Radiative corrections to anti-neutrino proton scattering at low energies

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MOTIVATION
Neutrino oscillations:

- Atmospheric neutrinos —— high energies
- Solar neutrinos —— low energies ($E_\nu^{max} < 20$ MeV)
- Reactor neutrinos —— very low energies: KAMLAND, Chooz, Daya Bay, RENO

Anti-neutrinos from nuclear reactors are ideal in order to determine the neutrino mixing parameter $\sin^2(2\Theta_{13})$.

The mixing angle Θ_{13} is the last one to be determined.
It is non-zero.
Experimental values published in Phys. Rev. Letters **108**, 2012.

The reactor anti-neutrinos hit a H_2 target.
Measure the positron yield Y_{e^+} for a given reactor $\bar{\nu}_e$ flux $n_\nu(E_\nu)$:

$$Y_{e^+} = n_\nu(E_\nu)\ \sigma(\bar{\nu}_e + p \to e^+ + n)$$

NEUTRINO OSCILLATIONS

Neutrino mixing proposed by
Pontecorvo, Sov. Phys. JEPT bf 6,429 (1957) and **26**, 984 (1968)
and
Maki, Nakagawa and Sakata, Prog. Theor. Phys. **28**, 870 (1962)

Three angles and one phase determine the mixing of the three neutrino species.

The weak interactions act on the three neutrino flavors
ν_e, ν_μ and ν_τ.

When Θ_{13} is "large" one could expect to be able to measure
the leptonic CP-violating phase

$$V_{13} \propto \sin\Theta_{13}\, e^{-i\delta_{CP}}$$

An accurate evaluation of $\sigma(\bar{\nu}_e + p \to e^+ + n)$,
requires knowledge of the RADIATIVE and nucleon RECOIL corrections.

Effective Field Theory

allows a systematic evaluation of these corrections
within a unified approach to electro-weak and strong processes at low energies.
in a gauge-invariant and model-independent way.

- Long wavelength probes are insensitive to short distance details.
- Low-energy-constants (LECs) parametrize short-distance physics.
- The effective degrees of freedom are hadrons.
- The interactions among hadrons reflect the symmetries and symmetry-breakings of QCD.

We use Heavy Baryon Chiral Perturbation Theory (HBChPT)

Baryons treated non-relativistically.

Leptons treated relativistically.

The expansion parameters: $\left(\frac{Q}{\Lambda_\chi}\right) \simeq \left(\frac{\alpha}{2\pi}\right) \sim 10^{-3}$; $\Lambda_\chi \sim 1$ GeV.

“The Lowest Order (LO)” lagrangian:

$$\mathcal{L}_{QED} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}_e\,(i\gamma\cdot D)\psi_e - m_e\bar{\psi}_e\psi_e + \bar{\psi}_\nu\,(i\gamma\cdot\partial)\,\psi_\nu,$$

$$\mathcal{L}_{NN\gamma} = \bar{N}\,(iv\cdot D)\,N,$$

$$\mathcal{L}_{LO} = -\left(\frac{G_F V_{ud}}{\sqrt{2}}\right)\{\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_\nu\} \times \{\bar{N}\tau^+[v^\mu - 2\,g_A\,S^\mu]\,N\}\,.$$

The lagrangian contains several LECs assumed known from other processes, e.g. the Fermi constant G_F and axial coupling constant g_A.

The "Next to Leading Order (NLO)" interaction lagrangian includes nucleon recoil :

$$\begin{aligned}
\mathcal{L}_{NLO} \;=\; & \left(\frac{\alpha}{4\pi}e_1\right)\bar{\psi}_e\,(i\gamma\cdot\partial)\psi_e + \left(\frac{\alpha}{8\pi}e_2\right)\bar{N}(1+\tau_3)\,(iv\cdot\partial)\,N + \\
& -\frac{(G_F V_{ud})}{\sqrt{2}}\{\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_\nu\}\times\left\{\bar{N}\tau^+\left[\left(\frac{\alpha}{4\pi}e_V\right)v^\mu - 2\,g_A\left(\frac{\alpha}{4\pi}e_A\right)S^\mu\right]N+\right.\\
+\; & \left(\frac{1}{2m_N}\right)\left\{\bar{N}\tau^+\left[i(v^\mu v^\nu - g^{\mu\nu})(\overleftarrow{\partial}-\overrightarrow{\partial})_\nu - 2i\mu_V[S^\mu, S\cdot(\overleftarrow{\partial}+\overrightarrow{\partial})]\right.\right.\\
& \qquad \left.\left.\left. -2i\;g_A\;v^\mu\;S\cdot(\overleftarrow{\partial}-\overrightarrow{\partial})\right]N\right\}\right\}
\end{aligned}$$

Only one linear combination of the radiative LECs is relevant:

$$e_V^{(R)}(\mu=m_N) \;=\; e_V - \frac{1}{2}(e_1+e_2) + \frac{3}{2}\left[\frac{2}{4-d} - \gamma_E + \ln(4\pi) + 1\right]$$

The LEC μ_V is the nucleon isovector magnetic moment.

