Determination of SU(2) ChPT LECs from 2+1 flavor staggered lattice simulations

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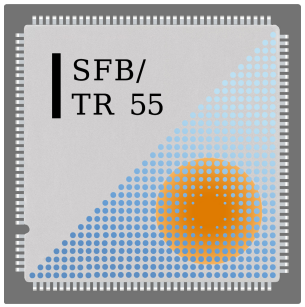

in collaboration with
S. Borsanyi, S. Dürr, Z. Fodor,
S. Krieg, A. Schäfer, K.K. Szabo

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- SU(2)-ChPT: effective meson-lagrangian for pion-interactions
- LECs $\bar{\ell}_3$, $\bar{\ell}_4$ describe quark-mass dependence of f_π, M_π^2; decay constant in chiral limit: f
- used in phenomenological models, FV-corrections of lattice data, . . .

value from lattice-QCD simulations

- ChPT often used to extrapolate lattice results at (slightly) too high $m_{\text{up,down}}$ to physical masses

systematic error?

- compare fits including and excluding the physical point
- 2+1 flavor staggered QCD-simulation with meson-masses: $\approx 135\,\text{MeV} \rightarrow \approx 435\,\text{MeV}$

S. Borsanyi, S. Dürr, Z. Fodor, S. Krieg, A. Schäfer, EES, K.K. Szabo
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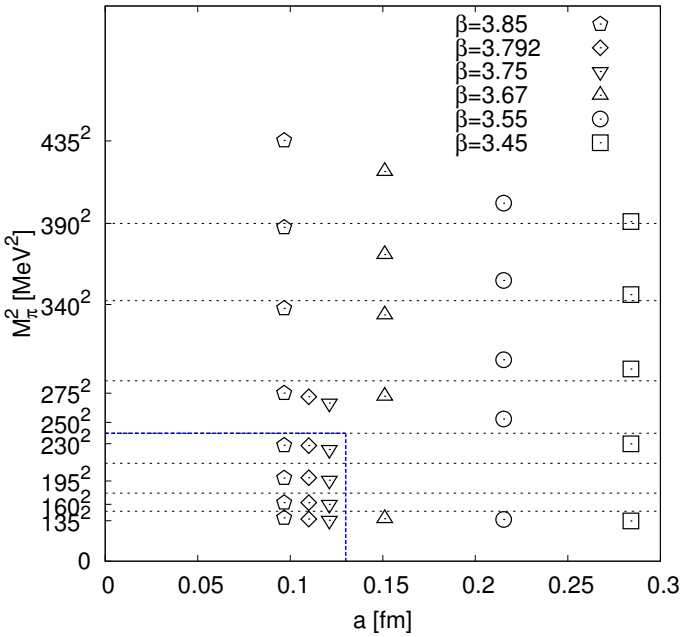
2+1 flavour staggered simulations

- tree-level Symanzik improved gauge action
- 2-level stout-smeared staggererd fermions
- 6 diff. gauge coupl. β: $a \approx 0.097$ – 0.284 fm
- vary light quark mass: $135\,\text{MeV} < M_\pi < 435\,\text{MeV}$
- m_h fixed to m_{strange}: tuned via $M_{\bar{s}s}/M_\phi$
  $(M_{\bar{s}s}^2 = 2M_K^2 - M_\pi^2)$
- measured M_π, f_π
- finite volume corrections: resummed Lüscher-formula (2-, 3-loop) Colangelo, Dürr, Haefeli (2005)
- minimal $M_\pi L \approx 3.2$
- $L^3 \approx (4.3\,\text{fm})^3$ – $(6.8\,\text{fm})^3$
- R_M: 0.1 – 2.7 per-mille, $-R_f$: 0.2 – 7.5 per-mille

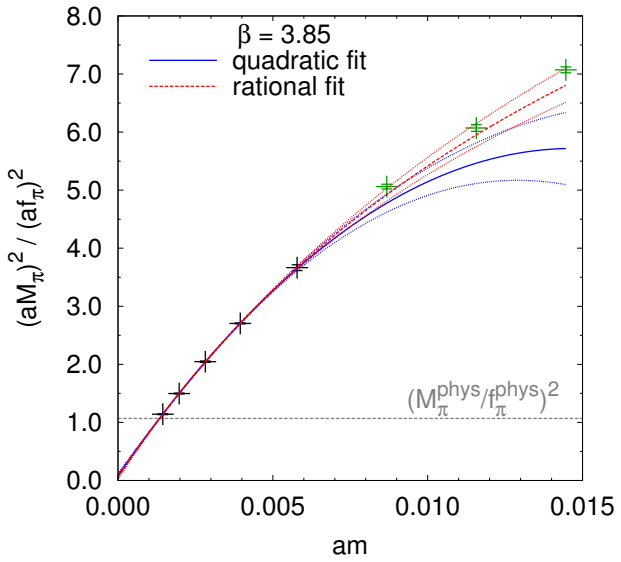


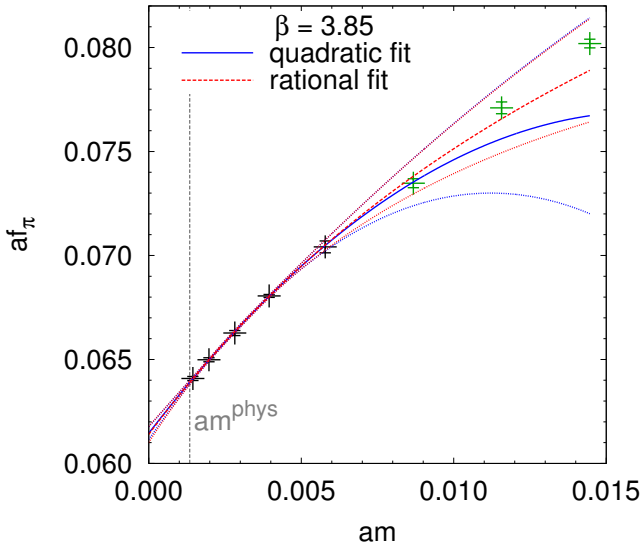
$$M_\pi^\infty = (1 - R_m)\, M_\pi^{FV}$$
$$f_\pi^\infty = (1 - R_f)\, f_\pi^{FV}$$

setting the scale

- at each β
  - fit ratio $(aM_\pi)^2/(af_\pi)^2$, from $(M_\pi^{\text{phys}}/f_\pi^{\text{phys}})^2$:
    physical point $am_\beta^{\text{phys}} = (am_u + am_d)/2$
  - fit (af_π) from $(af_\pi)|_{am_{\text{phys}}} = a_\beta\, f_\pi^{\text{phys}}$:
    lattice scale $1/a_\beta$
- quadratic and rational fit (3 param.)
- typically include 4 or 5 lightest data points
- combined results

β	3.45	3.55	3.67
$1/a$ [GeV]	0.6947(07)	0.9165(12)	1.3063(25)
β	3.75	3.792	3.85
$1/a$ [GeV]	1.6288(15)	1.7935(22)	2.0410(35)





green points excluded from fit

NLO-SU(2) ChPT fits

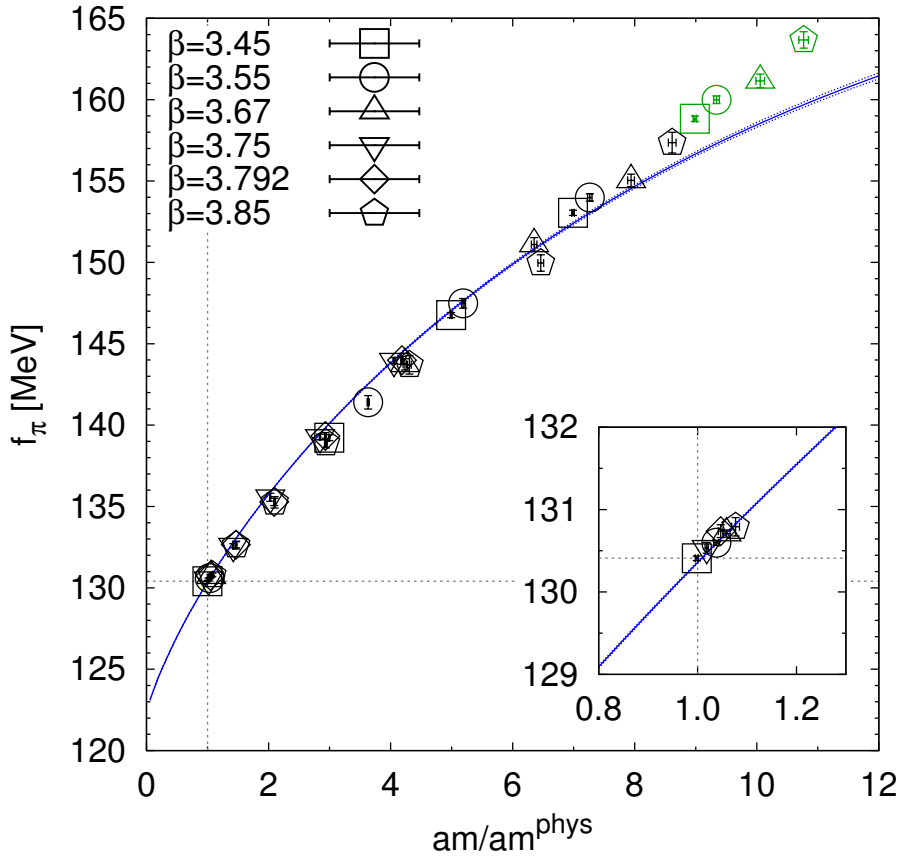
$$M_\pi^2 = \left(\frac{1}{a}\right)^2 (aM_\pi)^2 = \chi\left[1 + \frac{\chi}{16\pi^2 f^2}\log\frac{\chi}{\Lambda_3^2}\right]$$

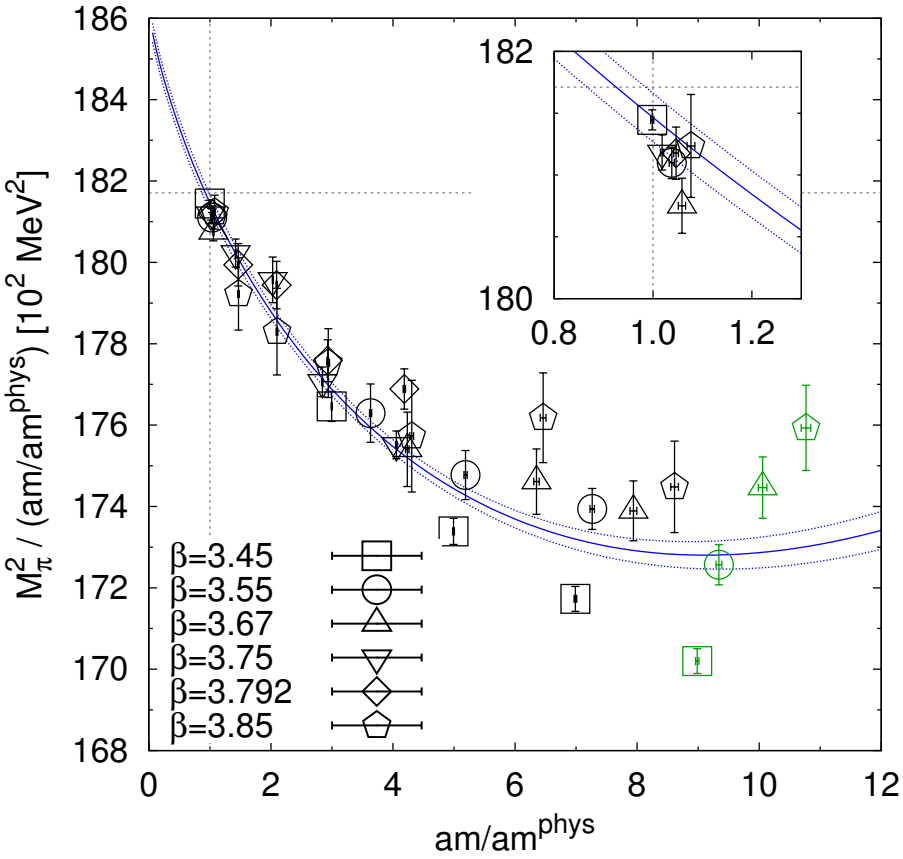
$$f_\pi = \frac{1}{a}(af_\pi) = f\left[1 - \frac{\chi}{8\pi^2 f^2}\log\frac{\chi}{\Lambda_4^2}\right]$$

$$\chi = 2B\,m = (2Bm^{\text{phys}})\frac{am}{am^{\text{phys}}} = \chi^{\text{phys}}\frac{am}{am^{\text{phys}}}$$

