Chiral Extrapolation of the Nucleon-Nucleon S-wave scattering lengths

Jaume Tarrús Castellà

Departament d'Estructura i Constituents de la Matèria
and
Institut de Ciències del Cosmos
Universitat de Barcelona
Work with: Joan Soto
tarrus@ecm.ub.es

August 2012

Objective:

- To study the quark mass dependence of the NN S-wave scattering lengths in NNEFT with dibaryon fields.
- To use lattice data to determine the relevant (combinations) of low–energy constants.

NNEFT with dibaryon fields Soto, JTC (08); Soto, JTC (10)

- Two dibaryon fields an isovector $({}^1S_0)$ and an isoscalar $({}^3S_1)$, coupled to pions and nucleons according to Chiral Symmetry.
- All low–energy constants of natural size.
- Perturbative pions.
- Dimensional regularization and $\overline{MS}$, so all scales are explicit.
- Integrating out the dibaryons we obtain a theory with enhanced contact interactions similar to KSW. Bedaque, Grießhammer (99)

NNEFT Lagrangian

- The pion and one nucleon sectors as well as the nucleon–pion interaction are the usual ones:

$\mathcal{L}_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \ldots$ Gasser, Leutwyler (84) $\quad N_f = 2$

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}\left[D^\mu U^\dagger D_\mu U + \chi^\dagger U + U^\dagger \chi\right], \quad \chi = 2B_0\hat{m}\mathbf{1}, \quad m_\pi^2 = 2B_0\hat{m}, \quad \hat{m} = \frac{m_u + m_d}{2}$$

$$U = e^{i\,2T\cdot\pi/F_0}, \quad \boldsymbol{\pi} \equiv \pi^\alpha\tau^\alpha = 2\pi^\alpha T^\alpha = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)}$

$$\mathcal{L}_{\pi N}^{(1)} = N^\dagger\left(iD_0 - \frac{g_A}{2}\vec{\sigma}\cdot\vec{u}\right)N\,,$$

$$\mathcal{L}_{\pi N}^{(2)} = N^\dagger\left(\frac{\vec{D}^2}{2m_N} + \frac{ig_A}{4m_N}\{\vec{\sigma}\cdot\vec{D}, u_0\} + c_1\mathrm{Tr}\left[\chi_+\right] + c_2 u_0^2 - c_3\vec{u}\cdot\vec{u}\right.$$
$$\left. + i\frac{c_4}{2}\epsilon^{ijk}\sigma^k u_i u_j + c_5\tilde{\chi}_+ - \frac{c_6}{8m_N}\epsilon^{ijk}F_{ij}^+\sigma_k - \frac{c_7}{8m_N}\epsilon^{ijk}\mathrm{Tr}\left[F_{ij}^+\right]\sigma_k\right)N\,.$$

The Dibaryon-Pion interaction

The LO Lagrangian:

$$\mathcal{L}_D^{(1)} = \frac{1}{2}\mathrm{Tr}\left[D_s^\dagger\left(-id_0+\delta'_{m_s}\right)D_s\right] + \vec{D}_v^\dagger\left(-i\partial_0+\delta'_{m_v}\right)\vec{D}_v + ic_{sv}\left(\vec{D}_v^\dagger \mathrm{Tr}\left[\vec{u}D_s\right] - h.c.\right) ,$$

with $D_s = D_s^a\tau^a$, and $d_0 D_s = \partial_0 D_s + \frac{1}{2}[[u,\partial_0 u], D_s]$. The NLO Lagrangian

$$\begin{aligned}\mathcal{L}_D^{(2)} =& s_1\mathrm{Tr}\left[D_s(u\mathcal{M}^\dagger u + u^\dagger\mathcal{M}u^\dagger)D_s^\dagger\right] + s_2\mathrm{Tr}\left[D_s^\dagger(u\mathcal{M}^\dagger u + u^\dagger\mathcal{M}u^\dagger)D_s\right] + \\ &+ v_1\vec{D}_v^\dagger\cdot\vec{D}_v\mathrm{Tr}\left[u^\dagger\mathcal{M}u^\dagger + u\mathcal{M}^\dagger u\right] + \ldots \quad \mathcal{M} = \hat{m}\mathbf{1}\end{aligned}$$

