Theorerical studies of muon capture on light nuclei

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August 7, 2012

Marcucci *et al.*, PRC **83**, 014002 (2011)
Marcucci *et al.*, PRL **108**, 052502 (2012)
Marcucci, IJMPA **27**, 1230006 (2012)

Outline

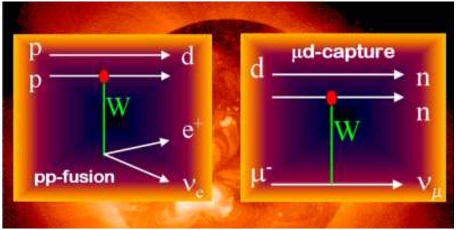
- Introduction: observables of interest and experimental situation [see P. Kammel's talk]
- Theoretical calculations since 2010
- Comparison with previous theoretical work and experiment
- Final remarks and conclusions

Reactions of interest ($A \leq 3$)

- $\mu^- + d \to n + n + \nu_\mu$
- $\mu^- + {}^3\mathrm{He} \to {}^3\mathrm{H} + \nu_\mu$ (70%)
- $\mu^- + {}^3\mathrm{He} \to n + d + \nu_\mu$ (20%)
- $\mu^- + {}^3\mathrm{He} \to n + n + p + \nu_\mu$ (10%)

Motivations

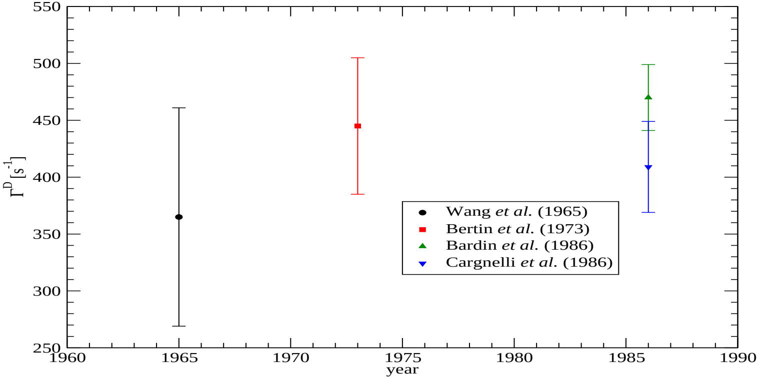
- "Indirect" access to reactions of astrophysical interst
- Extraction of $G_{PS}(q^2) \to$ validate χPT



$$j^\mu = \overline{u_p}\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N} - G_A(q^2)\gamma^\mu\gamma^5 - G_{PS}(q^2)\frac{q^\mu\gamma^5}{2M_N}\right]u_n$$

Experimental situation: $\mu^- + d \rightarrow n + n + \nu_\mu$

Two hyperfine states: 1/2 and 3/2 $\Rightarrow$ Γ^D and Γ^Q but only Γ^D



Experimental situation: $\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu$

Two hyperfine states: $P(f, f_z) = (1; \pm 1, 0)$ and $(0; 0)$

$$\cos\theta = \hat{\mathbf{z}} \cdot \hat{\mathbf{q}} \leftarrow \text{momentum transfer of the lepton pair}$$

$$\frac{d\Gamma}{d(\cos\theta)} = \frac{1}{2}\Gamma_0\left[1 + A_v P_v \cos\theta + A_t P_t\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) + A_\Delta P_\Delta\right]$$

$$P_v = P(1,1) - P(1,-1)$$

$$P_t = P(1,1) + P(1,-1) - 2P(1,0)$$

$$P_\Delta = P(1,1) + P(1,-1) + P(1,0) - 3P(0,0) \equiv 1 - 4P(0,0)$$

$\Gamma_0 =$ total capture rate

$A_{v,t,\Delta} =$ angular correlation parameters

Ackerbauer *et al.*, PLB **417**, 224 (1998): $\Gamma_0 = 1496(4)\ \mathrm{s}^{-1}$

Souder *et al.*, NIMA **402**, 311 (1998): $A_v = 0.63 \pm 0.09\ (\mathrm{stat.})^{+0.11}_{-0.14}\ (\mathrm{syst.})$

Theoretical studies: "main ingredients" (I)

