Hyperon-nucleon Interaction in Chiral EFT

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Chiral Dynamics, Newport News, August 6, 2012

Outline

Chiral Effective Field Theory

- *NN* interaction
  - S. Weinberg, PLB 251 (1990) 288; NPB 363 (1991) 3

$$V_{\rm eff} \equiv V_{\rm eff}(Q, g, \mu) = \sum_{\nu} (Q/\Lambda)^{\nu} \mathcal{V}_{\nu}(Q/\mu, g)$$

  - C. Ordoñez, L. Ray, and U. van Kolck, PRC 53 (1996) 2086
  - D.R. Entem and R. Machleidt, PRC 68 (2003) 041001 (N^3LO)
  - E. Epelbaum, W. Glöckle, U.-G. Meißner, NPA 747 (2005) 362 (N^3LO)
- *YN* interaction
  - C.K. Korpa et al., PRC 65 (2001) 015208
  - S.R. Beane et al., NPA 747 (2005) 55
  - H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 24
- *YN* interaction at NLO
  - J.H., N. Kaiser, U.-G. Meißner, A. Nogga, S. Petschauer, W. Weise, in preparation

YN in chiral effective field theory

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three baryon forces and external current operators in a consistent way

Obstacle: YN data base is rather poor
($\rightarrow$ impose $SU(3)_f$ constraints)

YN data

- about 35 data points, all from the 1960s
- 10 new data points, from the KEK-PS E251 collaboration (from $\approx$ 2000)
  (cf. $>$ 4000 NN data for $E_{lab} < 350$ MeV!)
- constraints from hypernuclei

Contact terms for BB

e.g., LO contact terms for BB:

$$\mathcal{L} = C_i \left(\bar{N}\Gamma_i N\right)\left(\bar{N}\Gamma_i N\right) \quad \Rightarrow \quad \begin{aligned} \mathcal{L}^1 &= \tilde{C}_i^1 \left\langle \bar{B}_a \bar{B}_b \left(\Gamma_i B\right)_b \left(\Gamma_i B\right)_a \right\rangle , \\ \mathcal{L}^2 &= \tilde{C}_i^2 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle , \\ \mathcal{L}^3 &= \tilde{C}_i^3 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \right\rangle \left\langle \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle \end{aligned}$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

a, b ... Dirac indices of the particles

$$\Gamma_1 = 1\,,\ \Gamma_2 = \gamma^\mu\,,\ \Gamma_3 = \sigma^{\mu\nu}\,,\ \Gamma_4 = \gamma^\mu\gamma_5\,,\ \Gamma_5 = \gamma_5$$

Contact terms for BB

spin-momentum structure of the contact term potential:

BB contact terms without derivatives (LO):

$$V^{(0)}_{BB\to BB} = C_{S,BB\to BB} + C_{T,BB\to BB}\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

BB contact terms with two derivatives (NLO):

$$\begin{aligned} V^{(2)}_{BB\to BB} &= C_1 \vec{q}^{\,2} + C_2 \vec{k}^{\,2} + (C_3 \vec{q}^{\,2} + C_4 \vec{k}^{\,2})(\vec{\sigma}_1 \cdot \vec{\sigma}_2) \\ &+ iC_5 (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{q} \times \vec{k}) + C_6 (\vec{q} \cdot \vec{\sigma}_1)(\vec{q} \cdot \vec{\sigma}_2) \\ &+ C_7 (\vec{k} \cdot \vec{\sigma}_1)(\vec{k} \cdot \vec{\sigma}_2) + iC_8 (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot (\vec{q} \times \vec{k}) \end{aligned}$$

note: $C_i \to C_{i,BB\to BB}$

$\vec{q} = \vec{p}\,' - \vec{p}$; $\vec{k} = (\vec{p}\,' + \vec{p})/2$

$SU(3)$ symmetry

10 independent spin-isospin channels in NN and YN (for L=0)
(NN (I=0), NN (I=1), ΛN, ΣN (I=1/2), ΣN (I=3/2), $\Lambda N \leftrightarrow \Sigma N$)

$\Rightarrow$ in principle (at LO), 10 low-energy constants

$SU(3)$ symmetry $\Rightarrow$ only 5 independent low-energy constants

$SU(3)$ structure for scattering of two octet baryons:
direct product:

$$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$$

$C_{S,i}$, $C_{T,i}$, $C_{1,i}$, etc., can be expressed by the coefficients corresponding to the $SU(3)_f$ irreducible representations:
C^1, C^{8_a}, C^{8_s}, C^{10^*}, C^{10}, C^{27}

