The Two Nucleon System in Chiral Effective Field Theory: Searching for the Power Counting

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Chiral Dynamics, JLab, August 2012

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MPV PRC 83, 044002 (2011); PRC 84, 064002 (2011)

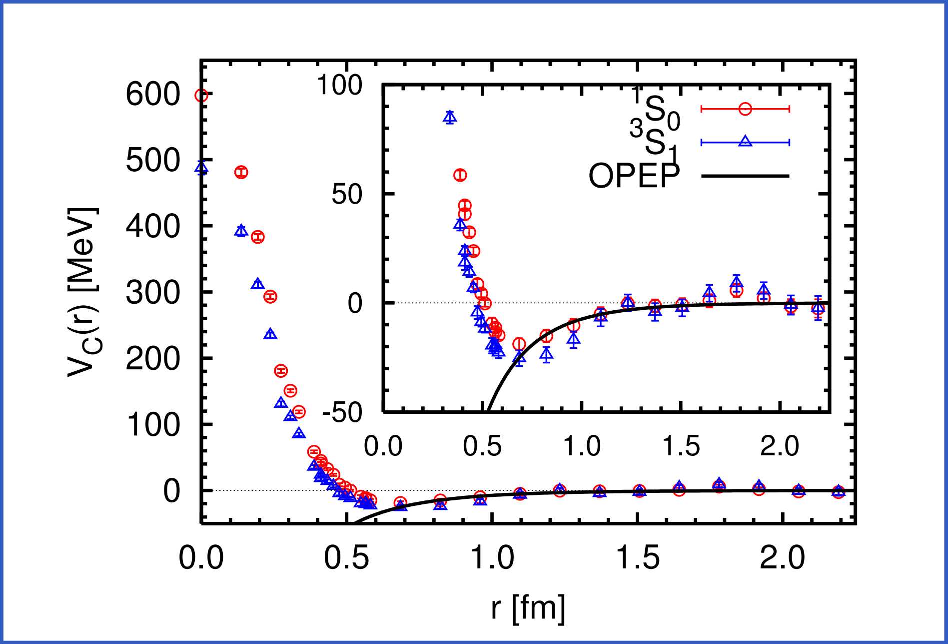
Deriving Nuclear Forces from QCD

The nuclear force is the fundamental problem in nuclear physics

- Many phenomenological descriptions available which are, however, not grounded in QCD.
- The Goal: a QCD based description of the nuclear force

Deriving Nuclear Forces from QCD

- Strategy 1: Lattice QCD (talks this morning) will eventually do it



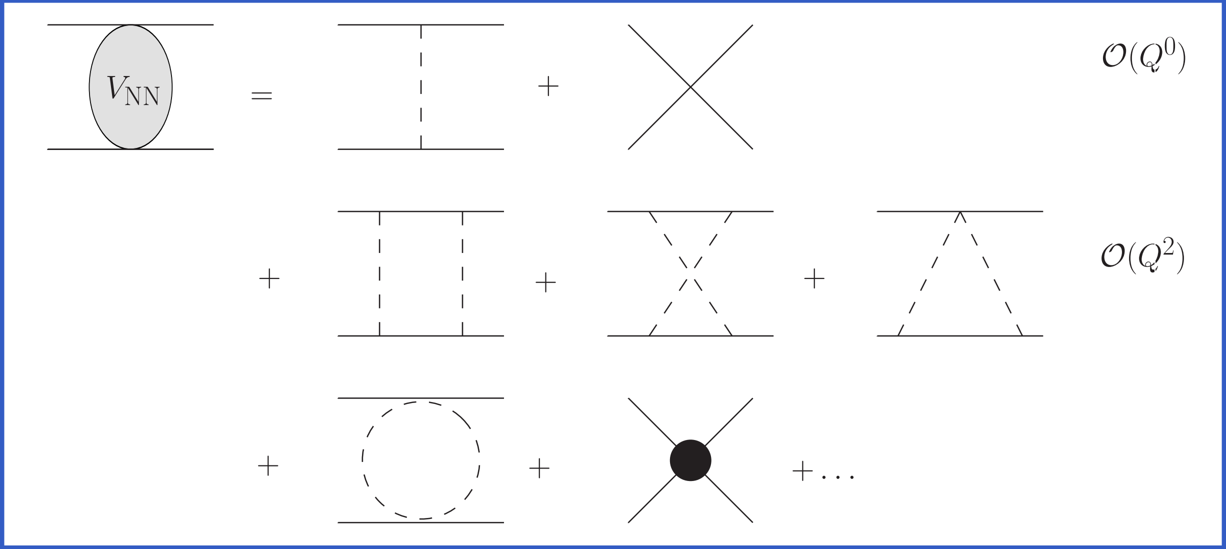
Ishii, Aoki, Hatsuda 06 (with $m_\pi \simeq 0.53\,\mathrm{GeV}$, $m_N \simeq 1.34\,\mathrm{GeV}$).

- Strategy 2: Low energy EFT of nuclear forces incorporating known low energy symmetries of QCD (if you can't wait or you don't have a supercomputer)

The Nucleon-Nucleon Chiral Potential (I)

Here we construct a nuclear effective field theory

- Chiral perturbation theory is the starting point: the πN interaction constrained by broken chiral symmetry (the QCD remnant).
- Nucleons are heavy ($M_N \sim \Lambda_\chi$): we can define a non-relativistic potential (the Weinberg proposal) that admits an expansion



Weinberg (90); Ray, Ordoñez, van Kolck (93,94); etc.

Power Counting (I)

It's important, so I repeat, there are two essential ingredients:

- Chiral symmetry provides the connection with QCD.
- Power counting makes the EFT systematic: it orders the infinite number of chiral symmetric diagrams.
    - In EFT we have a separation of scales:

$$\underbrace{|\vec{q}| \sim p \sim m_\pi \sim}_{\text{the known physics}} \quad Q \ll \Lambda_0 \quad \underbrace{\sim m_\rho \sim M_N \sim 4\pi f_\pi}_{\text{the unknown physics}}$$

    - Then the idea is to expand amplitudes as powers of Q/Λ_0:

$$T = \sum_{\nu=\nu_{\rm min}}^{\nu_{\rm max}} T^{(\nu)} + \mathcal{O}\left(\frac{Q}{\Lambda_0}\right)^{\nu_{\rm max}+1}$$

    - Power counting refers to the set of rules from which we construct this kind of low energy expansion.

Power Counting (II)

What is power counting useful for? What are its consequences?

