Nucleon-Nucleon Chiral Two Pion Exchange potential vs Coarse grained interactions

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Motivation

- No unique determination of the nucleon-nucleon interaction
- Potentials are designed to make nuclear structure calculations
    - Reproduce the same experimental data
    - Different predictions (e.g. Triton binding energy)
    - No one has done them with the PWA
- Different phenomenological NN potentials [Nijmegen, ArgonneV18, CD-Bonn, Covariant Spectator]
    - High accuracy, $\chi^2/\mathrm{d.o.f.} \leq 1$
    - OPE as a long range interaction
    - ~ 40 parameters for the short and intermediate range
    - Repulsive core for most of them

Motivation

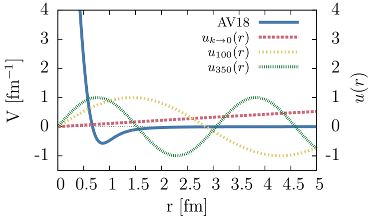
- pp Partial Wave Analysis with chiral TPE [Rentmeester, et al. (1999)]
  - Long and intermediate range interaction: OPE + TPElo + TPEso
  - Smaller χ^2
  - Less boundary condition parameters

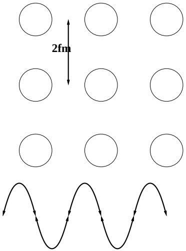
Motivation

- Effective coarse graining
  - Oscillator Shell Model
  - Euclidean Lattice EFT
  - V_{lowk} interaction
- Characteristic distance $\sim 0.5 - 1.0$ fm
- Nyquist Theorem
  - Optimal sampling
  - Finite Bandwidth

$$\Delta r \Delta k \sim 1$$

  - de Broglie wavelength of the most energetic particle
  - Sampling resolution determined





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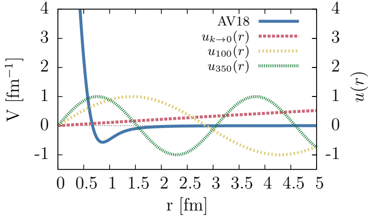
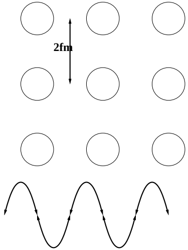


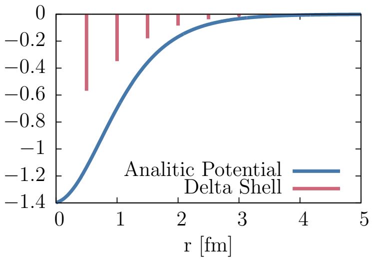


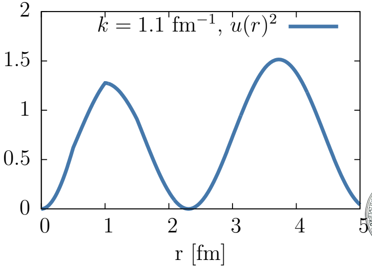
Delta Shell Potential

- A sum of delta functions [Aviles 1972]

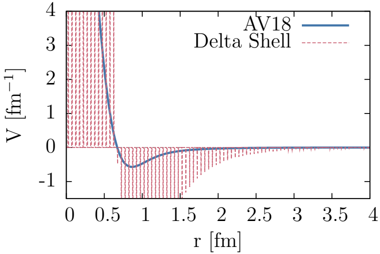
$$V(r) = \sum_i \frac{\lambda_i}{2\mu}\delta(r - r_i)$$

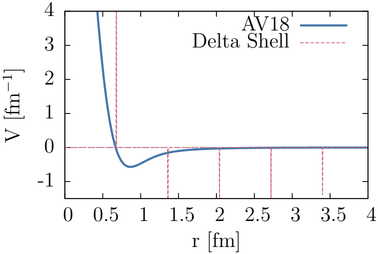
- Optimal and minimal sampling of the nuclear interaction
- Pion production threshold $\Delta k = 2$ fm^{-1}
- Optimal sampling around 0.5 fm