$SU(3)$ structure of contact terms for BB

	Channel	I	V_{1S0}, $V_{3P0,3P1,3P2}$	V_{3S1}, $V_{3S1-3D1}$, V_{1P1}	$V_{1P1-3P1}$
$S = 0$	$NN \to NN$	0	–	C^{10^*}	–
	$NN \to NN$	1	C^{27}	–	–
$S = -1$	$\Lambda N \to \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{-1}{\sqrt{20}} C^{8_s 8_a}$
	$\Lambda N \to \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(-C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}} C^{8_s 8_a}$
					$\frac{-1}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \to \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27} + 9C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \to \Sigma N$	$\frac{3}{2}$	C^{27}	C^{10}	–

Number of contact terms:

NN: 2 (LO) 7 (NLO)
YN: +3 (LO) +11 (NLO)
YY: +1 (LO) + 4 (NLO)
$\Rightarrow C^1$ contributes only to $I = 0$, $S = -2$ channels!!

Pseudoscalar-meson exchange

SU(3)$_f$-invariant pseudoscalar-meson-baryon interaction Lagrangian:

$$\mathcal{L} = -\left\langle \frac{g_A(1-\alpha)}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\{\partial_\mu P, B\} + \frac{g_A\alpha}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5[\partial_\mu P, B]\right\rangle$$

$$f = g_A/(2F_\pi); \quad g_A \simeq 1.26, \quad F_\pi = 92.4 \text{ MeV}$$

$$\alpha = F/(F+D) \text{ with } g_A = F+D$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$\begin{array}{llllll}
f_{NN\pi} = & f & f_{NN\eta_8} = & \frac{1}{\sqrt{3}}(4\alpha-1)f & f_{\Lambda NK} = & -\frac{1}{\sqrt{3}}(1+2\alpha)f \\
f_{\Xi\Xi\pi} = & -(1-2\alpha)f & f_{\Xi\Xi\eta_8} = & -\frac{1}{\sqrt{3}}(1+2\alpha)f & f_{\Xi\Lambda K} = & \frac{1}{\sqrt{3}}(4\alpha-1)f \\
f_{\Lambda\Sigma\pi} = & \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma\Sigma\eta_8} = & \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma NK} = & (1-2\alpha)f \\
f_{\Sigma\Sigma\pi} = & 2\alpha f & f_{\Lambda\Lambda\eta_8} = & -\frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Xi\Sigma K} = & -f
\end{array}$$

Pseudoscalar-meson (boson) exchange

- One-pseudoscalar-meson exchange (V^{OBE}) [LO]

$$V^{OBE}_{B_1 B_2 \to B_1' B_2'} = -f_{B_1 B_1' P} f_{B_2 B_2' P} \frac{\left(\vec{\sigma}_1 \cdot \vec{q}\right)\left(\vec{\sigma}_2 \cdot \vec{q}\right)}{\vec{q}^{\,2} + m_P^2}$$

$f_{B_1 B_1' P}$ ... coupling constants
m_P ... mass of the exchanged pseudoscalar meson

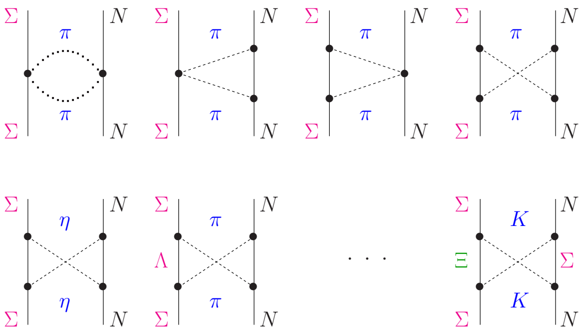
- dynamical breaking of $SU(3)$ symmetry due to the mass splitting of the ps mesons
  (m_π = 138.0 MeV, m_K = 495.7 MeV, m_η = 547.3 MeV)
  taken into account already at LO!

Details:

(H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244)
(H. Polinder, J.H., U.-G. Meißner, PLB 653 (2007) 29)

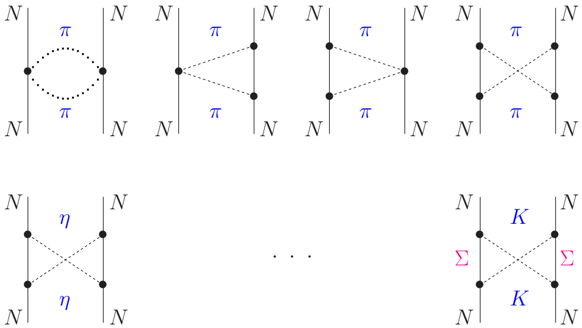
Two-pseudoscalar-meson exchange diagrams

- Two-pseudoscalar-meson exchange diagrams (V^{TBE}) [NLO]



Two-pseudoscalar-meson exchange diagrams

- Two-pseudoscalar-meson exchange diagrams (V^{TBE}) [NLO]



