# The DSE Perspective on Chiral Dynamics in Hadron Physics

Peter C. Tandy

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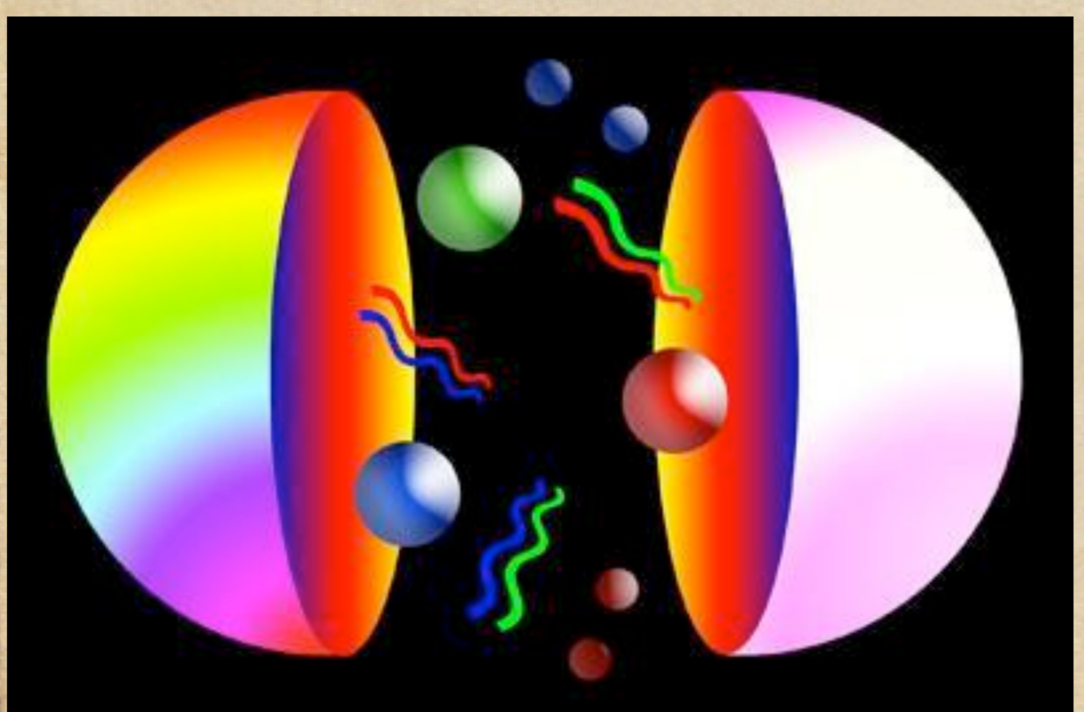
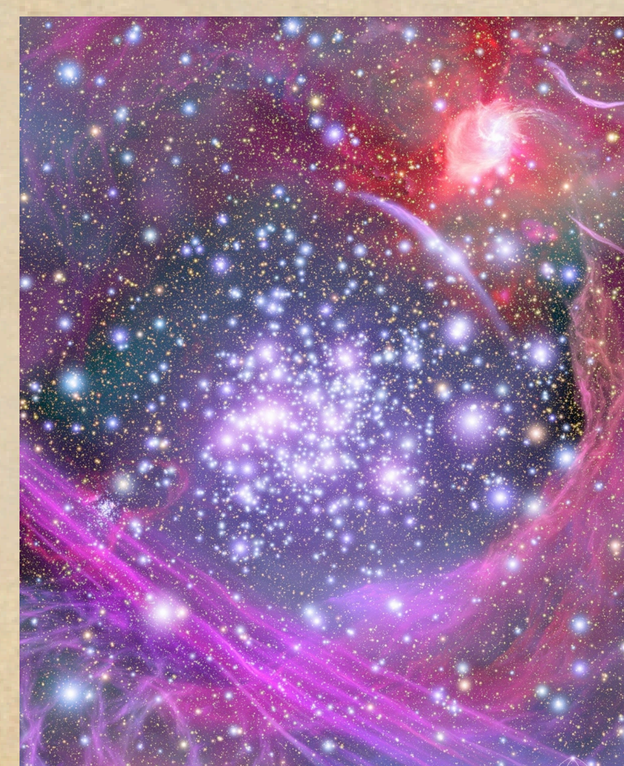
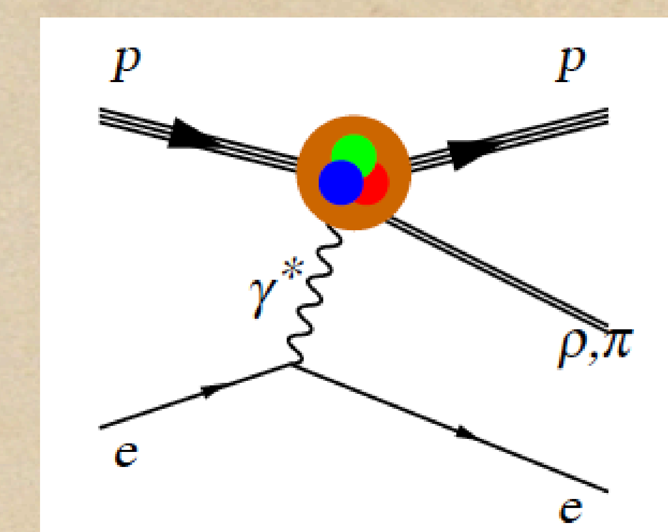
$$\langle 0 \,|\, \bar{q}q \,|\, 0 \rangle$$







## Collaborators:

- Stan Brodsky, SLAC, Stanford Univ
- Robert Shrock, CN Yang Inst for Th Phys, Stony Brook
- Craig Roberts, Theory, Phys Div, Argonne National Lab
- Lei Chang, Peking Univ & Julich
- Konstantin Khitrin, Kent State University
- Javier Cobos-Martinez, Kent State University

Some recent reviews:

4.

**Selected highlights from the study of mesons.**
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**Collective perspective on advances in Dyson-Schwinger Equation QCD.**
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e-Print: **arXiv:1201.3366 [nucl-th] PDF**

# Outline

- Overview of QCD modeling based on Dyson-Schwinger Eqns of QCD
- Emphasis is on small scales (quark-gluon content) as well as large scales
- Model dependence unavoidable; still some exact results
- Chiral symmetry at the level of quark dynamics---a Goldberger-Trieman relation, Goldstone theorem, consequences for hadron structure
- Meson and Baryon masses, decay constants......
- Elastic and transition form factors
- Valence quark PDF of pion and kaon, pion distribution amplitude

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## Lattice-QCD and DSE-based modeling

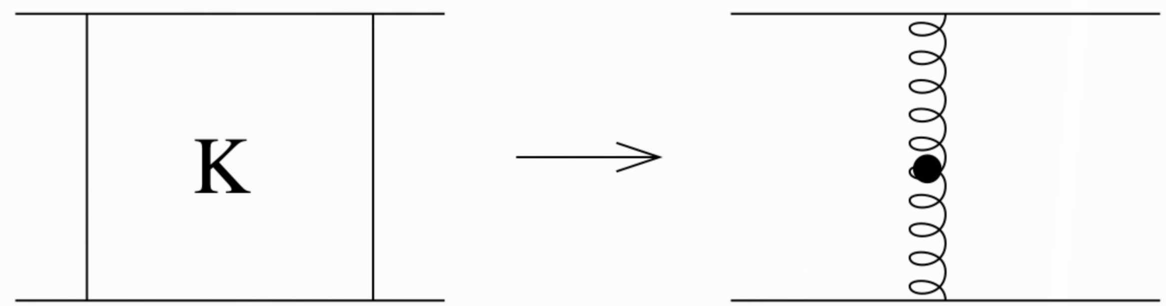
- Lattice: $\langle \mathcal{O} \rangle = \int D\bar{q}qG \;\; \mathcal{O}(\bar{q}, q, G) \;\; e^{-\mathcal{S}[\bar{q},q,G]}$
  - Euclidean metric, x-space, Monte-Carlo
  - Issues: lattice spacing and vol, sea and valence $m_q$, fermion Det
  - Large time limit $\Rightarrow$ nearest hadronic mass pole
- EOMs (DSEs): $0 = \int D\bar{q}qG \;\; \frac{\delta}{\delta q(x)} \; e^{-\mathcal{S}[\bar{q},q,G]+(\bar{\eta},q)+(\bar{q},\eta)+(J,G)}$
  - Euclidean metric, p-space, continuum integral eqns
  - Issues: truncation and phenomenology—not full QCD
  - Analtyic contin. $\Rightarrow$ nearest hadronic mass pole
  - Can be quick to identify systematics, mechanisms, $\cdots$

# DSE-based modeling of Hadron Physics

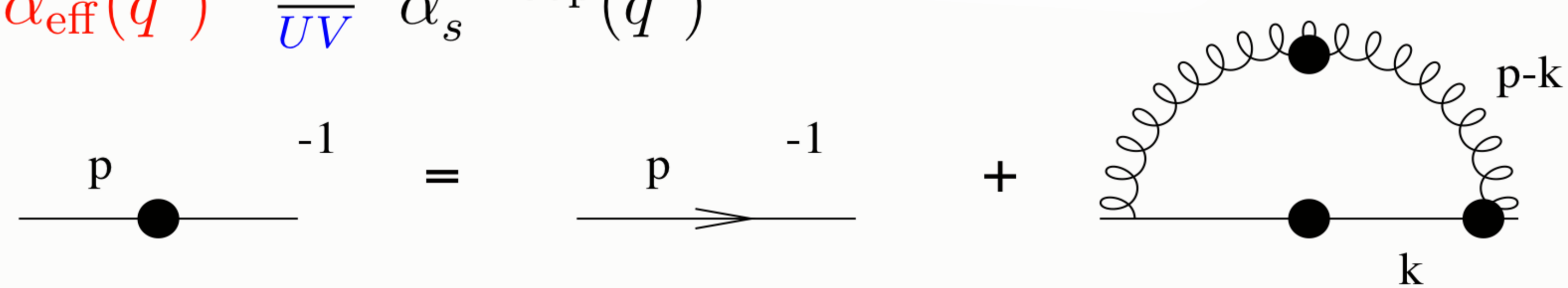
- Soft physics: truncate DSEs to min: 2-pt, 3-pt fns
- Should be relativistically covariant—-convenient for decays, Form Factors, etc
  - No boosts needed on wavefns of recoiling bound st.
  - $\infty$ d.o.f. $\rightarrow$ few quasi-particle effective d.o.f.
- Do not make a 3-dimensional reduction
- Preserve 1-loop QCD renorm group behavior in UV
- Preserve global symmetries, conserved em currents, etc
- Preserve PCAC $\Rightarrow$ Goldstone's Thm
- Can't preserve local color gauge covariance—-just choose Landau gauge [RG fixed pt]
- Parameterize the deep infrared (large distance) QCD coupling

## *Ladder-Rainbow Model*

Landau gauge only

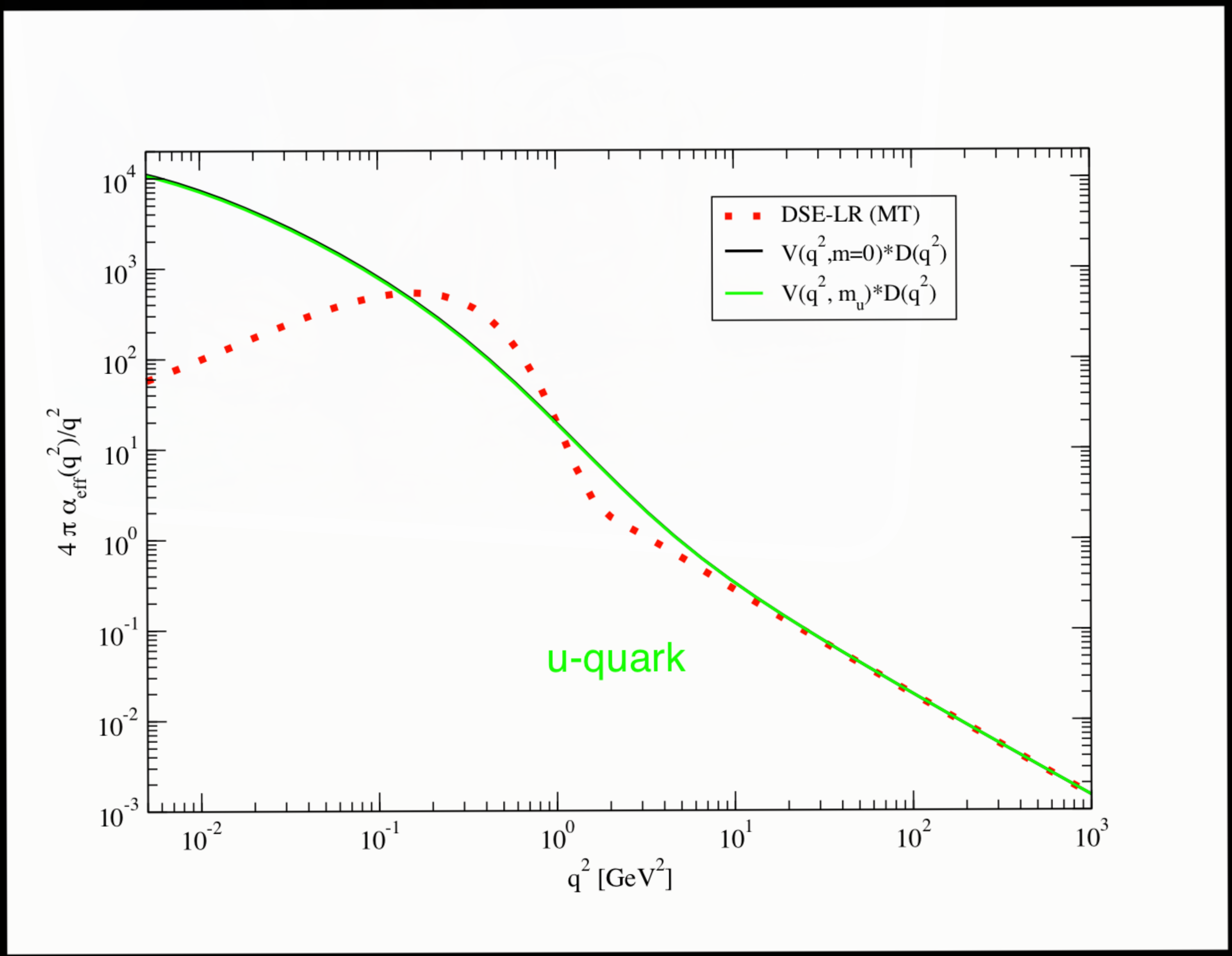



- $K_{\rm BSE} \to -\gamma_\mu \frac{\lambda^a}{2} \; 4\pi\alpha_{\rm eff}(q^2) \; D^{\rm free}_{\mu\nu}(q) \; \gamma_\nu \frac{\lambda^a}{2}$
- $\alpha_{\rm eff}(q^2) \;\; \overrightarrow{IR} \;\; \langle \bar{q}q \rangle_{\mu=1\ {\rm GeV}} = -(240\,\text{MeV})^3$ , incl vertex dressing
- $\alpha_{\rm eff}(q^2) \;\; \overrightarrow{UV} \;\; \alpha_s^{1-{\rm loop}}(q^2)$




- P. Maris & P.C. Tandy, PRC60, 055214 (1999)
  $M_\rho, M_\phi, M_{K^\star}$ good to 5%, $f_\rho, f_\phi, f_{K^\star}$ good to 10%

# Quenched lattice ⇒ $m_q$ Depn of DSE Kernel




Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

# Modern Context for Ladder-Rainbow Kernel

$\mathbf{Landau\ gauge},\ \color{red}{\mathbf{lattice - QCD\ gluon\ propagator}},$
$\mathbf{I.L.Bogolubisky}\ \mathit{etal.}, \mathbf{PosLAT2007, 290\ (2007)}$

$\mathbf{DSE\ Studyw/\ modern\ n - pt\ fns}$
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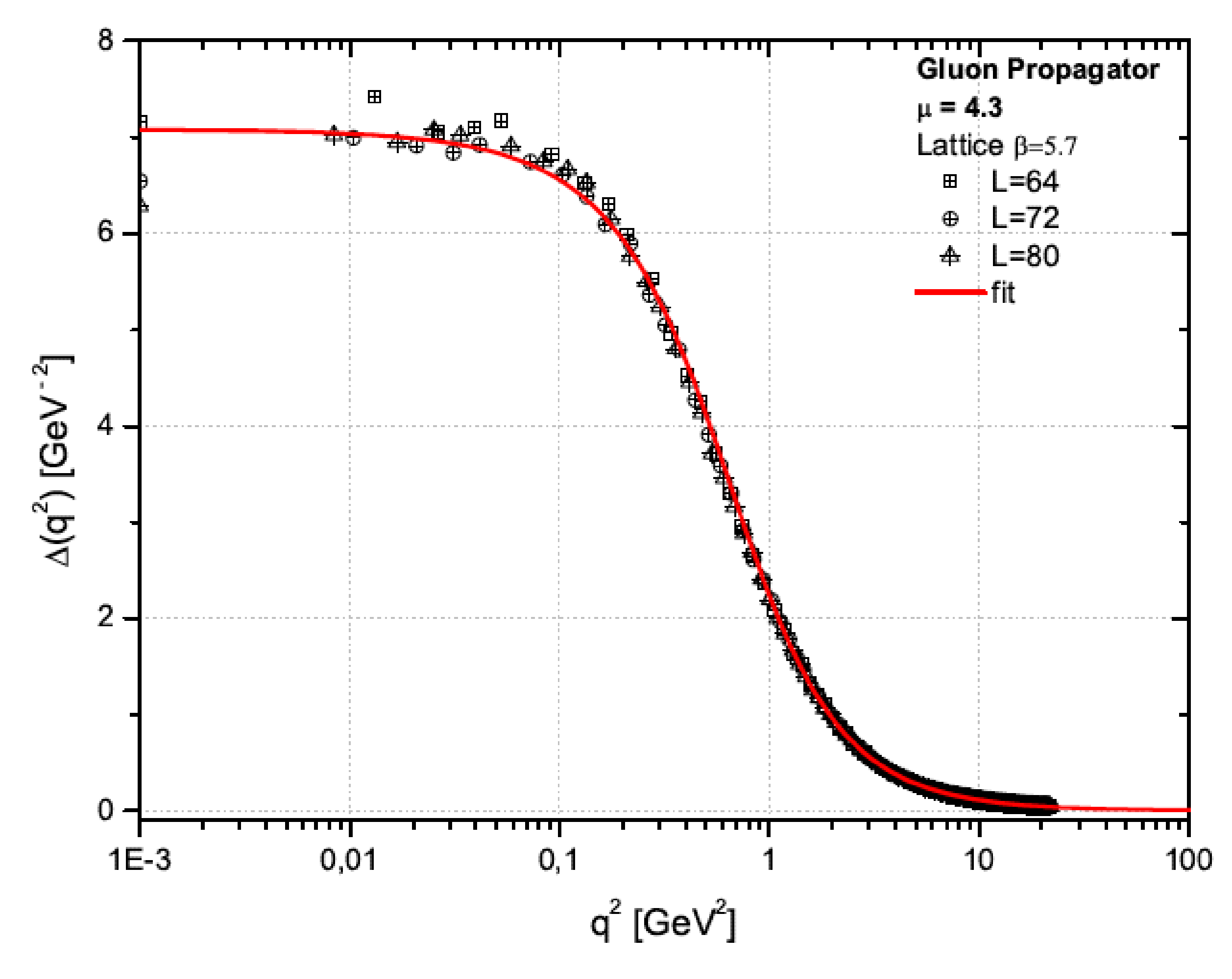


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$\Rightarrow m_G(k^2)$ $\qquad$ $\mathrm{m_G}(0) \sim 0.38$ GeV

