Chiral effective field theory for electromagnetic currents and form factors

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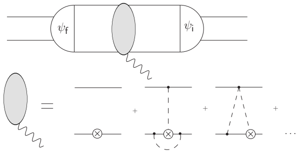
Motivation/Framework

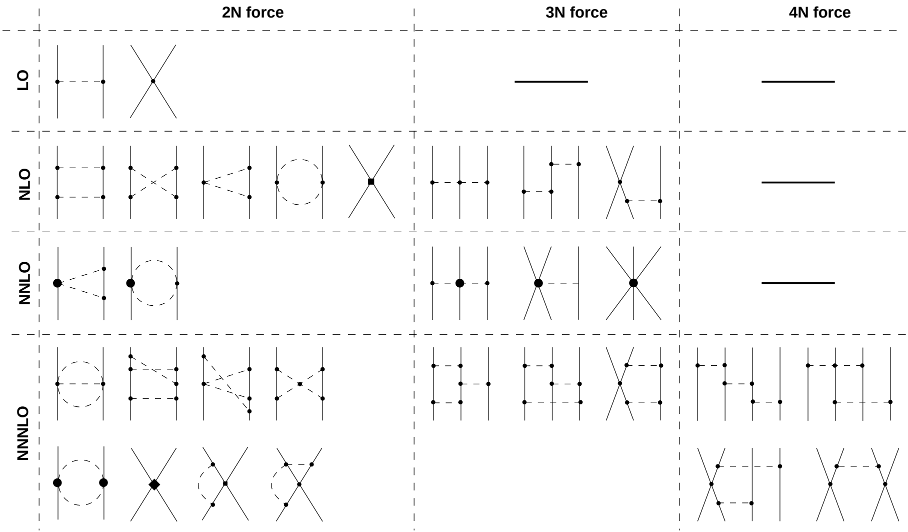
- Application of chiral EFT to the few-nucleon sector → success!
- Extend formalism to cover electromagnetic observables.
- In this talk, look at magnetic moment of the deuteron and the charge ff of ^{3}He.



- Treat em-interaction as perturbation
- Convolute between wave-functions, obtained from chiral potentials.

- Define effective potential with V_{eff} unitary transformation *Okubo'57*
- Calculate V_{eff} by a chiral expansion order by order.
- Solve Lippmann-Schwinger equation for V_{eff} with different cutoffs $\Lambda \sim 400 - 700$ MeV.
- Unitary transformation ensures that only the irreducible part of a diagram is iterated.
- Define effective current $\eta J_{\text{eff}} \eta = \eta U^\dagger J U \eta$, →CE: $\vec{\nabla} \cdot \vec{J}_{\text{eff}} = -i[V_{\text{eff}}, \rho]$.

Chiral potential schematically



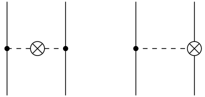
Current operators

- Expansion of the current operator starts with single nucleon current.

$$\vec{J}_{\mathrm{SN}} = F_1(q^2) \times (\vec{p} + \vec{q}/2)/m_N + \frac{i}{2m_N} G_M(q^2)\,[\vec{\sigma} \times \vec{q}]$$

- In principle, the single nucleon form factors F and G can be calculated in chiral EFT.
- Limits our description of few-nucleon observables.
- We use the parameterization of Belushkin, Hammer, Meißner instead.
- Leading-order two-nucleon currents:

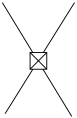
LO contribution:



$$\vec{J}^{(1)}_{1\pi} = e \frac{g_A^2\, i}{4F_\pi^2} [\vec{\tau}_1 \times \vec{\tau}_2]^3 \left[\vec{q}_1 \frac{\vec{q} \cdot \vec{\sigma}_1}{q_1^2 + M_\pi^2} \frac{\vec{q}_2 \cdot \vec{\sigma}_2}{q_2^2 + M_\pi^2} + \vec{\sigma}_2 \frac{(\vec{q}_1 \cdot \vec{\sigma}_1)}{q_1^2 + M_\pi^2} \right] + (1 \leftrightarrow 2)$$

Contact currents

- NLO contributions to the two-nucleon current:

