Axial couplings and strong decay widths of heavy hadrons

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Heavy-hadron chiral perturbation theory

Heavy hadrons: here, mesons and baryons containing a **single heavy quark** Q, with $m_Q \gg \Lambda_{\mathrm{QCD}}$

HHχPT: effective field theory combining **heavy-quark symmetry** and **chiral symmetry** [M. Wise 1992, T. M. Yan et al. 1992, G. Burdman and J. F. Donoghue 1992, P. Cho 1992]

- heavy-light mesons with $s_l = 1/2$:

$$(P^i) = \begin{pmatrix} B^+ \\ B^0 \end{pmatrix}, \quad (P^{*i})_\mu = \begin{pmatrix} B^{*+} \\ B^{*0} \end{pmatrix}_\mu$$

combined into a single field

$$H^i = \left[-P^i \gamma_5 + P^{*i}_\mu \gamma^\mu \right] \frac{1 - \not{v}}{2}$$

Heavy-hadron chiral perturbation theory

- heavy-light baryons with $s_l = 1$:

$$(B^{ij}) = \begin{pmatrix} \Sigma_b^+ & \frac{1}{\sqrt{2}}\Sigma_b^0 \\ \frac{1}{\sqrt{2}}\Sigma_b^0 & \Sigma_b^- \end{pmatrix}, \quad (B^{*ij})_\mu = \begin{pmatrix} \Sigma_b^{*+} & \frac{1}{\sqrt{2}}\Sigma_b^{*0} \\ \frac{1}{\sqrt{2}}\Sigma_b^{*0} & \Sigma_b^{*-} \end{pmatrix}_\mu$$

  combined into a single field

$$S_\mu^{ij} = \sqrt{\frac{1}{3}}(\gamma_\mu + v_\mu)\gamma_5 B^{ij} + B_\mu^{*ij}$$

- heavy-light baryons with $s_l = 0$:

$$(T^{ij}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \Lambda_b \\ -\Lambda_b & 0 \end{pmatrix}$$

- pions:

$$\xi = \sqrt{\Sigma} = \exp(i\Phi/f)$$

Heavy-hadron chiral perturbation theory

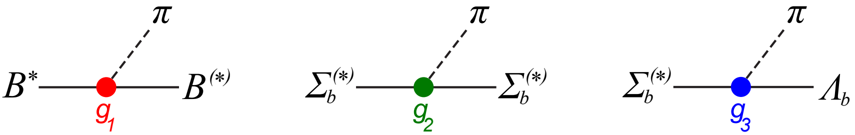
At leading order in chiral- and heavy-quark expansion ($m_Q = \infty$),

$$\mathcal{L} = \frac{f^2}{8}(\partial^\mu \Sigma^\dagger)_{ij}\partial_\mu \Sigma^{ji} - i\mathrm{Tr}\left(\overline{H}_i v\cdot\mathscr{D} H^i\right) - i\overline{S}^\mu_{ij} v\cdot\mathscr{D} S^{ij}_\mu + \Delta \overline{S}^\mu_{ij} S^{ij}_\mu + i\overline{T}_{ij} v\cdot\mathscr{D} T^{ij}$$
$$+g_1 \mathrm{Tr}\left(\overline{H}_i (\mathscr{A}^\mu)^i_j \gamma_\mu\gamma_5 H^j\right)$$
$$-i g_2 \epsilon_{\mu\nu\sigma\lambda} \overline{S}^\mu_{ki} v^\nu (\mathscr{A}^\sigma)^i_j (S^\lambda)^{jk}$$
$$+\sqrt{2} g_3 \left[\overline{S}^\mu_{ki}(\mathscr{A}_\mu)^i_j T^{jk} + \overline{T}_{ki}(\mathscr{A}^\mu)^i_j S^{jk}_\mu\right],$$

where

$$\mathscr{A}^\mu = \frac{i}{2}\left(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger\right) = -\frac{1}{f}\partial^\mu \Phi + \ldots$$

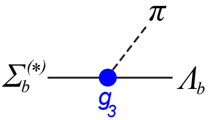
This gives the vertices



$[g_1 \sim g \sim \hat{g} \sim g_\pi \sim g_{B^* B\pi}]$

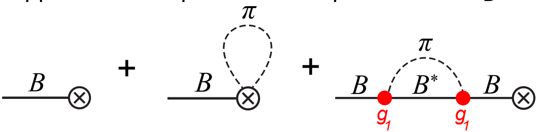
Heavy-hadron chiral perturbation theory

Example application 1: strong decay $\Sigma_b \to \Lambda_b \pi$



$$\Gamma(\Sigma_b^+ \to \Lambda_b \pi^+)_{\mathrm{LO}} = \frac{g_3^2}{6\pi f^2} |\mathbf{p}_\pi|^3$$

Example application 2: quark-mass dependence of f_B in $SU(2)$ χPT:



$$f_B = A\left[1 + \frac{3}{4}(1+3g_1^2)\frac{m_\pi^2}{(4\pi f)^2}\log\left(\frac{m_\pi^2}{(4\pi f)^2}\right)\right] + B\, m_\pi^2$$

Previous knowledge of g_1, g_2, g_3:

Reference	Method	g_1	g_2	g_3
Yan *et al.*, 1992	Nonrelativistic quark model	1	2	$\sqrt{2}$
Colangelo *et al.*, 1994	Relativistic quark model	1/3	...	...
Bećirević, 1999	Quark model with Dirac eq.	0.6 ± 0.1	...	...
Guralnik *et al.*, 1992	Skyrme model / large N_c	...	1.6	1.3
Colangelo *et al.*, 1994	Sum rules	0.15 - 0.55	...	...
Belyaev *et al.*, 1994	Sum rules	0.32 ± 0.02	...	...
Dosch and Narison, 1995	Sum rules	0.15 ± 0.03	...	...
Colangelo and Fazio, 1997	Sum rules	0.09 - 0.44	...	...
Pirjol and Yan, 1997	Sum rules	...	$< \sqrt{6 - g_3^2}$	$< \sqrt{2}$
Zhu and Dai, 1998	Sum rules	...	$1.56 \pm 0.30 \pm 0.30$	$0.94 \pm 0.06 \pm 0.20$
Cho and Georgi, 1992	$\mathcal{B}[D^* \to D\,\pi, D\,\gamma]$	0.34 ± 0.48	...	...
Arnesen *et al.*, 2005	$\mathcal{B}[D^*_{(s)} \to D_{(s)}\pi, D_{(s)}\gamma]$	0.51	...	...
Li *et al.*, 2010	$d\Gamma[B \to \pi\ell\nu]$	< 0.87	...	...
Cheng, 1997	$\Gamma[\Sigma_c^* \to \Lambda_c\,\pi]$, NRQM	0.70 ± 0.12	1.40 ± 0.24	0.99 ± 0.17
De Divitiis *et al.*, 1998	$n_f = 0$ Lattice QCD	0.42 ± 0.09	...	...
Abada *et al.*, 2004	$n_f = 0$ Lattice QCD	0.48 ± 0.11	...	...
Negishi *et al.*, 2007	$n_f = 0$ Lattice QCD	0.517 ± 0.016	...	...
Ohki *et al.*, 2008	$n_f = 2$ Lattice QCD	0.516 ± 0.052	...	...
Bećirević *et al.*, 2009	$n_f = 2$ Lattice QCD	$0.44 \pm 0.03^{+0.07}_{-0.00}$	...	...
Bulava *et al.*, 2010	$n_f = 2$ Lattice QCD	0.51 ± 0.02	...	...

This work

- Complete calculation of g_1, g_2, g_3 from **lattice QCD** with $n_f = 2 + 1$, controlling all systematic uncertainties (excited-state contamination, chiral extrapolation, continuum extrapolation, finite-volume effects, renormalization)

- To allow the extraction of g_1, g_2, g_3 from the lattice data, we also performed a one-loop calculation in partially-quenched, finite-volume HHχPT

- Use result for g_3 to compute heavy-baryon strong decay widths

References:

PRD 84, 094502 [arXiv:1108.5594]
PRL 108, 172003 [arXiv:1109.2480]
PRD 85, 114508 [arXiv:1203.3378]

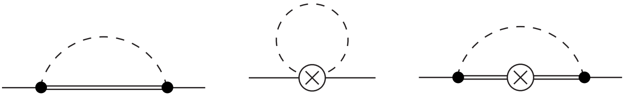
How can we calculate g_1, g_2, g_3 from QCD?

- We compute suitable hadronic observables both in HHχPT and in lattice QCD.
- The expressions derived from HHχPT are then fitted to the lattice data, and in these fits the axial couplings are parameters.
- The simplest quantities that depend on the axial couplings *already at leading order* are the **matrix elements of the axial current** between single-hadron states.

$$
\begin{aligned}
\langle P^{*d}|A_\mu|P^u\rangle &= -2\,(g_1)_{\rm eff}\ \varepsilon^*_\mu, \\
\langle S^{dd}|A_\mu|S^{du}\rangle &= -(i/\sqrt{2})\,(g_2)_{\rm eff}\ v^\sigma\,\epsilon_{\sigma\mu\nu\rho}\,\overline{U}^\nu U^\rho, \\
\langle S^{dd}|A_\mu|T^{du}\rangle &= -(g_3)_{\rm eff}\ \overline{U}_\mu\,\mathcal{U}
\end{aligned}
$$

- At leading order, $(g_i)_{\rm eff} = g_i$
- Lattice calculation done at $m_{u,d} \neq 0$, therefore need NLO

Axial current matrix elements at NLO in HHχPT



Simplest case: $SU(2)$, $m_u = m_d$

$$
\begin{aligned}
(g_1)_{\text{eff}} &= g_1 \left[1 - \frac{2}{f^2} I(m_\pi) + \frac{4 g_1^2}{f^2} H(m_\pi, 0) + \text{analytic terms} \right], \\
(g_2)_{\text{eff}} &= g_2 \left[1 - \frac{2}{f^2} I(m_\pi) + \frac{3 g_2^2}{2 f^2} H(m_\pi, 0) \right. \\
&\quad \left. + \frac{g_3^2}{f^2} \Big\{ H(m_\pi, -\Delta) - 2K(m_\pi, -\Delta, 0) \Big\} + \text{analytic terms} \right], \\
(g_3)_{\text{eff}} &= g_3 \left[1 - \frac{2}{f^2} I(m_\pi) + \frac{g_2^2}{f^2} \Big\{ -2H(m_\pi, \Delta) + H(m_\pi, 0) \Big\} \right. \\
&\quad + \frac{g_3^2}{2 f^2} \Big\{ H(m_\pi, -\Delta) + 9H(m_\pi, \Delta) - 2K(m_\pi, \Delta, 0) \Big\} \\
&\quad \left. + \text{analytic terms} \right]
\end{aligned}
$$

see PRD 84, 094502 for $SU(4|2)$, $SU(6|3)$ and finite volume

Lattice calculation of axial-current matrix elements

Use the interpolating fields

$$\begin{aligned} P^i &= (\gamma_5)_{\alpha\beta}\,\overline{Q}_{a\alpha}\,\tilde{q}^i_{a\beta}, \\ P^{*i}_{\mu} &= (\gamma_\mu)_{\alpha\beta}\,\overline{Q}_{a\alpha}\,\tilde{q}^i_{a\beta}, \\ S^{ij}_{\mu\,\alpha} &= \epsilon_{abc}\,(C\gamma_\mu)_{\beta\gamma}\,\tilde{q}^i_{a\beta}\,\tilde{q}^j_{b\gamma}\,Q_{c\alpha}, \\ T^{ij}_{\alpha} &= \epsilon_{abc}\,(C\gamma_5)_{\beta\gamma}\,\tilde{q}^i_{a\beta}\,\tilde{q}^j_{b\gamma}\,Q_{c\alpha}, \\ A_\mu &= Z_A\,\overline{d}_{a\alpha}(\gamma_\mu\gamma_5)_{\alpha\beta}u_{a\beta}. \end{aligned}$$

Note that g_1, g_2, g_3 are **defined in the static limit** $m_Q = \infty$. Use static Eichten-Hill action for Q

