A scrutiny of hard pion chiral perturbation theory

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Outline

- Hard pion ChPT: factorization of leading chiral logarithms
- Pion vector and scalar form factors for $M_\pi^2 \ll s$: dispersive representation
- Elastic contributions (intermediate 2 pions): factorization valid to all orders
- Inelastic contributions: factorization violated at 3 loops (intermediate 4 pions)

G. Colangelo, MP, L. Rothen, R. Stucki and J. Tarrús Castellà
arXiv: 1208.0498

Hard pion ChPT

- Flynn and Sachrajda (2008)

  $K \to \pi$ form factors in SU(2) ChPT: predictions for leading chiral logarithms even for $q^2 = 0$ (i.e. $E_\pi \simeq M_K/2$)

- Bijnens and Celis (2009), Bijnens and Jemos (2010, 2011)

  predictions for leading chiral logarithms in a variety of processes with hard pions in the final state: $K \to \pi\pi$ decays, $B \to \pi$ and $D \to \pi$ form factors at $q^2 < q^2_{\rm max}$, pion form factors $F_{V,S}(s, M_\pi^2)$ for $M_\pi^2 \ll s$

Vector and scalar pion form factors

- Hard pion ChPT predicts that, for $M_\pi^2/s \ll 1$, the leading chiral logarithm factorizes from the energy dependence in the chiral limit:

$$F_{V,S}(s,M^2) = \overline{F}_{V,S}(s)\left[1+\alpha_{V,S}\,L\right] + \mathcal{O}(M^2)$$

with

$$L \equiv \frac{M^2}{(4\pi F)^2}\ln\frac{M^2}{s}\,, \qquad M_\pi^2 = M^2 + \mathcal{O}(M^4)\,, \qquad F_\pi^2 = F + \mathcal{O}(M^2)$$

- Expanding the two-loop standard SU(2) ChPT result in Bijnens, Colangelo and Talavera (1998) for $M_\pi^2/s \ll 1$ one obtains the factorized form predicted by Hard pion ChPT with

$$\alpha_S = -\frac{5}{2}\,, \qquad \alpha_V = -1$$

Quantitative explanation of this factorization property ?

Still valid beyond two loops ?

Dispersive representation of the FFs

- Analyticity :

$$F(s) = 1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \, \frac{\mathrm{Im}\, F(s')}{s'(s'-s)}$$

with $F_{V,S}(0) = 1$

- Unitarity :

$$\mathrm{Im}\, F(s) = \sigma(s)\, F(s)\, t^*(s) + \text{inelastic terms}$$

$$\sigma(s) = \sqrt{1 - \frac{4M_\pi^2}{s}}$$

two-pion phase space

ππ partial wave with the appropriate quantum numbers

Dispersive representation of the FFs

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- Our notation (diagrammatic definition):

$$F(s) = F_{\mathrm{el}}(s) + F_{\mathrm{inel}}(s)$$

↑ (F_{el}):

2 pion intermediate states also for the ππ partial wave

the only contribution up to 2 loops

Dispersive representation of the FFs

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$$\mathrm{Im}\, F(s) = \sigma(s)\, F(s)\, t^*(s) + \text{inelastic terms}$$

- ChPT provides a perturbative solution to the dispersion relation, allows us to argue recursively applying the chiral counting:

one loop → $\mathrm{Im}\, F^{(2)}(s) = \sigma(s)\, t^{(2)*}(s)$ ← tree level

$$\mathrm{Im}\, F^{(2)}(s) = \sigma(s)\, t^{(2)*}(s)$$

$$\mathrm{Im}\, F^{(4)}(s) = \sigma(s) \left[t^{(4)*}(s) + F^{(2)}(s)\, t^{(2)}(s) \right]$$

$$\vdots \qquad \vdots$$

Dispersive representation of the FFs

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$$\vdots \qquad\qquad \vdots$$

two loops → $\mathrm{Im}\, F^{(4)}(s)$

one loop → $t^{(4)*}(s)$

Dispersive representation of the FFs

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$$F(s) = 1 + \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \, \frac{\mathrm{Im}\, F(s')}{s'(s'-s)}$$ with $$F_{V,S}(0) = 1$$

- The leading chiral logarithms can arise :

1. from an integrand which does not contain a log of the pion mass and this is produced by the integration over s'

2. if the integrand itself contains a chiral log

Chiral logs from integration (ELASTIC)