- combined (M_π^2, f_π) global (different lattice spacings) fits
- using $1/a$, am^{phys} from scale-setting
- unconstrained fit: 4 fit parameters: f, χ^{phys}, Λ_3, Λ_4
- parameter-reduced fit: 2 fit parameters: f, χ^{phys}
  and 2 constraints: $M_\pi^2\big|_{m=m^{\text{phys}}} = (M_\pi^{\text{phys}})^2$, $f_\pi\big|_{m=m^{\text{phys}}} = f_\pi^{\text{phys}}$ $(\rightarrow\ \Lambda_3,\ \Lambda_4)$

unconstr. NLO fit, all β, $135\,\text{MeV} < M_\pi < 390\,\text{MeV}$

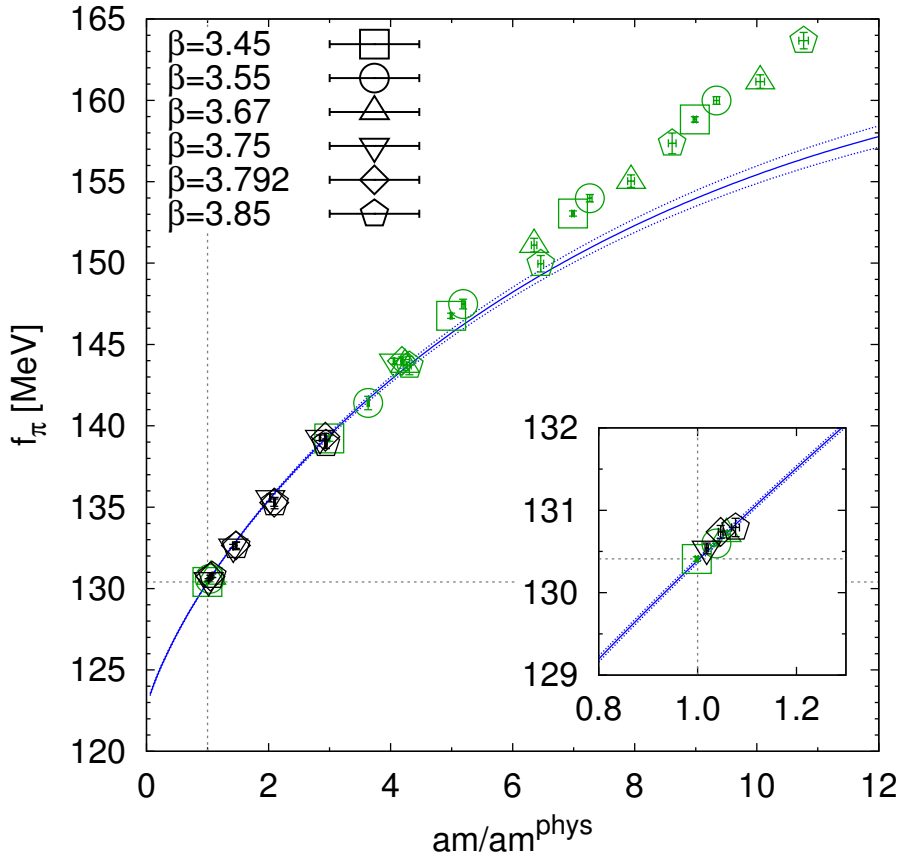


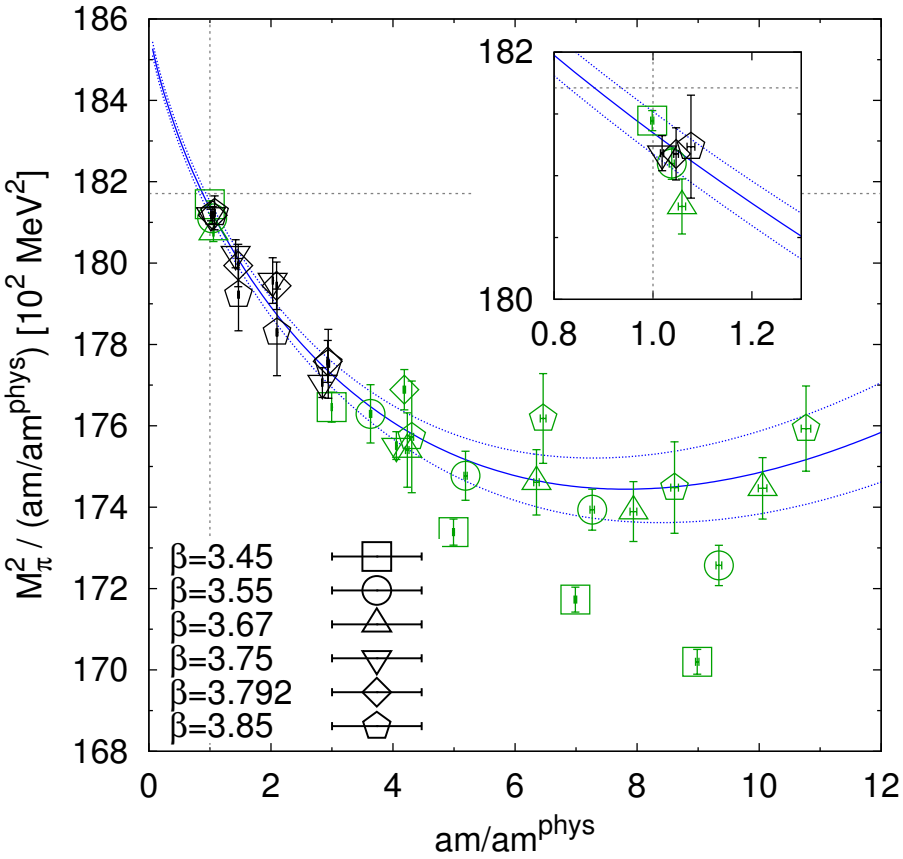


green points are excluded from fit

$\chi^2/\text{d.o.f.} \approx 4.1 \quad \rightarrow \quad$ reduce fit ranges in M_π, $1/a$

unconstr. NLO fit, $1/a > 1.6$ GeV, 135 MeV $< M_\pi <$ 240 MeV

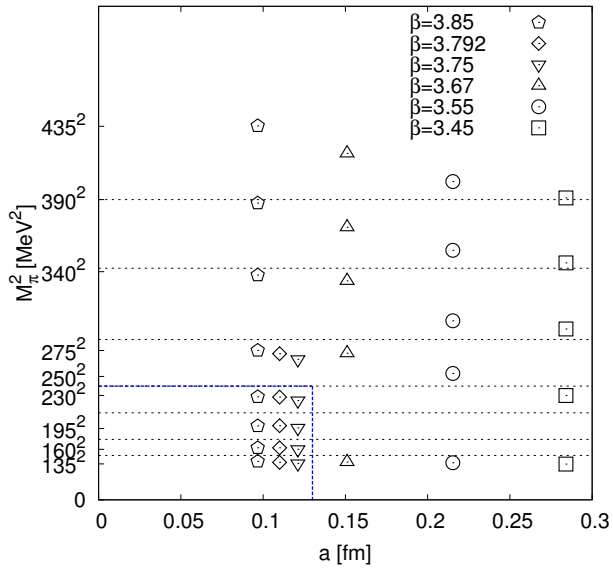


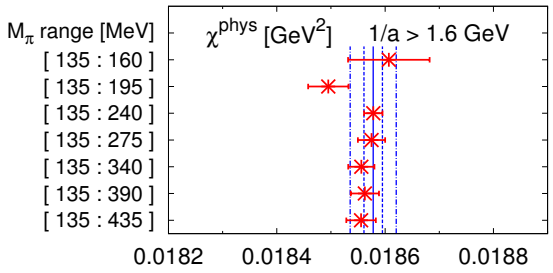


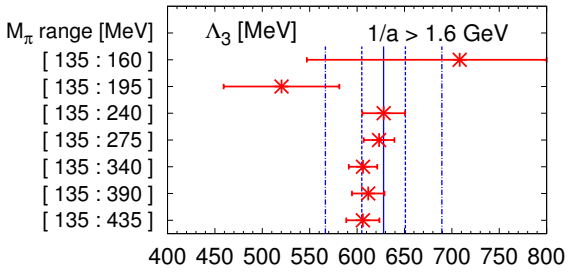
green points are excluded from fit

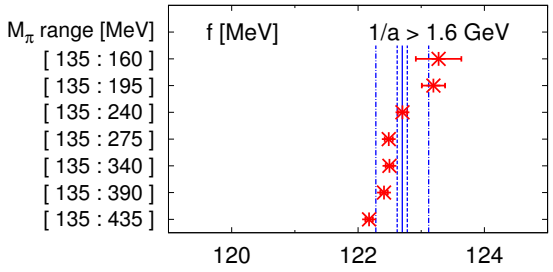
$$\chi^2/\text{d.o.f.} \approx 1.4$$

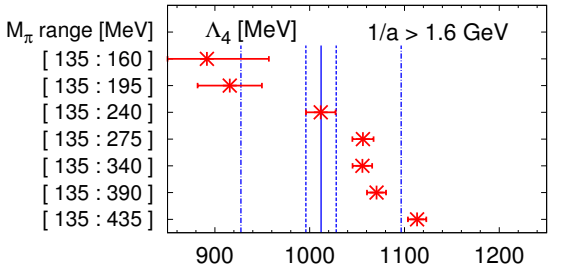
unconstrained NLO fits: different M_π-ranges

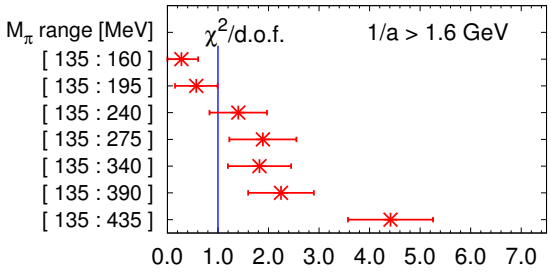








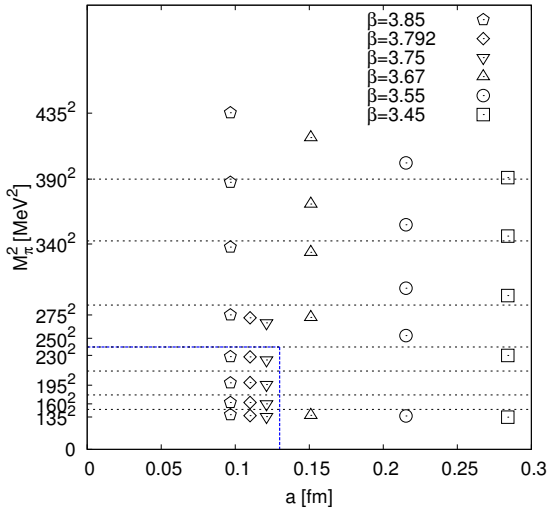


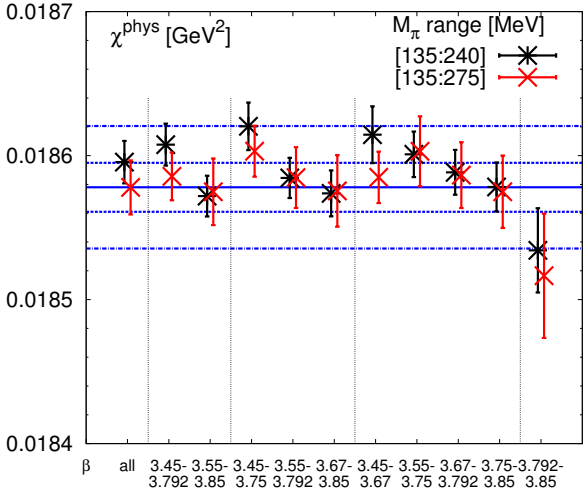


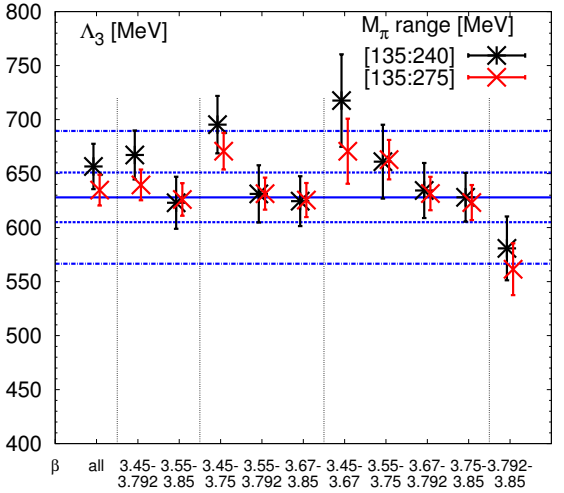
$$\chi^{\text{phys}} = 0.018578(17)(39)\ \text{GeV}^2 \qquad \Lambda_3 = 628(23)(57)\ \text{MeV}$$