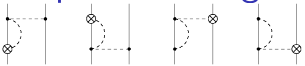


$$\vec{J}_{\mathrm{C}}^{(3)} = -ieL_1 \left(\tau_1^3 - \tau_2^3\right) \left[\vec{k} \times (\vec{\sigma}_1 - \vec{\sigma}_2)\right] + ieL_2 \left[(\vec{q}_1 - \vec{q}_2) \times (\vec{\sigma}_1 - \vec{\sigma}_2)\right],$$

$$\vec{J}_{NN}^{(3)} = i[\vec{\tau}_1 \times \vec{\tau}_2]^3 \Big\{ (C_2 + 3C_4 + C_7) \frac{e}{16} (\vec{q}_1 - \vec{q}_2) - (-C_2 + C_4 + C_7) \frac{e}{16} (\vec{\sigma}_1 \cdot \vec{\sigma}_2) (\vec{q}_1 - \vec{q}_2)$$
$$+ C_7 \frac{e}{16} \left(\vec{\sigma}_1 \, (\vec{q}_1 - \vec{q}_2) \cdot \vec{\sigma}_2 + \vec{\sigma}_2 \, (\vec{q}_1 - \vec{q}_2) \cdot \vec{\sigma}_1\right) \Big\} + C_5 \, i \, \frac{e}{16} \left(\tau_1^3 - \tau_2^3\right) \left[(\vec{q}_1 - \vec{q}_2) \times (\vec{\sigma}_1 + \vec{\sigma}_2)\right].$$

NLO One-pion exchange

Class 1:



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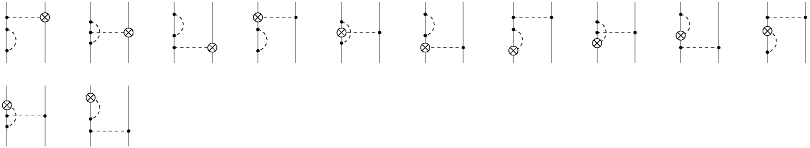

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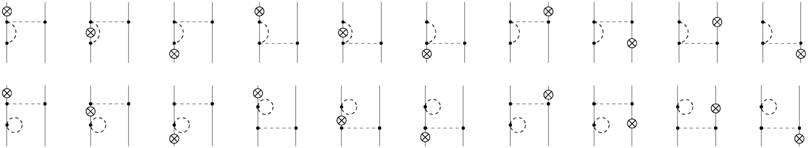
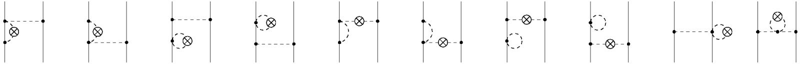
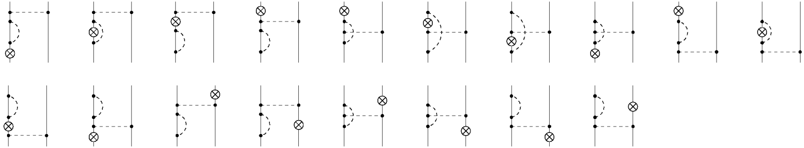


Class 5:

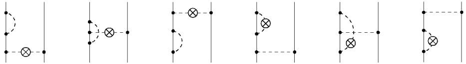


NLO One-pion exchange Ctd.

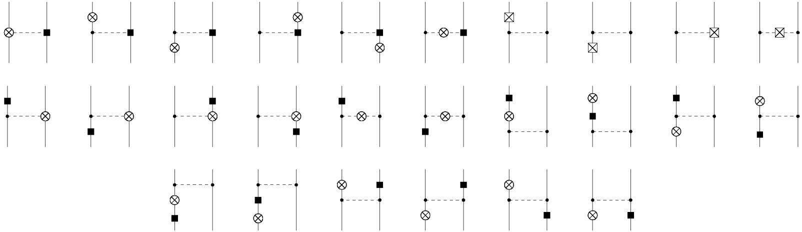
Class 6:



Class 7:



Counterterms:



Results NLO One-pion exchange

$$\vec{J}^{(3)}_{\rm LEC} = -2e\frac{g_A\, i}{F_\pi^2}\left(d_8\tau_2^3 + d_9(\vec{\tau}_1\cdot\vec{\tau}_2)\right)\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}[\vec{q}_2\times\vec{q}] + e\frac{g_A\, i}{4F_\pi^2}[\vec{\tau}_1\times\vec{\tau}_2]^3\Big\{(2d_{21}-d_{22})\frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}[\vec{q}\times[\vec{q}_1\times\vec{\sigma}_2]]$$
$$+ d_{22}\frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}\left(\vec{k}\,\vec{\sigma}_2\cdot\vec{k} - \vec{\sigma}_2\, k^2\right) + \vec{\sigma}_2\frac{(\vec{q}_1\cdot\vec{\sigma}_1)}{q_1^2+M_\pi^2}\left[-4M_\pi^2\, d_{18}\right]$$
$$+ \vec{q}_1\frac{\vec{q}\cdot\vec{\sigma}_1}{q_1^2+M_\pi^2}\frac{\vec{q}_2\cdot\vec{\sigma}_2}{q_2^2+M_\pi^2}\left[-4M_\pi^2\, d_{18} + g_A\, k^2 l_6 - g_A\, l_6\left(q_1^2-q_2^2\right)\right]\Big\} + (1\leftrightarrow 2)\,.$$