- Produced at the lower integration boundary $s' \sim 4M_\pi^2$:

  use standard ChPT to analyze the integrand for $s' \sim M^2 \ll s$

$$F(s) = 1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \, \frac{\mathrm{Im}\, F(s')}{s'(s'-s)} = 1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \, \frac{\sigma(s')}{s'(s'-s)} \Big(c_1 \, M^2 + c_2 \, s' + \mathcal{O}(p^4) \Big)$$

- The numerical constants c_1 and c_2 are related to the leading chiral contributions to the ππ scattering lengths and effective ranges

- The leading chiral logarithm generated by the dispersive integration is

$$16\pi F^2 \, (c_1 - 2\, c_2) \, L \equiv \alpha_{V,S} \, L$$

$$\rightarrow \quad \alpha_S = -\frac{5}{2} \, , \qquad \alpha_V = -1$$

in agreement with Bijnens, Colangelo, Talavera (1998), Bijnens and Jemos (2011)

Chiral logs in the integrand (ELASTIC)

- Contribution to the form factors at 2 loops :

$$\mathrm{Im}\, F^{(4)}(s) = \sigma(s) \left[t^{(4)*}(s) + F^{(2)}(s)\, t^{(2)}(s) \right]$$

$$F^{(2)}(s)\, t^{(2)}(s) = \left(\overline{F}^{(2)}(s) + \alpha L \right) \overline{t}^{(2)}(s) + \mathcal{O}(M^2)$$

$$t^{(4)}(s) = \overline{t}^{(4)}(s) + \beta\, sL + \mathcal{O}(M^2)$$

- Using Roy equations for ππ partial waves, we show that $\beta = 0$, which implies that factorization is valid up to 2 loops :

$$F(s) = \left(1 + \overline{F}^{(2)}(s) \right) (1 + \alpha L) + \overline{F}^{(4)}(s) + \mathcal{O}(M^2) + \mathcal{O}(p^6)$$

in agreement with Hard pion ChPT

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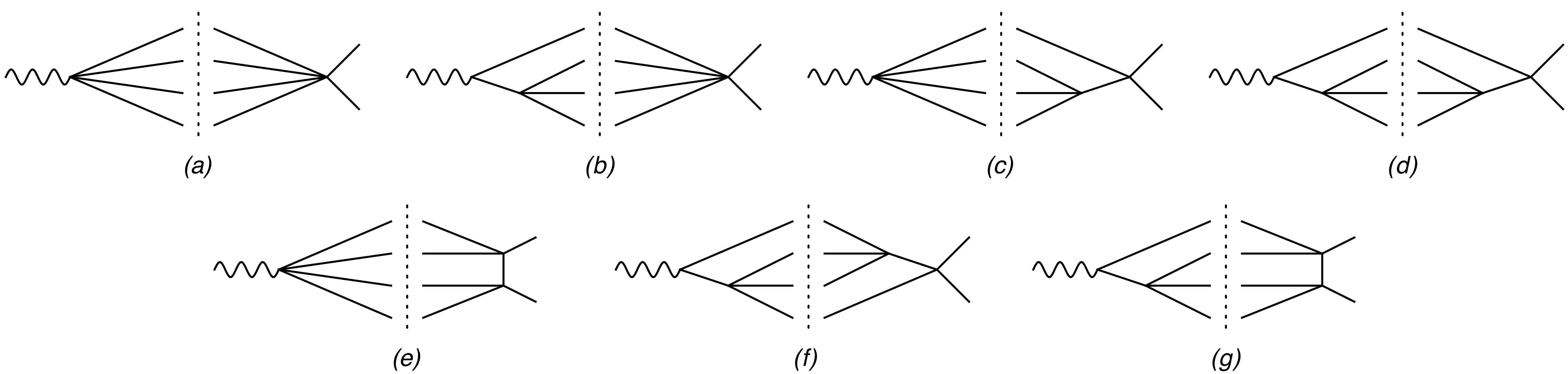
- By induction, we prove that all terms $s^{n-1}L$ in $t^{(2n)}(s)$ are absent: the elastic part (subclass of diagrams: 2 pion intermediate states) of the form factors factorizes to all orders in the chiral counting