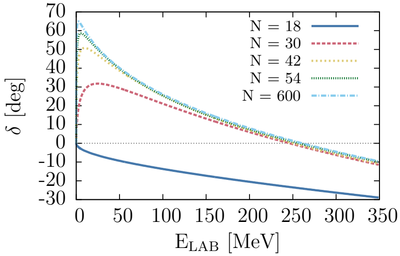


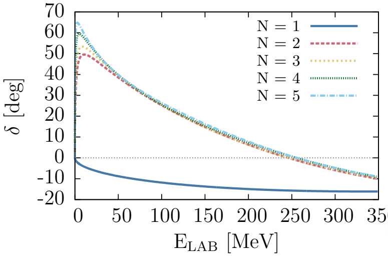


Coarse Graining the AV18 potential



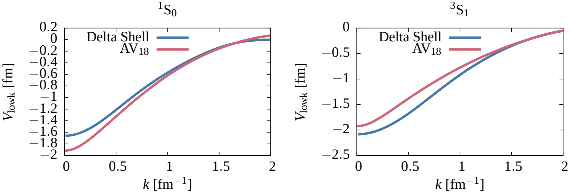




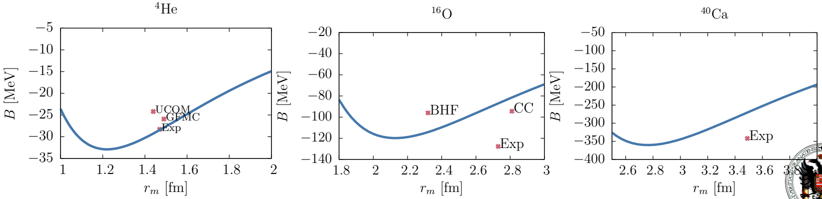


Delta Shell Potential

- Comparison with V_{lowk}



- Nuclear structure calculations



Delta Shell Potential

- 3 well defined regions
- Innermost region $r \leq 0.5$ fm
  - Short range interaction
  - No delta shell (No repulsive core)
- Intermediate region $0.5 \leq r \leq r_c$
  - Unknown interaction
  - λ_i parameters fitted to scattering data
- Outermost region $r \geq r_c$
  - Long range interaction
  - Described by OPE and χ TPE
  - Sampled with delta shells as well

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Fitting np Phase shifts

- Several HQ potentials with similar $\chi^2/\mathrm{d.o.f.}$ but different predictions
- Example: Triton Binding Energy

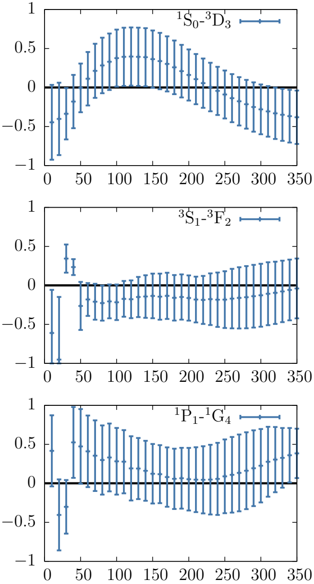
$$V_i \to B_i = (8.00, 7.62, 7.63, 7.62, 7.72, 8.50)\mathrm{MeV}$$

- Similar but not identical (systematic uncertainty)
- IF the potentials are independent then

$$\Delta B(A)^2 = \frac{1}{N-1}\sum_i (B^{(i)}(A) - \bar{B}(A))^2$$

- $B(A) = 7.85 \pm 0.35$ MeV

Fitting np Phaseshifts



- Can the potentials be treated as independent?
- Correlation coefficient

$$r_{x,y} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{n\sigma_x \sigma_y}$$

