Life between KSW and Weinberg

Renormalization and power counting of chiral nuclear forces

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A modest goal

Chiral EFT for few-nucleon system:

- interesting as chiral NON-perturbation theory
- useful for low-energy nuclear physics
    - for most nuclei, binding momenta $Q \lesssim 140$ MeV
    - light-nucleus reactions relevant in fusion physics, big-bang nucleosynthesis ... :
      $d + {}^3H \to {}^4He + n(14 \text{ MeV})$
      $d + d \to {}^3He + n(2.5 \text{ MeV})$
      $\ldots$

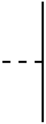
First step: NN scattering for $k_{\rm CM} \lesssim 200$ MeV within first few orders

Guidelines

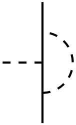
- One Lagrangian, many power countings (PCs)
  Renormalization-group invariance help to rule out PCs, but there still may be several self-consistent PCs

- Power counting at the level of on-shell amplitude

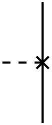
Paradigm of power counting



$\sim \frac{Q}{f_\pi}$



$\sim \frac{Q}{f_\pi}\left(\frac{Q}{4\pi f_\pi}\right)^2 \sim$



$= \frac{g}{f_\pi}\left(\partial^3\pi\right) N^\dagger N$

$\rightarrow g \sim \frac{1}{M_{\text{hi}}^2}$

$M_{\text{hi}} \sim 4\pi f_\pi \simeq 1.2$ GeV

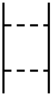
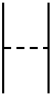
- long-range physics: non-analytic part of loops, contributed by momenta near mass shell
- short-range physics: by naturalness, counterterms comparable to the non-analytic part $\rightarrow$ naive dimensional analysis (NDA) can be used

$$[g] = mass^{-2} \rightarrow g \sim \frac{1}{M_{\text{hi}}^2}$$

! difficult to capture numerical factors: better not take 4π too seriously

! LEC may be less suppressed due to fine-tuning

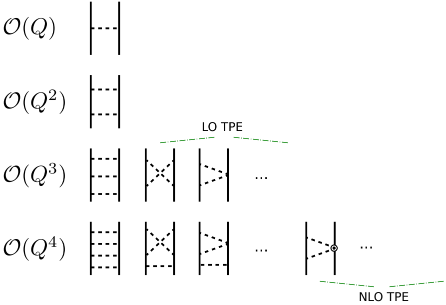
Loss of power in counting?

 $\sim \frac{m_N}{4\pi f_\pi} \frac{k_{CM}}{\# f_\pi} \times$ 

NN cut $\to$ m_N appearing in numerator

- KSW: $\# f_\pi \sim 300$ MeV $\to$ expansion in $Q/300$ MeV (Kaplan, Savage & Wise , 1998).
  Too optimistic for spin-triplet channels (Fleming, Mehen & Stewart, 2000)
- Weinberg: $M_{\rm lo} \equiv \# f_\pi \sim f_\pi \simeq 92$ MeV
- Two mass scales $\to$ naive dimensional analysis no longer powerful
  $m_{\rm lo} \sim 100$ MeV, $m_{\rm hi} \sim 1000$ MeV
- Solution: nonperturbative pion in lower partial waves and perturbative pion in higher partial waves (Nogga et al, 2005)

Perturbative pion in higher partial waves

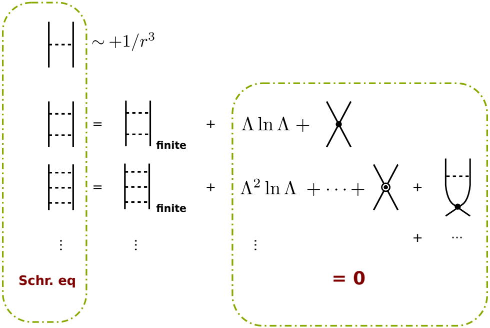


$\mathcal{O}(1)$ being OPE-resummed lower partial waves

- Reducible pion loops less suppressed (KSW)
- Peripheral partial waves may need revisit

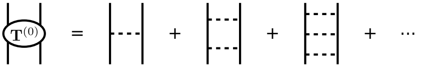
Nonperturbative pion: triplet channels

Eg.

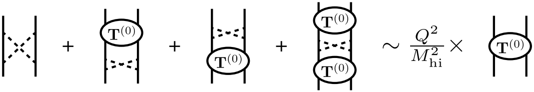


- Resumming OPE $\approx$ imposing at LO correlations among counterterms
- More nontrivial in attractive triplet channels (Beane et al, 2002; Nogga et al, 2005; Valderrama et al, 2005 - 2007)
  $\rightarrow$ $^3P_0$ counterterm at LO: two-derivative operator not suppressed!