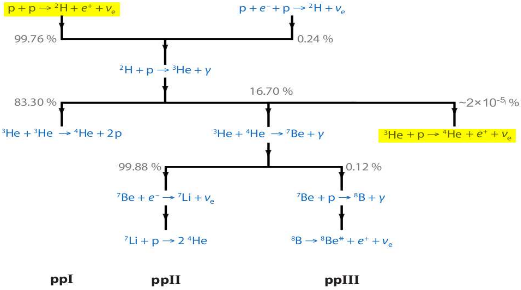
- Accurate nuclear wave functions
  - Realistic Hamiltonian: AV18(+UIX) or N3LO(+N2LO)$\leftarrow\chi$EFT
  - Accurate techniques to solve the Schrödinger equation ($A = 3 \rightarrow$ HH method)

$A = 2$	AV18	N3LO	Exp.
B_d (MeV)	2.22457	2.22456	2.224574(9)
a_{nn} (fm)	–18.487	–18.900	–18.9(4)
${}^1a_{np}$ (fm)	–23.732	–23.732	–23.740(20)
${}^3a_{np}$ (fm)	5.412	5.417	5.419(7)
$A = 3$	AV18/UIX	N3LO/N2LO	Exp.
$B_{^3\mathrm{H}}$ (MeV)	8.479	8.474	8.482
$B_{^3\mathrm{He}}$ (MeV)	7.750	7.733	7.718
${}^2a_{nd}$ (fm)	0.590	0.675	0.645(10)
${}^4a_{nd}$ (fm)	6.343	6.342	6.35(2)

Theoretical studies: "main ingredients" (II)

- Nuclear weak transition operators: $[\rho^{(A,V)}, \mathbf{j}^{(A,V)}]$
  - Standard Nuclear Physics Approach - SNPA[1]
  - Hybrid Chiral Effective Field Theory Approach - χEFT*[2]

  SNPA and χEFT* used for $p + p \to d + e^+ + \nu_e$ and $p + {}^3\text{He} \to {}^4\text{He} + e^+ + \nu_e$



  - "Less hybrid" Chiral Effective Field Theory Approach - χEFT[3]

[1] Schiavilla *et al.*, PRC **58**, 1263 (1998); Marcucci *et al.*, PRC **63**, 015801 (2000)
[2] Park *et al.*, PRC **67**, 055206 (2003)
[3] Marcucci *et al.*, PRL **108**, 052502 (2012)

Nuclear transition operators: SNPA

- One-body operators: NRR of $j_i^\mu \rightarrow O(1/m^2)$
- Two-body $\rho^{(V)}$ and $\mathbf{j}^{(V)}$: CVC $\rightarrow$ EM operators (MEC+$\underline{\mathbf{j}^{(V)}(\Delta)}$)

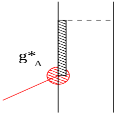
	$\mu(^3\mathrm{H})$	$\mu(^3\mathrm{He})$
1b	2.5745	-1.7634
Full	**2.9525**	**-2.1299**
Exp.	2.9790	-2.1276

AV18/UIX
$\Rightarrow$ Full=1b+2b+3b [1]

[1] Marcucci *et al.*, PRC **72**, 014001 (2005)

- Two-body $\rho^{(A)}$: PCAC + low-energy theorem $\rightarrow$ π-exchange and short-range terms
- Two-body $\mathbf{j}^{(A)}$: π- and ρ-exchange, $\pi\rho$ mechanism, and $\underline{\mathbf{j}^{(A)}(\Delta)}$

Largest contribution to $\mathbf{j}^{(A)}(\Delta)$ from



g_A^* fit to observable: $\underline{GT_{exp}\text{ of tritium }\beta\text{-decay}}$

Nuclear transition operators: χEFT at N^{3}LO

- One-body operators $\equiv$ SNPA
- Two-body $\rho^{(A)}$: soft π-exchange dominant
- Two-body $\rho^{(V)} = 0$ at N^{3}LO
- Two-body $\mathbf{j}^{(V)}$: CVC $\rightarrow$ EM current: $1\pi \leftarrow$ *pp* & *hep*
  here $1\pi + 1\pi C + 2\pi +$ CT[1] $\rightarrow$ two LECs (g_{4S} & g_{4V}) $\Rightarrow$
  from $\mu(^3\mathrm{H} - {}^3\mathrm{He})$
- Two-body $\mathbf{j}^{(A)}$: $1\pi +$ CT $\rightarrow$ one LEC (d_R) $\Rightarrow$
  from GT_{exp} of tritium β-decay

