Pion-pion scattering lengths determination from kaons decays

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Outline

1.Scattering lengths as a test of Chiral Perturbation Theory (ChPT).

2.Scattering lengths from $K^+ \to \pi^0\pi^0\pi^+$ decay.

3.Scattering lengths from K_{e4} decay
$K^+ \to \pi^+\pi^-e^+\nu; \quad K^+ \to \pi^0\pi^0e^+\nu$.

Scattering lengths as ChPT test.

In massless theory (chiral limit) scattering lengths $\pi\pi \to \pi\pi$ with isospin I=0,2 $a_0 = a_2 = 0$

S.Weinberg,1966; J.Gasser,H.Leutwyler, 1982; J.Gasser et al.1996

$$\begin{aligned} a_0 &= \frac{7m^2}{32\pi F_\pi^2}[1 + c_1 m^2 + c_2 m^4 + O(m^6)] \\ &= 0.156(1 + 0.28 + 0.11 + O(m^6)) = 0.22 \pm 0.005; \\ a_2 &= -0.044 \end{aligned}$$

Hadronic atoms in the standard model, Dubna, 1998:

Experiment:

1) Batusov, Bunyatov, Sidorov, Yarba, Rochester, 1960: Final state interaction in $\pi N \to \pi\pi N$

2) Decay $K^\pm \to \pi^+\pi^- e^\pm \nu$; Rosselet et al. 1977

Result: $a_0 = 0.26 \pm 0.05$; $a_2 = -0.028 \pm 0.012$

Experiment Dirac PS,CERN, accepted in 1995

$\Gamma_{atom} = \frac{1}{\tau} \approx \alpha^3 \mid a_0 - a_2 \mid^2$

The primary goal: Determination of pionium lifetime with $\approx$ 10% accuracy gives for $a_0 - a_2 \approx 5\%$

At present Adeva et al., 2011:

$\mid a_0 - a_2 \mid = 0.2533 \pm 0.008_{st} \pm 0.007_{syst}$

Scattering lengths from NA48/2 (2006-2009):

$K^+ \to \pi^0\pi^0\pi^+; \quad K^+ \to \pi^+\pi^- e^+\nu$

$a_0 - a_2 = 0.264 \pm 0.0021$

Direct CP violation. NA48/2, SPS CERN

Primary goal of NA48/2: Measurement of the Dalitz plot linear slopes with precision better than previous one (one order better).

$$T(K^\pm \to \pi^\pm\pi^0\pi^0) = N_0(1 + g^\pm \frac{(M^2 - s_0)}{2m_c^2})$$

$$T(K^\pm \to \pi^\pm\pi^+\pi^-) = N_+(1 + h^\pm \frac{(M^2 - s_0)}{2m_c^2})$$

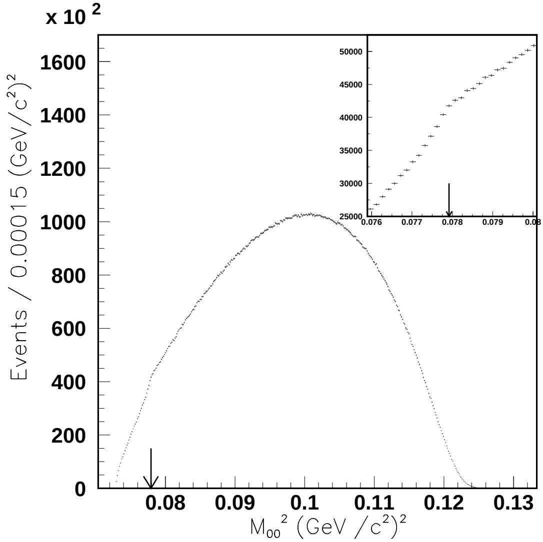
Charge asymmetries of Dalitz plot slopes:

$$A_h = \frac{h^+ - h^-}{h^+ + h^-} = (-1.5 \pm 2.2)10^{-4}$$

$$A_g = \frac{g^+ - g^-}{g^+ + g^-} = (-1.8 \pm 1.8)10^{-4}$$

Standard model predicts direct CP violation on the level $A_{sm} \approx 10^{-5}$ Batley et al.,EPJ,2007