- If we express the NN potential as a low energy expansion:

$$V_{\mathrm{EFT}} = V^{(0)}(\vec{q}) + V^{(2)}(\vec{q}) + V^{(3)}(\vec{q}) + \mathcal{O}(\frac{Q^4}{\Lambda_0^4})\,,$$

  we appreciate that the potential should convergence quickly at low energies / large distances (and diverge at high energies).

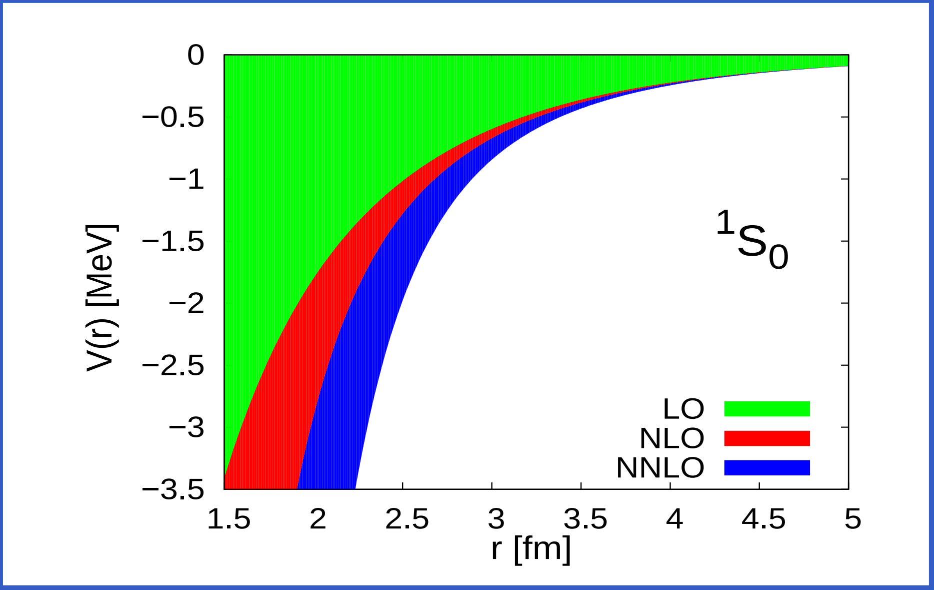
- Apart, we can know in advance how the potential diverges:

$$V^{(\nu)}(\vec{q}) \propto \frac{|\vec{q}\,|^{\,\nu}}{\Lambda_0^{\nu+2}}\, f(\frac{|\vec{q}|}{m_\pi}) \underbrace{\longrightarrow}_{\mathcal{F}} V^{(\nu)}(\vec{r}) \propto \frac{1}{\Lambda_0^{\nu+2} r^{\nu+3}}\, f(m_\pi r)\,.$$

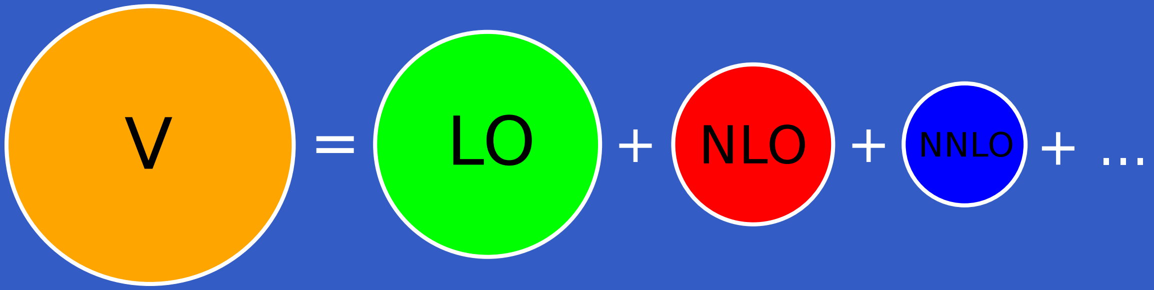
  This means that regularization and renormalization are required: we will have a cut-off Λ.

The Nucleon-Nucleon Chiral Potential (II)

The NN chiral potential in coordinate space:

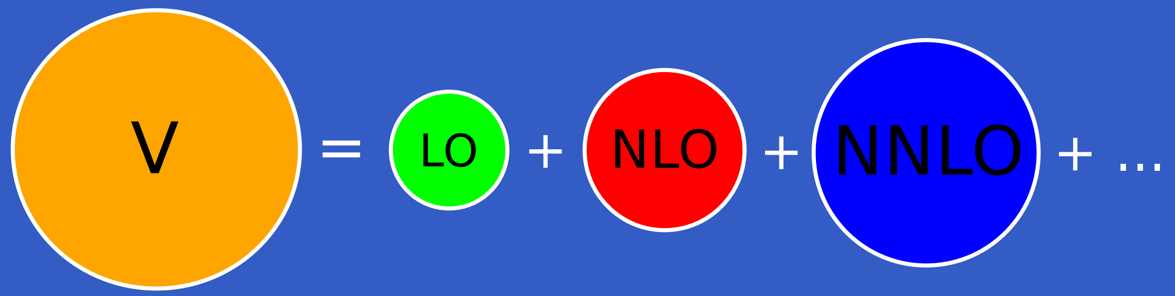


At long distances power counting implies:

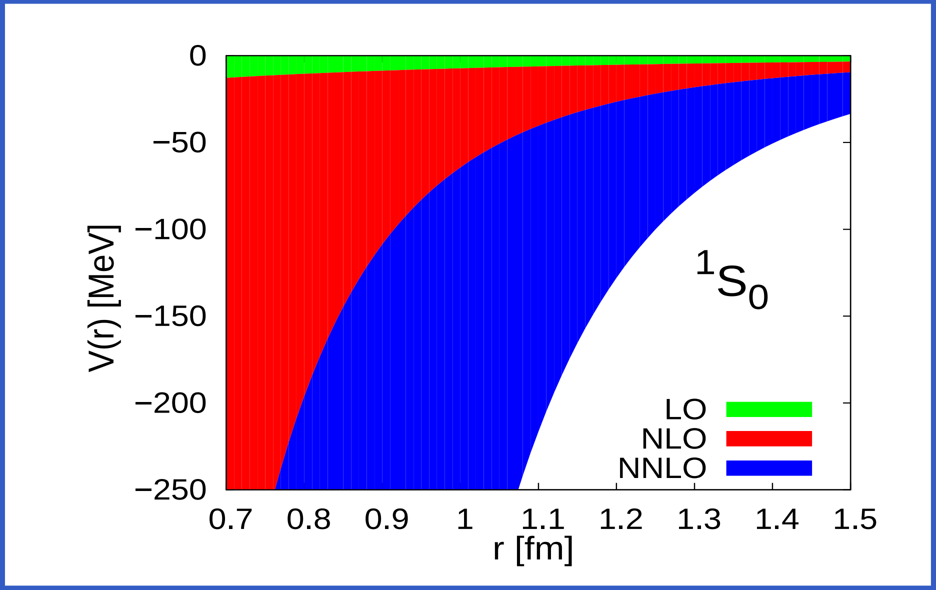


The Nucleon-Nucleon Chiral Potential (III)