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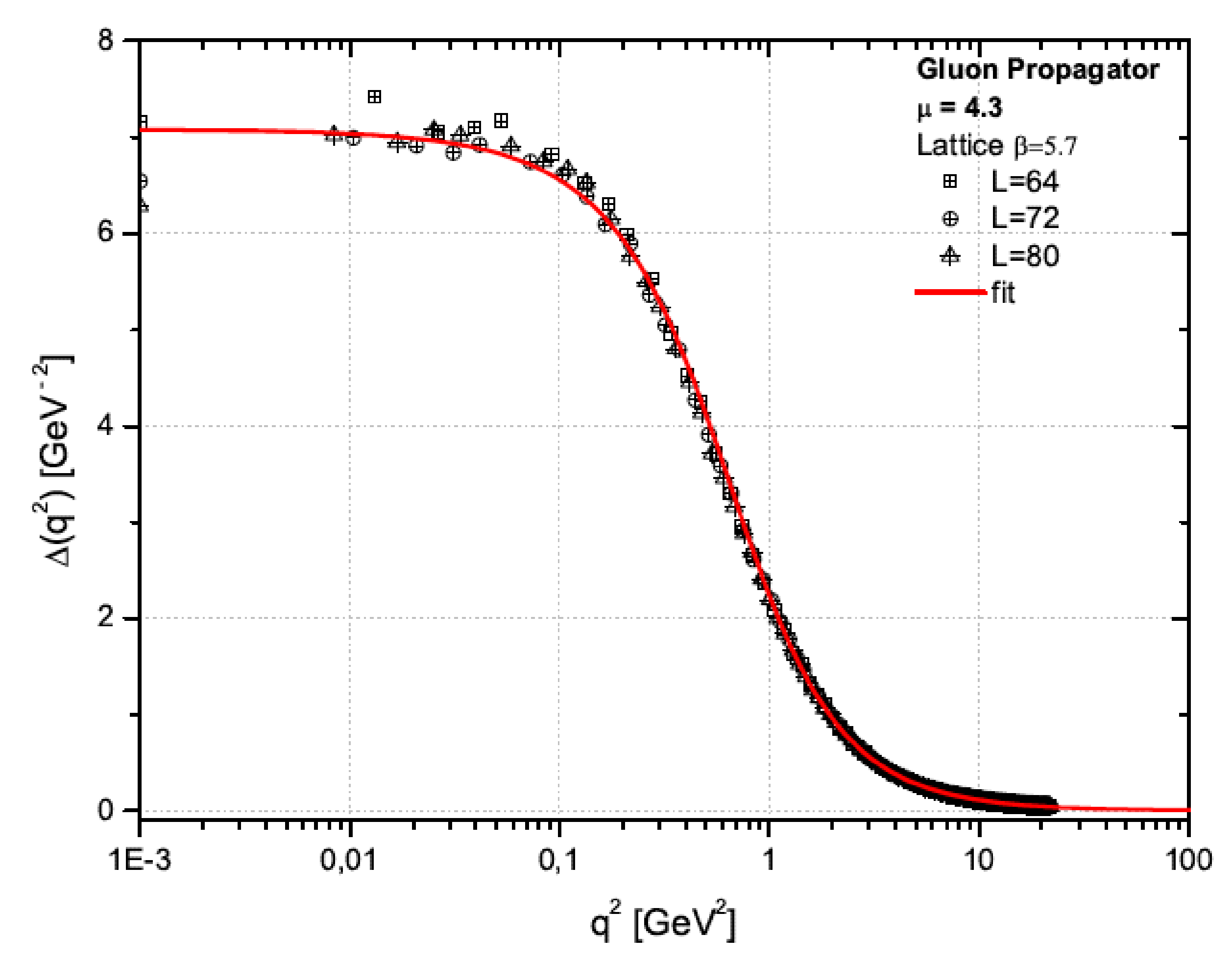


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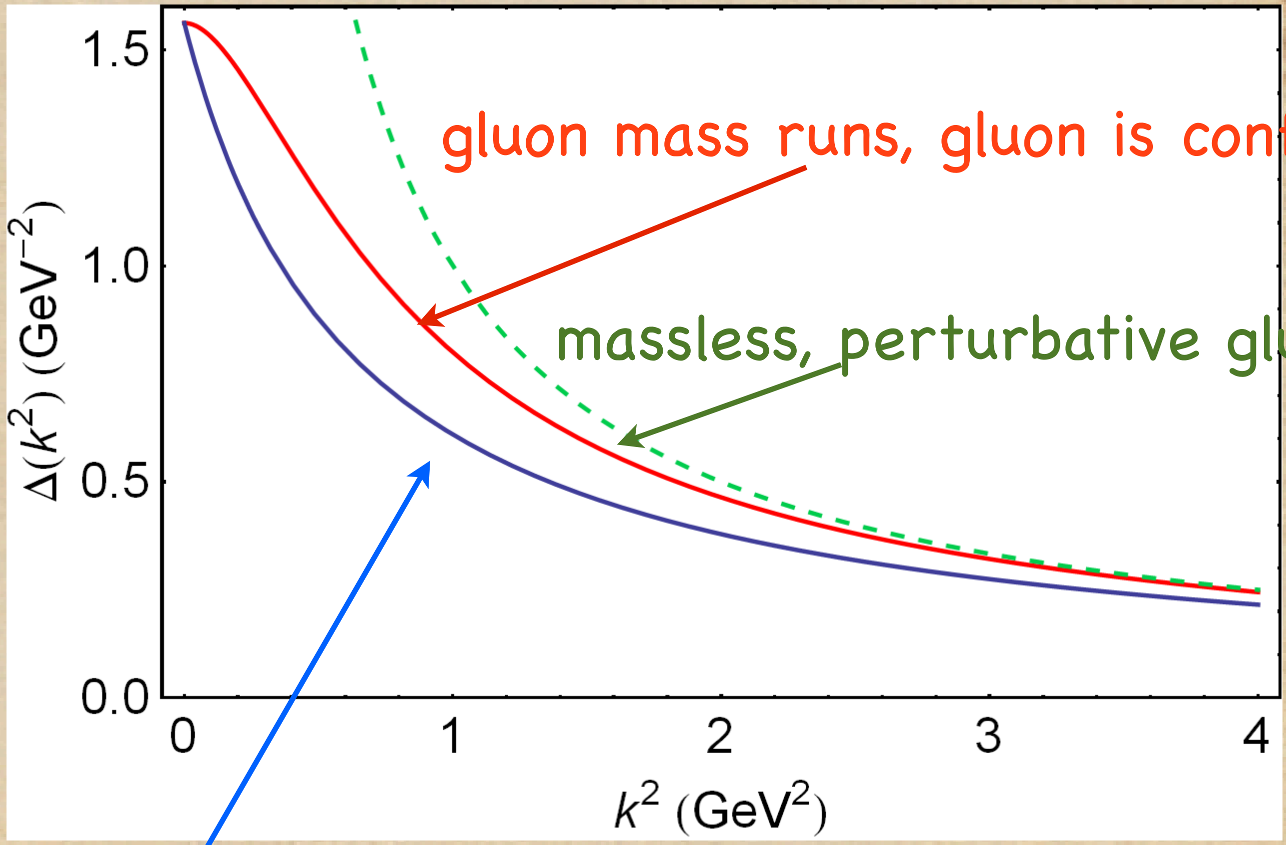
$$\mathbf{K^{RL}_{BSE}} = \frac{4\pi\hat{\alpha}_{\rm eff}(\mathbf{q^2})}{\mathbf{m_G^2(q^2) + q^2}}$$

$$\Rightarrow \frac{\hat{\alpha}_{\rm eff}(\mathbf{0.1})}{\pi} \approx \mathbf{3 - 4}$$



## Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

| | expt. | calc. |
|---|---|---|
| $-\langle \bar{q}q \rangle_\mu^0$ | $(0.236 \text{ GeV})^3$ | $(0.241^\dagger)^3$ |
| $m_\pi$ | 0.1385 GeV | $0.138^\dagger$ |
| $f_\pi$ | 0.0924 GeV | $0.093^\dagger$ |
| $m_K$ | 0.496 GeV | $0.497^\dagger$ |
| $f_K$ | 0.113 GeV | 0.109 |

Charge radii (PM, Tandy, PRC62, 055204)

| | | |
|---|---|---|
| $r_\pi^2$ | 0.44 fm$^2$ | 0.45 |
| $r_{K^+}^2$ | 0.34 fm$^2$ | 0.38 |
| $r_{K^0}^2$ | -0.054 fm$^2$ | -0.086 |

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

| | | |
|---|---|---|
| $g_{\pi\gamma\gamma}$ | 0.50 | 0.50 |
| $r_{\pi\gamma\gamma}^2$ | 0.42 fm$^2$ | 0.41 |

Weak $K_{l3}$ decay (PM, Ji, PRD64, 014032)

| | | |
|---|---|---|
| $\lambda_+(e3)$ | 0.028 | 0.027 |
| $\Gamma(K_{e3})$ | $7.6 \cdot 10^6 \text{ s}^{-1}$ | 7.38 |
| $\Gamma(K_{\mu 3})$ | $5.2 \cdot 10^6 \text{ s}^{-1}$ | 4.90 |

Vector mesons (PM, Tandy, PRC60, 055214)

| | | |
|---|---|---|
| $m_{\rho/\omega}$ | 0.770 GeV | 0.742 |
| $f_{\rho/\omega}$ | 0.216 GeV | 0.207 |
| $m_{K^*}$ | 0.892 GeV | 0.936 |
| $f_{K^*}$ | 0.225 GeV | 0.241 |
| $m_\phi$ | 1.020 GeV | 1.072 |
| $f_\phi$ | 0.236 GeV | 0.259 |

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

| | | |
|---|---|---|
| $g_{\rho\pi\pi}$ | 6.02 | 5.4 |
| $g_{\phi KK}$ | 4.64 | 4.3 |
| $g_{K^*K\pi}$ | 4.60 | 4.1 |

Radiative decay (PM, nucl-th/0112022)

| | | |
|---|---|---|
| $g_{\rho\pi\gamma}/m_\rho$ | 0.74 | 0.69 |
| $g_{\omega\pi\gamma}/m_\omega$ | 2.31 | 2.07 |
| $(g_{K^*K\gamma}/m_K)^+$ | 0.83 | 0.99 |
| $(g_{K^*K\gamma}/m_K)^0$ | 1.28 | 1.19 |

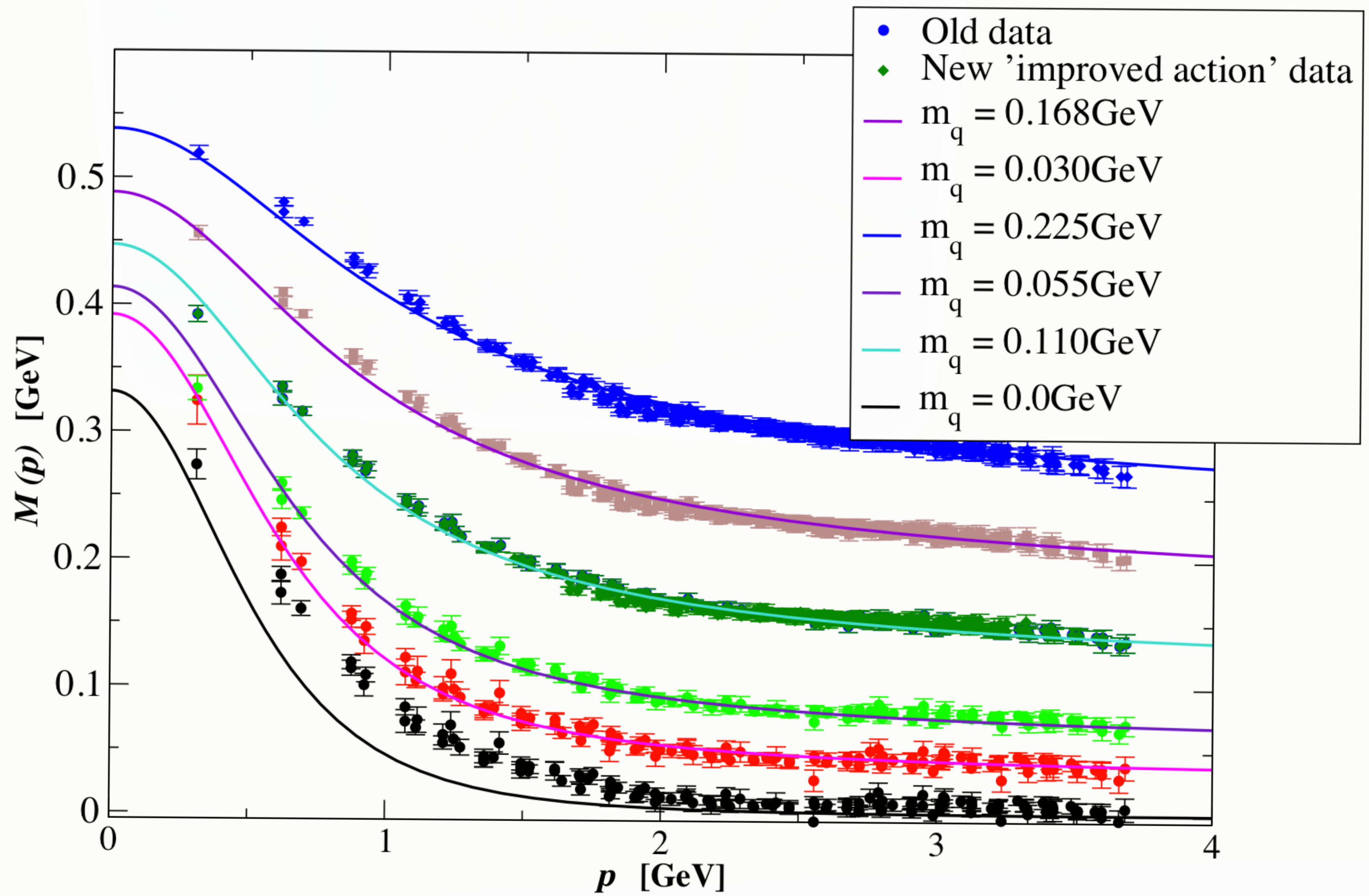
Scattering length (PM, Cotanch, PRD66, 116010)

| | | |
|---|---|---|
| $a_0^0$ | 0.220 | 0.170 |
| $a_0^2$ | 0.044 | 0.045 |
| $a_1^1$ | 0.038 | 0.036 |

**In summary: 31 exptl data @ RMS error of 15%**

# Qu-lattice $S(p), D(q)$ mapped to a DSE kernel

$$S(p) = Z(p)\,[i\,\not{p} + M(p)]^{-1}$$



# Exact Mass Relation for Flavor Non-Singlet PS Mesons

$\mathrm{Maris, Roberts, PCT, Phys.\ Lett.\ B420, 267(1998)}$ $\mathrm{--an\ exact\ result\ in\ QCD}$

$$\mathbf{PCAC} \;\Rightarrow\; \langle\, \bar{\mathbf{q}}(\mathbf{x})\mathbf{q}(\mathbf{y})\, (\partial_\mu \mathbf{J}_{5\,\mu} = \mathbf{2m_q}\, \mathbf{J_5})\, \rangle \;\Rightarrow \mathbf{AV - WTI}:$$

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$$-\mathbf{i}\mathbf{P}_\mu\, \Gamma_{5\mu}(\mathbf{k};\mathbf{P}) = \mathbf{S}^{-1}(\mathbf{k}+\mathbf{P}/2)\, \gamma_5 \frac{\tau}{2} + \gamma_5 \frac{\tau}{2}\, \mathbf{S}^{-1}(\mathbf{k}-\mathbf{P}/2) - 2\mathbf{m_q}\, \Gamma_5(\mathbf{k};\mathbf{P})$$

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- $\mathbf{m}_\mathbf{q} = 0, \mathbf{P} = 0 \;\Rightarrow\; \mathbf{GT}_\mathbf{q}: \;\; \Gamma_\pi(\mathbf{k}^2;0) = \mathbf{i}\gamma_5\, \tau\, \dfrac{\frac{1}{4}\mathrm{tr}\, \mathbf{S}_0^{-1}(\mathbf{k})}{\mathbf{f}_\pi^0} + \cdots$

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$$\mathbf{\Gamma}_\pi(\mathbf{k};\mathbf{P})\;\frac{\mathbf{f}_\pi\;\mathbf{P}_\mu}{\mathbf{P^2}+\mathbf{m}_\pi^2} \qquad\qquad \mathbf{\Gamma}_\pi(\mathbf{k};\mathbf{P})\;\frac{\mathbf{i}\;\rho_\pi}{\mathbf{P^2}+\mathbf{m}_\pi^2}$$

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$$\Gamma_\pi(k;P)\, \frac{f_\pi\, P_\mu}{P^2 + m_\pi^2} \qquad\qquad \Gamma_\pi(k;P)\, \frac{i\, \rho_\pi}{P^2 + m_\pi^2}$$

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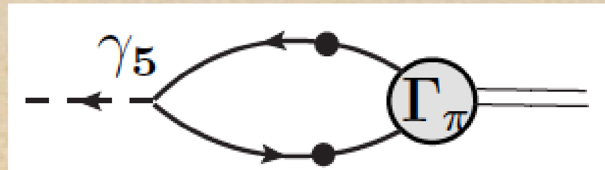
- $m_q \neq 0: \;\Rightarrow\; f_\pi\, m_\pi^2 = 2\, m_q\, \rho_\pi(m_q)$ [for all $m_q$, all ps mesons]
- $\rho_{ps}(\mu) = -\langle 0\,|\, \bar{q}\, \gamma_5\, q\,|\, ps \rangle$



Gen axial vertex $\Gamma_{5\mu}(k;P) = \gamma_5 \left\{ \gamma_\mu F_R(k;P) + \not{k}\, k_\mu G_R(k;P) - \sigma_{\mu\nu} k_\nu H_R(k;P) \right\}$

$$+ \tilde{\Gamma}_{5\mu}(k;P) + \Gamma_\pi(k;P) \frac{2 f_\pi P_\mu}{P^2 + m_\pi^2}$$

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chiral+soft pi limits of AV-WTI give: **DCSB**

$$S(k) = \frac{1}{i \not{k} A(k^2) + B(k^2)}$$

$$f_\pi E_\pi(k; P=0) = B(k^2)$$

$$F_R(k;0) + 2 f_\pi F_\pi(k;0) = A(k^2)$$

$\Rightarrow g_A^q(F) \approx 0.81\ (RL) + 6\%(BRL)$

$g_A^N$ Roberts, Chang, Schmidt, 2012

$$G_R(k;0) + 2 f_\pi G_\pi(k;0) = 2A'(k^2)$$

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Gen axial vertex $$\Gamma_{5\mu}(k;P) = \gamma_5 \left\{ \gamma_\mu F_R(k;P) + \not{k}\, k_\mu\, G_R(k;P) - \sigma_{\mu\nu} k_\nu\, H_R(k;P) \right\}$$
$$+ \tilde{\Gamma}_{5\mu}(k;P) + \Gamma_\pi(k;P)\, \frac{2f_\pi P_\mu}{P^2 + m_\pi^2}$$

Gen pion BS ampl $$\Gamma_\pi(k;P) = \gamma_5 \left\{ iE_\pi(k;P) + \not{P}\, F_\pi(k;P) + \not{k}\, k\cdot P\, G_\pi(k;P) + \sigma : k P\, H_\pi(k;P) \right\}$$

chiral+soft pi limits of AV-WTI give: **DCSB**

$$S(k) = \frac{1}{i\not{k}\, A(k^2) + B(k^2)}$$

- $f_\pi\, E_\pi(k;P=0) = B(k^2)$
- $F_R(k;0) + 2f_\pi\, F_\pi(k;0) = A(k^2)$

$\Rightarrow g_A^q(F) \approx 0.81\ (RL)\ + 6\%(BRL)$

$g_A^N$ Roberts, Chang, Schmidt, 2012

$$G_R(k;0) + 2f_\pi\, G_\pi(k;0) = 2A'(k^2)$$

$$H_R(k;0) + 2f_\pi\, H_\pi(k;0) = 0$$

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# Beyond LR

- Deficiencies of ladder-rainbow truncation:
- Axial vector (a1, b1) and scalar mesons (L >0) are too light
- This does not bode well for exotic/hybrid hadron states
- Eg, $\pi_1(1^{-+})$ exotic soln of LR-BSE is at 0.9 GeV. Expected to have L >0, so fix that.
- Craig Roberts & Lei Chang have developed a semi-phenomenological extension of LR BSE kernel that is a major advance and performs well for L>0 u/d quark mesons.
- No hadronic decay widths of states--must calc independently
- Any important chiral loop effects have to be added later---(however LR can't be characterized as purely quenched either)

# DCSB-driven Transv q-g Vertex & BSE Kernel

- Three dressed non-pert terms are explored in vertex
- Including Quark Anom color magn mom $\int d^4x \, \frac{1}{2} q \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$
- Zero for massless quarks, but DCSB makes it finite

Standard "Ball-Chiu" term from STI

$$\lambda^3_\mu(p,q) = 2(p+q)_\mu \Delta_B(p,q)$$

$$\Delta_F(p,q) = \frac{F(p^2)-F(q^2)}{p^2-q^2}$$

Anom color mag. mom. term, strength comparable to lattice-QCD: Skullerud, et al., hep-ph/0303176

$$\Gamma^5_\mu(p,q) = \eta \, \sigma_{\mu\nu}(p-q)_\nu \Delta_B(p,q)$$

New term, recovers pQCD--Davydyshev, et al., PRD63, 014022 (2000). Adds to acm.

$$\Gamma^4_\mu(p,q) = [\ell^{\mathrm{T}}_\mu \gamma \cdot k + i\gamma^{\mathrm{T}}_\mu \sigma_{\nu\rho} \ell_\nu k_\rho] \tau_4(p,q)$$

$$\tau_4(p,q) = \mathcal{F}(z) \left[ \frac{1-2\eta}{M_E} \Delta_B(p^2,q^2) - \Delta_A(p^2,q^2) \right]$$

$\mathcal{F}(z) = (1-\exp(-z))/z$, $z = (p_i^2 + p_f^2 - 2M_E^2)/\Lambda^2_{\mathcal{F}}$, $\Lambda_{\mathcal{F}} = 1\,\mathrm{GeV}$, Simplifies numerical analysis;

$M_E = \{s | s > 0, s = M^2(s)\}$ is the Euclidean constituent-quark mass

An Ansatz to build BSE kernel from analytic Ansatz for q-g vertex has been developed and used: Lei Chang, C.D. Roberts, PRL103 081601 (2009)

# Beyond Ladder-Rainbow Truncation

Full kernel/vertex with quark acm,
(one parameter)

## Meson Spectrum

| | Experiment | Rainbow-ladder | One-loop corrected | Ball-Chiu | Full vertex |
|---|---|---|---|---|---|
| a1 | 1230 | 759 | 885 | 1128 | 1270 |
| ρ | 770 | 644 | 764 | 919 | 790 |
| Mass splitting | 455 | 115 | 121 | 209 | 480 |

