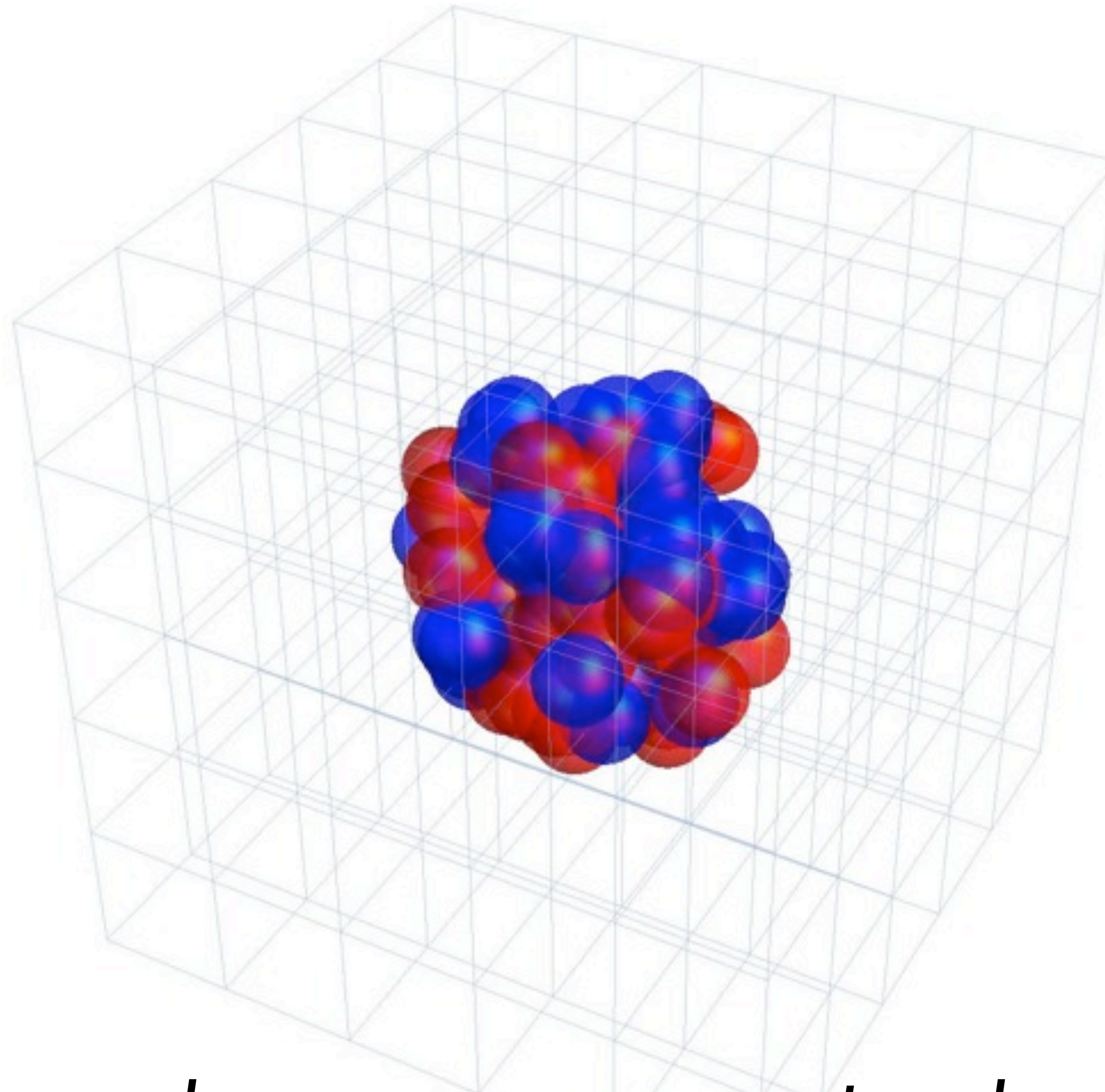
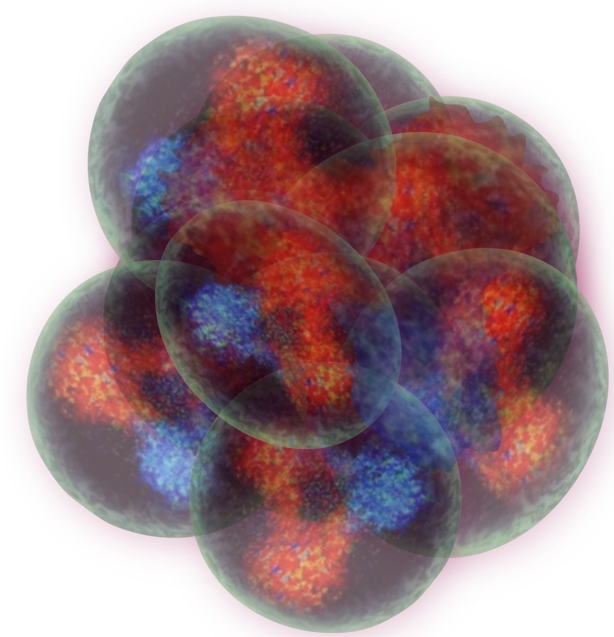


Few body systems in lattice QCD

William Detmold

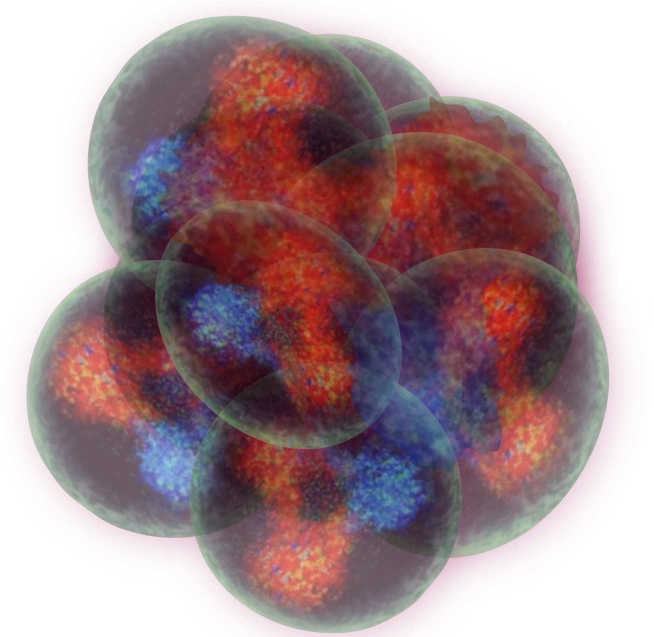
WILLIAM & MARY   Jefferson Lab   Massachusetts Institute of Technology

From quarks to nuclei



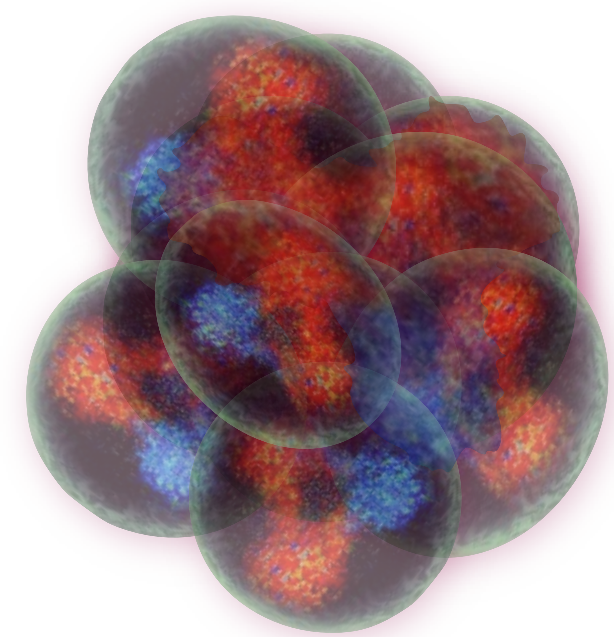
From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model



From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
- *How do nuclei emerge from QCD?*



From quarks to nuclei

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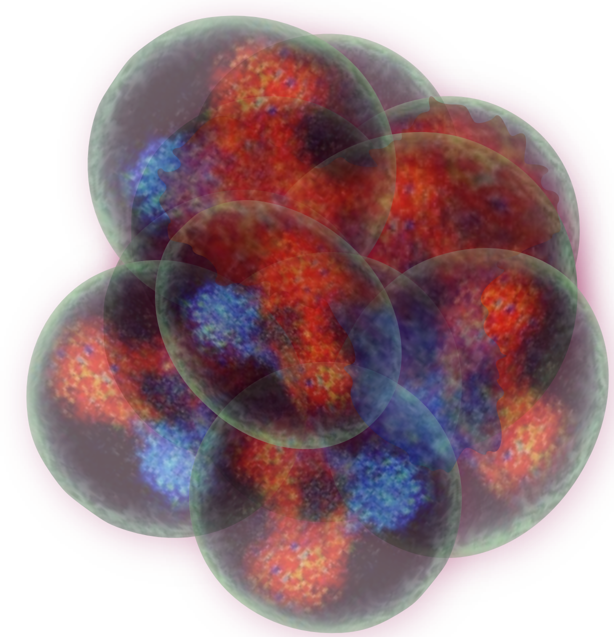

From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
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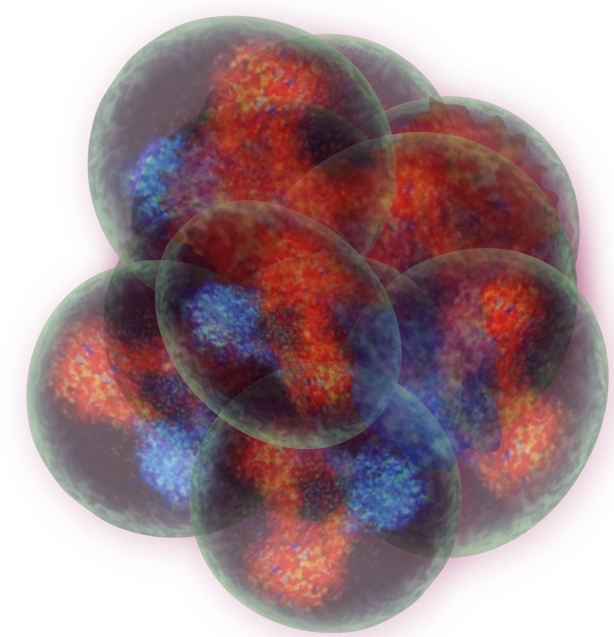

From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
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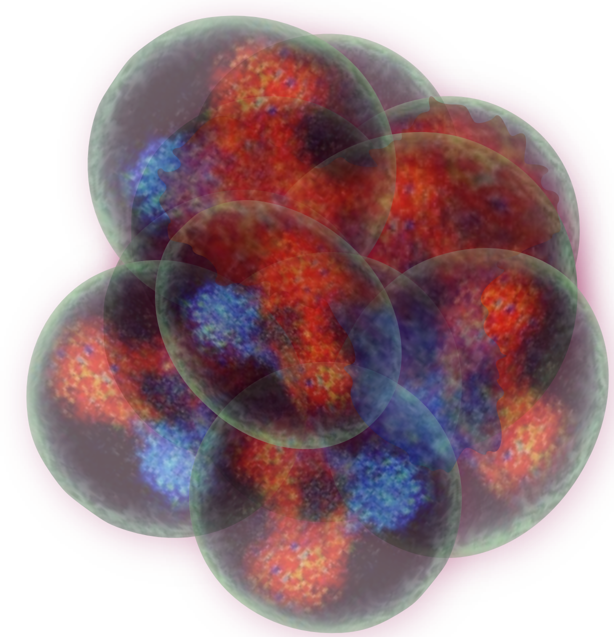
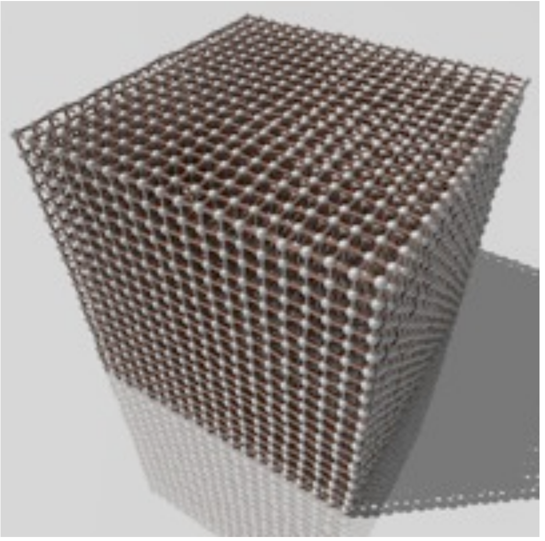
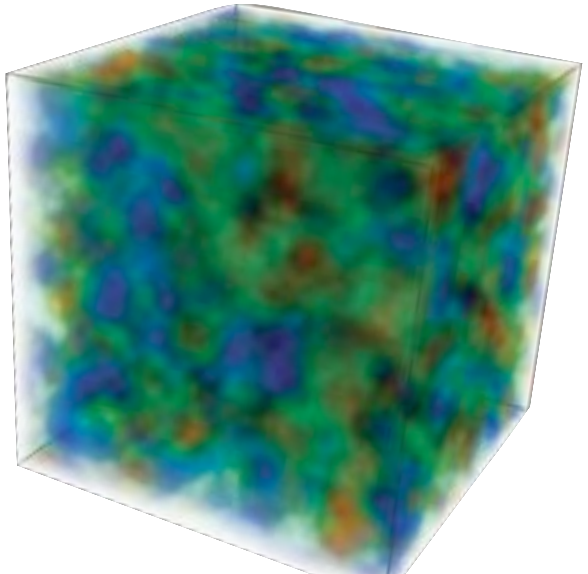
  - Issues
  - Recent progress
  - Future directions



Quantum chromodynamics

- Lattice QCD: quarks and gluons
  1. Formulate problem as functional integral over gluonic degrees of freedom on R^4
  2. Discretise and compactify system
  3. Integrate via importance sampling (average over important gluon cfgs)
  4. *Undo the harm done in previous steps*
- Major computational challenge ...

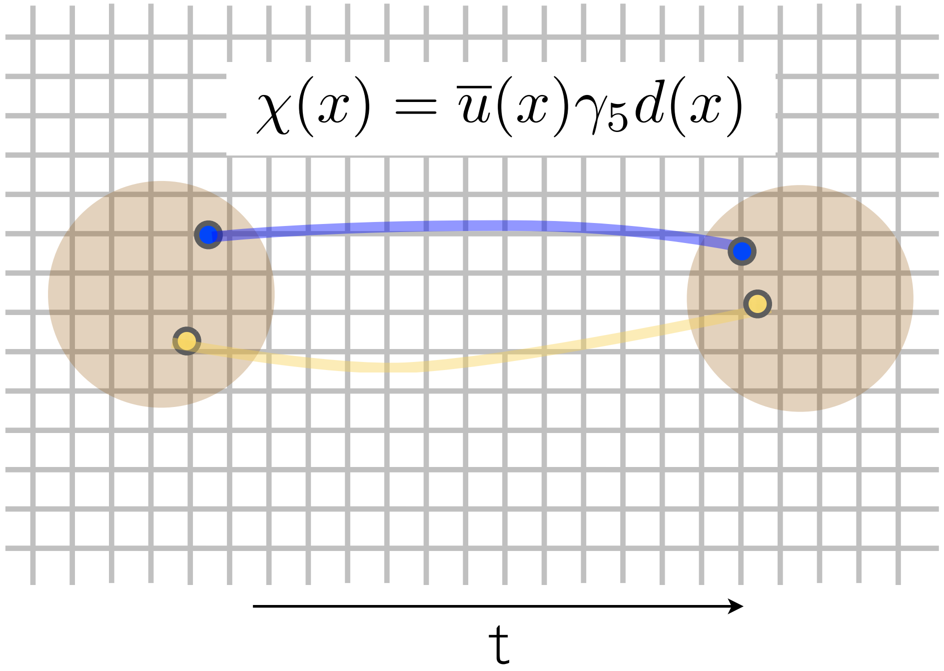




QCD Spectroscopy

- Measure correlator (χ = object with q# of hadron)

$$C_2(t) = \sum_{\mathbf{x}} \langle 0 | \chi(\mathbf{x}, t) \overline{\chi}(\mathbf{0}, 0) | 0 \rangle$$



- Unitarity: $\sum_n |n\rangle\langle n| = 1$

$$= \sum_{\mathbf{x}} \sum_n \langle 0 | \chi(\mathbf{x}, t) | n \rangle \langle n | \overline{\chi}(\mathbf{0}, 0) | 0 \rangle$$

- Hamiltonian evolution

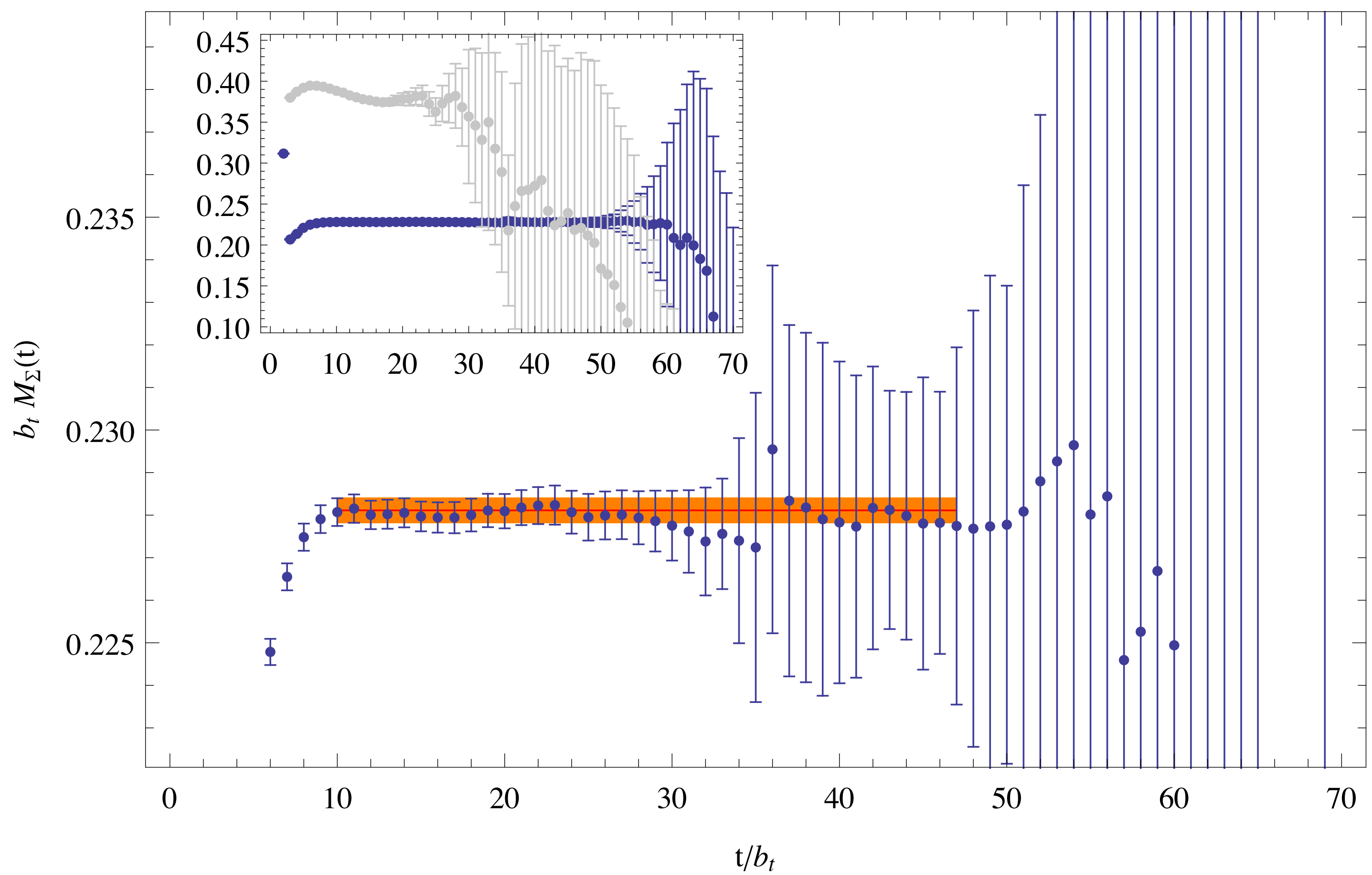
$$= \sum_{\mathbf{x}} \sum_n e^{-E_n t} e^{i\mathbf{p}_n \cdot \mathbf{x}} \langle 0 | \chi(\mathbf{0}, 0) | n \rangle \langle n | \overline{\chi}(\mathbf{0}, 0) | 0 \rangle$$

- Long times only ground state survives

$$\xrightarrow{t \to \infty} e^{-E_0(\mathbf{0})t} |\langle \mathbf{0}; 0 | \overline{\chi}(\mathbf{x_0}, t) | 0 \rangle|^2 = Z\, e^{-E_0(\mathbf{0})t}$$

Effective mass

- Construct $M(t) = \ln\left[C_2(t)/C_2(t+1)\right] \stackrel{t\to\infty}{\longrightarrow} M$
  - Plateau corresponds to energy of ground state



- Fancier techniques able to resolve multiple eigenstates

Nuclear physics from LQCD

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- Can we compute the mass of ^{208}Pb in QCD?

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- Yes

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$$\langle 0 | T q_1(t) \ldots q_{624}(t) \overline{q}_1(0) \ldots \overline{q}_{624}(0) | 0 \rangle$$

Nuclear physics from LQCD

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$$\langle 0|T q_1(t) \ldots q_{624}(t)\overline{q}_1(0) \ldots \overline{q}_{624}(0)|0\rangle$$

- Long time behaviour gives ground state energy up to EW effects

$$\stackrel{t\to\infty}{\longrightarrow} \# \exp(-M_{Pb} t)$$

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- But...

An (exponentially hard)^2 problem?

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- Complexity: number of Wick contractions = $(A+Z)!(2A-Z)!$

$$a_i^\dagger(t_1)a_j^\dagger(t_1)a_j(t_1)a_i(t_1)a_i^\dagger(t_2)a_j^\dagger(t_2)a_j(t_2)a_i(t_2)$$

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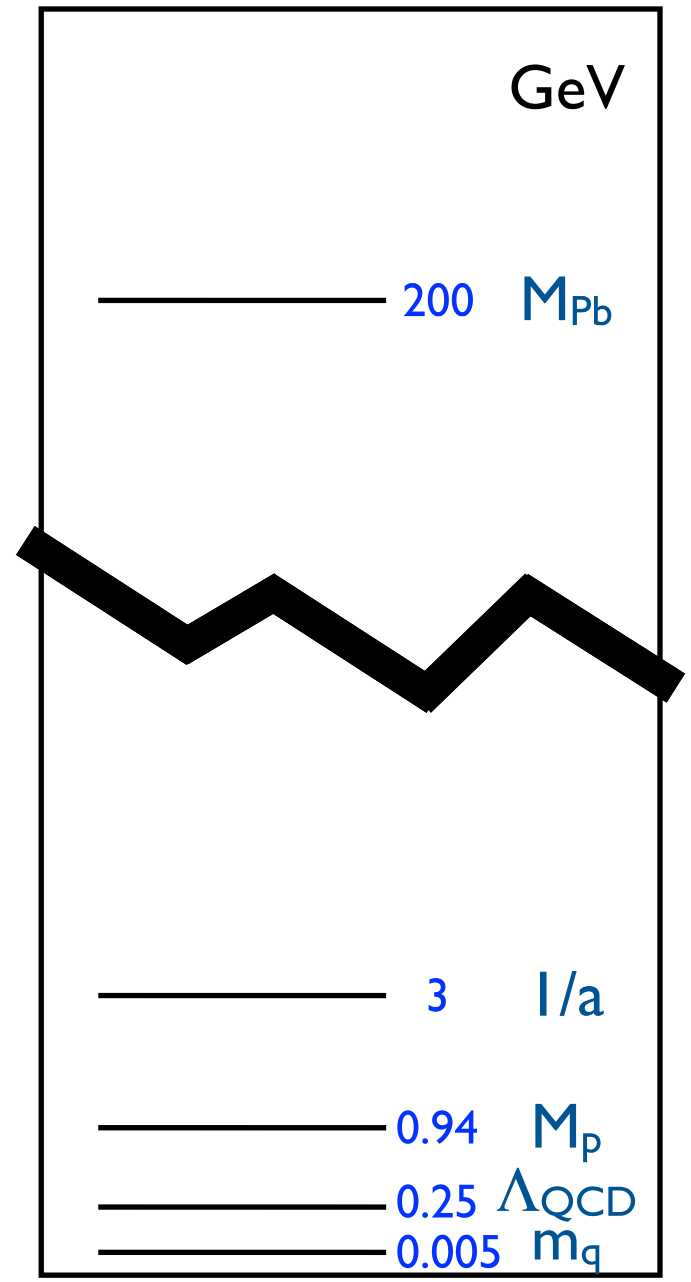
- Dynamical range of scales (numerical precision)



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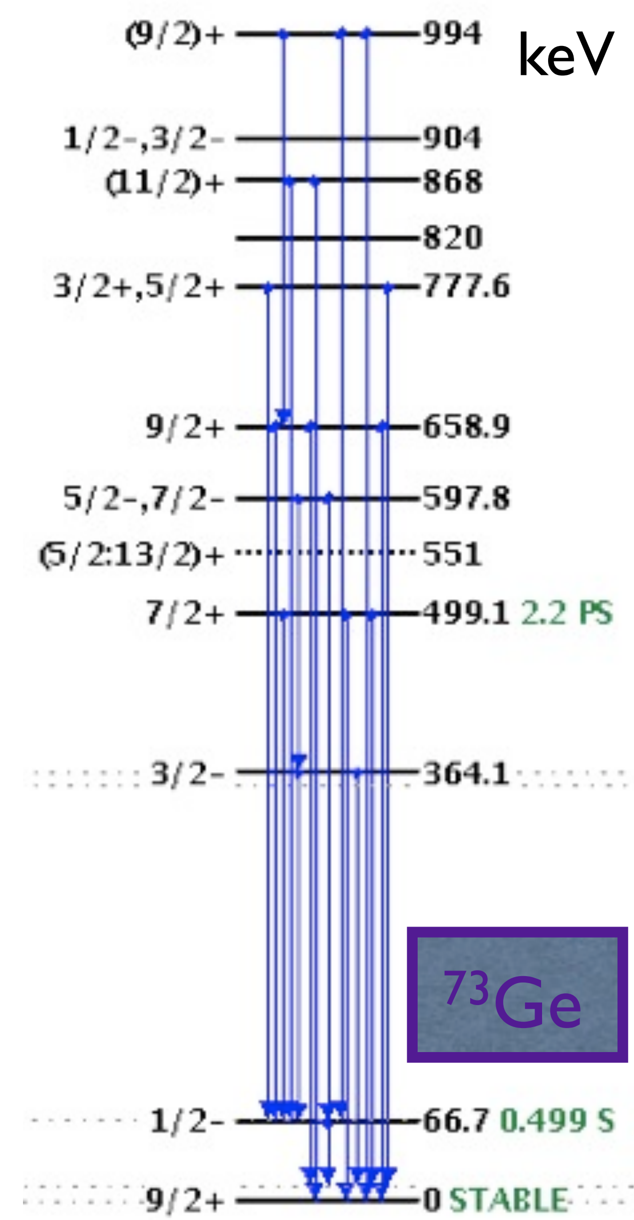
- Dynamical range of scales (numerical precision)