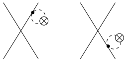
$$\vec{J}^{(3)}_{1\pi,\rm Loop} = -e\frac{i\, g_A^4}{64F_\pi^4\pi^2}[\vec{\tau}_1\times\vec{\tau}_2]^3\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}[\vec{q}\times[\vec{q}_2\times\vec{\sigma}_1]](2L(q)-1)$$
$$+ e\frac{i\, g_A^4}{1536F_\pi^4\pi^2}[\vec{\tau}_1\times\vec{\tau}_2]^3\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}(\vec{q}_1-\vec{q}_2)\left[(4M_\pi^2+q^2)L(q) - \frac{1}{6}(5q^2+24M_\pi^2)\right]$$
$$+ e\frac{i\, g_A^4}{768F_\pi^4\pi^2 q^2}[\vec{\tau}_1\times\vec{\tau}_2]^3\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_2\cdot\vec{q}_2\,(\vec{q}_1+\vec{q}_2)\frac{1}{q_2^2+M_\pi^2}\left((4M_\pi^2+q^2)L(q) - \frac{1}{6}(5k^2+24M_\pi^2)\right)$$
$$+ (1\leftrightarrow 2)\,,$$

with $$L(q) = \frac{1}{2}\frac{\sqrt{4M_\pi^2+q^2}}{q}\log\left(\frac{\sqrt{4M_\pi^2+q^2}+q}{\sqrt{4M_\pi^2+q^2}-q}\right)\,.$$

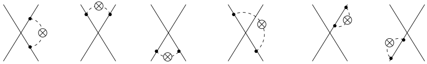
- Only the LECs d_9 and L_1 contribute to D the magnetic moment!

NLO One-pion exchange with contact interactions

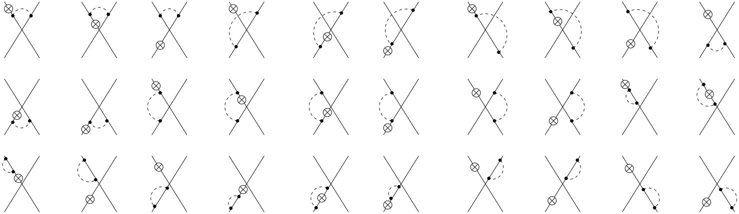
Class 10:



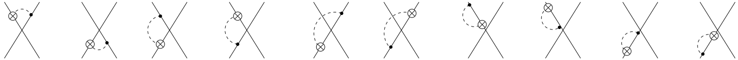
Class 11:



Class 12:

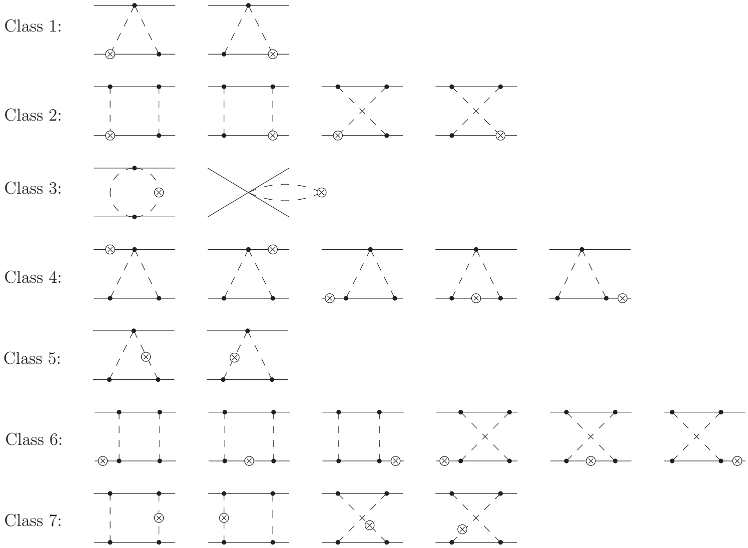


Class 13:



- There is no contribution to the current.
- However, there is a contribution to the charge density.

