The role of spin-3/2 field in chiral nuclear forces

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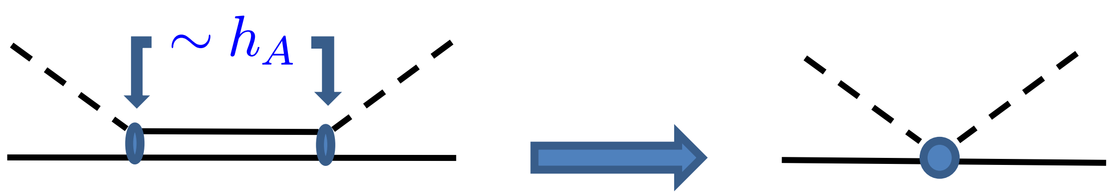
Outline

- Why explicit delta?
- Small scale expansion and explicit decoupling
- Convergence of NN-forces
- Pion-nucleon scattering up to ε^3
- N^3LO three-nucleon force with explicit delta
- Summary & Outlook

Why explicit delta?

- Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293 \text{ MeV}$
- Small scale expansion: $Q \sim M_\pi \sim \Delta \ll \Lambda_\chi$ *(Hemmert, Holstein & Kambor '98)*
- Delta contributions encoded in LECs
  (Bernard, Kaiser & Meißner ´97)

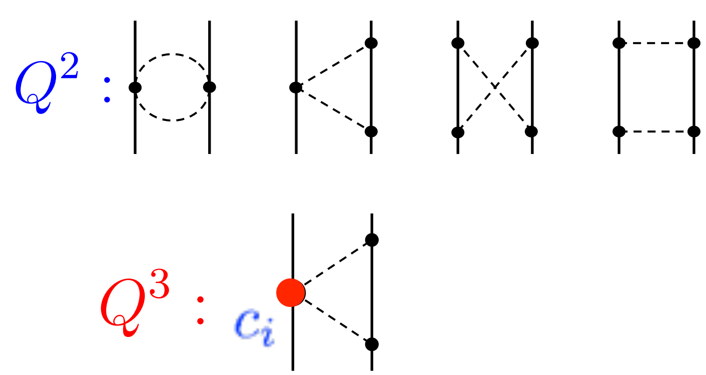


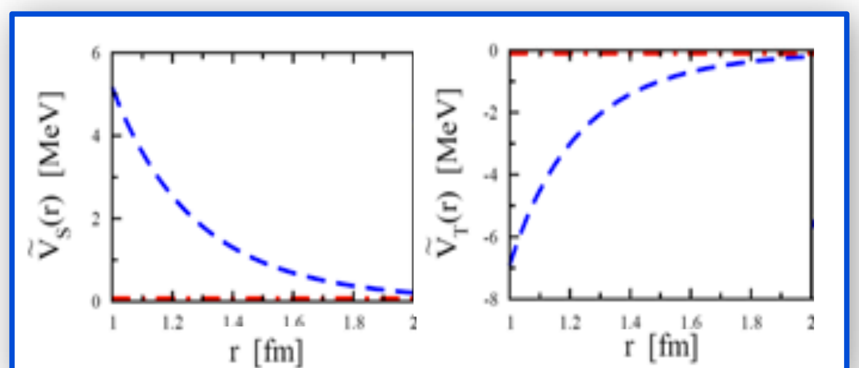
Delta-resonance saturation

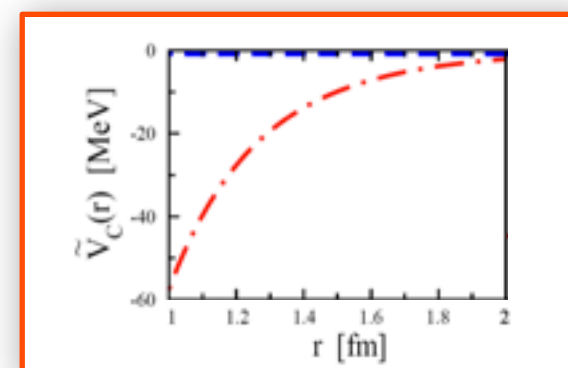
$$c_3 = -2c_4 = c_3(\not{\Delta}) - \boxed{\frac{4h_A^2}{9\Delta}}$$

Enlargement due to Delta contribution

- Convergence of EFT potential







The subleading contributions are larger than the leading one!

Expectation from inclusion of Δ explicitely

- more natural size of LECs
- better convergence
- applicability at higher energies

Explicit decoupling

Do the positive powers of Δ not spoil the convergence?

Small scale expansion parameter $\Delta/\Lambda_\chi \sim \frac{1}{3}$ is not that small!

Manifest decoupling through the choice of renormalization conditions (no positive powers of Δ)

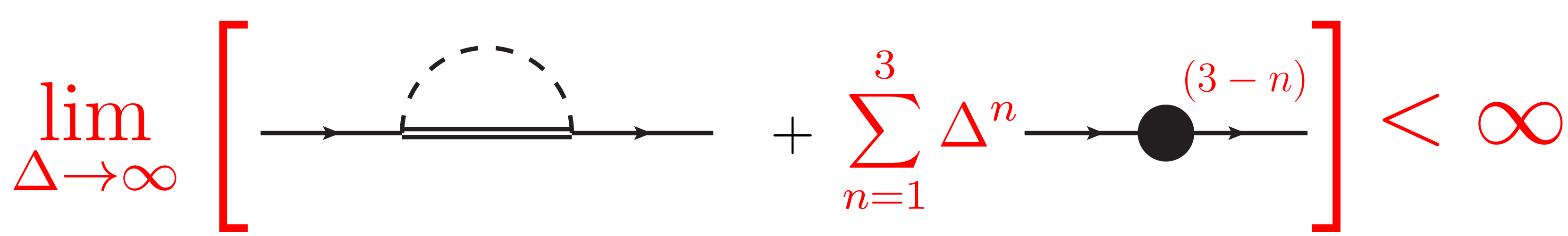
Decoupling theorem due to Appelquist & Carrazone Phys. Rev. 11 (1974) 2856

$$\mathcal{L}^{\rm SSE}_{piN} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \Delta\mathcal{L}^{(1)}_{\pi N} + \Delta\mathcal{L}^{(2)}_{\pi N} + \Delta^2\mathcal{L}^{(1)}_{\pi N} + \mathcal{O}(\epsilon^4)$$