However, at short distances the situation is just the opposite:



... as can be checked in coordinate space:



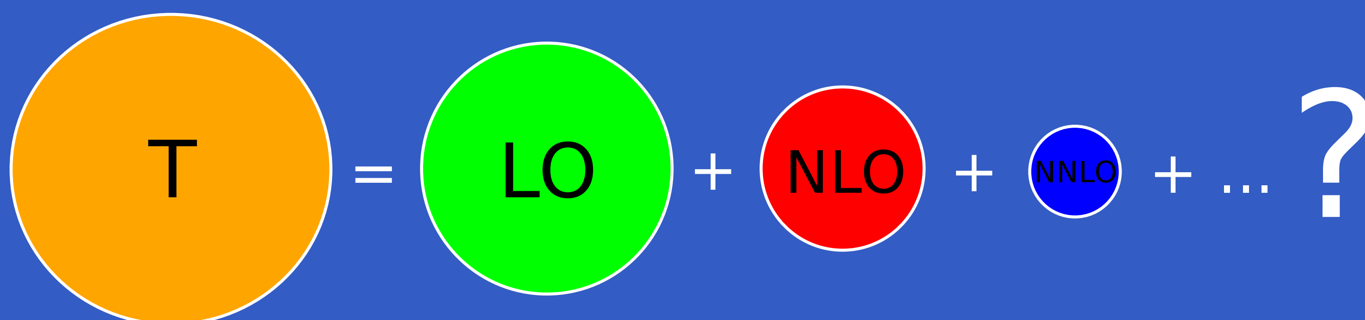
Scattering Observables (I)

What about scattering observables? The naive answer is as follows:

- We plug the potential into the Lippmann-Schwinger equation

$$T = V + V\,G_0\,T$$

- We check that we preserve power counting in T:



However, this is far from trivial.

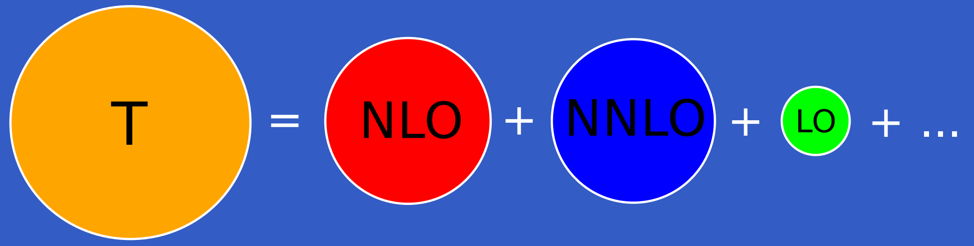
Scattering Observables (II)

What can fail in the power counting of the scattering amplitude?

We are iterating the full potential. Subleading interactions may dominate the calculations if:

- We are using a too hard cut-off, $\Lambda \geq \Lambda$.
- We are not including enough contact range operators to guarantee the preservation of power counting in T.

In either case we can end up with something in the line of:



that is, an anti-counting. Lepage (98); Epelbaum and Gegelia (09). This could be happening to the $\mathrm{N}^3\mathrm{LO}$ potentials!

Scattering Observables (III)

Let's start all over again, but now we will be careful.

There is a fool proof way of respecting power counting in T:

- We begin with $T = V + V\,G_0\,T$
- But now, we re-expand it according to counting, that is, we treat the subleading pieces of V as a perturbation.

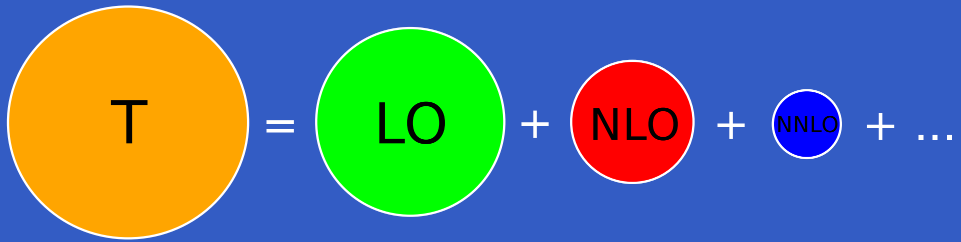
$$\begin{aligned} T^{(0)} &= V^{(0)} + V^{(0)}\,G_0\,T^{(0)}\,, \\ T^{(2)} &= (1 + T^{(0)}\,G_0)\,V^{(2)}\,(G_0\,T^{(2)} + 1)\,, \text{ etc.} \end{aligned}$$

- Perturbations are small, so we expect power counting to hold.

And now we can give a general recipe for constructing a power counting for nuclear EFT...

Constructing a Power Counting

The Power Counting Algorithm (simplified version):

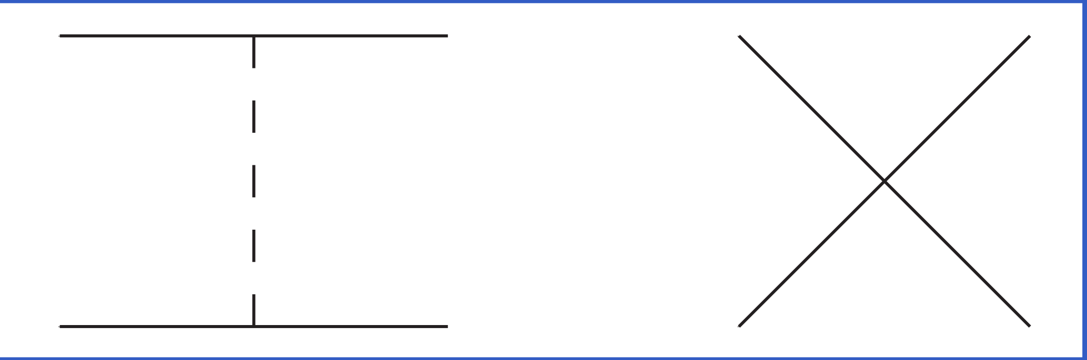


- Choose a minimal set of diagrams (the lowest order potential): this is the only piece of the potential we iterate!
- Higher order diagrams enter as perturbations
- At each step check for cut-off independence
  - If not, include new counterterms to properly the results.
  - Once cut-off independence is achieved, we are finaly done!

    (Well, actually not. There are additional subtleties I didn’t mention.)