Two-Pion exchange currents



Two-Pion exchange currents in configuration-space

$$\vec{J}_{c1}(\vec{r}_{10},\vec{r}_{20}) = e\,\frac{g_A^2 M_\pi^7}{128\pi^3 F_\pi^4}\left[\vec{\nabla}_{10}[\vec{\tau}_1\times\vec{\tau}_2]^3 + 2\left[\vec{\nabla}_{10}\times\vec{\sigma}_2\right]\tau_1^3\right]\delta(\vec{x}_{20})\,\frac{K_1(2x_{10})}{x_{10}^2} + (1\leftrightarrow 2)\,,$$

$$\vec{J}_{c2}(\vec{r}_{10},\vec{r}_{20}) = -\,e\,\frac{g_A^4 M_\pi^7}{256\pi^3 F_\pi^4}\left(3\nabla_{10}^2 - 8\right)\left[\vec{\nabla}_{10}[\vec{\tau}_1\times\vec{\tau}_2]^3 + 2\left[\vec{\nabla}_{10}\times\vec{\sigma}_2\right]\tau_1^3\right]\delta(\vec{x}_{20})\,\frac{K_0(2x_{10})}{x_{10}}$$
$$+\,e\,\frac{g_A^4 M_\pi^7}{32\pi^3 F_\pi^4}\left[\vec{\nabla}_{10}\times\vec{\sigma}_1\right]\tau_2^3\,\delta(\vec{x}_{20})\,\frac{K_1(2x_{10})}{x_{10}^2} + (1\leftrightarrow 2)\,,$$

$$\vec{J}_{c3}(\vec{r}_{10},\vec{r}_{20}) = -\,e\,\frac{M_\pi^7}{512\pi^4 F_\pi^4}[\vec{\tau}_1\times\vec{\tau}_2]^3(\vec{\nabla}_{10}-\vec{\nabla}_{20})\,\frac{K_2(x_{10}+x_{20}+x_{12})}{(x_{10}\,x_{20}\,x_{12})(x_{10}+x_{20}+x_{12})} + (1\leftrightarrow 2)\,,$$

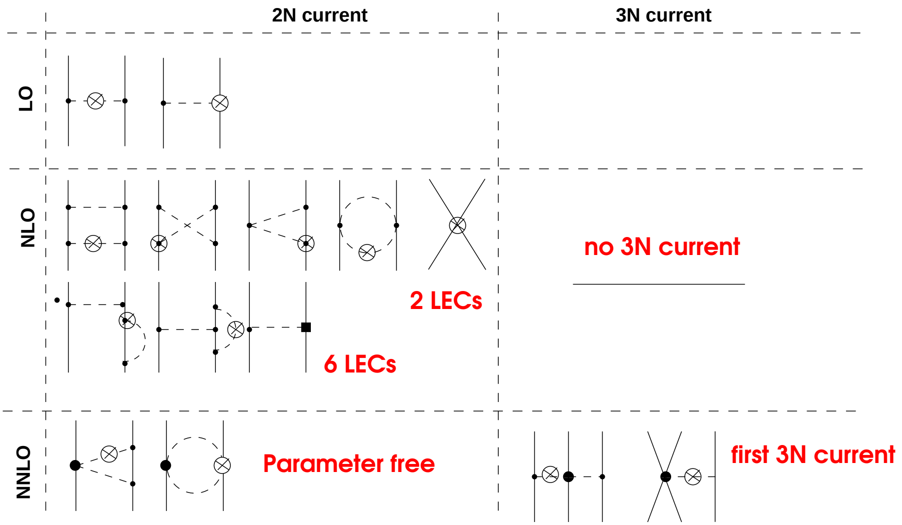
$$\vec{J}_{c5}(\vec{r}_{10},\vec{r}_{20}) = -\,e\,\frac{g_A^2 M_\pi^7}{256\pi^4 F_\pi^4}\left(\vec{\nabla}_{10}-\vec{\nabla}_{20}\right)\left[[\vec{\tau}_1\times\vec{\tau}_2]^3\vec{\nabla}_{12}\cdot\vec{\nabla}_{20} - 2\tau_1^3\vec{\sigma}_2\cdot\left[\vec{\nabla}_{12}\times\vec{\nabla}_{20}\right]\right]$$
$$\times\,\frac{K_1(x_{10}+x_{20}+x_{12})}{(x_{10}\,x_{20}\,x_{12})} + (1\leftrightarrow 2)\,,$$

$$\vec{J}_{c7}(\vec{r}_{10},\vec{r}_{20}) = e\,\frac{g_A^4 M_\pi^7}{512\pi^4 F_\pi^4}\left(\vec{\nabla}_{10}-\vec{\nabla}_{20}\right)\Big[[\vec{\tau}_1\times\vec{\tau}_2]^3\vec{\nabla}_{12}\cdot\vec{\nabla}_{10}\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{20} + 4\tau_2^3\vec{\sigma}_1\cdot\left[\vec{\nabla}_{12}\times\vec{\nabla}_{10}\right]$$
$$\times\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{20}\Big]\,\frac{x_{10}+x_{20}+x_{12}}{x_{10}\,x_{20}\,x_{12}}K_0(x_{10}+x_{20}+x_{12}) + (1\leftrightarrow 2)$$