Subleading orders of triplet channels



Power counting of pion exchanges not affected much: TPE still suppressed by two orders



- Subleading counterterms are those necessary to renormalize insertions of TPE

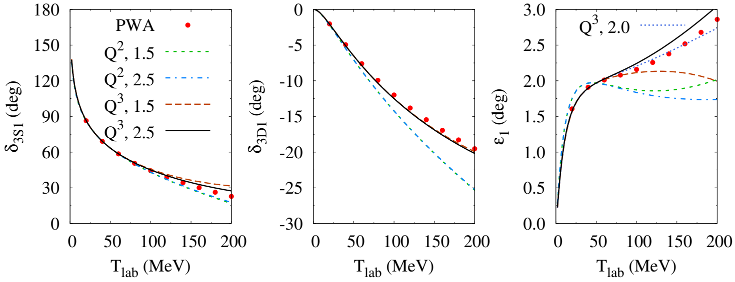
$$\mathcal{L}_{3P0} = D_0 (N^\dagger \partial^2 N)(N^\dagger N) + D_2 (N^\dagger \partial^4 N)(N^\dagger N) + \cdots$$

$$D_0 \propto \frac{1}{M_{\rm lo}^2}\,, D_2 \propto \frac{1}{M_{\rm lo}^2 M_{\rm hi}^2}\,, D_4 \propto \frac{1}{M_{\rm lo}^2 M_{\rm hi}^4} \cdots$$

(BwL & Yang 2011; Valderrama 2011)

${}^3S_1 - {}^3D_1$ phase shifts

(BwL & Yang, 2011)



Q^2: leading TPE, Q^3: subleading TPE. "1.5": $\Lambda = 1.5$ GeV

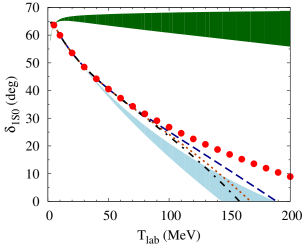
- Plots not shown here: ${}^3P_0$ and ${}^3P_1$ (BwL & Yang, 2011)
- Overall good fit up to $T_{\text{lab}} \sim 100$ MeV ($k_{\text{CM}} \simeq 200$ MeV). Can be improved by tuning $\pi\pi NN$ couplings, especially in ${}^3P_1$.

Saga of ${}^1S_0$

- ${}^1S_0$ LO = Yukawa + $C_{^1S_0}$
- ${}^1S_0$ LO residual cutoff dependence $= \mathcal{O}(Q/\Lambda) > \text{TPE} = \mathcal{O}(Q^2/M_{hi}^2)$
  - $\mathcal{O}(Q)$ does not vanish, unlike Weinberg's
  - subleading counterterms more important than TPE
- Pionless-theory-like power counting (KSW, 1998; Birse et al, 1999)

$$\mathcal{L}_{1S0} = C_0(N^\dagger N)^2 + C_2(N^\dagger \partial^2 N)(N^\dagger N) + \cdots$$

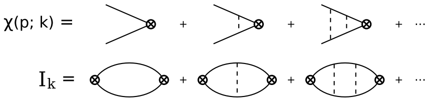
$$C_0 \propto \frac{1}{M_{lo}}, C_2 \propto \frac{1}{M_{lo}^2 M_{hi}}, C_4 \sim \frac{1}{M_{lo}^3 M_{hi}^2} \ldots$$



(BwL & Yang, 2012)

- Green: $\mathcal{O}(1)$ with $\Lambda = 0.5 - 2.0$ GeV
- Blue: $\mathcal{O}(Q)$ with $\Lambda = 0.5 - 2.0$ GeV
- Lines: $\mathcal{O}(Q^2)$. Dashed - 0.5, dotted - 1.0, and dot-dashed 2.0 GeV

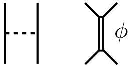
Improve LO of $^1S_0$



- Only C_0 at LO (Kaplan et al, 1996):

$$V^{(0)} = V_{Yukawa} + C_0\,,\ \ T^{(0)}_{1S0} = T_{Yukawa} + \frac{\chi^2(k;k)}{\frac{1}{C_0} - I_k}\,,\ \ I_k \sim \#\Lambda + \#\ln\Lambda$$

- To introduce energy dependence in LO counterterm, use spin-0, di-baryon field ϕ (Kaplan, 1996, with a bit of change):



$$V^{(0)} = V_{Yukawa} + \frac{\sigma y^2}{E+\Delta}\,, \qquad T^{(0)}_{1S0} = T_{Yukawa} + \frac{\chi^2(k;k)}{\frac{E+\Delta}{\sigma y^2} - I_k}$$

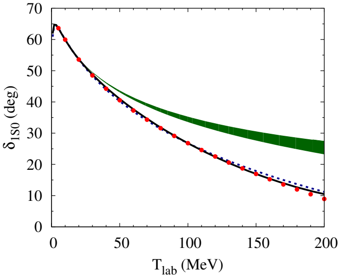
$$\sigma = -1\,, \quad \Delta_R \sim 13\text{MeV}$$

$^1S_0$ Dibaryon

- ϕ an auxiliary field to facilitate resummation, not physical d.o.f.
- Lagrangian with dibaryon not most general $\rightarrow$ at a given order the number of c.c. not increased

	with dibaryon	w/o dibaryon
$\mathcal{O}(1)$	$\frac{\sigma y^2}{E+\Delta}$ + Yukawa	$C_0 + C_2 p^2$ + Yukawa
$\mathcal{O}(Q)$	C_0	$C_4\, p^4$
$\mathcal{O}(Q^2)$	$C_2\, p^2$+ leading TPE	$C_6\, p^6$+ leading TPE

$^1S_0$ phase shifts



- Green: $\mathcal{O}(1)$ with $\Lambda = 0.6 - 1.6$ GeV
- Blue: $\mathcal{O}(Q)$ with $\Lambda = 1.6$ GeV
- Black: $\mathcal{O}(Q^2)$ with $\Lambda = 1.6$ GeV

Conclusion

- Nuclear EFT: power counting done at the level of on-shell diagrams
- RG invariance may allow several power countings
- Need to revisit the lower bound of $J(L)_{pert.pion}$
- Dibaryon field improves convergence of $^1S_0$