Inelastic contributions to the DR

- Start with 4 intermediate pions (3-loop diagrams)

$$F_{\rm inel}(s) = \frac{s}{\pi}\int_{16M_\pi^2}^{\infty} ds' \frac{{\rm Im}\,F_{\rm inel}(s')}{s'(s'-s)} \,, \quad {\rm Im}\,F_{\rm inel}(s) = \frac{1}{2}\int d\Phi_4(s;p_1,p_2,p_3,p_4)\, F_{4\pi}\cdot T^*_{6\pi}$$

- For the scalar form factor:



- Chiral logs are produced by integrations over intermediate momenta with pion-mass-dependent boundaries

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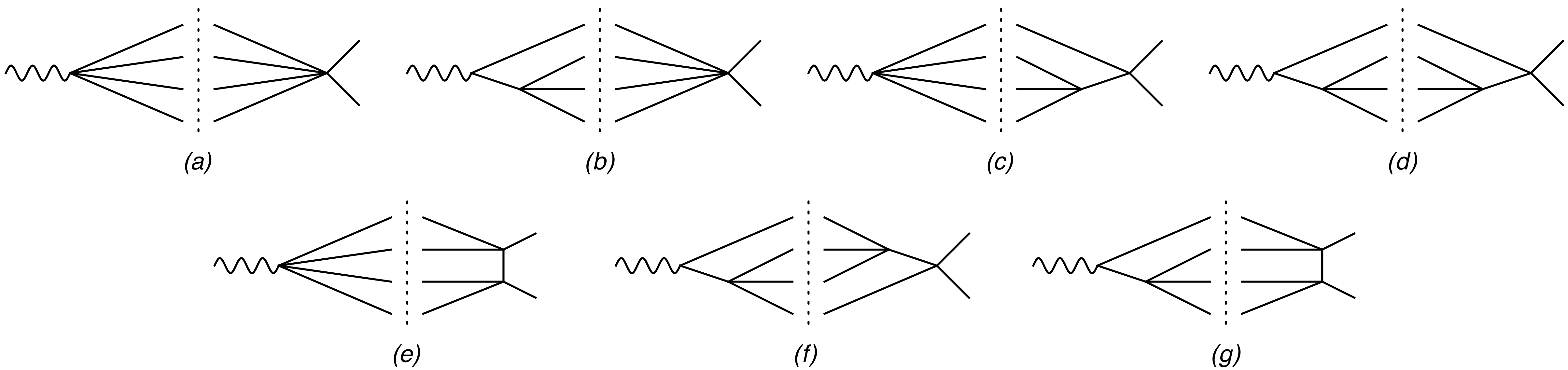


- Analytical results for the chiral limit values and coefficients of the leading chiral log for graphs (a), (b), (c), (d) and numerical results for (e), (f) and (g)

Inelastic contributions to the DR

- Factorization is not valid at three loops:

$$F(s) = \left(1 + \overline{F}^{(2)}(s) + \overline{F}^{(4)}(s)\right)(1 + \alpha L) + \alpha_{\rm inel}(s) L + \overline{F}^{(6)}(s) + \mathcal{O}(M^2) + \mathcal{O}(p^8)$$