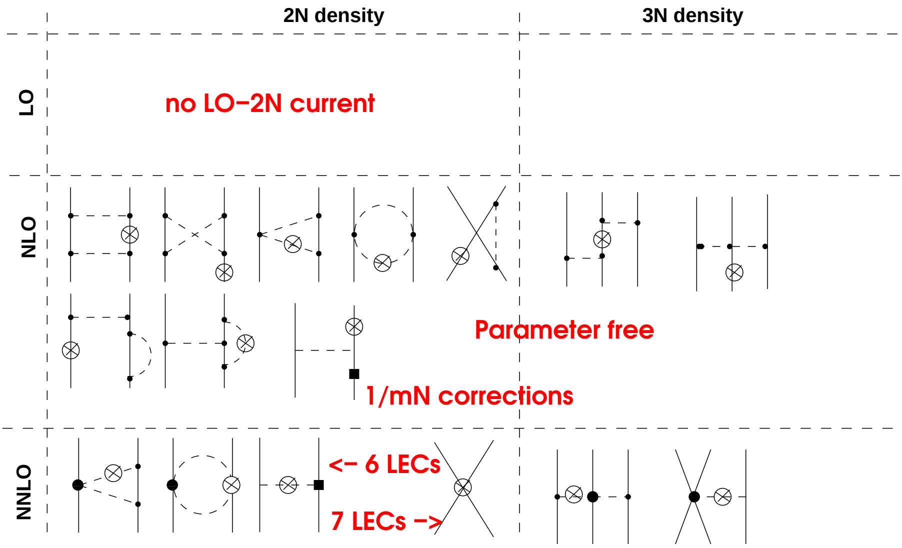
$$\vec{J}_{c4}(\vec{r}_{10},\vec{r}_{20}) = \vec{J}_{c6}(\vec{r}_{10},\vec{r}_{20}) = 0\,,$$

with $\vec{r}_{1/2/0}$ the positions of the first/second nucleon/the photon, and $\vec{x}_{10} = M_\pi\,(\vec{r}_1 - \vec{r}_0)$, $\vec{x}_{20} = M_\pi\,(\vec{r}_2 - \vec{r}_0)$, $\vec{x}_{12} = M_\pi\,(\vec{r}_1 - \vec{r}_2)$ and $\vec{\nabla}_{ij} \equiv \partial/\partial x_{ij}$ and $x_{ij} \equiv |\vec{x}_{ij}|$.
All derivatives have to be evaluated as if the variables were independent.

Currents schematically



Charge density schematically



Elastic Electron deuteron scattering experiment

Spin-1 nucleus $\rightarrow$ three structures

$$\langle \vec{P}\,', i|\rho|\vec{P}, j\rangle = |e|\left[G_C\left(q^2\right)\delta_{ij} + \frac{1}{2}G_Q\left(q^2\right)\left(\vec{q}_i\vec{q}_j - \frac{1}{3}q^2\delta_{ij}\right)\right]$$

$$\langle \vec{P}\,', i|J_k|\vec{P}, j\rangle = \frac{|e|}{2M_d}\left[G_C\left(q^2\right)\delta_{ij}\left(\vec{P}+\vec{P}\,'\right)_k + G_M\left(q^2\right)\left(\delta_{jk}\vec{q}_i - \delta_{ik}\vec{q}_j\right) + \frac{1}{2}G_Q\left(q^2\right)\left(\vec{q}_i\vec{q}_j - \frac{1}{3}q^2\delta_{ij}\right)\left(\vec{P}+\vec{P}\,'\right)_k\right]$$

$$G_C(0) = 1\,,\qquad \frac{m_N}{M_d}G_M(0) = \mu_d = 0.857\mu_N\,,\qquad G_Q(0) = Q_d = 0.286\ \mathrm{fm}^2\,,$$