The Leading Order Potential

What to iterate? Two (a posteriori obvious) candidates:



- a) The bound (virtual) state happen at momenta of $\gamma = 45\,\mathrm{MeV}$ ($8\,\mathrm{MeV}$), much smaller than $m_\pi = 140\,\mathrm{MeV}$.
- b) There is an accidental low energy scale in tensor OPE

$$\Lambda_T = \frac{16\pi\, f_\pi^2}{3M_N g^2} \simeq 100\,\mathrm{MeV}$$

Kaplan, Savage, Wise (98); van Kolck (98); Gegelia (98); Birse et al. (98); Nogga, Timmermans, van Kolck (06); Birse (06); Valderrama (11); Long and Chen (11).

Check for Renormalizability (I)

The next step is to check cut-off dependence:

Nogga, Timmermans, van Kolck (06); Valderrama, Arriola (06); Epelbaum, Gegelia (12)

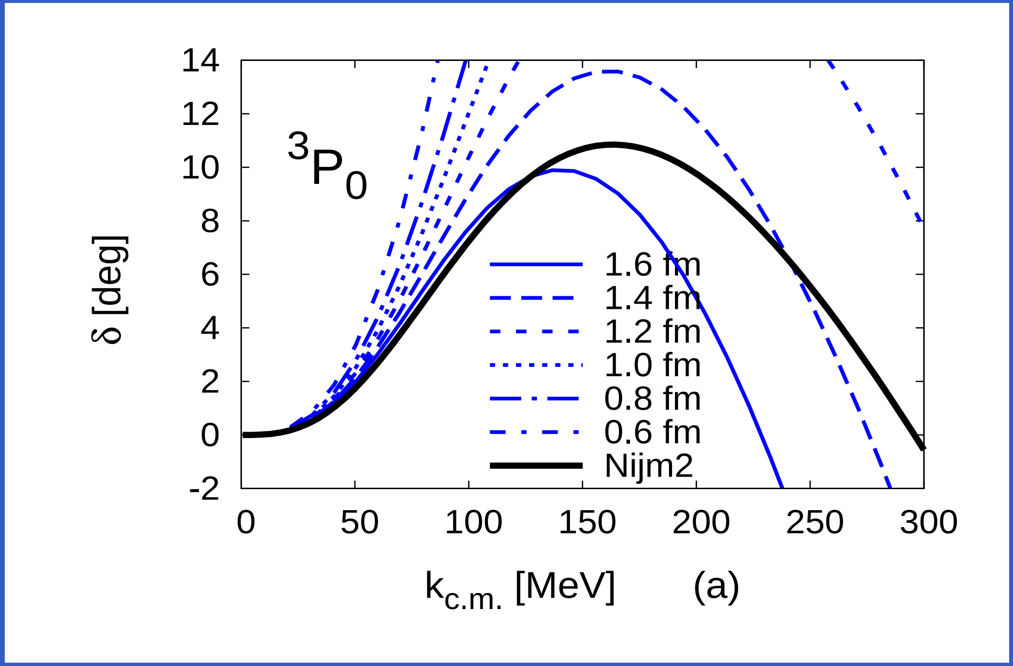
- S-waves:
  - $^1S_0$: everything's working fine.
  - $^3S_1$: everything's working fine too.
- P-waves:
  - $^1P_1$, $^3P_1$: again, everything's working fine.
  - $^3P_2$: hmmm... looks fine, unless the cut-off's really high.
  - $^3P_0$: definitively, something's wrong with this wave.
- D-waves and higher:
  - a few hmmm...'s, but generally OK.

So it seems that we are not done with the leading order!

Check for Renormalizability (II)

Nogga, Timmermans, van Kolck (06); Valderrama, Arriola (06); Epelbaum, Gegelia (12)

The $^3P_0$ shows a strong cut-off dependence:



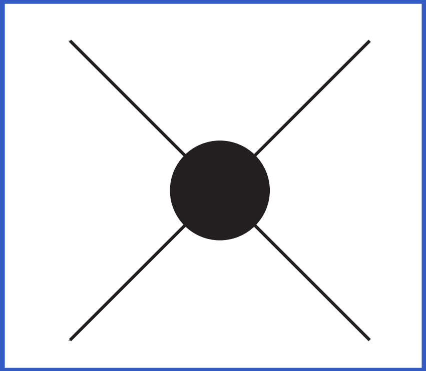
actually is cyclic, but we have only shown the first cycle.

Check for Renormalizability (III)

Nogga, Timmermans, van Kolck (06); Valderrama, Arriola (06); Epelbaum, Gegelia (12)

How to solve this issue? Easy: we include a P-wave counterterm at LO

- In principle we should have



$$C_{^3P_0}\,\vec{p}\cdot\vec{p}\,' \underbrace{\longrightarrow}_{Q\to\lambda Q} \lambda^2\, C_{^3P_0}\,\vec{p}\cdot\vec{p}\,'$$

i.e. order Q^2, which is true as far as $C_{^3P_0}(\lambda Q) = C_{^3P_0}(Q)$.

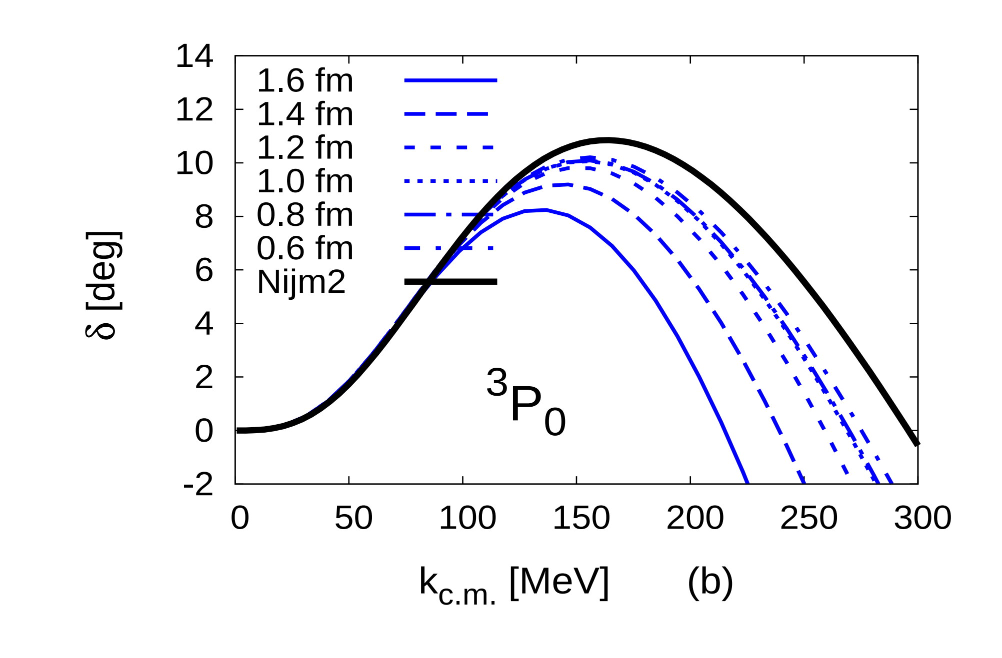
- But cut-off dependence at soft scales indicates that actually:

$$C_{^3P_0}(\lambda Q) = \frac{1}{\lambda^3}\,C_{^3P_0}(Q) \quad \text{or} \quad C_{^3P_0} \propto \frac{1}{\Lambda_0 Q^3} \text{ with } Q = \Lambda_T$$