The Dibaryon-Nucleon interaction

The LO Lagrangian:

$$\mathcal{L}_{DN}^{(1)} = \frac{A_s}{\sqrt{2}}(N^\dagger\sigma^2\tau^a\tau^2N^*)D_{s,a} + \frac{A_s}{\sqrt{2}}(N^\top\sigma^2\tau^2\tau^aN)D_{s,a}^\dagger + \\ + \frac{A_v}{\sqrt{2}}(N^\dagger\tau^2\vec{\sigma}\sigma^2N^*)\cdot\vec{D}_v + \frac{A_v}{\sqrt{2}}(N^\top\tau^2\sigma^2\vec{\sigma}N)\cdot\vec{D}_v^\dagger\,,$$

the NLO Lagrangian:

$$\mathcal{L}_{DN}^{(2)} = \frac{B_s}{\sqrt{2}}(N^\dagger\sigma^2\tau^a\tau^2\vec{D}^2N^*)D_{s,a} + \frac{B_s}{\sqrt{2}}(N^\top\sigma^2\tau^2\tau^a\vec{D}^2N)D_{s,a}^\dagger + \\ + \frac{B_v}{\sqrt{2}}(N^\dagger\tau^2\vec{\sigma}\sigma^2\vec{D}^2N^*)\cdot\vec{D}_v + \frac{B_v}{\sqrt{2}}(N^\top\tau^2\sigma^2\vec{\sigma}\vec{D}^2N)\cdot\vec{D}_v^\dagger \\ + \frac{B_v'}{\sqrt{2}}(D_iN^\dagger\tau^2\sigma^i\sigma^2D_jN^*)D_v^j + \frac{B_v'}{\sqrt{2}}(D_iN^\top\tau^2\sigma^2\sigma^iD_jN)D_v^{j\dagger}\,.$$

The Dibaryon-Nucleon interaction

The LO Lagrangian:

$$\mathcal{L}_{DN}^{(1)} = \frac{A_s}{\sqrt{2}}(N^\dagger\sigma^2\tau^a\tau^2N^*)D_{s,a} + \frac{A_s}{\sqrt{2}}(N^\top\sigma^2\tau^2\tau^aN)D_{s,a}^\dagger + \\ + \frac{A_v}{\sqrt{2}}(N^\dagger\tau^2\vec{\sigma}\sigma^2N^*)\cdot\vec{D}_v + \frac{A_v}{\sqrt{2}}(N^\top\tau^2\sigma^2\vec{\sigma}N)\cdot\vec{D}_v^\dagger\,,$$

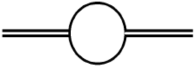
the NLO Lagrangian:

$$\mathcal{L}_{DN}^{(2)} = \frac{B_s}{\sqrt{2}}(N^\dagger\sigma^2\tau^a\tau^2\vec{D}^2N^*)D_{s,a} + \frac{B_s}{\sqrt{2}}(N^\top\sigma^2\tau^2\tau^a\vec{D}^2N)D_{s,a}^\dagger + \\ + \frac{B_v}{\sqrt{2}}(N^\dagger\tau^2\vec{\sigma}\sigma^2\vec{D}^2N^*)\cdot\vec{D}_v + \frac{B_v}{\sqrt{2}}(N^\top\tau^2\sigma^2\vec{\sigma}\vec{D}^2N)\cdot\vec{D}_v^\dagger \\ + \frac{B_v'}{\sqrt{2}}(D_iN^\dagger\tau^2\sigma^i\sigma^2D_jN^*)D_v^j + \frac{B_v'}{\sqrt{2}}(D_iN^\top\tau^2\sigma^2\sigma^iD_jN)D_v^{j\dagger}\,.$$