Choose finite part of these LECs such that

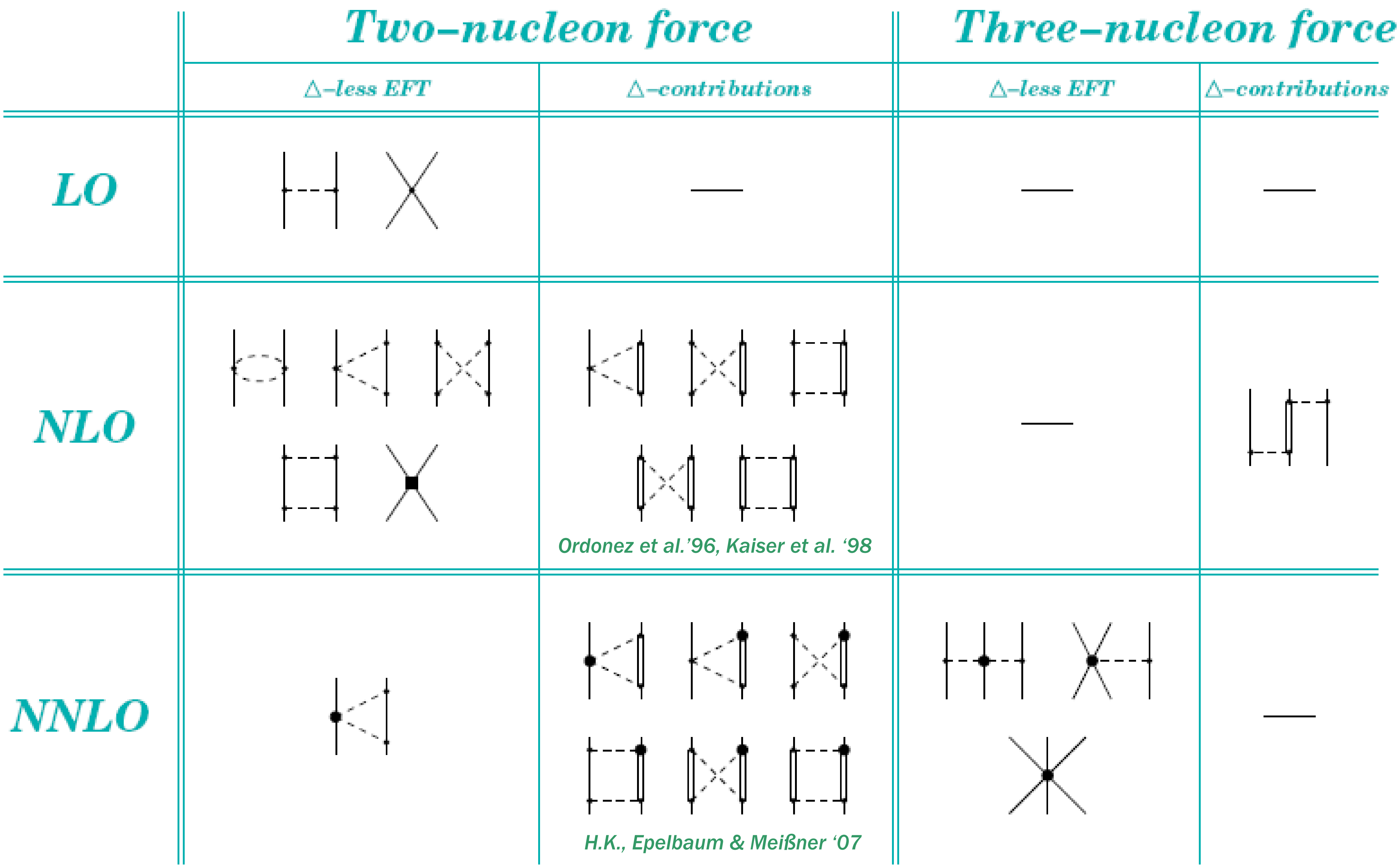
$$\lim_{\Delta\to\infty} \text{Green Function} < \infty$$