L Chang, C. D. Roberts, arXiv: 1104.4821

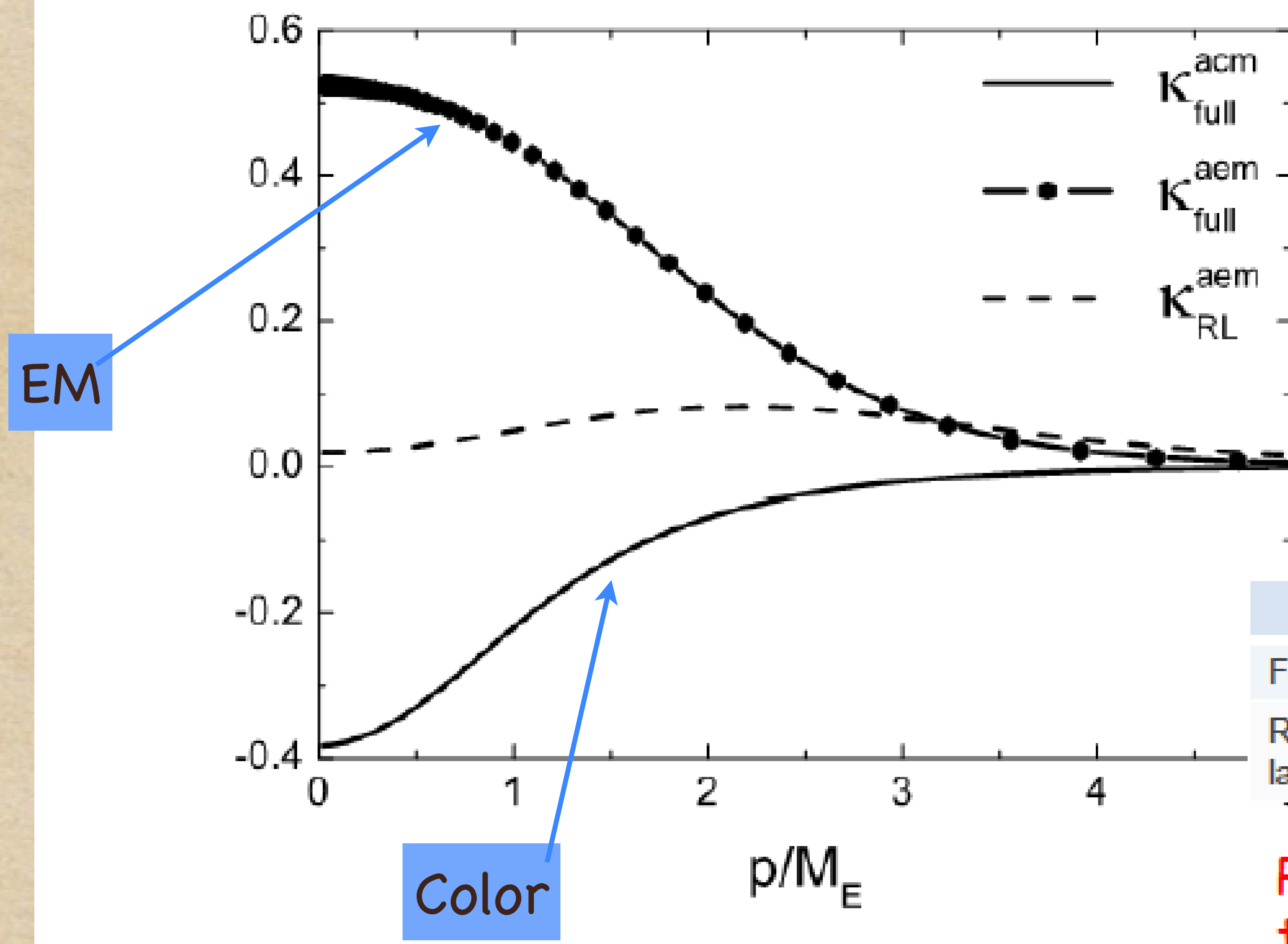
Rainbow-Ladder

- Same physics is expected to be important for hybrids/exotics, radial excitations......

# Quark Anomalous magnetic moments

- Confined quarks do not have a mass-shell

Can not unambiguously define magnetic moments, but can define magnetic moment distribution




ACM and AEM have similar magnitude and opposite sign.

| | $M^E$ | $\kappa^{ACM}$ | $\kappa^{AEM}$ |
|---|---|---|---|
| Full vertex | 0.44 | -0.22 | 0.45 |
| Rainbow-ladder | 0.35 | 0 | 0.048 |

Potentially important for transition form factor

---Lei Chang, Yuxin Liu, C.D. Roberts, PRL106 072001 (2011)

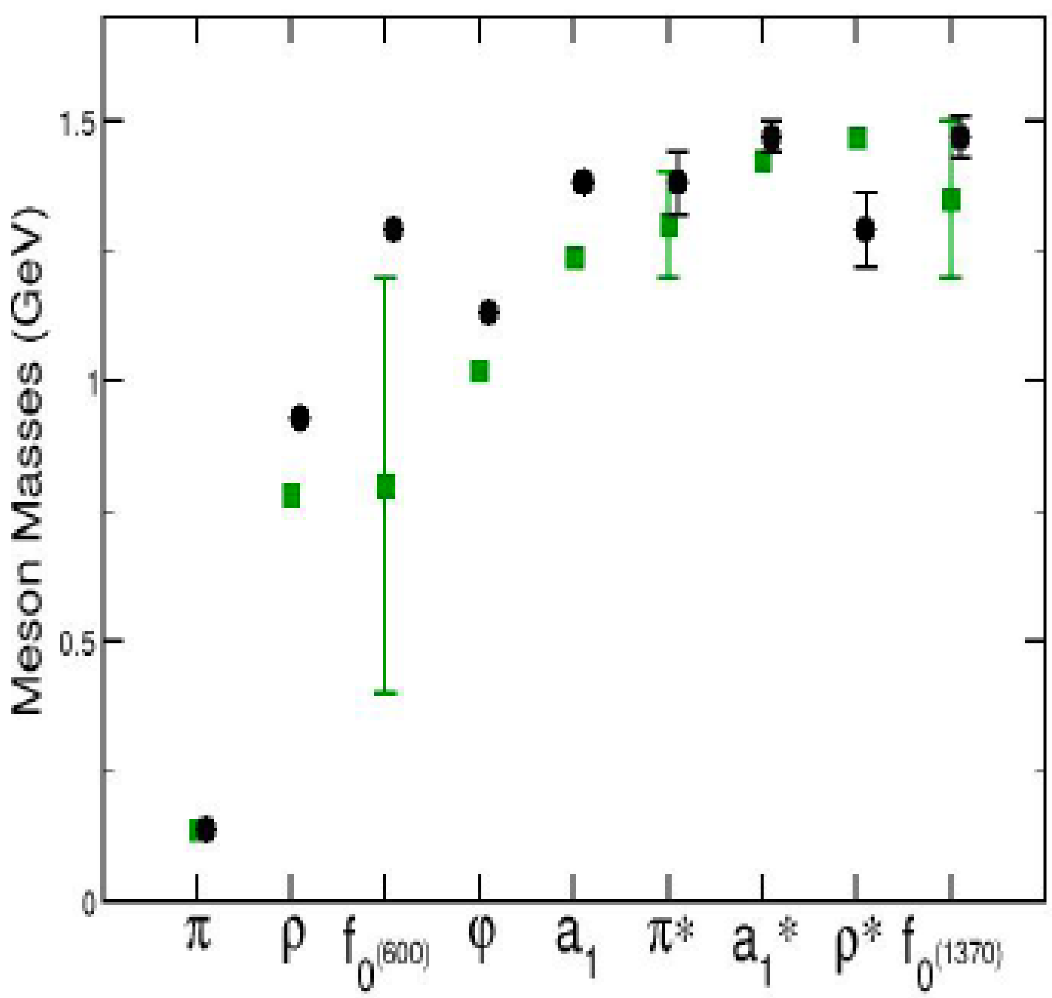
# Hadron Spectrum

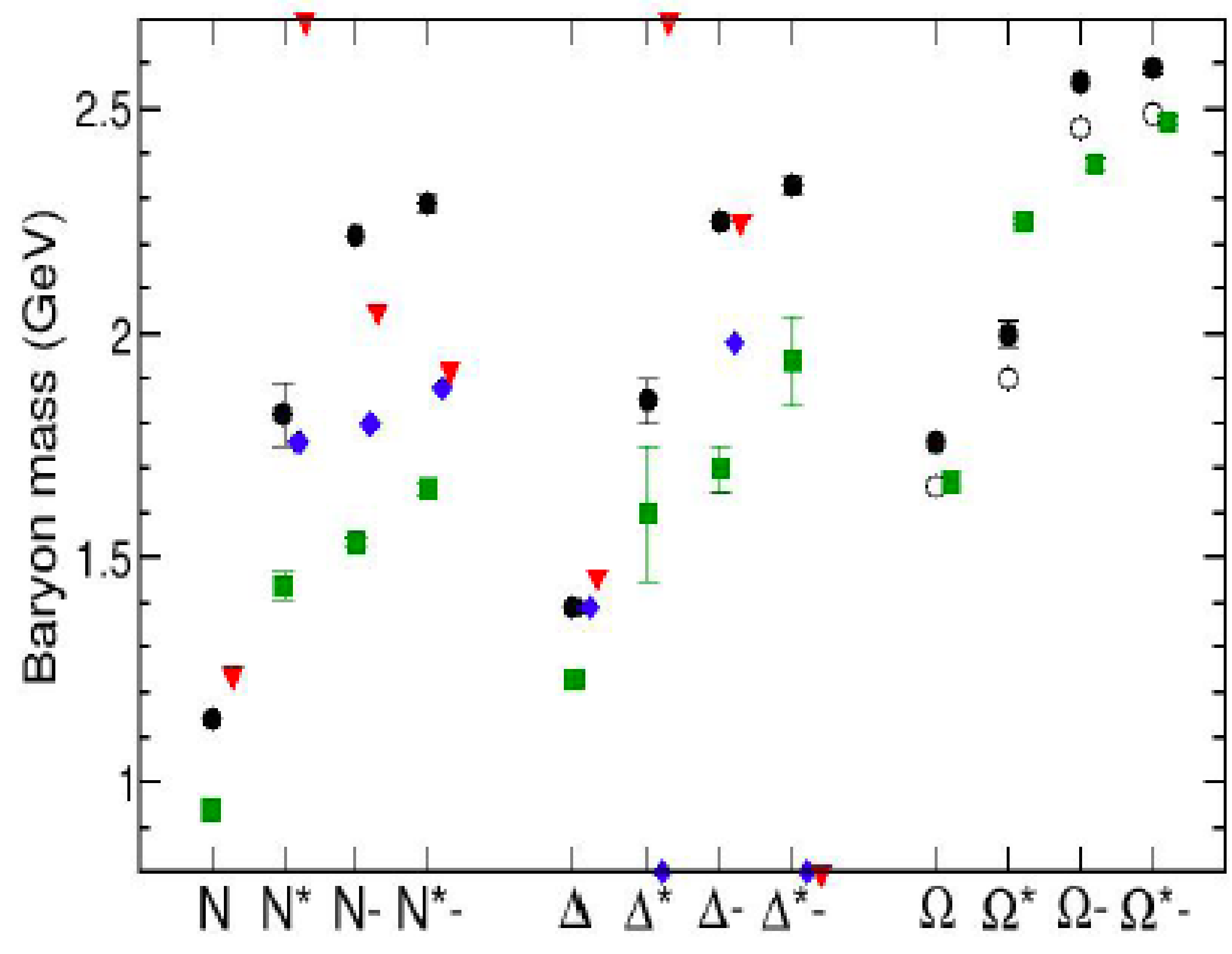
Legend:

- Particle Data Group
- H.L.L. Roberts *et al.* nucl-th/1102.4376
- EBAC
- Jülich

DSE contact model

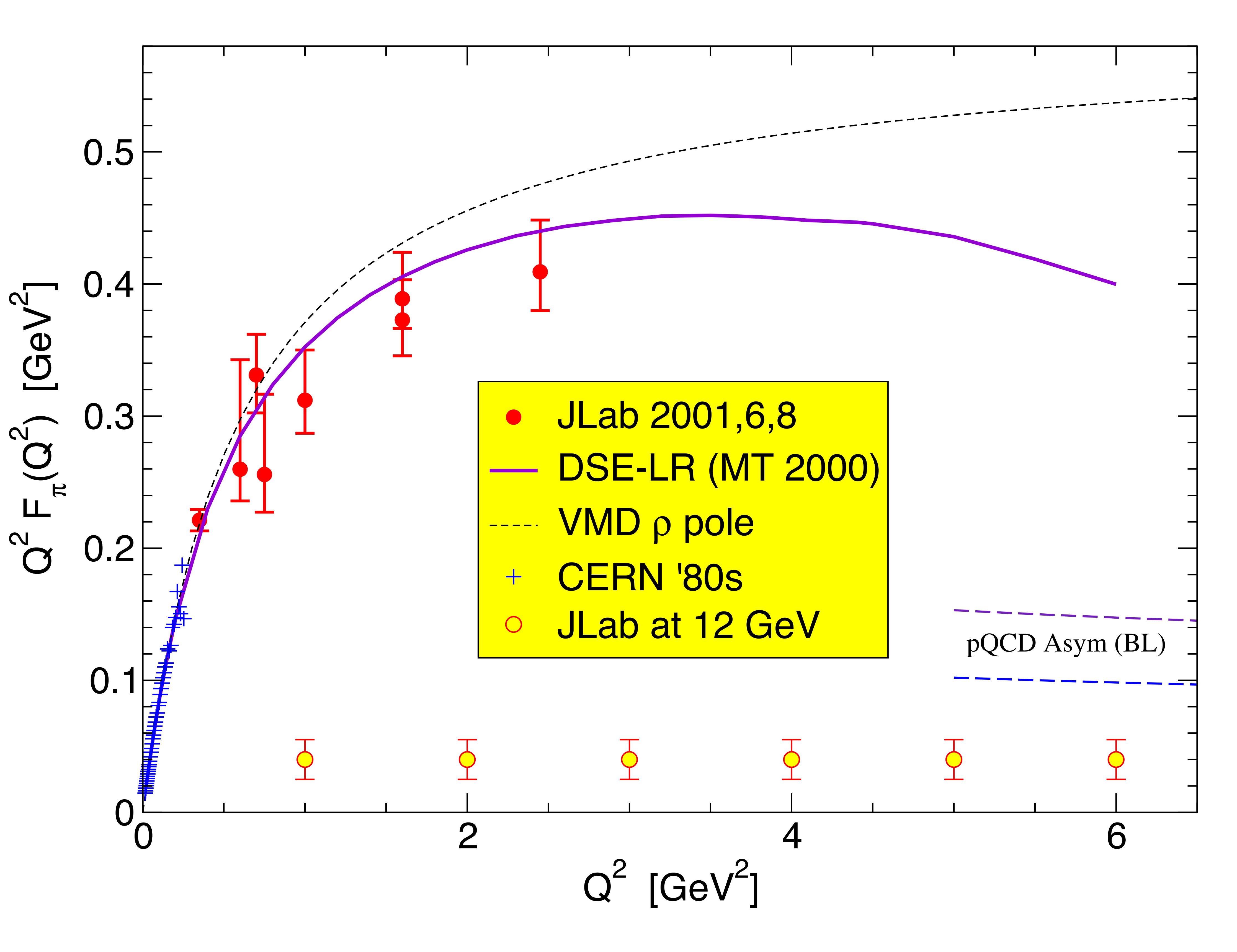
- Symmetry-preserving unification of the computation of meson & baryon masses
- rms-rel.err./deg-of-freedom = 13%
- PDG values (almost) uniformly overestimated in both cases - room for the pseudoscalar meson cloud?!



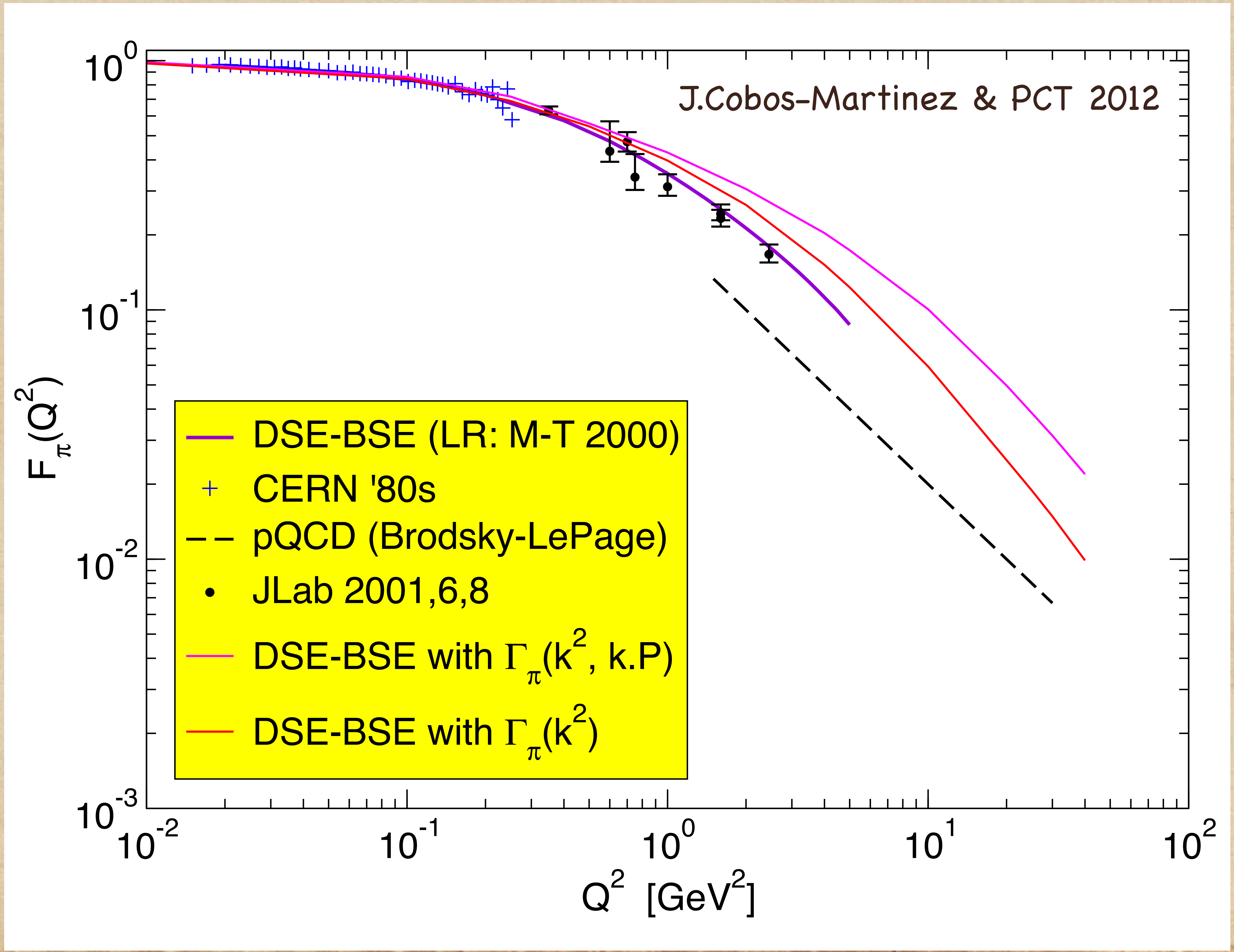
Now Back to Ladder-Rainbow....

PDFs and Form Factors....