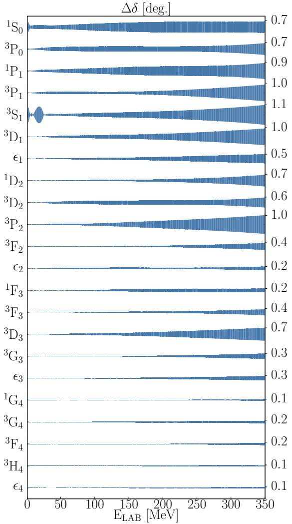
  - For every pair of Partial Waves
  - Taking the 6 HQ potentials and PWA as a sample
  - Mostly compatible with zero within error bars
- Then every potential can be considered an independent measurement of the interaction

Fitting np Phase shifts

- Several HQ potentials with similar χ^2/d.o.f. but different phase shifts
  - PWA
  - Nijm I
  - Nijm II
  - Reid93
  - Argonne V18
  - CD-Bonn
  - Covariant spectator

$$\Delta\delta_{\mathrm{PWA}} \lesssim \Delta\delta_{\mathrm{Allpotentials}}$$

- Systematic uncertainty *dominates*!
  - Systematic error propagation through fitting parameters
- → Today's poster session



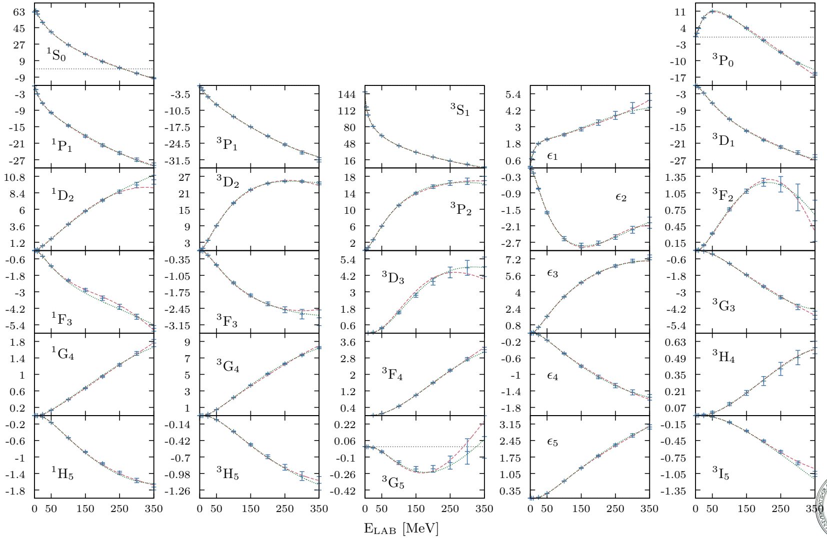
Fitting np Phaseshifts

- Fit for every partial wave with $j \leq 5$
- Strength coefficients λ_i as fit parameters
- Fixed and equidistant r_i, $\Delta r = 0.6$fm
- Long range interaction: OPE and χ TPE
  - $r_c = 3.0,\ 2.4,\ 1.8$ fm

Fitting Parameters

r_c [fm]	1.8		2.4		3.0	
	#p	χ^2/ν	#p	χ^2/ν	#p	χ^2/ν
OPE	37	2.1383	47	0.6470	51	0.4653
TPElo	40	2.0661	46	0.7361	52	0.5047
TPEso	32	0.5911	44	0.5225	51	0.3928