Check for Renormalizability (IV)

Nogga, Timmermans, van Kolck (06); Valderrama, Arriola (06); Epelbaum, Gegelia (12)

After the promotion of $C_{^3P_0}$ from Q^2 to Q^{-1}:



we recover approximate cut-off independence. A similar thing happens for the $^3P_2$ and $^3D_2$ partial waves.

Subleading Orders

Birse (06); Valderrama (11); Long and Chen (11).

We just follow the power counting recipe:

- 1) We include the subleading potential as a perturbation.
- 2) We check again for cut-off dependence.
- 3) And there is cut-off dependence:
  we include a few new counterterms.
- 4) We re-check for cut-off dependence,
  and now everything is working fine.

Of course, the actual calculations are fairly technnical,
but the underlying idea is fairly simple.

And we can summarize the results in a table.

Nuclear EFT: Power Counting

Partial wave	LO	NLO	N^2LO	N^3LO
$^1S_0$	1	3	3	4
$^3S_1 - {}^3D_1$	1	6	6	6
$^1P_1$	0	1	1	2
$^3P_0$	1	2	2	2
$^3P_1$	0	1	1	2
$^3P_2 - {}^3F_2$	1	6	6	6
$^1D_2$	0	0	0	1
$^3D_2$	1	2	2	2
$^3D_3 - {}^3G_3$	0	0	0	1
All	5	21	21	27
Weinberg	2	9	9	24

i) dependent on counterterm representation; ii) there are variations and fugues over this theme; iii) equivalent to Birse's RGA of 2006, modulo i) and ii).

Nuclear EFT: Phase Shifts

S, P and D-Waves

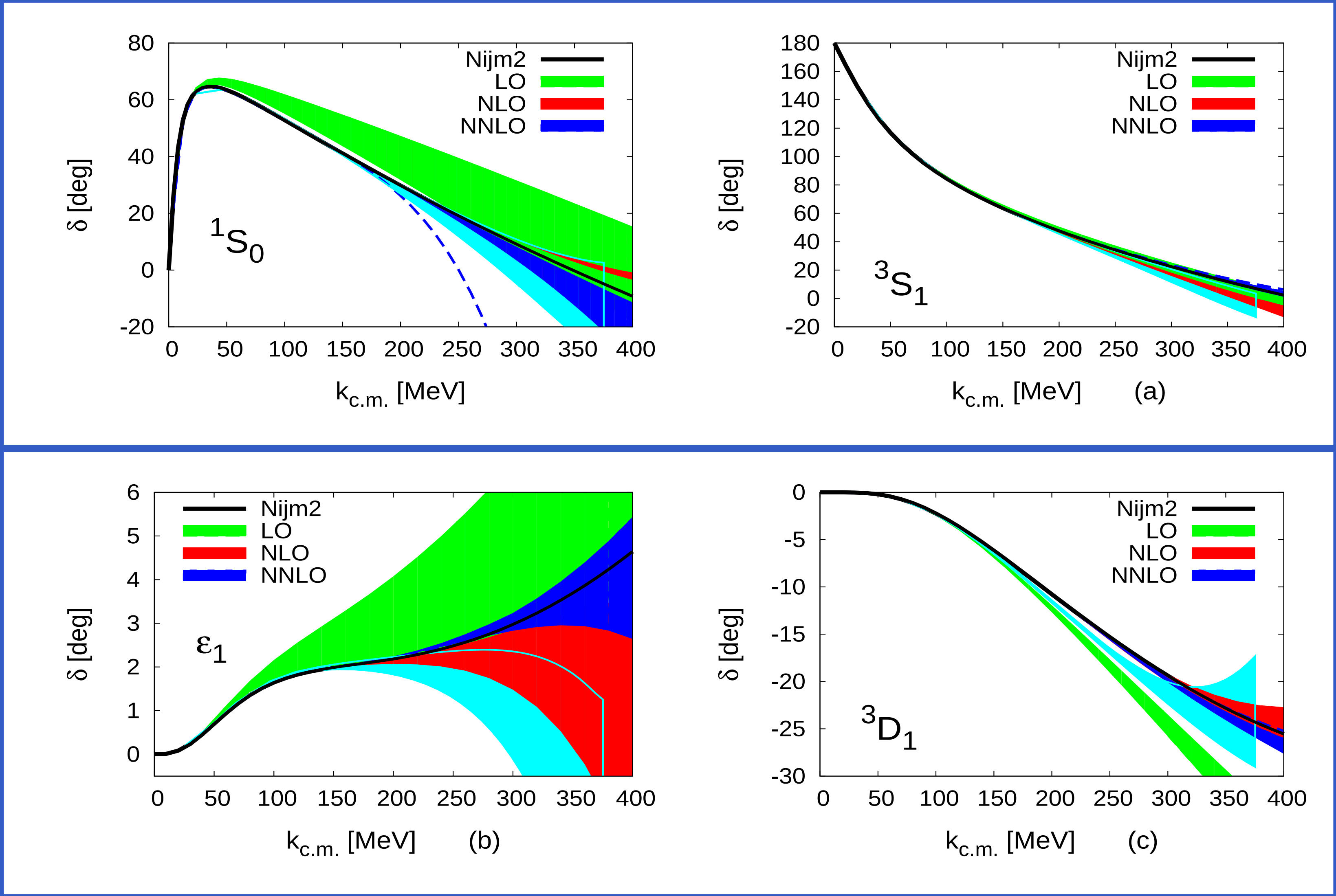
The following values have been taken:

$f_\pi = 92.4\,\mathrm{MeV}$, $m_\pi = 138.04\,\mathrm{MeV}$, $d_{18} = -0.97\,\mathrm{GeV}^2$
$c_1 = -0.81\,\mathrm{GeV}^{-1}$, $c_3 = -3.4\,\mathrm{GeV}^{-1}$, $c_4 = 3.4\,\mathrm{GeV}^{-1}$