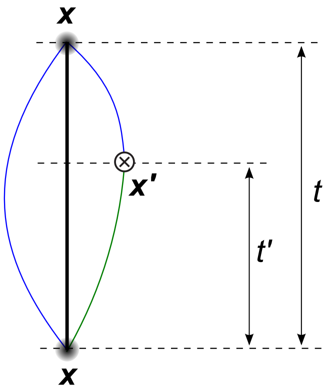
Light quarks u, d, s: **domain wall action**. Nonperturbative Z_A (thanks to chiral symmetry) from RBC/UKQCD [Aoki et al., 2011]

$$Z_A = \begin{cases} 0.7019(26) & \text{for } a = 0.112 \text{ fm}, \\ 0.7396(17) & \text{for } a = 0.085 \text{ fm} \end{cases}$$

Three-point correlators

For example

$$\langle S^{dd\,\mu}(\mathbf{x}, t)\ \sum_{\mathbf{x}'} A^{\mu}(\mathbf{x}', t')\ \overline{T}_{du}(\mathbf{x}, 0)\rangle$$



Correlator ratios

Simple ratios for mass-degenerate $P^* \to P$ and $S \to S$ matrix elements

$$R_1(t,\ t') \;=\; -\frac{1}{3}\frac{\sum_{\mu=1}^{3}\langle\, P^{*d\ \mu}(t)\ A^{\mu}(t')\ P_u^{\dagger}(0)\,\rangle}{\langle\, P^{u}(t)\ P_u^{\dagger}(0)\,\rangle} \quad \xrightarrow[t,t',|t-t'|\to\infty]{} \quad (g_1)_{\text{eff}},$$

$$R_2(t,\ t') \;=\; i\ \frac{\sum_{\mu,\nu,\rho=1}^{3}\epsilon_{0\mu\nu\rho}\,\langle\, S^{dd\ \mu}(t)\ A^{\nu}(t')\ \overline{S}^{\rho}_{du}(0)\,\rangle}{\sum_{\mu=1}^{3}\langle\, S^{dd\ \mu}(t)\ \overline{S}^{\mu}_{dd}(0)\,\rangle} \quad \xrightarrow[t,t',|t-t'|\to\infty]{} \quad (g_2)_{\text{eff}},$$

Double ratio for $S \to T$ transition matrix element

$$R_3(t,\ t') = \left[\frac{1}{3}\frac{\sum_{\mu,\nu=1}^{3}\langle\, S^{dd\ \mu}(t)\ A^{\mu}(t')\ \overline{T}_{du}(0)\,\rangle\langle\, T^{du}(t)\ A^{\nu\dagger}(t')\ \overline{S}^{\nu}_{dd}(0)\,\rangle}{\sum_{\mu=1}^{3}\langle\, S^{dd\ \mu}(t)\ \overline{S}^{\mu}_{dd}(0)\,\rangle\ \langle\, T^{du}(t)\ \overline{T}_{du}(0)\,\rangle}\right]^{1/2}$$

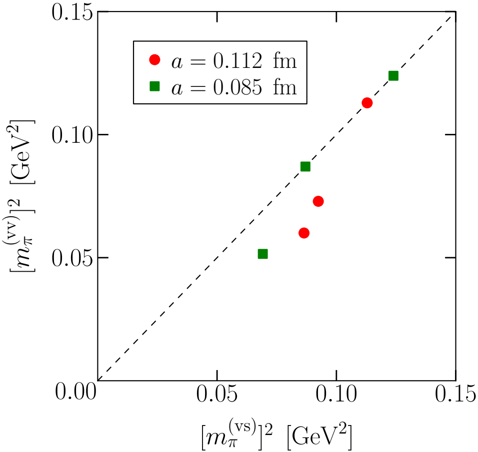
$$\xrightarrow[t,t',|t-t'|\to\infty]{} \quad (g_3)_{\text{eff}}$$

Lattice parameters

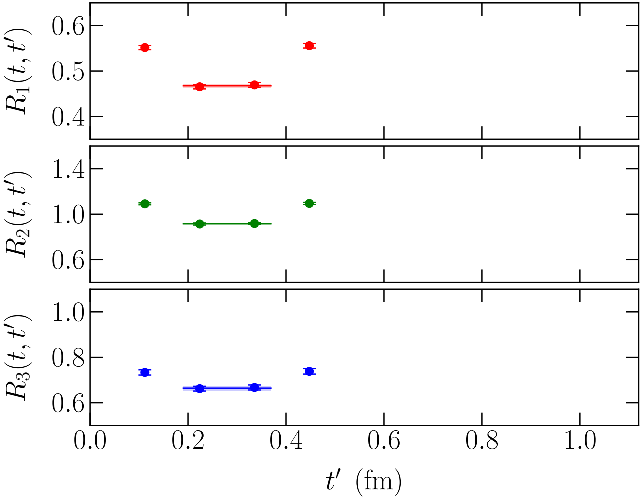
RBC/UKQCD ensembles with 2+1 flavors of domain wall fermions

$L^3 \times T$	$m_{u/d}^{(sea)}$	$m_{u/d}^{(val)}$	a (fm)	$m_\pi^{(vs)}$ (MeV)	$m_\pi^{(vv)}$ (MeV)
$24^3 \times 64$	0.005	0.005	0.1119(17)	336(5)	336(5)
$24^3 \times 64$	0.005	0.002	0.1119(17)	304(5)	270(4)
$24^3 \times 64$	0.005	0.001	0.1119(17)	294(5)	245(4)
$32^3 \times 64$	0.006	0.006	0.0848(17)	352(7)	352(7)
$32^3 \times 64$	0.004	0.004	0.0849(12)	295(4)	295(4)
$32^3 \times 64$	0.004	0.002	0.0849(12)	263(4)	227(3)