Fitting np Phaseshifts



Wolfenstein Parameters

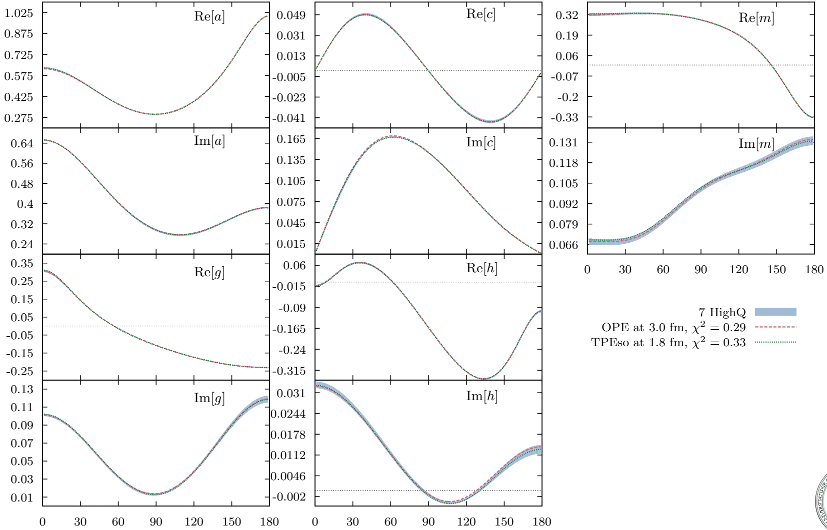
$$M(\mathbf{k}_f, \mathbf{k}_f) = a + m(\sigma_1, \mathbf{n})(\sigma_2, \mathbf{n}) + (g-h)(\sigma_1, \mathbf{m})(\sigma_2, \mathbf{m}) \\ +(g+h)(\sigma_1, \mathbf{l})(\sigma_2, \mathbf{l}) + c(\sigma_1 + \sigma_2, n)$$

- 7 High Quality potentials give a dispersion in Amplitude Parameters

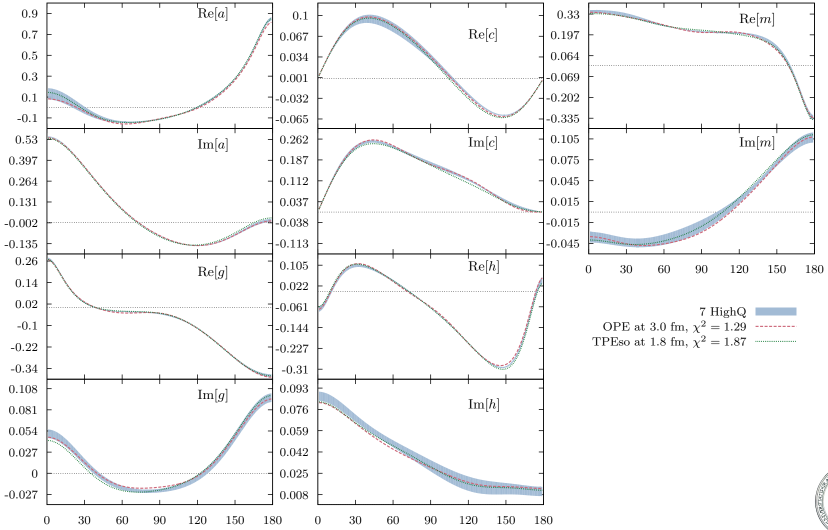
Comparing Fit with Potentials

r_c [fm]	1.8 χ^2/ν	2.4 χ^2/ν	3.0 χ^2/ν
OPE	2.45	0.56	0.47
TPElo	2.92	0.69	0.49
TPEso	0.54	0.70	0.41