Bernard, Fearing, Hemmert, Meißner NPA635 (1998) 121



Few-nucleon forces with the Delta

Isospin-symmetric contributions

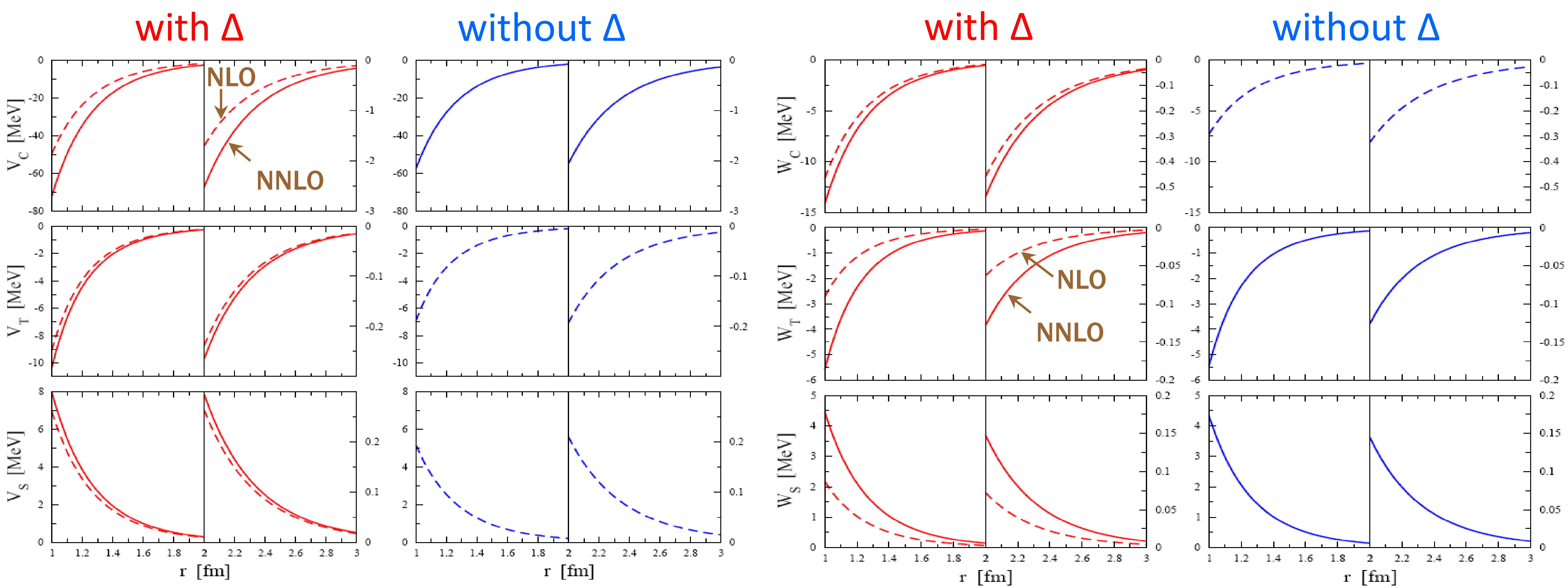


NN potential with explicit Δ

Epelbaum, H.K., Meißner, *Eur. Phys. J.* A32 (2007) 127

$$V_{\rm eff} = V_C + W_C \vec\tau_1\cdot\vec\tau_2 + [V_S + W_S \vec\tau_1\cdot\vec\tau_2]\vec\sigma_1\cdot\vec\sigma_2 + [V_T + W_T \vec\tau_1\cdot\vec\tau_2](3\,\vec\sigma_1\cdot\hat r\vec\sigma_2\cdot\hat r - \vec\sigma_1\cdot\vec\sigma_2)$$

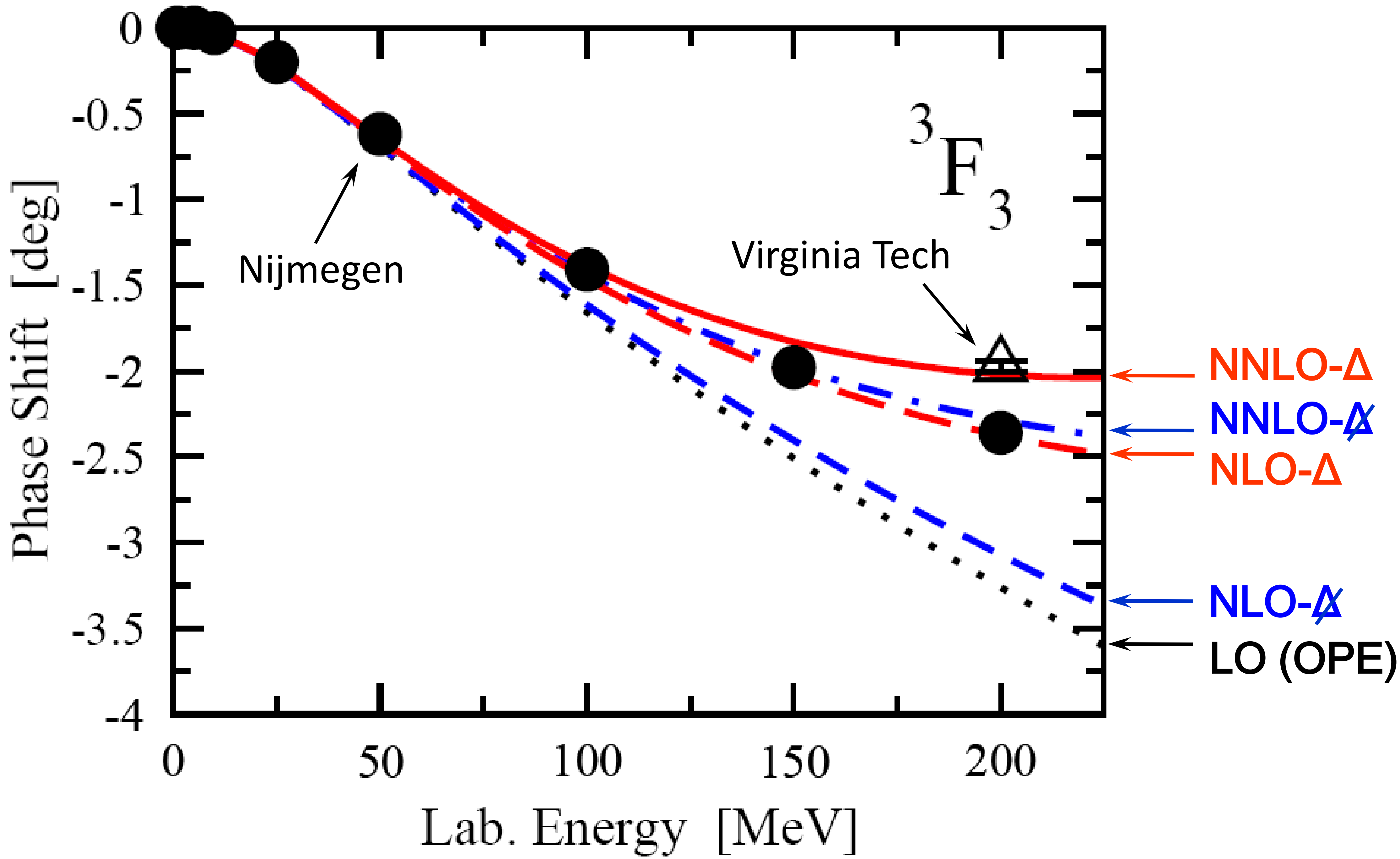
Chiral 2π- exchange potential up to NNLO