Lattice parameters

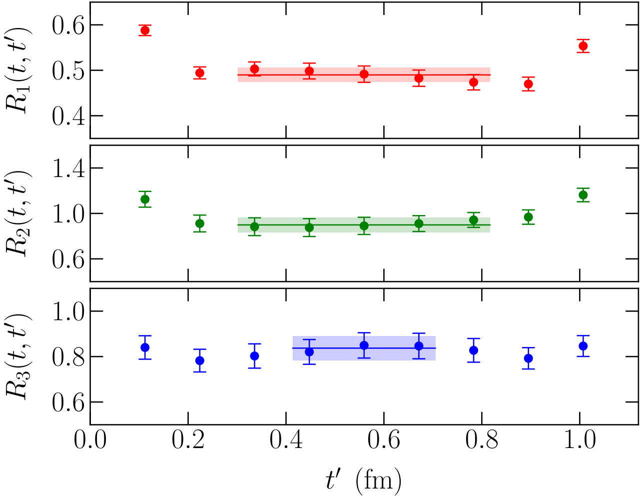


Examples of ratios



[$a = 0.112$ fm, $am_{u/d}^{(\text{val})} = 0.002$, $t/a = 5$]

Examples of ratios

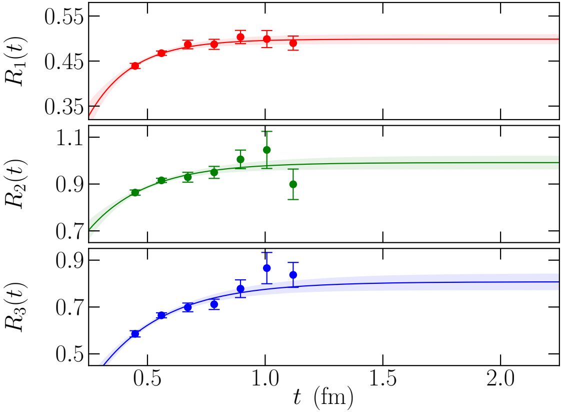


$[a = 0.112 \text{ fm}, \; am_{u/d}^{(\text{val})} = 0.002, \; t/a = 10]$

Extrapolation to infinite source-sink separation

Fit t-dependence including first excited-state contamination using

$$R_i(t) = (g_i)_{\rm eff} - A_i\, e^{-\delta_i\, t}$$



$[a = 0.112 \text{ fm},\ am_{u/d}^{(\rm val)} = 0.002]$

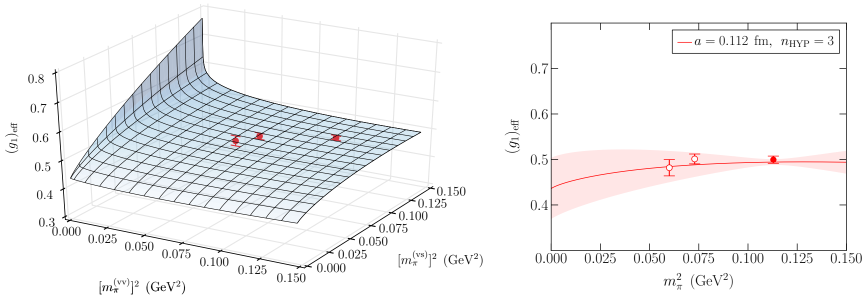
Chiral-continuum fits using $SU(4|2)$ HHχPT

$$
\begin{aligned}
(g_1)_{\rm eff} &= g_1\Big[1 + f_1(g_1, m_\pi^{(\rm vv)}, m_\pi^{(\rm vs)}, L) + d_{1,n_{\rm HYP}}\; a^2 \\
&\qquad + c_1^{(\rm vv)}\,[m_\pi^{(\rm vv)}]^2 + c_1^{(\rm vs)}\,[m_\pi^{(\rm vs)}]^2\Big],
\end{aligned}
$$

$$
\begin{aligned}
(g_2)_{\rm eff} &= g_2\Big[1 + f_2(g_2, g_3, m_\pi^{(\rm vv)}, m_\pi^{(\rm vs)}, \Delta, L) + d_{2,n_{\rm HYP}}\; a^2 \\
&\qquad + c_2^{(\rm vv)}\,[m_\pi^{(\rm vv)}]^2 + c_2^{(\rm vs)}\,[m_\pi^{(\rm vs)}]^2\Big], \\
(g_3)_{\rm eff} &= g_3\Big[1 + f_3(g_2, g_3, m_\pi^{(\rm vv)}, m_\pi^{(\rm vs)}, \Delta, L) + d_{3,n_{\rm HYP}}\; a^2 \\
&\qquad + c_3^{(\rm vv)}\,[m_\pi^{(\rm vv)}]^2 + c_3^{(\rm vs)}\,[m_\pi^{(\rm vs)}]^2\Big]
\end{aligned}
$$

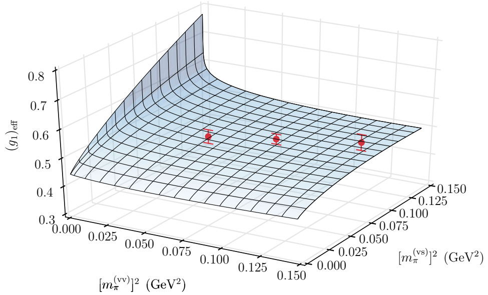
($\Delta = 200$ MeV)

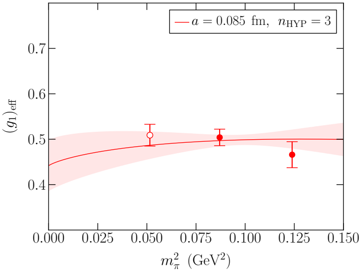
Fits using $SU(4|2)$ HHχPT



(data points shifted to and functions evaluated at $L = \infty$)

Fits using $SU(4|2)$ HHχPT





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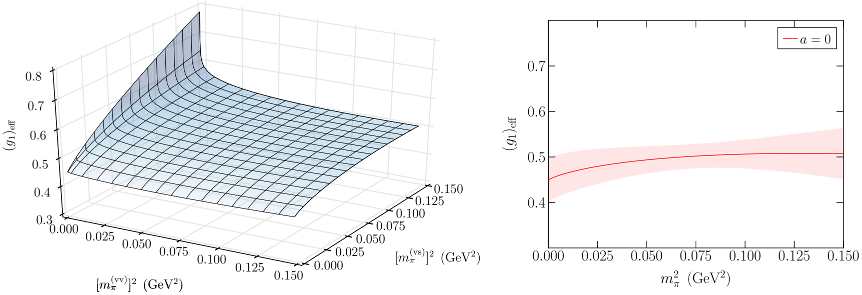
Fits using $SU(4|2)$ HHχPT