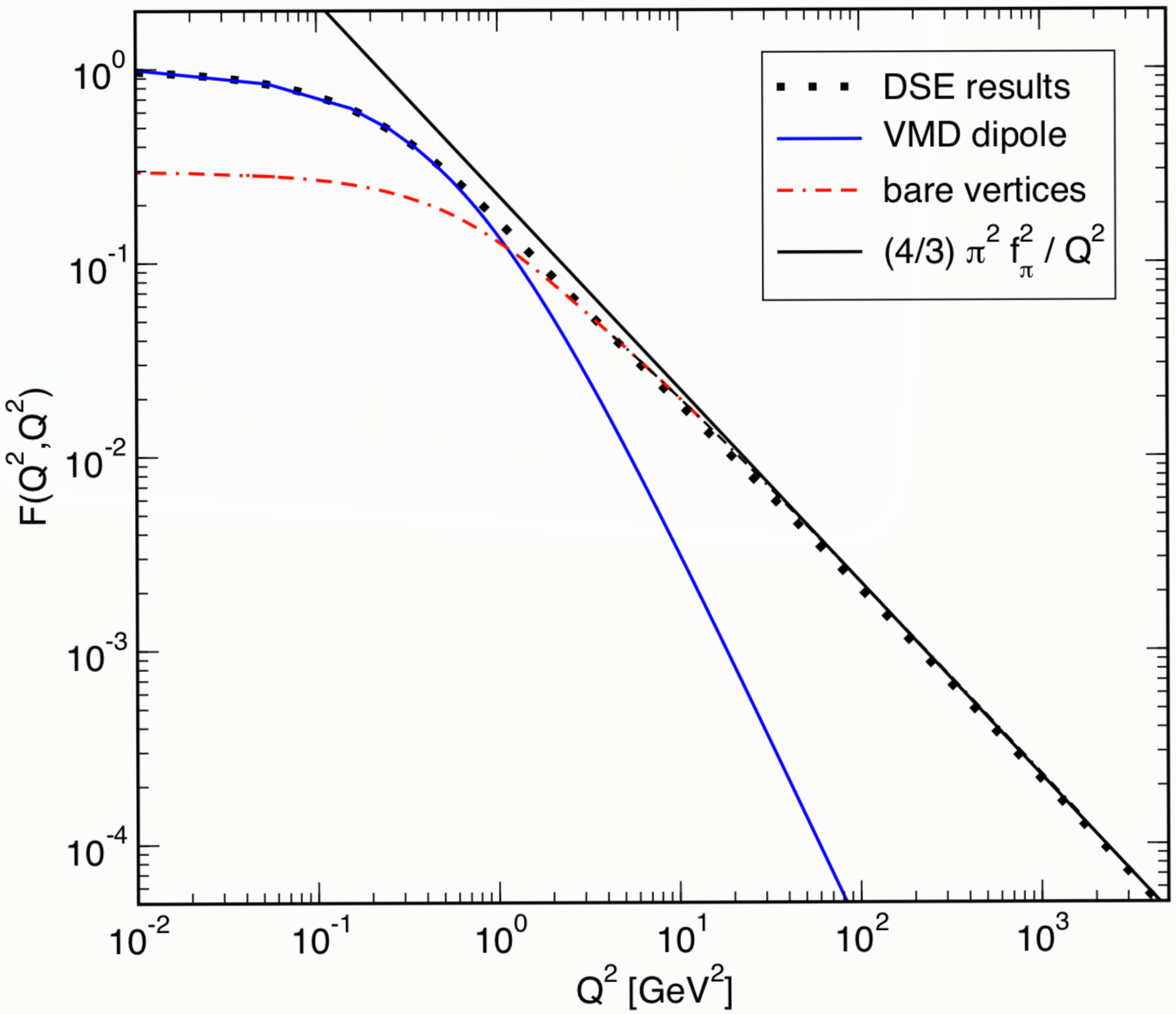
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
$Q^2 F_\pi(Q^2)$ [GeV$^2$]
$Q^2$ [GeV$^2$]
JLab 2001,6,8
DSE-LR (MT 2000)
VMD ρ pole
CERN '80s
JLab at 12 GeV
pQCD Asym (BL)

# DSE gives Sum of standard analytic Feynman integrals



# $\gamma^\star\pi\gamma^\star$ Asymptotic Limit

Lepage and Brodsky, PRD22, 2157 (1980): LC-QCD/OPE ⇒



# Asymptotic Pion Transition Form Factor
# (Brodsky, Lepage...)

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$\gamma(\mathbf{Q_1^2})\ \gamma(\mathbf{Q_2^2}) \to \pi^{\mathbf{0}}$

$$\mathbf{F}(\mathbf{Q_1^2}, \mathbf{Q_2^2}) \to \mathbf{4}\pi^{\mathbf{2}}\mathbf{f}_\pi^{\mathbf{2}}\left\{\frac{\mathbf{J}(\omega)}{\mathbf{Q_1^2}+\mathbf{Q_2^2}} + \mathcal{O}\left(\frac{\alpha_{\mathbf{s}}}{\pi}\ ,\ \frac{\mathbf{1}}{\mathbf{Q^2}}\right\}\right.$$

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$$\mathbf{J}(\omega) = \frac{\mathbf{4}}{\mathbf{3}} \int_\mathbf{0}^\mathbf{1} \mathbf{dx}\, \frac{\phi_\pi(\mathbf{x})}{\mathbf{1}-\omega^\mathbf{2}(\mathbf{2x}-\mathbf{1})}$$

$$\omega = \frac{\mathbf{Q_1^2}-\mathbf{Q_2^2}}{\mathbf{Q_1^2}+\mathbf{Q_2^2}}$$

# Asymptotic Pion Transition Form Factor (Brodsky, Lepage...)

$$\gamma(Q_1^2)\ \gamma(Q_2^2) \to \pi^0 \qquad \mathbf{F(Q_1^2, Q_2^2) \to 4\pi^2 f_\pi^2 \left\{ \frac{J(\omega)}{Q_1^2+Q_2^2} \ +\ \mathcal{O}\left(\frac{\alpha_s}{\pi}\ ,\ \frac{1}{Q^2}\right\}\right.}$$

$$\mathbf{J(\omega) = \frac{4}{3}\ \int_0^1\ dx \frac{\phi_\pi(x)}{1-\omega^2(2x-1)}} \qquad \omega = \frac{\mathbf{Q_1^2 - Q_2^2}}{\mathbf{Q_1^2 + Q_2^2}}$$

$$\phi_\pi(\mathbf{x}) \propto \int \frac{\mathbf{d}\lambda}{\mathbf{2}\pi}\ \mathbf{e}^{\mathbf{i}\lambda\mathbf{x}}\ \langle \mathbf{0}|\ \bar{\mathbf{q}}(\mathbf{0})\ \gamma\cdot\mathbf{n}\ \gamma_5\ \mathbf{q}(\lambda\mathbf{n})\ |\pi(\mathbf{P})\rangle \qquad \mathbf{n}^\nu\ \mathbf{n}_\nu = \mathbf{0}$$

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$$\phi_\pi^{\mathrm{asym}}(\mathbf{x}) \ \propto\ \mathbf{x}(1-\mathbf{x}) \quad \textbf{A finite size Bethe} - \textbf{Salpeter bound state}$$

$$\textbf{LC pQCD (BL)} \ \Rightarrow \mathbf{J}(1) = 2$$

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$\phi_\pi^{\mathrm{asym}}(\mathbf{x}) \propto \mathbf{x(1-x)}$ **A finite size Bethe – Salpeter bound state**

**LC pQCD (BL)** $\Rightarrow \mathbf{J(1) = 2}$ $\quad$ [ **DSE LR** $\Rightarrow \frac{\mathbf{7}}{\mathbf{8}} \times \mathbf{2}$ ]

# The 2009 BaBar data

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k-indepn gluon propagator : H.Roberts, C. Roberts, A. Bashir, L. Gutierrez-Guerrero, PCT: arXiv:1009.0067 (point-pion)

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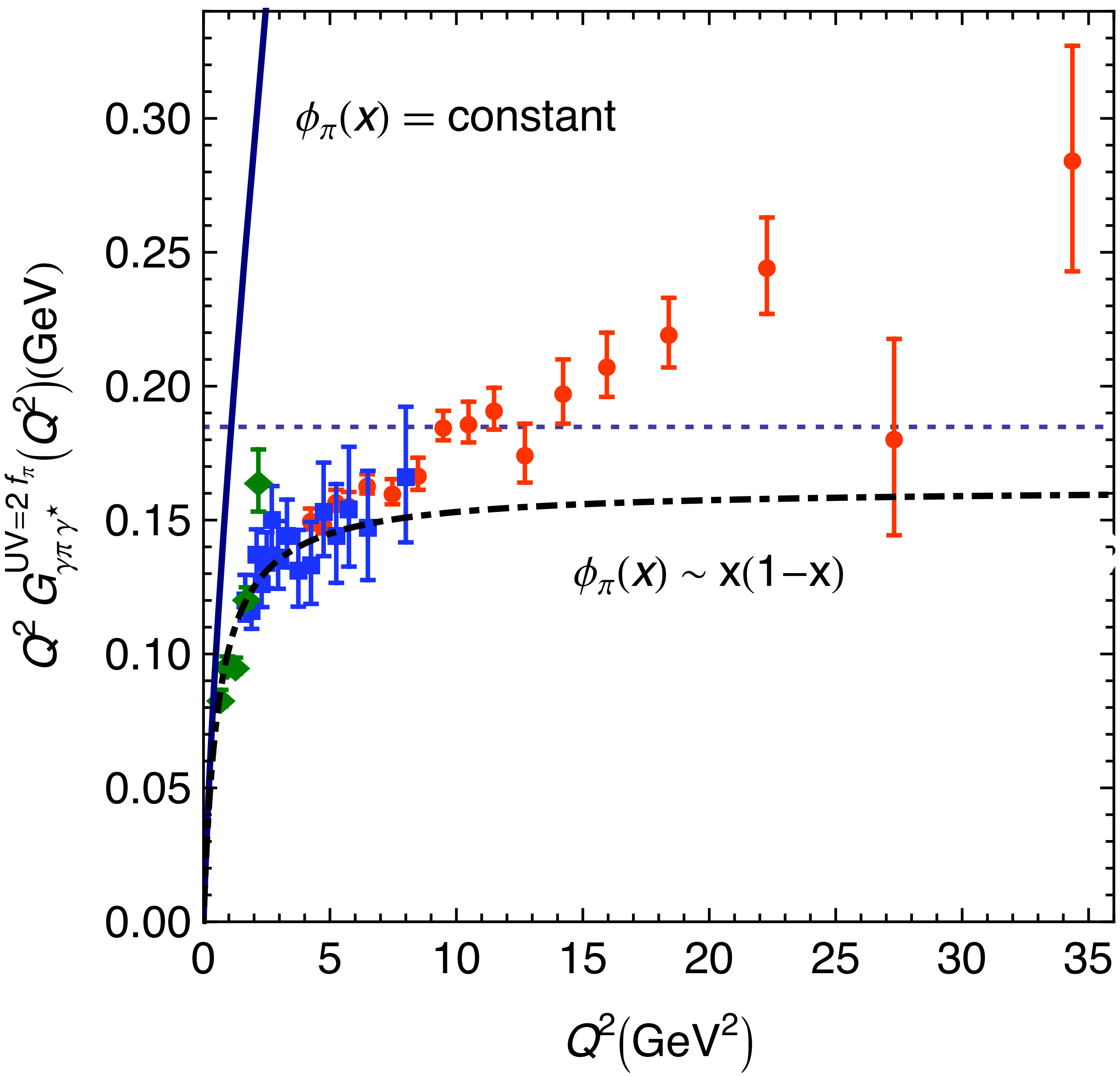


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pQCD(BL):

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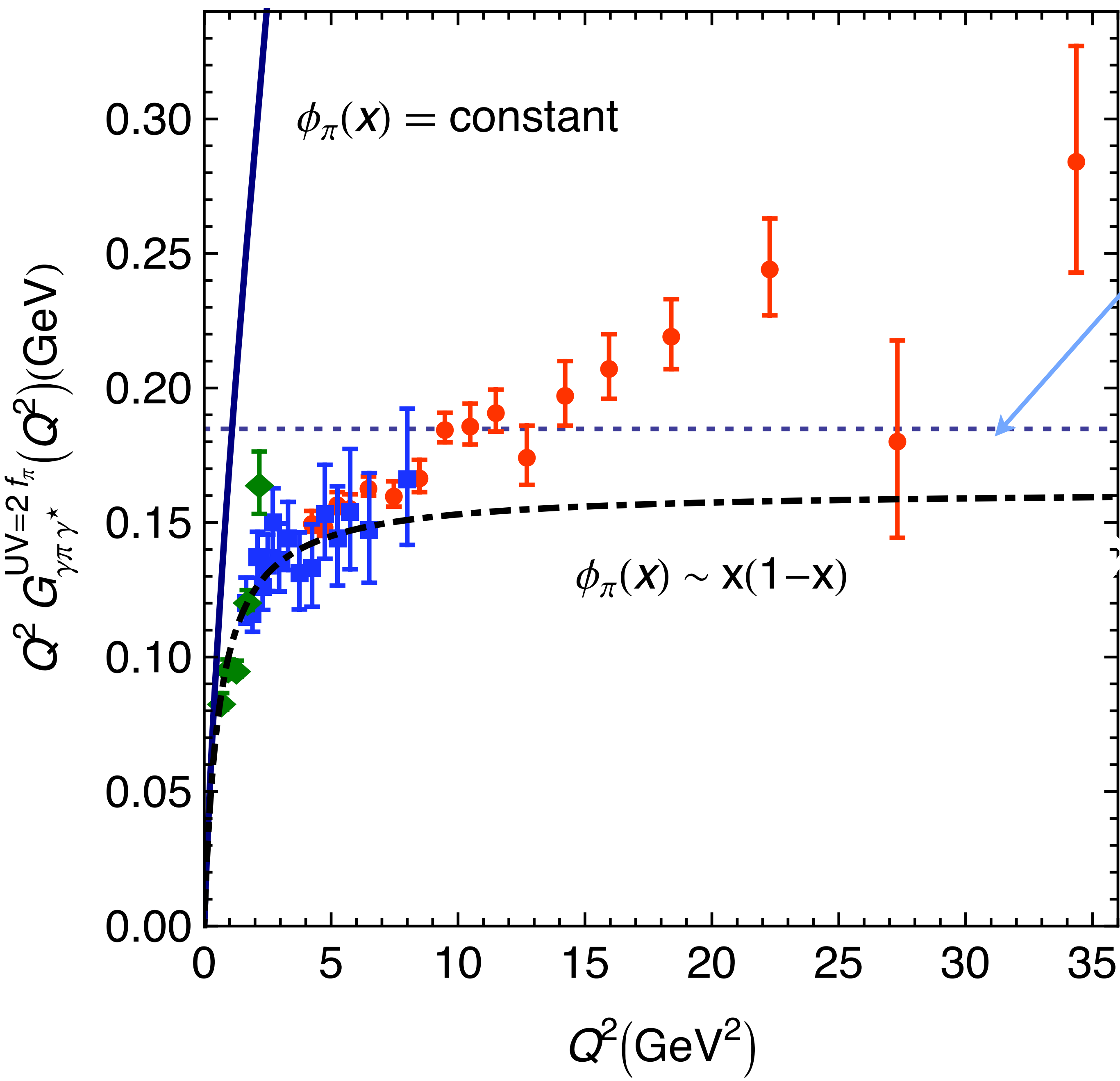


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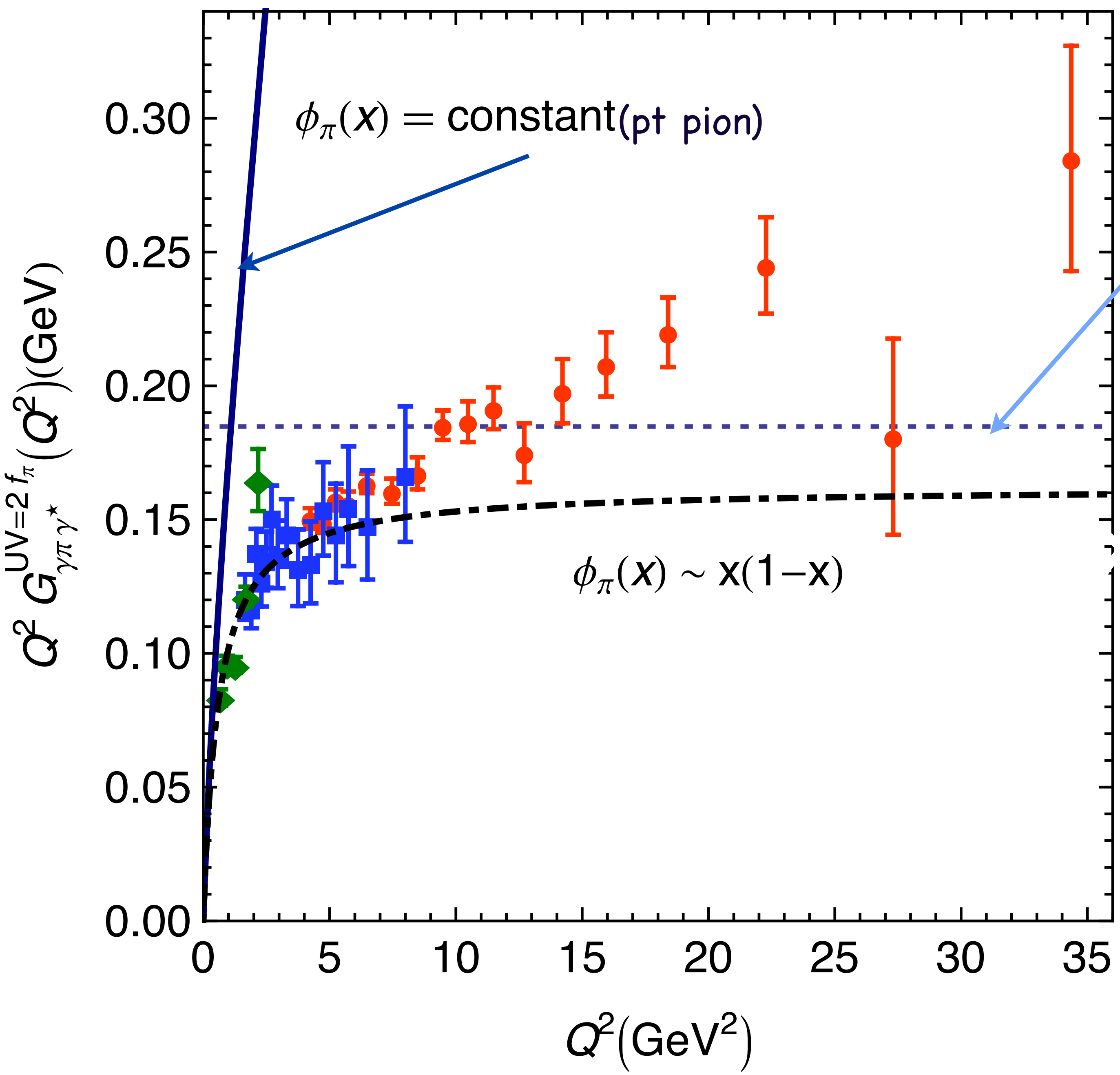


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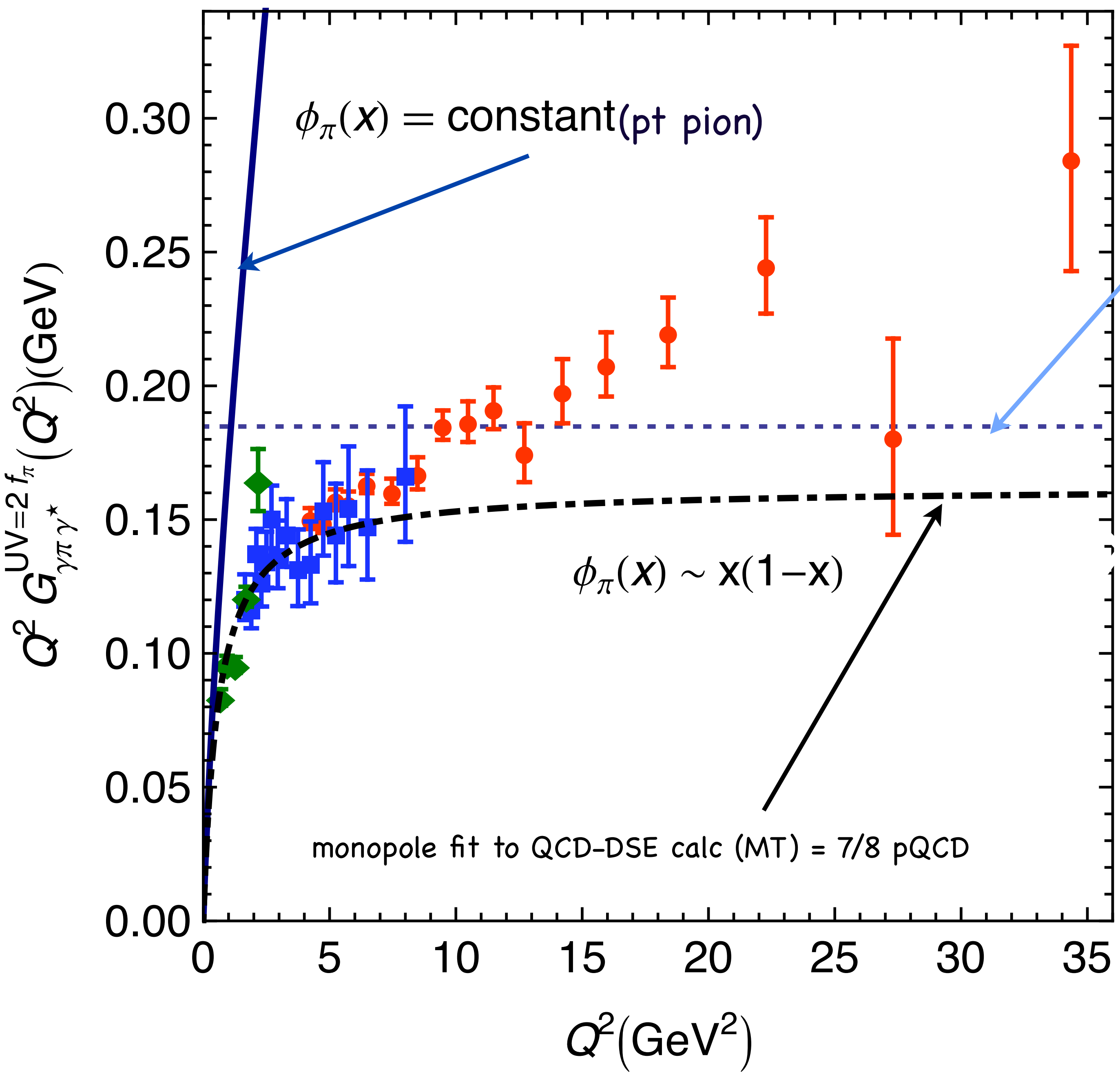