Advantages when Δ is included explicitly

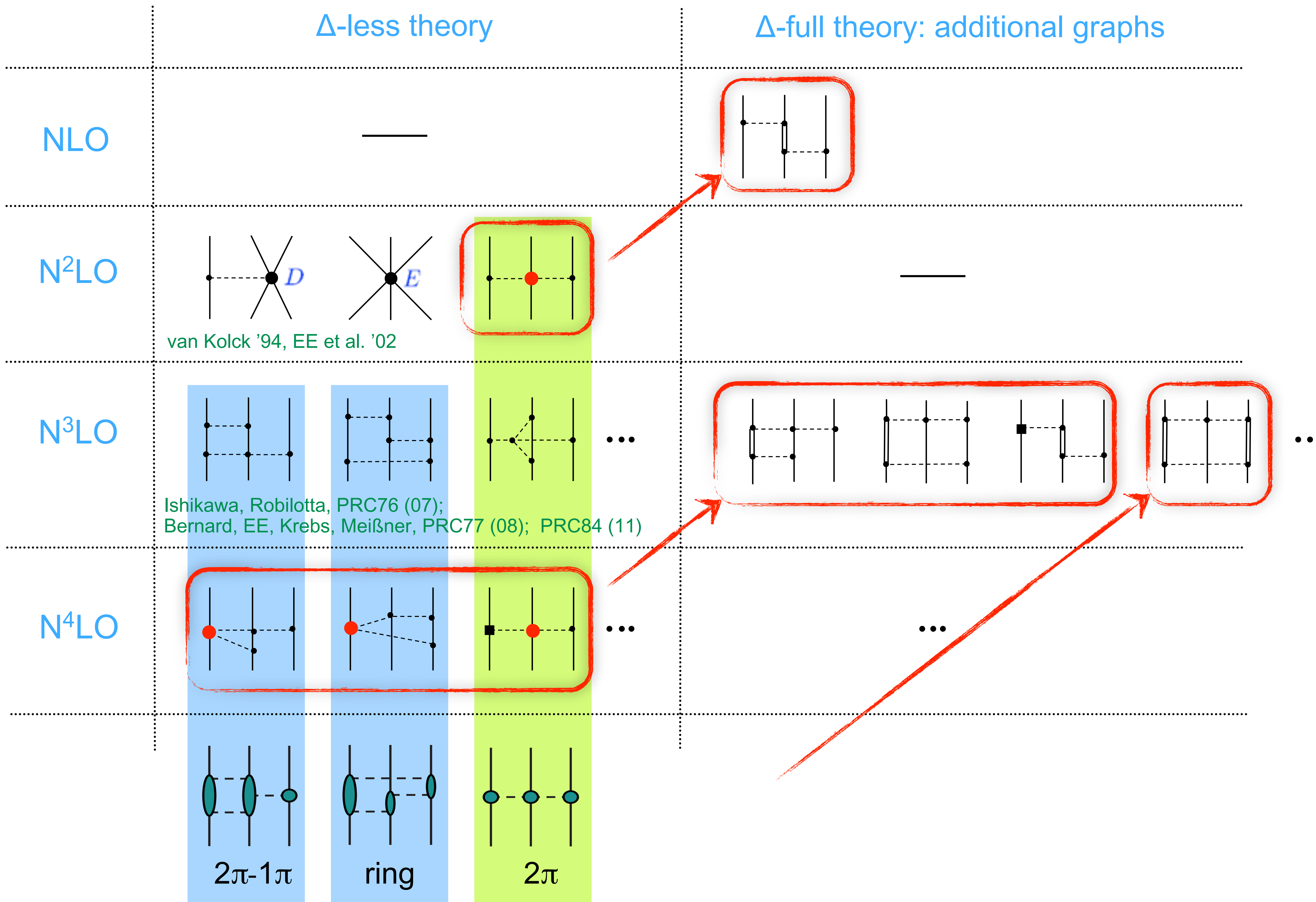
- Dominant contributions already at NLO
- Much better convergence in all potentials

