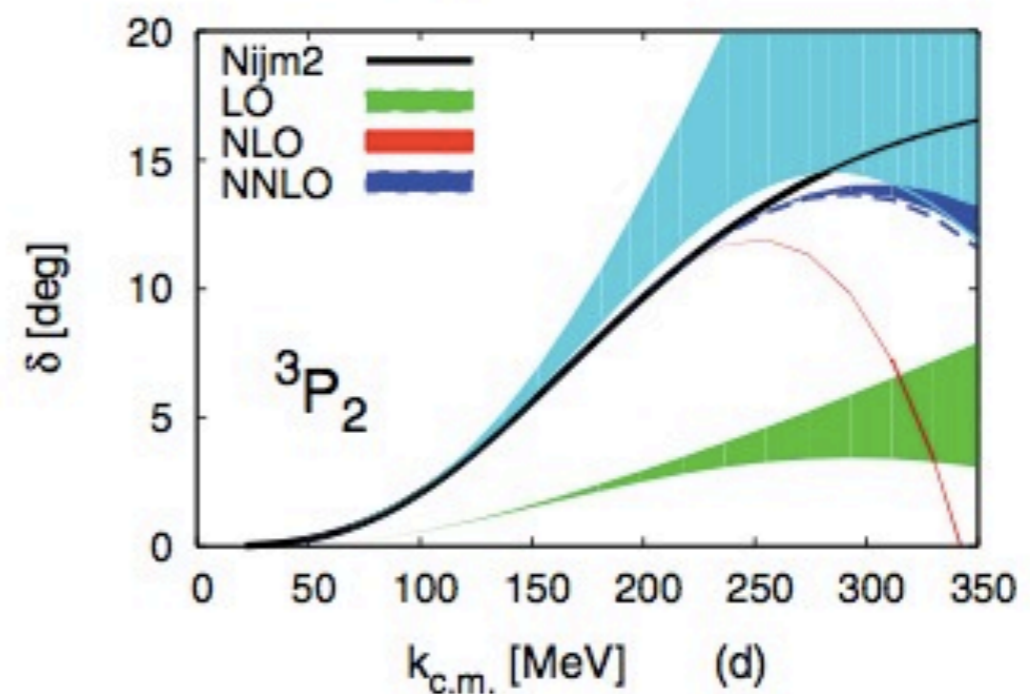
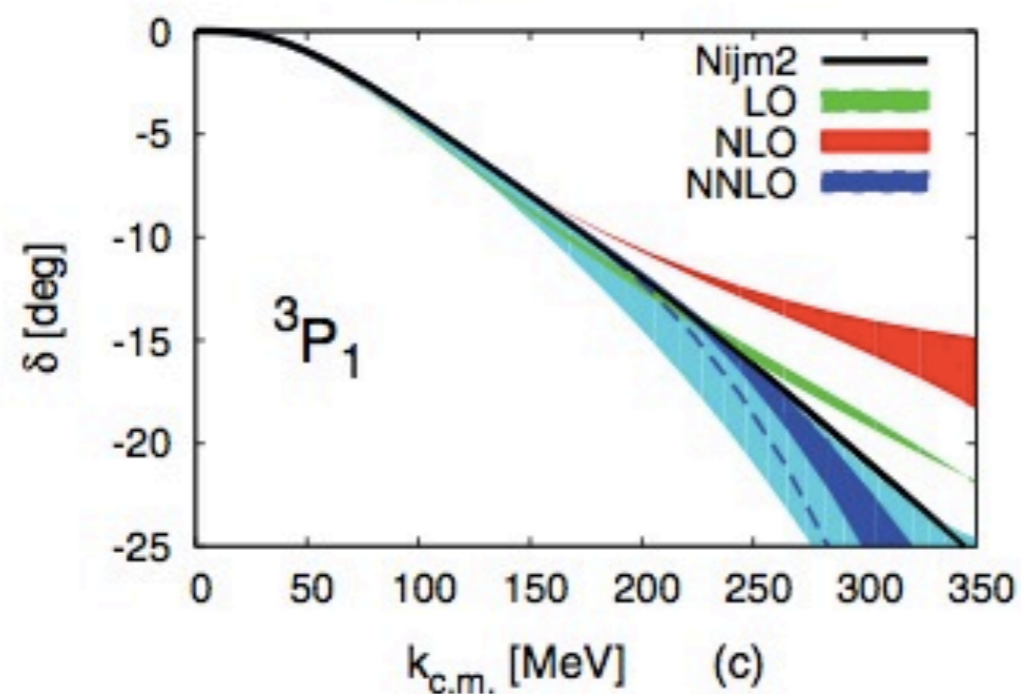
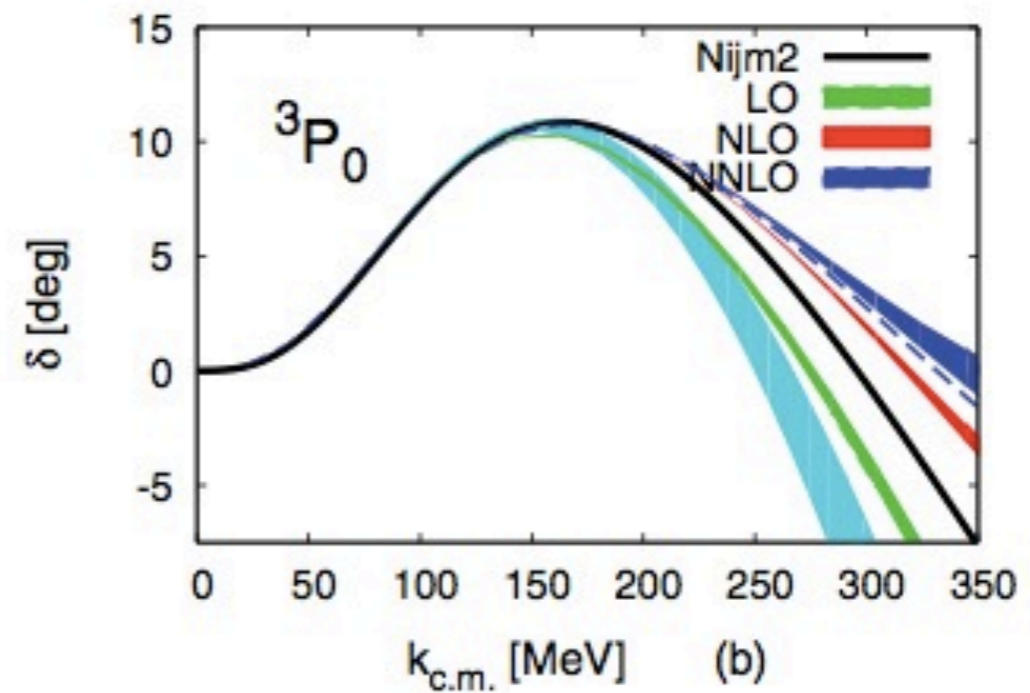
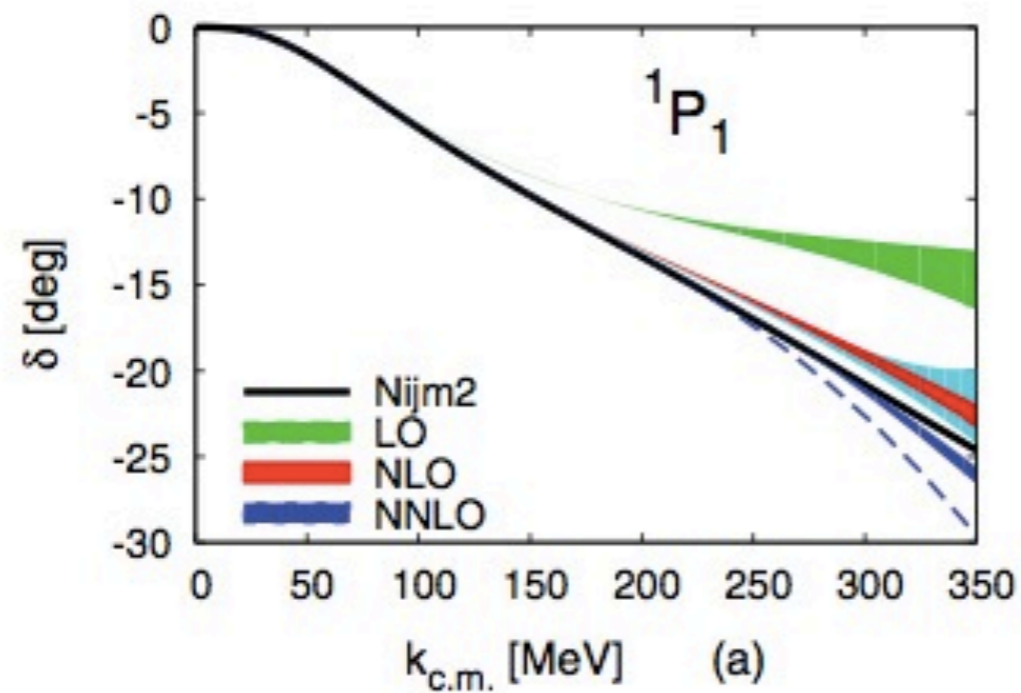
Summary of results III