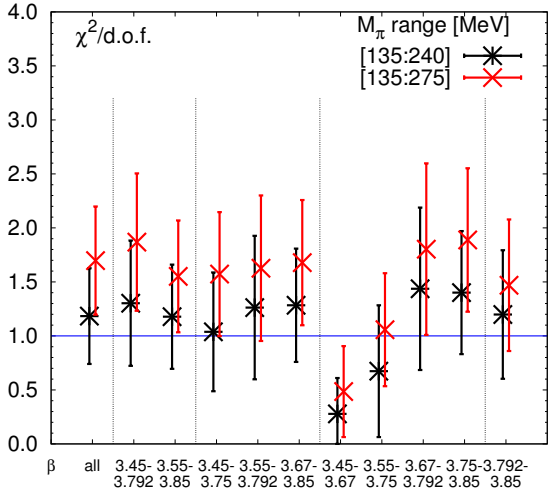
$$f = 122.70(08)(41)\ \text{MeV} \qquad \Lambda_4 = 1,012(16)(83)\ \text{MeV}$$

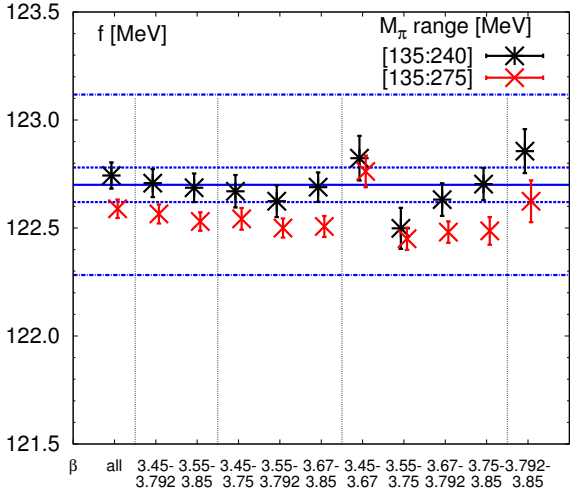
unconstrained NLO fits: different $1/a$-ranges

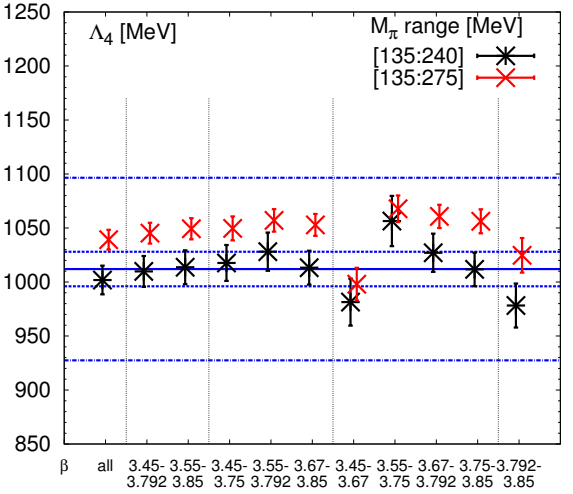












parameter-reduced NLO-fits

physical point constraints

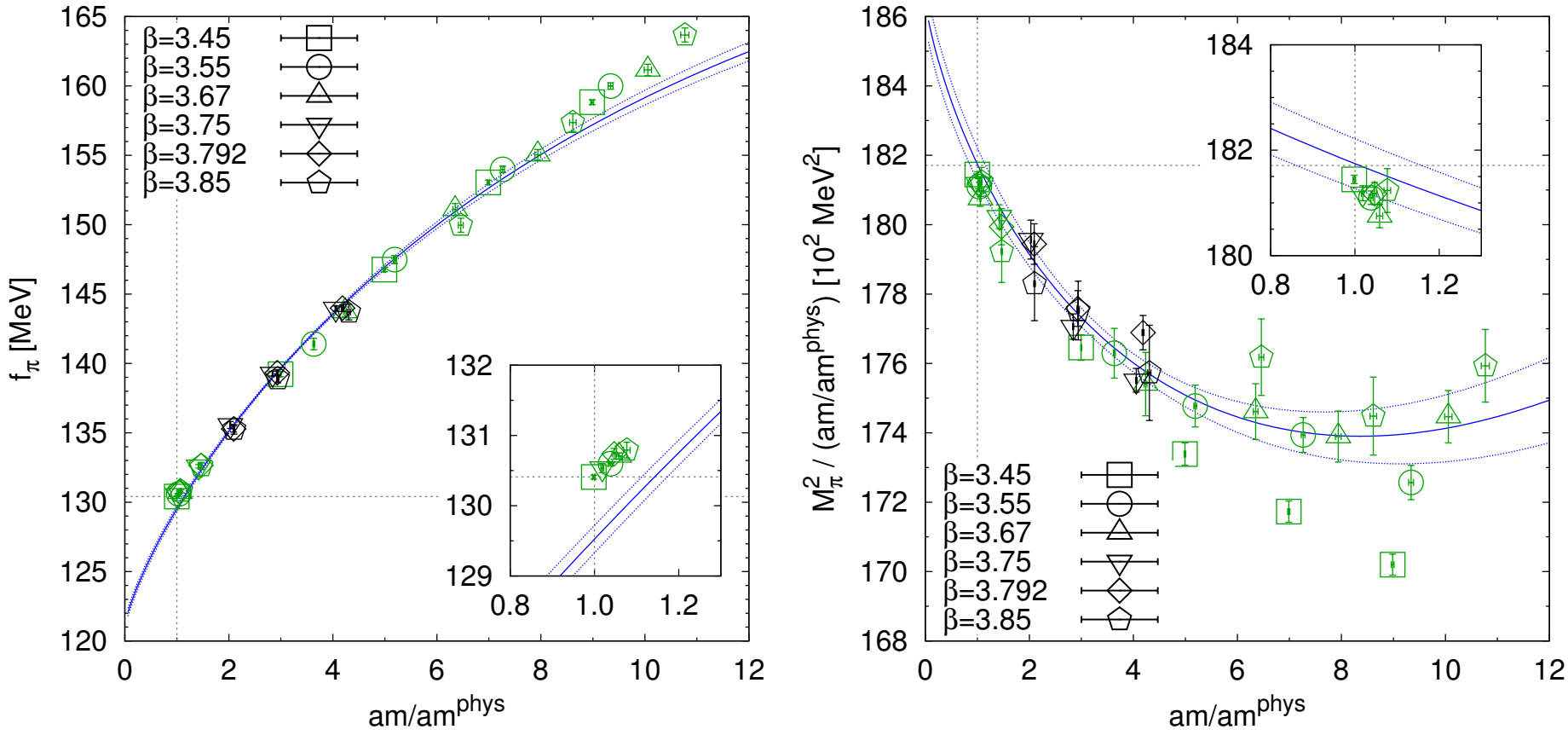
$$M_\pi^2\Big|_{m=m^{\text{phys}}} = (M_\pi^{\text{phys}})^2\,, \qquad f_\pi\Big|_{m=m^{\text{phys}}} = f_\pi^{\text{phys}}$$

repeated same analysis (only 2 fit-parameters left). . .

combined results from $1/a > 1.6\,\text{GeV}$, $135\,\text{MeV} < M_\pi < 240\,\text{MeV}$

	unconstrained	parameter-reduced	**final**
$\chi^{\text{phys}}/(10^{-2}\text{GeV}^2)$	$1.8578(17)(39)$	$1.8639(18)(44)$	$\mathbf{1.8609(18)(74)}$
f/MeV	$122.70(08)(41)$	$122.73(06)(28)$	$\mathbf{122.72(07)(35)}$
Λ_3/MeV	$628(23)(57)$	$678(40)(119)$	$\mathbf{653(32)(101)}$
Λ_4/MeV	$1,012(16)(83)$	$1,006(15)(71)$	$\mathbf{1,009(16)(77)}$
$\bar{\ell}_3$	$3.08(07)(19)$	$3.23(12)(30)$	$\mathbf{3.16(10)(29)}$
$\bar{\ell}_4$	$4.03(03)(17)$	$4.02(03)(14)$	$\mathbf{4.03(03)(16)}$
f_π^{phys}/f	$1.0627(05)(30)$	$1.0626(06)(24)$	$\mathbf{1.0627(06)(27)}$

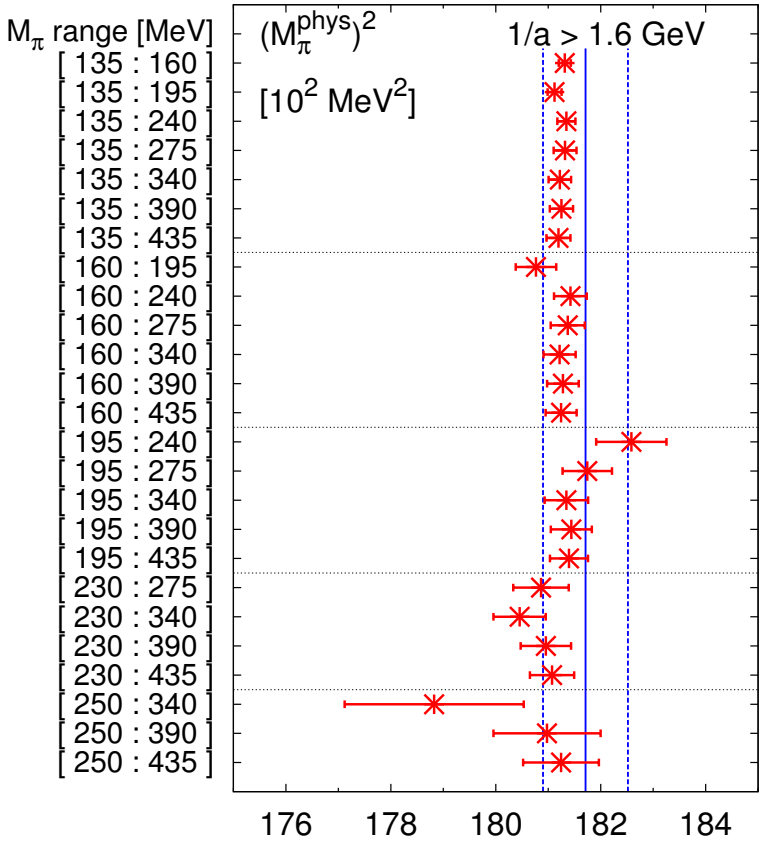
unconstrained NLO fit, $1/a > 1.6$ GeV, 195 MeV $< M_\pi <$ 275 MeV

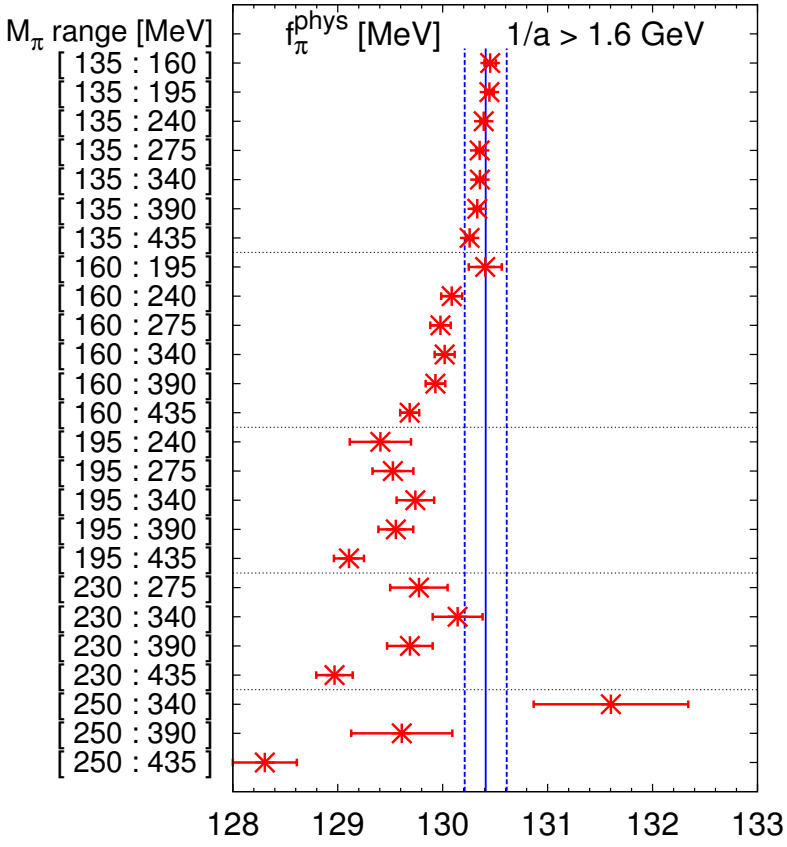


green points are excluded from fit

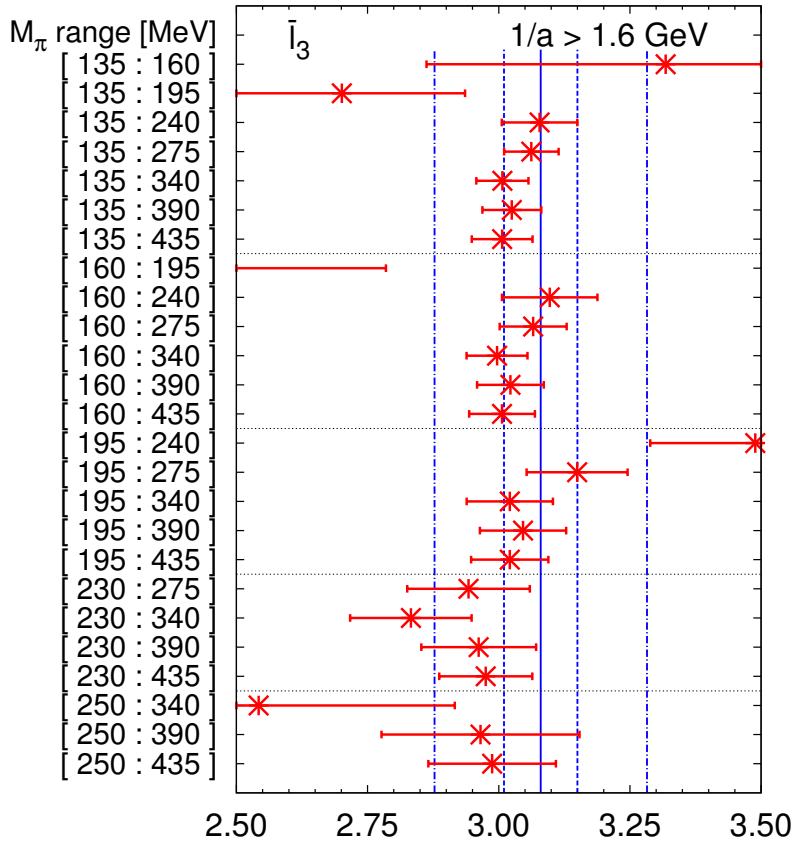
fit range now omits light pion masses. . .