$^3F_3$ partial waves up to NNLO with and without Δ

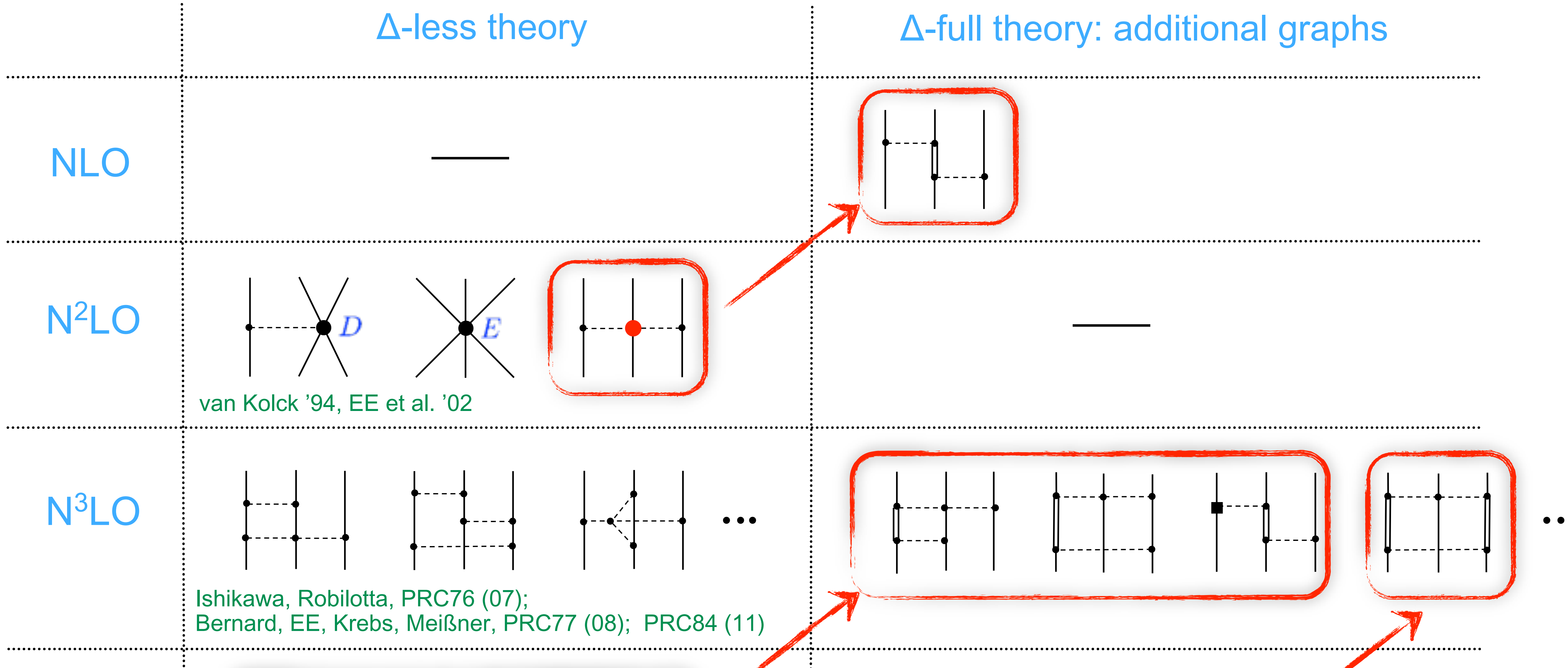


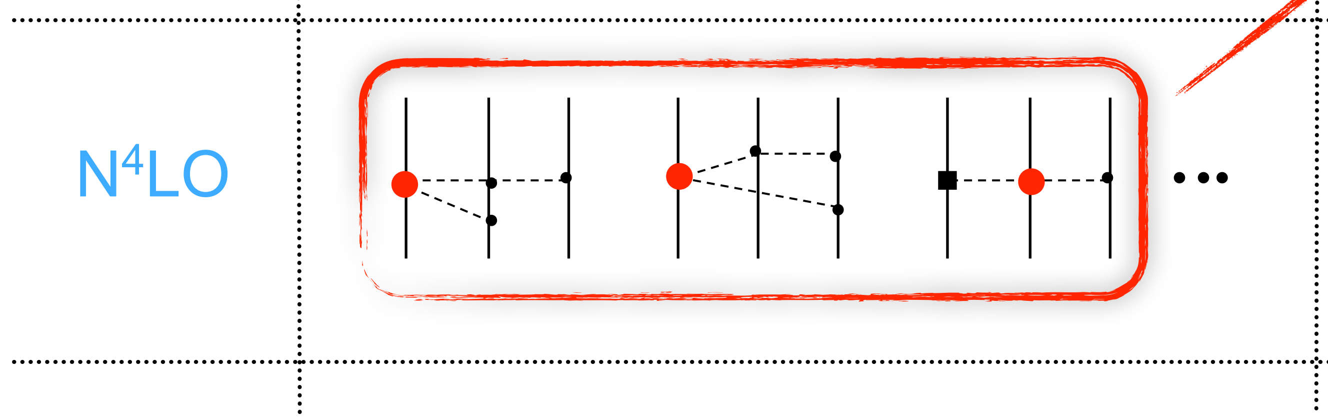
(calculated in the first Born approximation)

Small scale expansion of 3NF



Small scale expansion of 3NF





- no effect upto N²LO (modulo reshuffling)
- expect large contributions to the ring & 2π-1π-topologies saturating some of the $N^{4,5,6}LO$ graphs in the Δ-less theory
- What is more efficient: Δ-less N^4LO (and beyond?) vs Δ-full N^3LO ??

Computational strategy

- d-dim one loop tensor integrals by Passarino-Veltman reduction

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_{\mu_1}\dots l_{\mu_n}}{[l^2-M^2][(l+p)^2-M^2]} = T^{(1)}_{\mu_1\dots\mu_n}(p) f_1(p^2) \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2-M^2][(l+p)^2-M^2]} + T^{(2)}_{\mu_1\dots\mu_n}(p) f_2(p^2) \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2-M^2]}$$

Tensors in p

$f_1(p^2)$ and $f_2(p^2)$ include in general non-physical singularities which cancel in final result

- Dimensional-shift reduction Davydychev '91

Combinatorial factors

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_{\mu_1}\dots l_{\mu_n}}{[l^2-M^2][(l+p)^2-M^2]} = \sum_{ij} T^{(i)}_{\mu_1\dots\mu_n}(p) \int \frac{d^{d+2i} l}{(2\pi)^{d+2i}} \frac{c_{ij}}{[l^2-M^2]^{n_{ij}}[(l+p)^2-M^2]^{m_{ij}}}$$

- Partial integration techniques provide recursion relations

$$\int \frac{d^d l}{(2\pi)^d} \frac{\partial}{\partial l_\mu} f(l) = 0$$ ⟸ Connection betw. Dimensional-shift and Passarino-Veltman red.