All terms must have natural size!: $A_{s,v} \sim \Lambda_\chi^{-1/2}$, $B_{s,v} \sim \Lambda_\chi^{-5/2}$, $B_v' \sim \Lambda_\chi^{-5/2}$

Dibaryon Propagator

- Tree level $\frac{i}{-E+\delta'_{m_i}}$

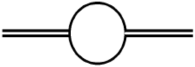


- One loop $\frac{i}{-E+\delta'_{m_i}+i\frac{A_i^2 m_N p}{\pi}}$

$\Longrightarrow$ E is always smaller than the self-energy term: $E \sim \frac{p^2}{\Lambda_\chi}$.

Dibaryon Propagator

- Tree level $\frac{i}{-E+\delta'_{m_i}}$



- One loop $\frac{i}{-E+\delta'_{m_i}+i\frac{A_i^2 m_N p}{\pi}}$

$\Longrightarrow$ E is always smaller than the self-energy term: $E \sim \frac{p^2}{\Lambda_\chi}$.

Size of δ'_{m_i}

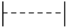
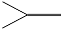
Matching the one loop propagator to the ERE: $\delta'_{m_i} \sim \frac{1}{\pi a_i} \lesssim \frac{m_\pi^2}{\Lambda_\chi} \qquad i = s, v.$

$$a^{{}^3S_1} = 5.38\text{fm}\left(\frac{1}{36.7\text{MeV}}\right) \quad a^{{}^1S_0} = -23.7\text{fm}\left(-\frac{1}{8.32\text{MeV}}\right)$$

Counting

There are two momentum regions with different counting:

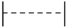
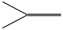
pNNEFT, $p \sim m_\pi$, $E \sim p^2/\Lambda_\chi$

-  $\Longrightarrow \frac{\pi}{A_i^2 m_N p} \sim \frac{1}{m_\pi}$ $\quad i = s, v.$
-  $\Longrightarrow i(-E + \delta_{m_i})$
-  Potential pions $q^0 \sim \vec{q}^2/m_N$
-  Dibaryon-Nucleon interactions

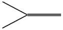
Counting

There are two momentum regions with different counting:

pNNEFT, $p \sim m_\pi$, $E \sim p^2/\Lambda_\chi$

-  $\Longrightarrow \frac{\pi}{A_i^2 m_N p} \sim \frac{1}{m_\pi}$ $\qquad i = s, v.$
-  $\Longrightarrow i(-E + \delta_{m_i})$
-  Potential pions $q^0 \sim \vec{q}^2/m_N$
-  Dibaryon-Nucleon interactions

$\not\pi$NNEFT, $p \lesssim \frac{m_\pi^2}{\Lambda_\chi}$, no dynamic pions

- $\Longrightarrow \frac{i}{\delta_{m_i} + i\frac{A_i^2 m_N p}{\pi}} \sim \frac{\Lambda_\chi}{m_\pi^2}$ $\qquad i = s, v.$
-  $\Longrightarrow -iE$
-  Dibaryon-Nucleon interactions

Chiral extrapolation of the scattering lengths

In $\not\pi$NNEFT the scattering lengths up to NLO are given by

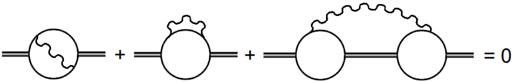
$$a_i^{-1} = \frac{\pi\delta_{m_i}}{m_N A_i^2} \approx \frac{\pi\delta_{m_i}^{LO}}{m_N A_i^2}\left(1-\Delta_{\rm NLO}\right) + \frac{\pi\delta_{m_i}^{NLO}}{m_N A_i^2}, \quad i = s(^1S_0)\,, v(^3S_1)\,.$$

with $\Delta_{\rm NLO} = A_{i,{\rm NLO}}^2/A_i^2 - 1$.