Wolfenstein Parameters E = 100 MeV

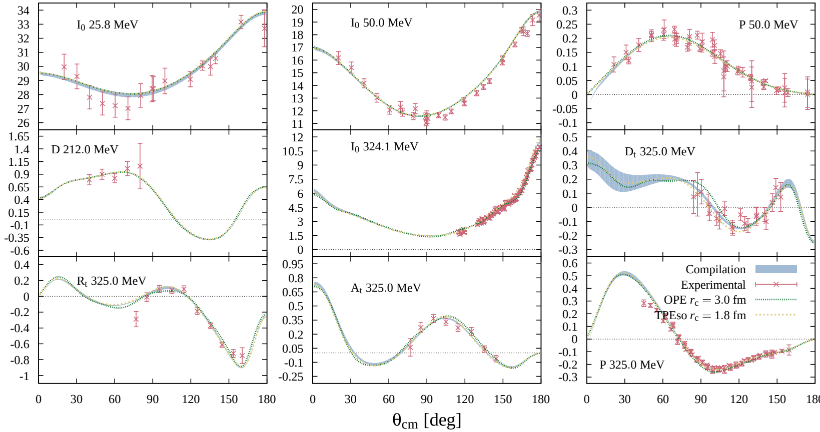


Wolfenstein Parameters E = 350 MeV



Scattering Observables

- Comparing with High Quality Potentials and Experimental data



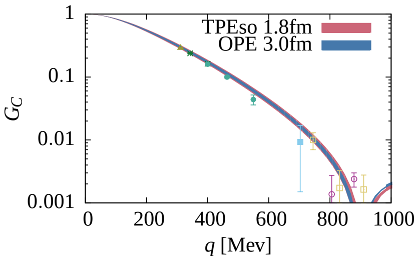
- Good agreement with data and other potentials

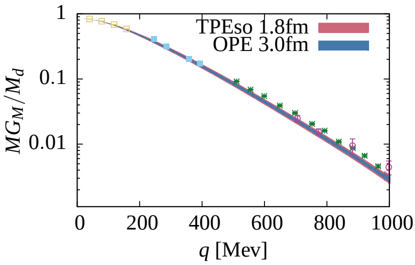
Deuteron Properties

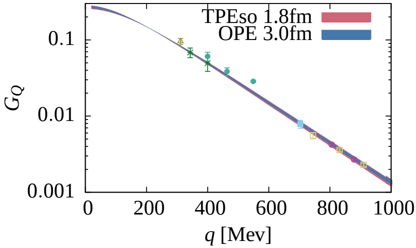
Potential	B_D MeV	η	A_S fm$^{1/2}$	P_D %	r_m fm	Q_D fm^2	$\langle r^{-1} \rangle$ fm^{-1}
$r_c = 3.0$							
OPE	-2.2(2)	0.025(2)	0.88(3)	5.7(2)	1.97(8)	0.272(9)	0.45(1)
TPElo	-2.2(2)	0.025(2)	0.88(3)	5.7(2)	1.97(8)	0.272(9)	0.45(1)
TPEso	-2.2(2)	0.025(2)	0.88(3)	5.7(2)	1.97(8)	0.272(9)	0.45(1)
$r_c = 2.4$							
OPE	-2.2(3)	0.025(2)	0.89(4)	5.6(3)	2.0(1)	0.27(1)	0.45(1)
TPElo	-2.2(3)	0.025(2)	0.89(4)	5.6(3)	2.0(1)	0.27(1)	0.45(1)
TPEso	-2.2(5)	0.025(4)	0.89(8)	5.5(6)	2.0(2)	0.27(2)	0.46(3)
$r_c = 1.8$							
OPE	-2.2(3)	0.025(3)	0.88(5)	5.7(4)	2.0(1)	0.27(1)	0.46(2)
TPElo	-2.2(6)	0.025(5)	0.88(9)	5.7(6)	2.0(2)	0.27(2)	0.46(3)
TPEso	-2.2(4)	0.025(3)	0.88(6)	5.6(4)	2.0(1)	0.27(2)	0.45(2)
Empirical	-2.2245(2)	0.0256(5)	0.8781(44)	5.67(4)	1.953(3)	0.2859(3)	

- No difference in central values, similar errors

Deuteron Form Factors







- Fairly good agreement with experimental data
- Room for improvement
  - Did someone say MEC's ?

Summary & Outlook

- Sampling of the NN interaction by a delta shell potential

$$1/\sqrt{m_\pi M} \lesssim \Delta r \lesssim 1/m_\pi$$

- 3 well defined regions
- Fit to the np phase shifts given by 7 High Quality Potentials
    - Every partial wave with $j \leq 5$
    - OPE, $r_c = 3.0$ fm, $\chi^2/\nu = 0.47$ and 51 parameters
    - OPE + χTPE, $r_c = 1.8$ fm, $\chi^2/\nu = 0.59$ and 32 parameters
- Deuteron wave function, properties and form factors
    - Good agreement with empirical values for all fits
- No substantial difference between OPE and χ TPE predictions
- Ongoing full scale fit to observables