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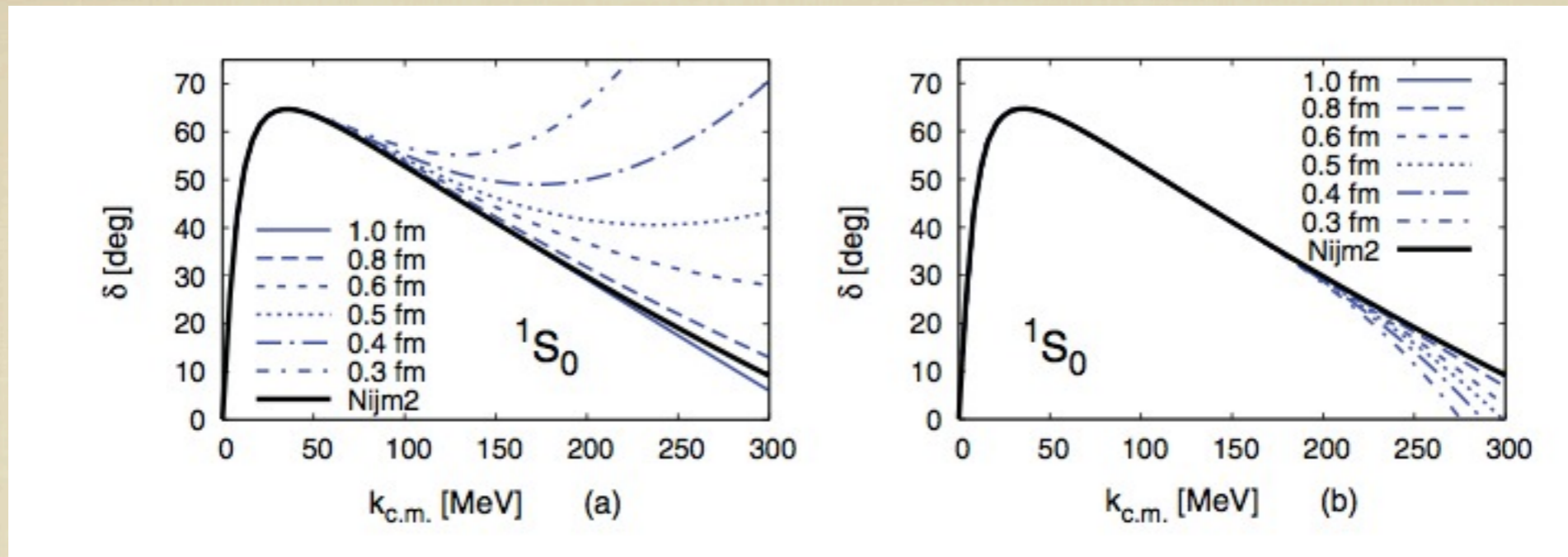
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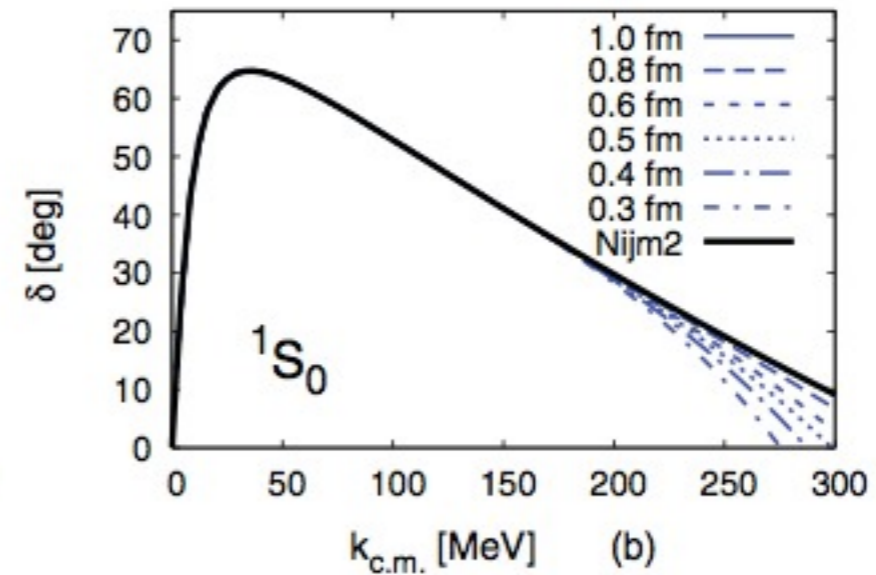
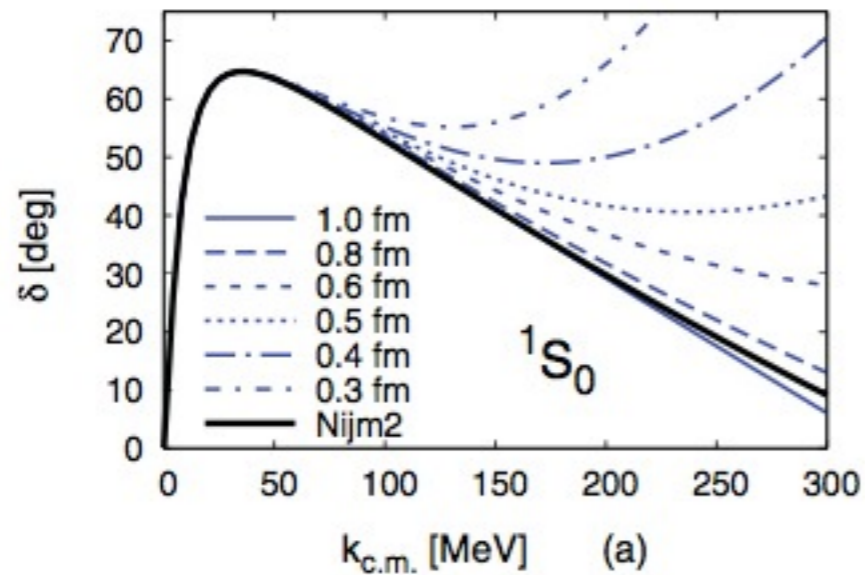
r_c dependence in “Weinberg” approach not under control in, e.g., 3P_0

Summary of results IV: 1S_0 phases

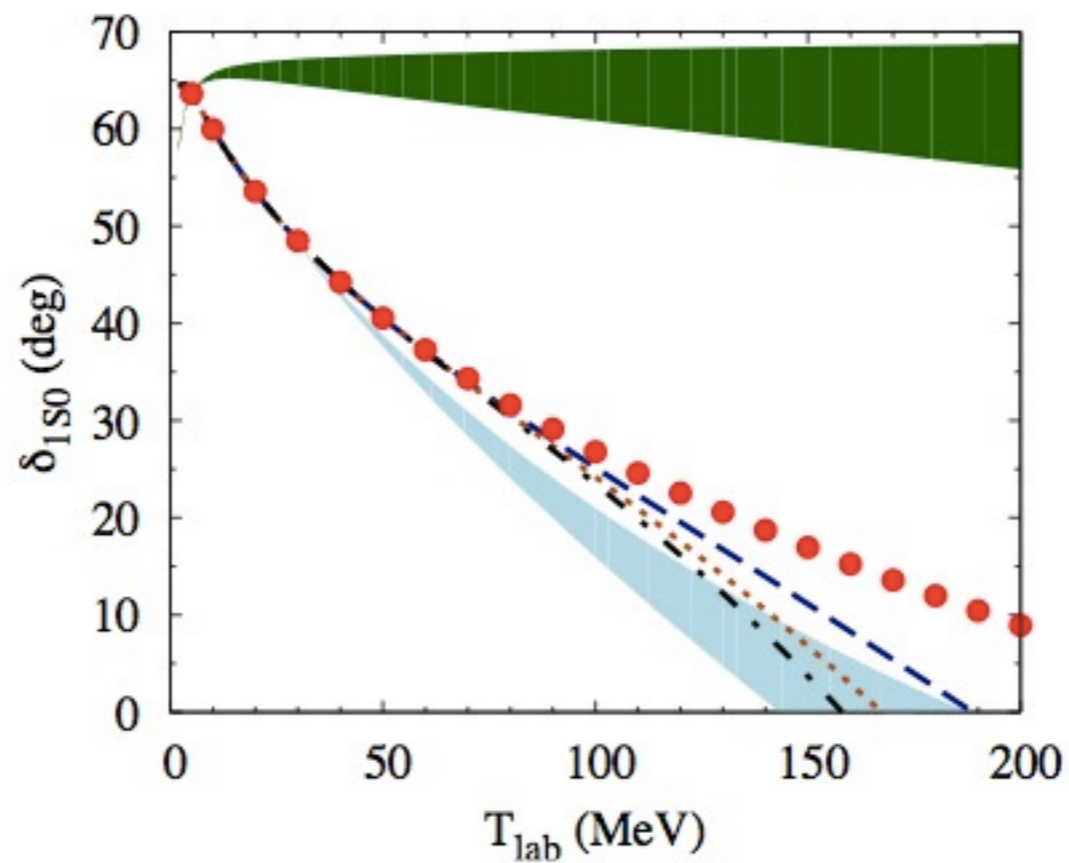


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Long and Yang (2012)