- Small energy splittings



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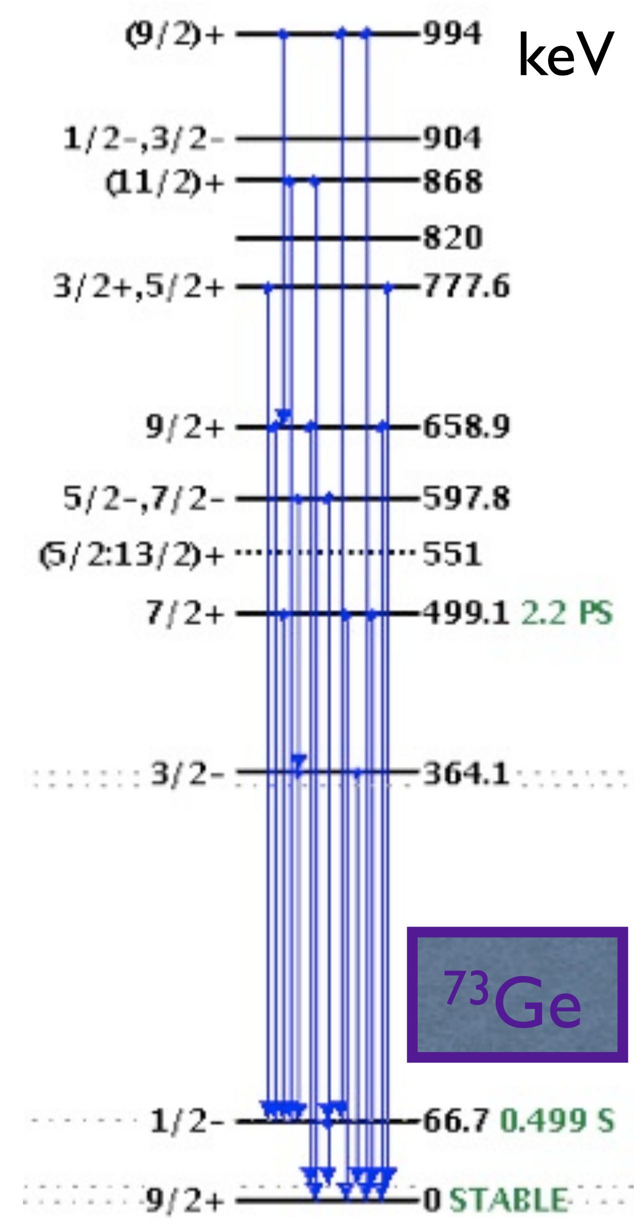
- Dynamical range of scales (numerical precision)
- Small energy splittings
- Importance sampling: statistical noise exponentially increases with A

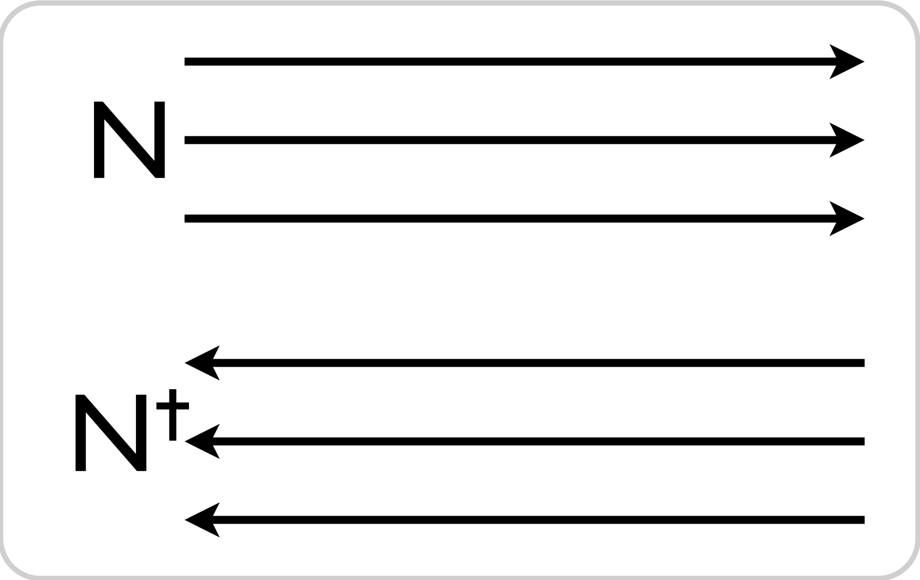


The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

- Variance in single nucleon correlator (C) determined by

$$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$



- For nucleon:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-(M_N - 3/2 m_\pi)t\right]$$

- For nucleus A:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2 m_\pi)t\right]$$

[Lepage '89]

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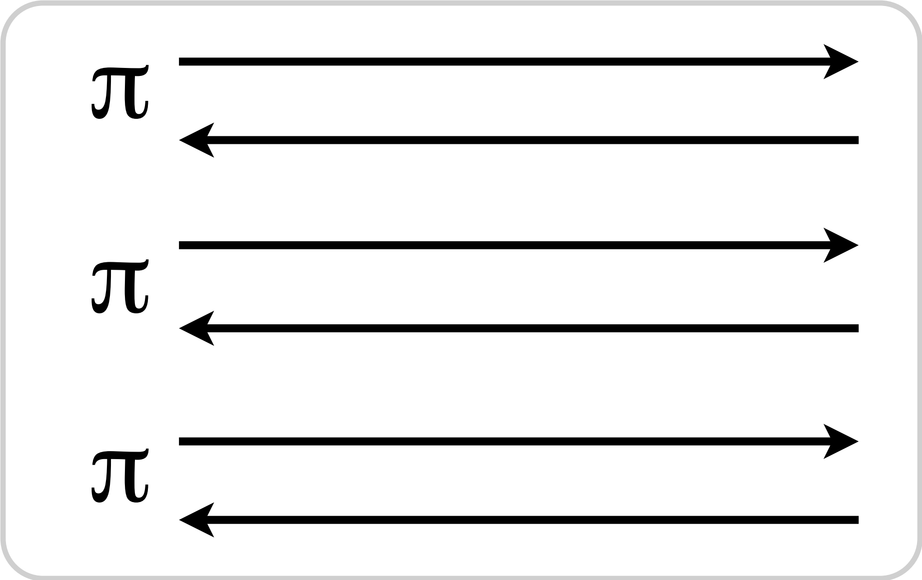

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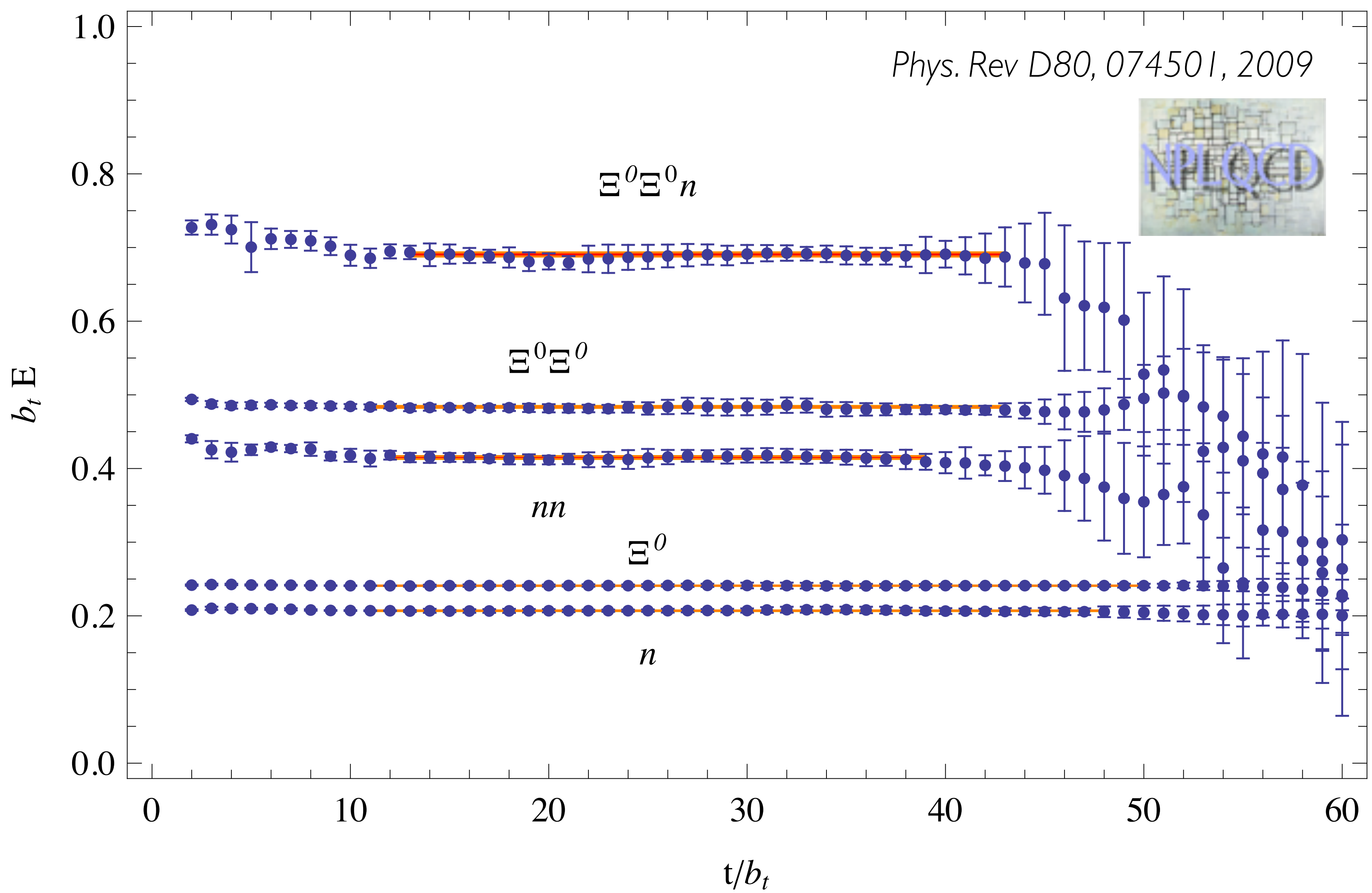
The trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)



@ m_π = 390 MeV

The trouble with baryons

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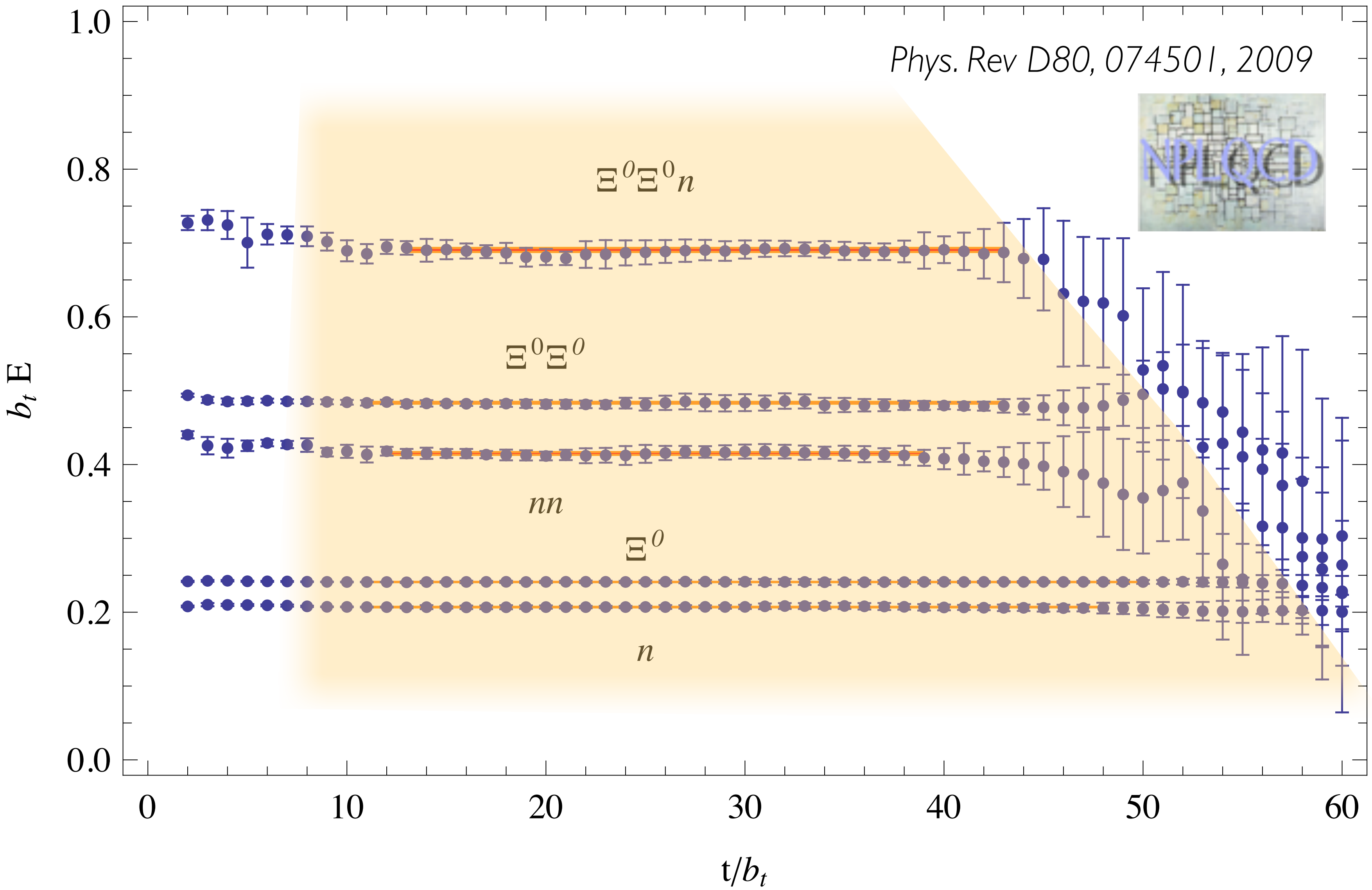

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Golden window of time-slices where signal/noise const

No? trouble with baryons

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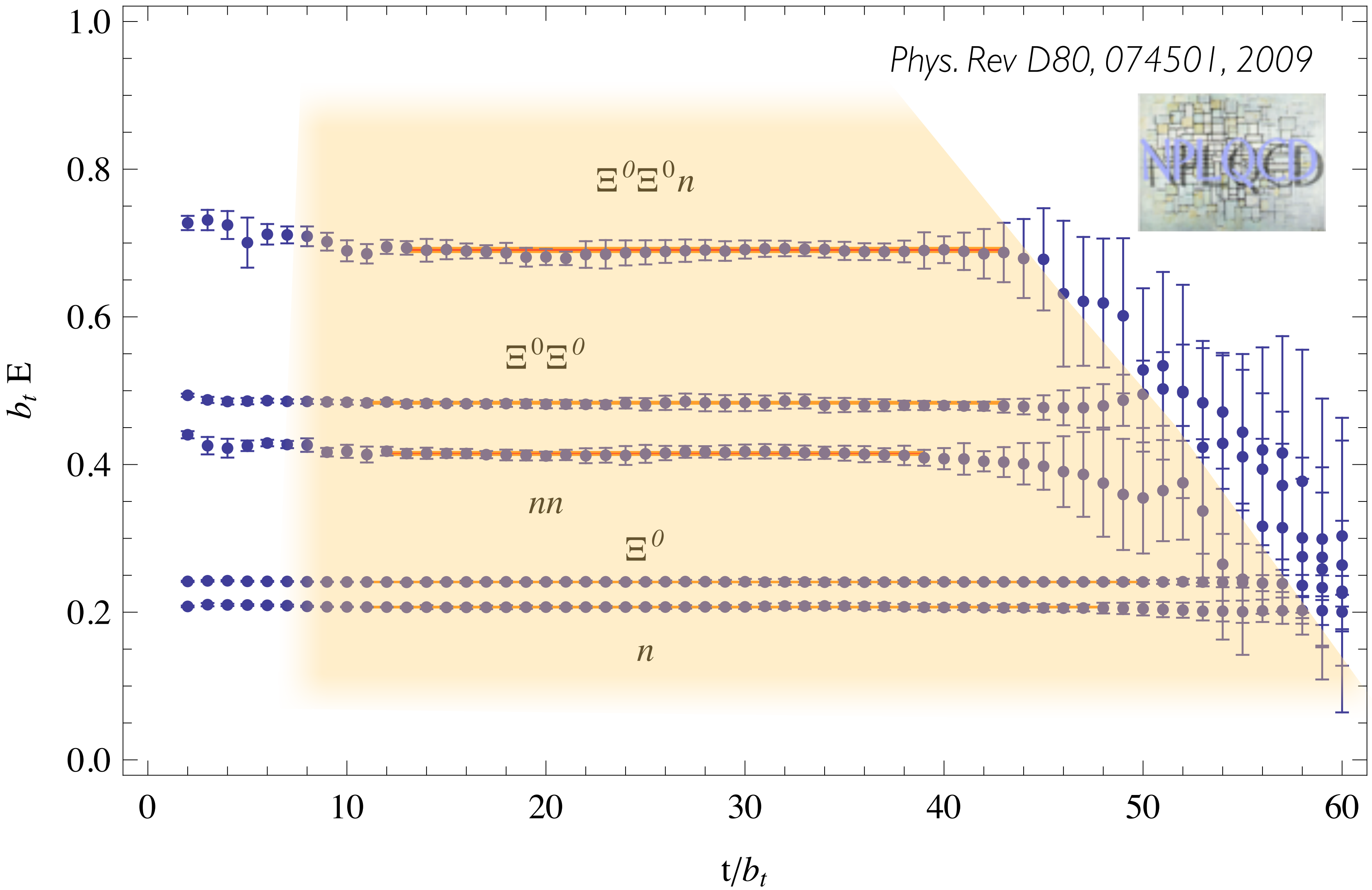

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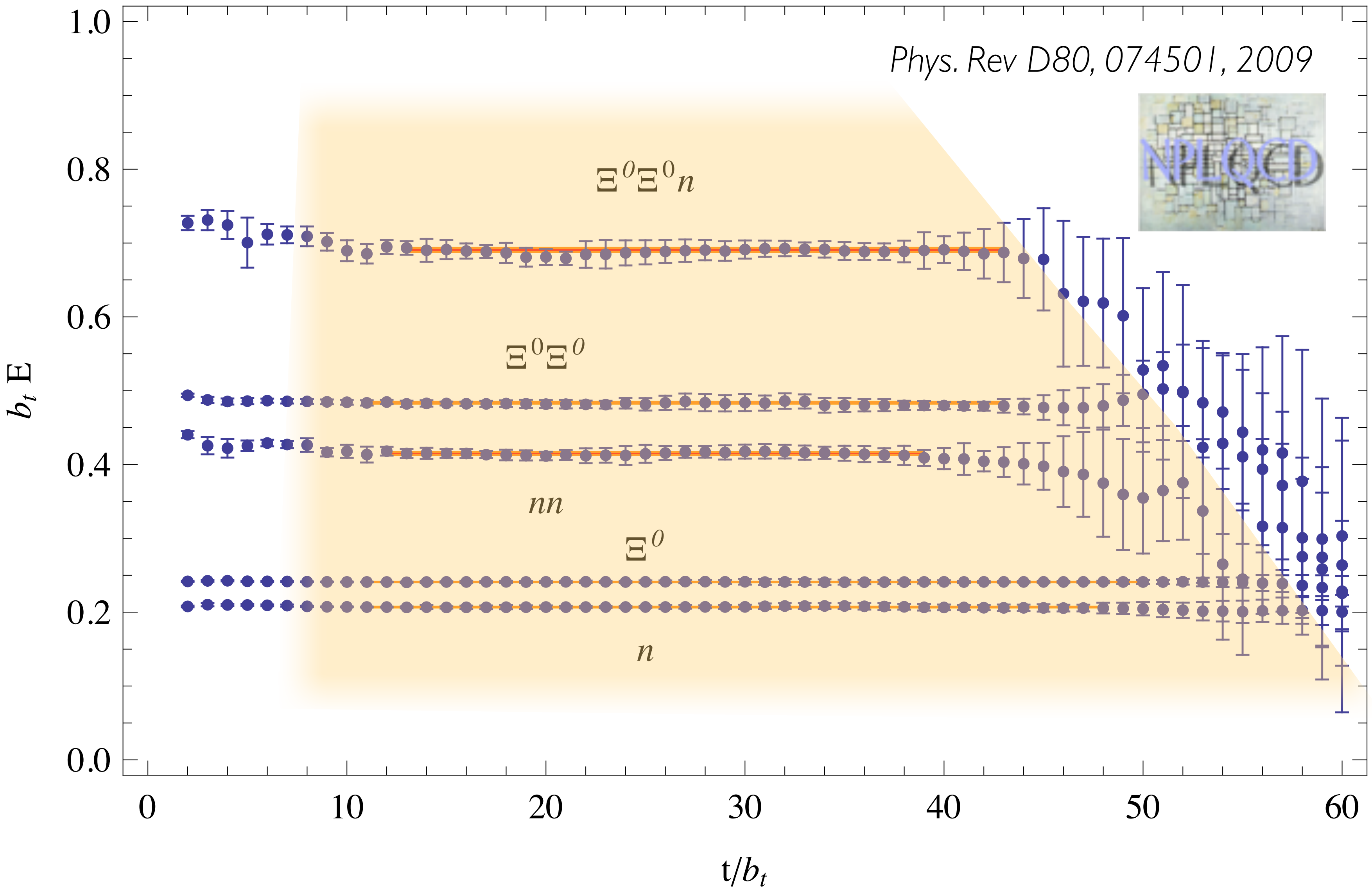

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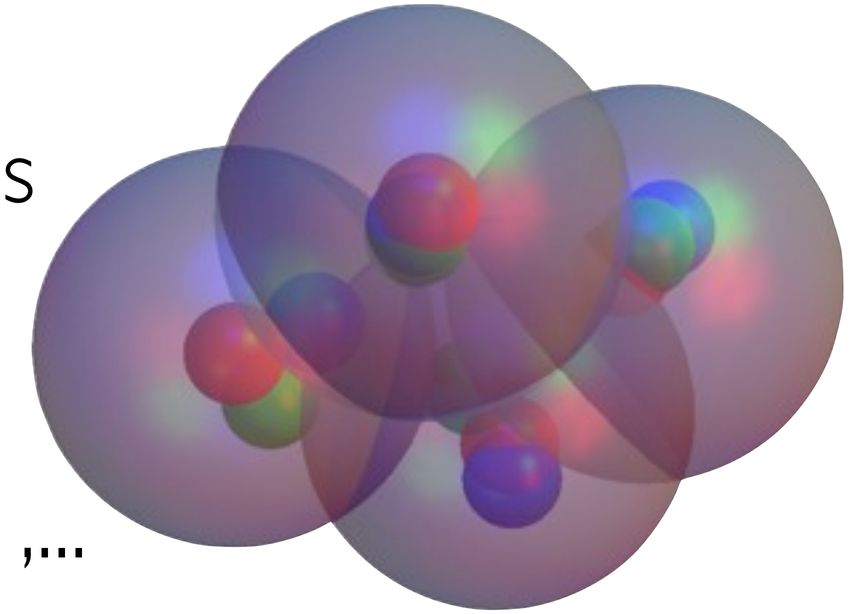
Golden window of time-slices where signal/noise const

Interpolator choice can be optimised to suppress noise

Multi-baryon systems

- Scattering and bound states
  - NB: Strong interaction bound states
- Dibaryons : H, deuteron, $\Xi\Xi$
- ^{3}H, ^{4}He and more exotic: $^{4}He_{\Lambda}$, $^{4}He_{\Lambda\Lambda}$,...
- Correlators for significantly larger A
- Caveat: at unphysical quark masses no electroweak interactions



Bound states at finite volume

- Two particle scattering amplitude in infinite volume

$$\mathcal{A}(p) = \frac{8\pi}{M}\frac{1}{p\cot\delta(p) - ip}$$

scattering phase shift

bound state at $p^2 = -\gamma^2$ when $\cot\delta(i\gamma) = i$

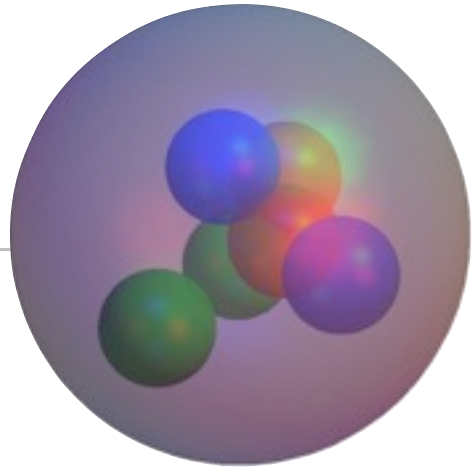
- Scattering amplitude in finite volume (Lüscher method)

$$\cot\delta(i\kappa) = i - i\sum_{\vec{m}\neq 0}\frac{e^{-|\vec{m}|\kappa L}}{|\vec{m}|\kappa L} \qquad \kappa \xrightarrow{L\to\infty} \gamma$$

  - Need multiple volumes

- More complicated for n>2 body bound states

H-dibaryon



- Jaffe [1977]: chromo-magnetic interaction

$$\langle H_m \rangle \sim \frac{1}{4}N(N-10) + \frac{1}{3}S(S+1) + \frac{1}{2}C_c^2 + C_f^2$$

most attractive for spin, colour, flavour singlet

  - H-dibaryon (uuddss) J=I=0, s=-2 most stable

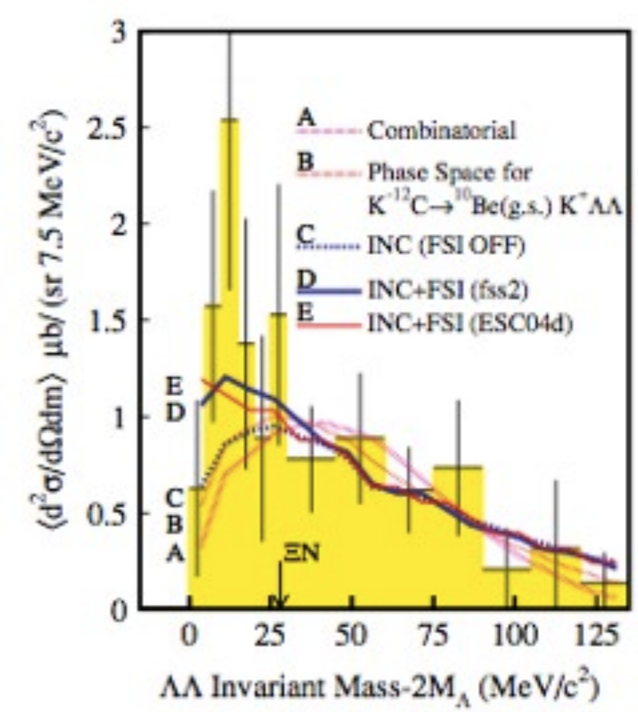
$$\Psi_H = \frac{1}{\sqrt{8}}\left(\Lambda\Lambda + \sqrt{3}\Sigma\Sigma + 2\Xi N\right)$$

  - Bound in a many hadronic models

- Experimental searches
  - Emulsion expts, heavy-ion, stopped kaons
  - No conclusive evidence for or against

KEK-ps (2007)

$K^- \, {}^{12}C \rightarrow K^+ \Lambda\Lambda \, X$



H dibaryon in QCD

- Early quenched studies on small lattices: mixed results
  [Mackenzie et al. 85, Iwasaki et al. 89, Pochinsky et al. 99, Wetzorke & Karsch 03, Luo et al. 07, Loan 11]

- Semi-realistic calculations

  - *"Evidence for a bound H dibaryon from lattice QCD"*
    PRL 106, 162001 (2011)
    $N_f=2+1$, $a_s=0.12$ fm, $m_\pi=390$ MeV, L=2.0, 2.5, 3.0, 3.9 fm

  - *"Bound H dibaryon in flavor SU(3) limit of lattice QCD"* *
    PRL 106, 162002 (2011)
    $N_f=3$, $a_s=0.12$ fm, $m_\pi=670, 830, 1015$ MeV, L=2.0, 3.0, 3.9 fm