Implement Heavy-Baryon extension of these techniques in Mathematica/FORM

Pion-nucleon scattering

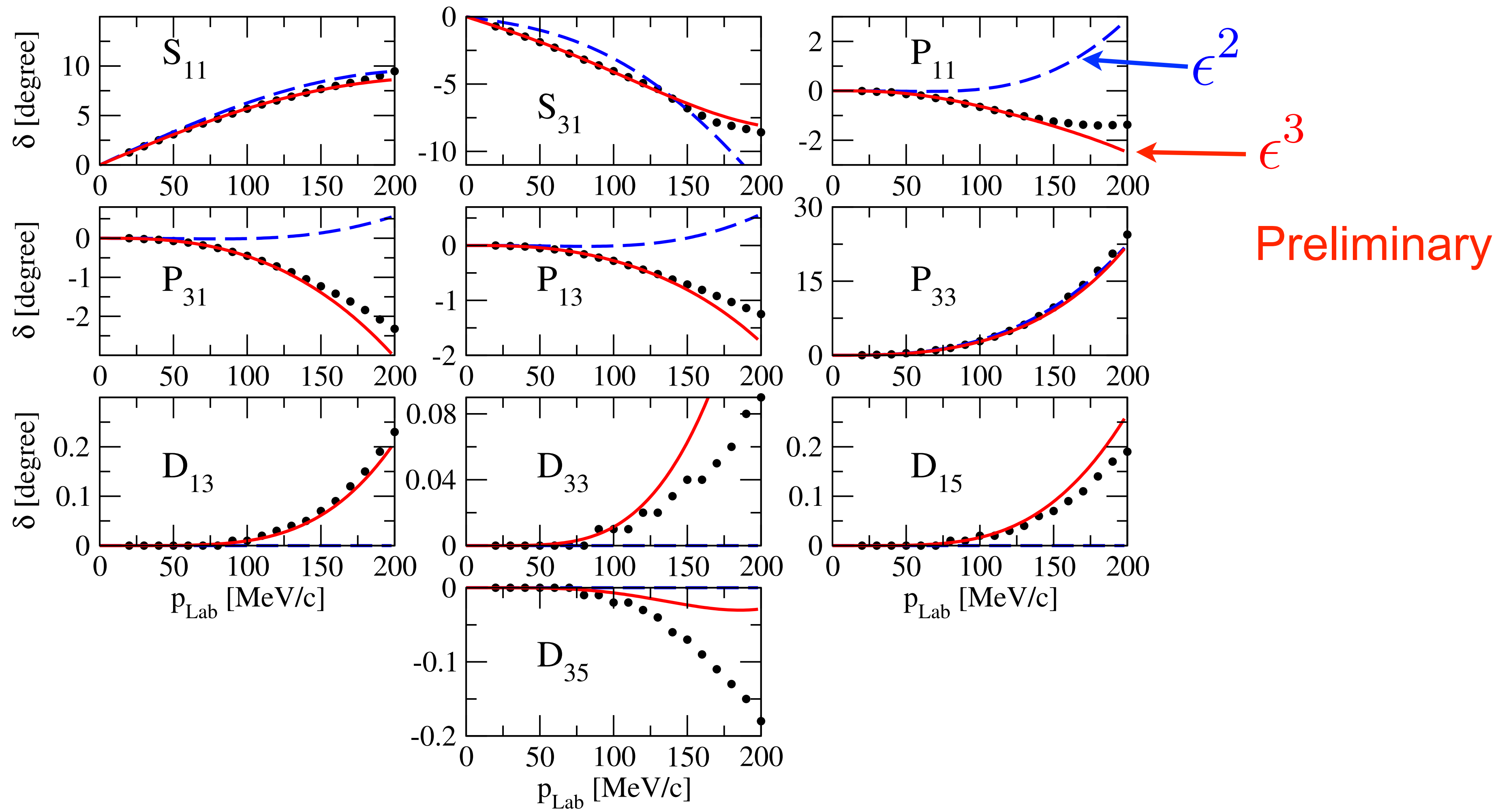
Heavy baryon SSE calculation up to ε^3 : *Fettes & Meißner NPA679 (2001) 629*

Recalculation needed due to different power-counting in 1/m

After renormalization of πNΔ-constant h_A and appropriate shift of c_i´s and d_i´s we do not find any dependence on new LECs from $\mathcal{L}^{(2)}_{\pi N\Delta}$ & $\mathcal{L}^{(3)}_{\pi N\Delta}$