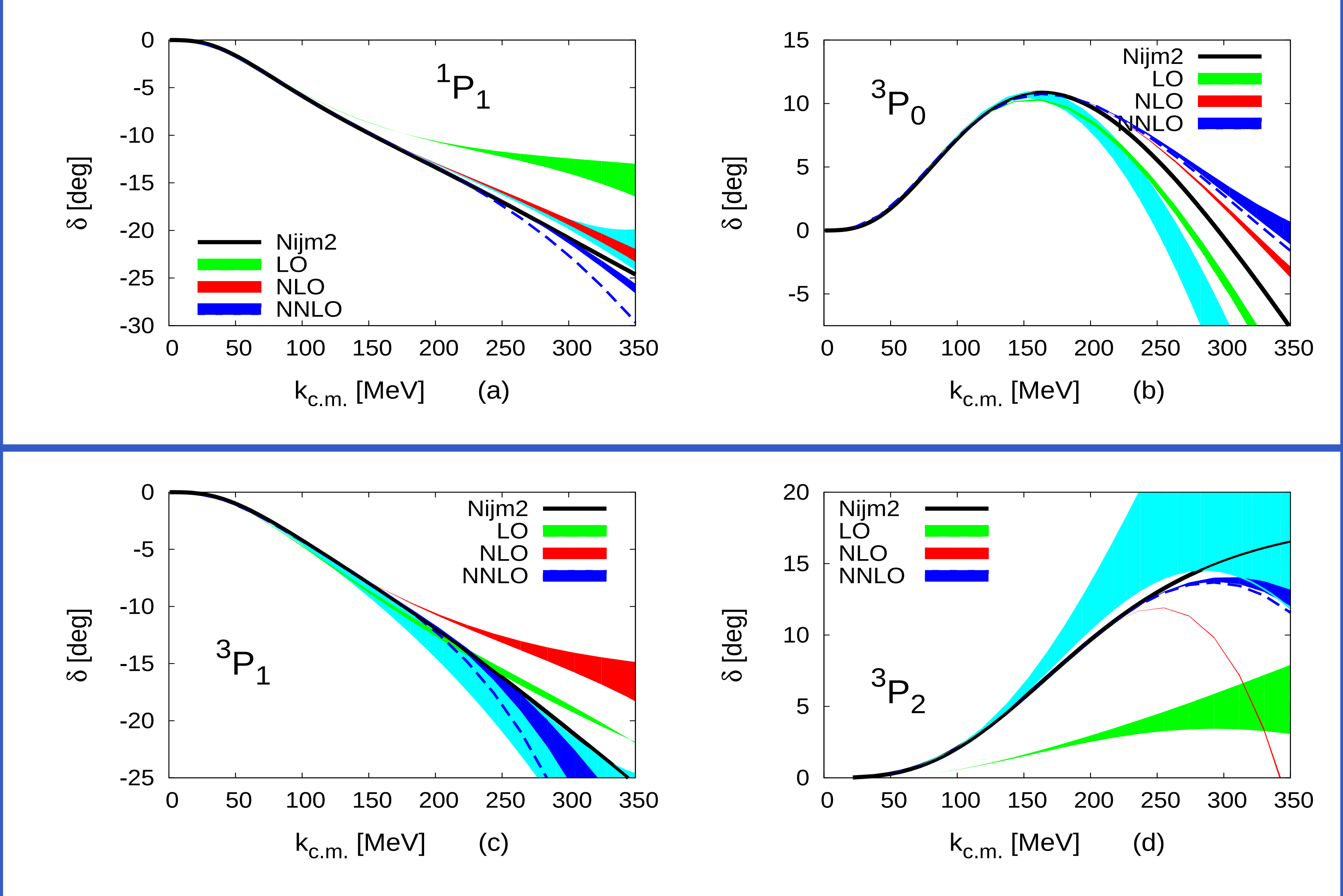
$1/M_N$ corrections included at $\mathrm{N^2LO}$

Comparison with $\mathrm{N^2LO}$ Weinberg results of Epelbaum and Meißner.