- NB: Quark masses unphysical, single lattice spacing

* use a somewhat different method

H dibaryon in QCD

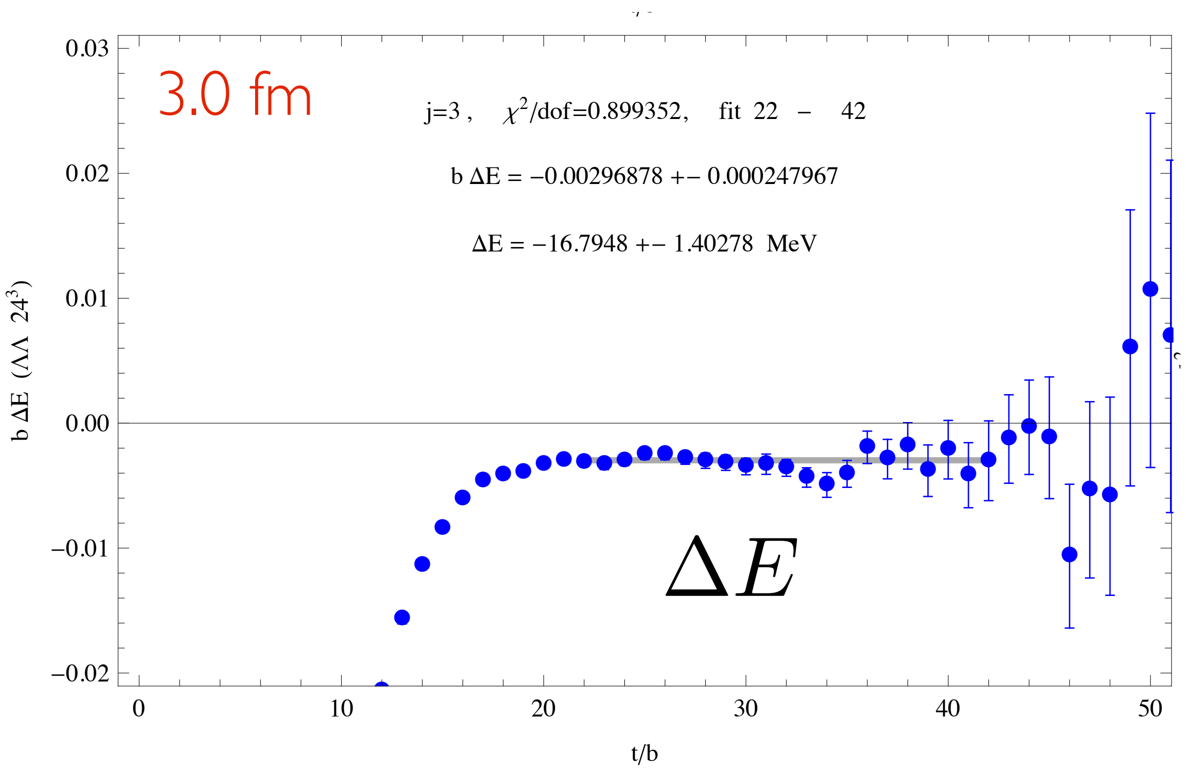
- Extract energy eigenstates from large Euclidean time behaviour of two-point correlators

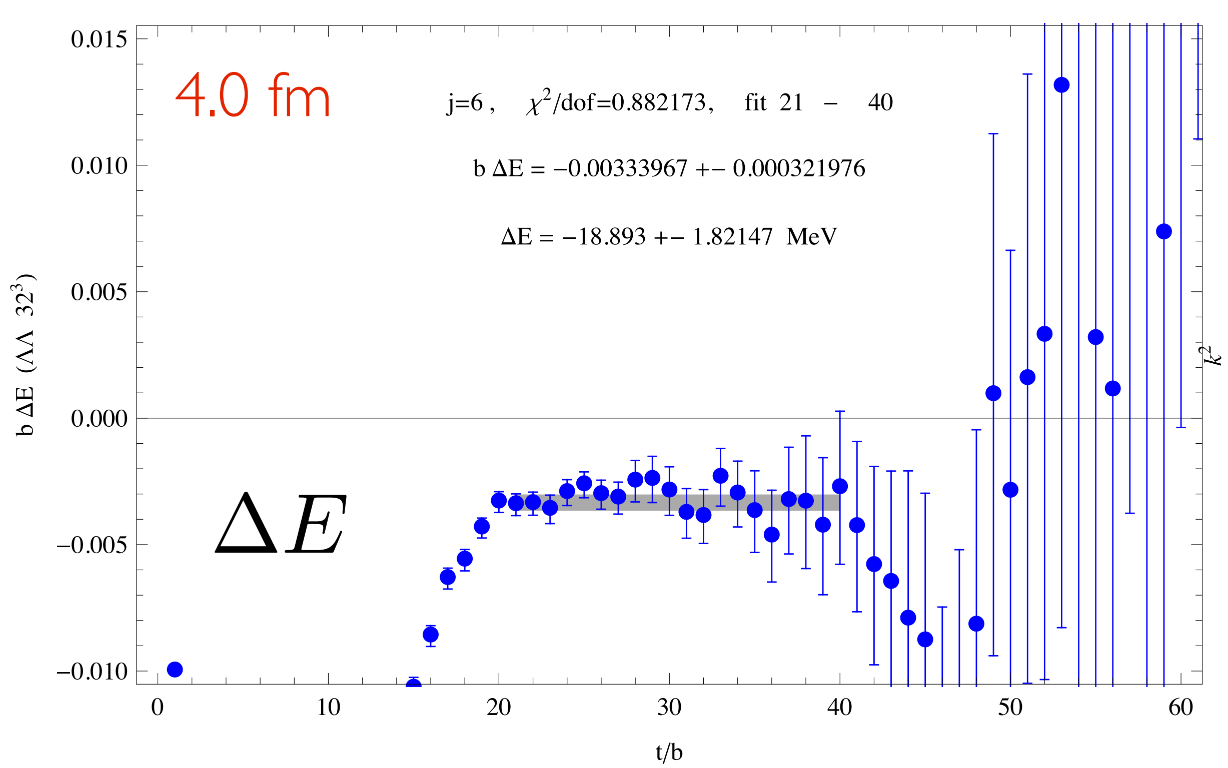
$$C_\Lambda(t) = \sum_{\mathbf{x}} \langle 0|\chi(\mathbf{x},t)\overline{\chi}(0)|0\rangle \stackrel{t\to\infty}{\longrightarrow} Z_\Lambda e^{-M_\Lambda t}$$

$$C_{\Lambda\Lambda}(t) = \sum_{\mathbf{x}} \langle 0|\phi(\mathbf{x},t)\overline{\phi}(0)|0\rangle \stackrel{t\to\infty}{\longrightarrow} Z_{\Lambda\Lambda} e^{-E_{\Lambda\Lambda} t}$$

$$R(t) = \frac{C_{\Lambda\Lambda}(t)}{C_\Lambda^2(t)} \stackrel{t\to\infty}{\longrightarrow} \widetilde{Z} e^{-\Delta E_{\Lambda\Lambda} t}$$

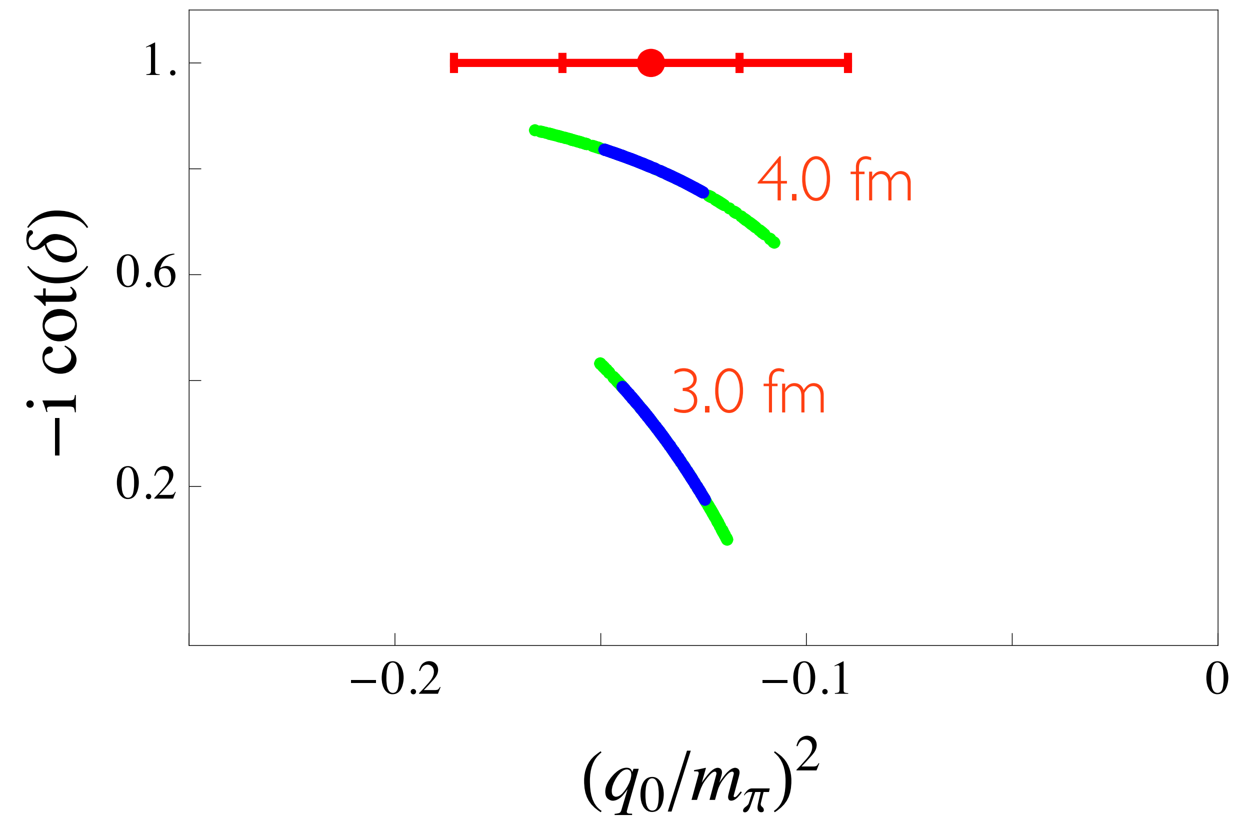
- Correlator ratio allows direct access to energy shift





Simple extrapolations

- After volume extrapolation H bound at unphysical quark masses



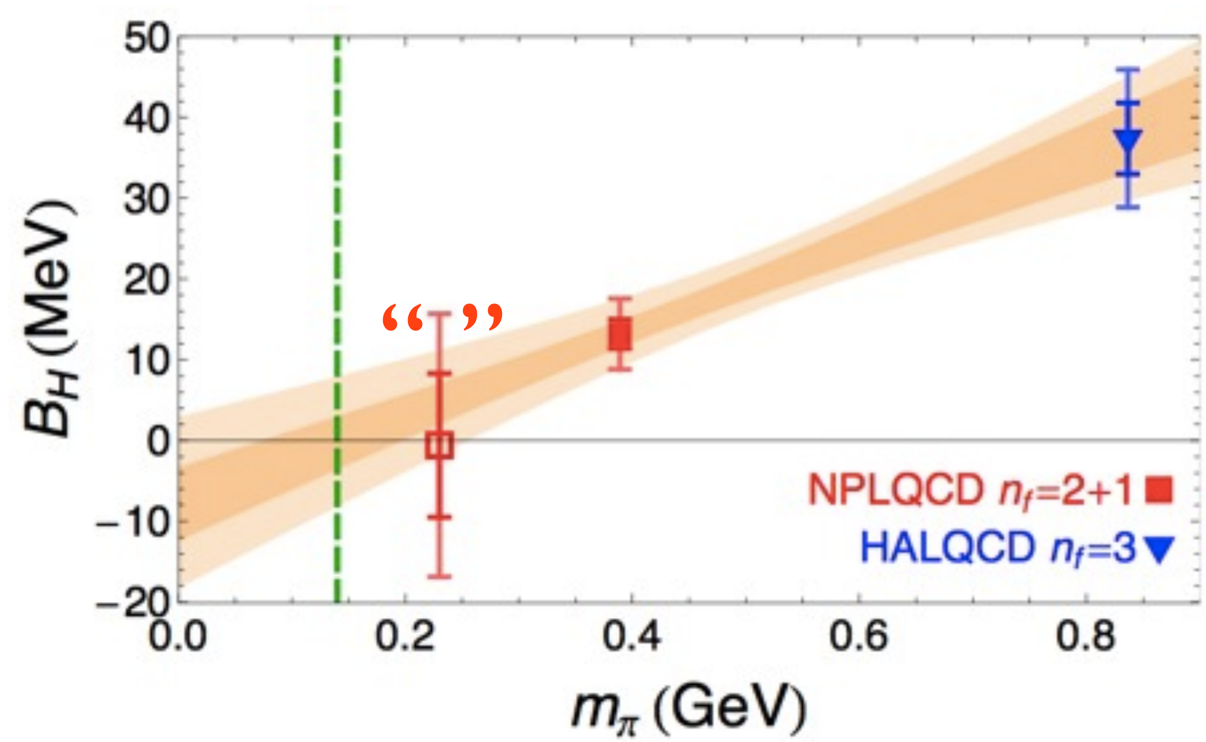
- Quark mass extrapolation is uncertain and unconstrained

$$B_H^{\rm quad} = +11.5 \pm 2.8 \pm 6.0 \ {\rm MeV}$$

$$B_H^{\rm lin} = +4.9 \pm 4.0 \pm 8.3 \ {\rm MeV}$$

- Other extrapolations, see [Shanahan, Thomas & Young PRL. 107 (2011) 092004, Haidenbauer & Meissner 1109.3590]

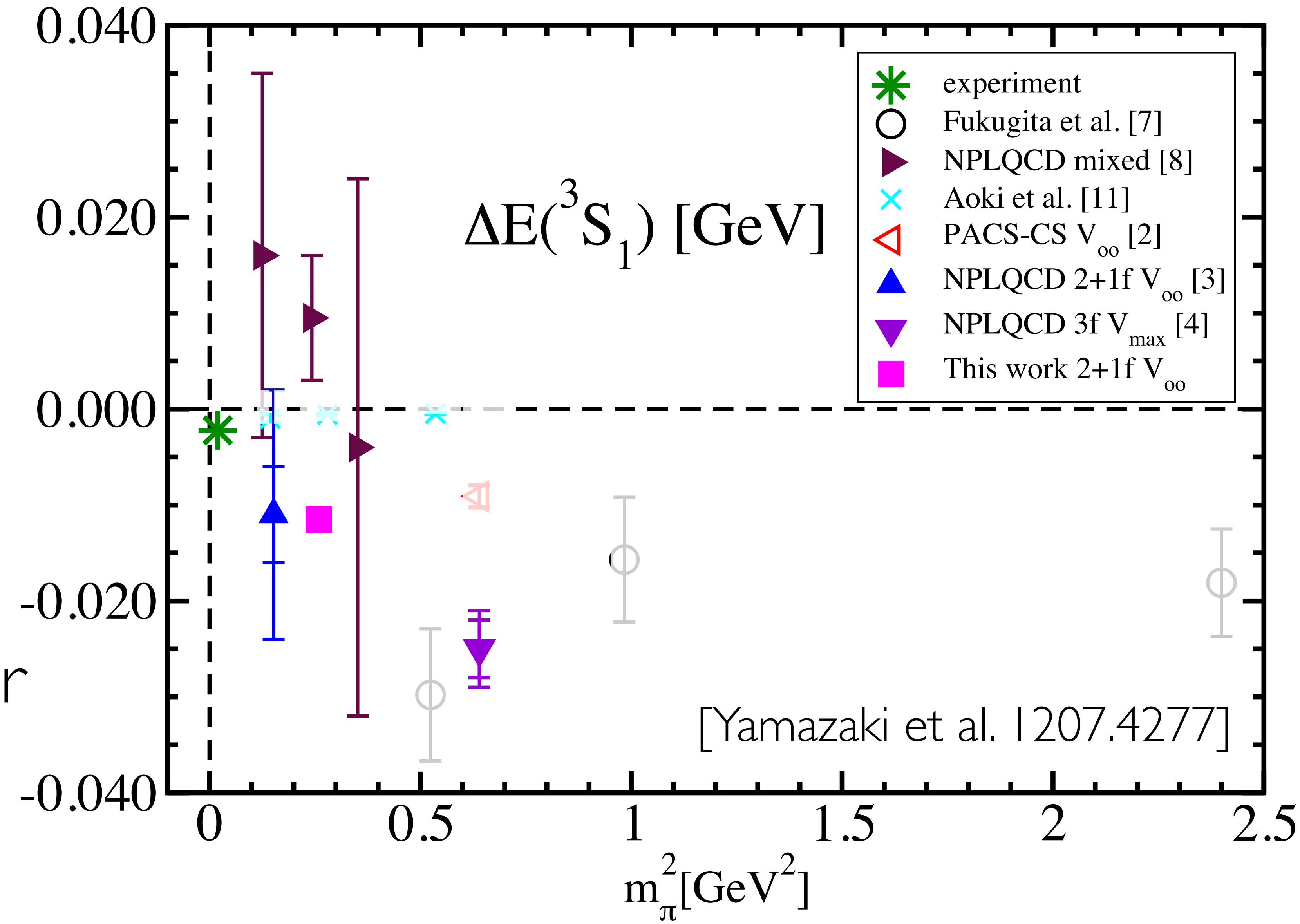
- Suggests H is weakly bound or just unbound



* 230 MeV point preliminary (one volume)

Deuteron

- Deuteron also investigated
  - NPLQCD
  - PACS-CS
- More work needed at lighter masses



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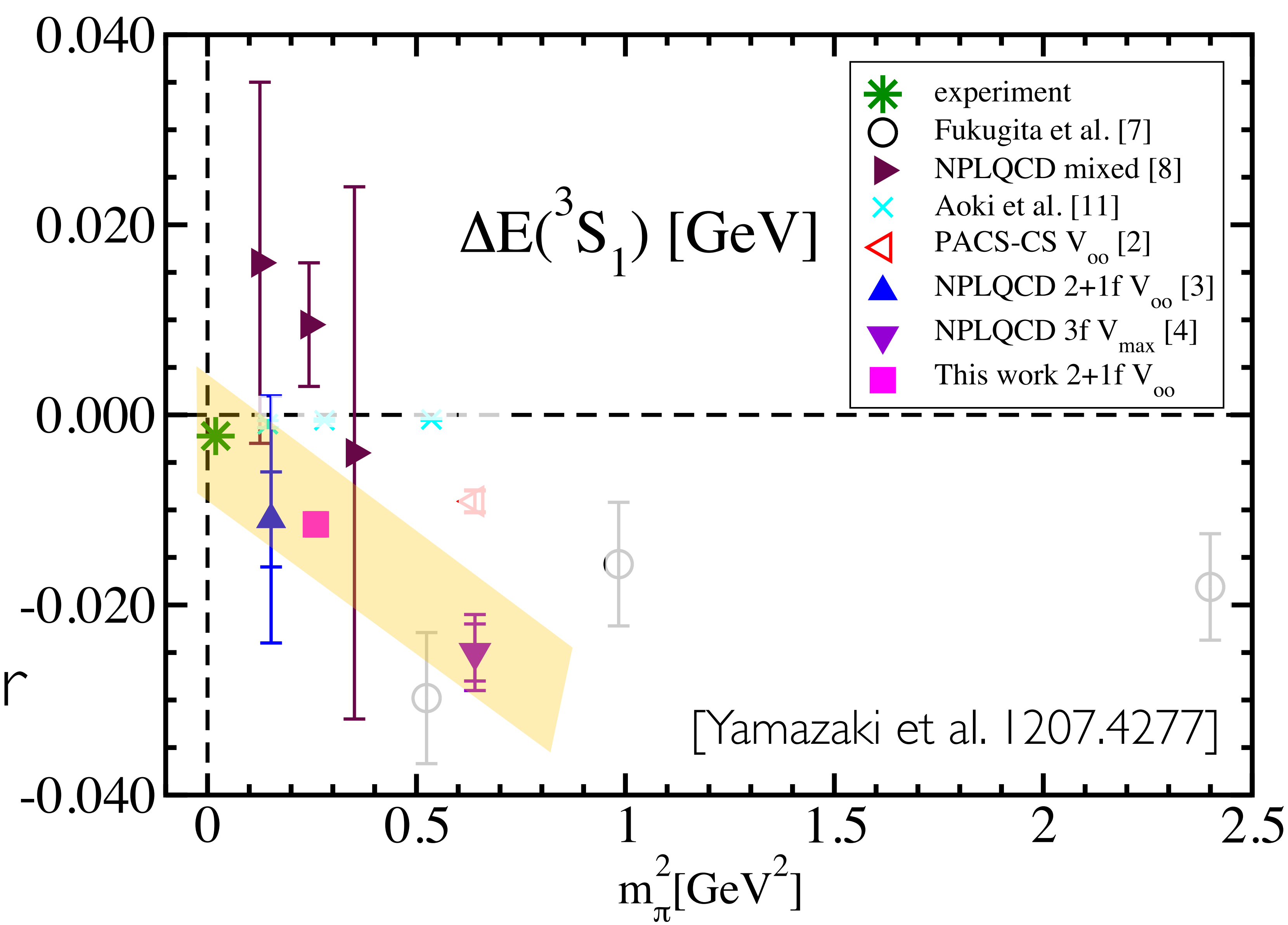

Many baryon systems

- New approach to many baryon correlator construction
- Interpolating fields – minimal expression as weighted sums

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

  - Automated generation of weights (symbolic c++ code) for given quantum numbers

- Baryon blocks (partly contracted at sink)

$$\mathcal{B}_b^{a_1,a_2,a_3}(\mathbf{p},t;x_0) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \sum_{k=1}^{N_{B(b)}} \tilde{w}_b^{(c_1,c_2,c_3),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,i_3} S(c_{i_1},x;a_1,x_0)S(c_{i_2},x;a_2,x_0)S(c_{i_3},x;a_3,x_0)$$

WD, Kostas Orginos, 1207.1452
see also Doi & Endres 1205.0585

Many baryon systems

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

WD, Kostas Orginos, 1207.1452

Many baryon systems

- Contractions

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

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$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})$$

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- Express in terms of blocks (quark-hadron)

WD, Kostas Orginos, 1207.1452

Many baryon systems

- Contractions

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$
$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$
$$= e^{-\mathcal{S}_{eff}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$
$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})$$

- Express in terms of blocks (quark-hadron)

- Or write as determinant (quark-quark)

$$\langle\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\mathcal{U}\; e^{-\mathcal{S}_{eff}} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times \det G(\mathbf{a}';\mathbf{a})$$

$$G(\mathbf{a}';\mathbf{a})_{j,i} = \begin{cases} S(a'_j;a_i) & a'_j \in \mathbf{a}' \text{ and } a_i \in \mathbf{a} \\ \delta_{a'_j,a_i} & \text{otherwise} \end{cases},$$

WD, Kostas Orginos, 1207.1452

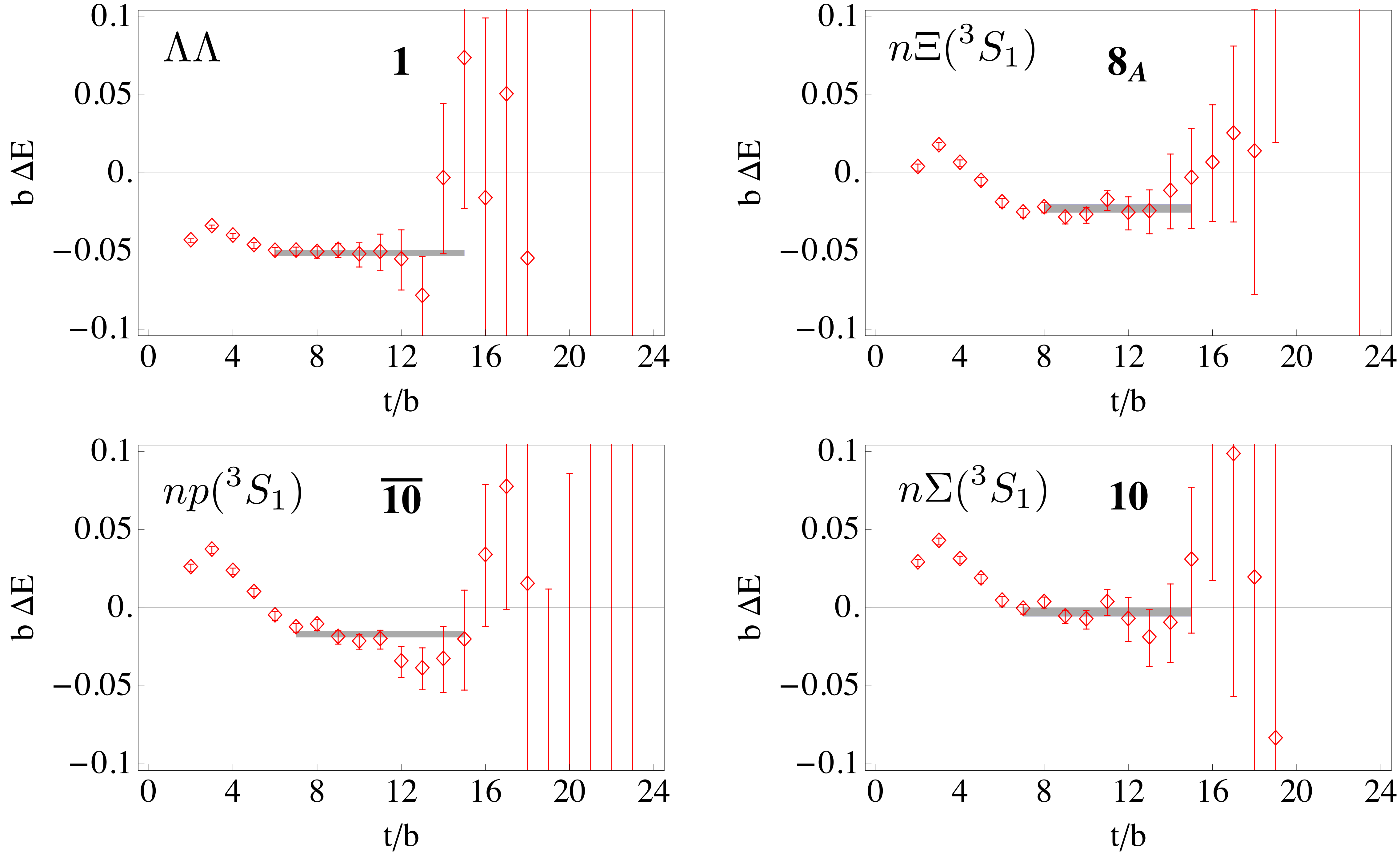
Nuclei

- Recent studies at SU(3) point (physical m_s)
    - Isotropic clover lattices
    - Single lattice spacing: 0.145 fm
    - Multiple volumes: 3.4, 4.5, 6.7 fm
    - High statistics

Label	L/b	T/b	β	$b\ m_q$	b [fm]	L [fm]	T [fm]	m_π [MeV]	$m_\pi\ L$	$m_\pi\ T$	N_{cfg}	N_{src}
A	24	48	6.1	-0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	48
B	32	48	6.1	-0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	24
C	48	64	6.1	-0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1212	32

Nuclei (A=2)