Additional LECs at ε^3 : πNΔ-constant h_A & πΔΔ-constant g_1

Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707

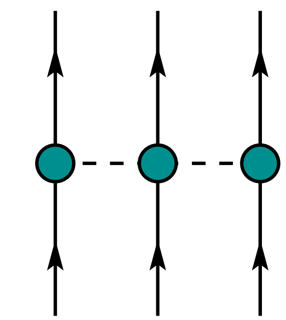


Two-pion-exchange 3NF

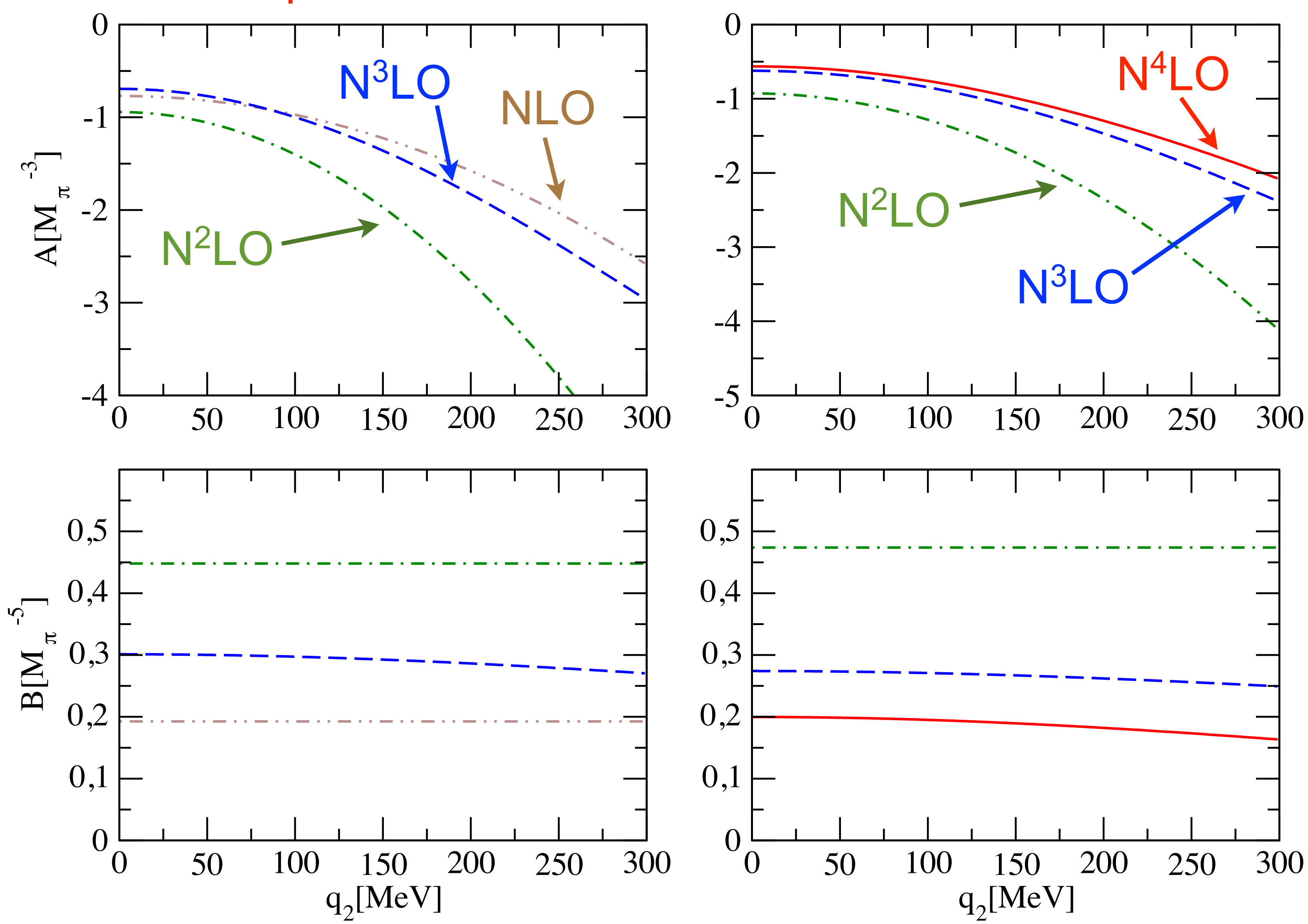
Preliminary

Epelbaum, Gasparyan, HK. forthcoming

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$



Explicit-Δ calc. | Δ-less calc.



- NLO with explicit Δ appears to be close to Δ-less N^{4}LO
- Similar results in both cases for N^{2}LO and N^{3}LO

Difference btw. N^{2}LO and N^{3}LO is given by loops

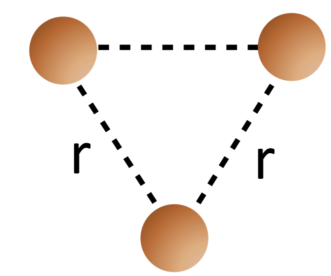
⇓

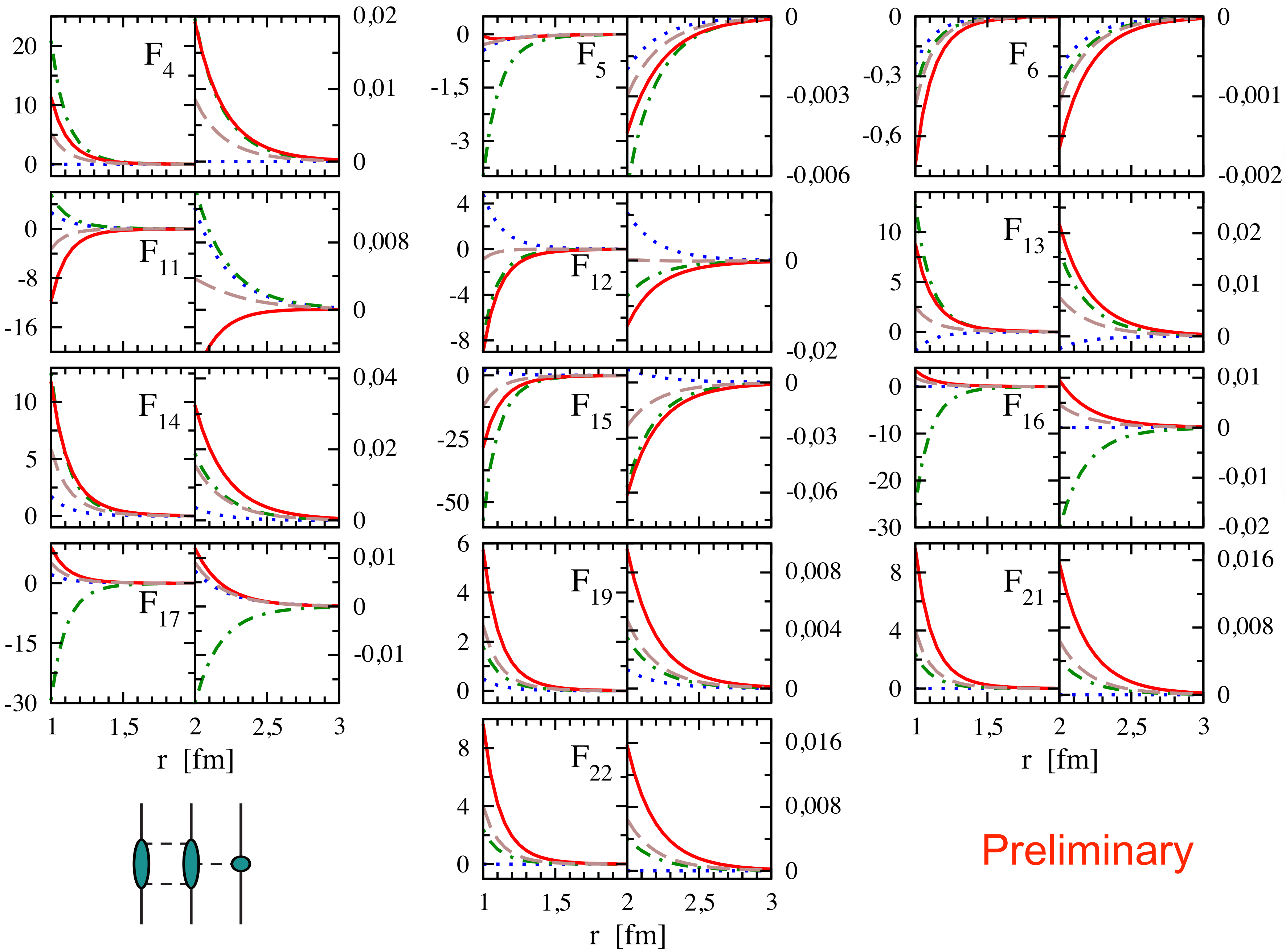
Loop contr. of Δ-dofs to two-pion-exchange 3NF are small

- Two-pion-exchange 3NF is already converged
- We expect small explicit-Δ N^{4}LO contributions to two-pion-exchange 3NF

Two-pion-one-pion-exchange 3NF

Coordinate space to discuss the long-range part at equilateral-triangle conf.