- $\delta_{m_i}^{LO}, \delta_{m_i}^{NLO}, \Delta_{\rm NLO}$ are obtained matching NNEFT→pNNEFT→ $\not\pi$NNEFT.

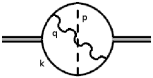
Matching NNEFT to pNNEFT: Radiation pions

pNNEFT is obtained by integrating out nucleons of and pions $E \gtrsim m_\pi$. Among the later there are the radiation pions $q^0 \sim \vec{q} \sim m_\pi$.



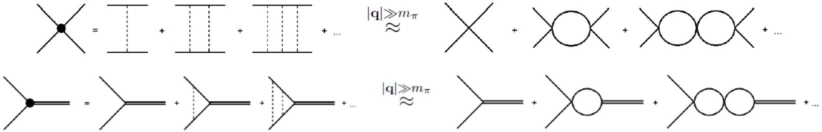
- Parametric suppression of $\mathcal{O}\left(\sqrt{m_\pi/\Lambda_\chi}\right)$, but numerically $\mathcal{O}(1)$.
- However they add up to zero, due to Wigner Symmetry. Fleming, Mehen, Stewart (00)

Further contributions are obtained introducing potential pions, $q^0 \sim \vec{q}^2/m_N$, inside the radiation pion loops.



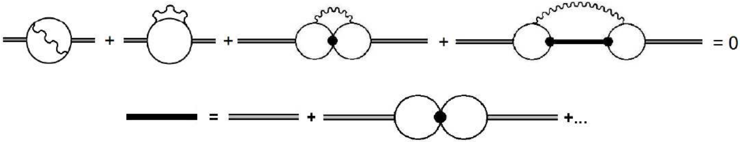
$$\vec{p} \sim \sqrt{m_\pi m_N} \Rightarrow \frac{\vec{p}^2}{\vec{p}^2 + m_\pi^2} \sim 1$$

As a consequence n potential pion exchanges can be simplified



- Each bubble should add a parametric suppression of $\sqrt{m_\pi/\Lambda_\chi}$ but in fact $\pi\sqrt{m_\pi/\Lambda_\chi} \sim 1.2!$.
- The resumation can be carried out in the $^1S_0$ channel ($^3S_1$ channel in the whole diagram).

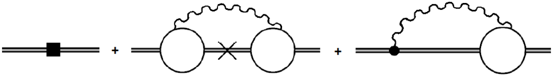
Summing up all this class of radiation pion diagrams for the $^3S_1$ channels adds up to zero!



- Reason: The contracted pion exchange, 4–nucleon vertex can be removed through field redefinitions.
- $^1S_0$ channel the resummation cannot be carried out, but the argument still applies to one potential pion insertion.
- Potentially large unknown contributions in the $^1S_0$ channel.

Matching NNEFT to pNNEFT

Residual Mass (δ_m) up to NLO

$\delta_{m_i} = \delta'_{m_i} + 2\delta m_N +$  , i=s,v.

δm_N are contributions to the matching to the nucleon mass reshuffled into δ_{m_i} through field redefinitions.

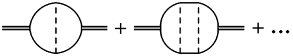
$$\delta m_N = -4c_1 m_\pi^2 - \frac{3g_A^2}{64\pi F_0^2} m_\pi^3 \,.$$

Matching pNNEFT$\rightarrow$ $\not\pi$NNEFT

$p \lesssim \frac{m_\pi^2}{\Lambda_\chi}$

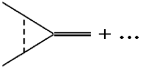
- Non-local potentials become local and can be organized in powers of p^2/m_π^2

Residual masses (δ_{m_i}) up to NLO



- Contribution to the residual mass and the time derivative.
- Contributions to the dibaryon time derivative can be reabsorbed by field redefinitions.

Dibaryon-Nucleon Interactions (A_i) up to NLO



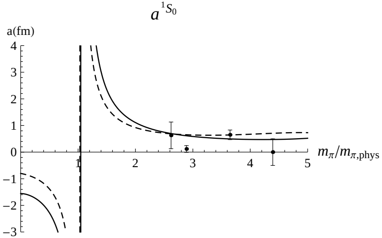
- The dibaryon-nucleon vertex gets a contribution from this vertex.