Quark-hadron contraction method



NPLQCD arXiv:1206.5219

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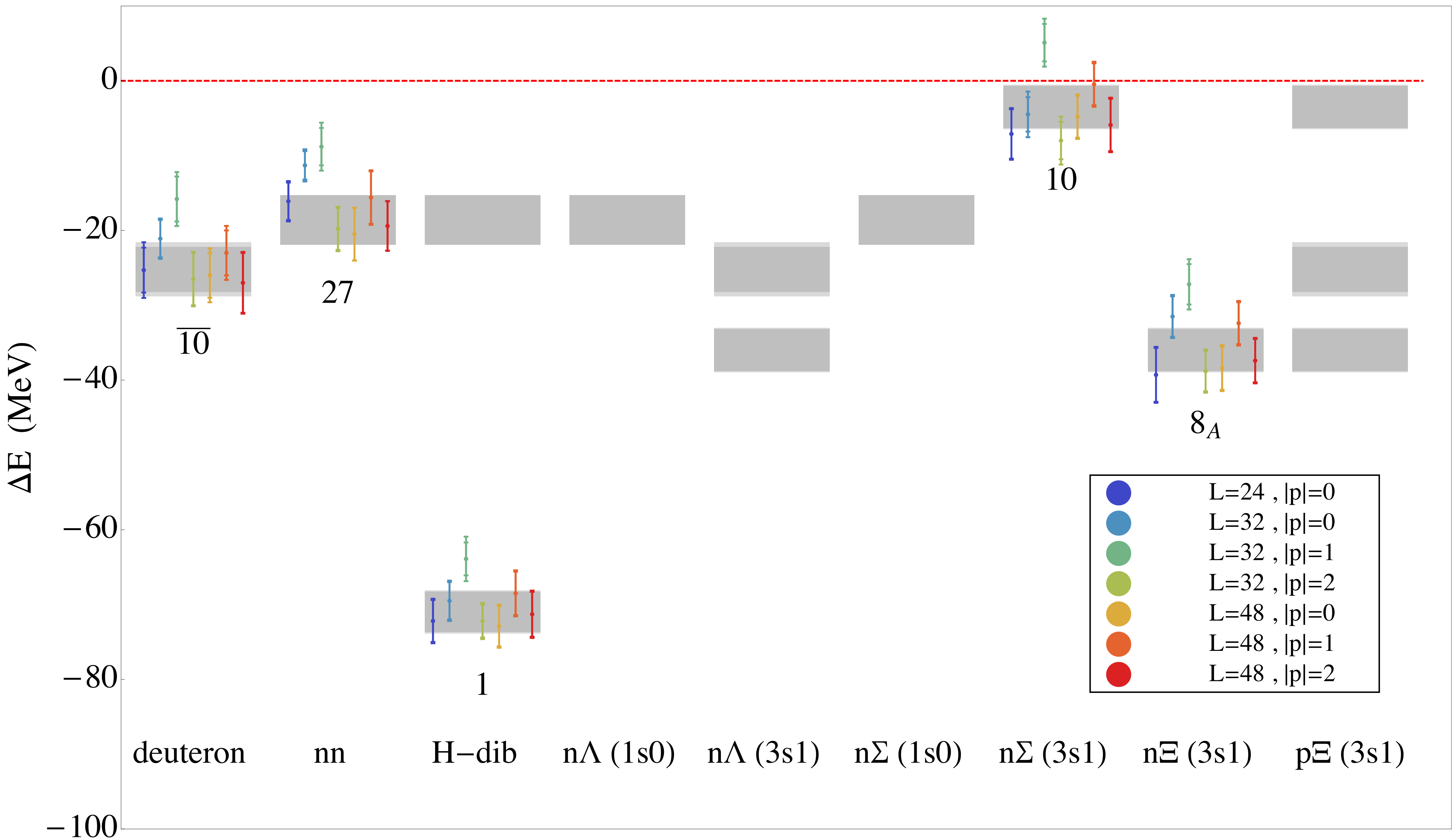

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Nuclei (A=2,3,4)

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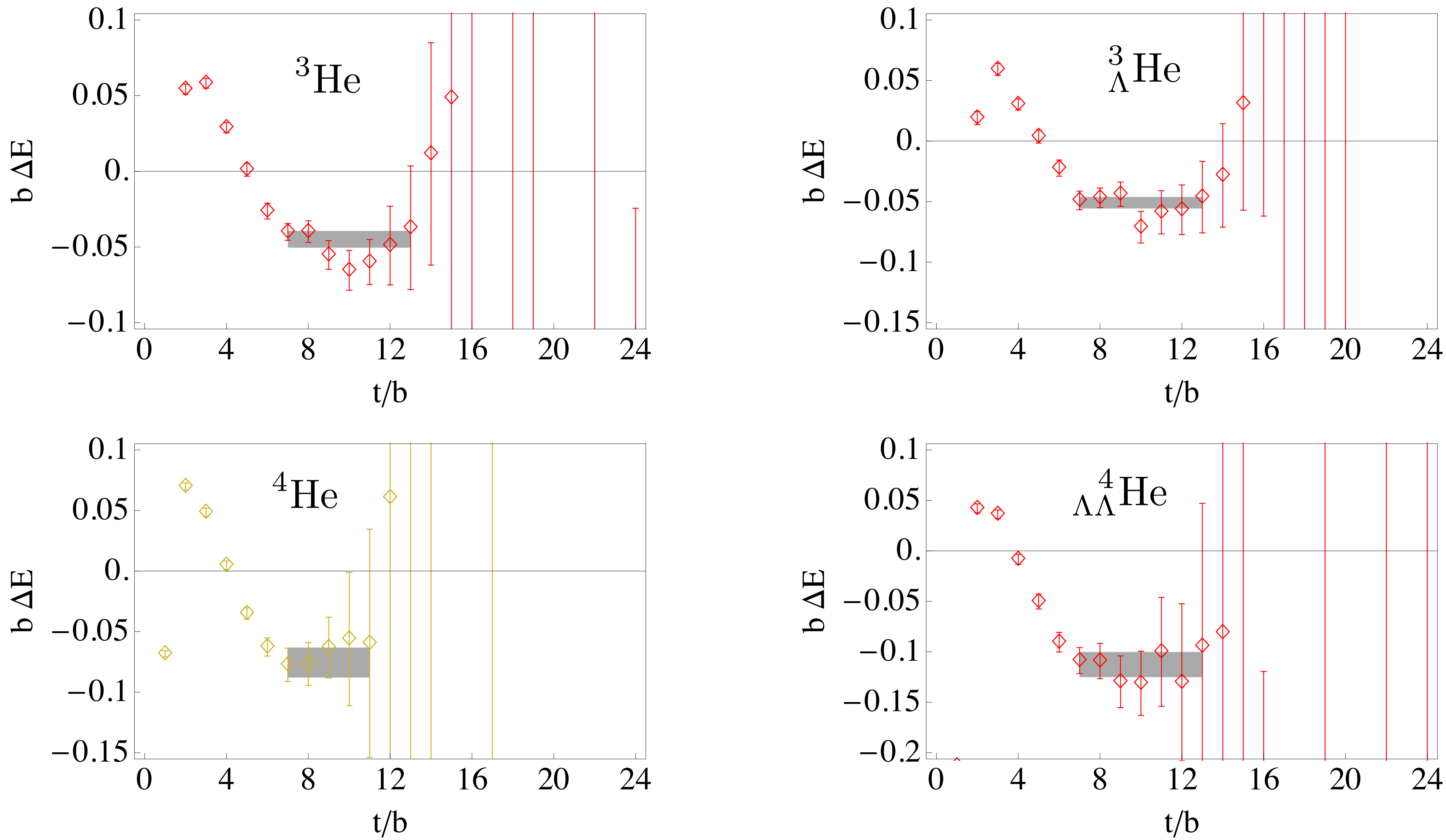

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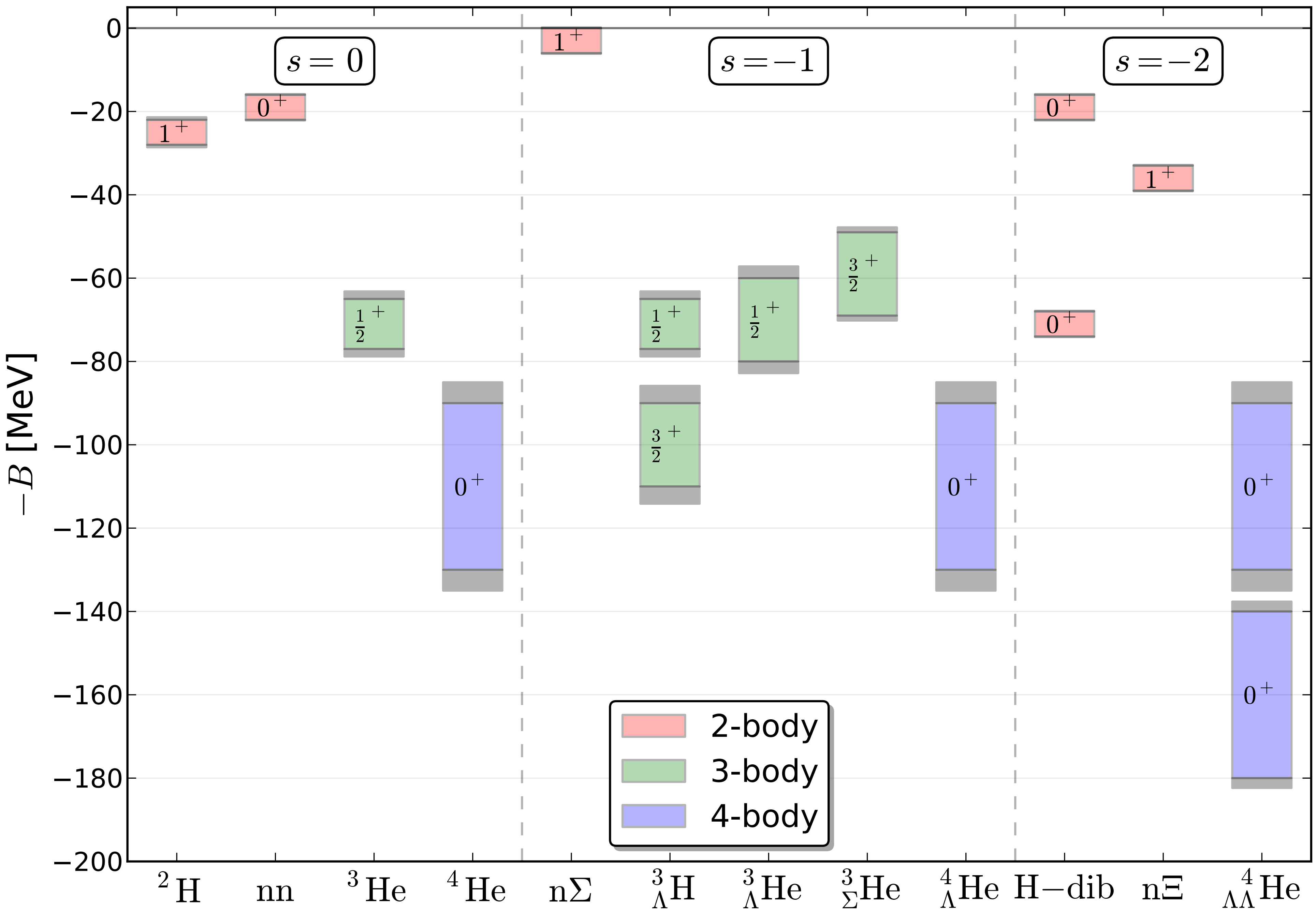
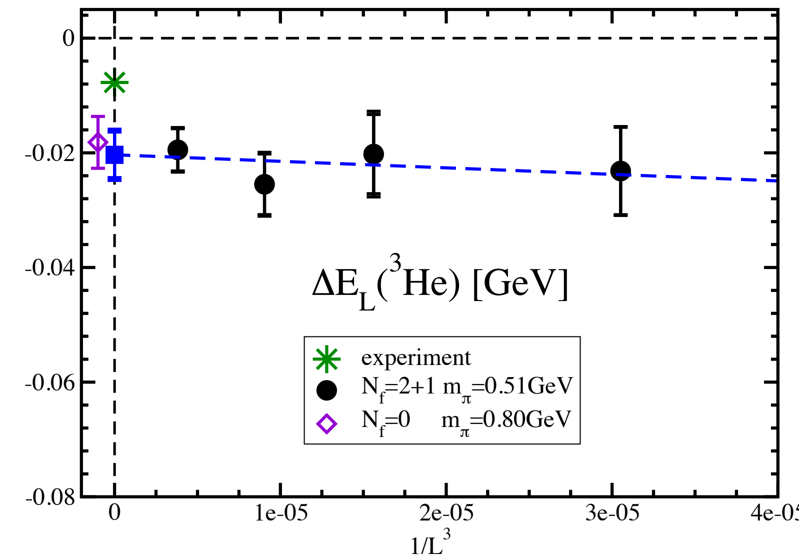
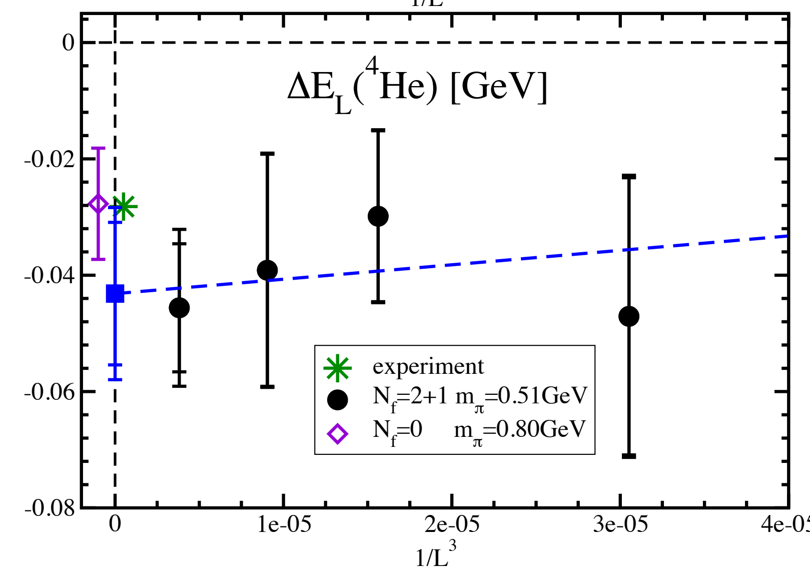


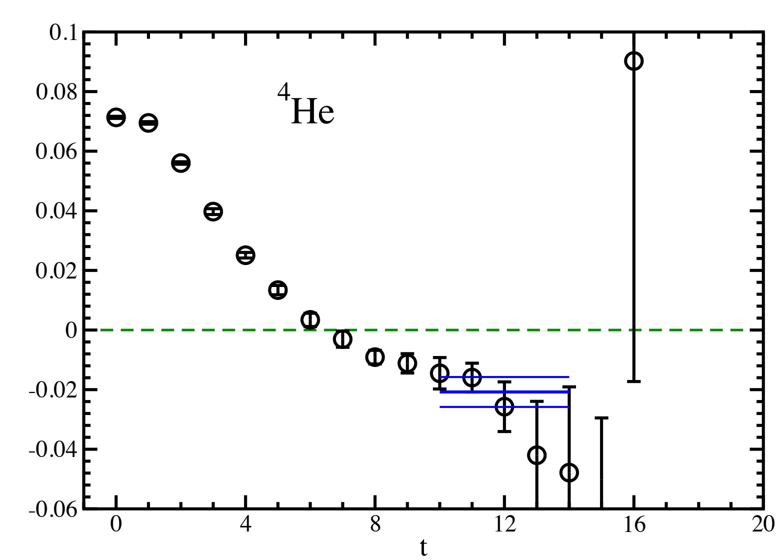
NPLQCD arXiv:1206.5219

d, nn, ^{3}He, ^{4}He

- PACS-CS: bound d,nn, ^{3}He, ^{4}He
  - Previous quenched work
  - Recent unquenched study at m_π=500 MeV
- HALQCD
  - Extract an NN potential
  - Strong enough to bind H, ^{4}He at m_{PS}=490 MeV SU(3) pt
  - d, nn not bound







[Yamazaki et al. 1207.4277]

Nuclei (A=4,...)

Quark-quark determinant contraction method

(low statistics, single volume)

WD, Kostas Orginos,1207.1452

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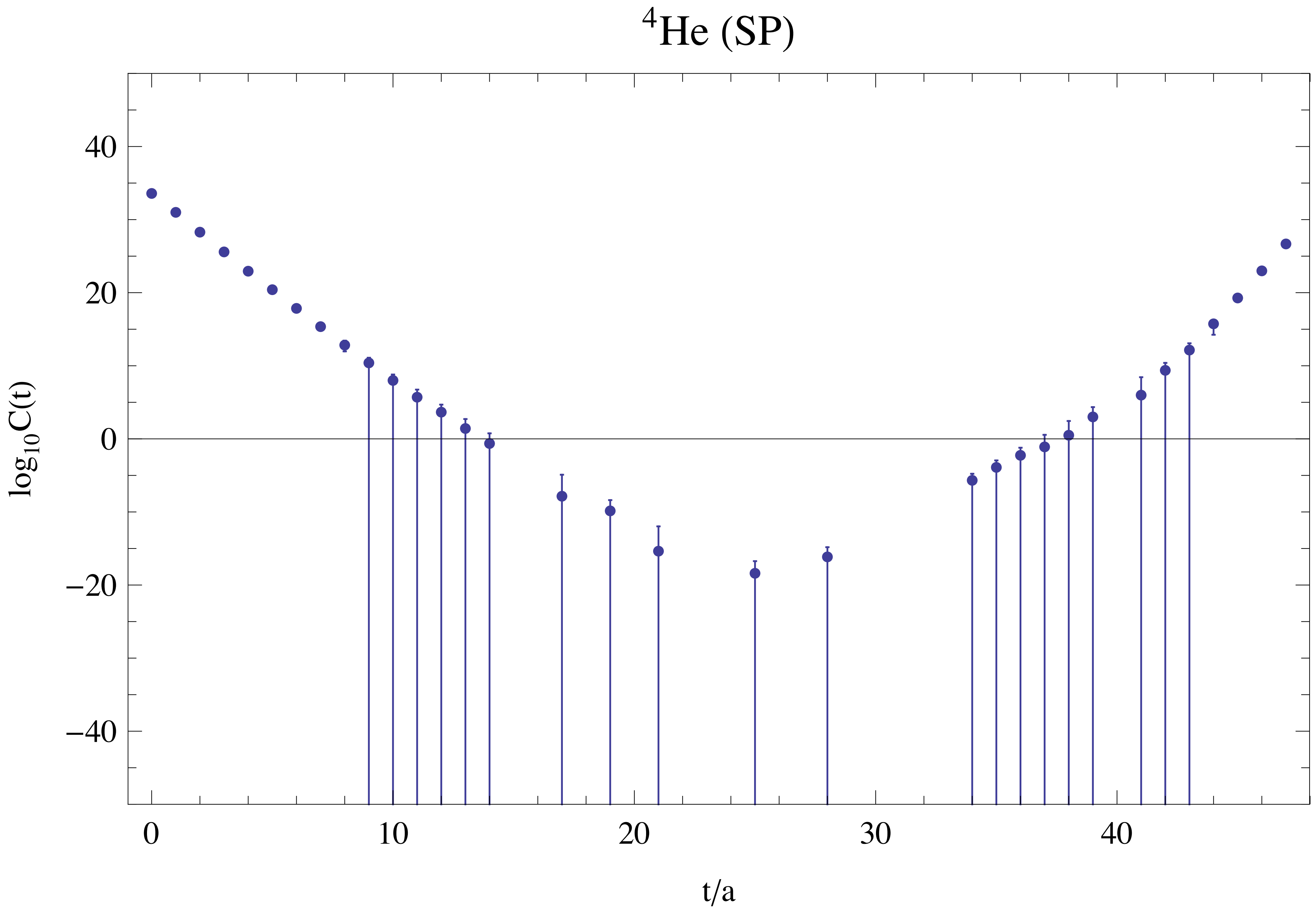

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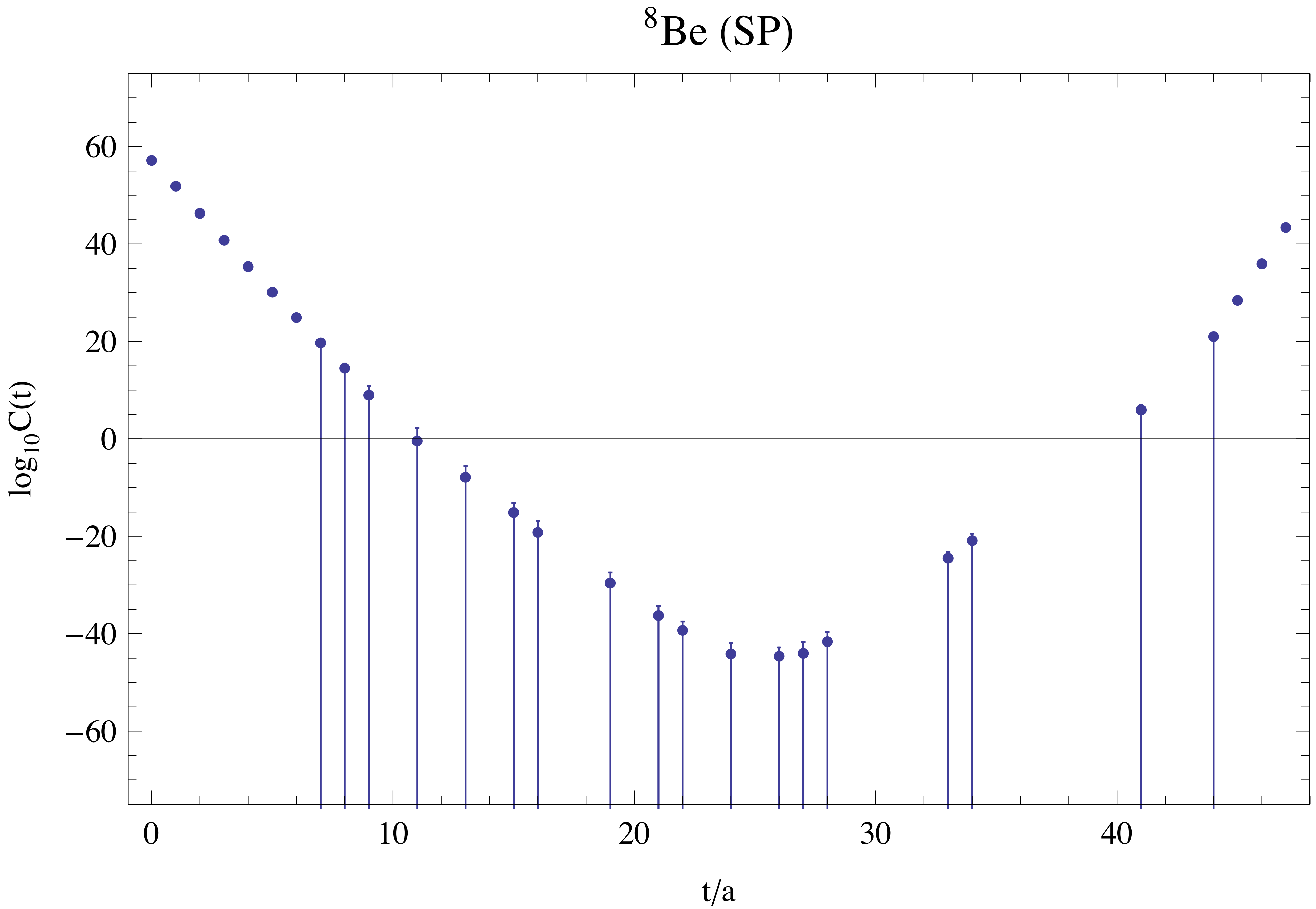

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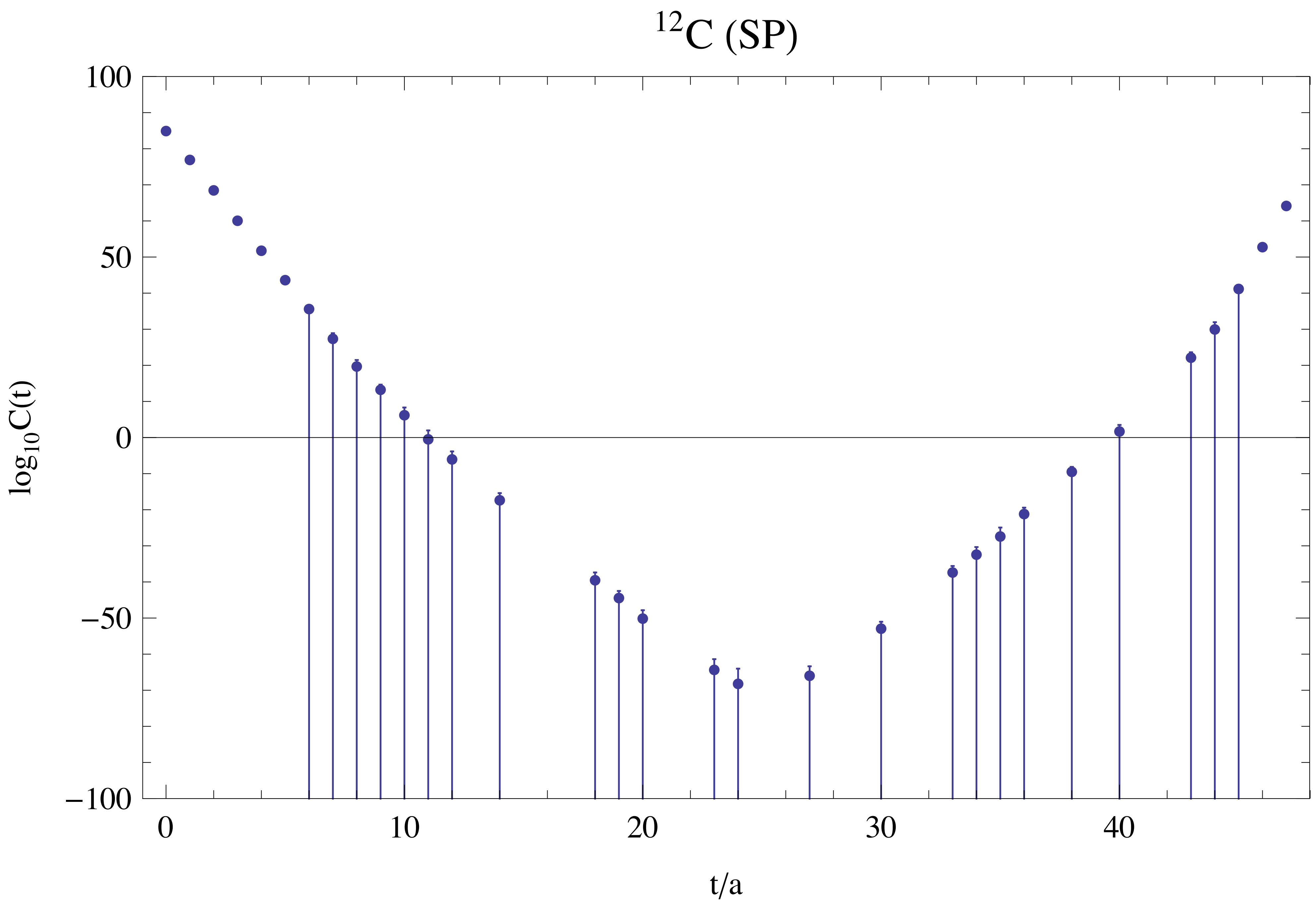