Two-pseudoscalar-meson exchange diagrams

example - Football diagram:

$$\begin{aligned} V^{TBE}(q) &= -\frac{N}{3072\pi^2 F_0^4}\left[-2(m_1^2+m_2^2) - \frac{(m_1^2-m_2^2)^2}{2q^2} - \frac{5}{6}q^2\right. \\ &- \frac{m_1^2-m_2^2}{2q^4}\left((m_1^2-m_2^2)^2 + 3(m_1^2+m_2^2)q^2\right)\ln\left(\frac{m_1}{m_2}\right) \\ &+ \left. w^2(q)\,L(q) \;+\; \ldots\right] \end{aligned}$$

$$\begin{aligned} w(q) &= \frac{1}{q}\sqrt{(q^2+(m_1+m_2)^2)(q^2+(m_1-m_2)^2)} \\ L(q) &= \frac{w(q)}{2q}\ln\frac{(qw(q)+q^2)^2-(m_1^2-m_2^2)^2}{4m_1 m_2 q^2} \end{aligned}$$

$$m_1 = m_2 = m:\;\; L(q) = \frac{1}{q}\sqrt{4m^2+q^2}\,\ln\frac{\sqrt{4m^2+q^2}+q}{2m}$$

Coupled channels Lippmann-Schwinger Equation

$$
\begin{aligned}
T_{\rho'\rho}{}^{\nu'\nu,J}(p',p) &= V_{\rho'\rho}{}^{\nu'\nu,J}(p',p) \\
&+ \sum_{\rho'',\nu''}\int_0^\infty \frac{dp''p''^2}{(2\pi)^3}\, V_{\rho'\rho''}{}^{\nu'\nu'',J}(p',p'')\,\frac{2\mu_{\nu''}}{p^2-p''^2+i\eta}\,T_{\rho''\rho}{}^{\nu''\nu,J}(p'',p)
\end{aligned}
$$

$\rho',\ \rho = \Lambda N,\ \Sigma N$

LS equation is solved for particle channels (in momentum space)

Coulomb interaction is included via the Vincent-Phatak method

The potential in the LS equation is cut off with the regulator function:

$$
V_{\rho'\rho}{}^{\nu'\nu,J}(p',p) \to f^\Lambda(p')V_{\rho'\rho}{}^{\nu'\nu,J}(p',p)f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^4}
$$

consider values Λ = 500 - 700 MeV

NLO Results

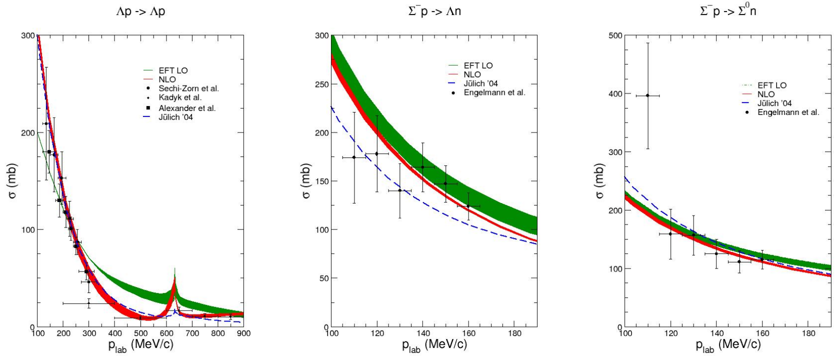
Pseudoscalar-meson exchange

- All one- and two-pseudoscalar-meson exchange diagrams are included
- $SU(3)$ symmetry is broken by using the physical masses of the pion, kaon, and eta
- $SU(3)$ breaking in the coupling constants is ignored
  $F_\pi = F_K = F_\eta = F_0 = 92.4$ MeV
- Correction to V^{OBE} due to baryon mass differences are ignored

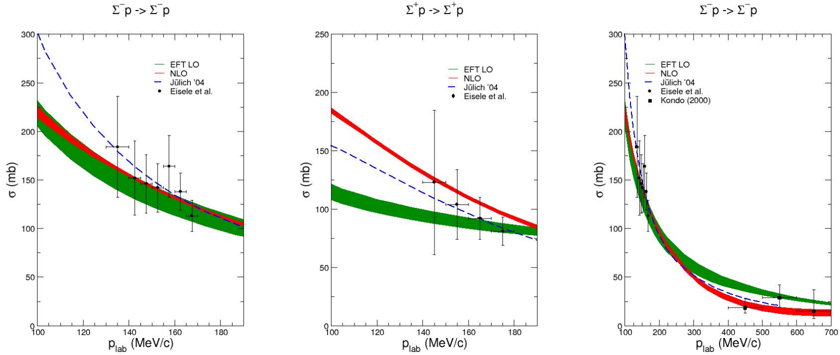
Contact terms

- $SU(3)$ symmetry is assumed
- (at NLO $SU(3)$ breaking corrections to the LO contact terms arise!)
- no SU(3) constraints from the *NN* sector are imposed
- 10 contact terms in S-waves
- 12 contact terms in P-waves and in ${}^3S_1 - {}^3D_1$
- 1 contact term in ${}^1P_1 - {}^3P_1$ (singlet-triplet mixing) is set to zero