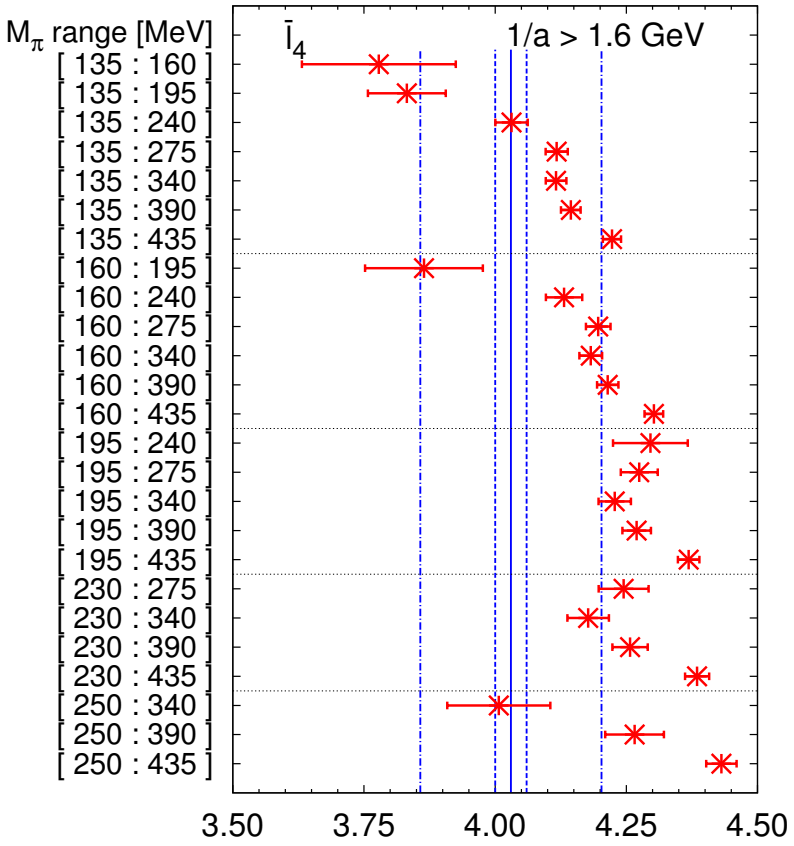
NLO-ChPT extrapolation to physical point





(blue lines indicate values quoted by PDG/FLAG)





(blue lines indicate central value, stat., and total error from our analysis incl. light masses)

SU(2)-ChPT to NNLO

$$M_\pi^2 = \chi \left[1 + \frac{\chi}{16\pi^2 f^2} \log \frac{\chi}{\Lambda_3^2} + \mathrm{NNLO}_{M^2} \right]$$

$$f_\pi = f \left[1 - \frac{\chi_l}{8\pi^2 f^2} \log \frac{\chi_l}{\Lambda_4^2} + \mathrm{NNLO}_f \right]$$

$$\chi = 2B\,m = (2Bm^{\mathrm{phys}}) \frac{am}{am^{\mathrm{phys}}} = \chi^{\mathrm{phys}} \frac{am}{am^{\mathrm{phys}}}$$

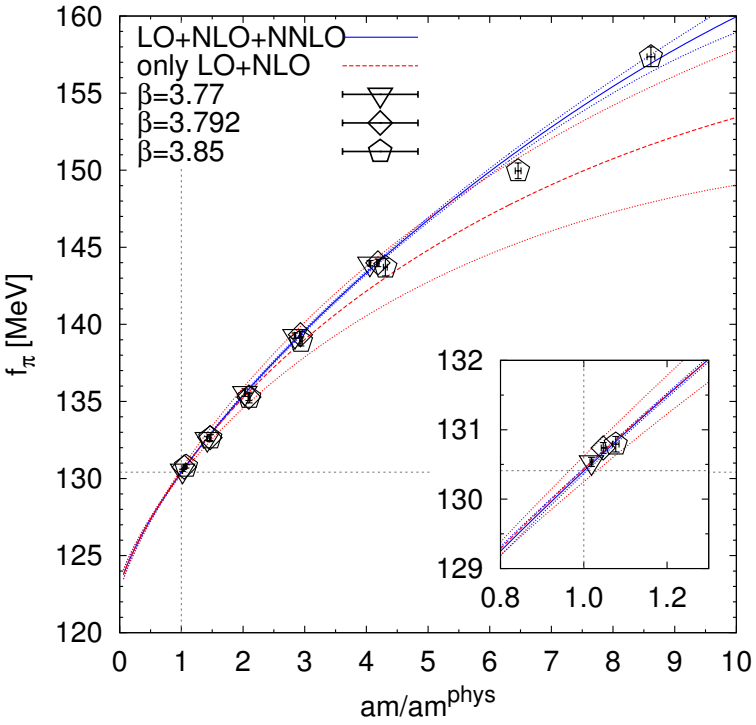
$$\mathrm{NNLO}_{M^2,f} : \quad \Lambda_{12}\,,\ k_{M^2}\,, k_f\,, \chi\,, f\,, \Lambda_3\,, \Lambda_4$$

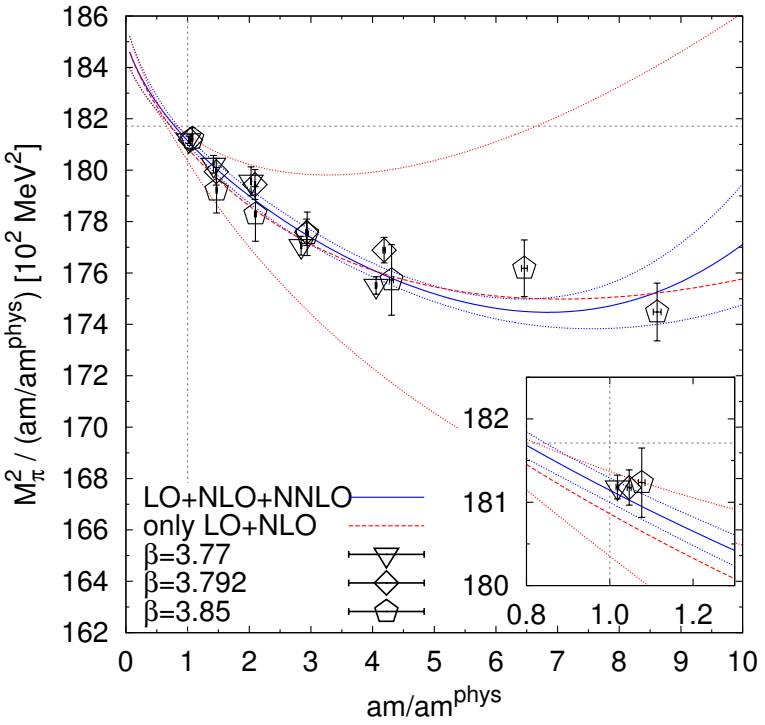
$$\log \Lambda_{12}^2 = \frac{7}{15} \log \Lambda_1^2 + \frac{8}{15} \log \Lambda_2^2 \quad (\mathrm{NLO} - \mathrm{LECs})$$

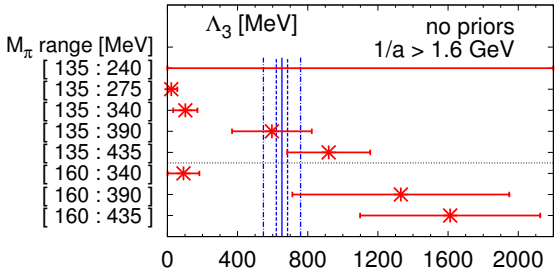
$$k_{M^2}, k_f : \quad \mathrm{NNLO} - \mathrm{LECs}$$

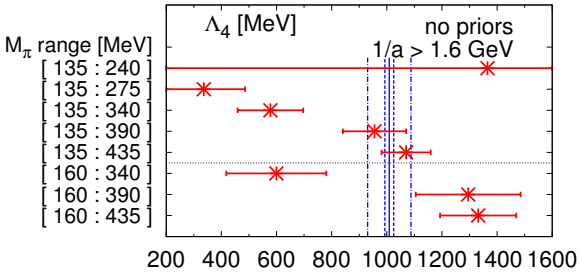
3 additional fit parameters

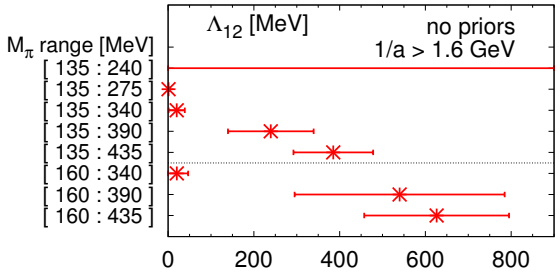
NNLO-fit, $1/a > 1.6$ GeV, 135 MeV $< M_\pi <$ 390 MeV





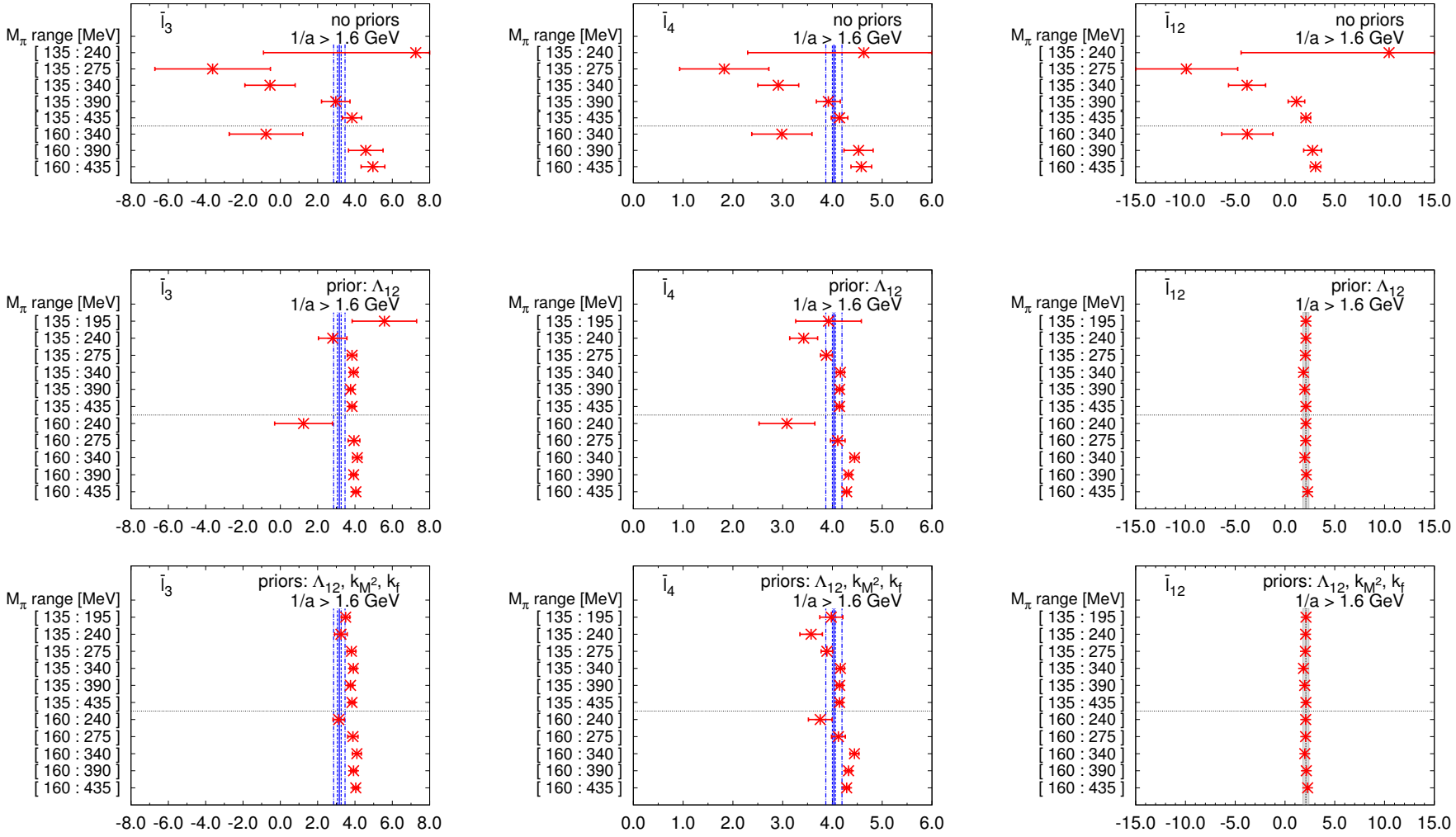






NNLO-fits with priors: $\bar{\ell}_{12}^{\text{prior}} = 2.1 \pm 0.3 \quad k_{M^2}^{\text{prior}} = 0 \pm 10 \quad k_f^{\text{prior}} = 0 \pm 10$