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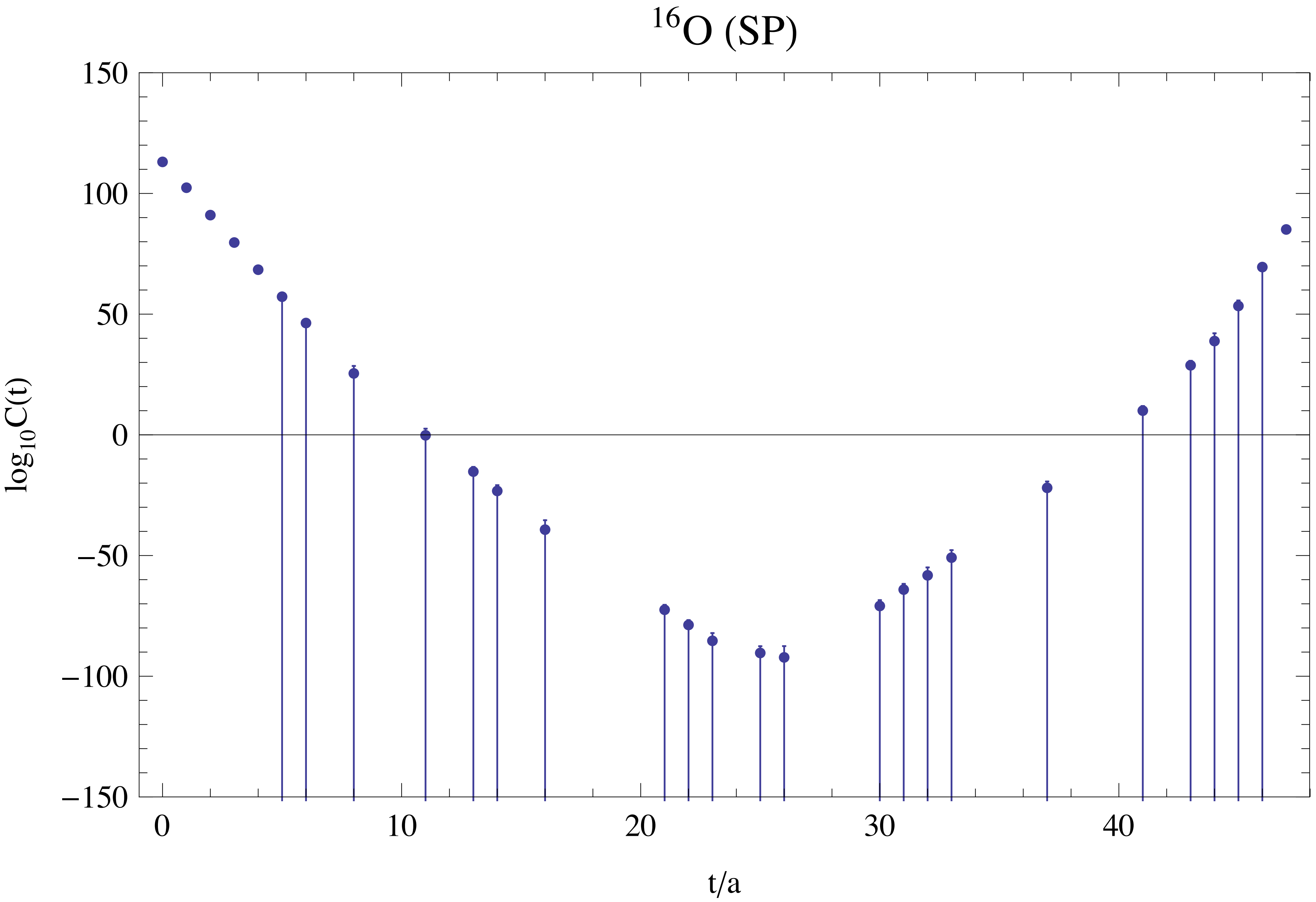

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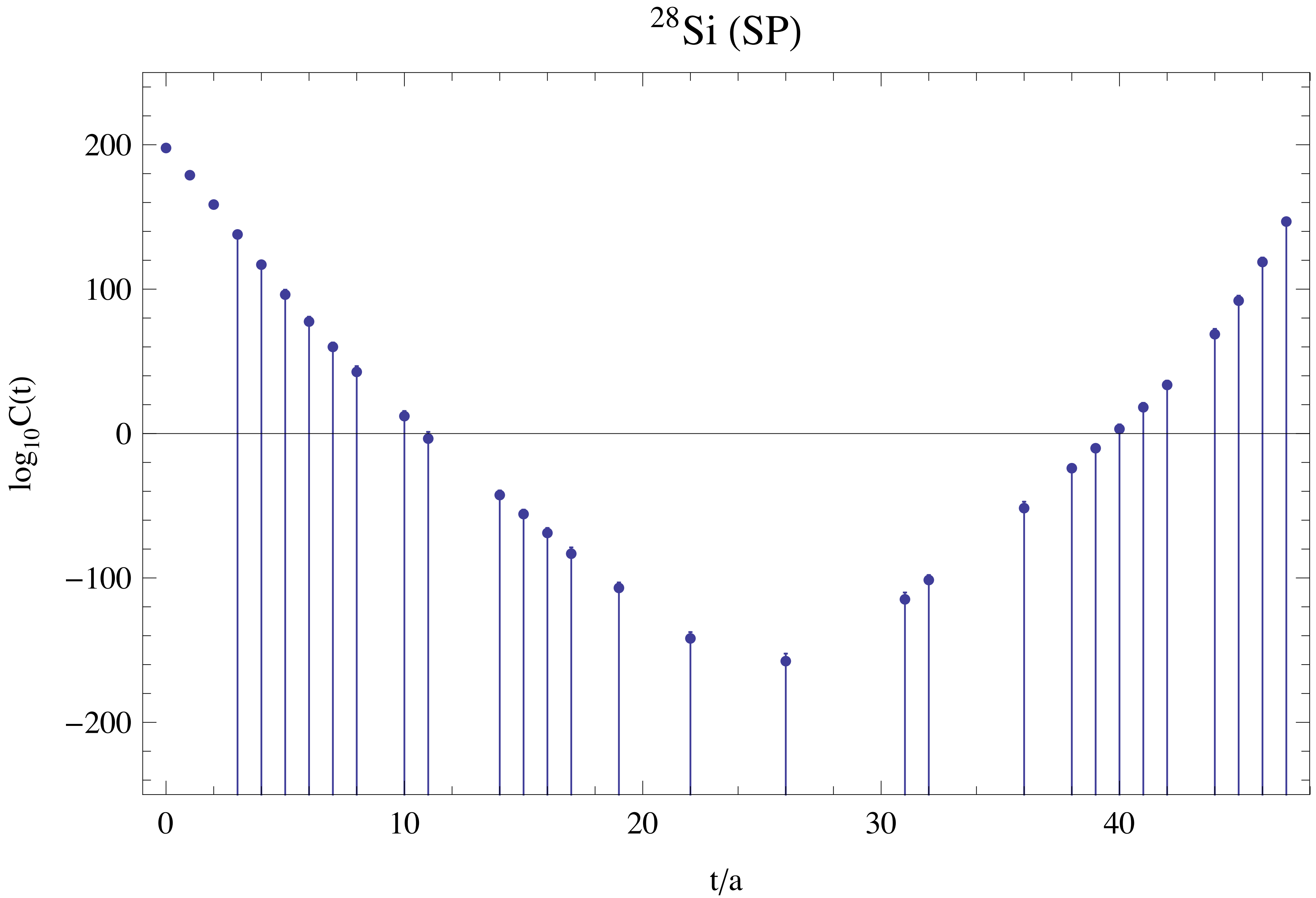

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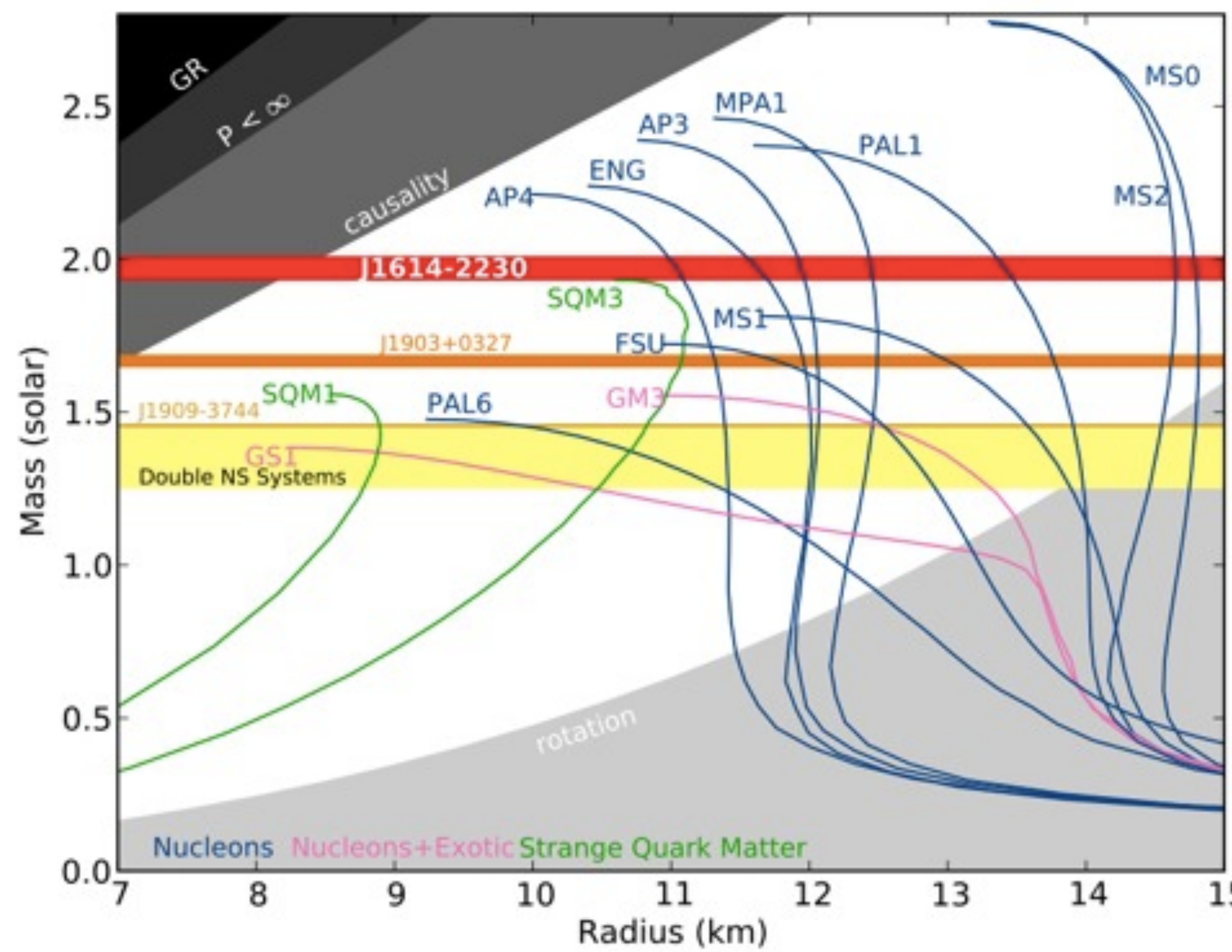
The road ahead...

Hyperon-nucleon interactions



Hyperon-nucleon interactions

- Observation of 1.97 $M_{\odot}$ n-star [Demorest et al., Nature, 2010]
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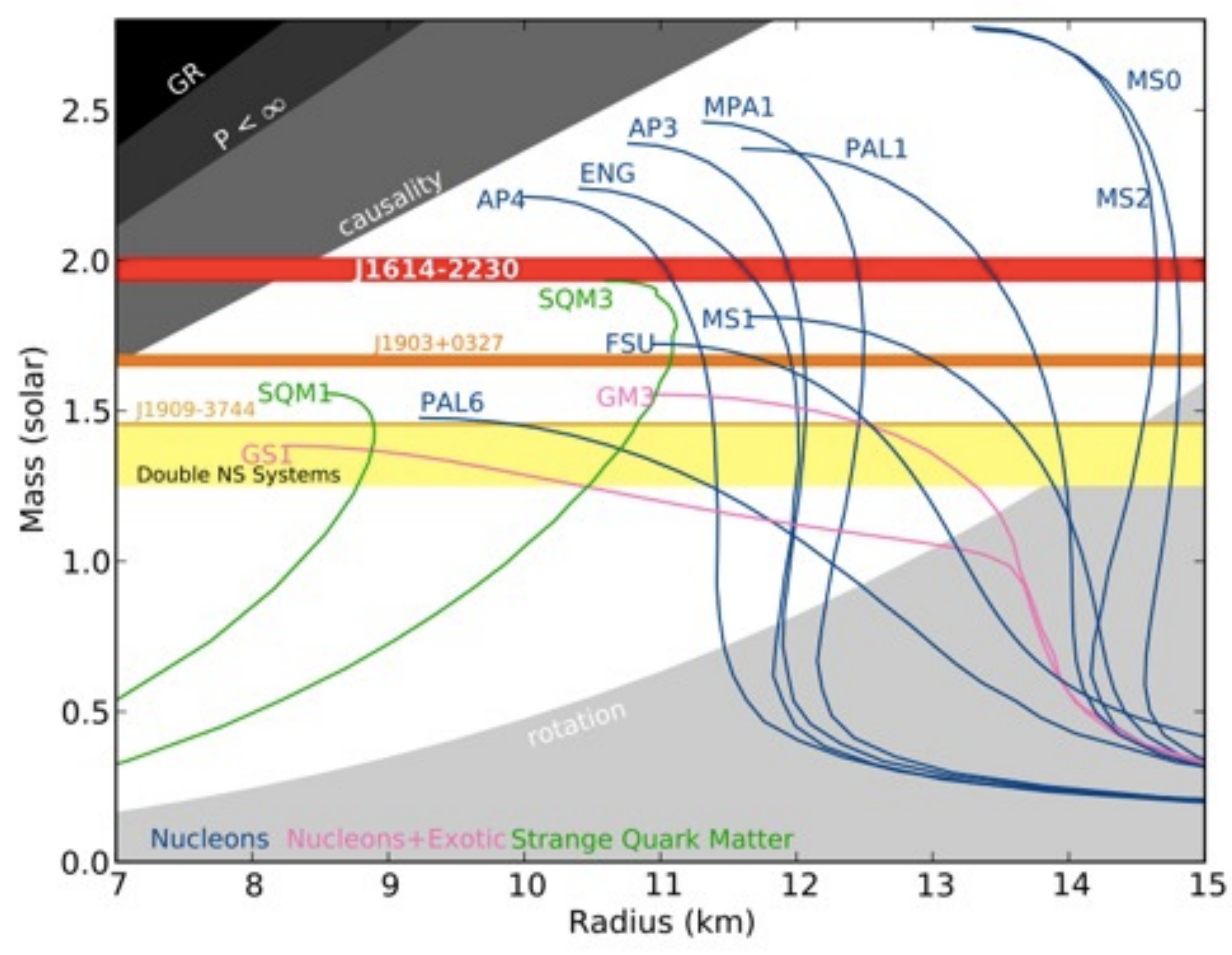

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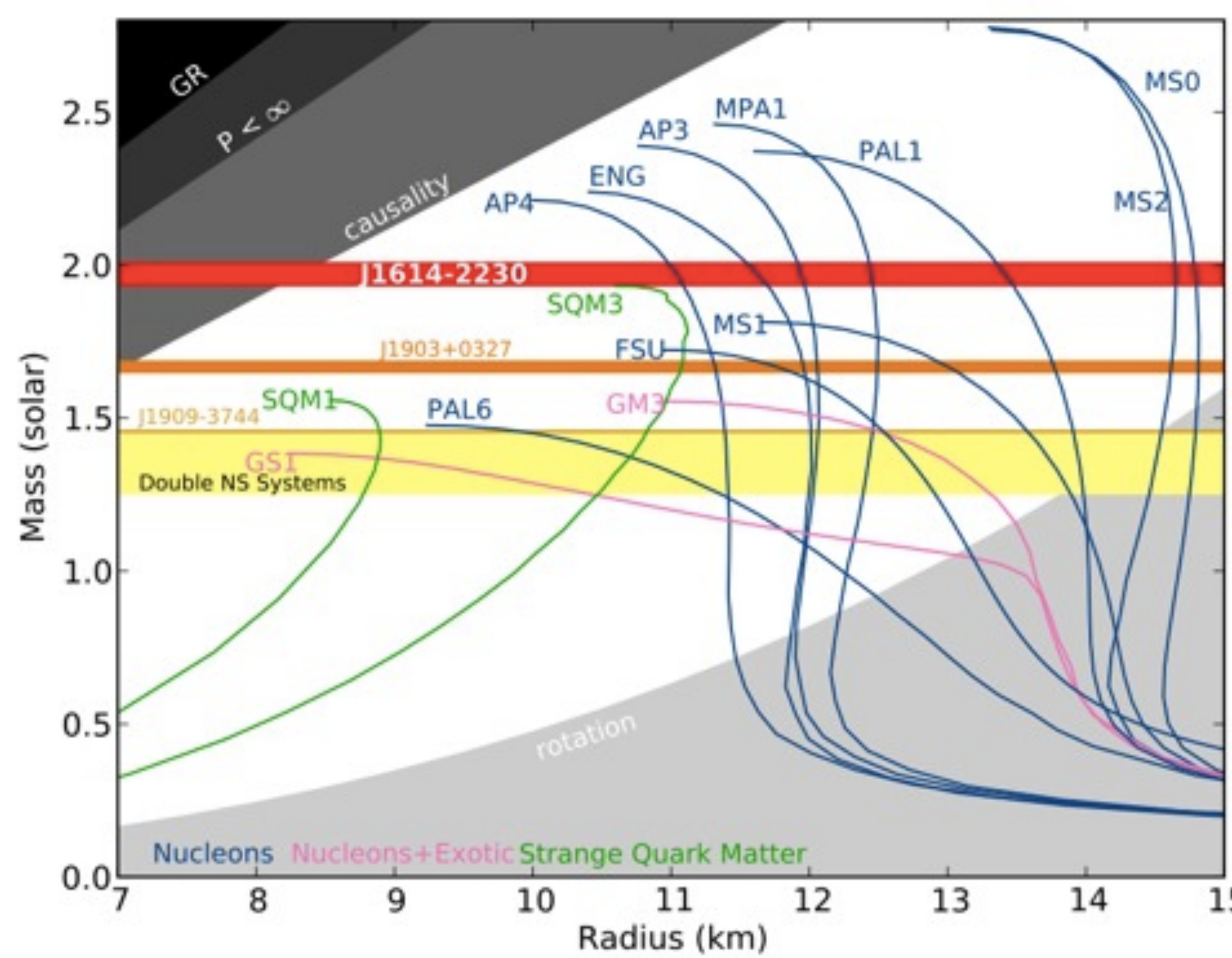

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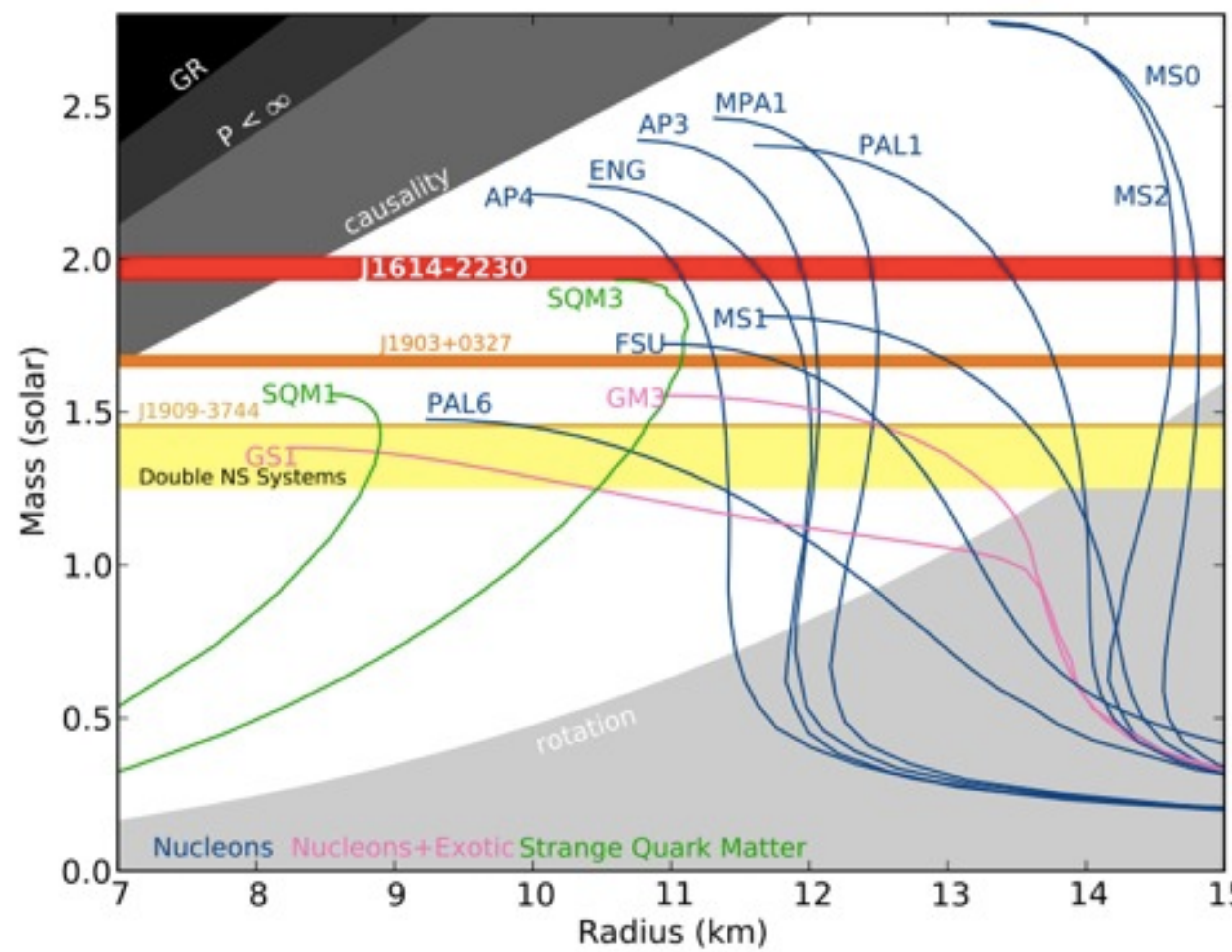
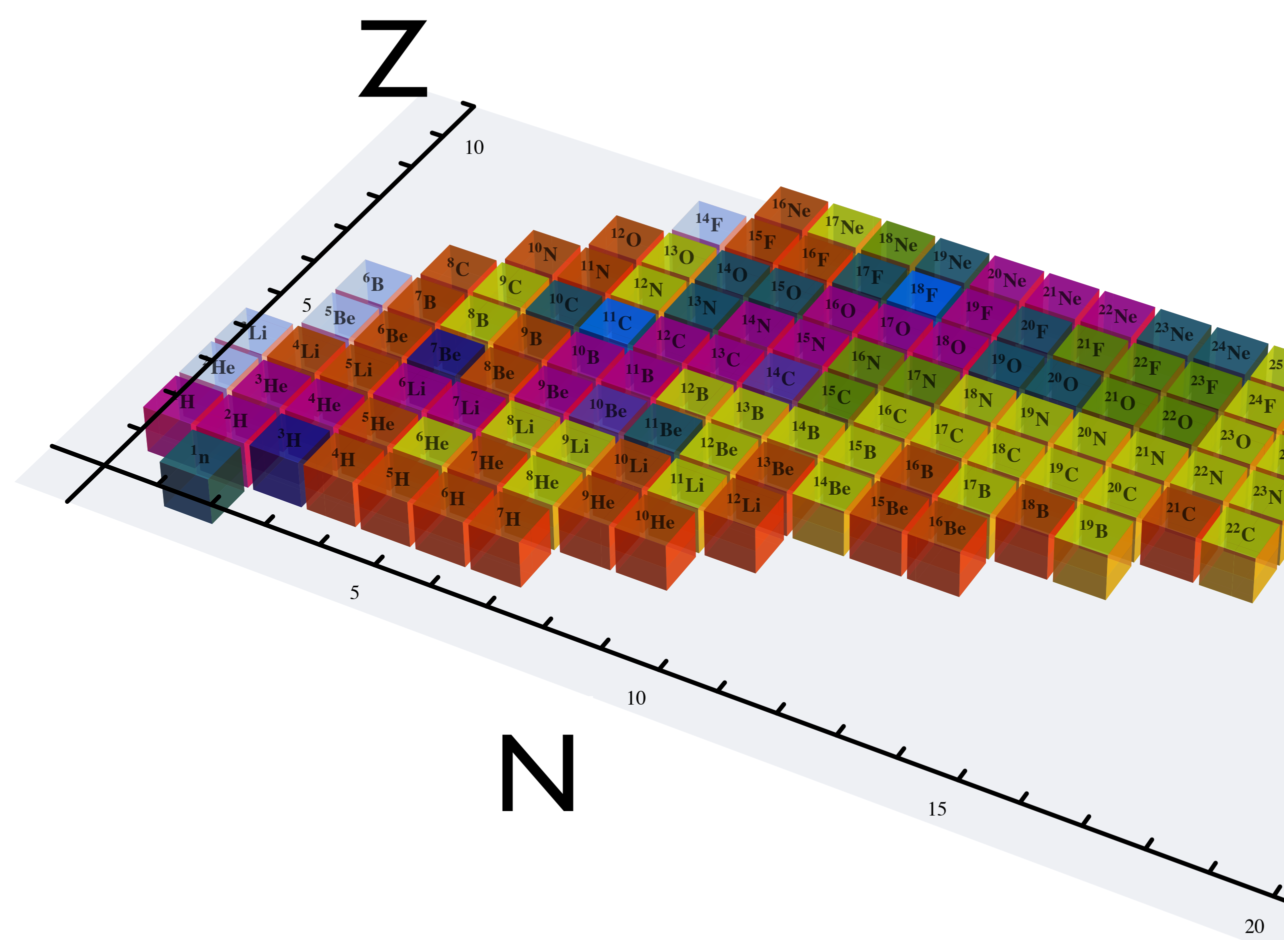
  - *nnn*, ...
- Calculable in QCD
  - 30% measurements would have impact