with

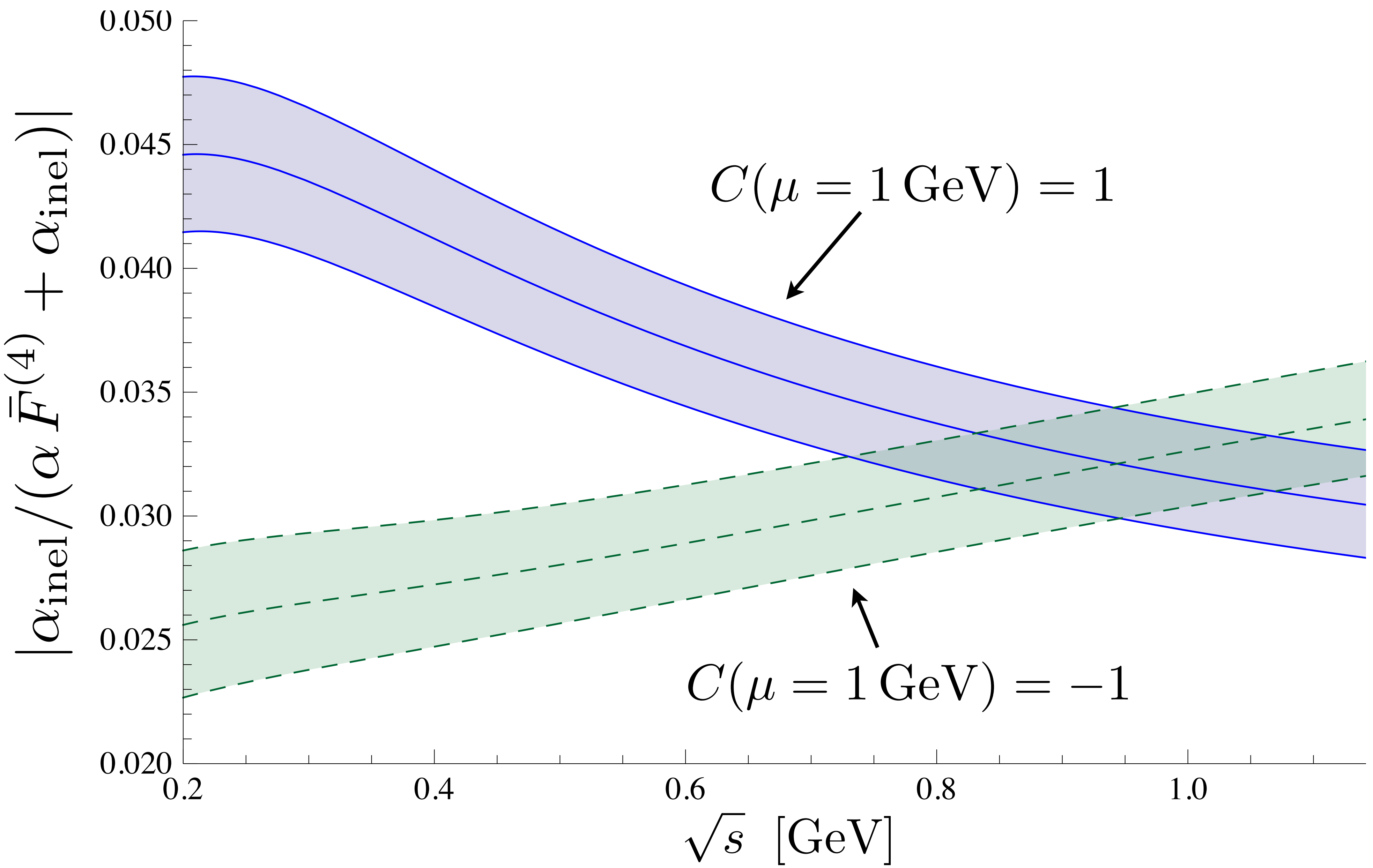
$$\alpha_{\rm inel}(s) = \left[C(\mu^2) + \delta \times \left(\ln\frac{\mu^2}{s} + i\pi\right)\right]\frac{s^2}{(4\pi F)^4}\,, \qquad \delta = -0.53 \pm 0.05$$

- The coefficient of the leading chiral log is not universal

- For values of $\sqrt{s}$ which are not small compared to Λ_χ the three-loop contribution is not suppressed compared to one and two loops

Pion scalar FF at 3 loops

$$F(s) = \left(1 + \overline{F}^{(2)}(s) + \overline{F}^{(4)}(s)\right)(1 + \alpha L) + \alpha_{\rm inel}(s) L + \overline{F}^{(6)}(s) + \mathcal{O}(M^2) + \mathcal{O}(p^8)$$



The 2-loop scalar FF in the chiral limit is extracted from Bijnens, Colangelo, Talavera (1998)