$\bar{\ell}_{12}^{\text{prior}}$ from $\pi\pi$-scattering: $\bar{\ell}_1 = -0.4 \pm 0.6$, $\bar{\ell}_2 = 4.3 \pm 0.1$ Colangelo et al. (2001)



- NLO-SU(2) chPT fits to staggered data, $135\,\text{MeV} \leq M_\pi \leq 240\,\text{MeV}$ ($\leq \ldots, 390, 435\,\text{MeV}$)
- LECs, decay constant ratio

$$2Bm^{\text{phys}} = 1.8609(18)_{\text{stat}}(74)_{\text{syst}} \cdot 10^{-2}\,\text{GeV}^2 \quad f = 122.72(07)_{\text{stat}}(35)_{\text{syst}}\,\text{MeV}$$

$$\bar{\ell}_3 = 3.16(10)_{\text{stat}}(29)_{\text{syst}} \quad \bar{\ell}_4 = 4.03(03)_{\text{stat}}(16)_{\text{syst}} \quad \frac{f_\pi^{\text{phys}}}{f} = 1.0627(06)_{\text{stat}}(27)_{\text{syst}}$$

$$\Lambda_3 = 653(32)_{\text{stat}}(101)_{\text{syst}}\,\text{MeV} \quad \Lambda_4 = 1,009(16)_{\text{stat}}(77)_{\text{syst}}\,\text{MeV}$$

- using average light quark mass from Dürr et al. (2011)

$$B^{\overline{\text{MS}},\mu=2\,\text{GeV}} = 2.682(36)_{\text{stat}}(39)_{\text{syst}}\,\text{GeV} \quad \Sigma^{\overline{\text{MS}},\mu=2\,\text{GeV}} = \left(272.3(1.2)_{\text{stat}}(1.4)_{\text{syst}}\,\text{MeV}\right)^3$$

- NNLO-SU(2) chPT

stable/reasonable results only obtained with external input (priors $\bar{\ell}_1$,$\bar{\ell}_2$)

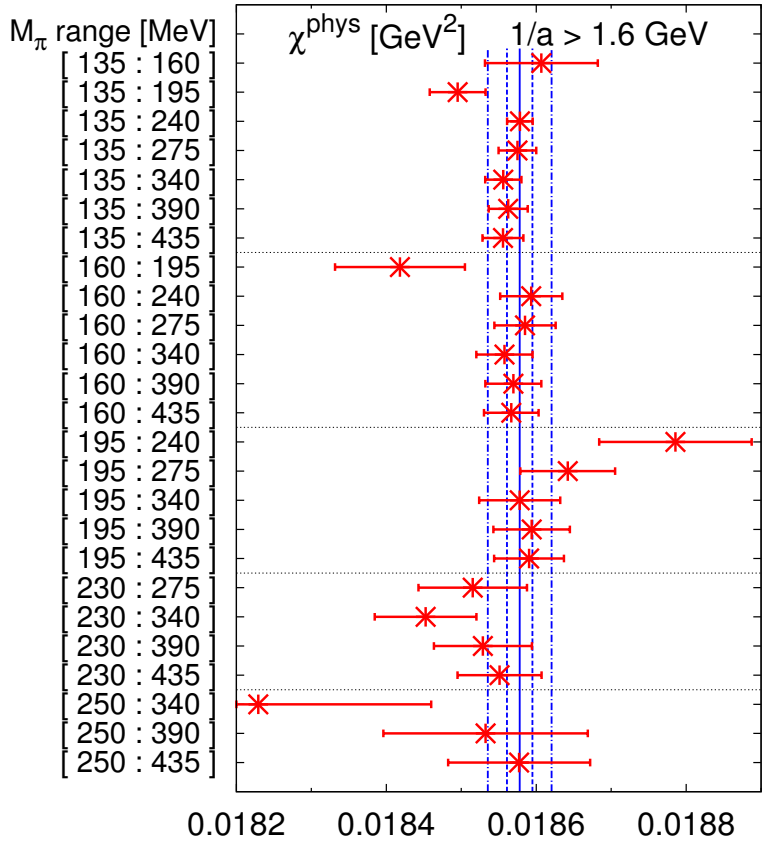
for more details:

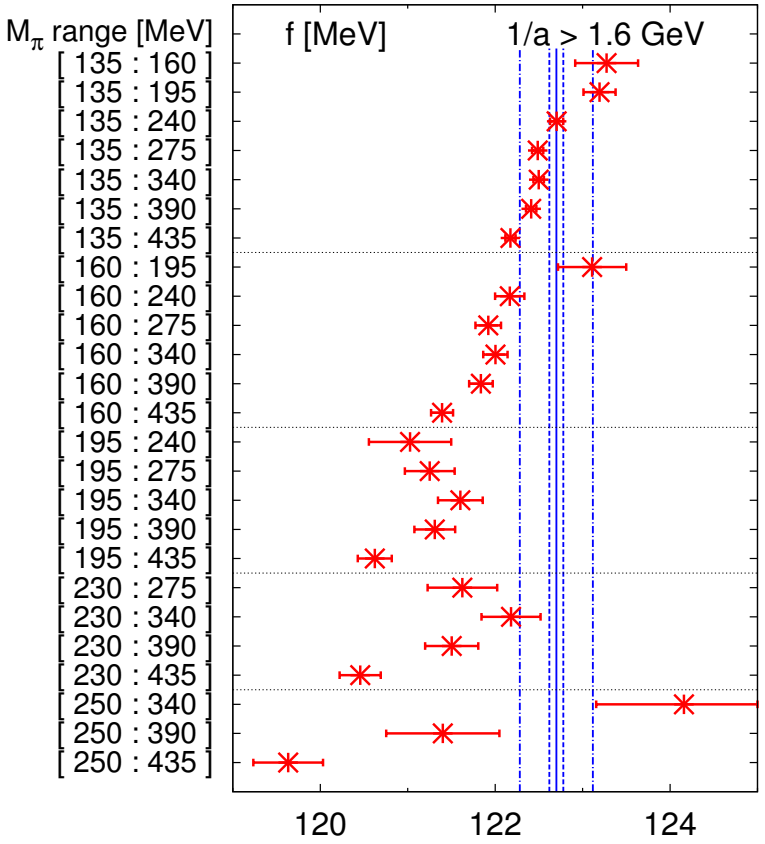
S. Borsanyi, S. Dürr, Z. Fodor, S. Krieg, A. Schäfer, EES, K.K. Szabo

arXiv:1205.0788[hep-lat]

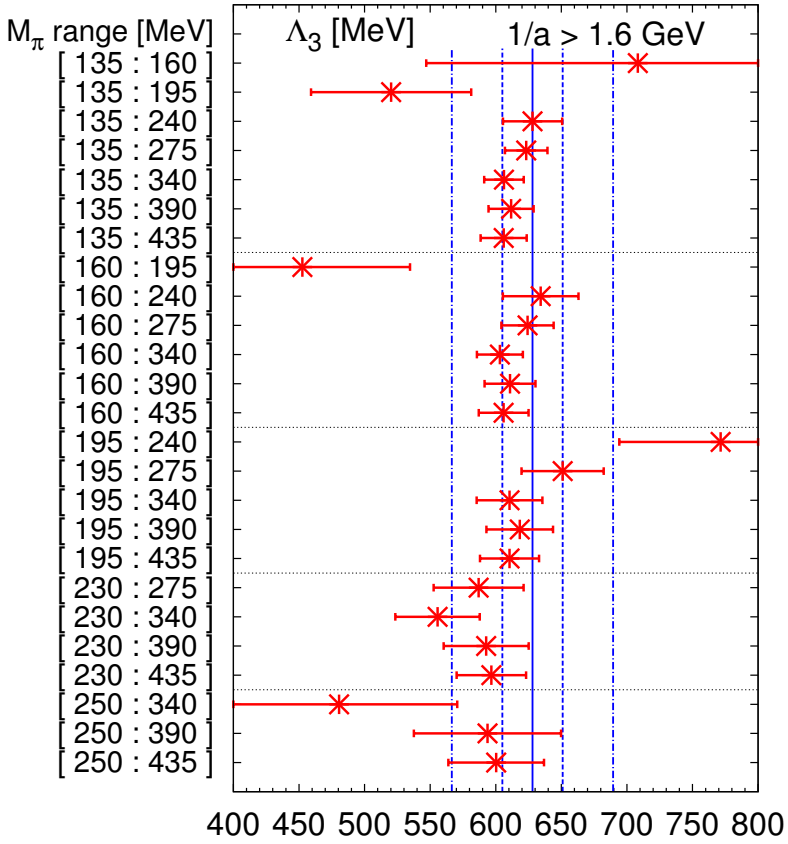
BACKUP

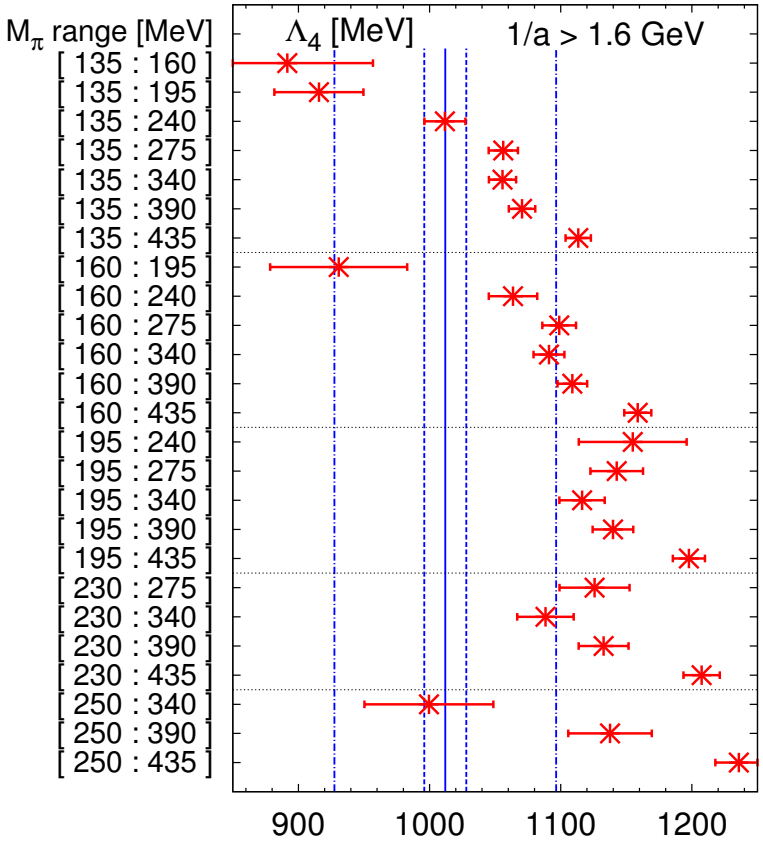
unconstrained NLO fits, $1/a > 1.6$ GeV I