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- Disagreement about counting for waves where one-pion exchange is repulsive (e.g. 3P_1); number of counterterms needed to stabilize 3P_2 - 3F_2
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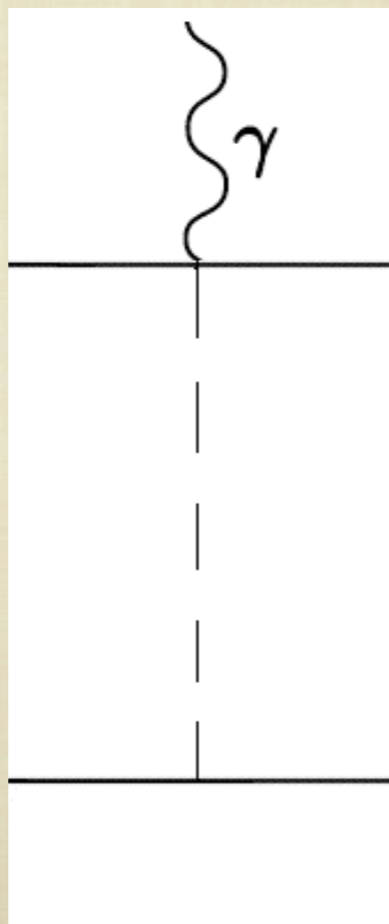
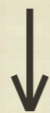
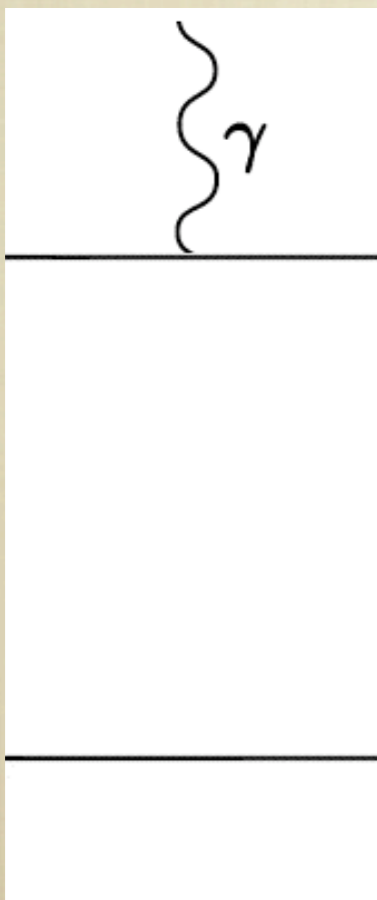
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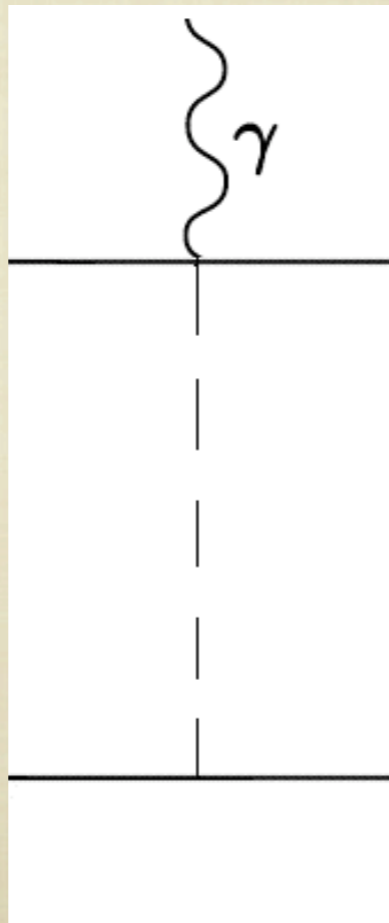
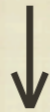
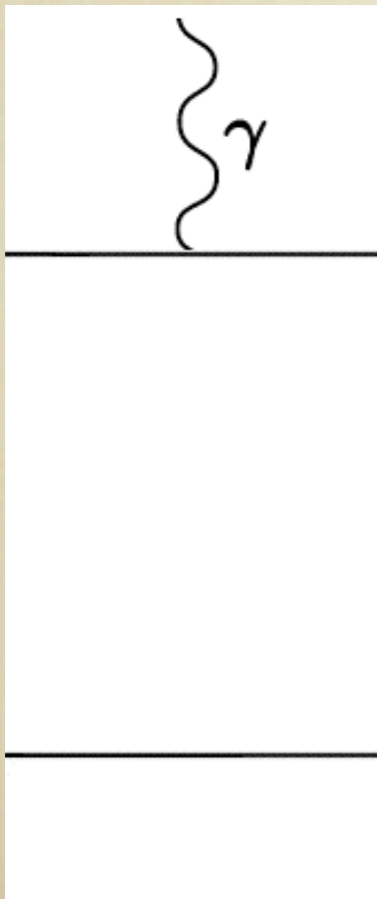
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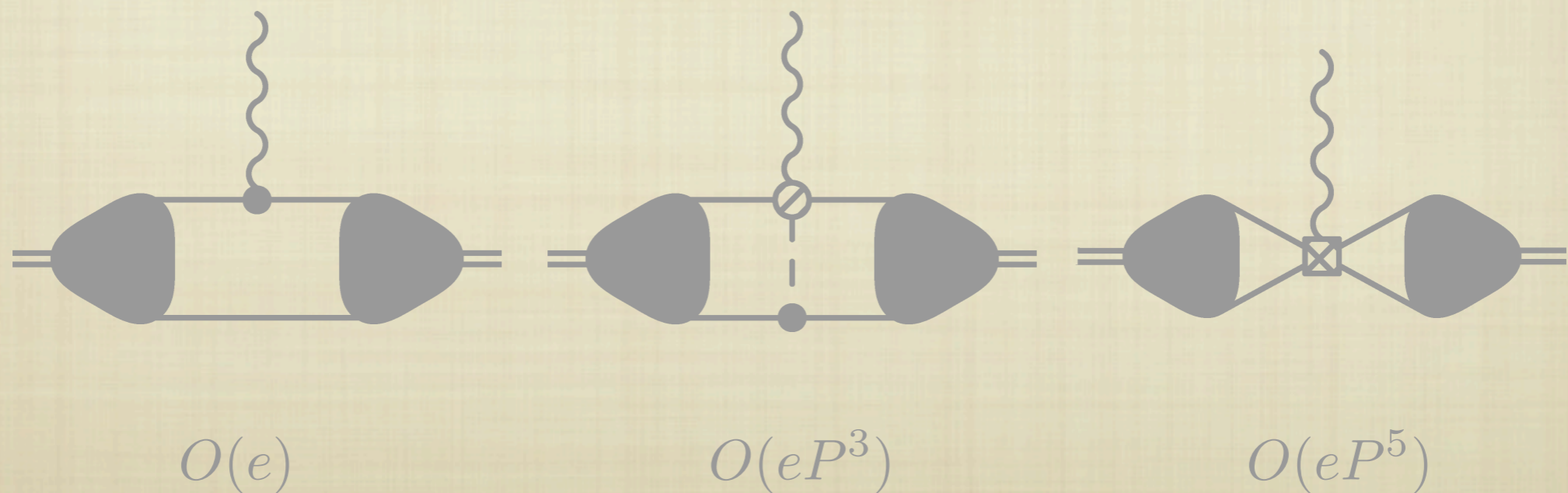
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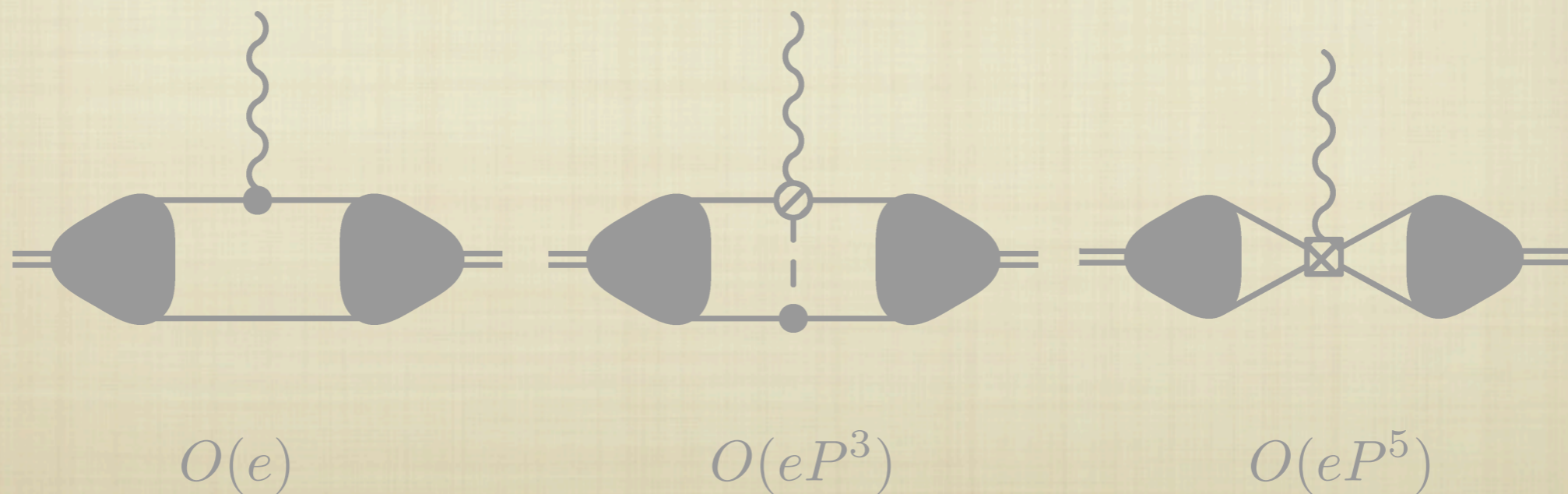
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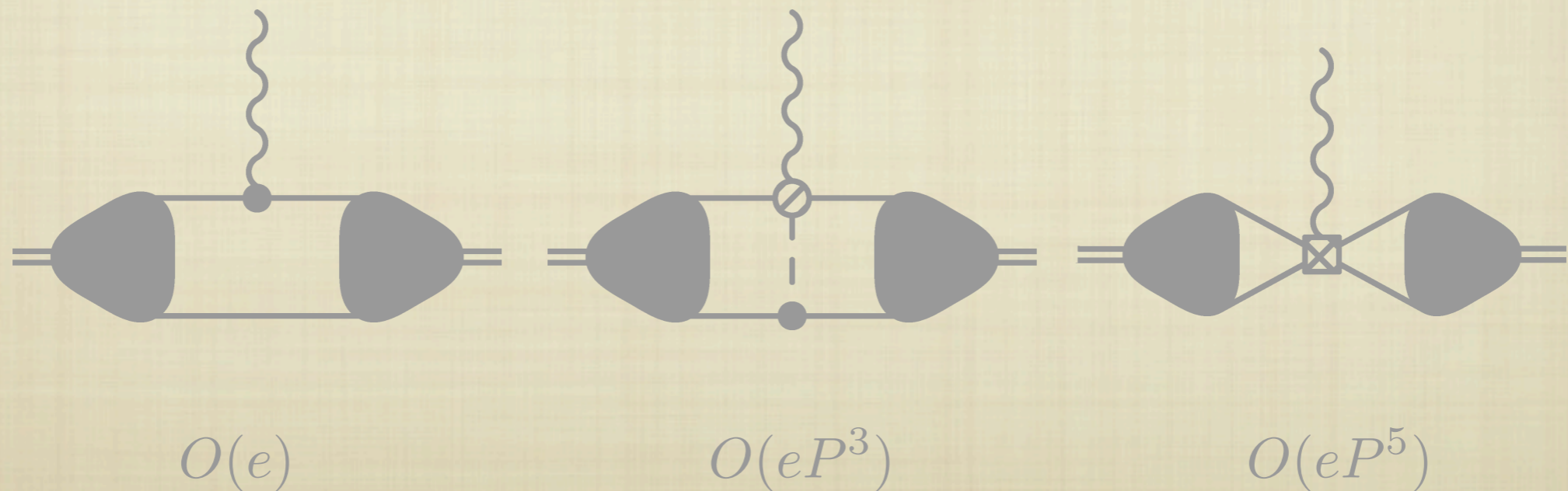
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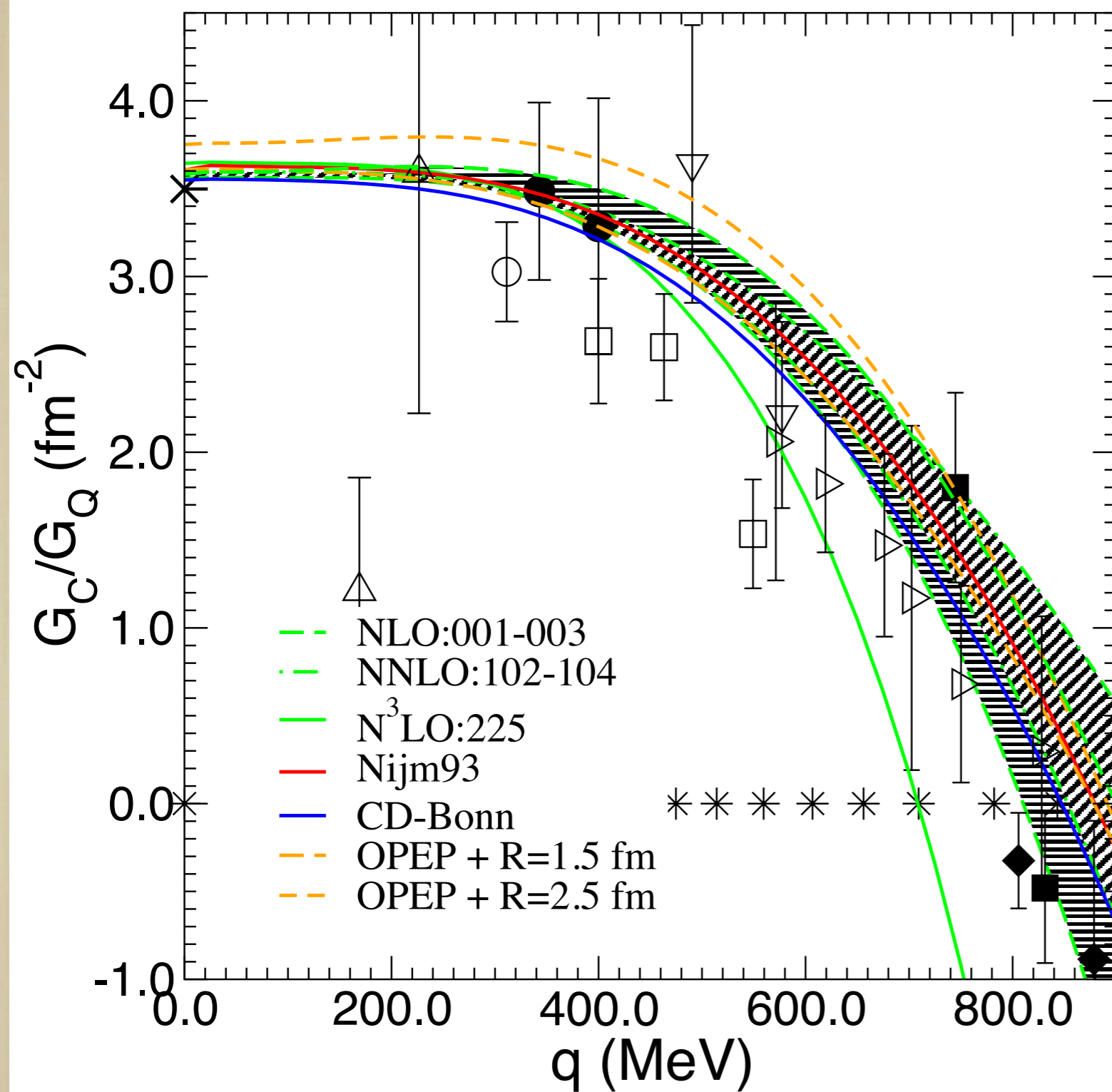
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IN NDA COUNTING FOR SHORT-DISTANCE OPERATORS