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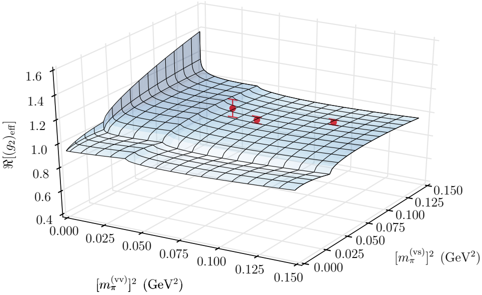

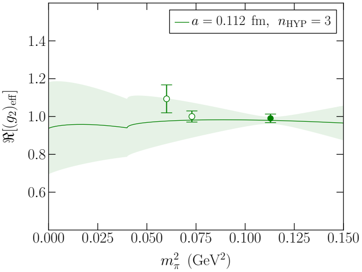

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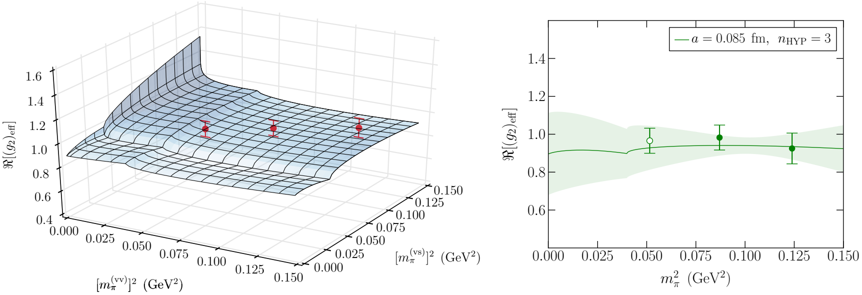
Fits using $SU(4|2)$ HHχPT



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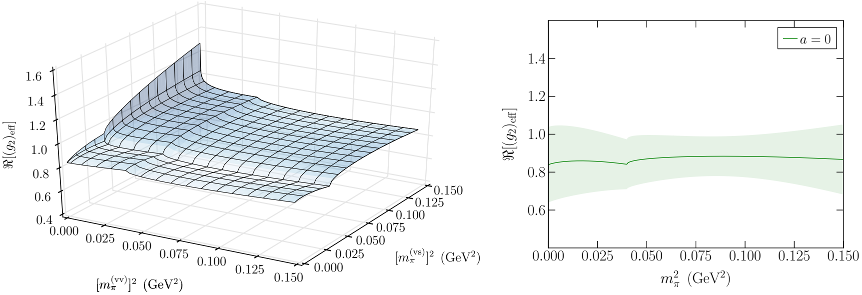

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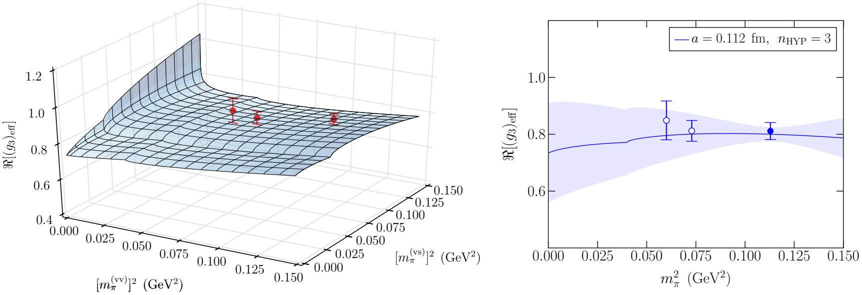
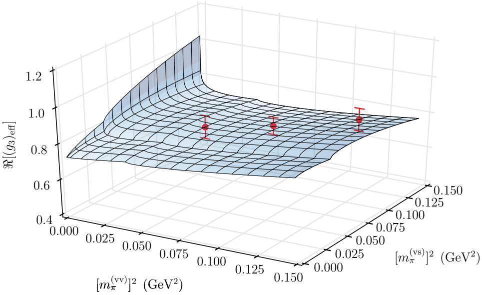
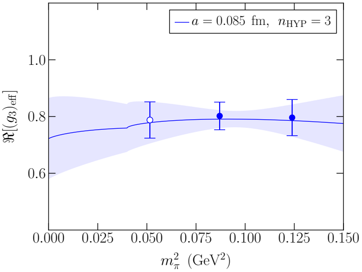
Fits using $SU(4|2)$ HHχPT



(data points shifted to and functions evaluated at $L = \infty$)

Fits using $SU(4|2)$ HHχPT





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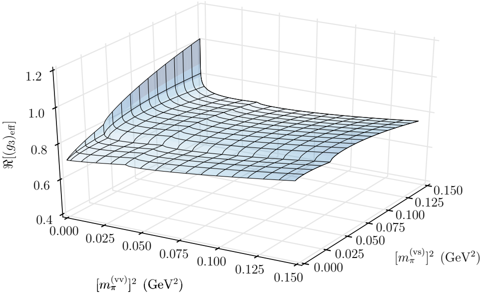
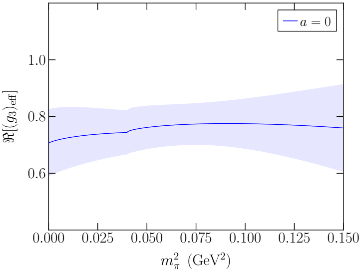
Fits using $SU(4|2)$ HHχPT

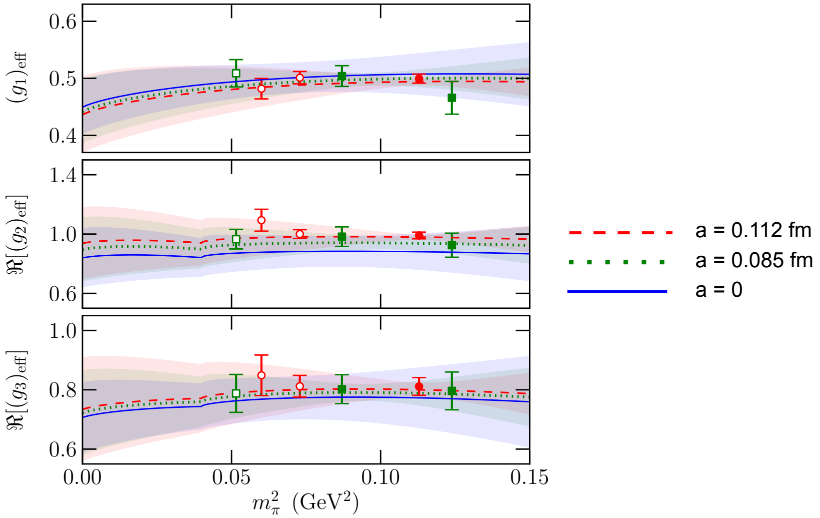




(functions evaluated at $L = \infty$)