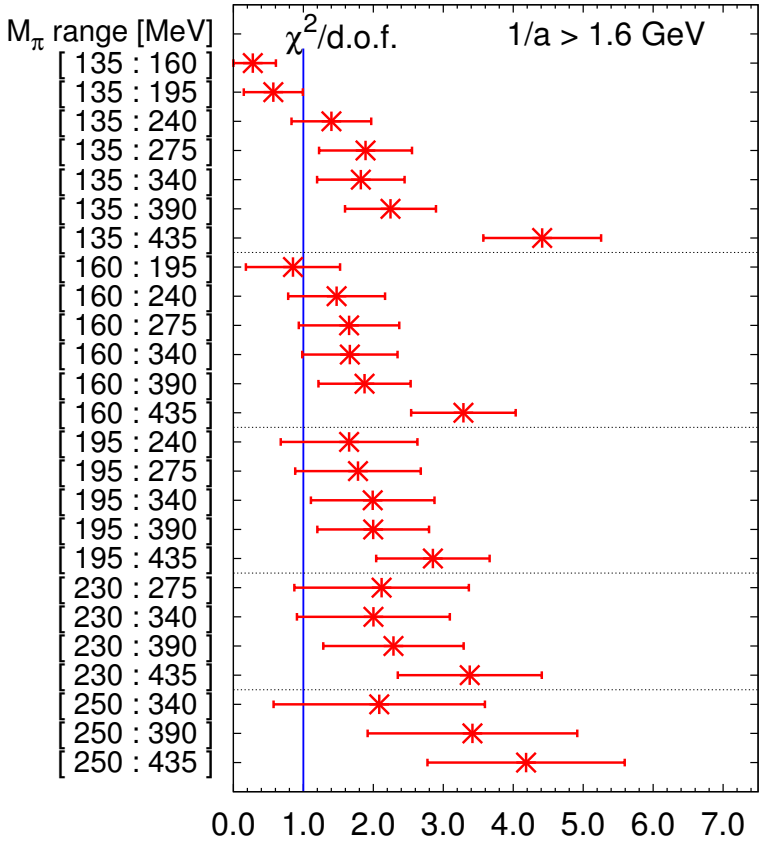


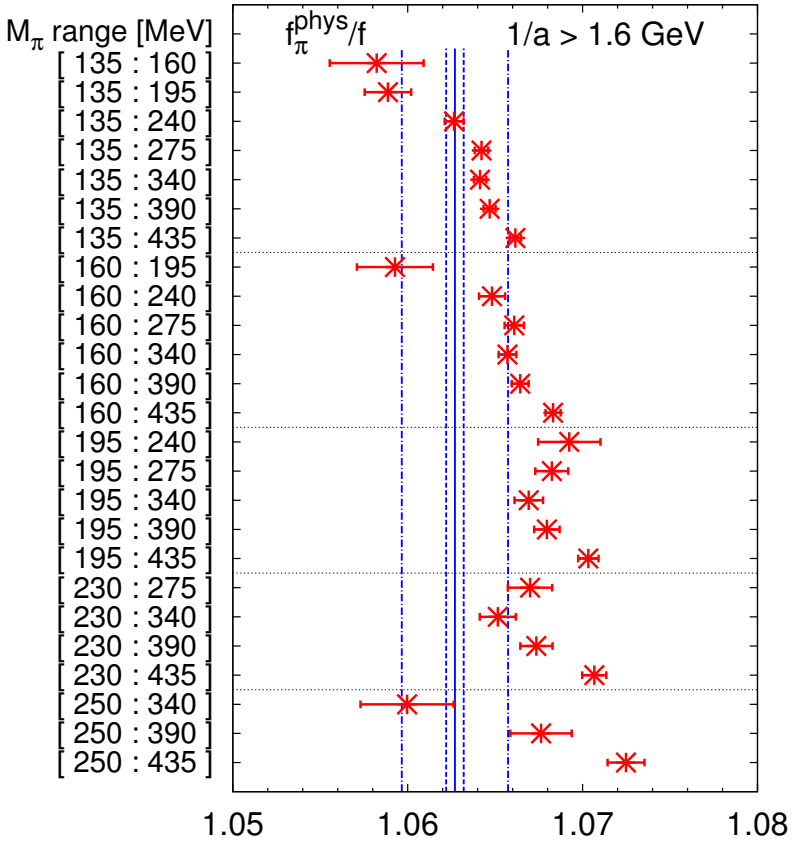
unconstrained NLO fits, $1/a > 1.6\,\mathrm{GeV}$ II



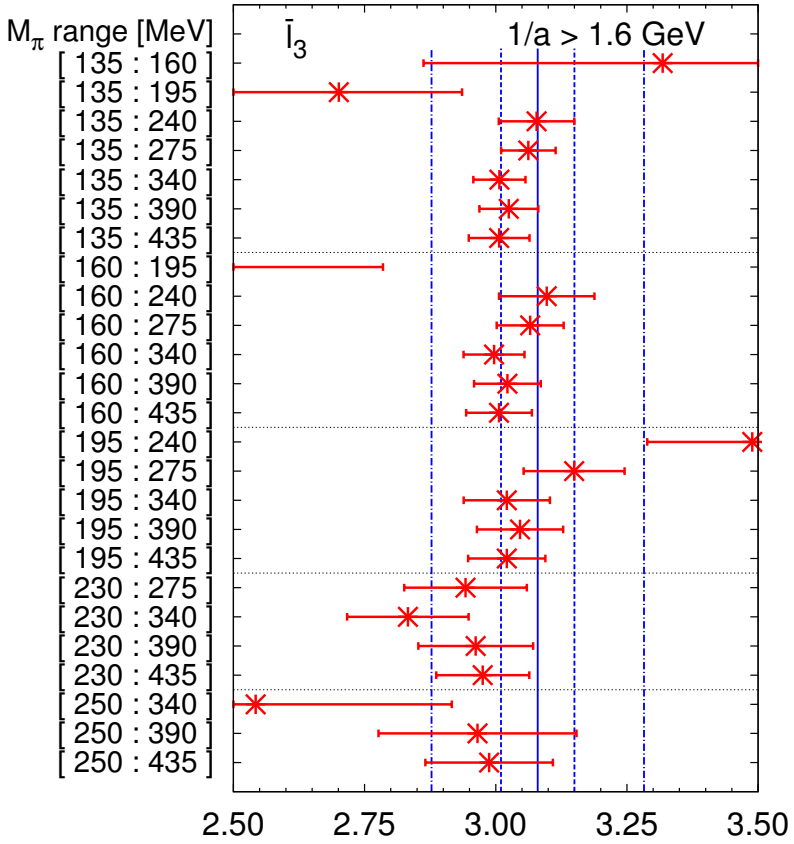


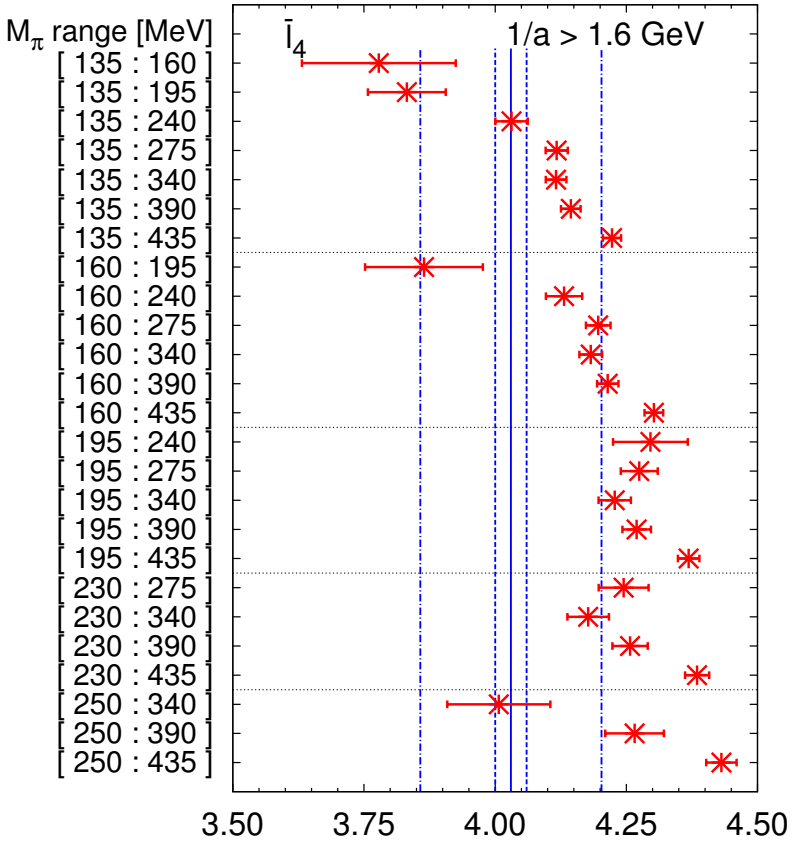
unconstrained NLO fits, $1/a > 1.6$ GeV III



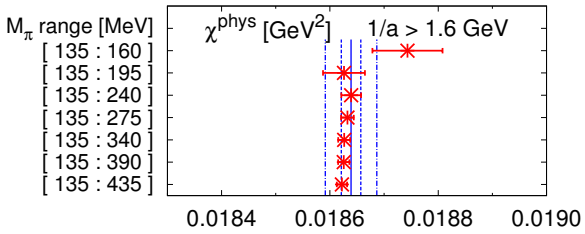


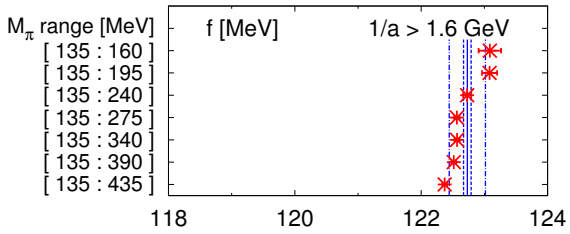
unconstrained NLO fits, $1/a > 1.6$ GeV IV

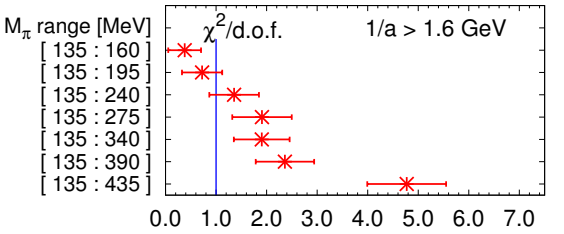


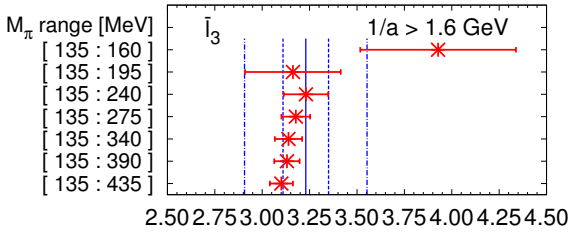


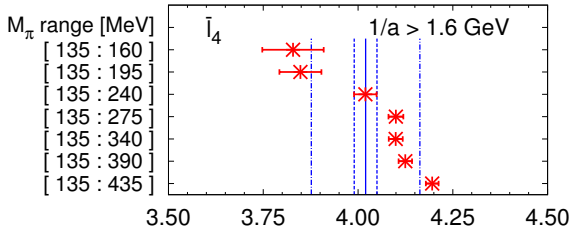
parameter-reduced NLO fits: different M_π-ranges