Hypernuclear Spectroscopy



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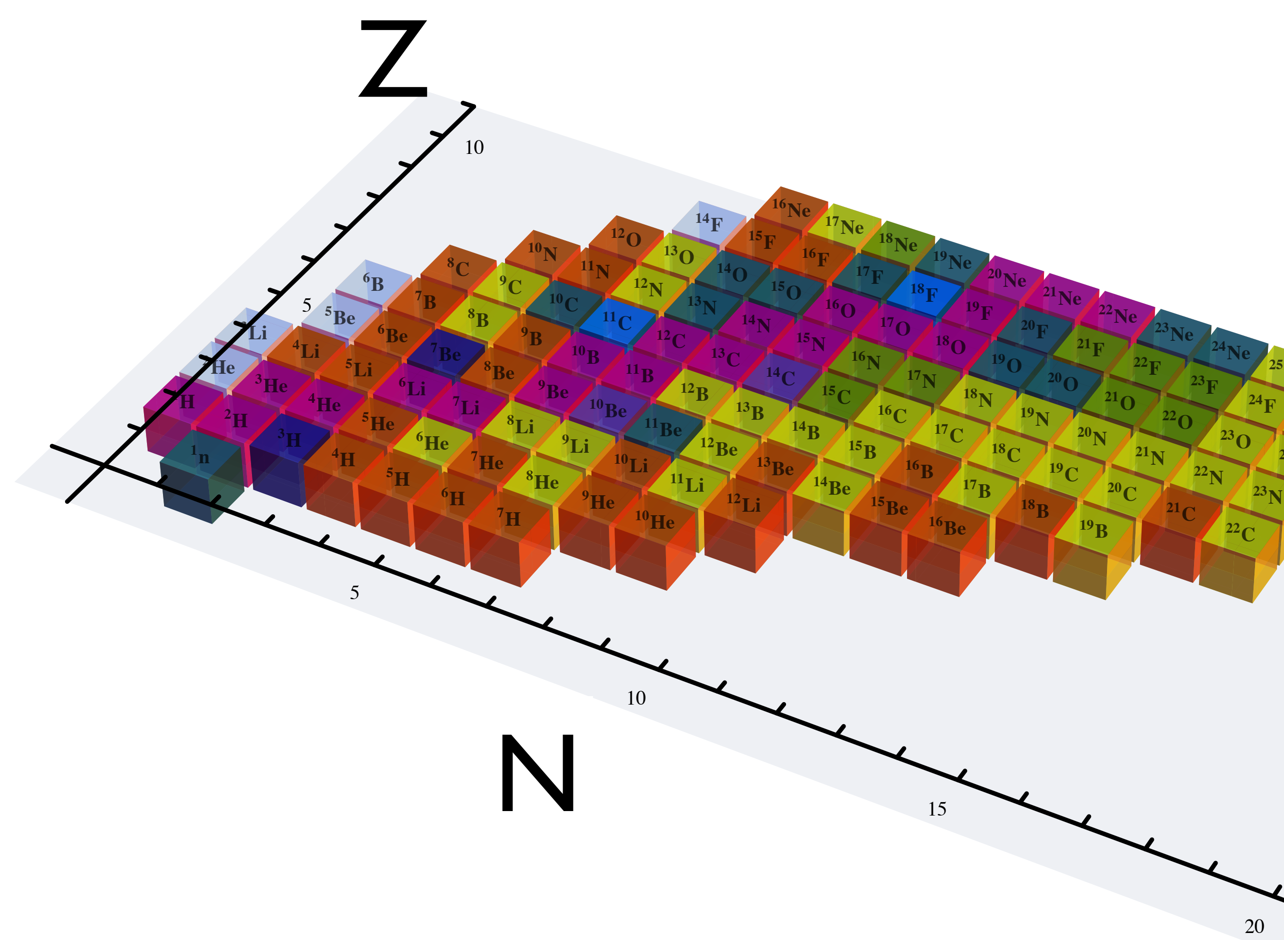
- Table of nuclides very well determined experimentally



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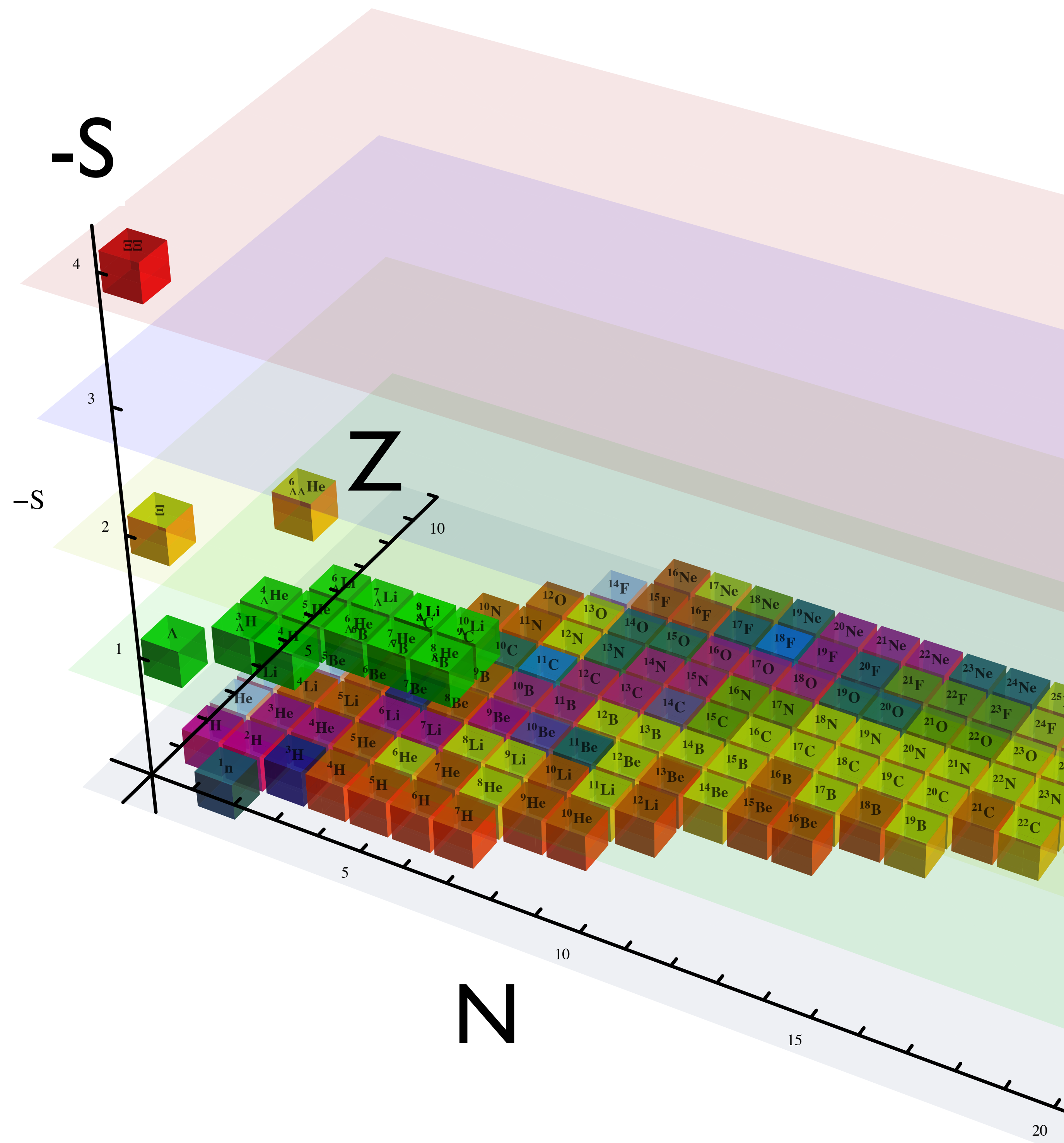

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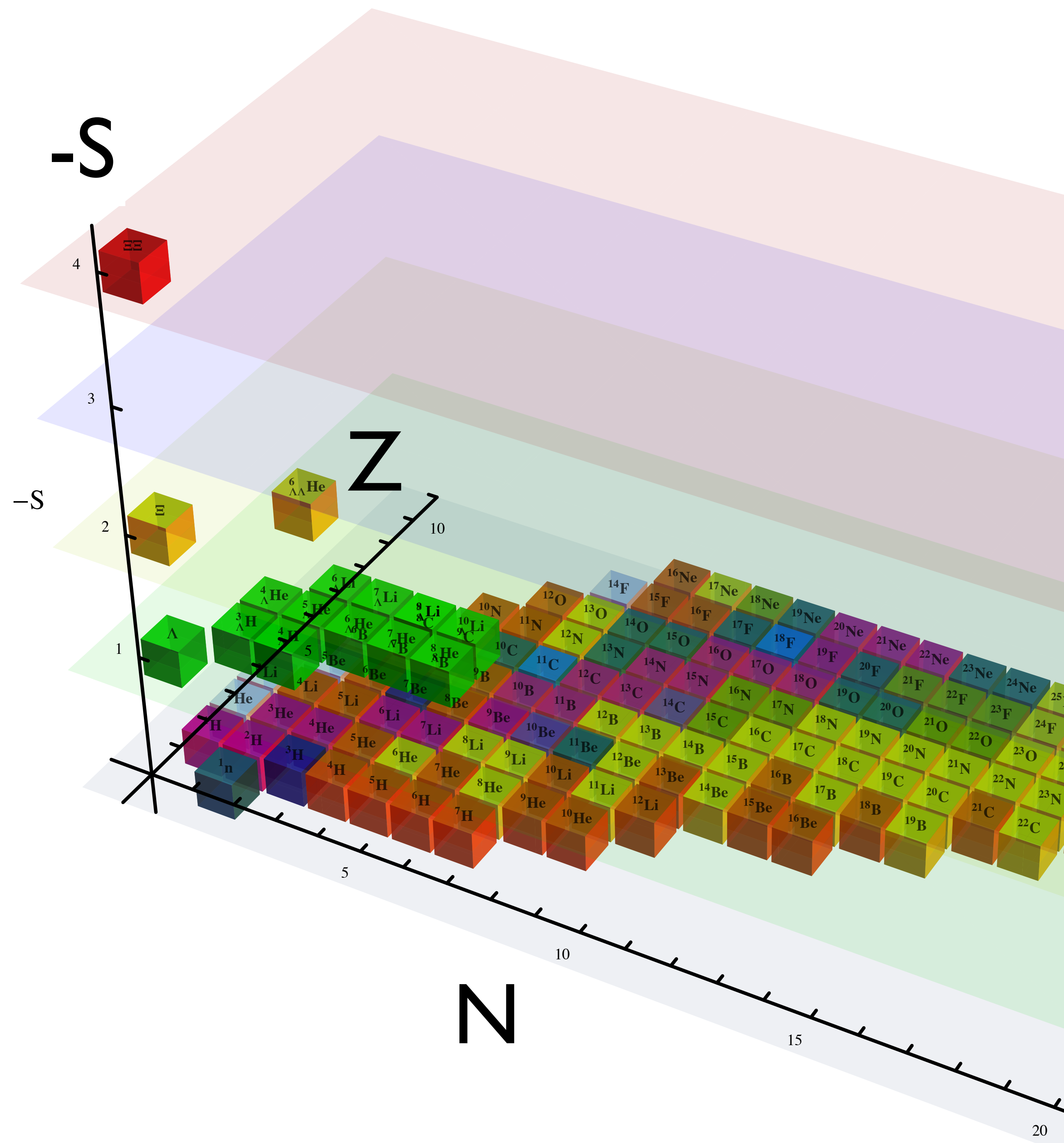

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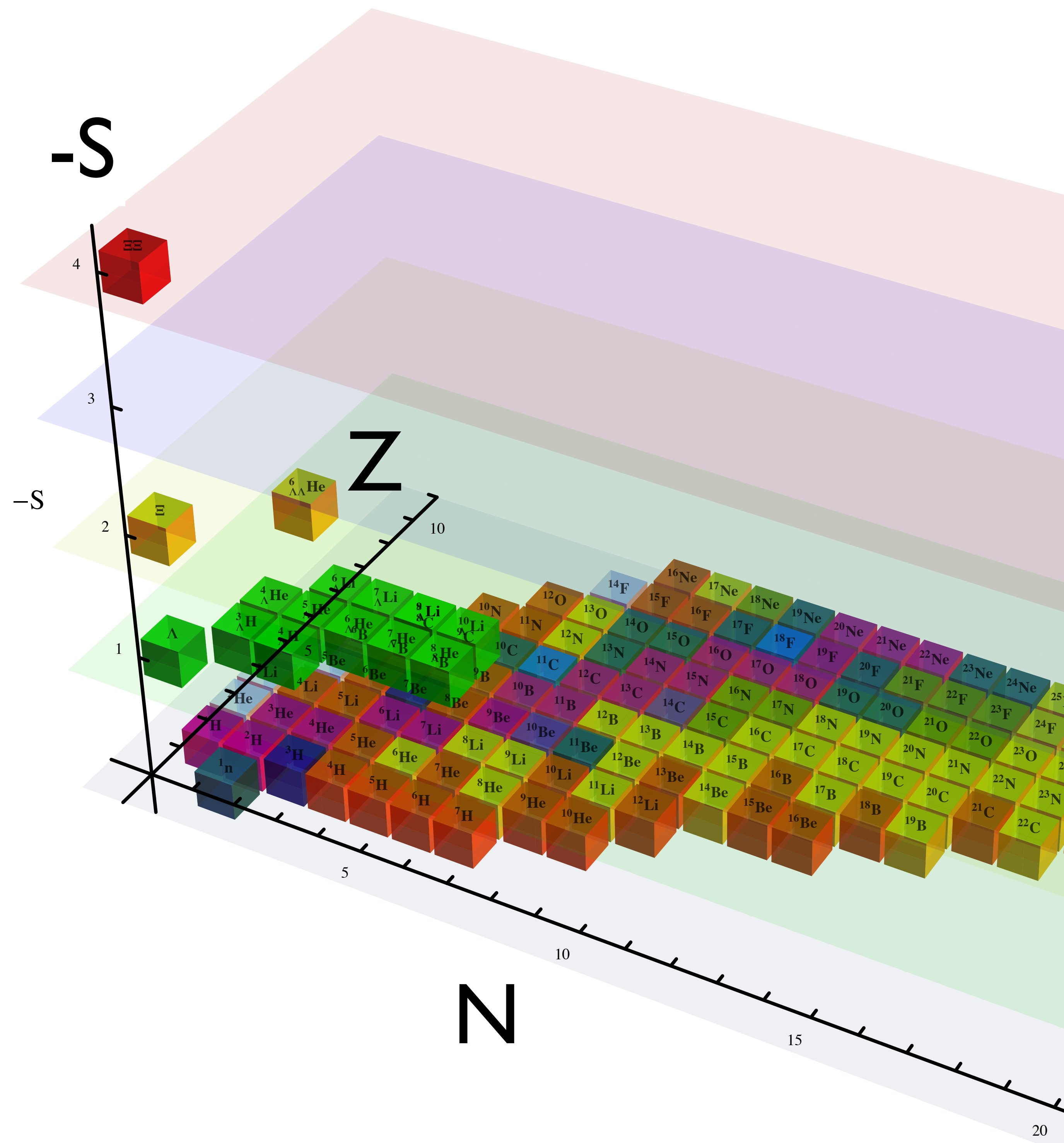

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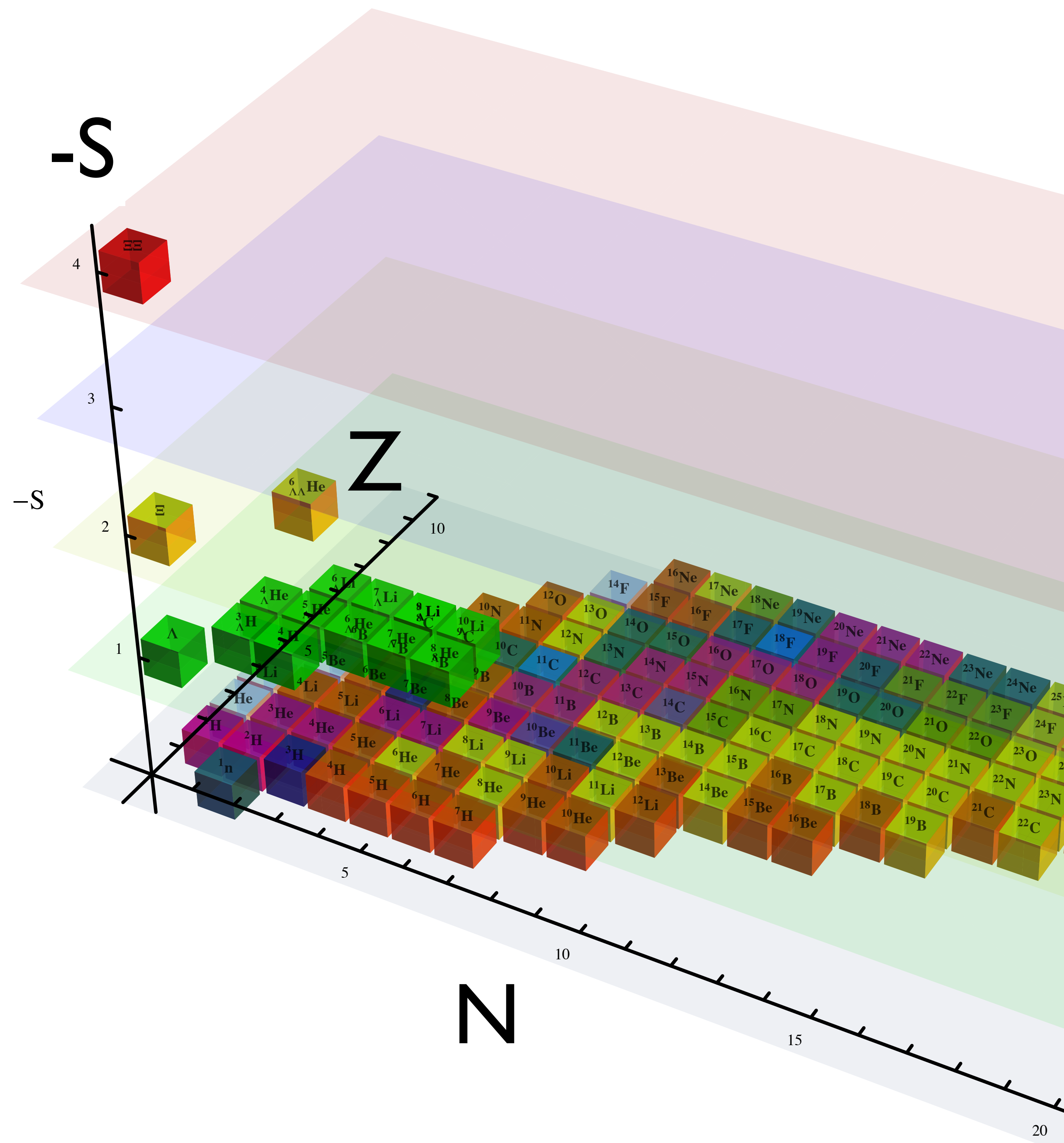

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1. Axial coupling to NN system
   - pp fusion: "Calibrate the sun"
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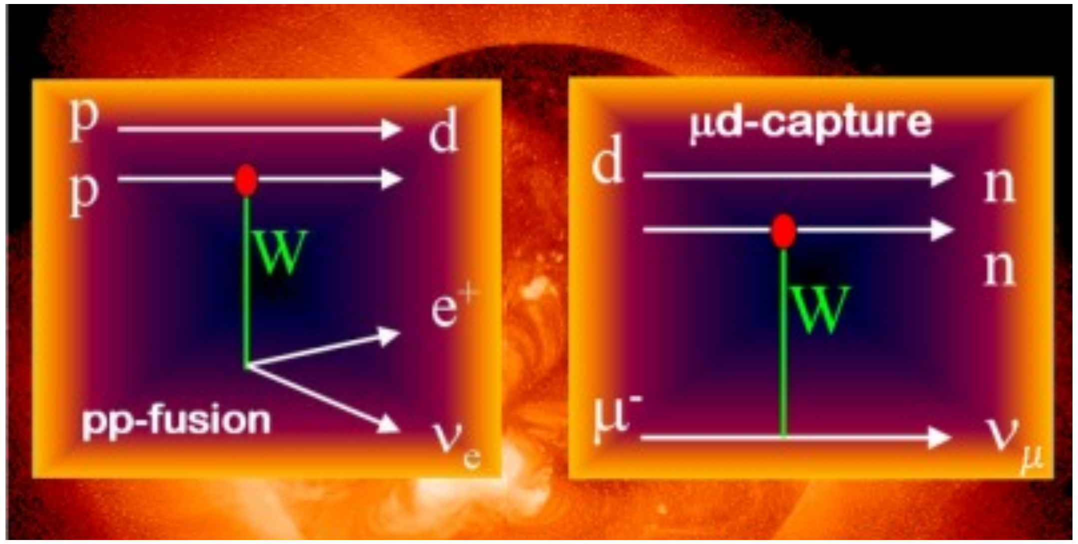

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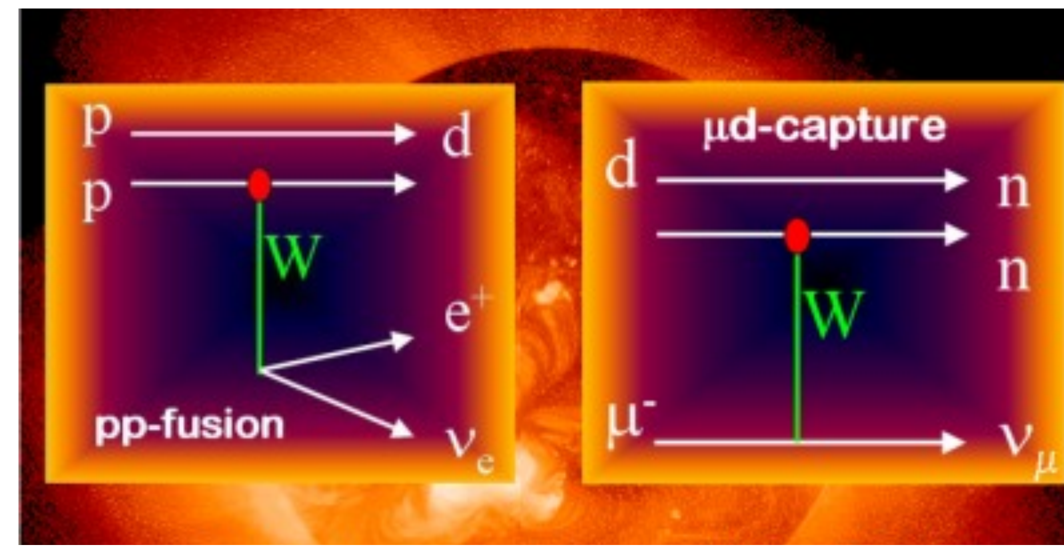

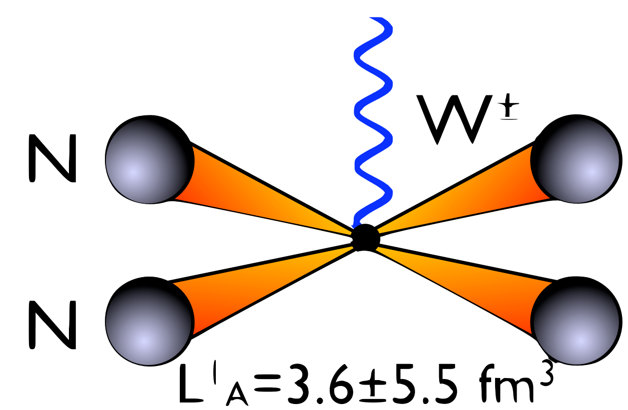

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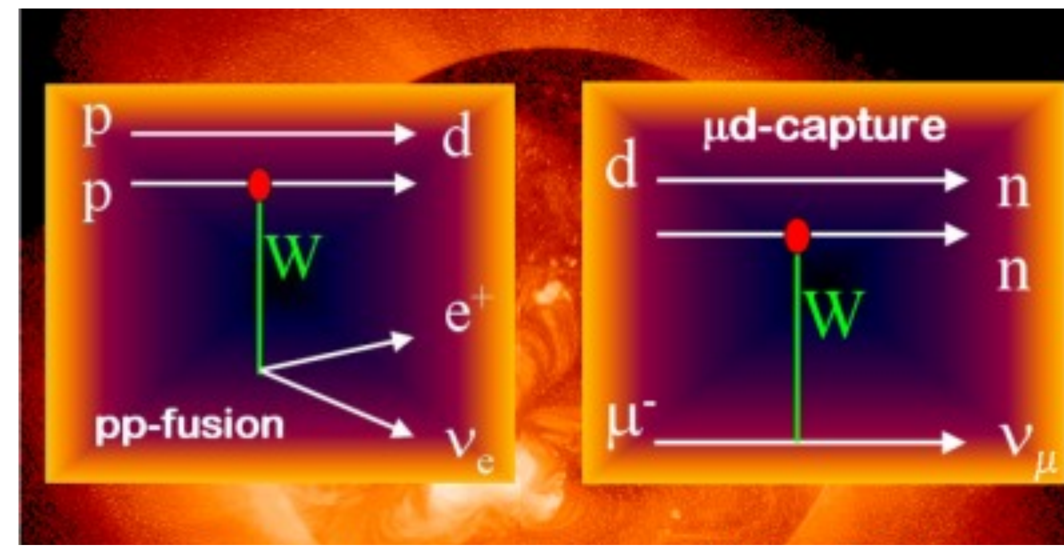

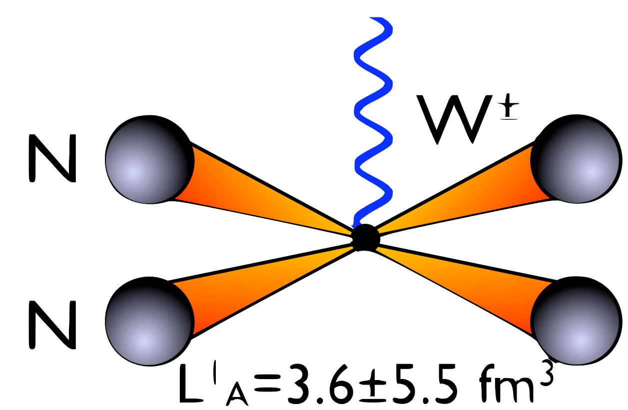

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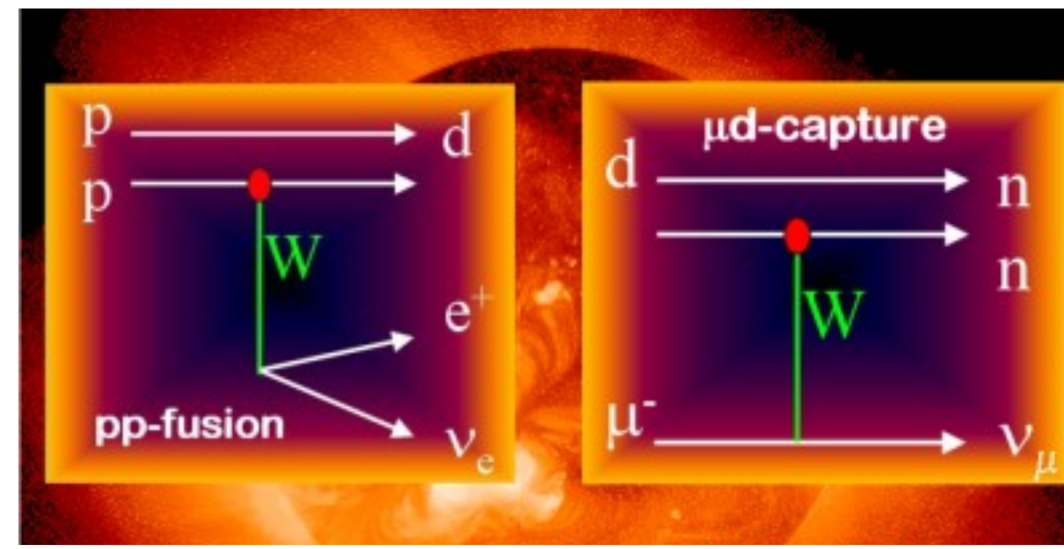

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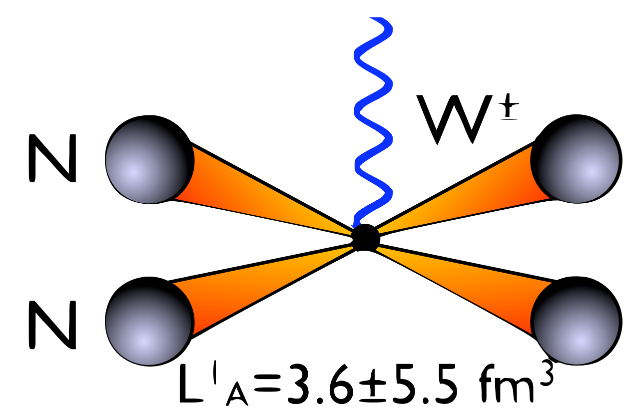
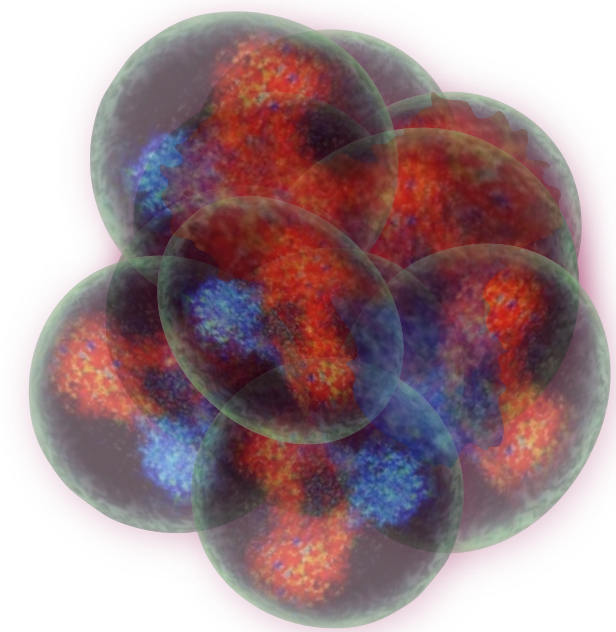