The lowest order differential cross section:

$$\frac{\mathrm{d}\sigma_{LO}}{\mathrm{d}(\cos\theta_e)} = \left(\frac{G_F V_{ud}}{\sqrt{2}}\right)^2 f_{LO}(E,\beta)\left[(1+3g_A^2)+(1-g_A^2)\beta\cos\theta_e\right]$$

Positron velocity: $\beta = \sqrt{E^2 - m_e^2}/E$

Phase-space factor: $f_{LO}(E,\beta) = \beta E^2/\pi$

NLO differential cross section:

$$\frac{\mathrm{d}\sigma_{NLO}}{\mathrm{d}(\cos\theta_e)} = \left(\frac{G_F V_{ud}}{\sqrt{2}}\right)^2 f_{NLO}(E,\beta,\theta_e)\left[(1+3g_A^2)\mathcal{G}_1(\beta)+(1-g_A^2)\beta\mathcal{G}_2(\beta)\cos\theta_e\right]$$

Phase-space factor:

$$f_{NLO}(E,\beta,\theta_e) = f_{LO}(E,\beta)\left[1-\frac{E}{m_N}\left(1-\frac{E_\nu}{\beta E}\cos\theta_e\right)\right]+\mathcal{O}(m_N^{-2})$$

Correction factors:

$$\mathcal{G}_i(\beta) = 1+\frac{\alpha}{2\pi}\mathcal{G}_i^{rad}(\beta)+\frac{1}{m_N}\mathcal{G}_i^{recoil}(\beta) \qquad i=1,2$$

The factors $\mathcal{G}_i(\beta)^{recoil}$ contain the "weak" magnetism interactions.

The pion-pole in the axial current and radiative corrections to the recoil terms are ignored

$$\left(\frac{Q}{m_\pi}\right)^2 \simeq 3\cdot 10^{-5} \quad \text{and} \quad \left(\frac{\alpha}{2\pi}\right)\mu_V\frac{Q}{2m_N} \simeq 10^{-6}$$

We use the standard definition of G_F, given by the purely leptonic muon decay

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu .$$

The elctro-weak $W^- \ Z^0$ box diagram is included in G_F,

i.e. G_F includes the factor:

$$1 + \frac{3\alpha}{4\pi} \ln\left(\frac{m_W}{m_Z}\right)$$

The LEC, e_V^R, represents the "inner" (short-distance) corrections.

A rough estimate, which is consistent with Marciano and Sirlin (1986),

$$\frac{\alpha}{2\pi} e_V^R(\mu = m_N) \simeq 2 \cdot 10^{-2}$$

The dominant part of these "inner" corrections are due to the eletro-weak loops.

In the standard theory approach

Sirlin and Marciano (2006), Fukugita and Kubota (2004)

and Vogel(1984) have derived these corrections.

The EFT approach allows a systematic improvement where, once e_V^R is determined, all low-energy corrections are uniquely determined.

We now compare neutron β-decay and $\bar{\nu}_e + p \to e^+ + n$.

Neutron β-decay recoil corrections (Ando et al. (2004):

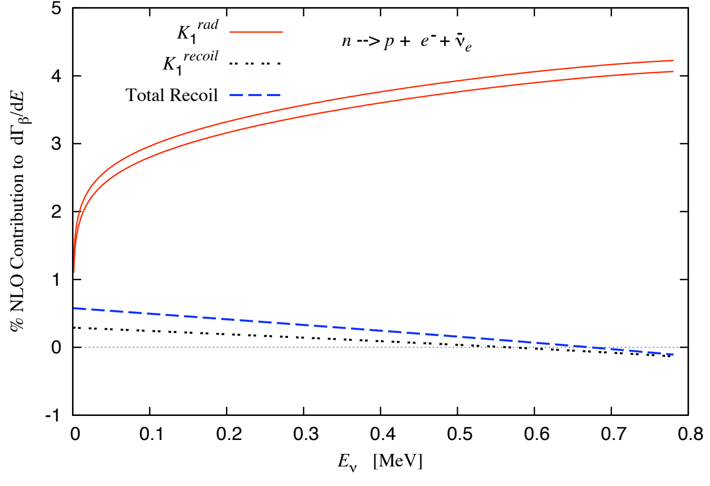
$$\mathcal{K}_1^{recoil} = \left(\frac{\beta^2 \; E}{1+3g_A^2}\right)\left[1+2g_A\mu_V+g_A^2\right] + \left(\frac{E_\nu}{1+3g_A^2}\right)\left[1-2g_A\mu_V+g_A^2\right] + \mathcal{O}(m_N^{-1})$$

where we used $E^{max} = E_\nu + E + \mathcal{O}(m_N^{-1})$.