Experiment:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}}\left[A(q^2) + B(q^2)\tan^2(\vartheta_e/2)\right]\,, \qquad T_{20}\left(q^2, \vartheta_e\right)$$

$$A = G_C^2 + \frac{2}{3}\eta G_M^2 + \frac{8}{9}\eta^2 M_d^2 G_Q^2\,, \qquad B = \frac{4}{3}\eta(1+\eta)G_M^2\,,$$

$$T_{20} = -\frac{1}{\sqrt{2}}\frac{1}{A + B\tan(\vartheta_e/2)}\left[\frac{8}{3}\eta G_C\,G_Q + \frac{8}{9}\eta^2 G_Q^2 + \frac{1}{3}\eta\left(1 + 2(1+\eta)\tan^2(\vartheta_e/2)\right)G_M^2\right],$$

Elastic Electron deuteron scattering theory

Compute form factors by (in the Breit frame):

$$G_C = \frac{1}{3|e|}\left(\langle 1|\rho|1\rangle + \langle 0|\rho|0\rangle + \langle -1|\rho|-1\rangle\right),$$

$$G_Q = \frac{1}{|e|\,q^2}\left(\langle 0|\rho|0\rangle - \langle 1|\rho|1\rangle\right), \qquad G_M = -\frac{1}{\sqrt{2\eta}|e|}\langle 1|J^+|0\rangle, \qquad \eta = \frac{q^2}{4M_d^2},$$

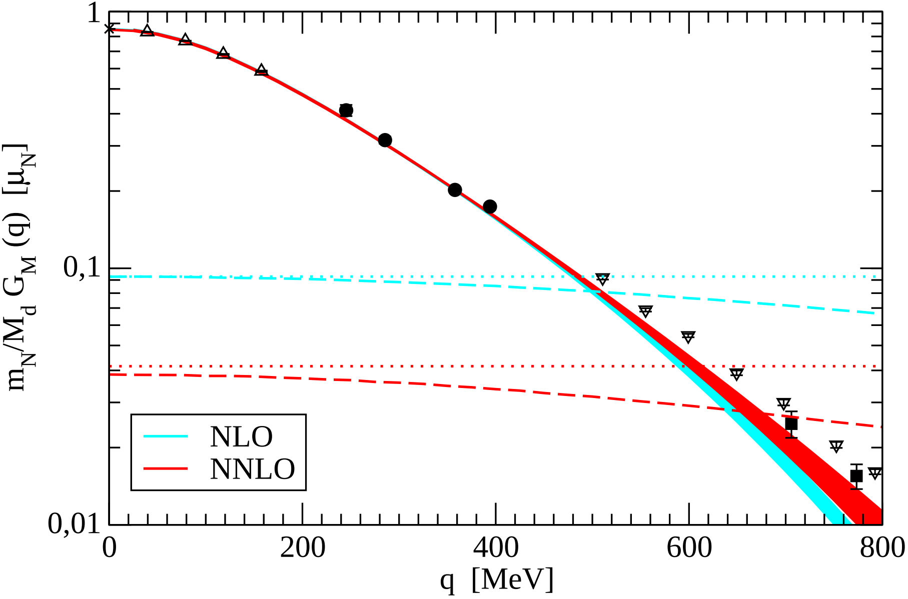
$$J^+ = J^1 + iJ^2 .$$

- in the following, we will concentrate on $G_M \Rightarrow \vec{J}$.
- IA-contribution given by:

$$\Rightarrow G_M \propto |e| \sum_{m_1,m_2,m_1',m_2'} \int d^3p\, \psi^*_{m_1',m_2'}(\vec{p}+\vec{q}/2)\; \psi_{m_1,m_2}(\vec{p})\; \vec{J}^+_{m_1',m_2',m_1,m_2,\mathrm{SN.}}$$

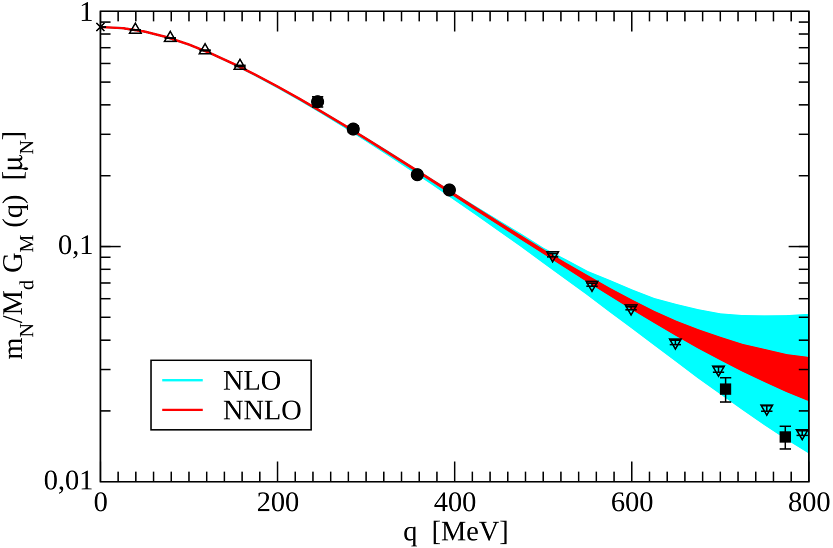
- with single nucleon form factors F and G.