Cusp in $K^+ \to \pi^0\pi^0\pi^+$; Data 2003-2004, NA48/2



Two step decay in $K^+ \to \pi^+\pi^0\pi^0$

Cabibbo, PRL 2004

$K^+ \to \pi^+\pi^+\pi^-; \quad \pi^+\pi^- \to \pi^0\pi^0; \quad K^+ \to \pi^+\pi^0\pi^0$

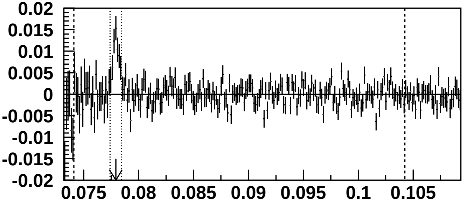
$$T = T_0 + 2ia_x m_c\sqrt{\frac{M^2 - 4m_c^2}{4m_c^2}}T_+; \quad a_x(\pi^+\pi^- \to \pi^0\pi^0) = \frac{a_0 - a_2}{3}$$

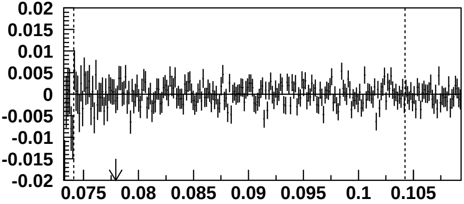
$T_0(K^+ \to \pi^+\pi^0\pi^0); \quad T_+(K^+ \to \pi^+\pi^-\pi^+)$ -unperturbed (without final state interaction) $k = \frac{1}{2}\sqrt{M^2 - 4m_c^2}$ - the momenta of charged pions in the reaction $\pi^+ + \pi^- \to \pi^0 + \pi^0$. Under threshold it becomes imaginary $k = i\kappa$ thus

$$|T|^2 = T_0^2 + 4\frac{(a_0 - a_2)^2 k^2}{9}T_+^2; M^2 > 4m_c^2$$

$$|T|^2 = T_0^2 + 4\frac{(a_0 - a_2)^2 \kappa^2}{9}T_+^2 - 4\frac{(a_0 - a_2)\kappa}{3}T_0 T_+; M^2 < 4m_c^2$$

$\Delta = \frac{(data - fit)}{data}$ vs M^2 in Cabibbo -Isidori approach.a)Without pionium atom; b) With pionium atom





Higher order terms and Electromagnetic effects in $K^+ \to \pi^0\pi^0\pi^+$

1.Under threshold $M_{\pi\pi} < 2m_c$: Pionium atom just under threshold.

2.Above threshold $M_{\pi\pi} > 2m_c$: Gamov factor.

Two unsolved challenges (2006) :

1.Higher order terms.

2.Electromagnetic effects.

To account for pionium effect NA48/2 multiplied the content of the bin centered at $M^2 = 4m_c^2$ by factor $(1 + f_{atom})$. The best fit value for f_{atom} corresponds to probability of pionium atom production $P = \frac{K^+ \to \pi^+ atom}{K^+ \to \pi^+\pi^-\pi^+} \approx (1.69 \pm 0.29) \times 10^{-5}$, whereas Silagadze,JETP letters,1994 $P = 0.8 \times 10^{-5}$

Strong interactions to all orders in $K \to 2\pi$

A.Tarasov,O.Voskresenskaya,S.G.
Eur. Phys. J. 2010

$$
\begin{aligned}
M_c(K \to \pi^+\pi^-) &= \int \Psi_c^+(r) M_0(r) d^3r; k_c = \frac{\sqrt{M^2 - 4m_c^2}}{2} \\
M_n(K \to \pi^0\pi^0) &= \int \Psi_n^+(r) M_0(r) d^3r; k_n = \frac{\sqrt{M^2 - 4m_n^2}}{2}
\end{aligned}
$$

$$
\begin{aligned}
-\Delta\Psi_c(r) + U_{cc}\Psi_c(r) + U_{cn}\Psi_n(r) &= k_c^2\Psi_c(r); \\
-\Delta\Psi_n(r) + U_{nn}\Psi_n(r) + U_{nc}\Psi_c(r) &= k_n^2\Psi_n(r)
\end{aligned}
$$

S-wave scattering, strong potential with sharp boundary $U_{ik} \gg k_{c(n)}$, asymptotic behavior of wave functions, unitarity constrains.