Fits using $SU(4|2)$ HHχPT

Plotted at $L = \infty$, $m_\pi^{(\mathrm{vs})} = m_\pi^{(\mathrm{vv})}$:



Final results for axial couplings

Final results:

$$g_1 = 0.449 \pm 0.047_{\,\text{stat}} \pm 0.019_{\,\text{syst}} = 0.449 \pm 0.051,$$
$$g_2 = 0.84 \pm 0.20_{\,\text{stat}} \pm 0.04_{\,\text{syst}} = 0.84 \pm 0.20,$$
$$g_3 = 0.71 \pm 0.12_{\,\text{stat}} \pm 0.04_{\,\text{syst}} = 0.71 \pm 0.13.$$

Systematic uncertainties (see extra slides):

Source	g_1	g_2	g_3
NNLO terms in fits of m_π- and a-dep	3.6%	2.8%	3.7%
Higher excited states in fits to $R_i(t)$	1.7%	2.8%	4.9%
Unphysical value of $m_s^{(\text{sea})}$	1.5%	1.5%	1.5%
Total	4.2%	4.3%	6.3%

g_1: comparison with previous lattice results

Reference	n_f, action	$[m_\pi^{(vv)}]^2$ (GeV^2)	g_1
De Divitiis *et al.*, 1998	0, clover	0.58 - 0.81	$0.42 \pm 0.04 \pm 0.08$
Abada *et al.*, 2004	0, clover	0.30 - 0.71	$0.48 \pm 0.03 \pm 0.11$
Negishi *et al.*, 2007	0, clover	0.43 - 0.72	0.517 ± 0.016
Ohki *et al.*, 2008	2, clover	0.24 - 1.2	$0.516 \pm 0.005 \pm 0.033 \pm 0.028$
Bećirević *et al.*, 2009	2, clover	0.16 - 1.2	$0.44 \pm 0.03^{+0.07}_{-0.00}$
Bulava *et al.*, 2010	2, clover	0.063 - 0.49	0.51 ± 0.02
This work	$2+1$, DW	0.052 - 0.12	$0.449 \pm 0.047_{\text{stat}} \pm 0.019_{\text{syst}}$

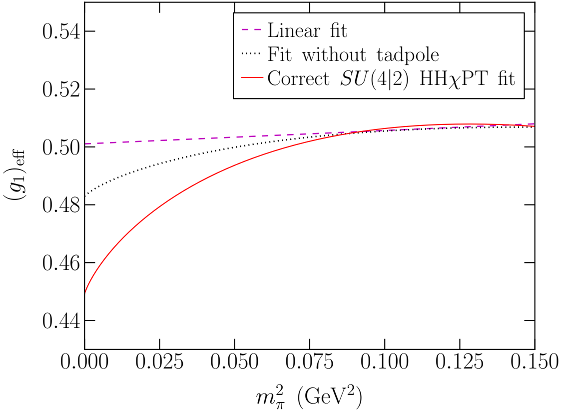
Note: none of the previous works use correct NLO χPT (linear extrapolation or tadpole missing)

Impact of NLO χPT corrections

In $SU(2)$:

Strong decay: $\mathcal{M}(P^* \to P\,\pi) \propto g_1 \left[1 - 4g_1^2 \frac{m_\pi^2}{(4\pi f_\pi)^2} \log \frac{m_\pi^2}{\mu^2} + \tilde{c}\, m_\pi^2\right]$.

Axial current m. elt.: $(g_1)_{\text{eff}} = g_1 \left[1 - (2 + 4g_1^2) \frac{m_\pi^2}{(4\pi f_\pi)^2} \log \frac{m_\pi^2}{\mu^2} + c\, m_\pi^2\right]$.



Comparison with other results

Reference	Method	g_1	g_2	g_3
Yan *et al.*, 1992	Nonrelativistic quark model	1	2	$\sqrt{2}$
Colangelo *et al.*, 1994	Relativistic quark model	1/3	...	...
Bećirević, 1999	Quark model with Dirac eq.	0.6 ± 0.1	...	...
Guralnik *et al.*, 1992	Skyrme model / large N_c	...	1.6	1.3
Colangelo *et al.*, 1994	Sum rules	0.15 - 0.55	...	...
Belyaev *et al.*, 1994	Sum rules	0.32 ± 0.02	...	...
Dosch and Narison, 1995	Sum rules	0.15 ± 0.03	...	...
Colangelo and Fazio, 1997	Sum rules	0.09 - 0.44	...	...
Pirjol and Yan, 1997	Sum rules	...	$< \sqrt{6 - g_3^2}$	$< \sqrt{2}$
Zhu and Dai, 1998	Sum rules	...	$1.56 \pm 0.30 \pm 0.30$	$0.94 \pm 0.06 \pm 0.20$
Cho and Georgi, 1992	$\mathcal{B}[D^* \to D\,\pi,\, D\,\gamma]$	0.34 ± 0.48	...	...
Stewart, 1998	$\mathcal{B}[D^*_{(s)} \to D_{(s)}\pi,\, D_{(s)}\gamma]$	$0.27^{+0.04+0.05}_{-0.02-0.02}$	...	...
Li *et al.*, 2010	$d\Gamma[B \to \pi\ell\nu]$	< 0.87	...	...
Cheng, 1997	$\Gamma[\Sigma_c^* \to \Lambda_c\,\pi]$, NRQM	0.70 ± 0.12	1.40 ± 0.24	0.99 ± 0.17
This work	Lattice QCD	0.449 ± 0.051	0.84 ± 0.20	0.71 ± 0.13