Renormalizing G_C/G_Q

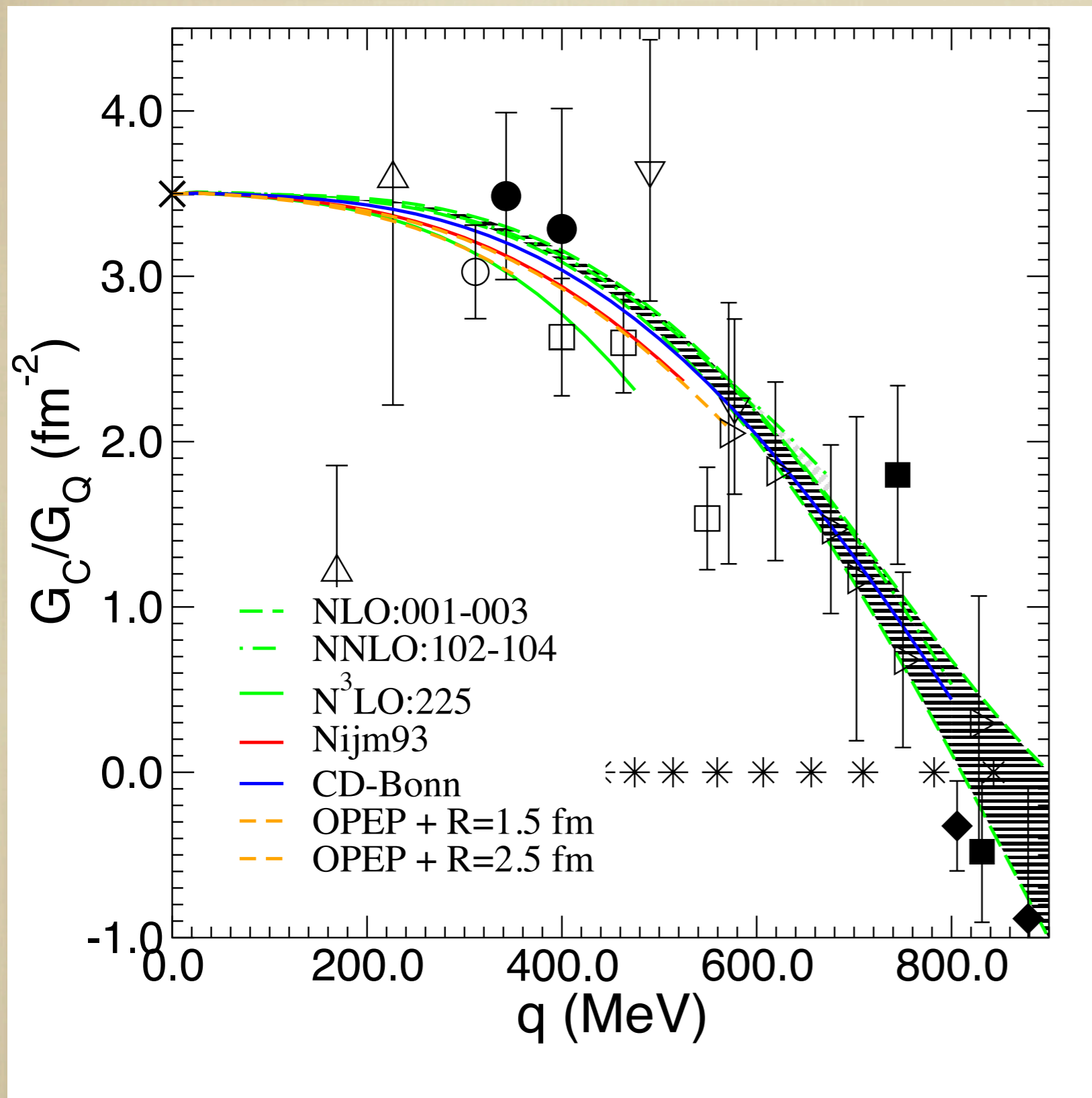
Phillips (2007)