Strong $\pi\pi$ interaction to all orders.

The decay amplitudes $K \to 2\pi$ are expressed through the "unperturbed" decay amplitudes M_{0c}, M_{0n} and $\pi\pi$ scattering amplitudes

$f_x(\pi^+\pi^- \to \pi^0\pi^0)$, $f_c(\pi^+\pi^- \to \pi^+\pi^-)$, $f_n(\pi^0\pi^0 \to \pi^0\pi^0)$

$$M_c = M_{0c}(1 + ik_c f_{cc}) + ik_n M_{0n} f_x; \; f_{cc} = \frac{a_{cc}(1 - ik_n a_{nn}) + ik_n a_x^2}{D}$$

$$M_n = M_{0n}(1 + ik_n f_{nn}) + ik_c M_{0c} f_x; \; f_{nn} = \frac{a_{nn}(1 - ik_c a_{cc}) + ik_c a_x^2}{D}$$

$$f_x = \frac{a_x}{D}; \; D = (1 - ik_c a_{cc})(1 - ik_n a_{nn}) + k_n k_c a_x^2$$

In the case of exact isospin symmetry in the vicinity of threshold:

$$a_x = \frac{a_0 - a_2}{3}; \quad a_{nn} = \frac{a_0 + 2a_2}{3}; \quad a_{cc} = \frac{2a_0 + a_2}{6}$$

Electromagnetic interaction in $\pi^+\pi^-$ system

A.Tarasov,O.Voskresenskaya,S.G.Phys.Lett., 2007

Receipt is known for many years Wigner(1948), Breit(1957),Baz (1957) $ik_c \rightarrow \tau = \left.\frac{d\log[G_0(kr)+iF_0(kr)]}{dr}\right|_{r=r_0}$ F_0, G_0 are the regular and irregular solutions of the Coulomb problem.

1.Under threshold $M_{\pi\pi} < 2m_c$: Pionium atom just under charged pions threshold.

D.Madigozhin,A.Tarasov,O.Voskresenskaya,S. G. Phys.Part.Nucl. Lett.5,85 (2008)

The contribution of pionium belong to the central bin and coincides with Silagadze prediction 0.8×10^{-5}. The excess is connected with $\pi\pi$ proper electromagnetic interaction.

2.Above threshold $M_{\pi\pi} > 2m_c$: Gamov-Sommerfield factor

$G = \frac{2\pi\omega}{1-e^{-2\pi\omega}}; \quad \omega = \frac{\alpha}{\upsilon}$

Decay with two charged pions in final state: $K^{\pm} \longrightarrow \pi^{+}\pi^{-}e^{\pm}\nu$

Rosselet et al.,CERN (1977); Pislak et al. BNL (2003)

NA48/2,CERN, Bloch-Devaux, Kaon workshops: December 2006;May,September 2007 $a_0^0 = 0.256 \pm 0.006 \pm 0.005$

ChPT $a_0^0 = 0.22 \pm 0.005$

A.Sissakian,A.Tarasov,H.Torosyan,O.Voskresenskaya,S.G. hep-ph 0704.2675; 07011; Yad.Phys.73 (2010)

1.The isospin symmetry breaking effects in K_{e4} decays.

2.The electromagnetic effects in K_{e4} decay.

Charge exchange in $K^{\pm} \longrightarrow \pi^{+}\pi^{-}e^{\pm}\nu$

We take into account the charge exchange (a la N.Cabibbo) $K \rightarrow \pi^0\pi^0 e\nu \rightarrow \pi^+\pi^- e\nu$ For dipion state I=l=0

$$M = M_c(1 + ik_c a_c) + ik_n a_x M_n; \, a_c = \frac{2a_0 + a_2}{3}(1 + \epsilon);$$

$$a_x = \frac{1}{3}(a_0 - a_2)(1 + \epsilon/3); \, \epsilon = \frac{m_c^2 - m_n^2}{m_c^2}$$

Cusp in $K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu$

Eur. Conference on High Energy Physics,Grenoble, July, 2011

45000 events; $q^2 = \frac{M^2}{4m_\pi^2} - 1$