G_M in Impulse Approximation



dotted: L_1 contribution, dashed: d_9 contribution Data: Sick, Simon et al., Auffret et al., Cramer et. al.

G_M with adjusted LECs



Fit data up to 400 MeV.
Results are compatible with $d_9 = 0 \pm .02\ \text{GeV}^{-2}$. Consistent with Pastore et al.

dotted: L_1 contribution, dashed: d_9 contribution, Data: Sick, Simon et al., Auffret et al., Cramer et. al.

Elastic Electron ^{3}He scattering

Spin-1/2 nucleus $\rightarrow$ two structures

$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{Z^2}{1+(k_0/m_N)(1-\cos(\vartheta_e))} \\
&\quad \times \left[F_C^2 + \frac{q^2}{4m_N^2} F_M^2 (1+\kappa) \left(1 + 2\left(1 + \frac{q^2}{4m_N^2}\tan^2(\vartheta_e/2)\right)\right)\right] \frac{1}{1+q^2/4m_N^2}\,, \\
F_C(0) &= 1 \qquad F_M(0) = 1\,,
\end{aligned}$$

with (in the Breit frame)

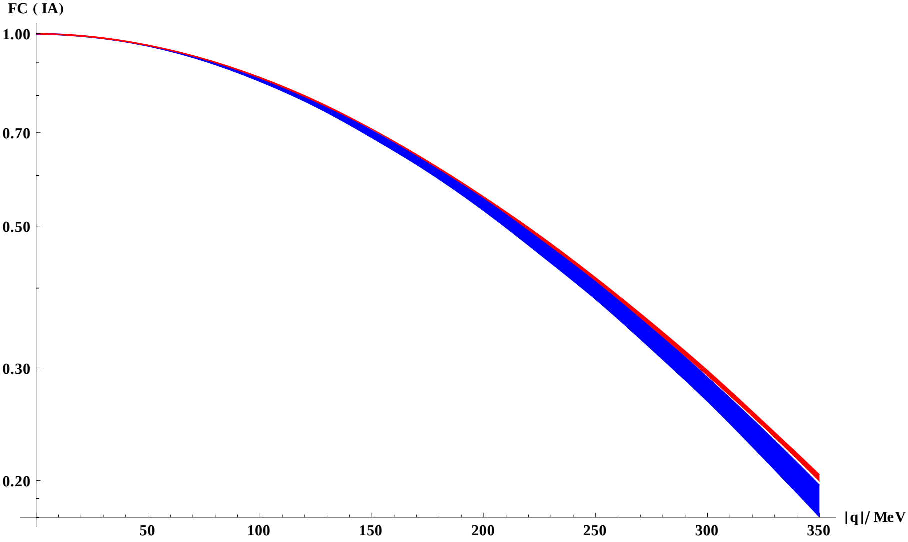
$$F_C\left(q^2\right) = \frac{1}{|e|}\left\langle \frac{1}{2}\middle|\rho\middle|\frac{1}{2}\right\rangle, \qquad F_M\left(q^2\right) = \frac{m_N}{|e|q}\left\langle \frac{1}{2}\middle|J^+\middle|-\frac{1}{2}\right\rangle.$$

F_C at NLO is free of LECs!

$$\begin{aligned}
F_C^{\text{IA}} &= \sum_{\alpha_t,\alpha_s} \int d^3p_{12}\, d^3p_3\, \psi^*\left(\vec{p}_{12}, \vec{p}_3 + 2\vec{q}/3, \alpha_t, \alpha_s\right)\left[G_E^N(q^2) + \tau_3^3 G_E^V(q^2)\right] \\
&\quad \times \psi\left(\vec{p}_{12}, \vec{p}_3, \alpha_t, \alpha_s\right)
\end{aligned}$$