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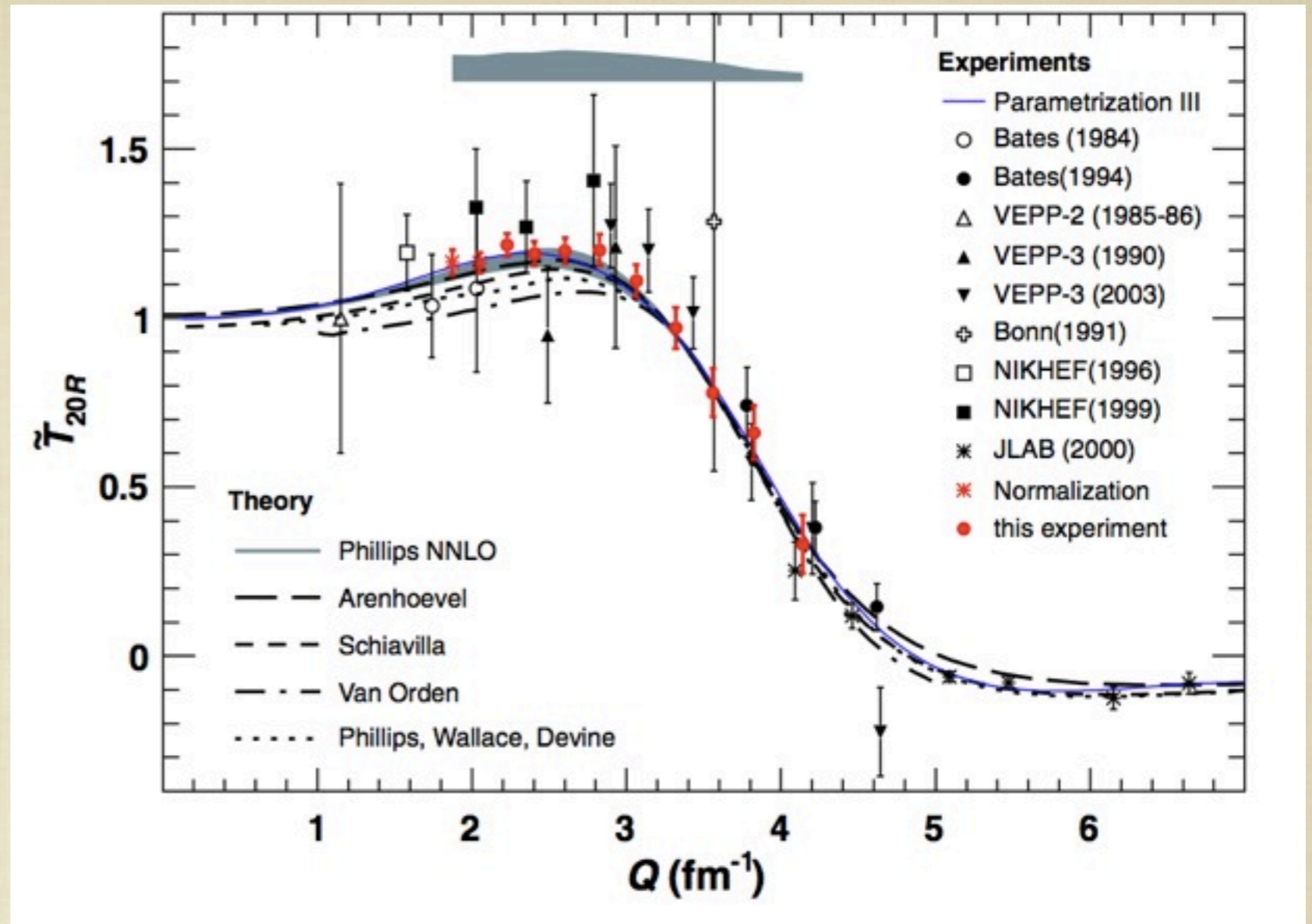
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- Ratio is largely independent of model for $q < 600$ MeV
- G_C/G_Q to 3% at $Q = 0.39$ GeV

Confronting experiment

Zhang et al., PRL (2011)