- LQCD: not much harder than spectroscopy

From quarks to nuclei



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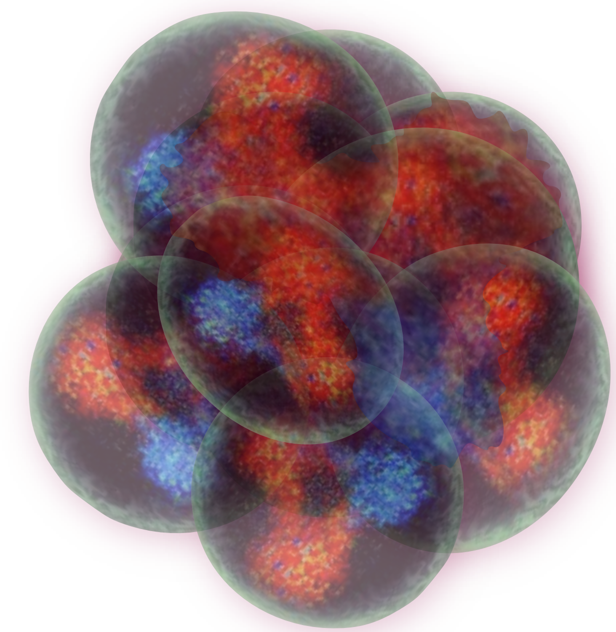

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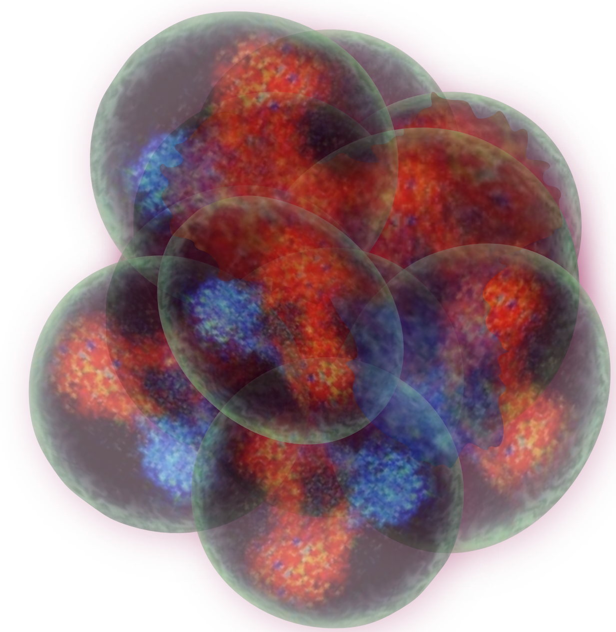

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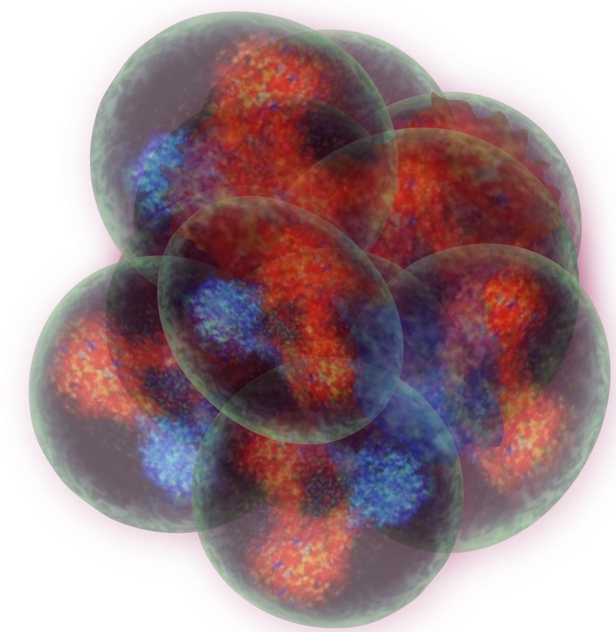

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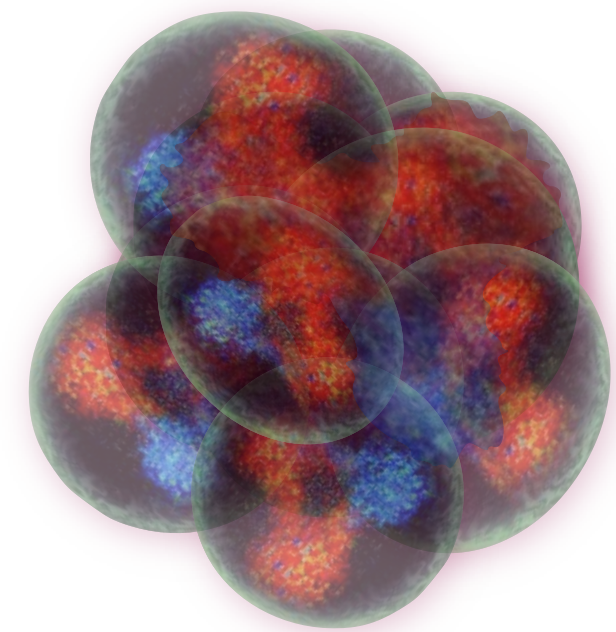

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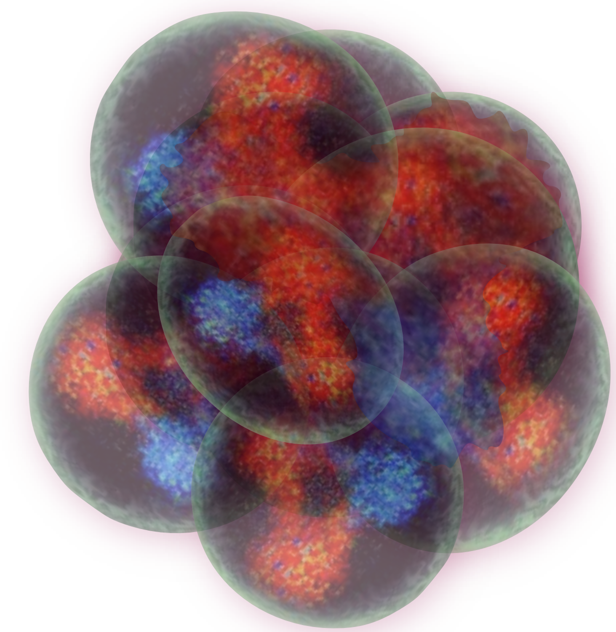

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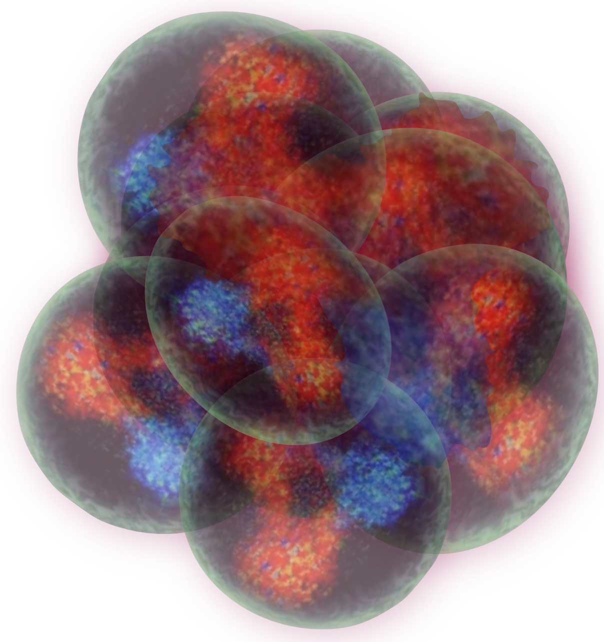

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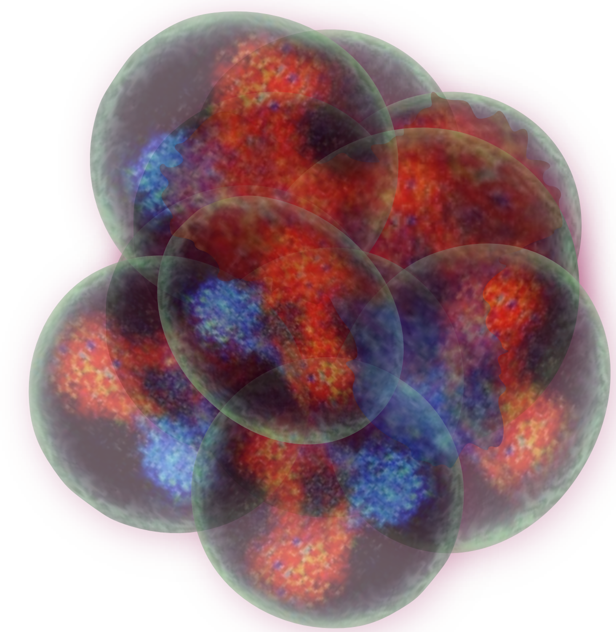

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[FIN]

thanks to

NPLQCD

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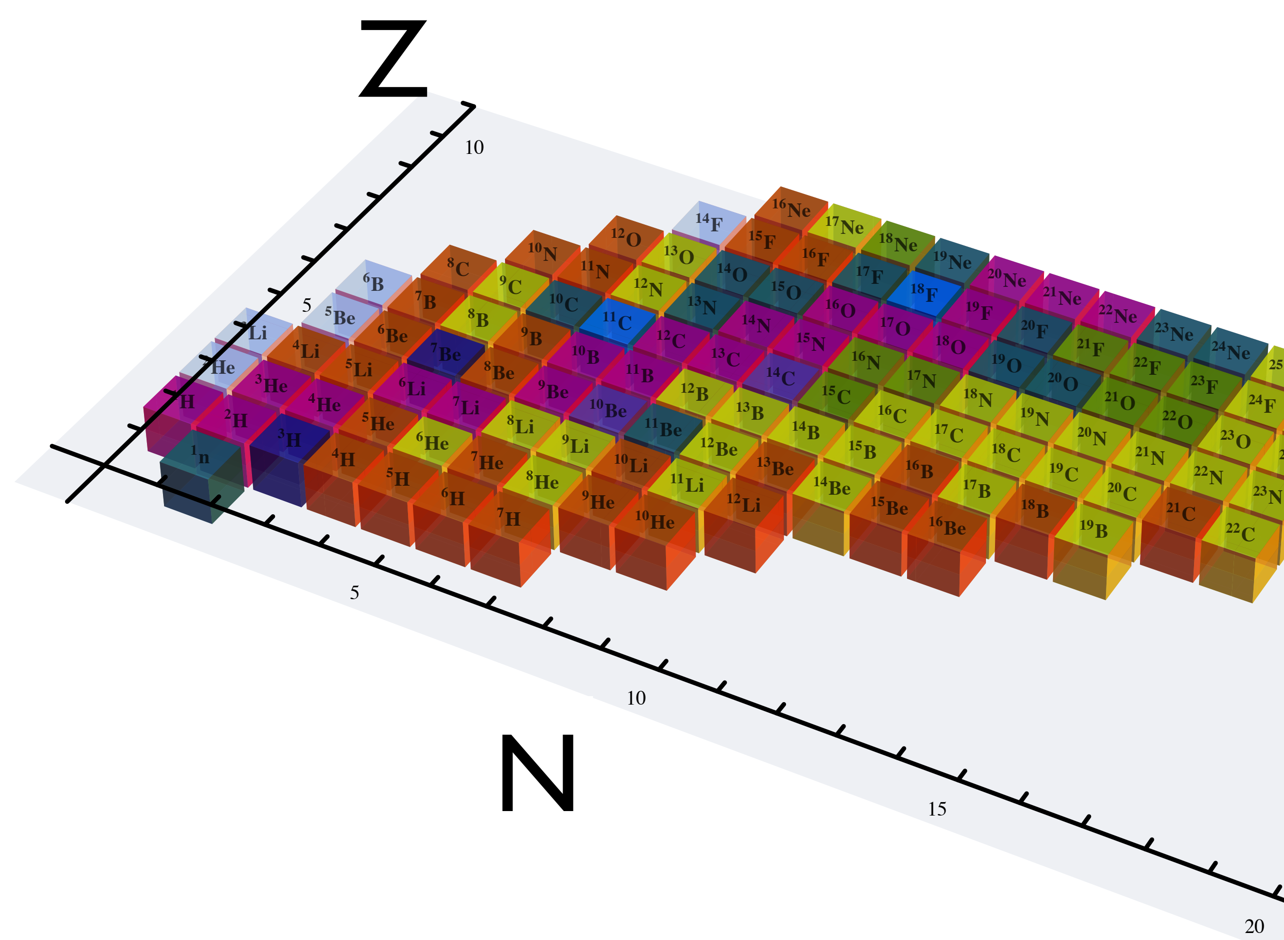

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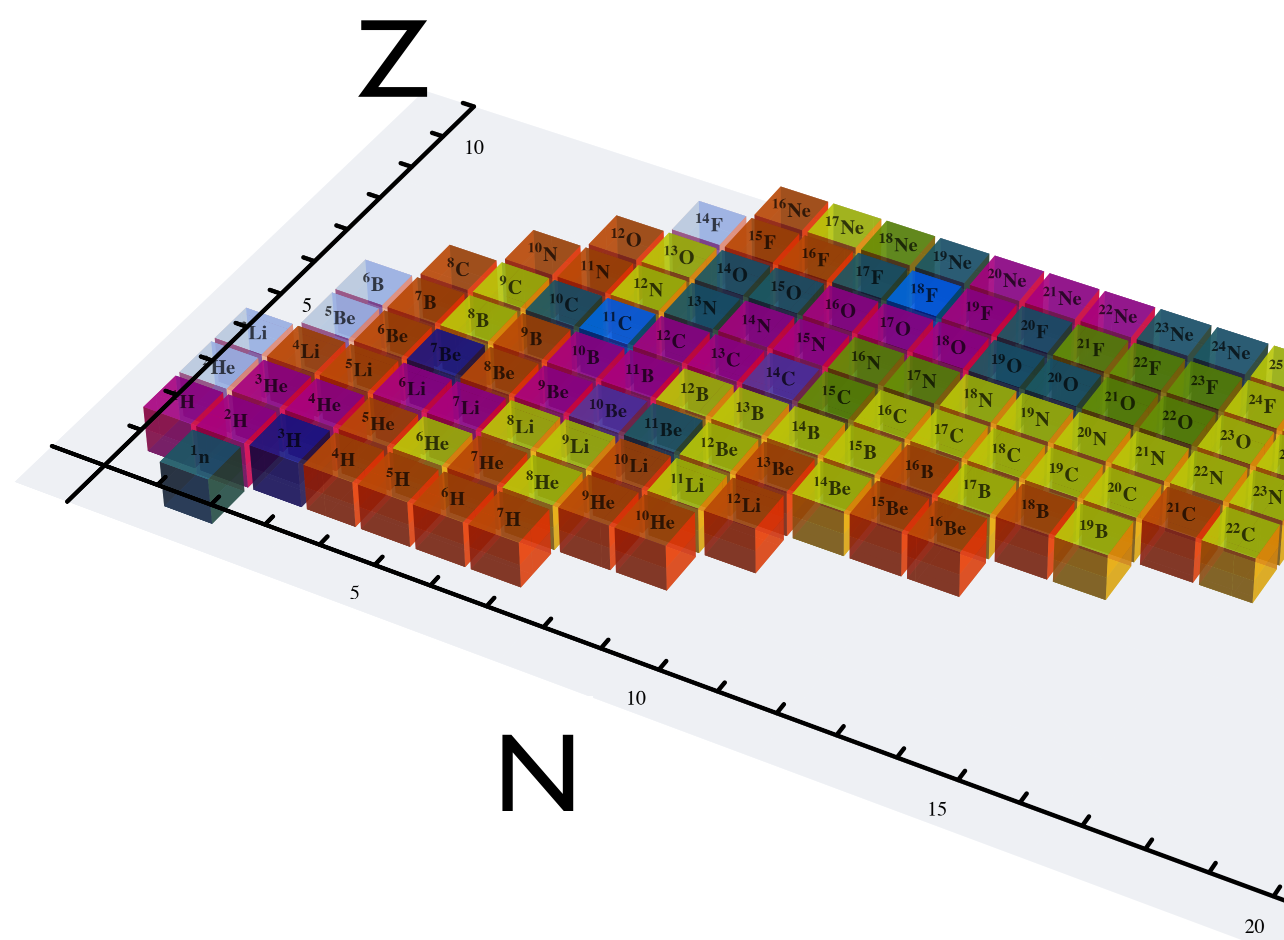

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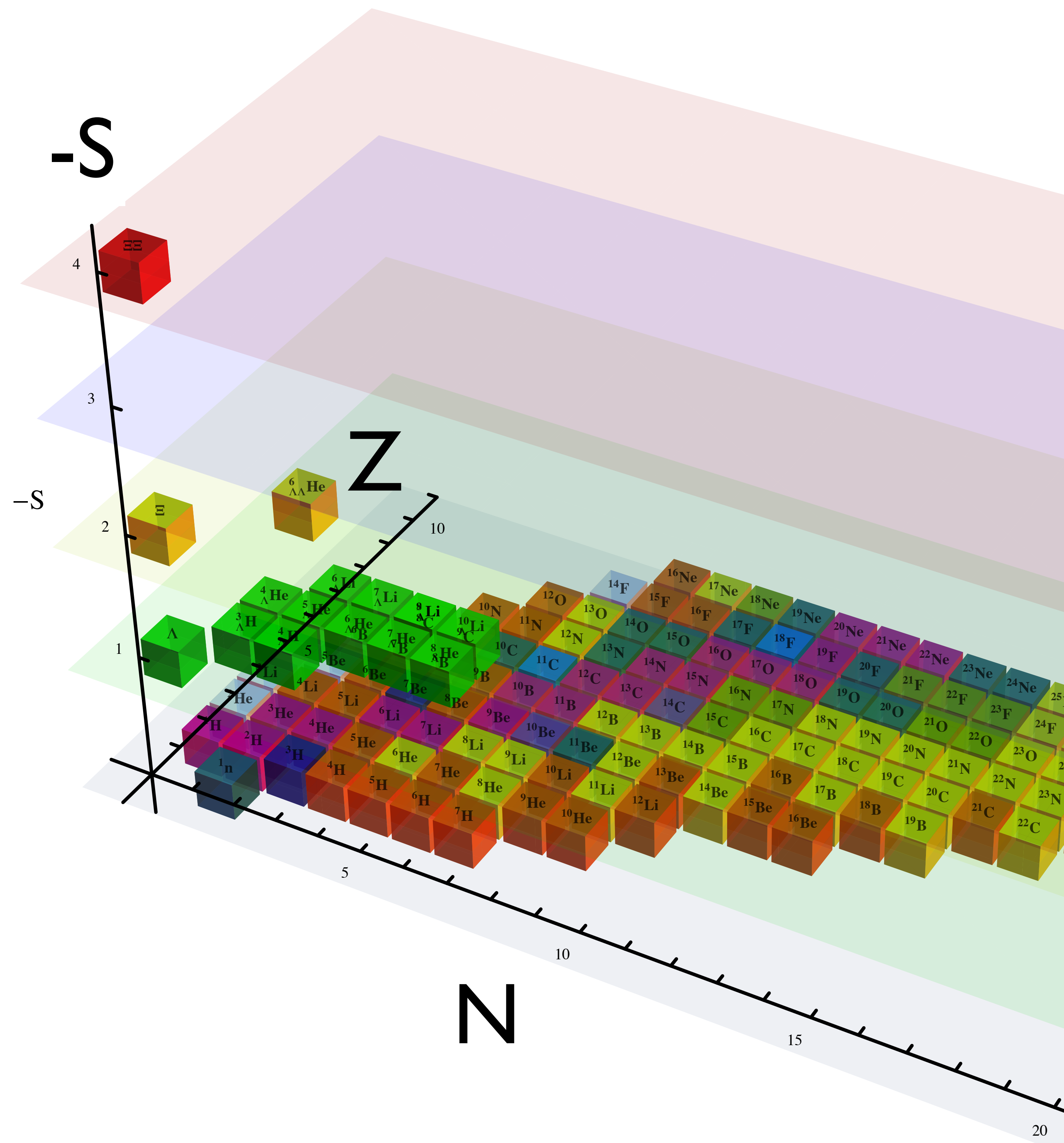

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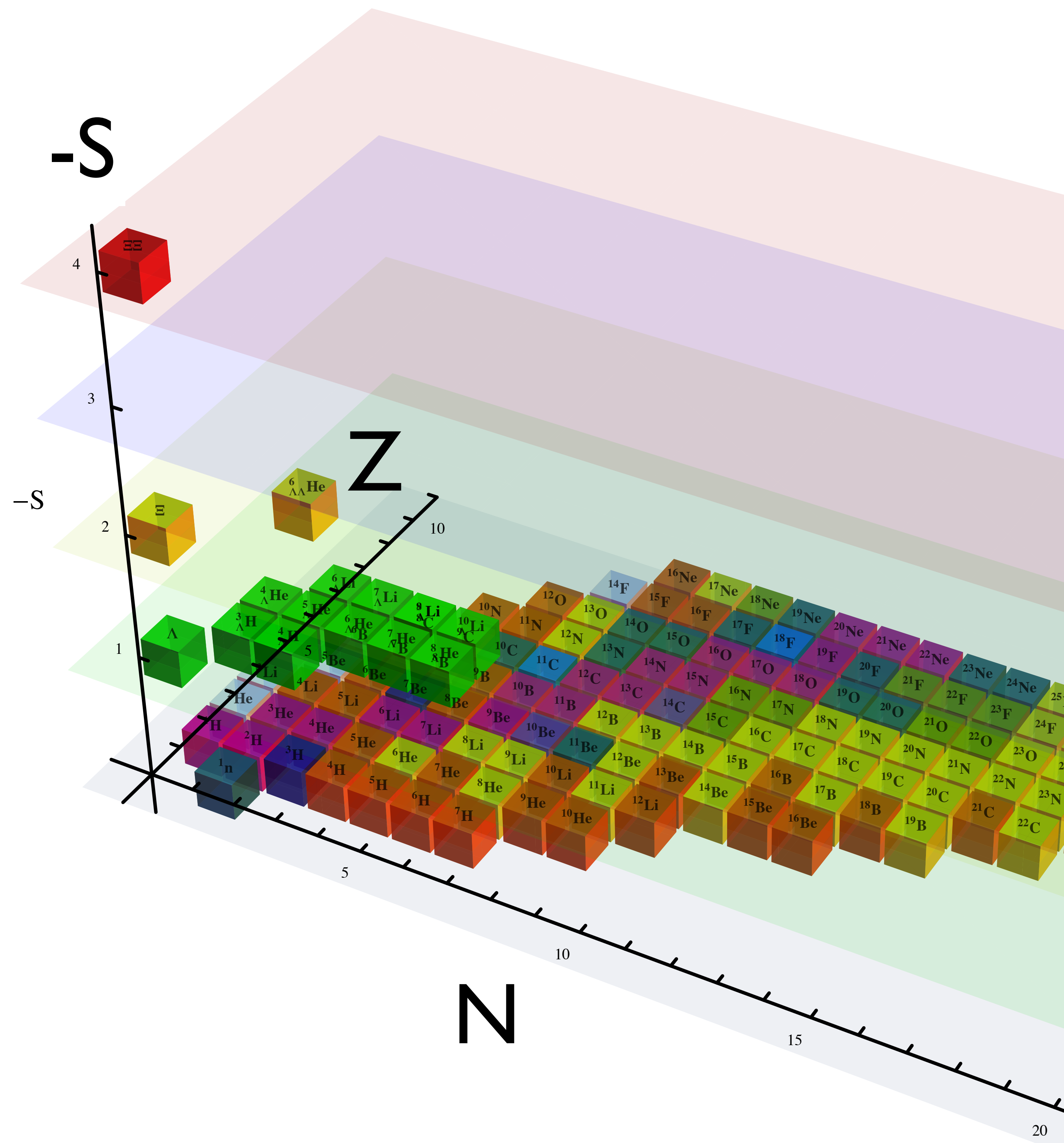

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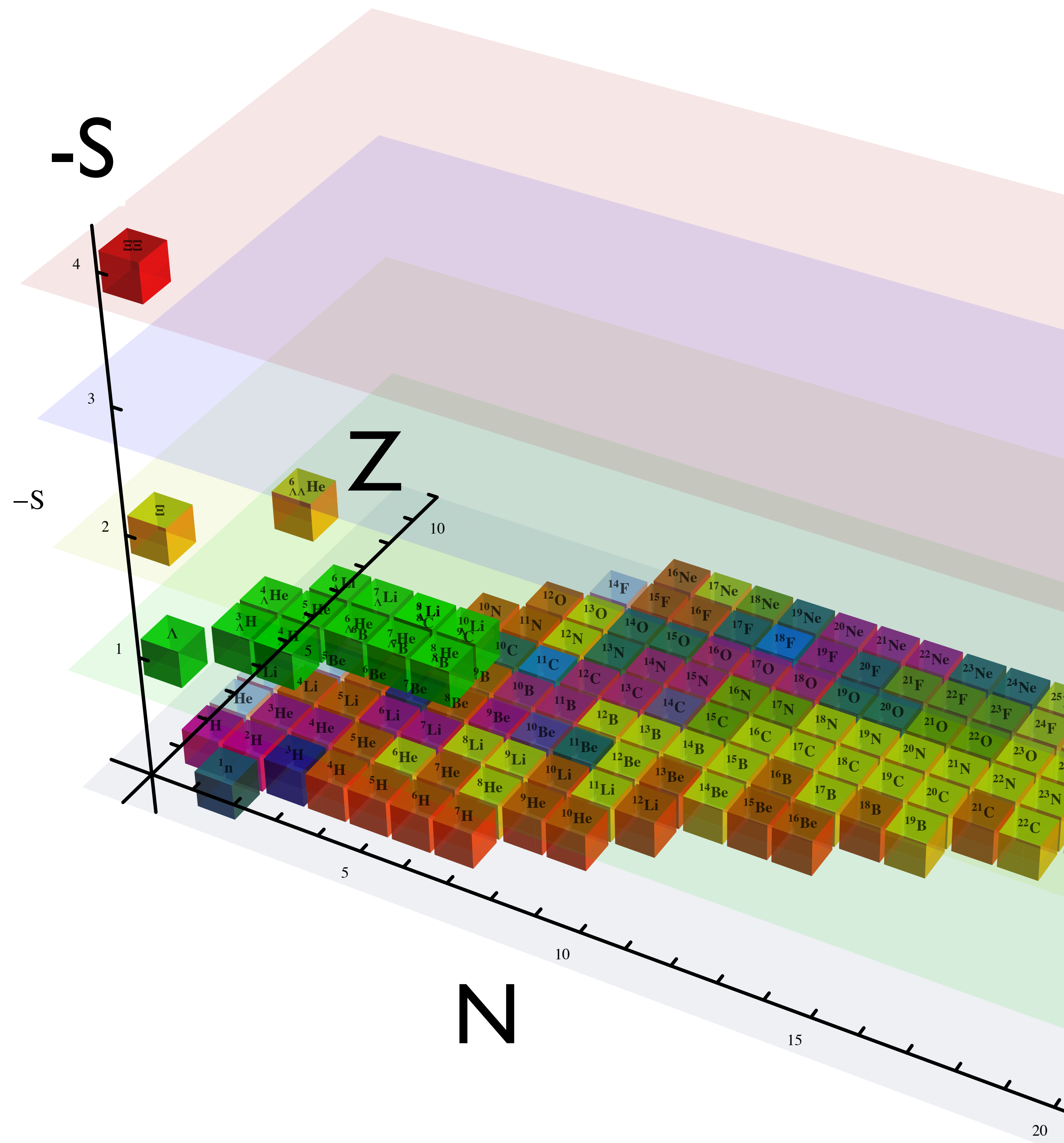