[1] Song *et al.*, PRC **79**, 064002 (2009)

χEFT* $\Rightarrow$ AV18/UIX [N3LO/N2LO] $\Rightarrow$ current and potentials "uncorrelated"

	$\Lambda = 500$ MeV	$\Lambda = 600$ MeV	$\Lambda = 800$ MeV
d_R	0.97(7)	1.75(8) [1.00(9)]	3.89(10)
g_{4S}	0.69(1)	0.55(1) [0.11(1)]	0.25(2)
g_{4V}	2.065(6)	0.793(6) [3.124(6)]	-1.07(1)

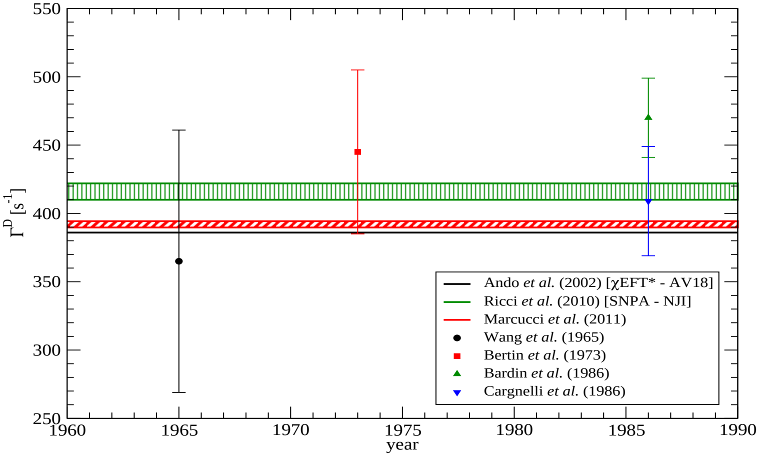
Results: $\Gamma^D(\mu^- + d)$ (SNPA and χEFT*)

SNPA(AV18)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
g_A=1.2654(42)	246.6(7)	20.1	46.7	71.6	4.5	0.9	390.4(7)
g_A=1.2695(29)	246.8(5)	20.1	46.8	71.8	4.5	0.9	390.9(7)
χEFT*(AV18)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
$\Lambda = 500$ MeV	250.0(8)	19.9	46.2	71.2	4.5	0.9	392.7(8)
$\Lambda = 600$ MeV	250.0(8)	19.8	46.3	71.1	4.5	0.9	392.6(8)
$\Lambda = 800$ MeV	249.7(7)	19.8	46.4	71.1	4.5	0.9	392.4(7)
χEFT*(N3LO)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
$\Lambda = 600$ MeV	250.5(7)	19.9	46.4	71.5	4.4	0.9	393.6(7)

$$\Rightarrow \boxed{\Gamma^D = 390 \div 394 \text{ s}^{-1}}$$

Marcucci *et al.*, PRC **83**, 014002 (2011)

Comparison with data and previous calculations



Results: $\Gamma_0(\mu^- + {}^3\mathrm{He})$ (SNPA and χEFT*)

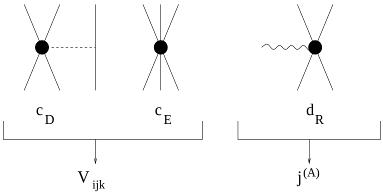
SNPA(AV18/UIX)	Γ_0
g_A=1.2654(42)	1486(8)
g_A=1.2695(29)	1486(5)
χEFT*(AV18/UIX)	Γ_0
$\Lambda = 500$ MeV	1487(8)
$\Lambda = 600$ MeV	1488(9)
$\Lambda = 800$ MeV	1488(8)
χEFT*(N3LO/N2LO; Λ=600 MeV)	1480(9)

$\Rightarrow$ $\Gamma_0 = 1484(13)\ \mathrm{s}^{-1}$

To be compared with:

- $\Gamma_0(\mathrm{exp}) = 1496(4)\ \mathrm{s}^{-1}$
- Marcucci *et al.*, PRC **66**, 054003 (2002) [SNPA–AV18/UIX] $\rightarrow$ 1484(8) s^{-1}
- Gazit, PLB **666**, 472 (2008) [χEFT*–AV18/UIX] $\rightarrow$ 1499(16) s^{-1}

"Less hybrid" χEFT calculation



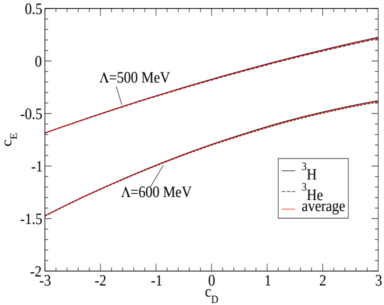
$$d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

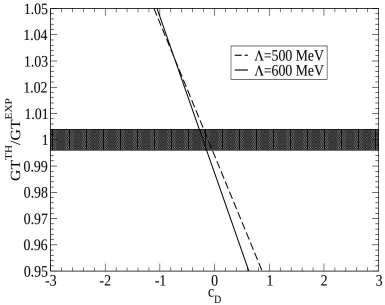
Gårdestig and Phillips, PRL **96**, 232301 (2006)

Gazit *et al.*, PRL **103**, 102502 (2009)

fit c_D and c_E to $B(A=3)$ and GT_{exp} with N3LO/N2LO

$\Rightarrow \{c_D; c_E\}_{\rm MAX}$ and $\{c_D; c_E\}_{\rm MIN}$





Remaining LEC's: g_{4S} and g_{4V} in the vector current $\Rightarrow$ fit to the $A = 3$ magnetic moments

	$\{c_D; c_E\}$	g_{4S}	g_{4V}
Λ=500 MeV	$\{-0.20; -0.208\}$	0.207 ± 0.007	0.765 ± 0.004
	$\{-0.04; -0.184\}$	0.200 ± 0.007	0.771 ± 0.004
Λ=600 MeV	$\{-0.32; -0.857\}$	0.146 ± 0.008	0.585 ± 0.004
	$\{-0.19; -0.833\}$	0.145 ± 0.008	0.590 ± 0.004

Radiative corrections[1] ARE included

[1] Czarnecki *et al.*, PRL **99**, 032003 (2007)

Results: $\Gamma^D(\mu^- + d)$ and $\Gamma_0(\mu^- + {}^3\mathrm{He}) \to \chi$EFT

	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	Γ^D	Γ_0
IA – $\Lambda = 500$ MeV	238.8	21.1	44.0	72.4	381.7	1362
IA – $\Lambda = 600$ MeV	238.7	20.9	43.8	72.0	380.8	1360
FULL – $\Lambda = 500$ MeV	254.4(9)	20.5	46.8	72.1	399.2(9)	1488(9)
FULL – $\Lambda = 600$ MeV	255(1)	20.3	46.6	71.6	399(1)	1499(9)

Γ^D =**399(3) s^{-1} & Γ_0 =1494(21) s^{-1}**

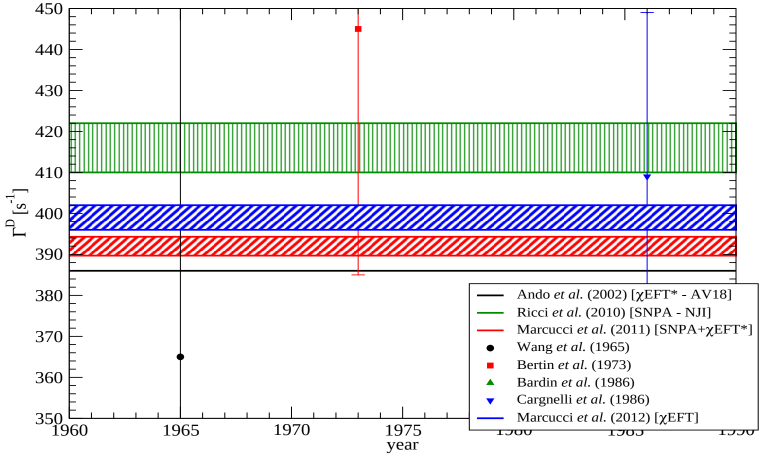
vs. Γ^D(exp) $\cdots$ & Γ_0(exp)=1496(4) s^{-1}