Our QCD results are MUCH smaller than the quark-model predictions (even for $g_A^{ud} = 0.75$, as needed to get correct nucleon g_A)

Heavy baryon decays

$S \to T$ strong decays: leading-order width

At LO in chiral and heavy quark expansion, HHχPT predicts

$$\Gamma[S \to T\,\pi] = c_{\rm f}^2\,\frac{1}{6\pi f_\pi^2}\,g_3^2\,\frac{M_T}{M_S}|\mathbf{p}_\pi|^3$$

where

$$c_{\rm f} = \begin{cases} 1 & \text{for} \quad \Sigma_Q^{(*)} \to \Lambda_Q\,\pi^\pm, \\ 1 & \text{for} \quad \Sigma_Q^{(*)} \to \Lambda_Q\,\pi^0, \\ 1/\sqrt{2} & \text{for} \quad \Xi_Q^{\prime(*)} \to \Xi_Q\,\pi^\pm, \\ 1/2 & \text{for} \quad \Xi_Q^{\prime(*)} \to \Xi_Q\,\pi^0, \end{cases}$$

and

$$|\mathbf{p}_\pi| = \frac{\sqrt{[(M_S - M_T)^2 - m_\pi^2][(M_S + M_T)^2 - m_\pi^2]}}{2M_S}$$

(kinematic factors not expanded in $1/m_Q$ here)

$S \to T$ strong decays: including $1/m_Q$ correction

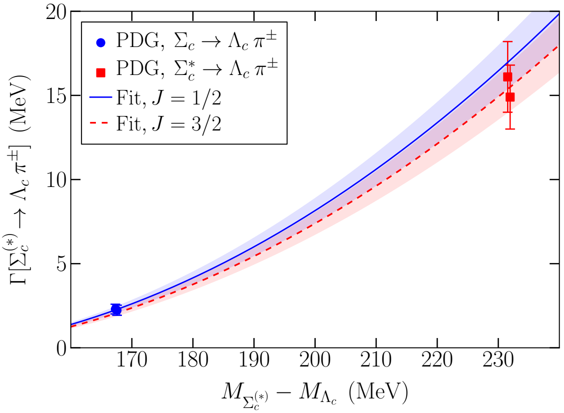
LO in chiral expansion is precice enough, because physical m_π is so small.

LO in HQ expansion is not good enough. Include generic $1/m_Q$ correction

$$\Gamma[S \to T\,\pi] = c_{\mathrm{f}}^2 \, \frac{1}{6\pi f_\pi^2} \left(g_3 + \frac{\kappa_J}{m_Q} \right)^2 \frac{M_T}{M_S} \, |\mathbf{p}_\pi|^3$$

Determine $\kappa_{1/2}$ and $\kappa_{3/2}$ by fitting experimental data for Σ_c^{++}, Σ_c^0 and Σ_c^{*++}, Σ_c^{*0} with g_3 constrained to LQCD value, and $m_Q = \frac{1}{2} M_{J/\psi}$.

$S \to T$ strong decays: including $1/m_Q$ correction

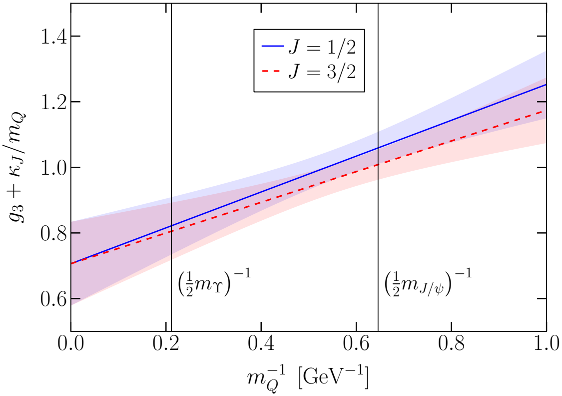


Result:

$$\kappa_{1/2} = 0.55(21)\ \text{GeV}, \quad \text{Cov}(\kappa_{1/2},\, g_3) = -0.025\ \text{GeV},$$
$$\kappa_{3/2} = 0.47(21)\ \text{GeV}, \quad \text{Cov}(\kappa_{3/2},\, g_3) = -0.025\ \text{GeV}.$$

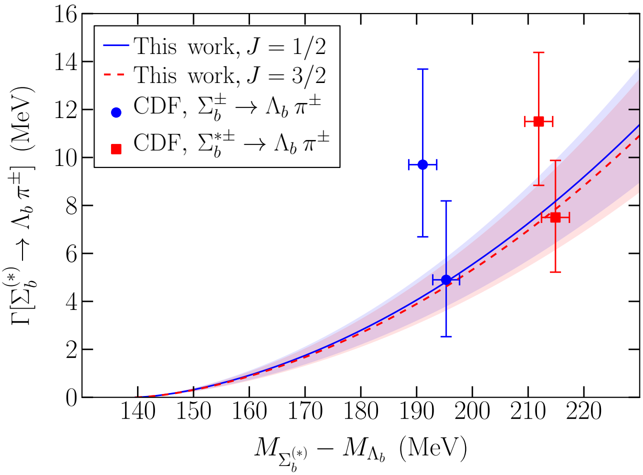
$S \to T$ strong decays: including $1/m_Q$ correction

Effective coupling vs $1/m_Q$:



$\to$ Evaluate for $m_Q = \frac{1}{2}M_\Upsilon$ to make predictions for bottom baryons

$S \to T$ strong decays: predictions for bottom baryons



Also: Ξ_b^{*0}, recently discovered by CMS

$\Gamma[\Xi_b^{*0}] = 2.1 \pm 1.7$ MeV (CMS), $\Gamma[\Xi_b^{*0}] = 0.51 \pm 0.26$ MeV (this work)