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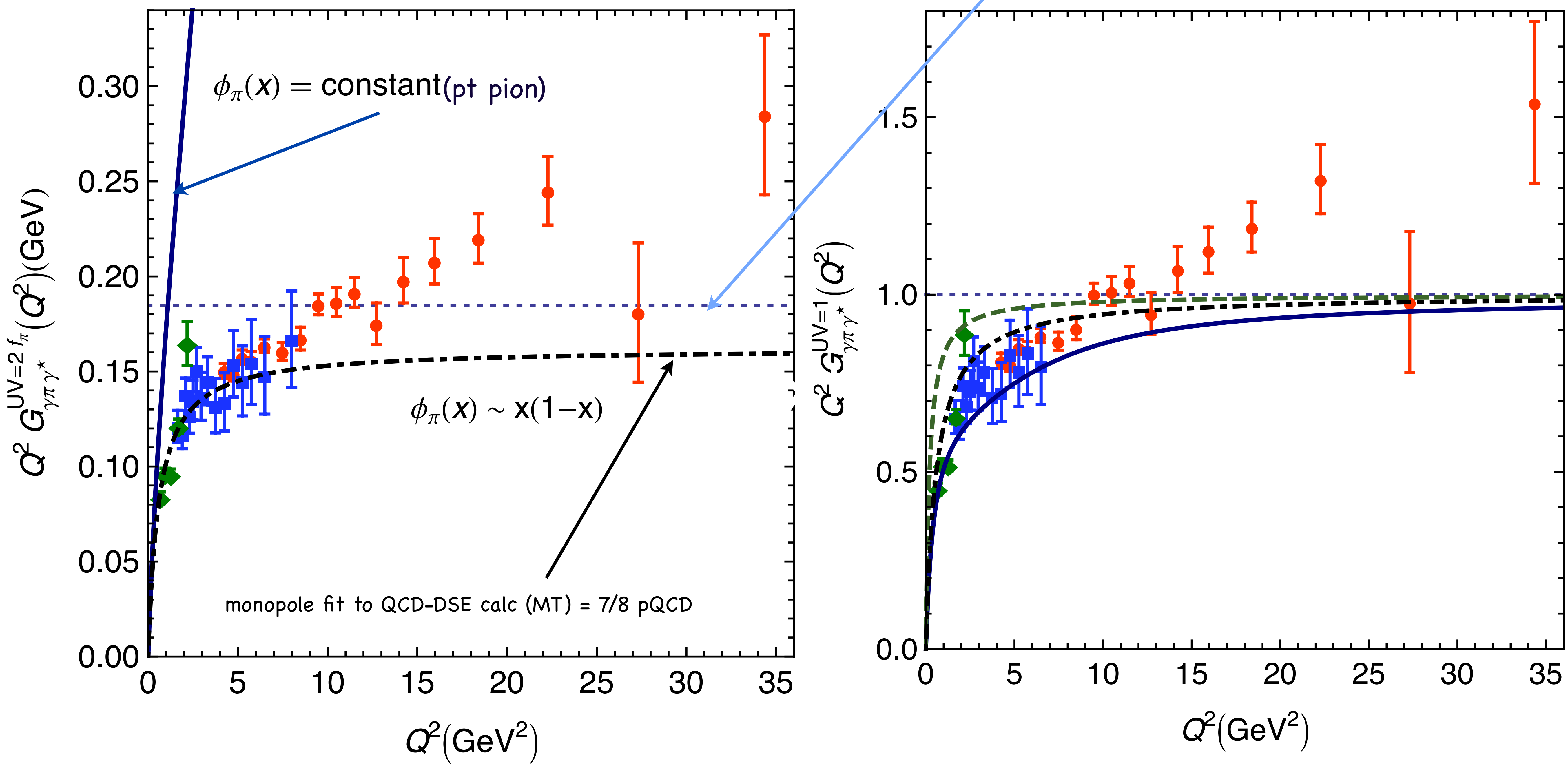


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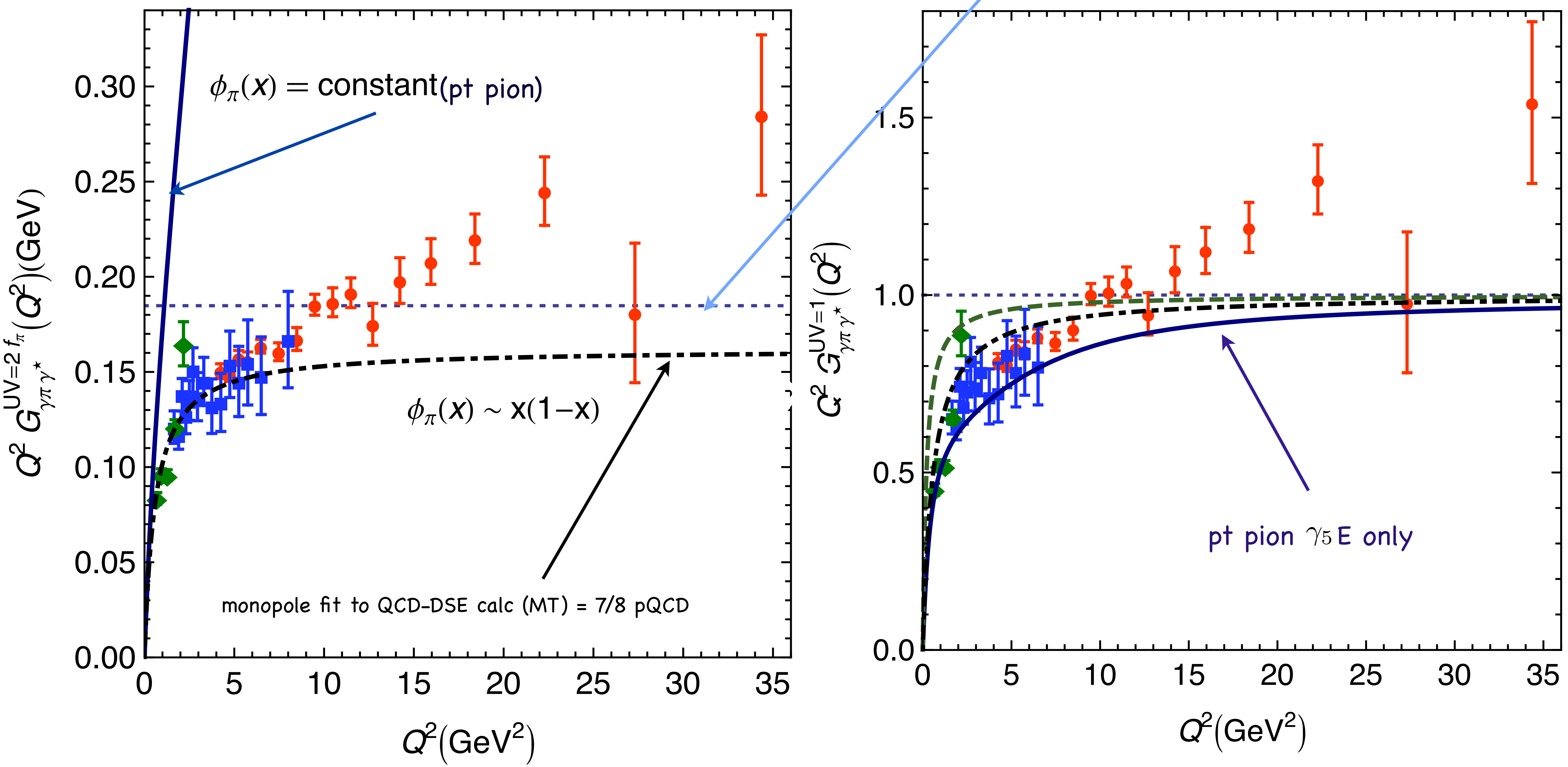


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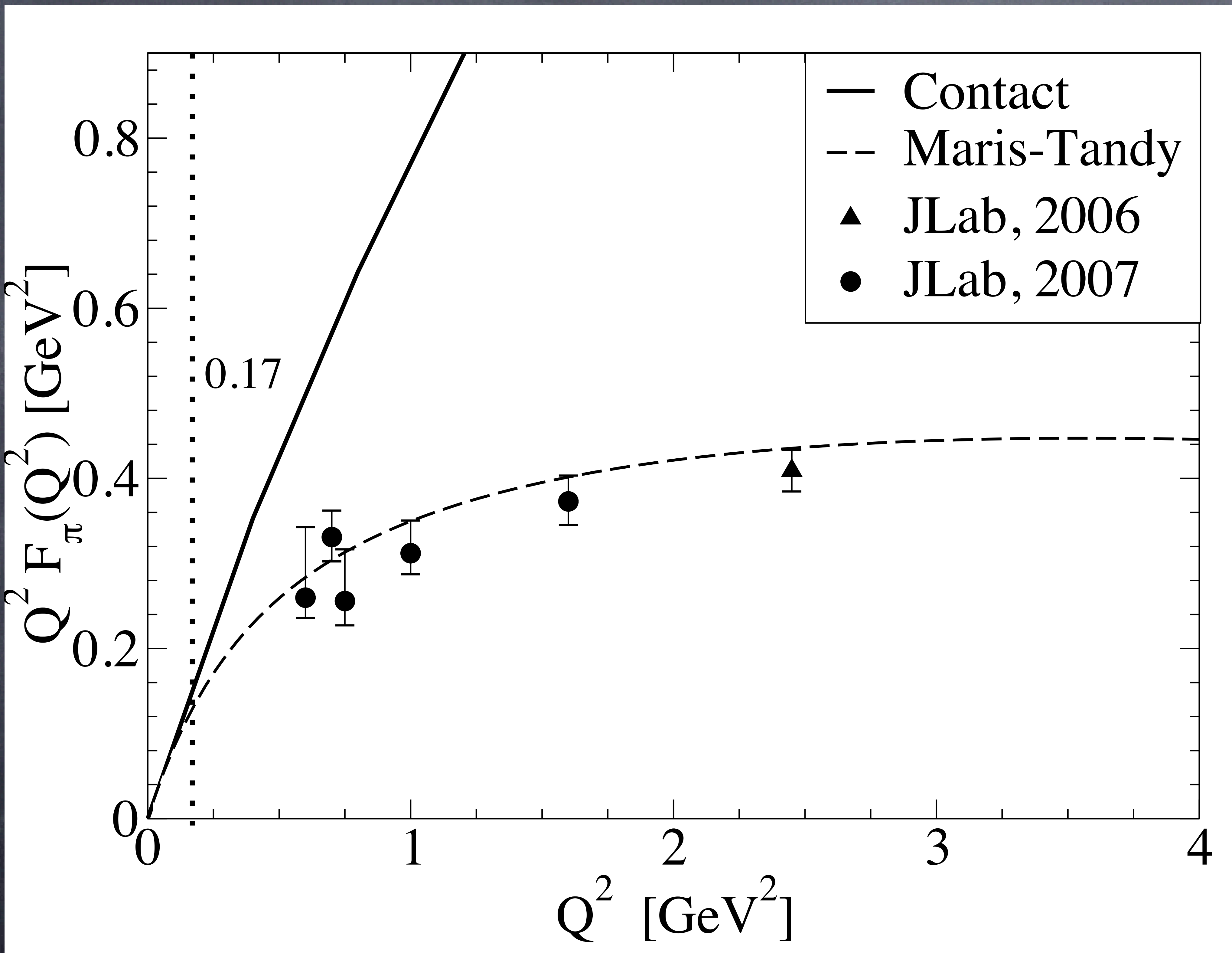
# Pion Charge Form Factor




$$= \frac{2\ \alpha_s(Q^2)}{\pi}\ [8\pi^2 f_\pi^2 \sim M_\rho^2]$$

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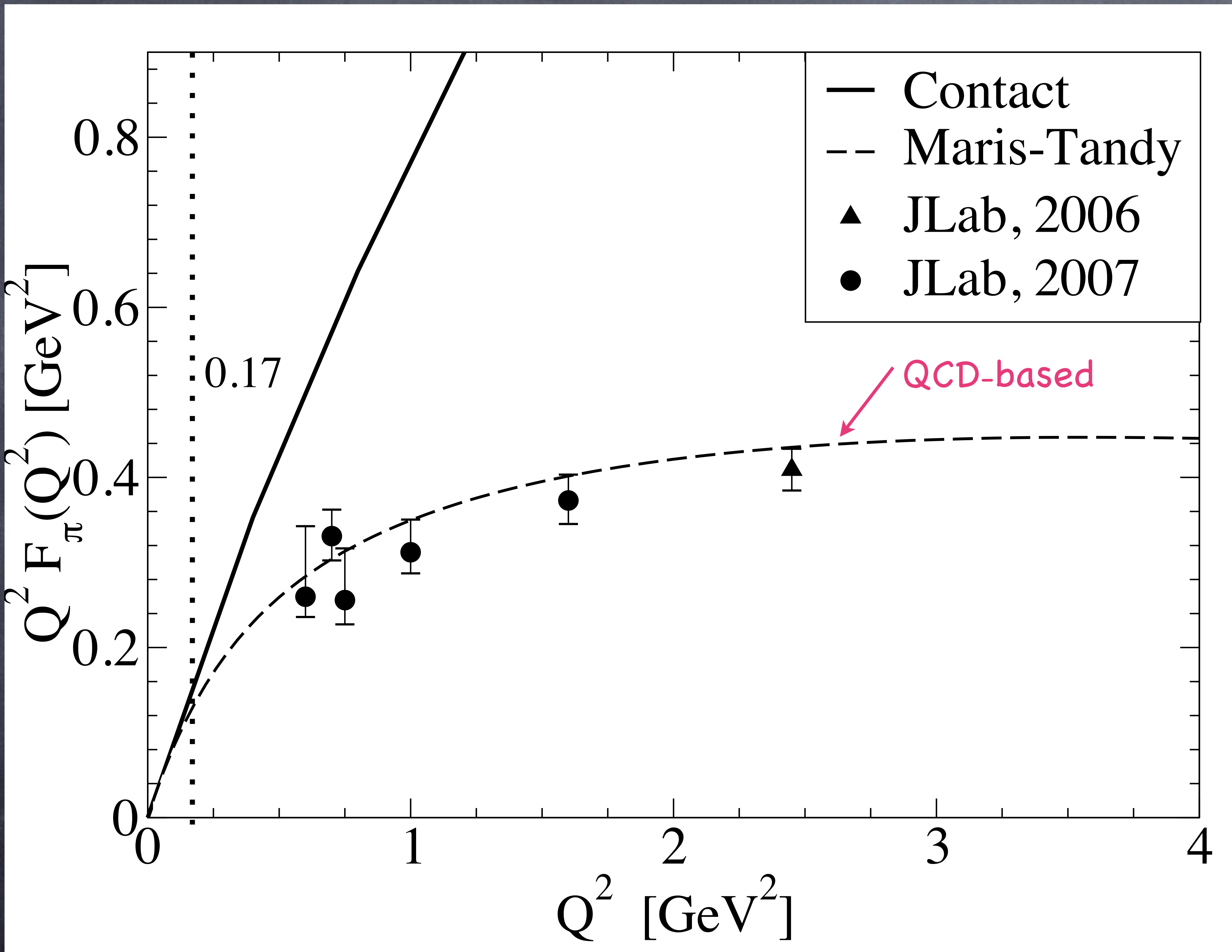


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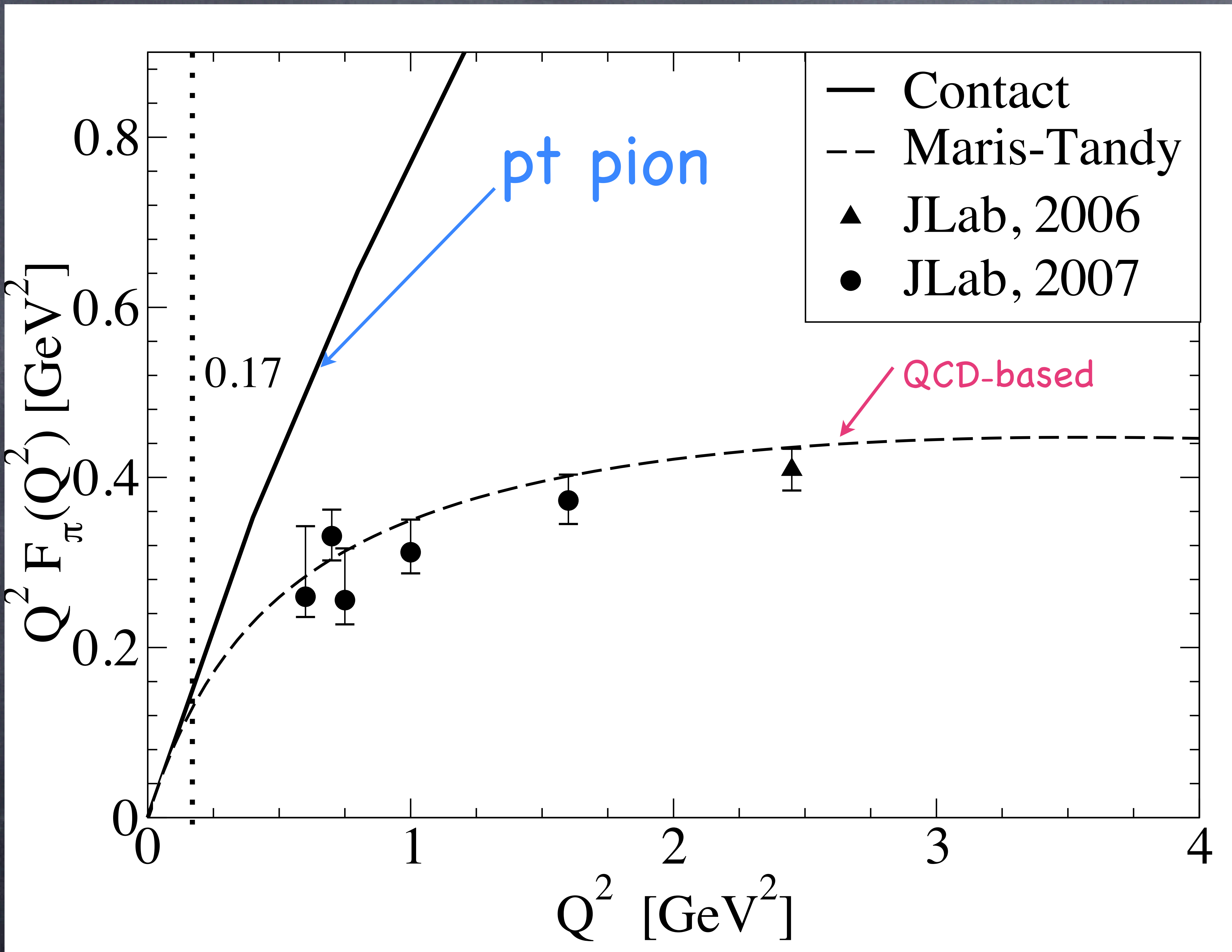