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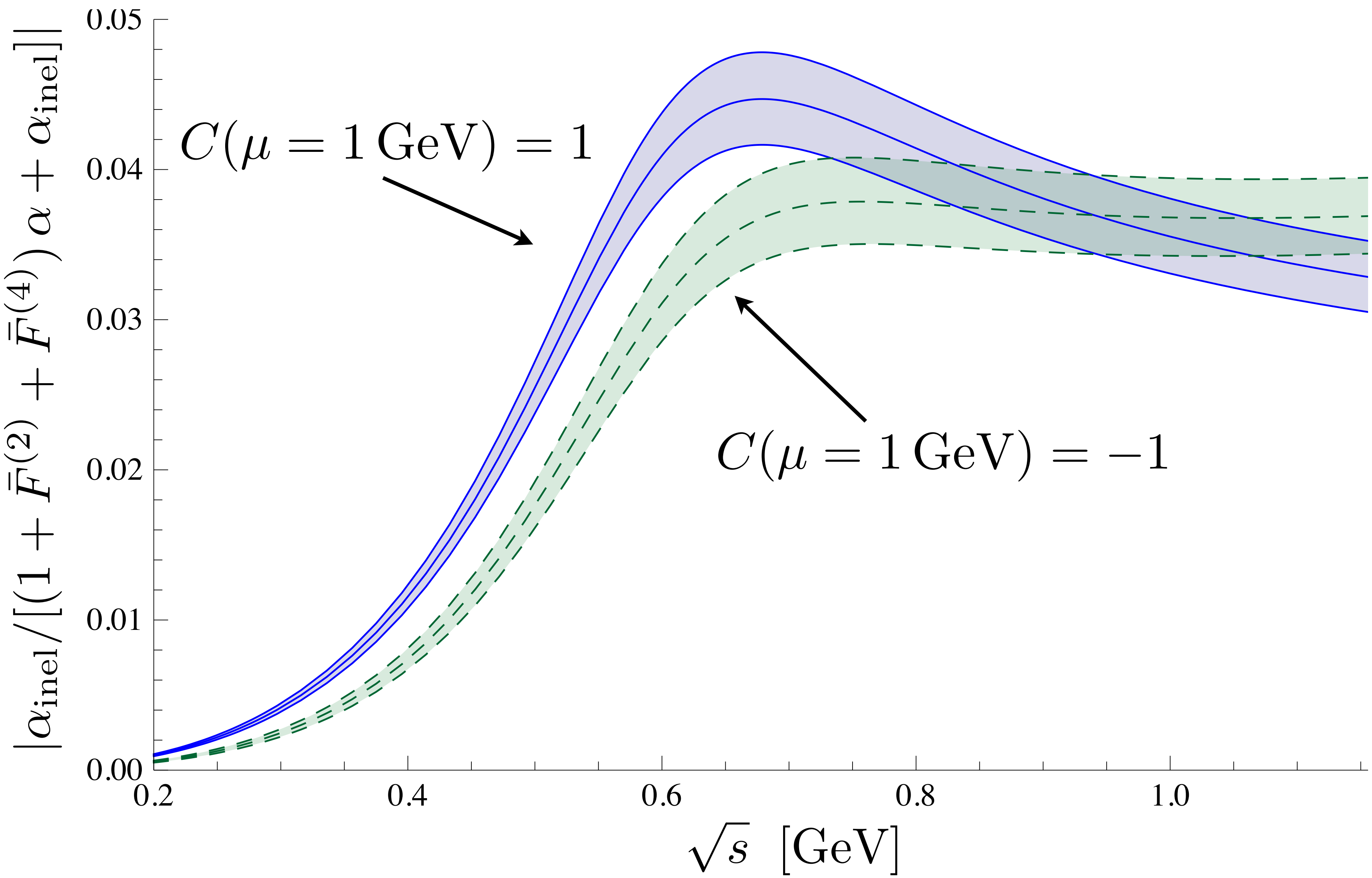

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Conclusions and outlook

- Factorization of leading chiral logs in the pion form factors for $M_\pi^2 \ll s$
- Dispersion relations and application of chiral counting: recursive analysis
- We show how factorization emerges at two loops and is valid for a whole subclass of diagrams to all orders (with 2 intermediate pions)
- Our calculation at 3 loops shows that factorization is broken by multipion contributions, which generate new leading chiral logs
- Factorization could be valid to a good approximation only if one remains in the low-energy regime, with very small quark masses
- Future work: extension of our analysis to heavy-light form factors

Additional slides

Chiral logs for asymptotic energies

- For asymptotically large values of s ,

$$F_V(s) = \frac{F_\pi^2}{s} \int_0^1 dx\, dy\, T(x,y,s)\, \phi_\pi(x)\phi_\pi(y) \times [1 + \mathcal{O}(\Lambda_{\rm QCD}^2/s, M_\pi^2/s)]$$

Brodsky and Lepage (1980)

- The leading chiral log is given just by the one in F_π

Chen and Stewart (2004)

- Hence the leading chiral log does factorize for $s \gg \Lambda_{\rm QCD}^2$ but $\alpha_V = -2$ while Hard pion ChPT predicts $\alpha_V = -1$ (valid only in the low-energy regime)