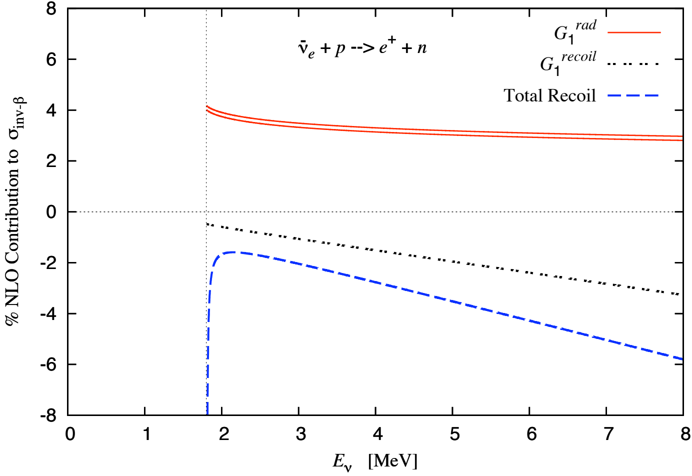
Recoil corrections for $\bar{\nu}_e + p \to e^+ + n$

$$\mathcal{G}_1^{recoil} = \left(\frac{\beta^2 \; E}{1+3g_A^2}\right)\left[1-2g_A\mu_V+g_A^2\right] - \left(\frac{E_\nu}{1+3g_A^2}\right)\left[1+2g_A\mu_V+g_A^2\right]$$

Observe the sign changes for the $2g_A\mu_V$ terms, which are the dominant ones.



K_1^{rad}
K_1^{recoil}
Total Recoil
$n \dashrightarrow p + e^- + \bar{\nu}_e$
% NLO Contribution to $d\Gamma_\beta/dE$
E_ν [MeV]
5
4
3
2
1
0
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8



$\bar{\nu}_e + p \to e^+ + n$
G_1^{rad}
G_1^{recoil}
Total Recoil
% NLO Contribution to $\sigma_{\text{inv-}\beta}$
E_ν [MeV]
8
6
4
2
0
-2
-4
-6
-8
0
1
2
3
4
5
6
7
8

The NLO total cross section

$$\sigma = (G_F V_{ud})^2 \frac{\tilde{E}^2 \tilde{\beta}}{\pi}(1+3g_A^2)\left(1+\frac{\alpha}{2\pi}\mathcal{G}_1^{rad}(\beta)\right)$$
$$\times \left\{1+\frac{1}{m_N}\left[\mathcal{G}_1^{recoil}(\beta) - \tilde{E} - \left(\frac{1+\tilde{\beta}^2}{\tilde{\beta}^2}\right)\left(E_\nu + \frac{(m_n-m_p)^2-m_e^2}{2\tilde{E}}\right) + \left(\frac{1-g_A^2}{1+3g_A^2}\right)E_\nu\right]\right\}$$

These EFT recoil corrections agree with what was derived by Vogel and Beacom (1999).

Here $\tilde{E} = E_\nu - (m_n - m_p) = E + \mathcal{O}(m_N^{-1})$

and $\tilde{\beta} = \sqrt{\tilde{E}^2 - m_e^2}/\tilde{E} = \beta + \mathcal{O}(m_N^{-1})$,

i.e. $\tilde{E}$ and $\tilde{\beta}$ ignore the neutron recoil.

As seen earlier the NLO phase-space factor $(E = E_\nu + m_p - E_n)$

$$f_{NLO}(E,\beta,\theta_e) = \frac{E^2\,\beta}{\pi}\left[1 - \frac{E}{m_N}\left(1-\frac{E_\nu}{\beta E}\cos\theta_e\right)\right] + \mathcal{O}(m_N^{-2})$$

has no singularity as the positron velocity $\beta \to 0$.

CONCLUSIONS

- The estimated value for the radiative LEC $e_V^R(m_N)$ is dominated by electro-weak physics.
- The recoil corrections (m_N^{-1}) are very small $(< 1\%)$ in neutron β-decay.
- The recoil corrections for $\bar{\nu}_e + p \to e^+ + n$ are significant (a few %) and as large as the radiative corrections.
  The energy dependences are very different for these two corrections.
- To extract an accurate value for the neutrino oscillating angle Θ_{13} from
  $$\bar{\nu}_e + p \to e^+ + n$$
  require attention to the energy dependences of the two corrections which are of opposite signs.