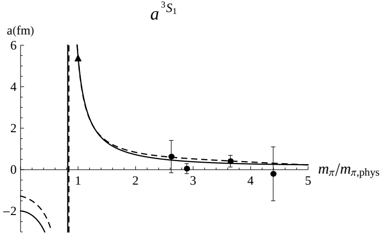
$$a_i^{-1} = \zeta_{i1}\left(1 - \frac{g_A^2 m_N}{16\pi F_0^2} m_\pi\right) + \left[\zeta_{i2} - \frac{g_A^2 m_N}{32\pi F_0^2}\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] m_\pi^2 + \zeta_{i3} m_\pi^3$$
$$+ \frac{1}{2}\left(\frac{g_A^2 m_N}{16\pi F_0^2}\right)^2 m_\pi^3 \ln\left(\frac{m_\pi^2}{\mu^2}\right) \quad , \quad i = s({}^1S_0)\,, v({}^3S_1)\,.$$

	ζ_1	ζ_2
${}^1S_0$	$\frac{\pi\delta'_{m_s}}{m_N A_s^2}$	$\frac{2\pi((s_1+s_2)/B_0-8c_1)}{m_N A_s^2}$
${}^3S_1$	$\frac{\pi\delta'_{m_v}}{m_N A_v^2}$	$\frac{2\pi(v_1/B_0-8c_1)}{m_N A_s^2}$

	ζ_3
${}^1S_0$	$\frac{g_A^2}{32 m_N F_0^2}\left(\frac{1}{A_v^2} + \frac{2c_{sv}}{g_A A_s A_v} - \frac{3}{A_s^2}\right) - \frac{g_A^2}{8F_0^2}\frac{(s_1+s_2)/B_0-8c_1}{A_s^2} + \left(\frac{g_A^2 m_N}{2F_0^2}\right)^2 \frac{\log(2)}{128\pi^2}$
${}^3S_1$	$\frac{g_A^2}{32 m_N F_0^2}\left(\frac{1}{A_s^2} + \frac{2c_{sv}}{g_A A_s A_v} - \frac{3}{A_v^2}\right) - \frac{g_A^2}{8F_0^2}\frac{(v_1/B_0-8c_1)}{A_v^2} + 5\left(\frac{g_A^2 m_N}{2F_0^2}\right)^2 \frac{6+13\log(2)}{256\pi^2}$

Lattice data from NPLQCD collaboration, Beane *et al.* (06)





Solid(dashed) line correspond to LO(NLO)

LO	$\chi^2_{d.o.f}$	$\zeta_1(MeV)$	$\zeta_2(MeV^{-1})$
$^1S_0$	3.74	-126	$0.67 \cdot 10^{-3}$
$^3S_1$	0.91	-98	$1.59 \cdot 10^{-3}$

NLO	$\chi^2_{d.o.f}$	$\zeta_1(MeV)$	$\zeta_2(MeV^{-1})$	$\zeta_3(MeV^{-2})$
$^1S_0$	2.4	-246	$4.56 \cdot 10^{-3}$	$9.21 \cdot 10^{-6}$
$^3S_1$	0.4	-155	$3.83 \cdot 10^{-3}$	$10.1 \cdot 10^{-6}$

Conclusions

- Contributions to the ${}^3S_1$ scattering length with one radiation pion and n potential pion are of $\mathcal{O}(1)$, but add up at to zero.
- In the ${}^1S_0$ channel it is not possible to compute such diagrams for an arbitrary number of potential pions, there are possible large contributions missing.
- We have given chiral extrapolation formulas for the scattering lengths up to $\mathcal{O}\left(m_\pi^3/\Lambda_\chi\right)$
- Lattice data favors an unbounded deuteron in the chiral limit.

Thank you for your attention