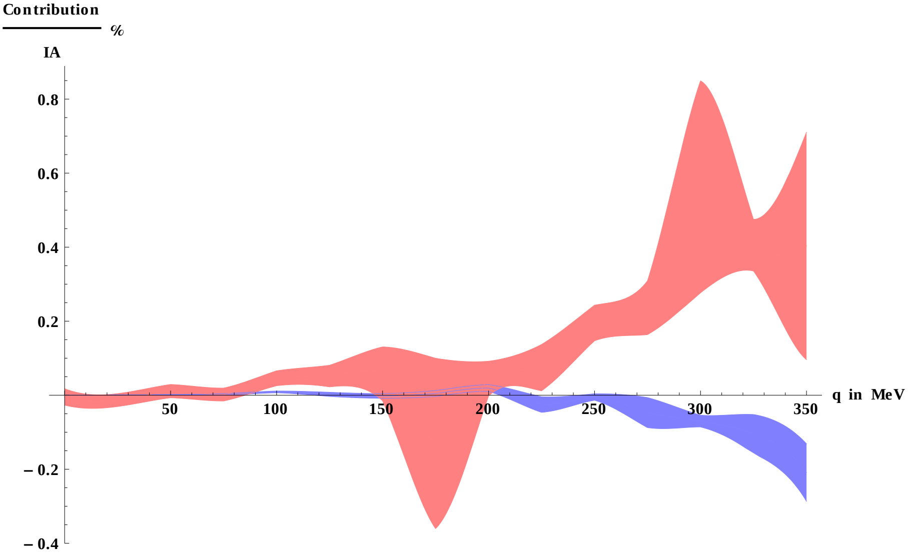
- With NLO two-nucleon currents $\rightarrow$ 9 dimensional integral.
- With NLO three-nucleon currents $\rightarrow$ 12 dimensional integral.
- Monte Carlo integration!

IA for F_C for ^{3}He



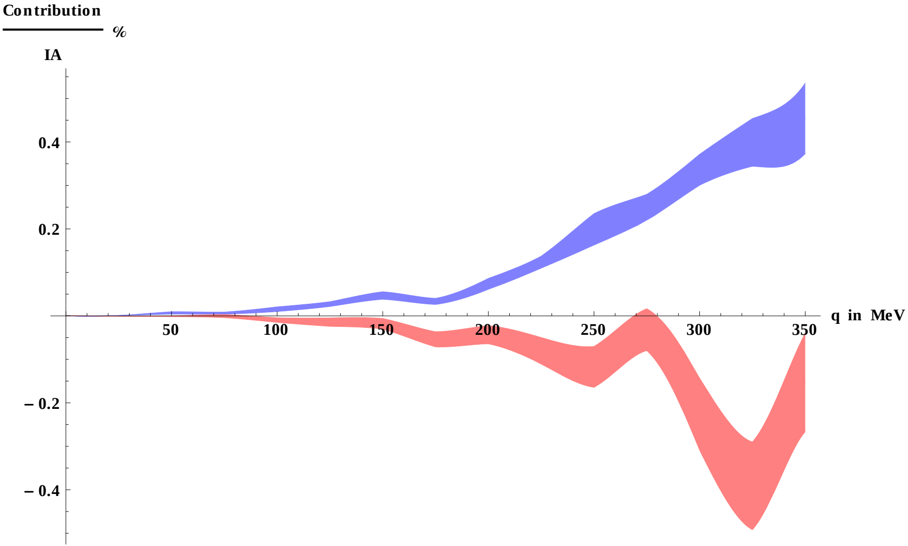
NLO: Blue, NNLO:Red

Relative Contributions OPE vs. 3N current



Blue: OPE, Red: 3N current for one NLO-WF, error are statistical errors from MC-integration

Relative Contributions TPE vs. Contact current



Blue: TPE, Red: Contact current for one NLO-WF, error are statistical errors from MC-integration

Conclusion and outlook

Summary

- Presented the whole two-nucleon current at NLO $\vec{J}$ and sketched ρ
- Only LECs L_2 and d_9 contribute to deuteron G_M.
- Presented calculations of deuteron form-factor, and started ^{3}He charge form factor.
- $\rightarrow$ Resulting $d_9 \approx -0.01 \ - \ 0.02$ GeV^{-2}. In good agreement with Pastore et. al '09.
- The charge form factor of ^{3}He is dominated by the IA. Higher order contributions are small and cancel each other.

Outlook

- Complete calculation of form factors & other electromagnetic reactions of other few-nucleon systems.
- Determination of LECs by fitting to pion electro-production data off a single nucleon.
- Inclusion of Δ-degrees of freedom
- Construction of a consistent axial current to describe weak interactions.
- Investigation of two-photon effects on the deuteron form factor.

Thank you!