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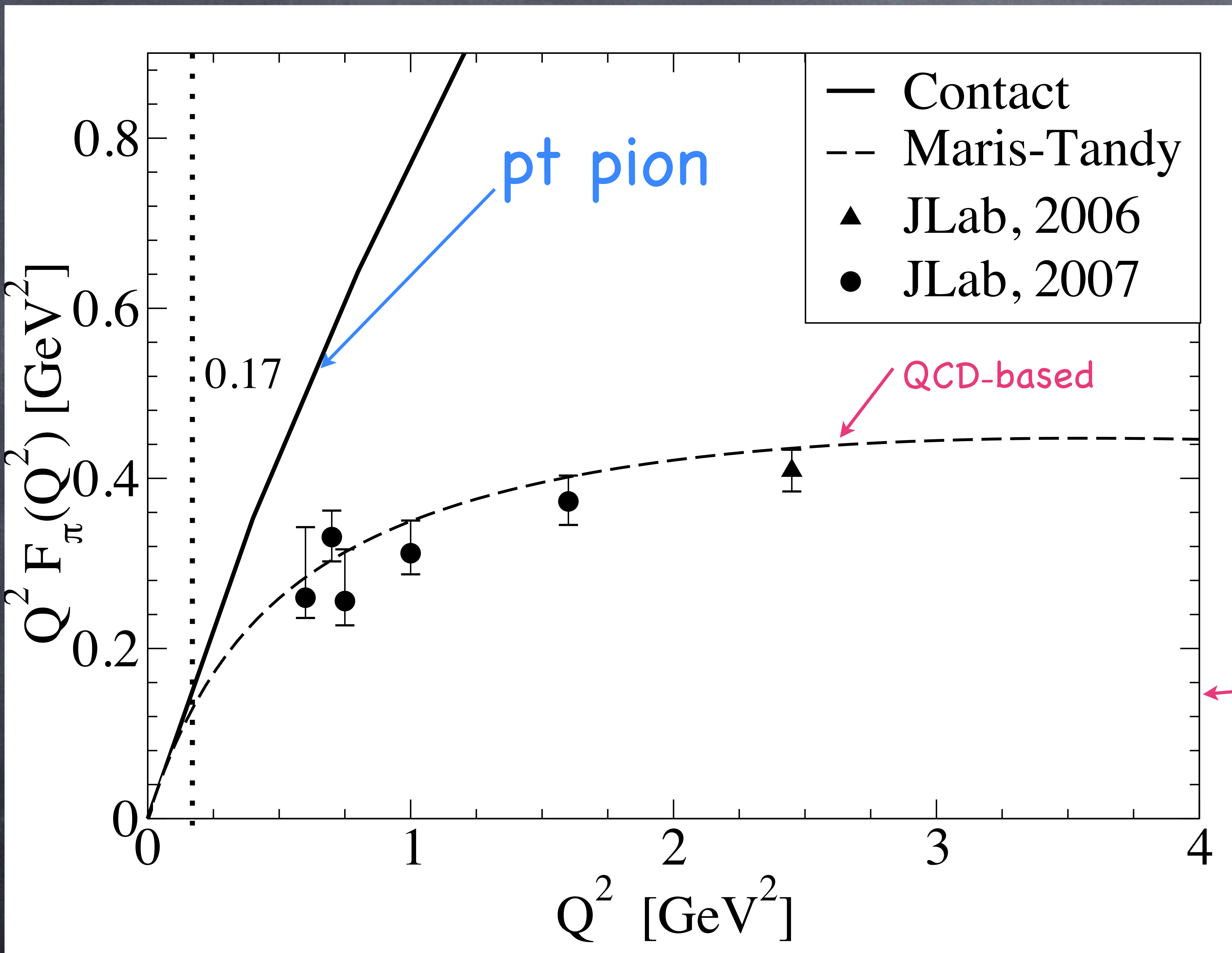


asympt pQCD

Farrar–Jackson

$$16\pi f_\pi^2\ \alpha_s(Q^2)$$

$$\sim 0.15 \text{ at } 10\ \mathrm{GeV}^2$$

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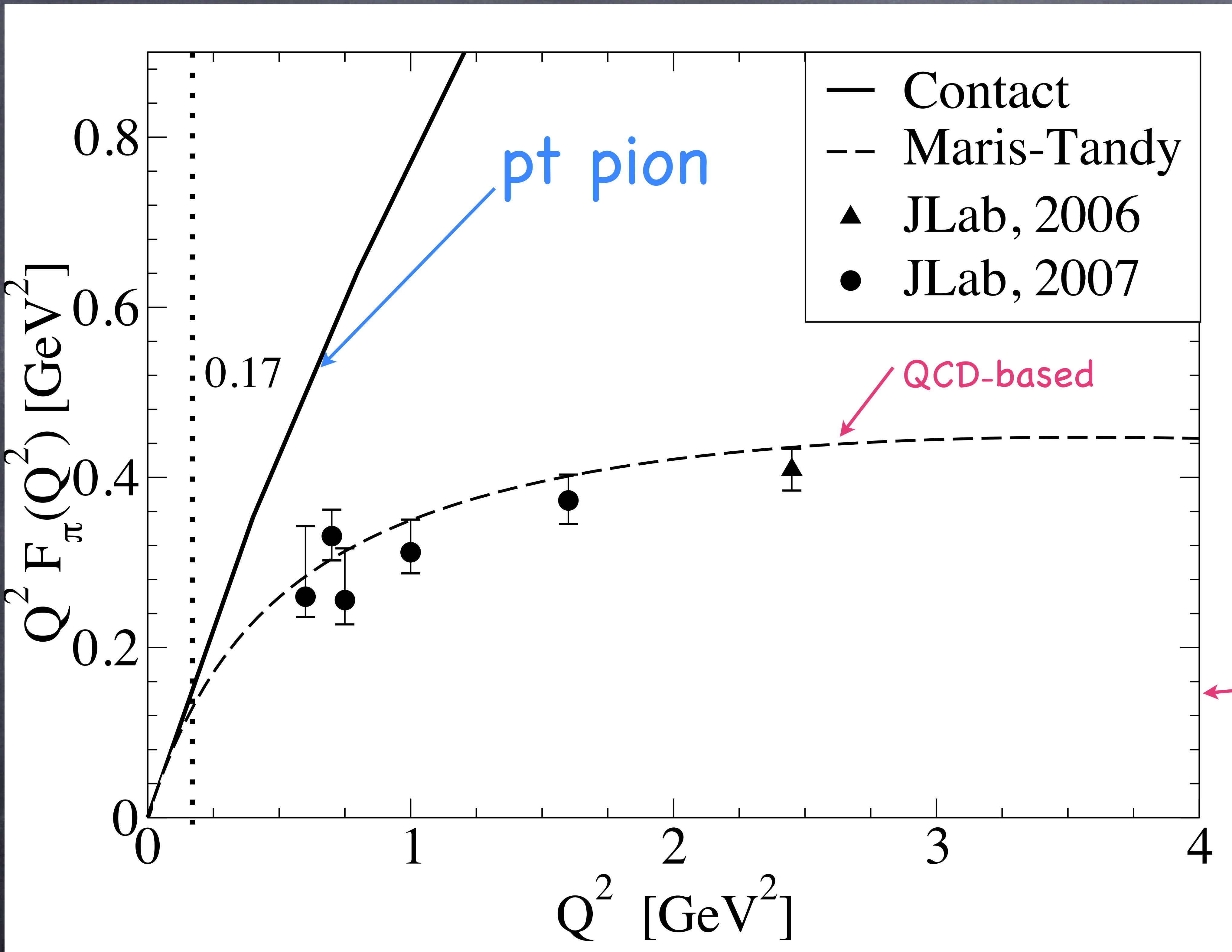


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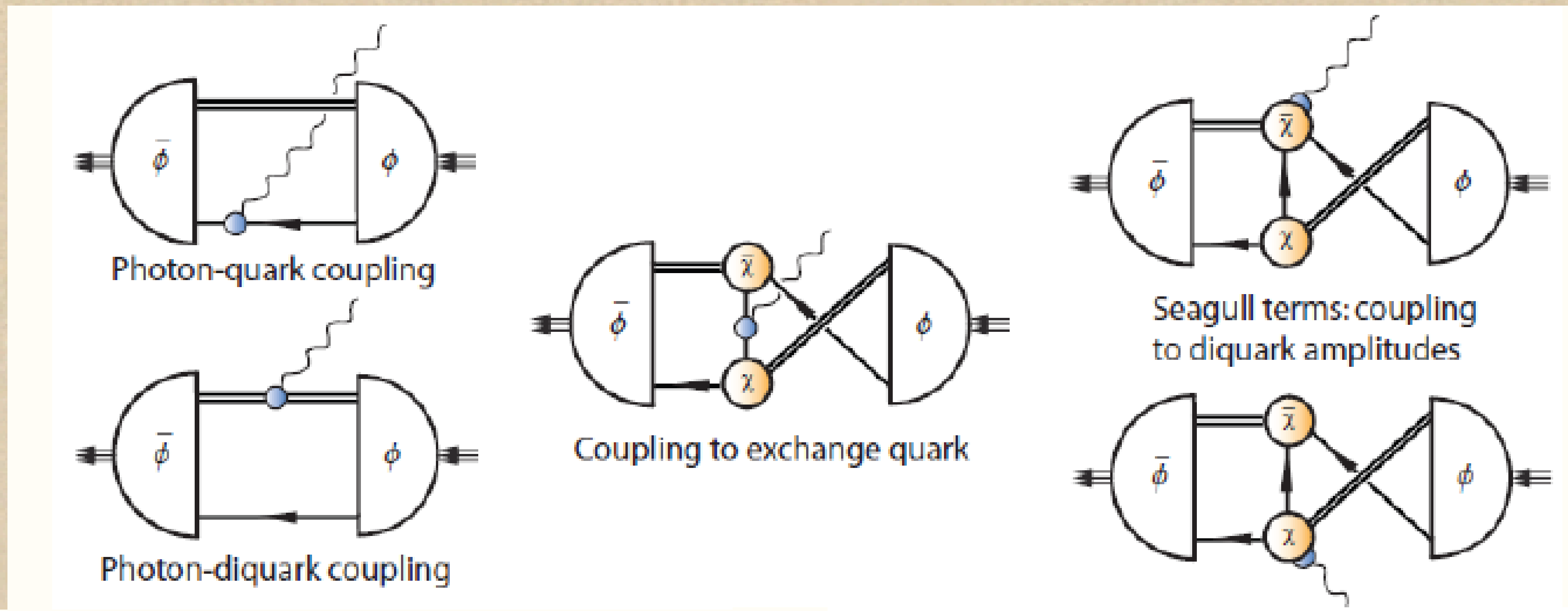
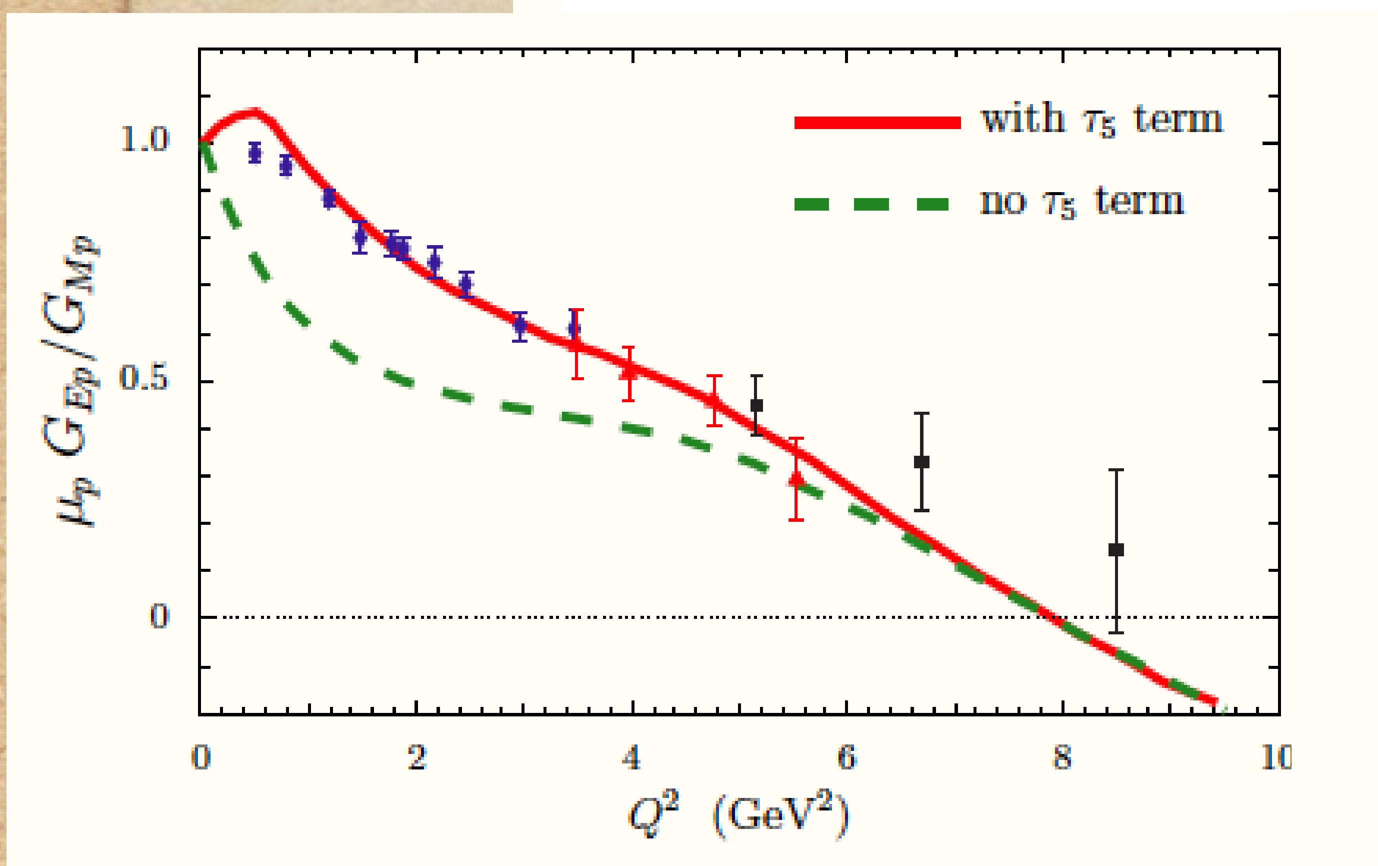
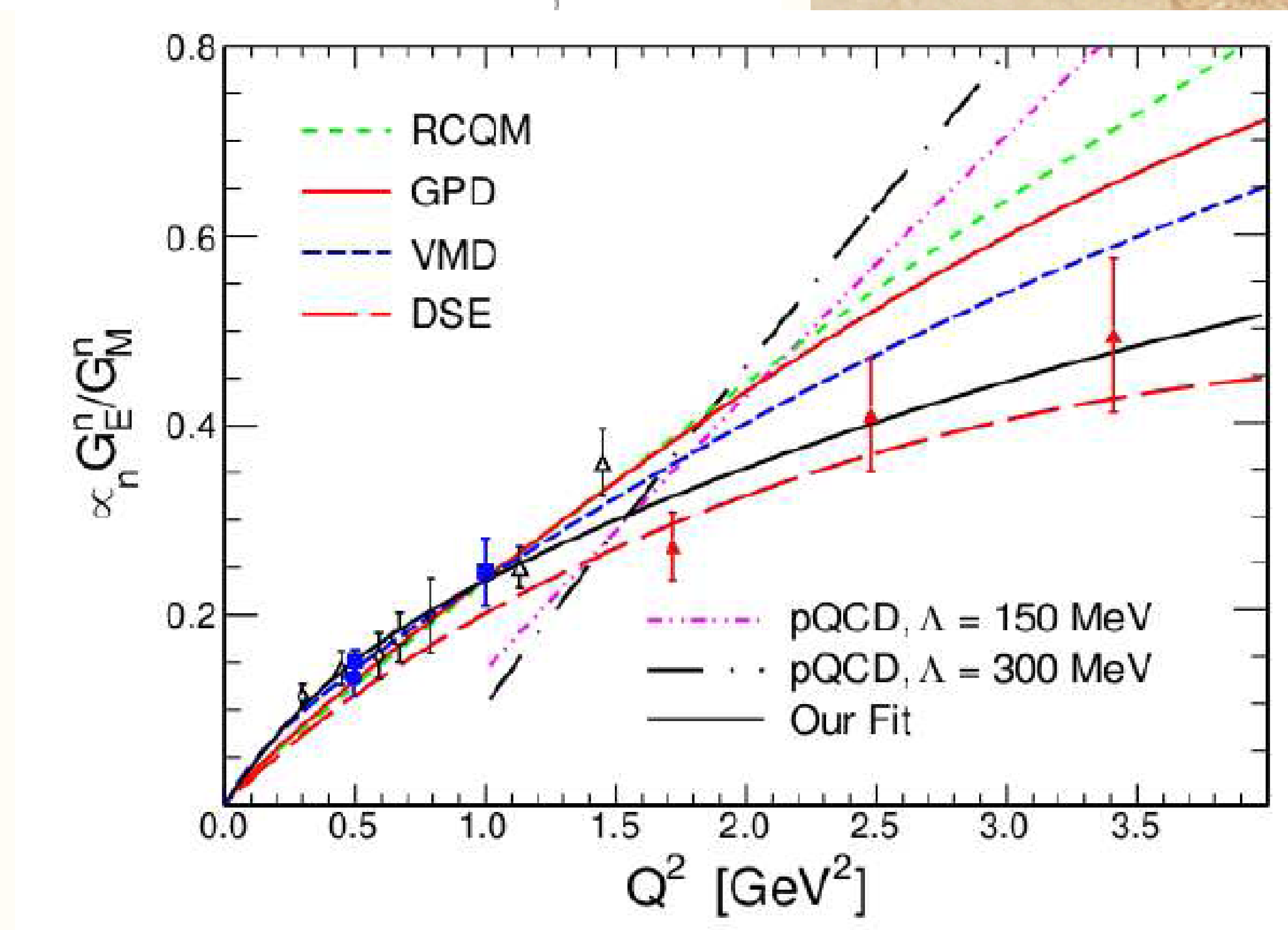
$\mathrm{VMD}:\ F(Q^2) \sim M^2/(Q^2 + M^2) \to M^2/Q^2\ ;\quad M_\rho^2 >> 0.15$

# Nucleon Form Factors



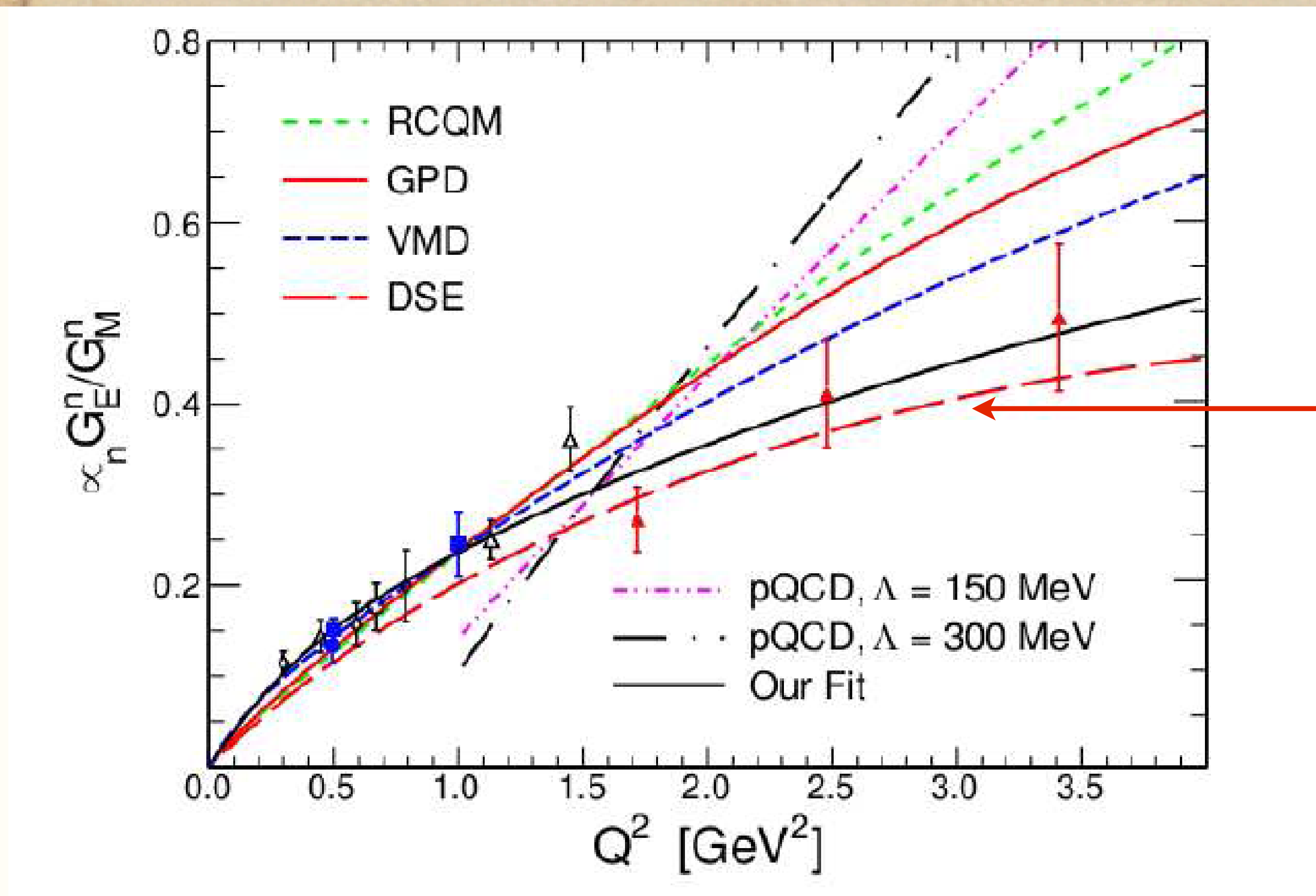




$\tau_5$

--I. Cloet et al (2011)

S. Riordan, *et al* Phys. Rev. Lett. **105**, 262302 (2010)

# DSE-Faddeev Result for Neutron Form Factors




--- Cloet, Roberts, et al (2010)

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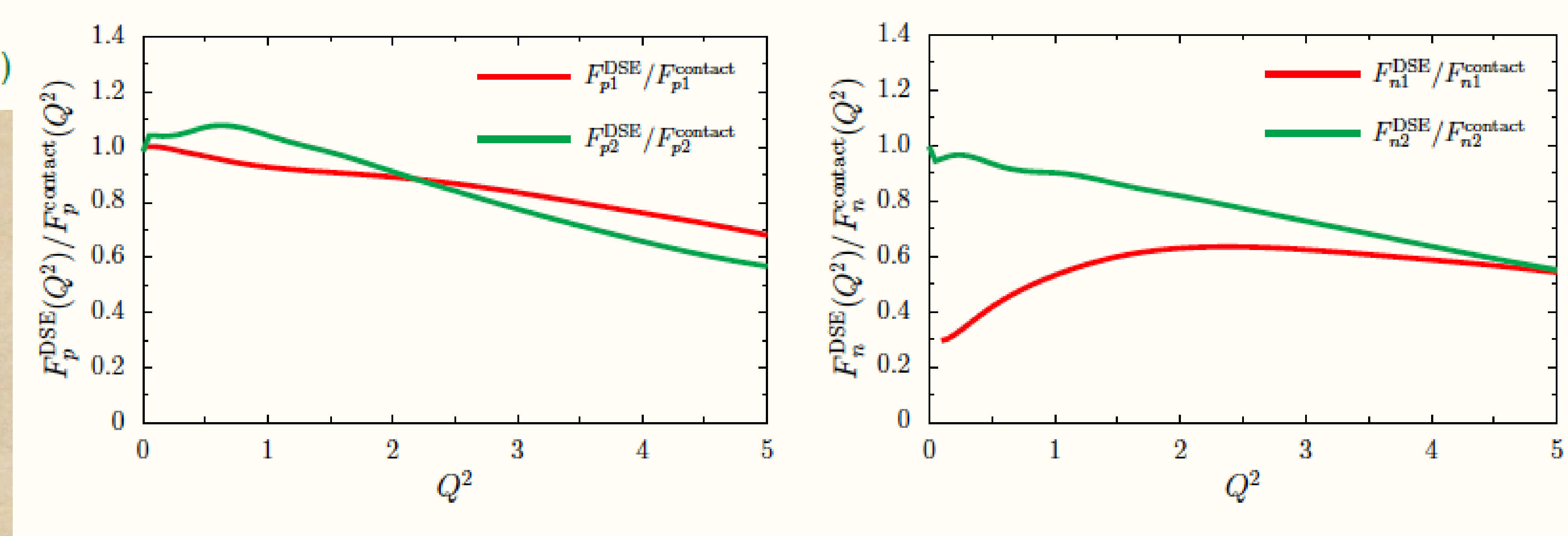
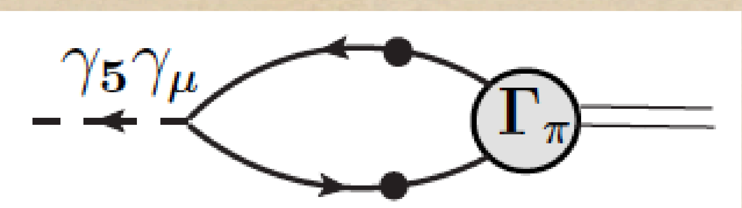