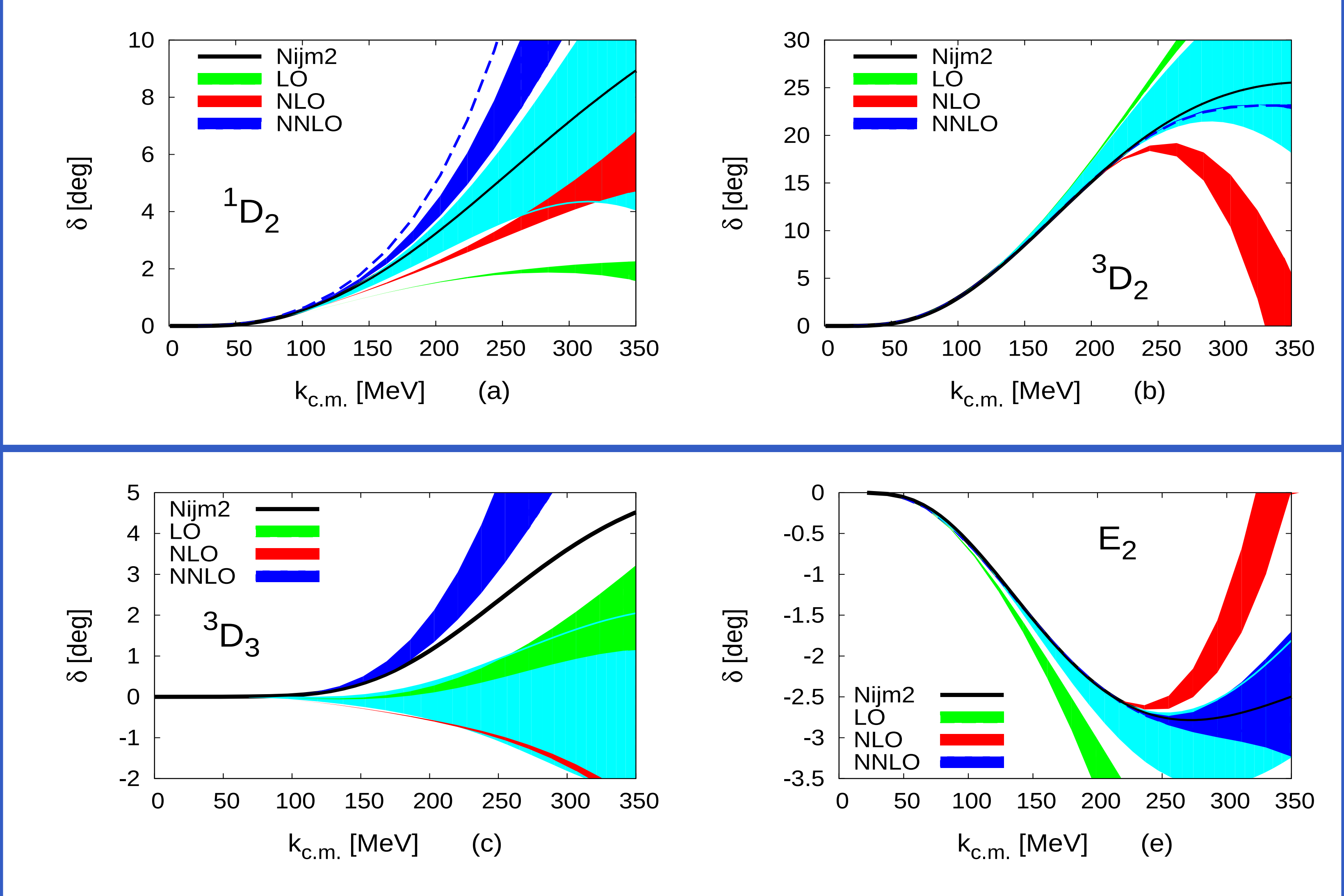
Nuclear EFT: S-Wave Phase Shifts



Nuclear EFT: P-Wave Phase Shifts



Nuclear EFT: D-Wave Phase Shifts



Nuclear EFT: Remarks

- S-waves are in general well-reproduced up to $k \sim 350 - 400\,\mathrm{MeV}$.
- P-waves tend to fail earlier (at $k \sim 300\,\mathrm{MeV}$).
  - There is a defined convergence pattern.
  - Results are very sensitive to the value of c_3 and c_4.
- Resulting power counting very similar to Birse's 06.
  (but a bit different from Long and Chen 11)
- However there are consistency reasons to prefer higher cut-offs: convergence of the perturbative series may require $r_c > 0.7\,\mathrm{fm}$.
  - Phenomenologically higher cut-offs are also preferred: the $r_c = 0.9 - 1.2\,\mathrm{fm}$ results are very similar to, and sometimes better than, the $r_c = 0.6 - 0.9\,\mathrm{fm}$ ones.

Formal Developments

What is the value of Λ_0 in nuclear EFT?

This interesting question is linked with the following observations:

- The cut-off is a separation scale: $Q \ll \Lambda \ll \Lambda_0$
- If the cut-off $\Lambda \geq \Lambda_0$ inconsistencies may happen.

(Well, this is actually a gross oversimplification. The real derivation is way too long.)

So we are going to look for a serious inconsistency that happens for a hard value of the cut-off.

Which one? A failure in the perturbative expansion!

Which is the Hardest Possible Cut-off?

If power counting is on a firm basis perturbation theory must converge and this condition imposes specific cut-off restrictions.

This condition holds for non-observables: if their perturbative expansion is not converging we are not using the right counting.

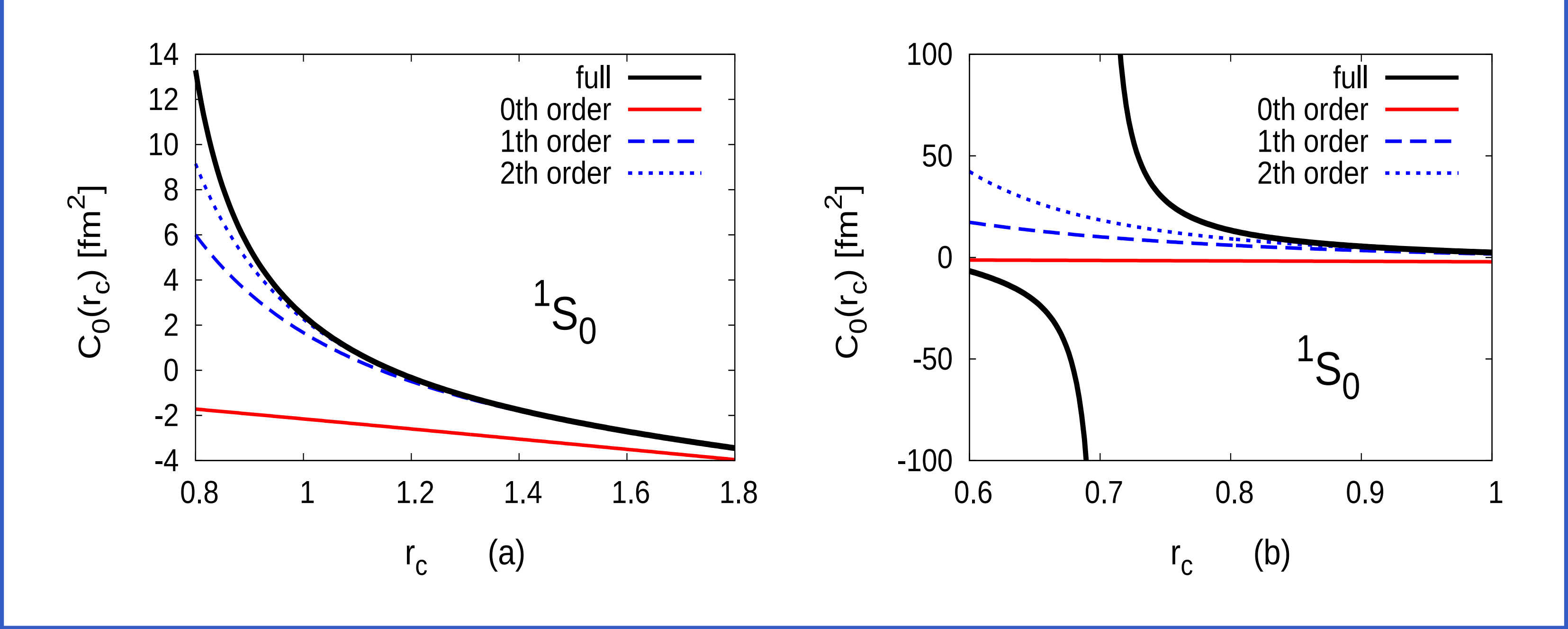
Example: the running of $C_0(r_c)$ at $\mathrm{N^2LO}$ in two schemes:

- Non-perturbatively, solving $C_0(r_c)$ for the full $\mathrm{N^2LO}$ potential.
- With TPE potential as a perturbation :
  - The 0th order is $C_0(r_c)$ plus non-perturbative OPE
  - The 1st order is $C_0(r_c)$ plus first order perturbative TPE
  - The 2nd order is $C_0(r_c)$ plus second order perturbative TPE

Then we compare perturbative versus the non-perturbative.