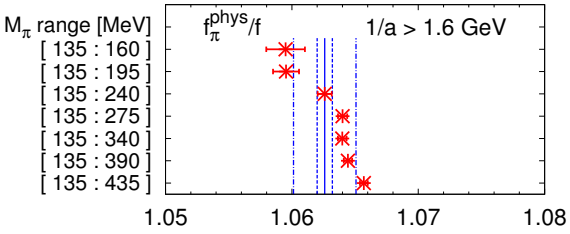


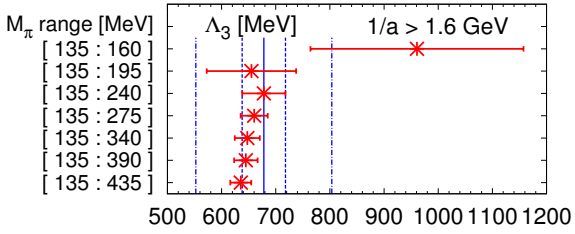


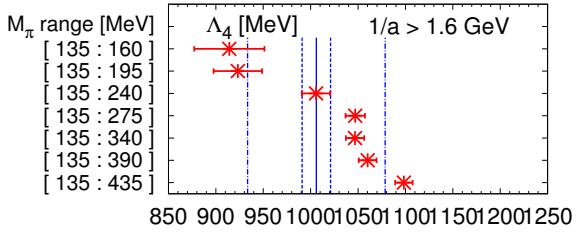




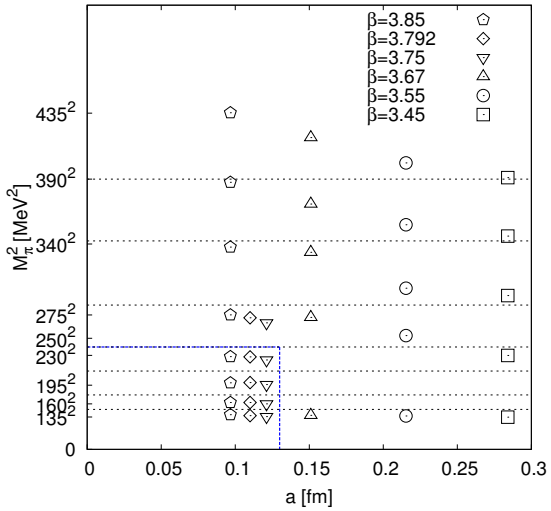


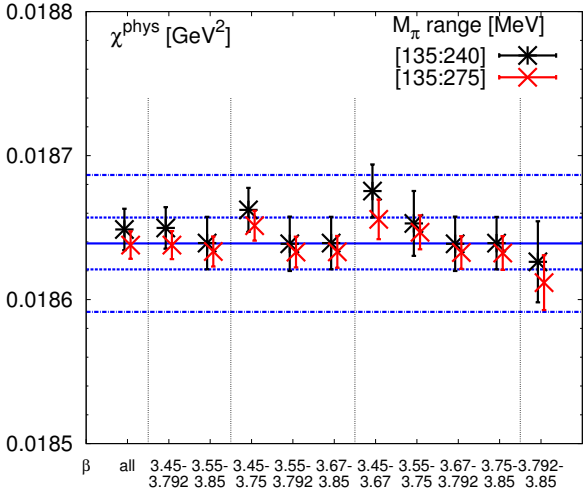


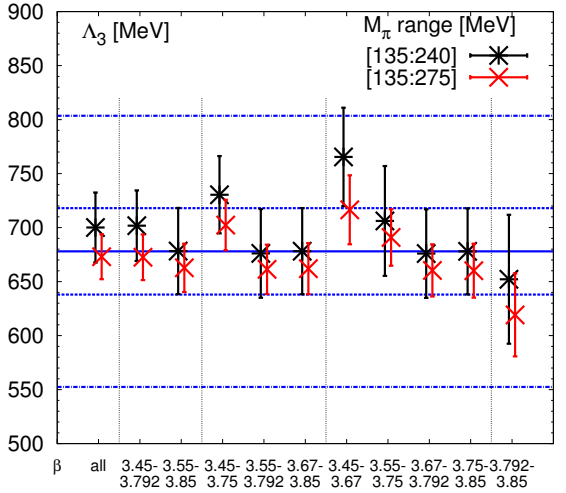


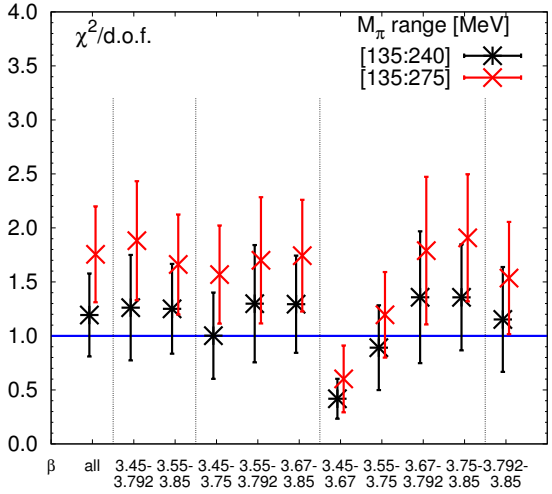


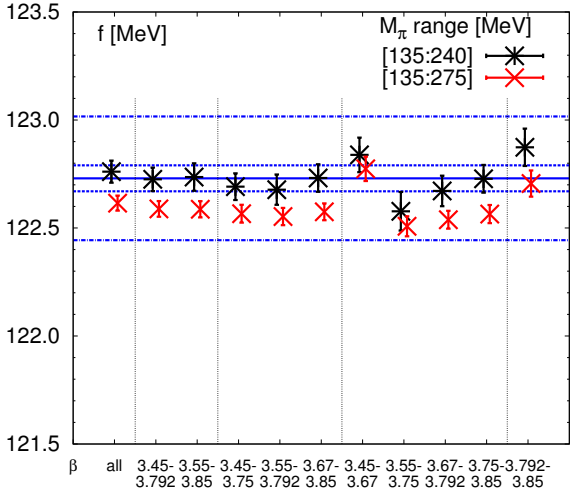
parameter-reduced NLO fits: different $1/a$-ranges

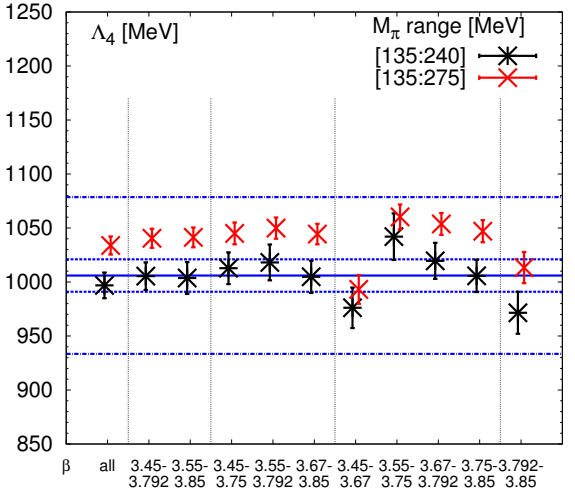




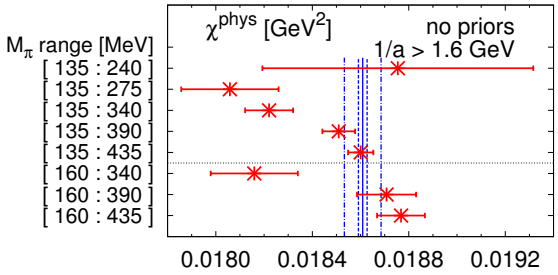


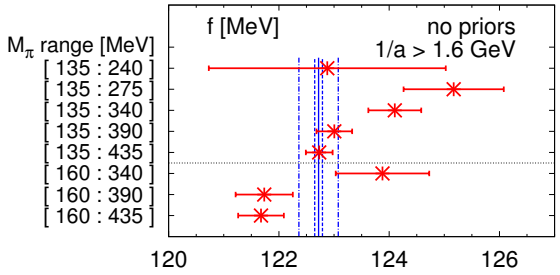


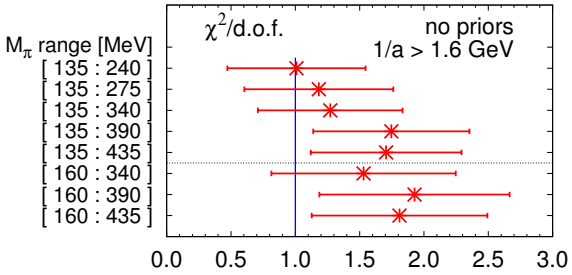


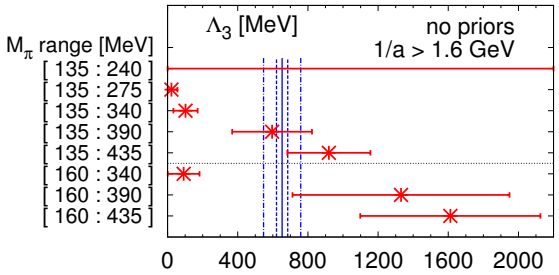


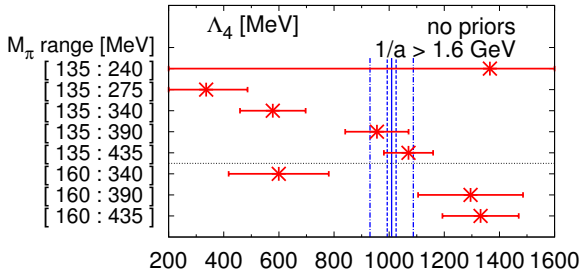
NNLO fits w/o priors: different M_π-ranges