Running mass effect on FFs
they fall faster:

# Pion Distribution Amplitude

$$\mathbf{f_\pi\, \phi_\pi(x) = \int \frac{d\lambda}{2\pi}\; e^{-ixP.n\lambda}\; \langle 0|\; \bar{q}(0)\; \gamma_5\, \not{n}\; q(\lambda n)\; |\pi(P)\rangle}$$




BS wavefn $\chi_\pi(\mathbf{k}-\frac{\mathbf{P}}{2})$

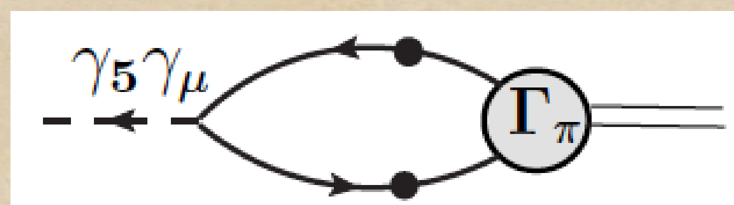
$$\mathbf{f_\pi\, \phi_\pi(x) = Z_2\, tr_{cd} \int_k \delta(k{\cdot}n - xP{\cdot}n)\; \gamma_5 \not{n}\; [\, S_u(k)\; \Gamma_\pi(k-\frac{P}{2};P)\; S_d(k-P)\,]}$$

$$\mathbf{f_\pi\, \langle x^m\rangle_\phi = \frac{Z_2 N_c}{P\cdot n}\; tr \int_k \left(\frac{k\cdot n}{P\cdot n}\right)^m \gamma_5 \not{n}\; [\, S(k)\; \Gamma_\pi(k-\frac{P}{2};P)\; S(k-P)\,]}$$

ERBL pQCD evolution $\quad \mathbf{\phi_\pi(x) = 6x(1-x)\;\{\, 1 + \Sigma_{n=2,4\cdots}\; a_n(\mu)\, C_n^{3/2}(2x-1)\,\}}$

# Pion Distribution Amplitude

$$\mathbf{f_\pi\,\phi_\pi(x) = \int \frac{d\lambda}{2\pi}\; e^{-ixP.n\lambda}\;\langle 0|\;\bar{q}(0)\;\gamma_5\,\not{n}\;q(\lambda n)\;|\pi(P)\rangle}$$

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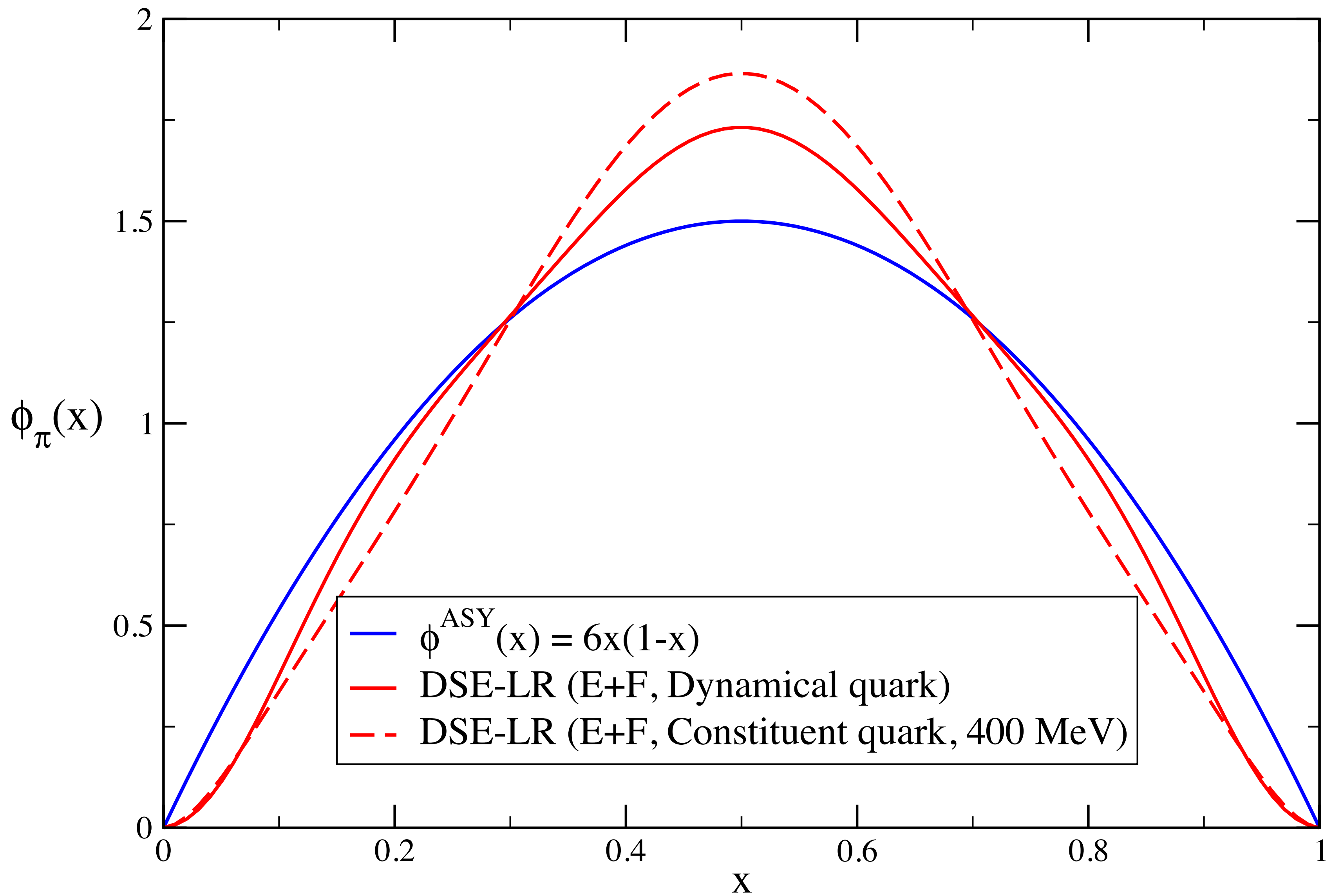
# Pion Distribution Amplitude

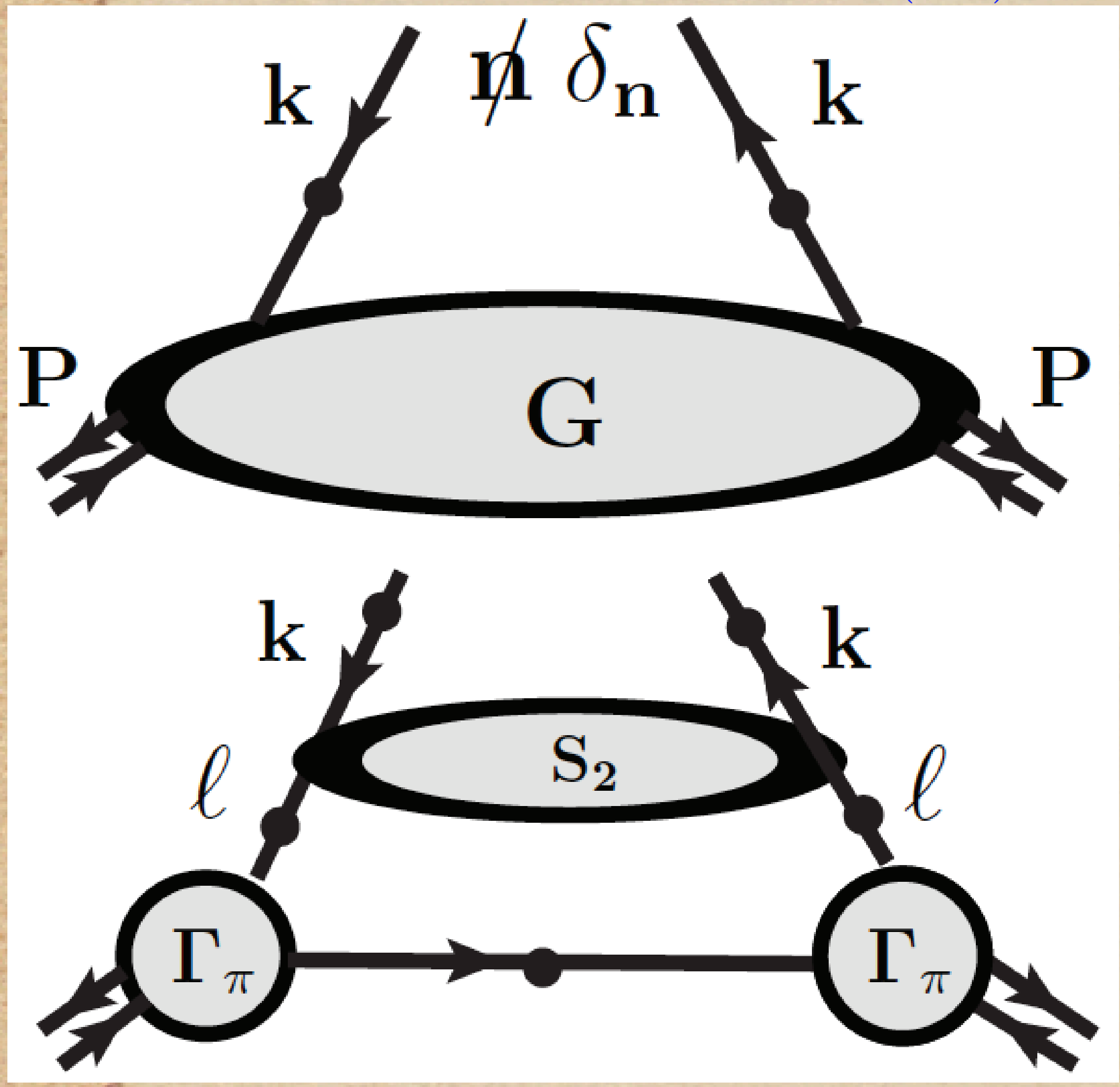



J. Cobos-Martinez, C.D. Roberts, L. Chang, PCT, 2012, in progress

# To Leading Order in OPE

$$\mathbf{q_f(x) = \frac{1}{4\pi} \int d\lambda\ e^{-ixP\cdot n\lambda}\ \langle \pi(P) |\ \bar{\psi}_f(\lambda n)\ \not{n}\ \psi_f(0)\ | \pi(P) \rangle_c}$$

$$\mathbf{q_f(x) = -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4}\ \delta(k\cdot n - xP\cdot n)\ \ \mathrm{tr_{cd}}[\,i\not{n}\ G(k,P)\,]}$$




An infinite subset of Fock space components enter via use of LR dressed objects

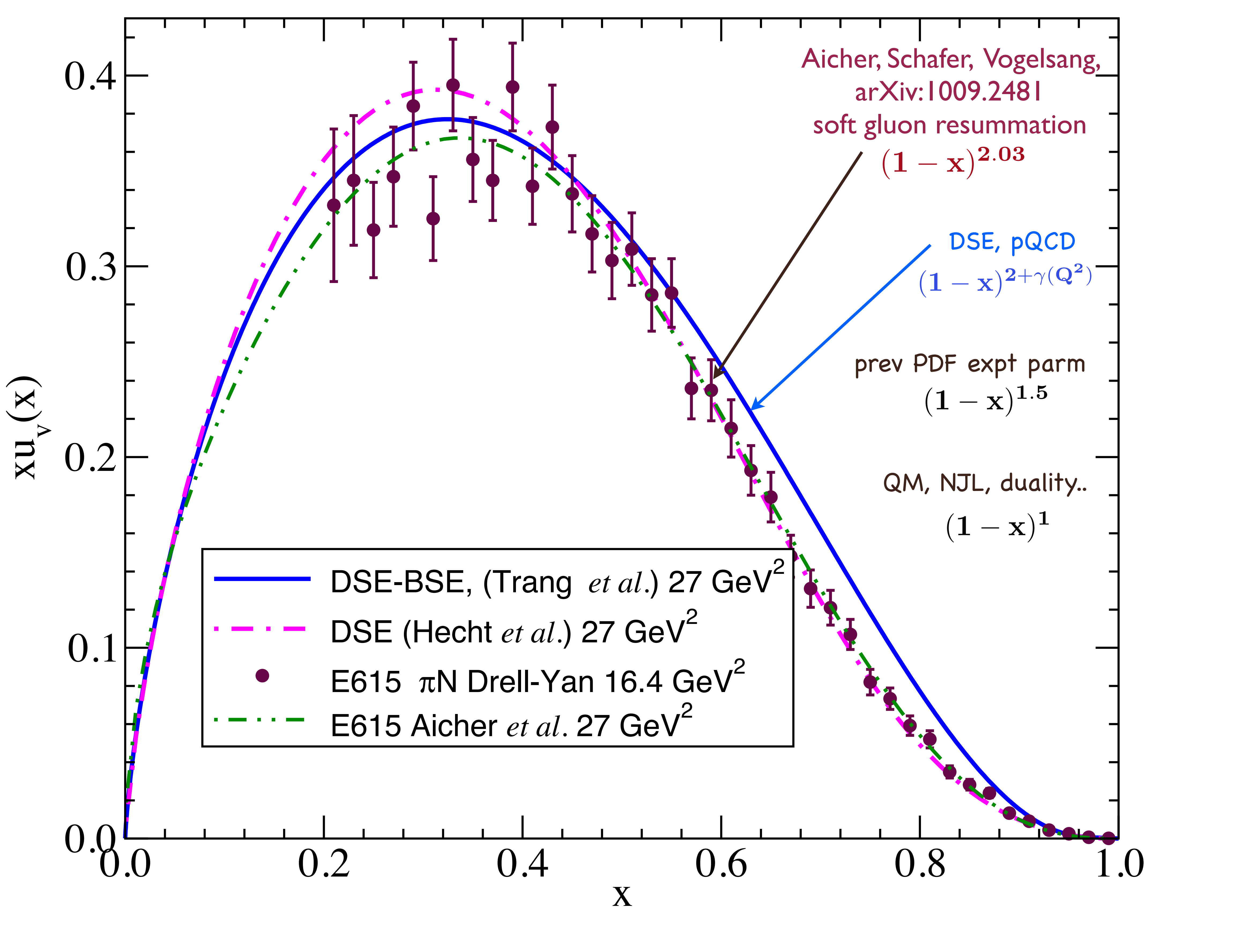
$$\mathbf{u_\pi(x) = -\frac{1}{2}\mathrm{tr_{cd}} \int_\ell \bar{\Gamma}_\pi\ S_u(\ell)\,\Gamma^n(\ell;x)\,S_u(\ell)\ \Gamma_\pi\ S_d(\ell-P)}$$

$$\mathbf{S_2\ \otimes\ i\not{n}\ \delta(k\cdot n - xP\cdot n) = S(\ell)\ \Gamma^n(\ell;x)\ S(\ell)}$$

$$\mathbf{\Gamma^n(\ell;x) = i\not{n}\ \delta(\ell\cdot n - xP\cdot n)\ + \cdots}$$

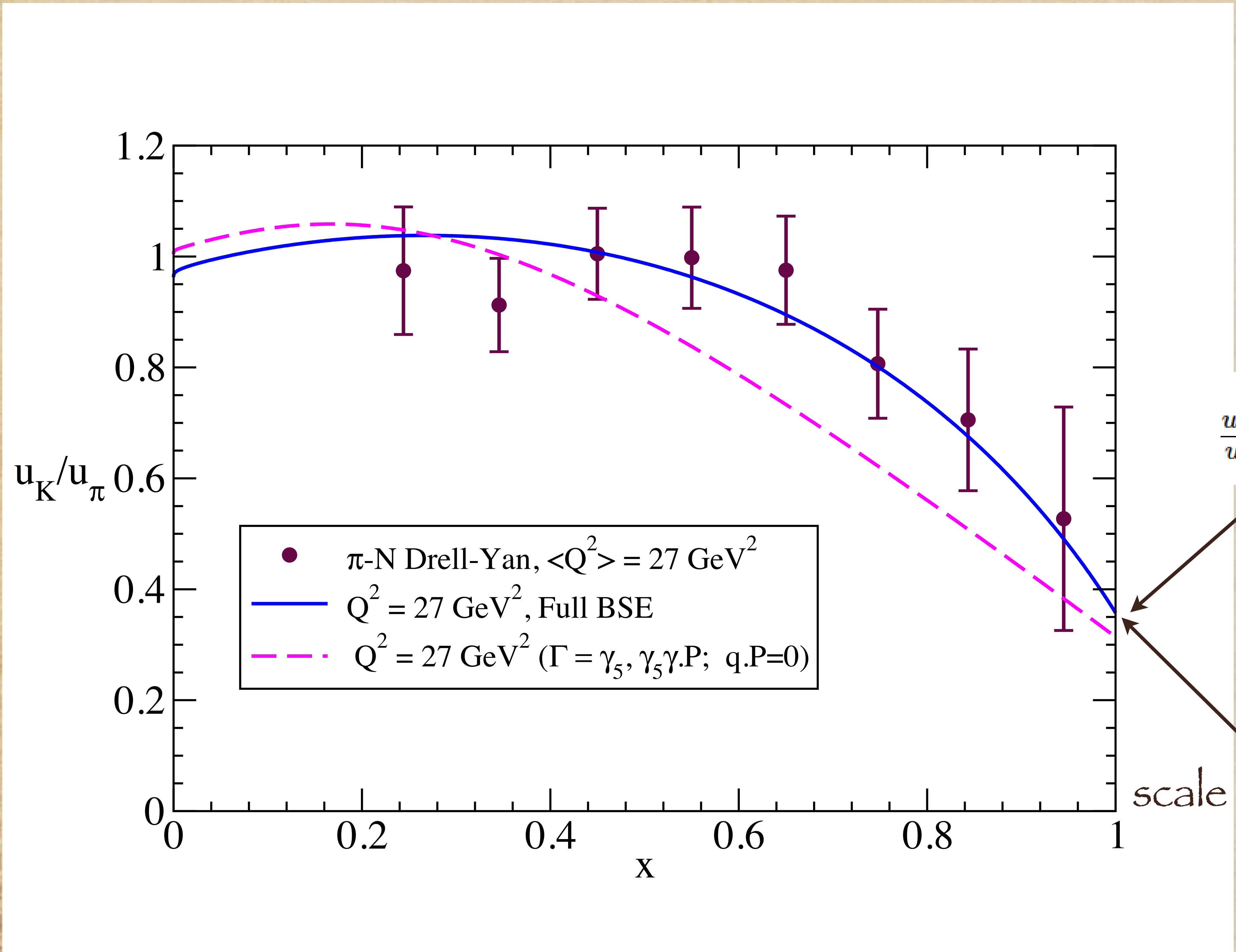
$$N_f^v = \int_0^1 dx\,[q_f(x) - q_{\bar{f}}(x)] = \frac{1}{2P^+}\ \langle \pi(P)|J^+(0)|\pi(P)\rangle_c = 1$$

Nguyen, Bashir, Roberts, PCT, PRC 83 062201 (2011); arXiv:1102.2448



# Environmental Dependence of Valence $u(x)$

Nguyen, Bashir, Roberts, PCT, arXiv : 1102.2448 (2011).