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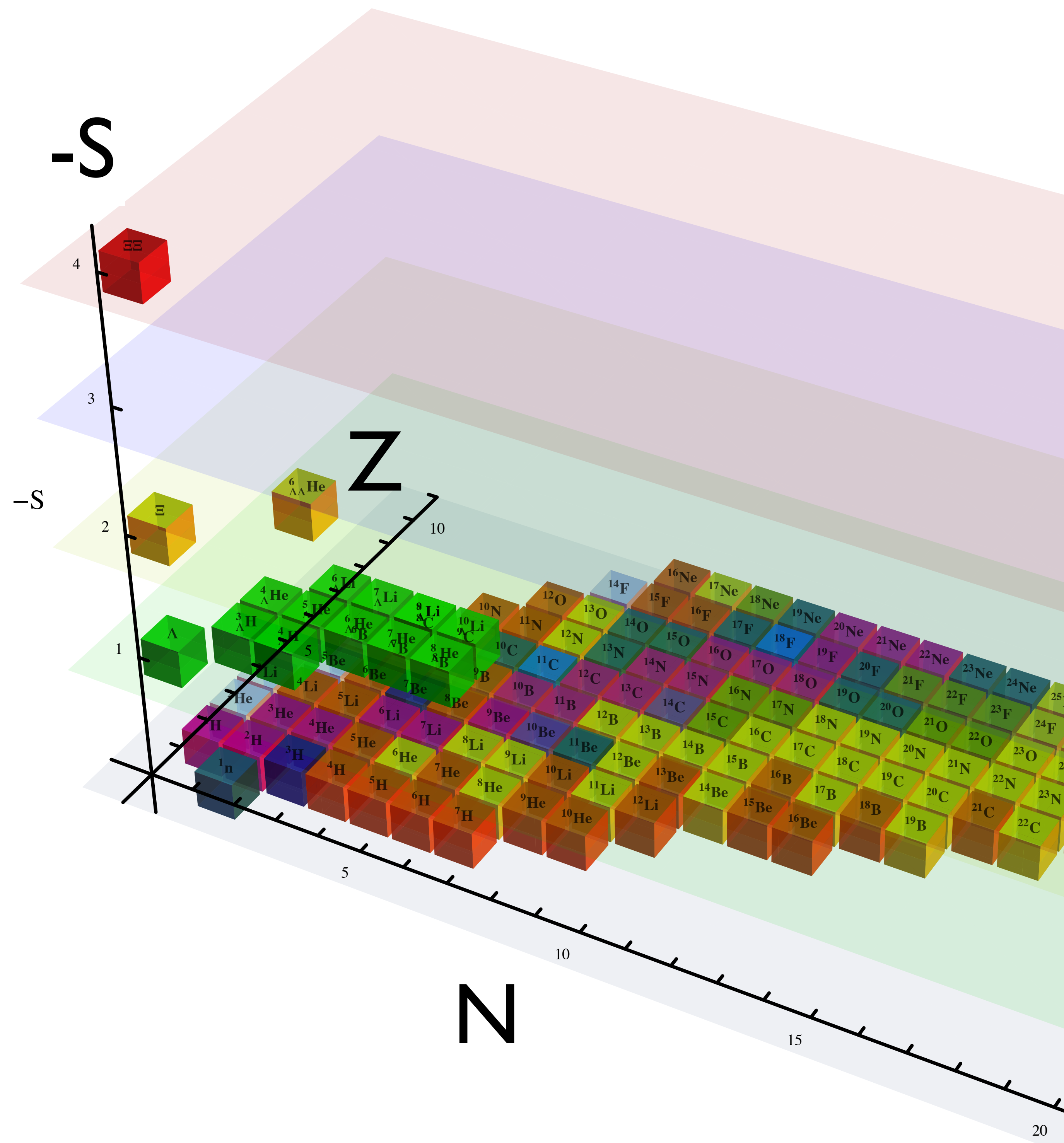

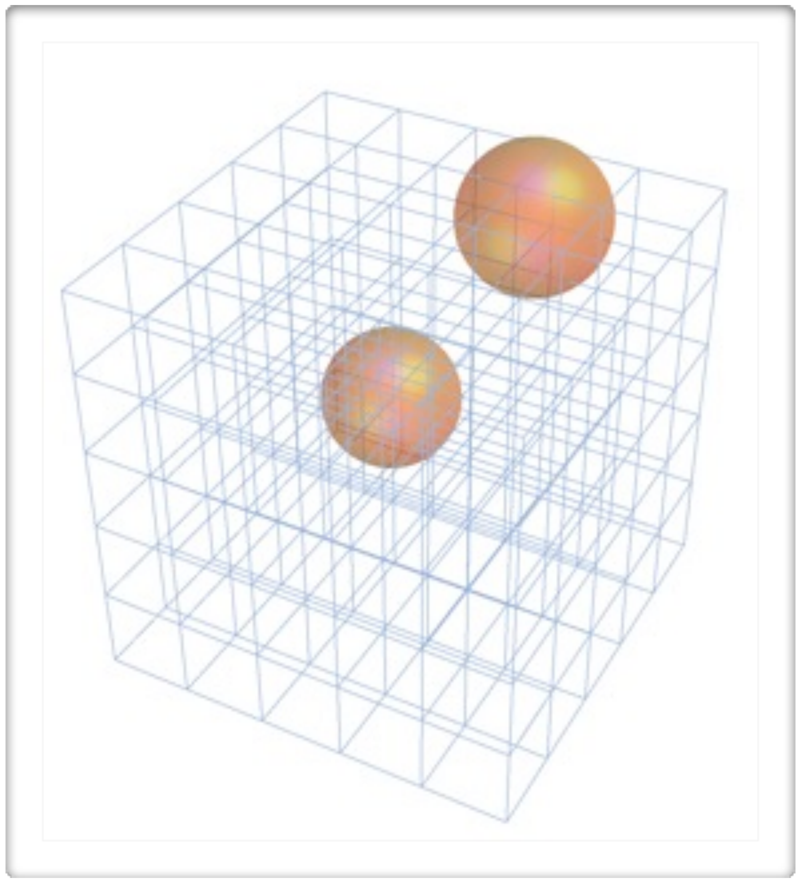

Hadron-hadron scattering

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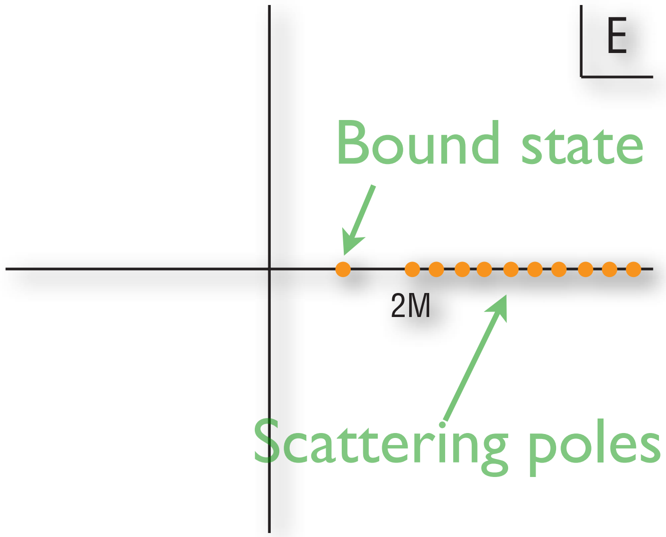

Scattering amplitude
at finite volume

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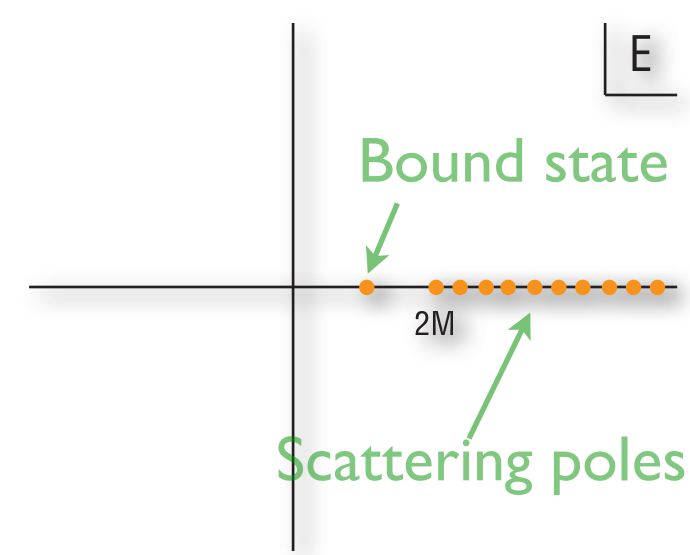
$$E^{(n)} \equiv \sqrt{|\mathbf{q}_{(n)}|^2 + m_A^2} + \sqrt{|\mathbf{q}_{(n)}|^2 + m_B^2}$$



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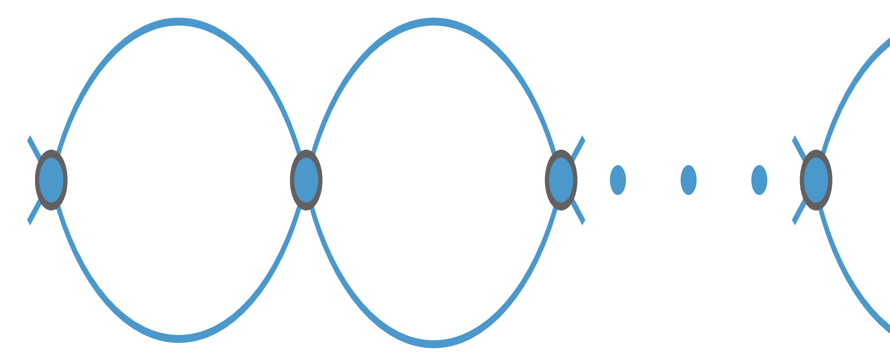
- Lüscher [86]: also true in relativistic field theory
- Volume dependence of two-particle energy levels ⇒ scattering phase-shift, δ(p), up to inelastic threshold

$$E^{(n)} \equiv \sqrt{|\mathbf{q}_{(n)}|^2 + m_A^2} + \sqrt{|\mathbf{q}_{(n)}|^2 + m_B^2}$$

$\mathcal{A} \sim$



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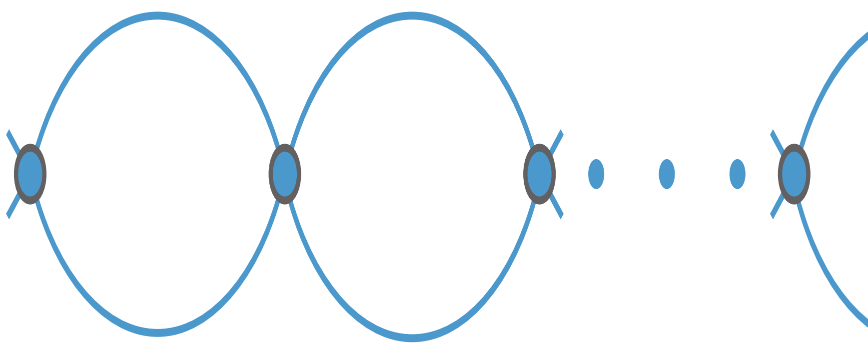
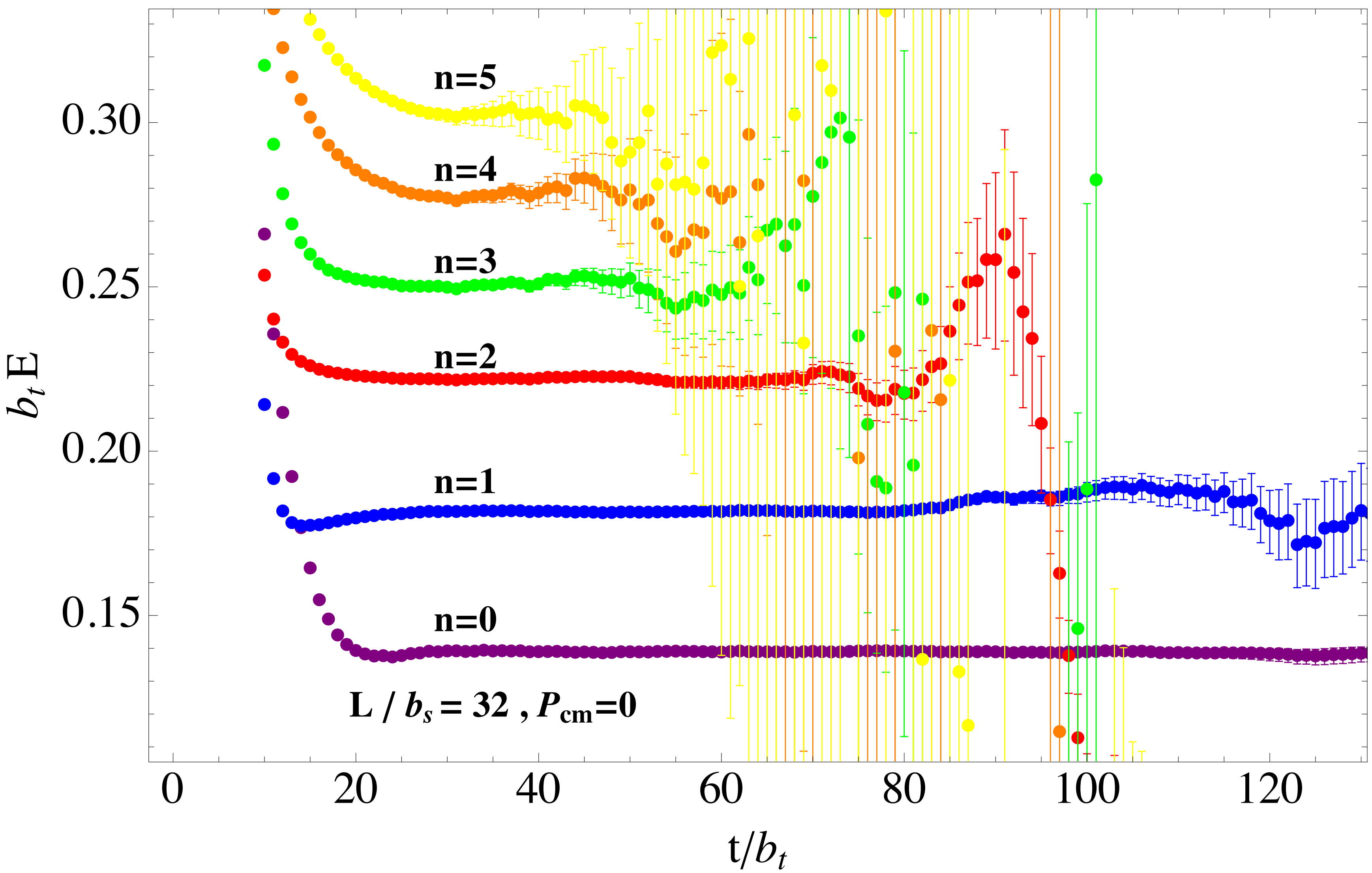
$$q_{(n)} \cot \delta(q_{(n)}) = \frac{1}{\pi \, L} S \left(\frac{q_{(n)} L}{2\pi} \right)$$

$$S(\eta) = \lim_{\Lambda \to \infty} \left[\sum_{\vec{n}}^{|\vec{n}|<\Lambda} \frac{1}{|\vec{n}|^2 - \eta^2} - 4\pi\Lambda \right]$$

$\mathcal{A} \sim$ 

Example: I=2 ππ

- Study multiple energy levels of two pions in a box for multiple volumes and with multiple P_{CM}



@ m_π = 390 MeV

NPLQCD

1107.5023 [prd]

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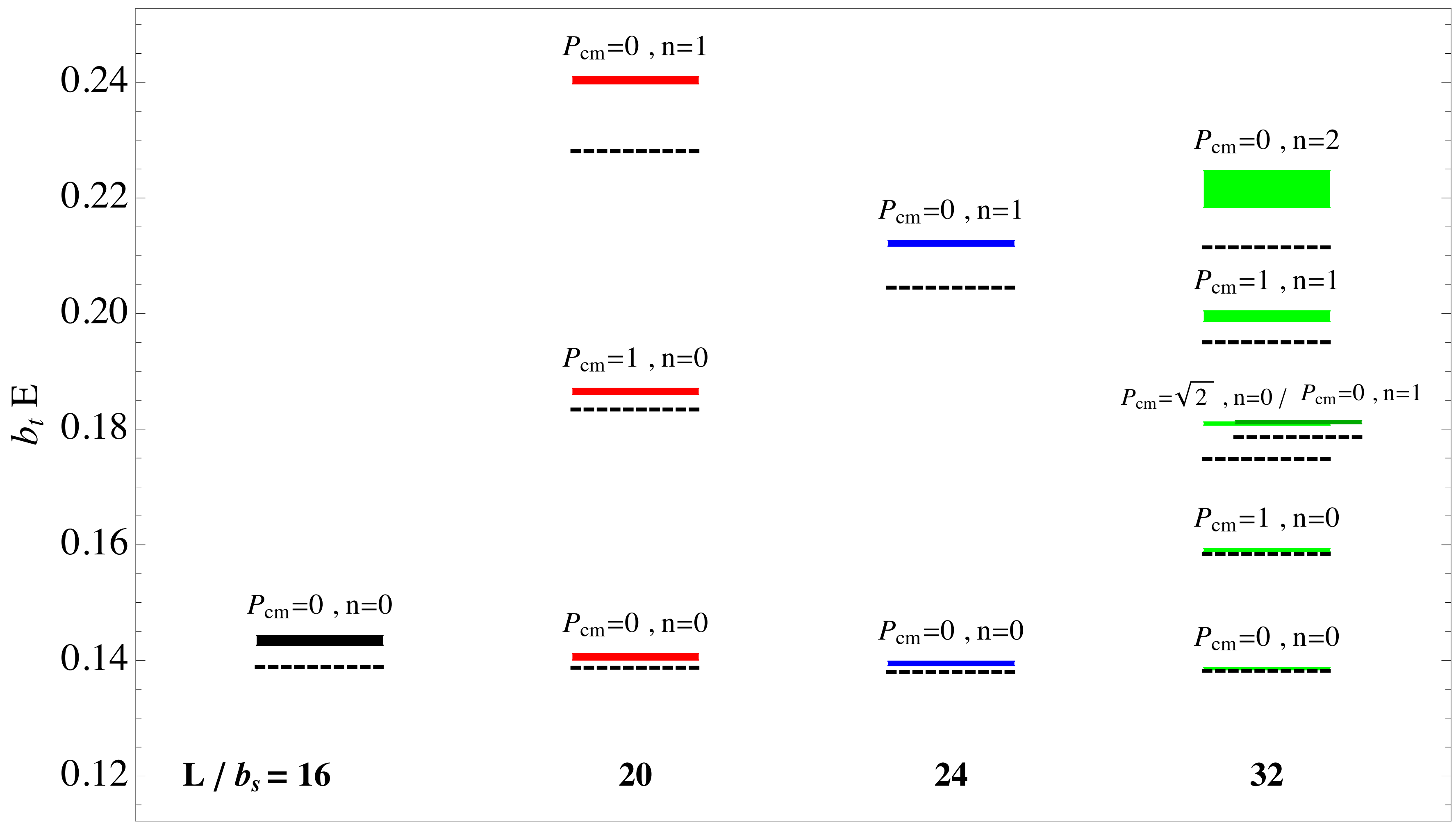

Dashed lines are non-interacting energy levels

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NPLQCD

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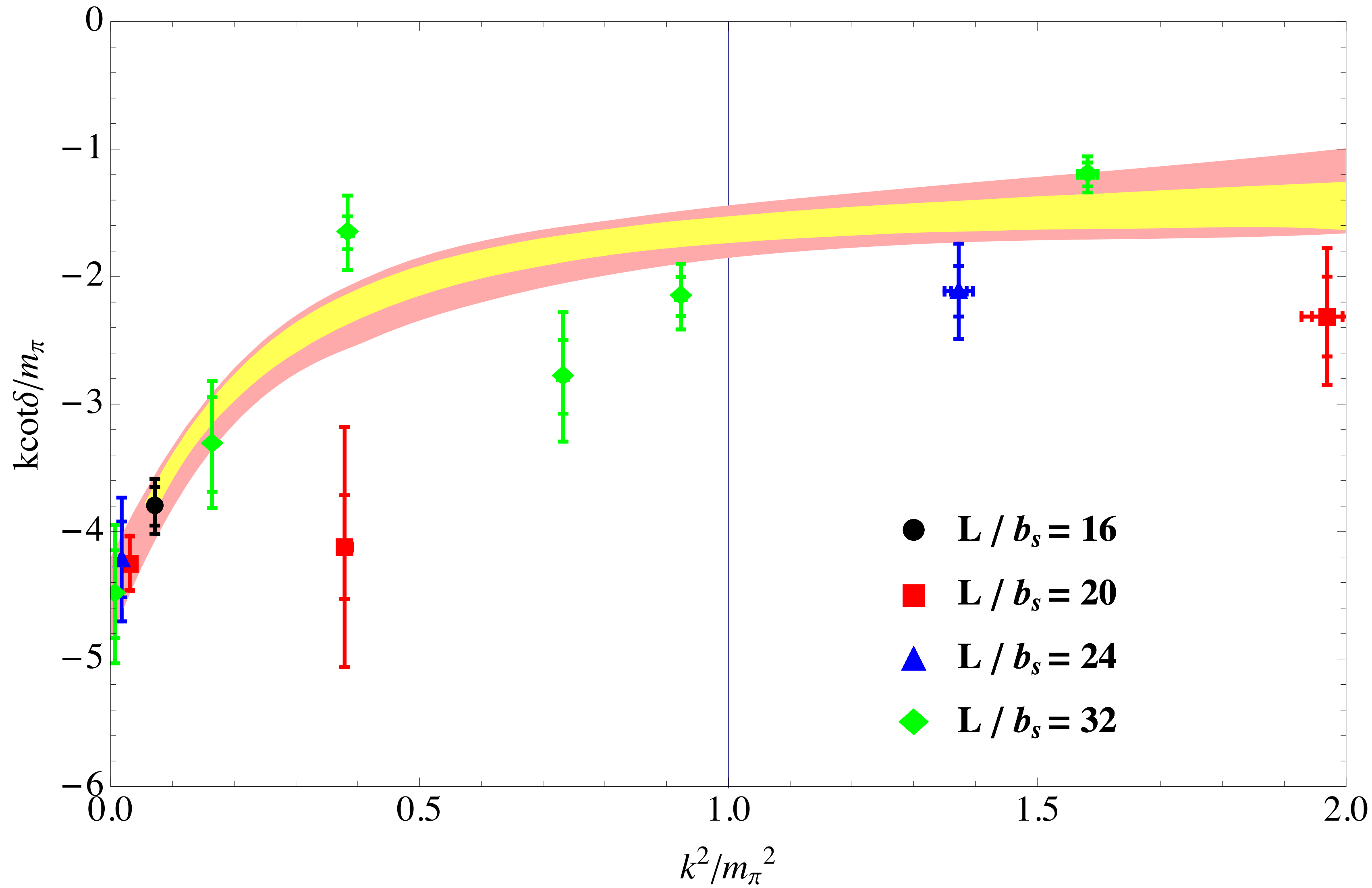
- Allows phase shift to be extracted at multiple energies



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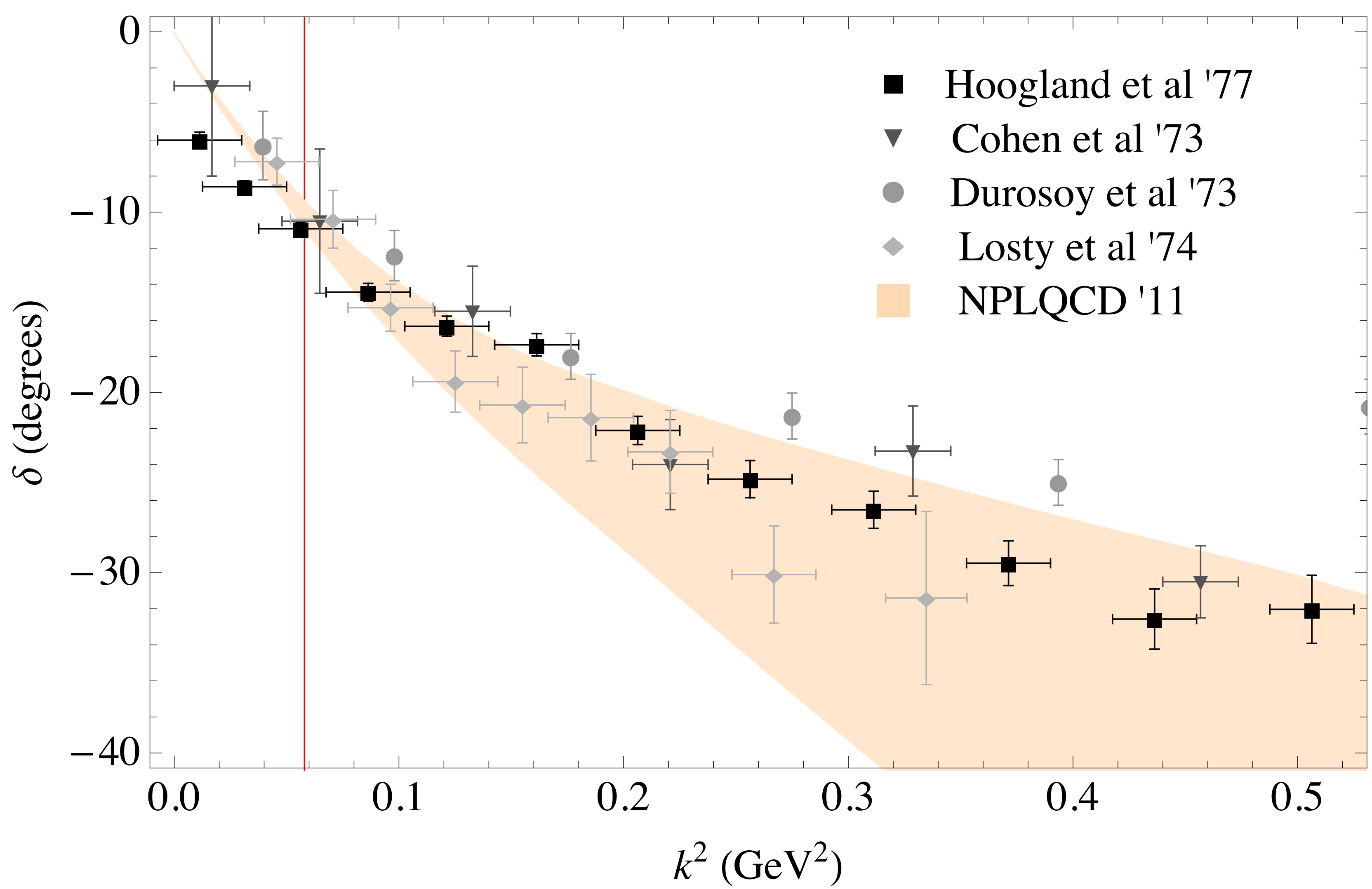
- Combine with chiral perturbation theory to interpolate to physical pion mass



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