YN integrated cross sections



YN integrated cross sections



YN scattering lengths [fm]

	EFT LO	EFT NLO	Jülich '04	NSC97f	experiment*
Λ [MeV]	$550 \cdots 700$	$500 \cdots 700$			
$a_s^{\Lambda p}$	$-1.90 \cdots -1.91$	$-2.88 \cdots -2.89$	-2.56	-2.51	$-1.8^{+2.3}_{-4.2}$
$a_t^{\Lambda p}$	$-1.22 \cdots -1.23$	$-1.59 \cdots -1.61$	-1.66	-1.75	$-1.6^{+1.1}_{-0.8}$
$a_s^{\Sigma^+ p}$	$-2.24 \cdots -2.36$	$-3.90 \cdots -3.83$	-4.71	-4.35	
$a_t^{\Sigma^+ p}$	$0.70 \cdots 0.60$	$0.51 \cdots 0.47$	0.29	-0.25	
χ^2	≈ 30	$18.0 \cdots 24.0$	≈ 25	16.7	
$({}^3_\Lambda\mathrm{H})$ E_B	$-2.34 \cdots -2.36$	$-2.31 \cdots -2.34$	-2.27	-2.30	-2.354(50)

* A. Gasparyan et al., PRC 69 (2004) 034006 $\Rightarrow$ extract from final-state interaction:

$pp \to K^+\Lambda p$ (COSY, Jülich)

$\gamma d \to K^+\Lambda n$ (SPring-8)

ΣN (I=3/2)

ΣN with maximal isospin ($\Sigma^+ p$, $\Sigma^- n$)

- no coupling to the ΛN system
- ${}^1S_0$: test for $SU(3)$ symmetry $\rightarrow V_{np} \equiv V_{\Sigma^- n}$
- ${}^3S_1 - {}^3D_1$: properties closely linked with the Σ properties in nuclear matter (attractive or repulsive)
- advantageous for lattice simulations
  $\rightarrow$ S.R. Beane et al. [NPLQCD], arXiv:1204.3606 [hep-lat]

ΣN (I=3/2)

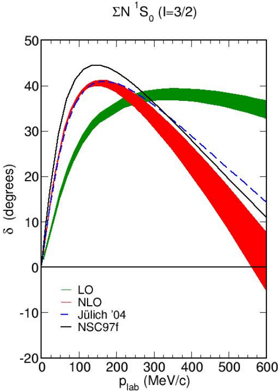
$^1S_0$

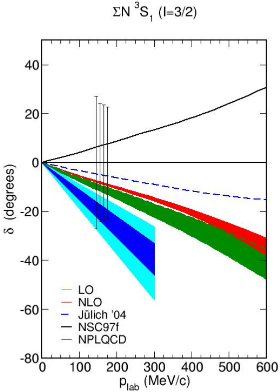
- description of pp phase shifts and $\sigma_{\Sigma^+ p}$ with LEC's that fulfill $SU(3)$ symmetry is not possible
- LEC's that are fitted to the pp $^1S_0$ phase shift produce a bound state in $\Sigma^+ p$
  $\rightarrow \sigma_{^1S_0} \approx 4 \times \sigma_{\Sigma^+ p}$
- simultaneous fit is possible if we assume that there is $SU(3)$ breaking in the LO contact term only.

$^3S_1 - {}^3D_1$:

- A description of all YN data is possible with an attractive as well as a repulsive $^3S_1 - {}^3D_1$ interaction
- However, the χ^2 is found to be slightly larger for a repulsive $^3S_1 - {}^3D_1$ interaction

ΣN (I=3/2) phase shifts





Summary

YN interaction based on chiral *EFT*

- approach is based on a modified Weinberg power counting, analogous to the NN case
- The potential (contact terms, pseudoscalar-meson exchanges) is derived imposing $SU(3)_f$ constraints
- Good description of the empirical YN data was achieved already at LO
  (with only 5 free parameters!)
- (Preliminary) Results at next-to-leading order (NLO) look very promising
- YN data are reproduced with a quality comparable to phenomenological models
- $SU(3)$ symmetry for the LEC's can be maintained in the YN system (ΛN, ΣN) but not between YN and NN