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$$\leftrightarrow G_C/G_Q$$

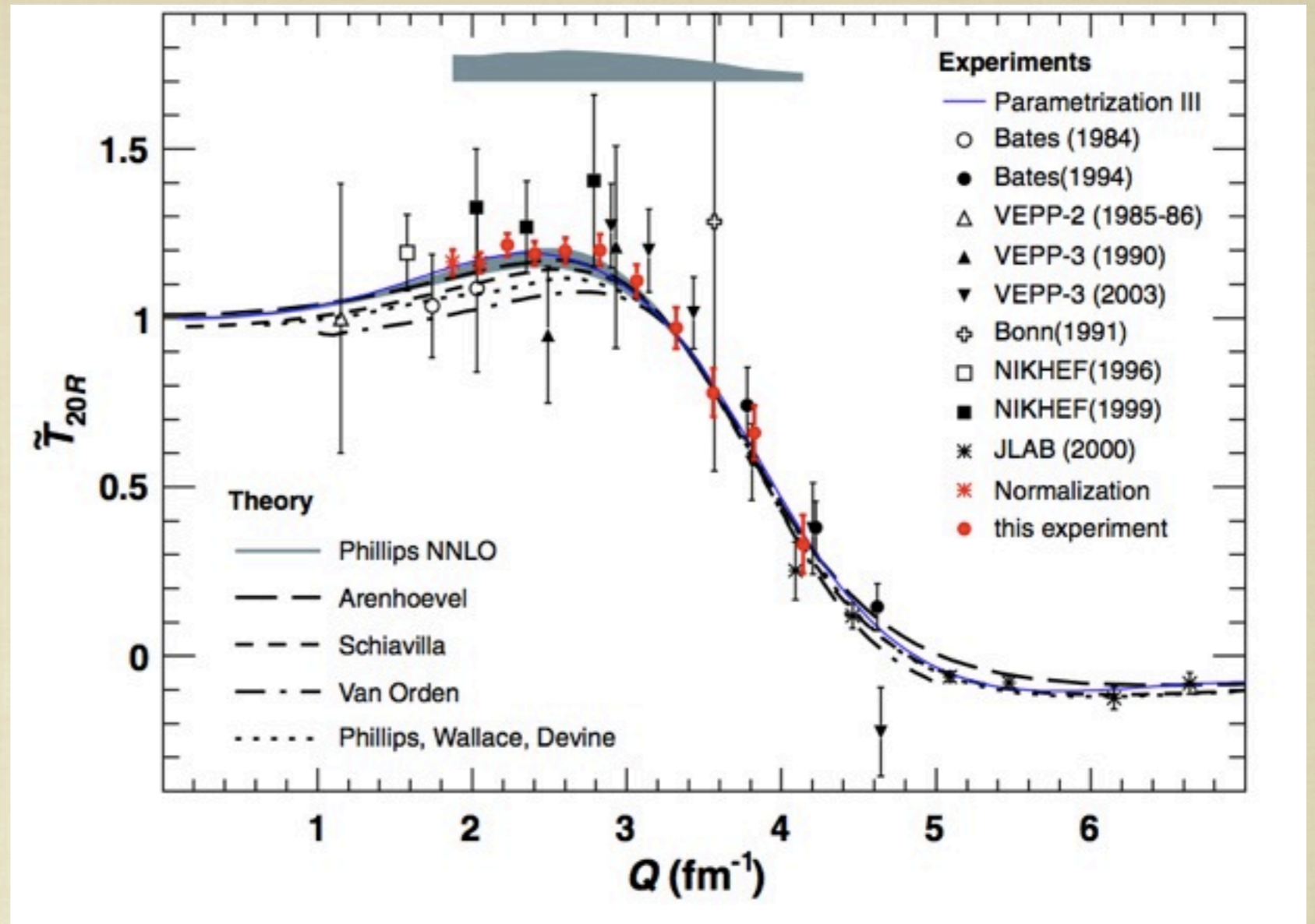


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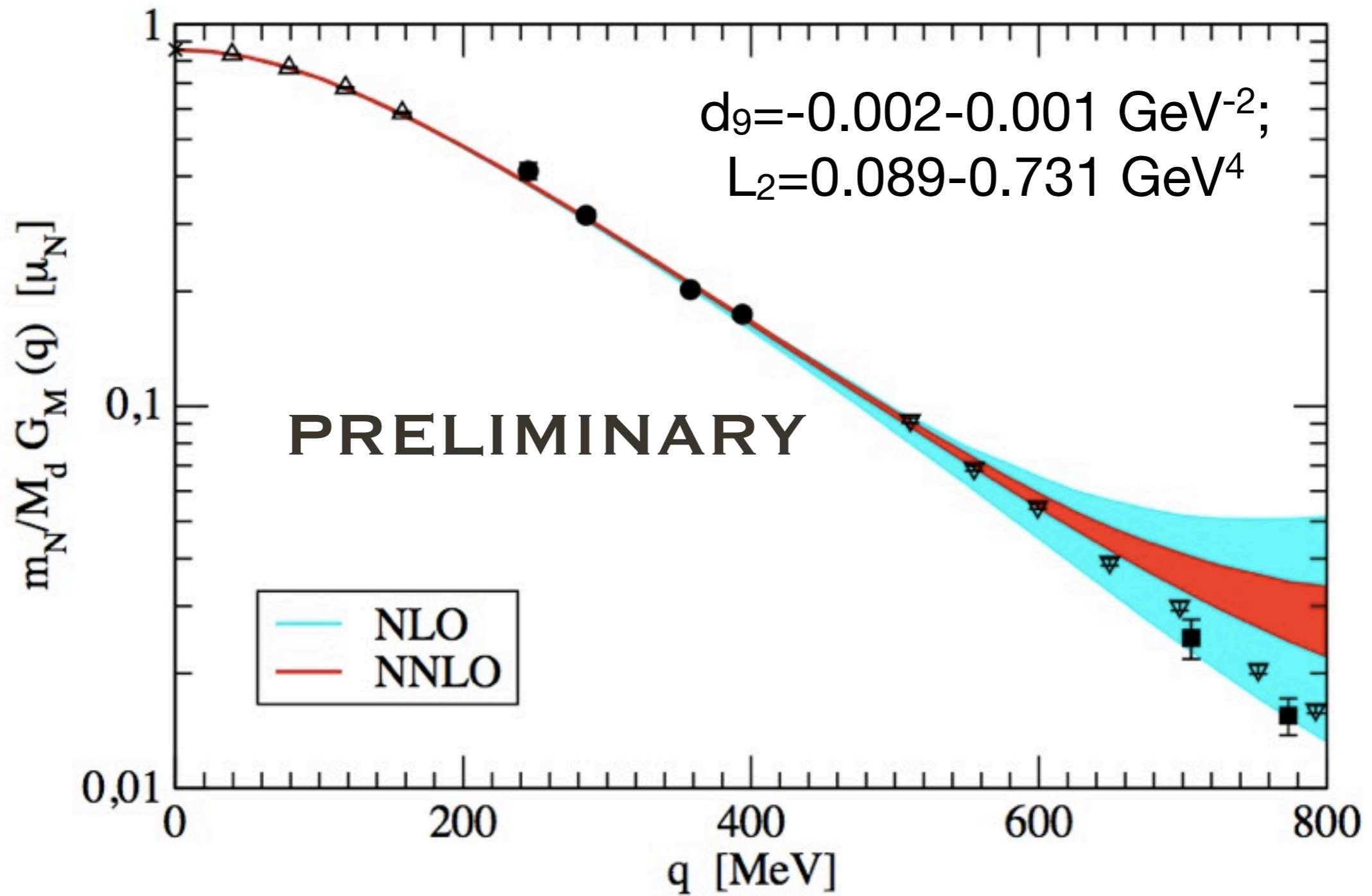


- Prediction for $G_C \leftrightarrow A$ at low Q^2 : Hall A experiment

Yang, Kaiser, Park, Phillips (in preparation)

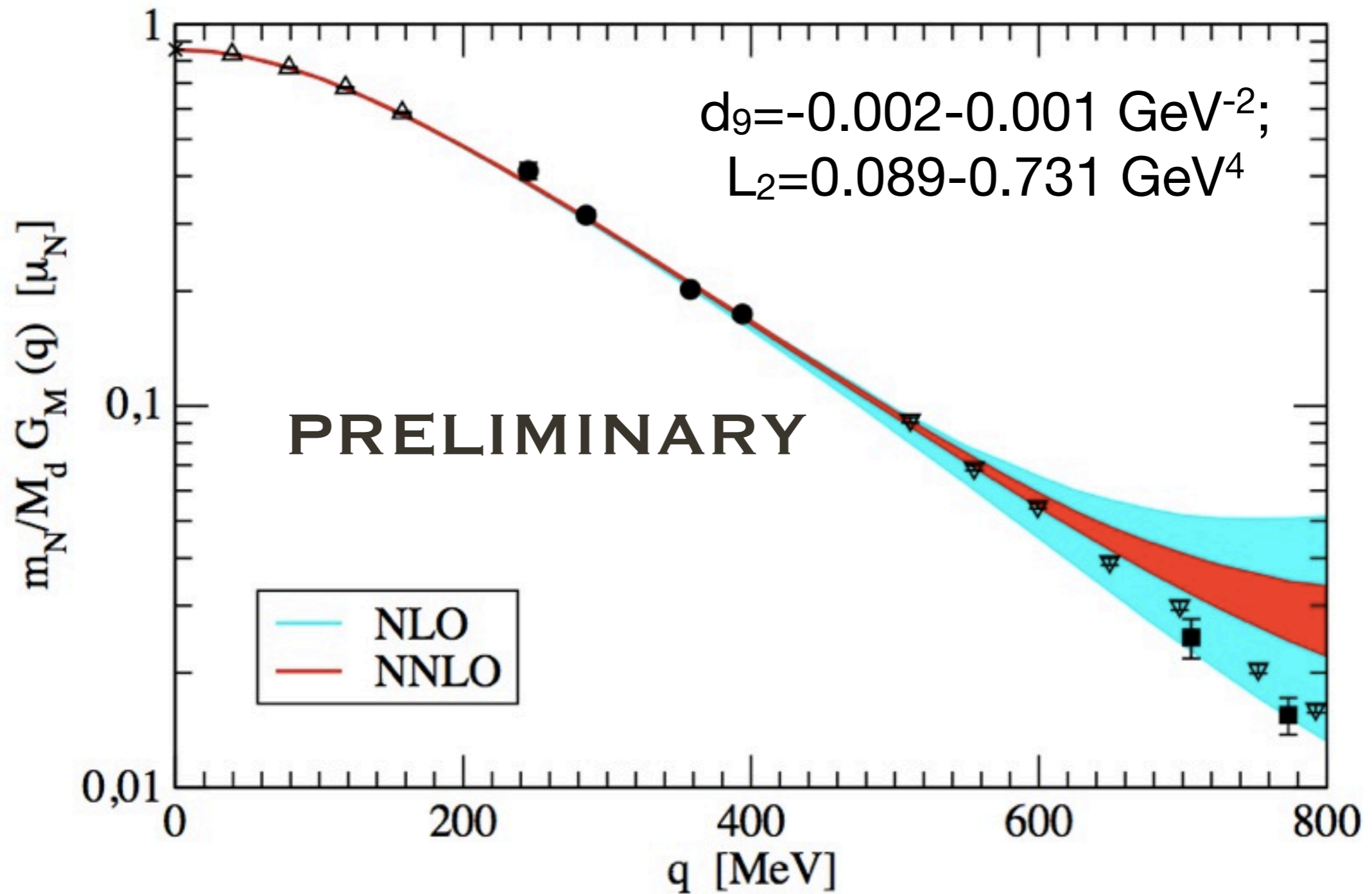
- Application to f_L in $(e, e'p)$

G_M to same order



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Koelling, Epelbaum, Phillips (in preparation)



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- Application of electroweak operators in $A=3, \dots$ ongoing
- Role of $\Delta(1232)$ in long-range part of V , electroweak operators