Which is the Hardest Possible Cut-off?

If power counting is on a firm basis perturbation theory must converge and this condition imposes specific cut-off restrictions.



At $r_c \simeq 0.7\,\mathrm{fm}$, C_0 changes sign $\Rightarrow$ first deeply bound state.
(Cannot be reproduced in perturbation theory)

The Breakdown Scale

- For transforming the $R_{\rm db}$ radius into a momentum scale we use

$$\Lambda_0 R_{\rm db} = \frac{\pi}{2} \, ,$$

(Entem, Arriola, Machleidt, Valderrama 07) yielding $\Lambda_0 \simeq 400 - 500\,{\rm MeV}$.

- The expected expansion parameter is:

$$\frac{Q}{\Lambda_0} \simeq \frac{1}{3} - \frac{1}{2}$$

for the more conservative estimation $\Lambda_0 = 300 - 400\,{\rm MeV}$.

- The breakdown scale could have been anticipated on sigma and rho exchange, yielding $\Lambda_{0,s} = m_\sigma / 2$ and $\Lambda_{0,t} = m_\rho / 2$.
- Not completely new: the KSW expansion parameter (NTvK is equivalent to KSW in the singlet), Birse's remarks from deconstruction, pole in the chiral potential by Baru et al. (12).

The Cut-off Window

The softest value of the cut-off is related to the maximum external momentum that we expect to describe within EFT ($k_{\max} \propto \Lambda$).

In r-space, the ideal cut-off window is given by:

$$0.7\,\mathrm{fm} \sim \frac{\pi}{2\,\Lambda_0} \leq r_c \leq \frac{\pi}{k_{\max}} \sim 1.4\,\mathrm{fm}$$

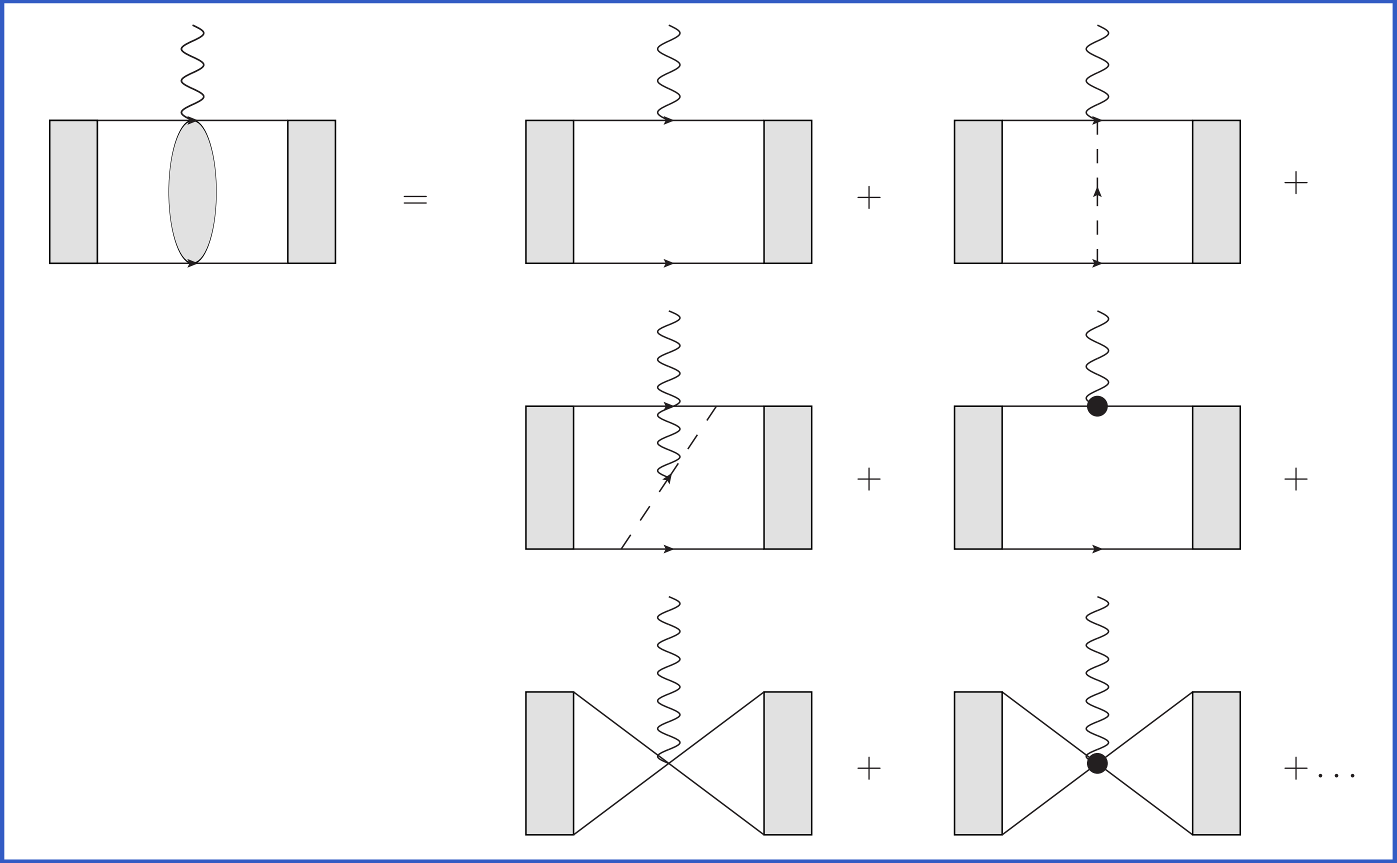
- The phase shifts can be described up to $k_{\max}$.
  - If we want to get the most from nuclear EFT, we set $k_{\max} = \Lambda_0$.
  - A softer cut-off will simply reduce $k_{\max}$.
- In momentum space, the conditions are more stringent:

$$k_{\max} \leq \Lambda \leq \Lambda_0$$

explaining the narrowness of usual cut-off windows.

External Probes and Power Counting

The previous ideas can be directly extended to deuteron reactions, in which case renormalizability controls the counting of counterterms:



Conclusions

- Nuclear EFT
  - There exist a well-defined power counting for two-body processes, and we know how to build it.

    Minor issues: How many counterterms? RGA of repulsive interactions.
  - Scattering Observables well-reproduced up to $k_{\mathrm{cm}} \simeq 300 - 400\,\mathrm{MeV}$.
  - Contact interactions are enhanced with respect to Weinberg.
  - As good as Weinberg, but without the consistency problems.
- Formal developments:
  - Determination of the expansion parameter
  - Extension to reactions on the deuteron
  - Other things underway: chiral extrapolations, three body systems, etc.