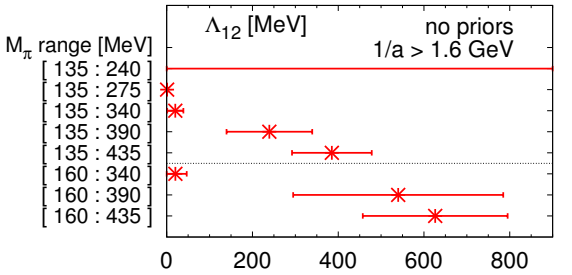


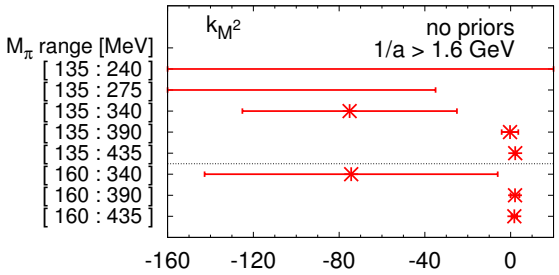


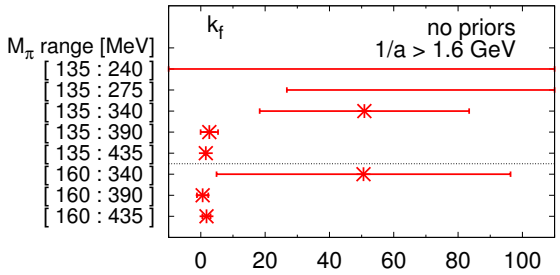


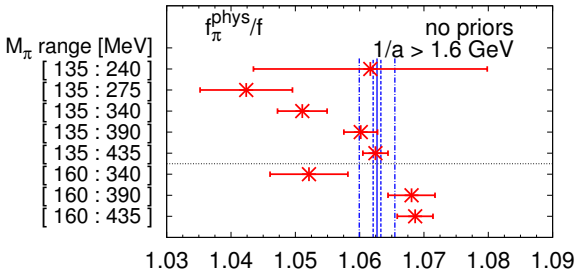




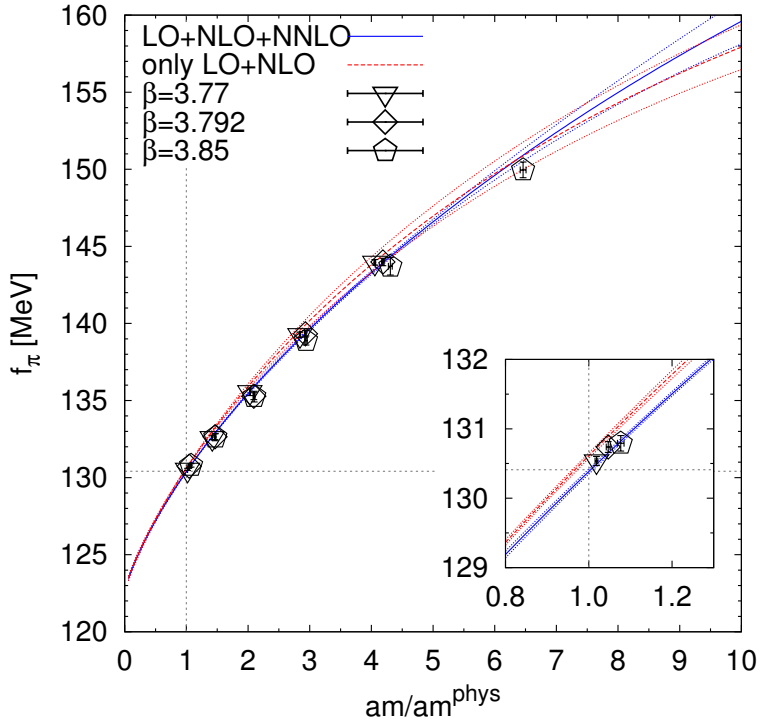


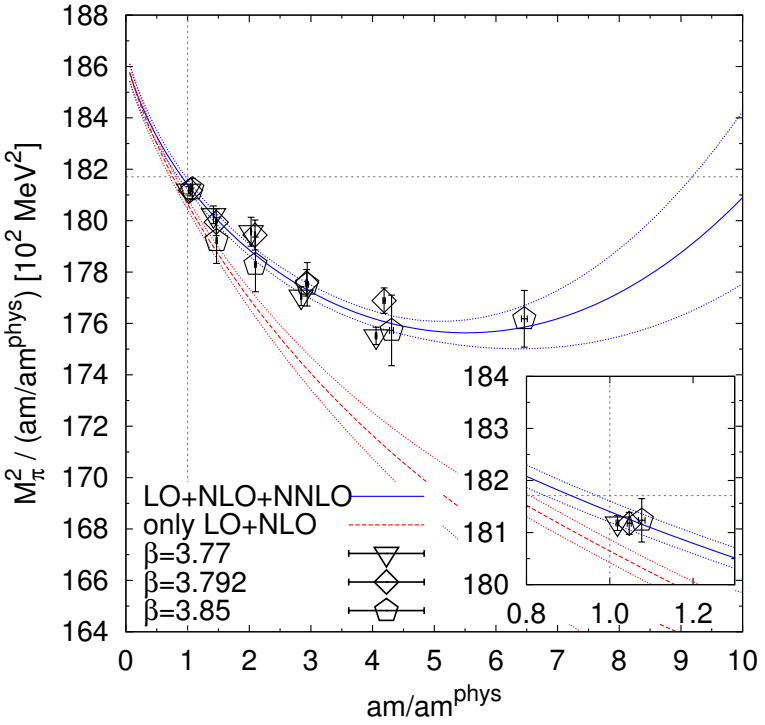


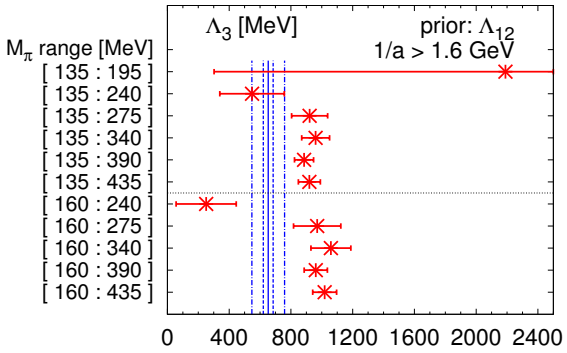


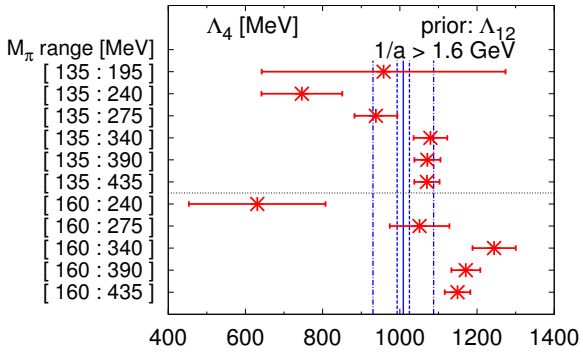


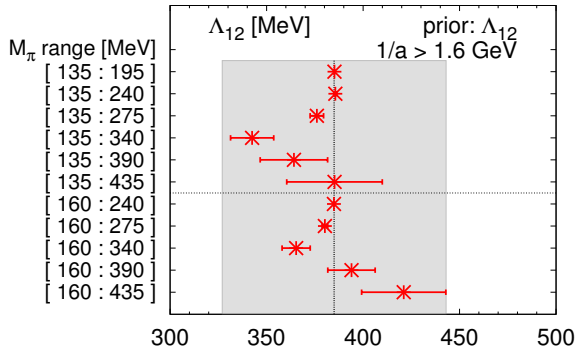
NNLO-fit with prior $\bar{\ell}_{12}$, $1/a > 1.6\,\text{GeV}$, $135\,\text{MeV} < M_\pi < 340\,\text{MeV}$



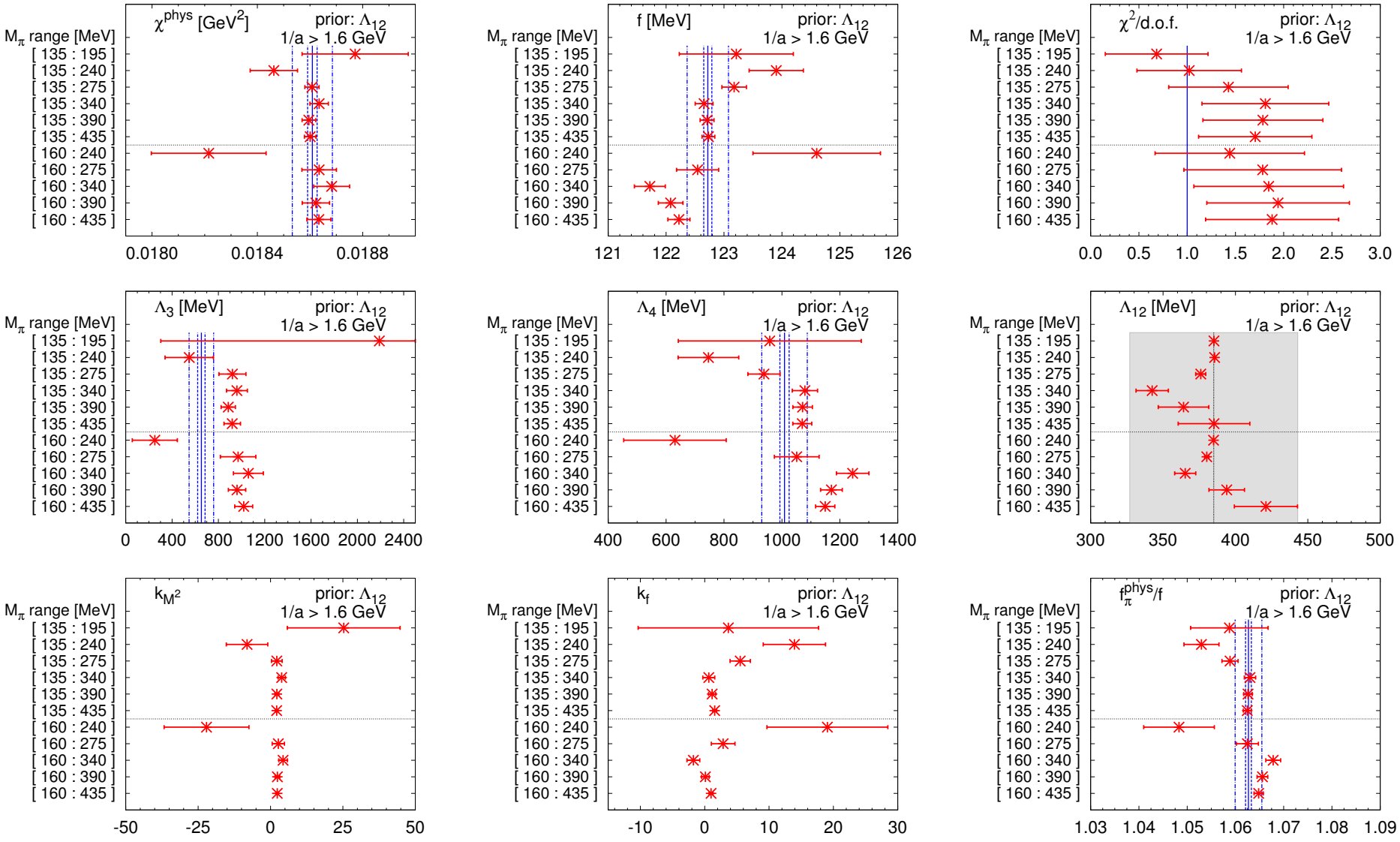




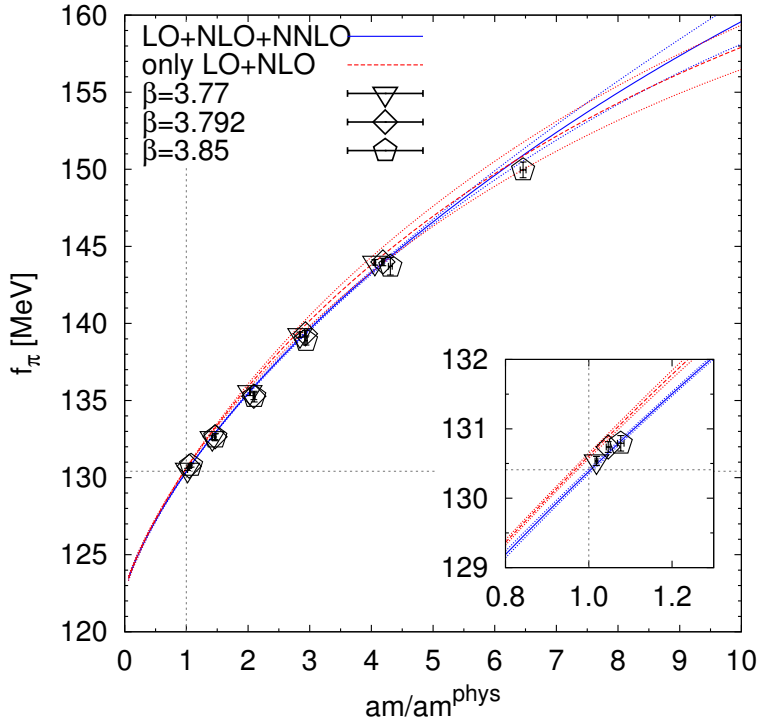


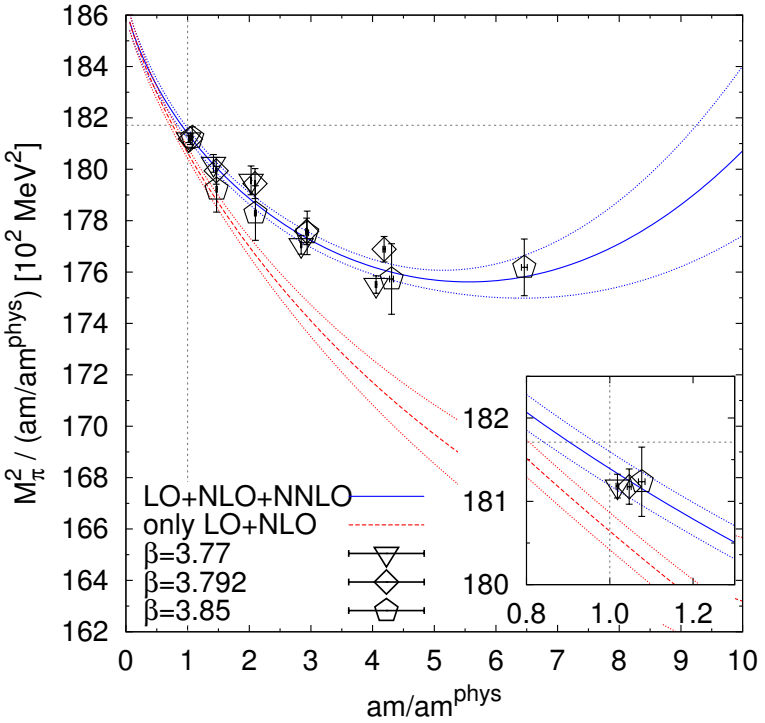


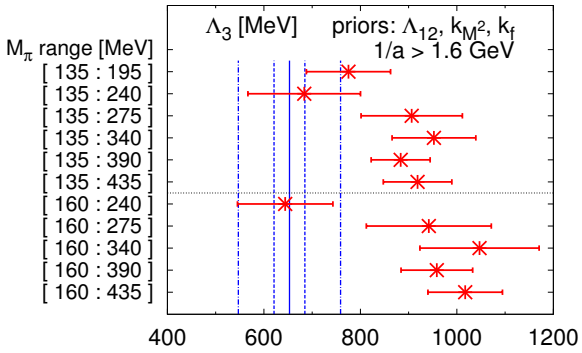
NNLO fits with prior $\bar{\ell}_{12}$: different M_π-ranges

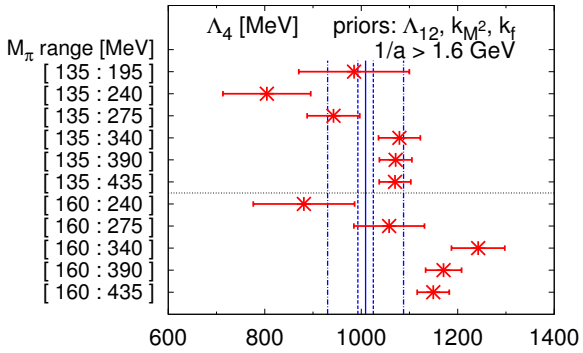


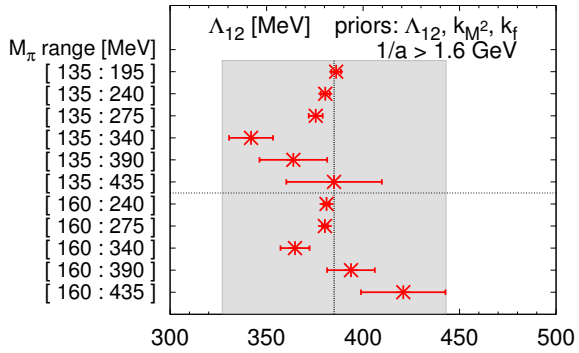
NNLO-fit, priors $\bar{\ell}_{12}$, k_{M^2}, k_f, $1/a > 1.6$ GeV, 135 MeV $< M_\pi <$ 340 MeV











NNLO fits with priors $\bar{\ell}_{12}$, k_{M^2}, k_f: different M_π-ranges