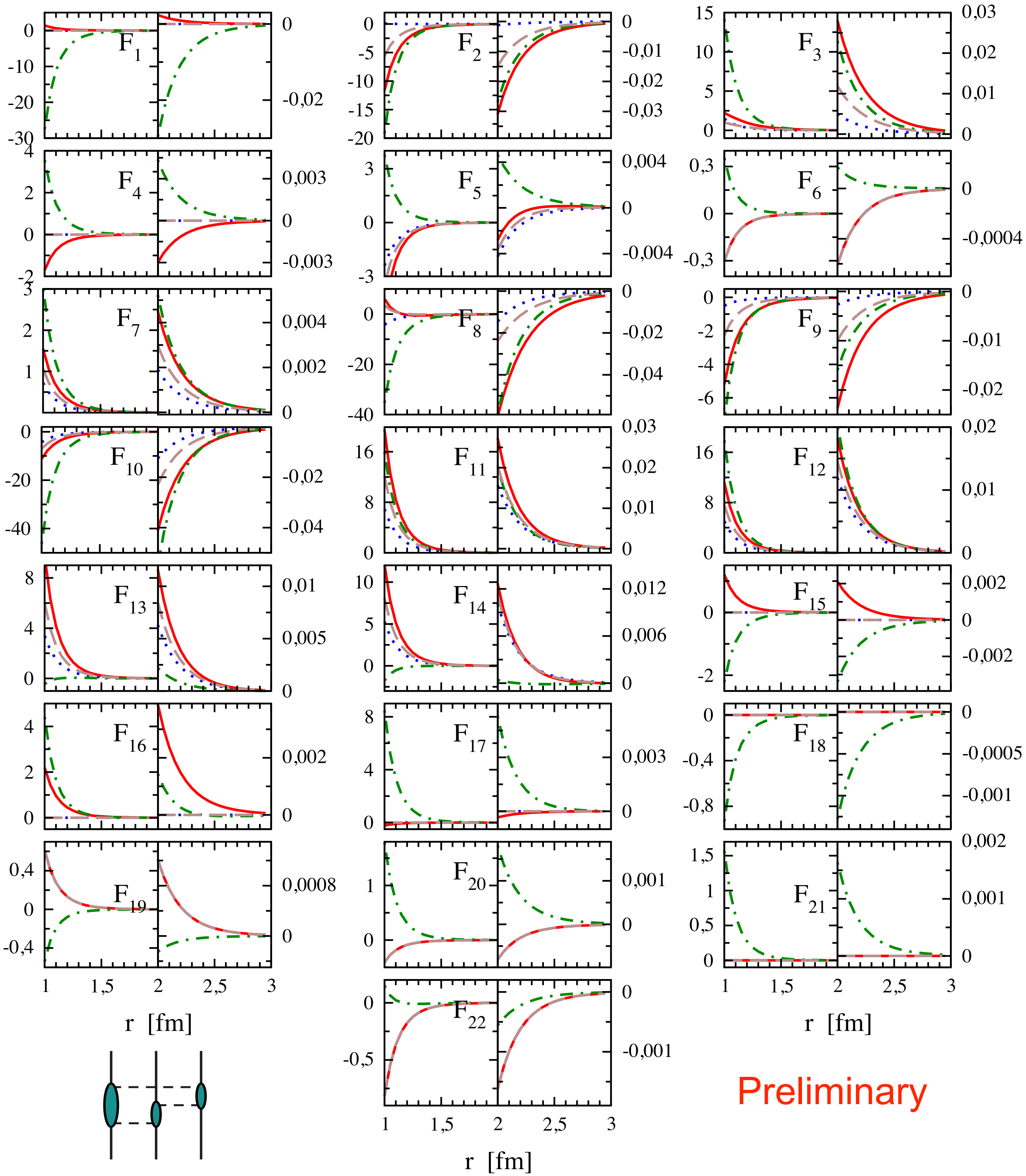
Preliminary

- ⋯⋯ N^3LO Δ-less
- – · – N^3LO Δ-full
- —— N^4LO Δ-less
- - - - - N4LO Δ-less with c_i´s from Δ-res. saturation

- Difference btw. res. sat. and N^4LO indicate contr. of c_i´s coming not from Δ-dofs
- Difference btw. res. sat. and explicit-Δ N^3LO indicates contr. ~$O(1/\Delta^2)$

- Structures with larger values like F_4 & F_{15} look similar for explicit-Δ N^3LO and Δ-less N^4LO
- There are sizable structures like F_{16} & F_{17} which in Δ-less N^4LO miss important Δ-cont.
- No statement about convergence of smaller structures like F_{11} can be made at this order

Ring-contributions to 3NF



- ······ N^3LO nucl-contr.
- – – N^3LO nucl- and Δ-contr.
- —— N^4LO full Δ-less result
- - - - - Resonance saturation

- Strong attractive central force coming from ~$O(1/\Delta^2)$ contr.
- Similar results btw. Δ-less N^4LO and explicit-Δ N^3LO results for structures with larger value like F_2, F_8, F_{10}, F_{11} & F_{12}
- No statement about convergence possible for smaller structures like F_4, F_5, F_6, F_{14} - F_{22} at this order
- Explicit-Δ N^4LO would be helpful to draw final conclusions about convergence of two-pion-exchange and ring contr. to 3NF

Summary

- Long-range part of 3NFs is analyzed up to N^3LO with explicit Δ-dof
- Small loop-contr. with Δ-dofs to two-pion-exchange part of 3NF
- Two-pion-exchange part seems to be converged
- Most of sizable F_i structures are similar in explicit-Δ N^3LO and Δ-less N^4LO calc.
- Some missing sizable Δ-contr. in N^4LO results like central attractive force ~$O(1/\Delta^2)$

Outlook

- Partial wave decomposition of N^3LO three-nucleon forces
- Explicit-Δ N^3LO calc. of shorter range part of 3NF
- N^4LO with explicit-Δ of long range part of 3NF

LECs in two-pion-exchange 3NF

Δ-less N^4LO

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
GW-fit	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.80	1.76	-0.58	0.96
KH-fit	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

explicit-Δ N^3LO

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$
GW-fit	-1.70	1.19	-3.52	1.85	0.10	-1.26	0.71	-1.17
KH-fit	-1.41	1.40	-3.43	1.80	0.45	-2.36	1.43	-2.18

LECs c_i & d_i become smaller once Δ-dofs are taken explicitly