Conclusions

- First complete lattice QCD determination of heavy-hadron axial couplings, Results are $g_1 = 0.449 \pm 0.047_{\text{stat}} \pm 0.019_{\text{syst}}$, $g_2 = 0.84 \pm 0.20_{\text{stat}} \pm 0.04_{\text{syst}}$, $0.71 \pm 0.12_{\text{stat}} \pm 0.04_{\text{syst}}$
- Systematic uncertainties very small, statistical uncertainties by far dominant
- g_1, g_2, g_3 are much smaller than NRQM prediction, but ratios close to NRQM
- Compare to $g_A \approx 1.26$, $|g_{N\Delta}| \sim 1.6$ and $g_{\Delta\Delta} \sim -1.9$
- $1/m_Q$ corrections in $\Gamma[\Sigma_Q^{(*)} \to \Lambda_Q \pi]$ are significant even for $Q = b$
- Calculated $\Gamma[\Sigma_b^{(*)} \to \Lambda_b \pi]$ widths agree with experiment
- Many future applications (e.g. chiral extrapolations for spectroscopy)

Extra slides

Systematic uncertainties from higher excited states

$$R_i(t) = (g_i)_{\text{eff}} - A_i \, e^{-\delta_i \, t}$$

Fit	$(g_1)_{\text{eff}}$	$\delta(g_1)_{\text{eff}}$	$(g_2)_{\text{eff}}$	$\delta(g_2)_{\text{eff}}$	$(g_3)_{\text{eff}}$	$\delta(g_3)_{\text{eff}}$
Original	0.499(11)	0	0.993(29)	0	0.810(36)	0
$t/a = 4$ removed	0.496(13)	0.0030(76)	0.975(35)	0.016(19)	0.783(43)	0.026(15)
$t/a = 4, 5$ removed	0.494(12)	0.0041(76)	0.984(41)	0.009(26)	0.807(54)	0.003(30)
Second exp. added	0.498(11)	0.0009(77)	0.988(30)	0.005(21)	0.796(40)	0.014(36)

Conservative estimate of uncertainty:

$$\sqrt{[\delta(g_i)_{\text{eff}}]^2 + [\sigma\delta(g_i)_{\text{eff}}]^2}$$

g_1 : 1.7%
g_2 : 2.8%
g_3 : 4.9%

Fits using $SU(4|2)$ HHχPT: estimates of systematic uncertainties

Add higher-order analytic terms.

$$
\begin{aligned}
(g_i)_{\rm eff}^{(\rm NLO+HO)}(a, m, n_{\rm HYP}) \\
= \quad & (g_i)_{\rm eff}^{(\rm NLO)}(a, m, n_{\rm HYP}) \\
& + g_i \Big[c_i^{(\rm vv,vv)}\,[m_\pi^{(\rm vv)}]^4 + c_i^{(\rm vs,vs)}\,[m_\pi^{(\rm vs)}]^4 + c_i^{(\rm vv,vs)}\,[m_\pi^{(\rm vv)}]^2[m_\pi^{(\rm vs)}]^2 \\
& \qquad + d_{i,\,n_{\rm HYP}}^{(\rm vv)}\,a^2\,[m_\pi^{(\rm vv)}]^2 + d_{i,\,n_{\rm HYP}}^{(\rm vs)}\,a^2\,[m_\pi^{(\rm vs)}]^2 + h_{i,\,n_{\rm HYP}}\,a^4 \Big]
\end{aligned}
$$

Fits using $SU(4|2)$ HHχPT: estimates of systematic uncertainties

Constrain with Gaussian priors. In terms of the natural scales,

$$
\begin{aligned}
c_i^{(\mathrm{vv,vv})} &= 0 \pm w/\Lambda_\chi^4, \\
c_i^{(\mathrm{vs,vs})} &= 0 \pm w/\Lambda_\chi^4, \\
c_i^{(\mathrm{vv,vs})} &= 0 \pm w/\Lambda_\chi^4, \\
d_{i,\,n_{\mathrm{HYP}}}^{(\mathrm{vv})} &= 0 \pm w\,\Lambda_{\mathrm{QCD}}^2/\Lambda_\chi^2, \\
d_{i,\,n_{\mathrm{HYP}}}^{(\mathrm{vs})} &= 0 \pm w\,\Lambda_{\mathrm{QCD}}^2/\Lambda_\chi^2, \\
h_{i,\,n_{\mathrm{HYP}}} &= 0 \pm w\,\Lambda_{\mathrm{QCD}}^4.
\end{aligned}
$$

w is the width

Fits using $SU(4|2)$ HHχPT: estimates of systematic uncertainties

w	g_1	$\delta\sigma(g_1)$	g_2	$\delta\sigma(g_2)$	g_3	$\delta\sigma(g_3)$
0	0.449(47)	0	0.84(20)	0	0.71(12)	0
1	0.449(47)	0.0020	0.84(20)	0.0023	0.71(12)	0.0045
5	0.452(48)	0.0089	0.84(20)	0.014	0.70(12)	0.017
10	0.455(50)	0.016	0.84(20)	0.024	0.70(12)	0.026
50	0.464(72)	0.054	0.82(22)	0.099	0.68(15)	0.094
100	0.452(94)	0.082	0.78(26)	0.17	0.63(21)	0.17

Conservative estimate: with $w = 10$, use

$$\delta\sigma(g_i) = \sqrt{\sigma^2(g_i)^{(\mathrm{NLO+HO})} - \sigma^2(g_i)^{(\mathrm{NLO})}}$$

g_1 : 3.6%
g_2 : 2.8%
g_3 : 3.7%

Fits using $SU(4|2)$ HHχPT: finite-volume corrections

$m_\pi^{(\mathrm{vs})}$ (MeV)	$m_\pi^{(\mathrm{vv})}$ (MeV)	$\frac{(g_1)_{\mathrm{eff}}^{(\infty)} - (g_1)_{\mathrm{eff}}^{(L)}}{(g_1)_{\mathrm{eff}}^{(\infty)}}$	$\frac{(g_2)_{\mathrm{eff}}^{(\infty)} - (g_2)_{\mathrm{eff}}^{(L)}}{(g_2)_{\mathrm{eff}}^{(\infty)}}$	$\frac{(g_3)_{\mathrm{eff}}^{(\infty)} - (g_3)_{\mathrm{eff}}^{(L)}}{(g_3)_{\mathrm{eff}}^{(\infty)}}$
294	245	0.0057	0.015	0.0074
304	270	0.0040	0.0070	0.0027
336	336	0.0016	0.00037	−0.00079
263	227	0.0072	0.028	0.013
295	295	0.0031	0.00027	−0.0012
352	352	0.0013	0.00033	−0.00071