Comparison between Γ_0 and Γ_0(exp) $\longrightarrow$

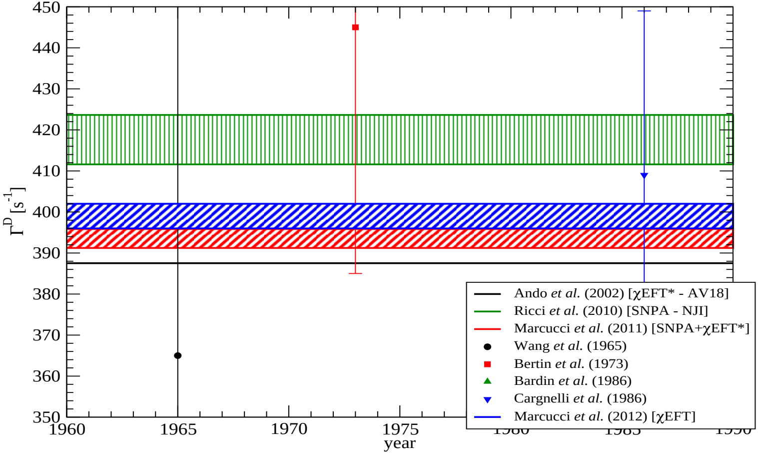
G_{PS} =8.2 $\pm$0.7 vs. $G_{PS}^{\chi\mathrm{PT}}$ =7.99$\pm$0.20

Marcucci *et al.*, PRL **108**, 052502 (2012)

Comparison with data and previous calculations

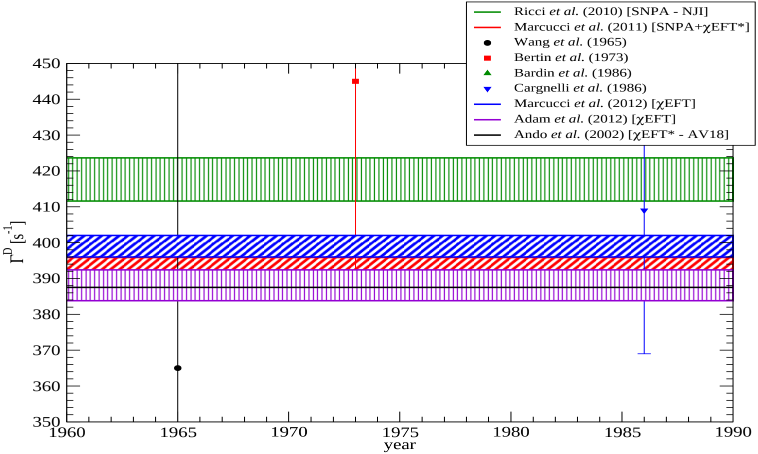


Comparison with data and previous calculations corrected including RC



Ricci *et al.* → χEFT calculation: Adam *et al.*, PLB **709**, 93 (2012)

- various χEFT potentials (EM^a, EM^b [N3LO], EGM(204) and EGM(205)) with different c_3 and c_4 values
- d_R from Gazit *et al.*, PRL **103**, 102502 (2009)
- vector current from SNPA
- Γ^D= 383.8–419.1 s^{-1} (383.8–392.4 excluding EGM(205))



Conclusions and outlook

- Extensive theoretical work on muon capture in SNPA, χEFT* and χEFT
- $\Gamma_0(\mu^- + {}^3\mathrm{He})$: nice agreement theory vs. experiment
- $\Gamma^D(\mu^- + d)$:
  - some discrepancies among different theoretical works
  - more accurate experimental results $\rightarrow$ MuSun
- In the future:
  - χEFT $\rightarrow$ $\mu^- + {}^3\mathrm{He} \rightarrow n + d + \nu_\mu$
    $\mu^- + {}^3\mathrm{He} \rightarrow n + n + p + \nu_\mu$
  - χEFT $\rightarrow$ reactions of astrophysical interest
    $p + p \rightarrow d + e^+ + \nu_e$
    $p + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + e^+ + \nu_e$
    $p + d \rightarrow^3 \mathrm{He} + \gamma$
    ....