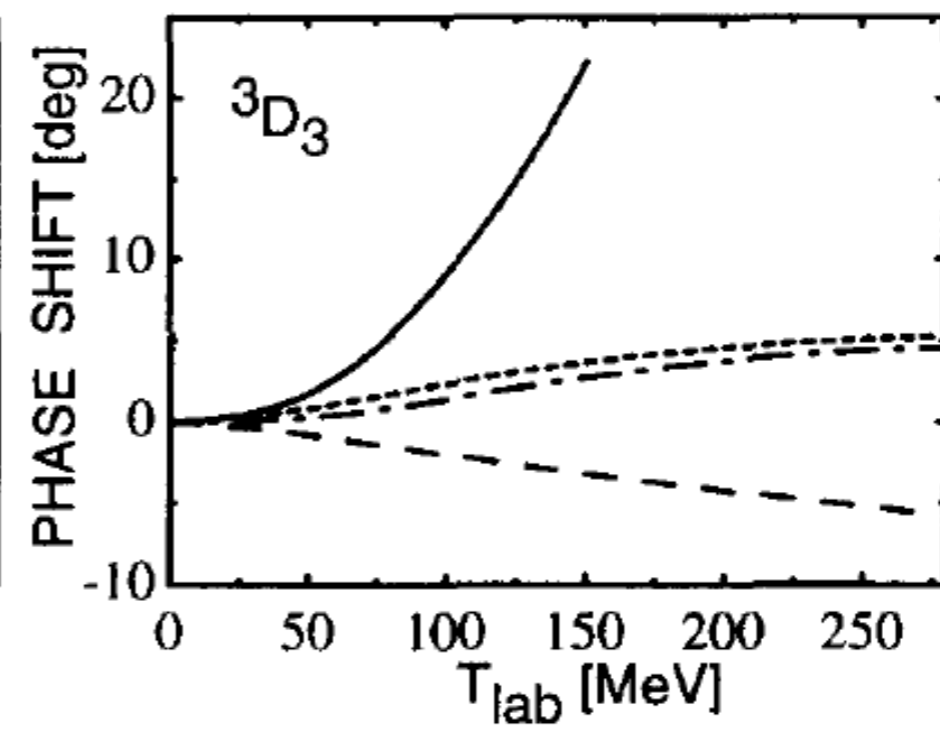
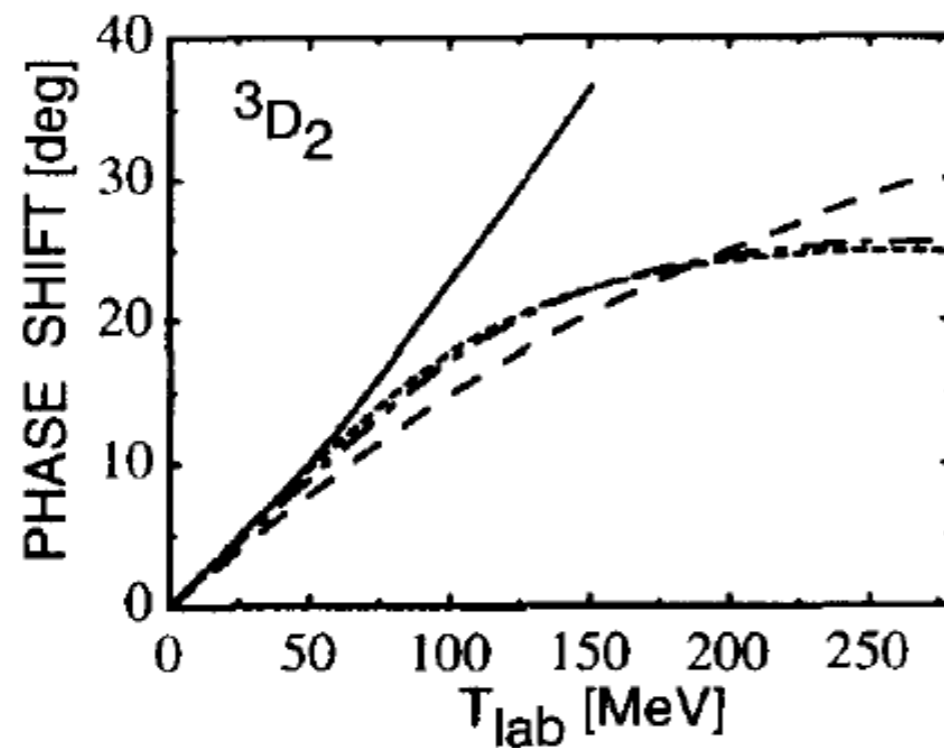
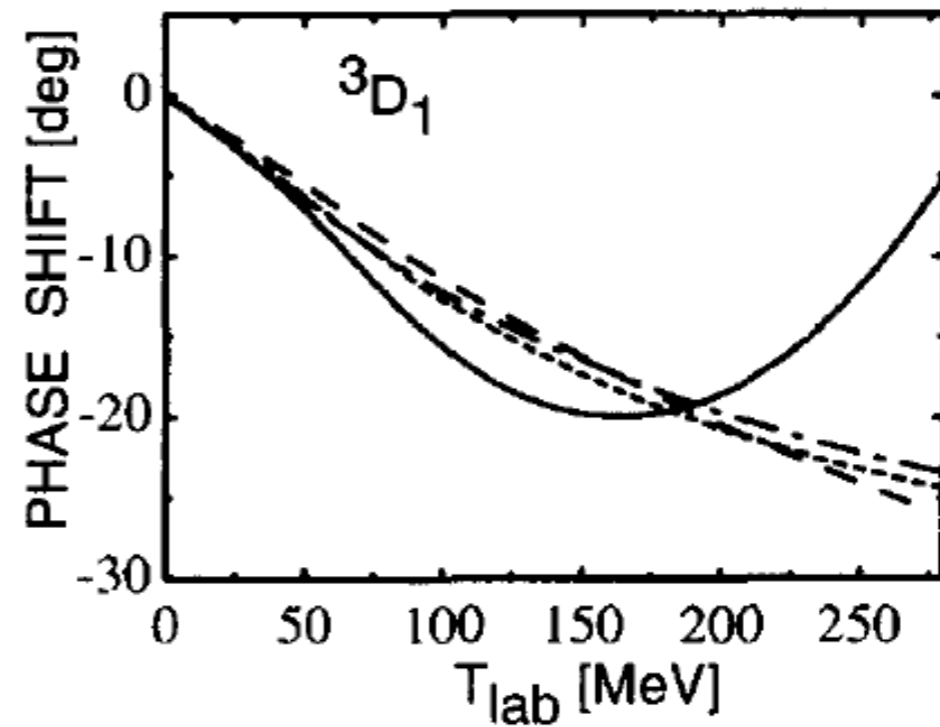
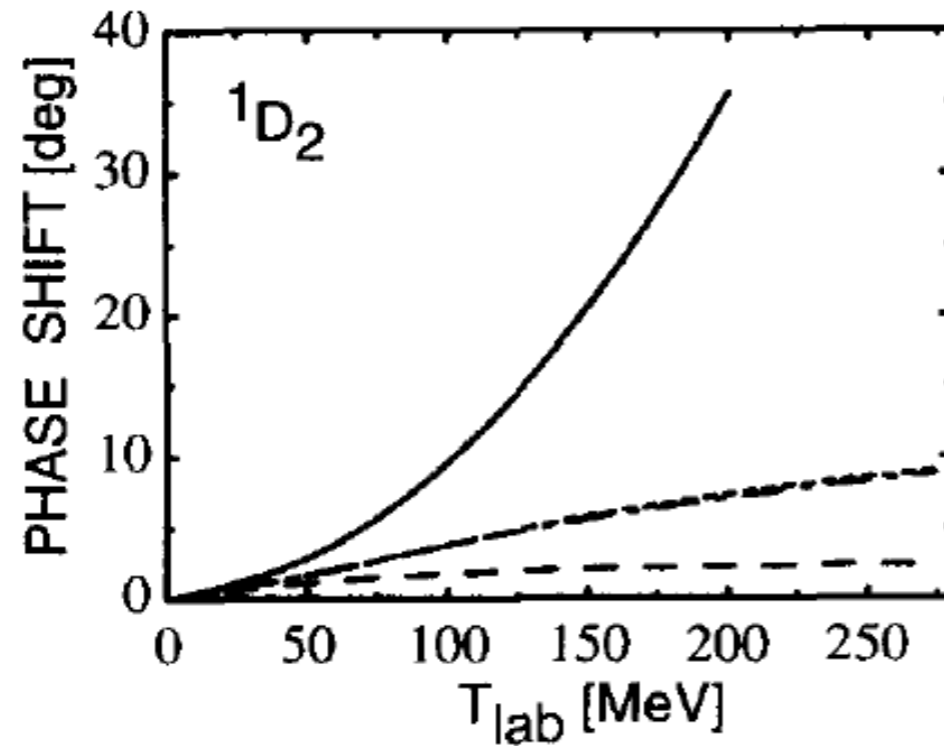
Building a better chiral V_{NN}

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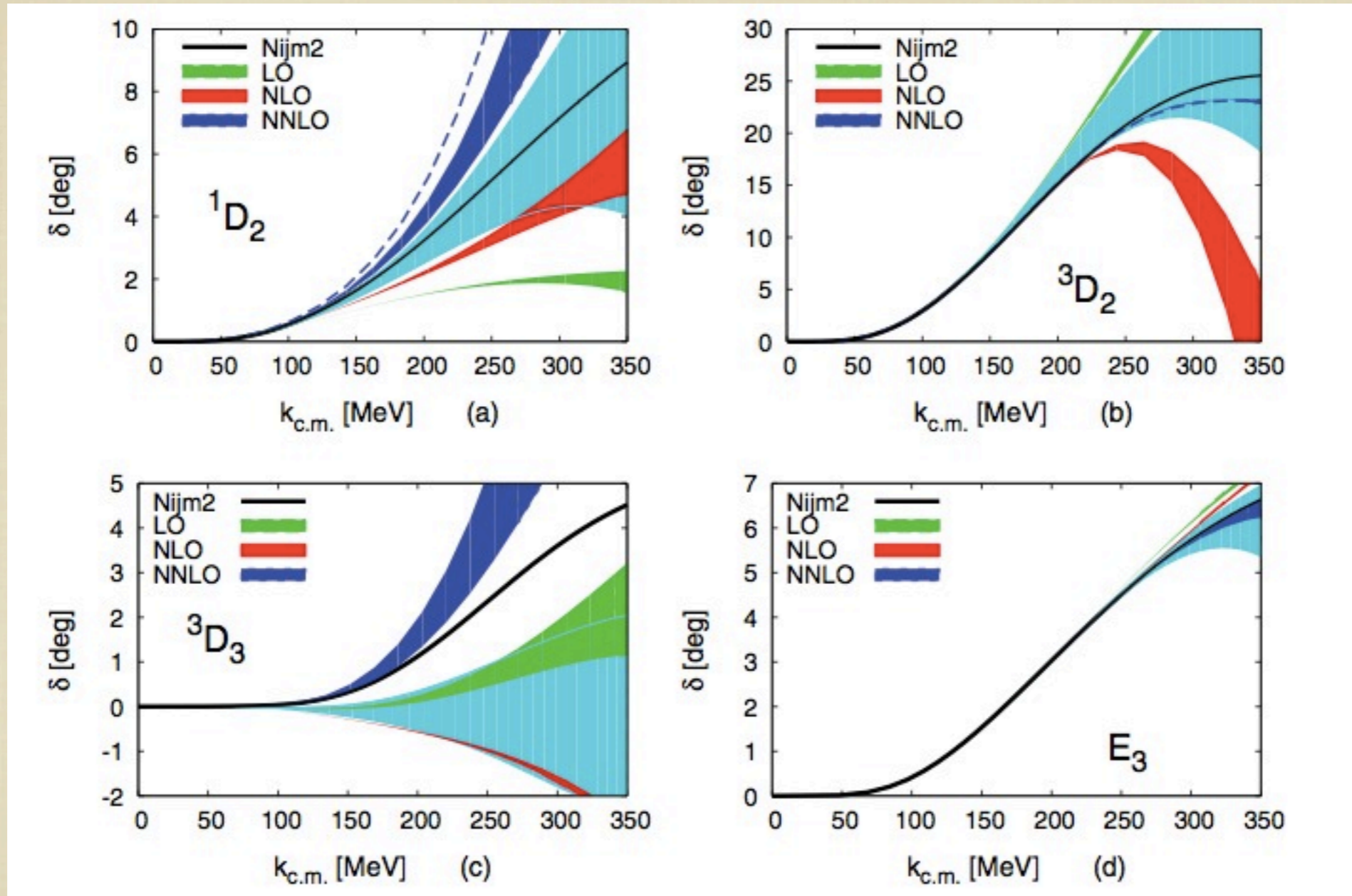
- Three-pion exchange less important than several short-distance operators
- For V_{NN} accurate to P^4 relative to leading include everything in “chiral potential” at NNLO plus second contact operators in 3P_0 and 3P_2 , third in 3S_1 and 1S_0 , and (?) 2 in 3D_2
- Most of these additional pieces are in N^3LO potential
- C.f. Nijmegen phase-shift analysis: chiral two-pion exchange plus similar number of short-distance operators
- Real problems with counting may come in 3NF: more short-distance operators enter if this counting is correct

BACKUP SLIDES

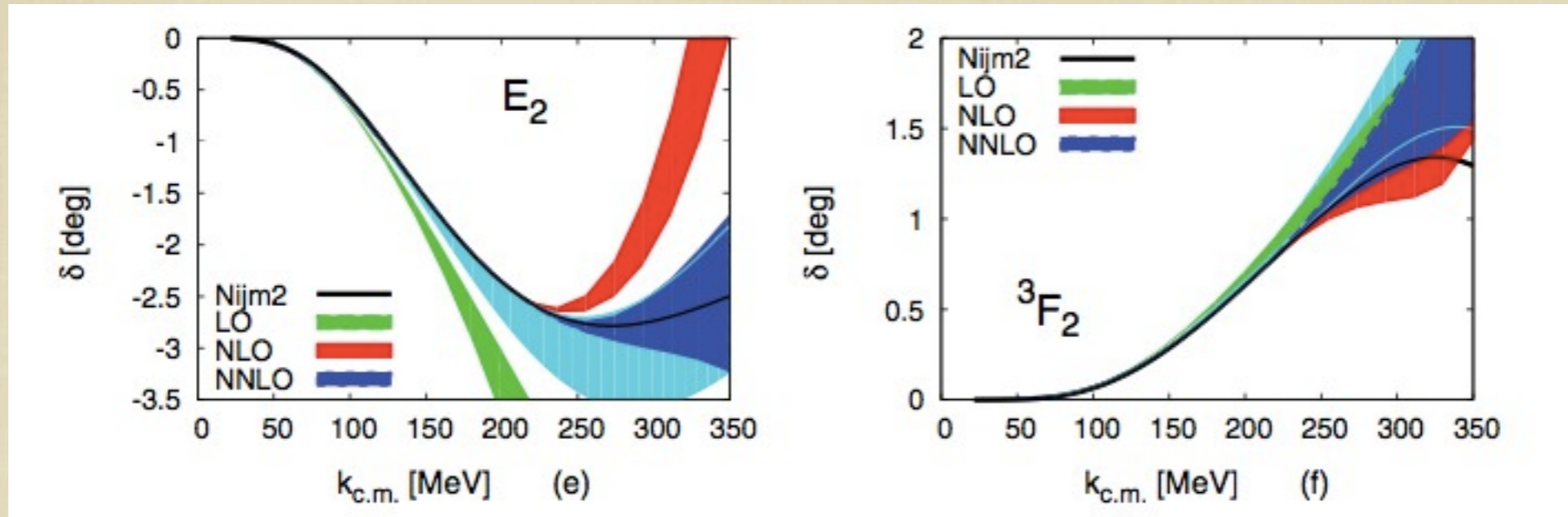
D Waves



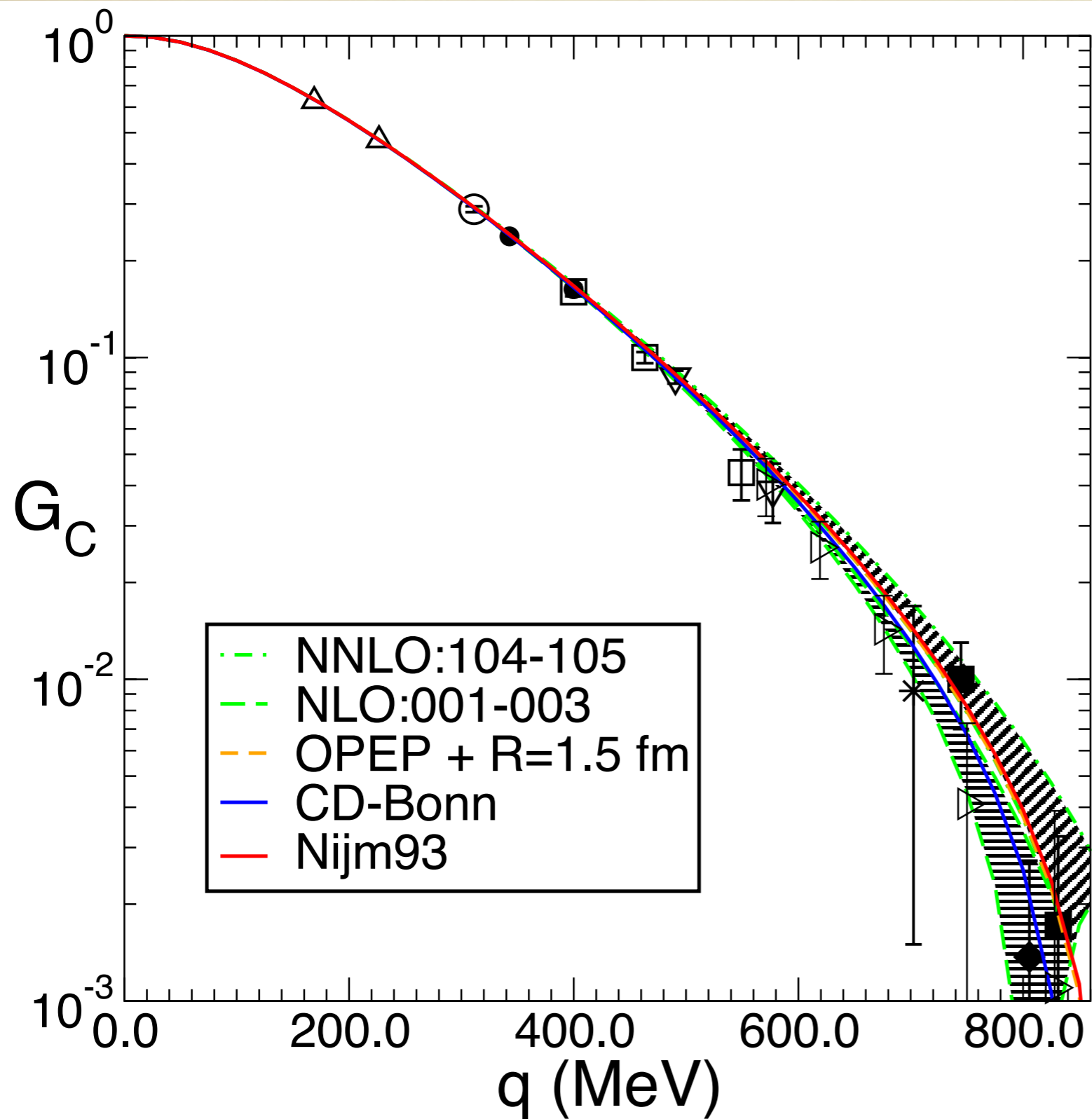
Supplementary phase-shift plots



Supplementary phase-shift plots



χ EFT for G_C up to $O(eP^4)$



- Good J_0 convergence
- G_C dominated by $r \sim 1/m_\pi$ physics in this q range
- But we need to constrain interplay of short-distance pieces of charge operator and pion-range physics

Static properties and renormalization

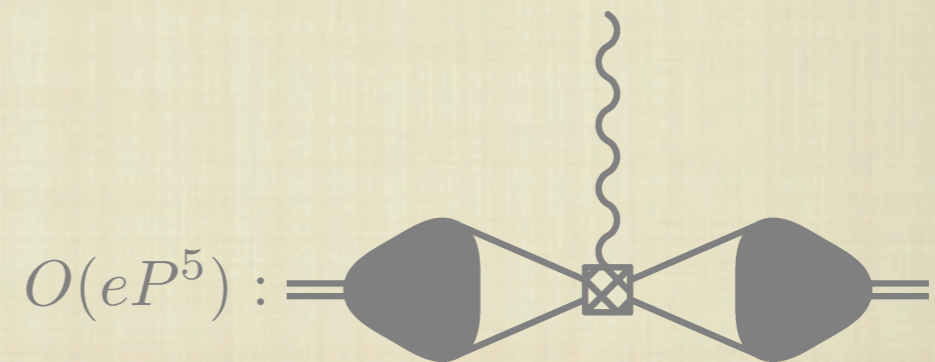
Static properties and renormalization

	Expt.	NNLO	N ³ LO	Nijm93
$\langle r_d^2 \rangle_{pt}$ (fm)	1.9753(10)	1.974- 1.976	1.979- 1.989	1.970
Q_d (fm ²)	0.2859(3)	0.279- 0.282	0.264- 0.268	0.276

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Two-body ${}^3S_1 \rightarrow {}^3S_1$ operator:



Chen, Rupak, and Savage (1999); DP (2007)

G_M beyond impulse approximation

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$$\mathbf{J}_{d_9}^{(s)} = -2e \frac{g_A^i}{f_\pi^2} d_9 \tau_1^a \tau_2^a \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2}{\mathbf{q}_2^2 + m_\pi^2} (\mathbf{q}_2 \times \mathbf{q}) + (1 \leftrightarrow 2)$$

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- d_9 poorly constrained from single-nucleon sector