DSE-QCD derivation

$$\frac{u_K(1)}{u_\pi(1)} \sim \frac{f_\pi^2}{f_K^2}\left(\frac{M_u}{M_s}\right)^4 \sim 0.3$$

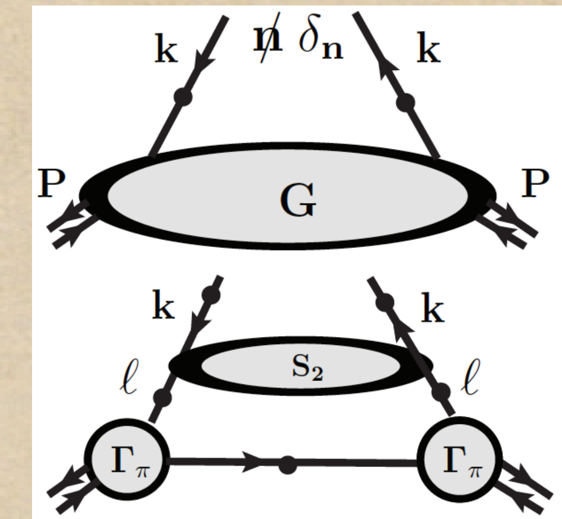
scale invariant pt

- CERN-SPS data: J. Badier et al, PLB **93**, 354 (1980)

(valence is not isolated)

# Valence PDFs via direct calcn of moments

---with K. Khitrin, C.D. Roberts, J. Cobos-Martinez




$$\mathbf{Euclidean, ladder - rainbow:}$$

$$\mathbf{unamputated\ vertex:}\quad \Gamma_{\mathbf{n}}(\ell) \approx -[(\ell \cdot \mathbf{n})^{\mathbf{m}}\, \partial_{\mathbf{n}} \mathbf{S}(\ell)]$$

$$\langle \mathbf{x}^{\mathbf{m}} \rangle_{\mathbf{v}}^{\mathrm{RL}} = \frac{-\mathbf{N_c}}{2\mathbf{P}\cdot\mathbf{n}} \mathrm{tr} \int_{\ell} \Gamma_\pi(\ell - \frac{\mathbf{P}}{2})\ [(\frac{\ell \cdot \mathbf{n}}{\mathbf{P}\cdot\mathbf{n}})^{\mathbf{m}}\, \partial_{\mathbf{n}} \mathbf{S}(\ell)]\ \Gamma_\pi(\ell - \frac{\mathbf{P}}{2})\ \mathbf{S}(\ell - \mathbf{P})$$

$$\mathbf{Only\ trivial\ analytic\ continuation\ needed:}\ (\mathbf{k}\cdot\mathbf{n})^{\mathbf{m}} = (-\mathbf{i}\mathbf{k_4} + \mathbf{k_3})^{\mathbf{m}}$$

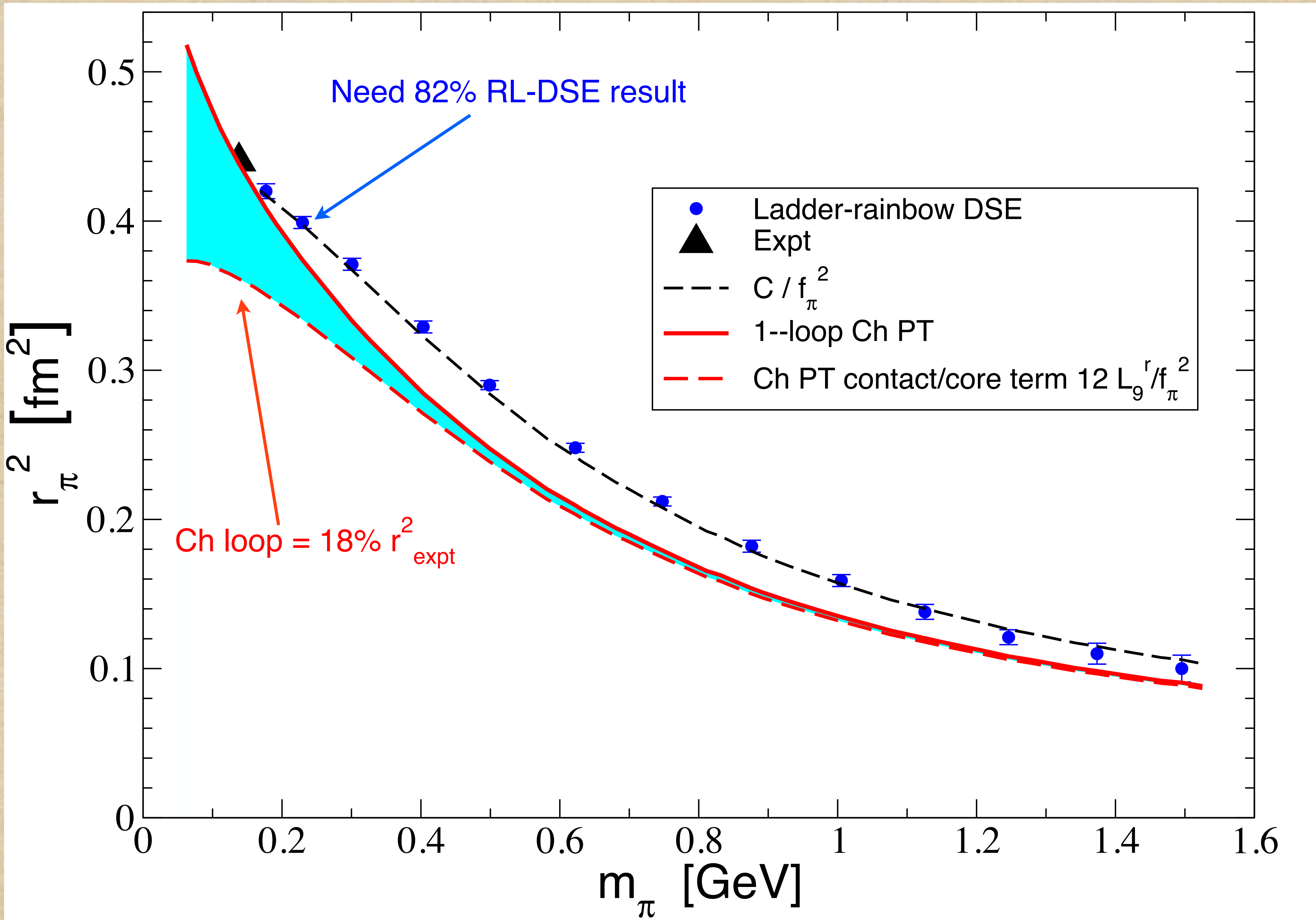
$$\mathbf{Method\ can\ easily\ exceed\ the\ Lattice-QCD\ practical\ limit:\ m = 3}$$

$$\mathbf{Note\ point\ limit:}\quad \Gamma_\pi \to \mathbf{const}\ \ \Rightarrow <\mathbf{x}^{\mathbf{m}}>_{\mathrm{pt}} = \frac{\mathbf{1}}{\mathbf{1}+\mathbf{m}}$$

# Results for Pion PDF moments

- Valence q only
- Rainbow-ladder results are good for valence-dominated x > 1/2
- Some deficiencies of pure rainbow-ladder truncation show up
- Not enough strength at low x:  u(x=0) = dbar(x=0) = 0
- Too much momentum in valence q:   2<x> = 2x0.47 = 94%
- Brings to mind an analogy with the pion charge form factor
- Another class of diagram, eg chiral loop contribution to r^2

# Chiral loop contribution to charge radius^2

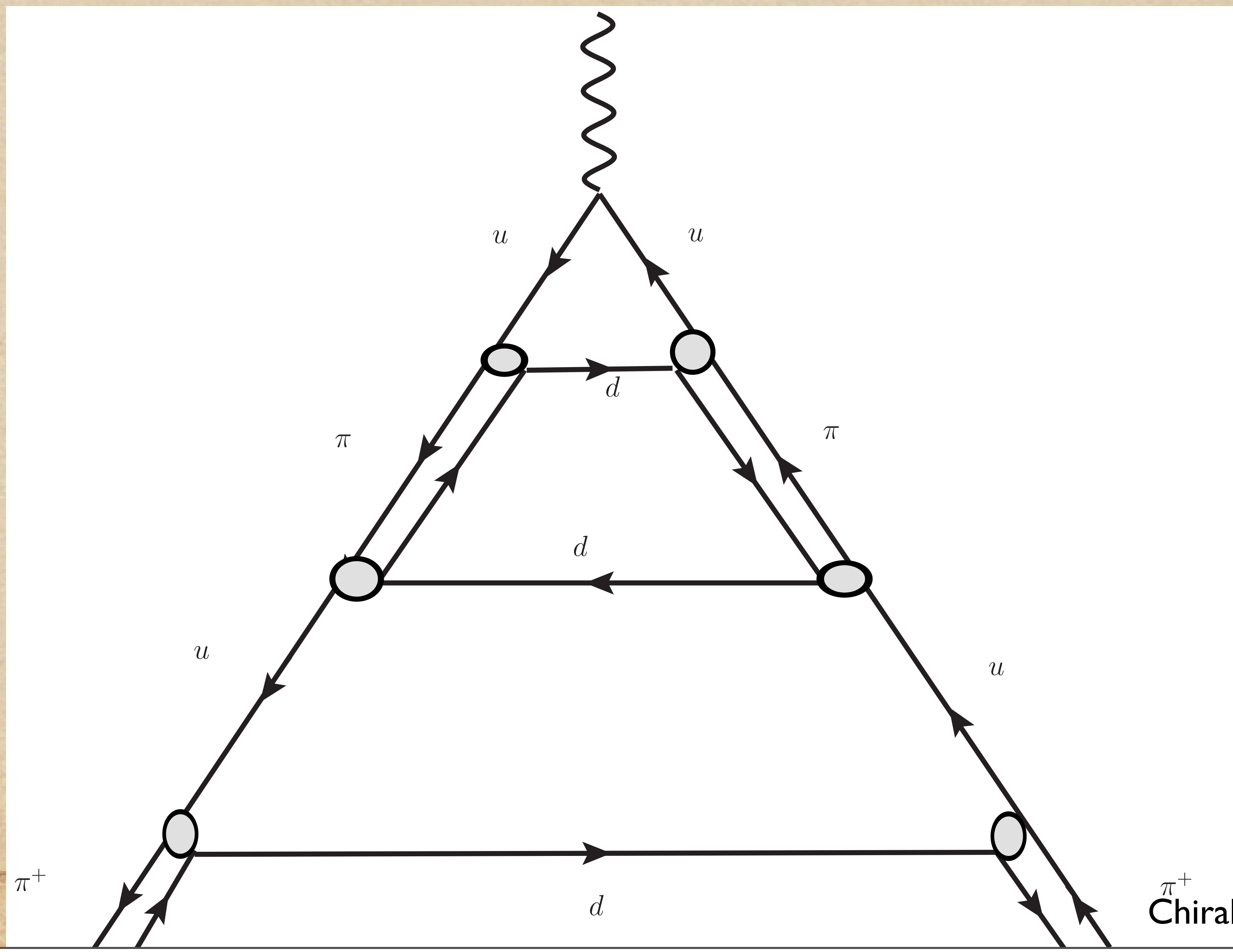


# Pion Loop in Pion Charge Form Factor

$$\mathbf{F}_\pi(\mathbf{Q^2}) = (\mathbf{1}-\mathbf{z})\mathbf{F}_\pi^{\mathrm{RL}}(\mathbf{Q^2}) + \mathbf{F}_\pi^{1-\mathrm{CPT}}(\mathbf{Q^2})$$

$$\mathbf{F}_\pi(\mathbf{Q^2}) = (\mathbf{1}-\mathbf{z})(\mathbf{1} - \frac{\mathbf{Q^2\, r^2_{\mathrm{RL}}}}{\mathbf{6}} + \cdots) + \mathbf{z}(\mathbf{1} - \frac{\mathbf{Q^2\, r^2_l}}{\mathbf{6}} + \cdots)$$

$$\mathbf{F}_\pi(\mathbf{Q^2}) = (\mathbf{1} - \frac{\mathbf{Q^2\, r^2_{\mathrm{TOT}}}}{\mathbf{6}} + \cdots), \qquad \mathbf{r^2_{\mathrm{TOT}}} = (\mathbf{1}-\mathbf{z})\mathbf{r^2_{\mathrm{RL}}} + [\mathbf{z\, r^2_l}]$$



# Analysis of Pion Parton Momentum Sum Rule

$$\mathbf{u_v(x) = (1-z)\, u_v^{RL}(x) + z\, u_v^{EX}(x)}$$

$$\mathbf{Preserve}\ \langle \mathbf{x^0} \rangle_\mathbf{v} = \mathbf{1}, \qquad \mathbf{minimize}\ \langle \mathbf{x^m} \rangle_\mathbf{ex}, \mathbf{m} > \mathbf{0}$$

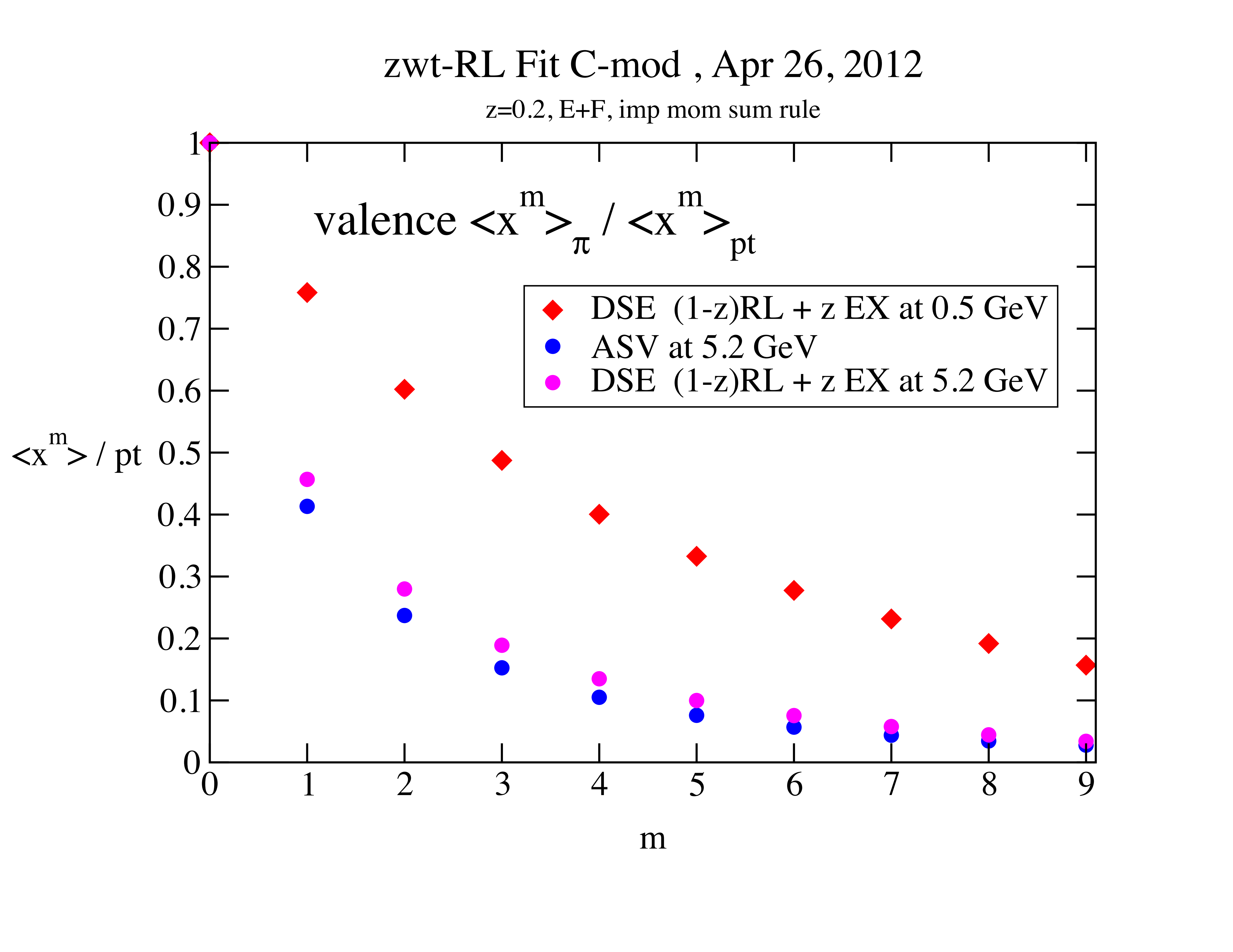
$$\langle \mathbf{x^1} \rangle_\pi = \int_0^1 \mathbf{dx\ x} \{ \mathbf{u_v(x) + u_s(x) + \bar{u}_s(x) + (u \to \bar{d}) + g(x)} \}$$

**Pion Parton Momentum Sum Rule** **using $\mathbf{z = 20\%}$**

| Analysis | Valence q | Sea q † | Gluon | Total |
|---|---|---|---|---|
| DSE RL+ (2012) | 0.717 | (0.026) | (0.264) | 1.00 |
| ASV (2010) | 0.649 | (0.052) | 0.300 | 1.00 |
| GRS (1999) | 0.559 | 0.141 | 0.300 | 1.00 |

†not well determined

# Feyn Integral Method: z=20%, q0 = Mq(0)=0.5 GeV



# Two In-Hadron Quantities within QCD:

$$if_\pi P_\mu = \langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle$$
$$= Z_2(\zeta,\Lambda)\,\mathrm{tr}_{CD}\int^\Lambda \frac{d^4q}{(2\pi)^4} i\gamma_5\gamma_\mu S(q_+)\Gamma_\pi(q;P)S(q_-), \quad (5)$$

$$i\rho_\pi = -\langle 0|\bar{q}i\gamma_5 q|\pi\rangle$$
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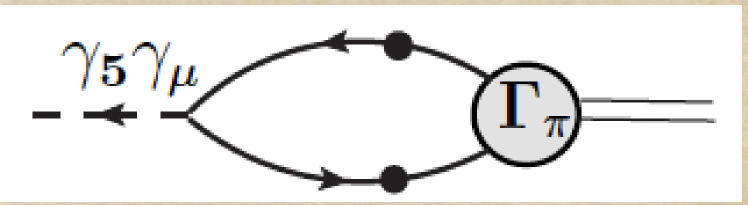


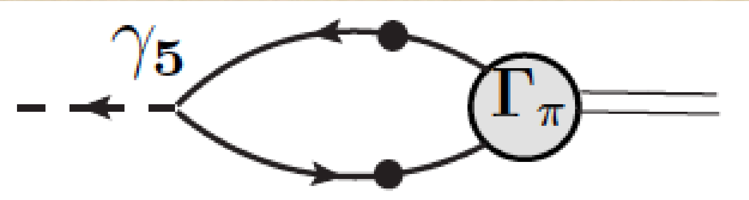


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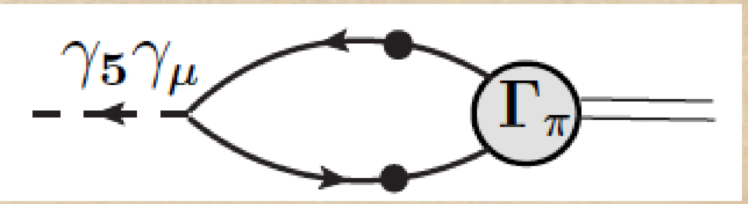


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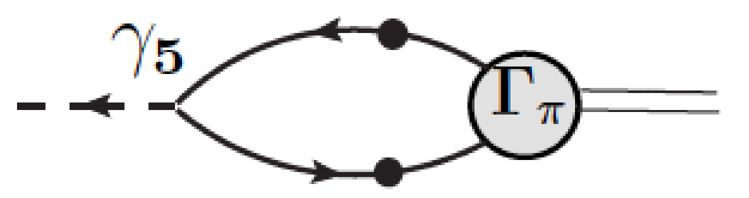


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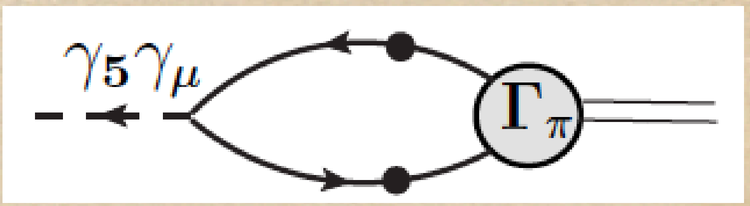


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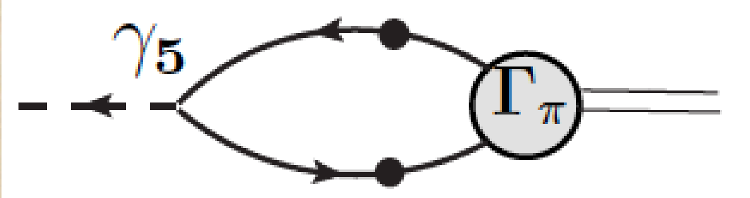


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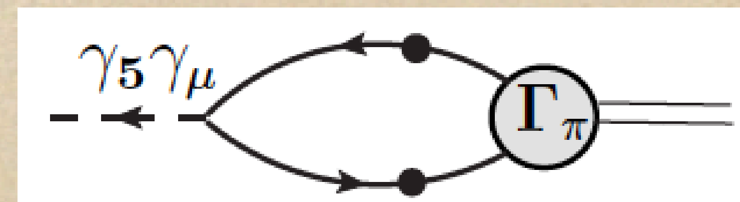


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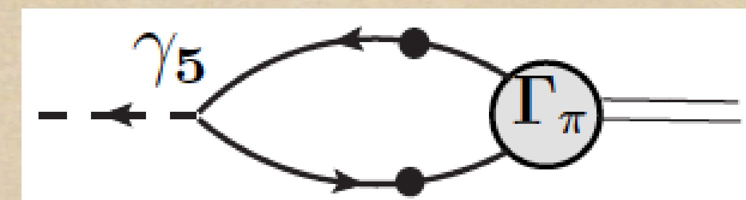


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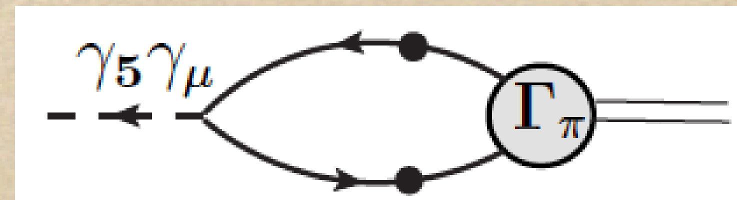


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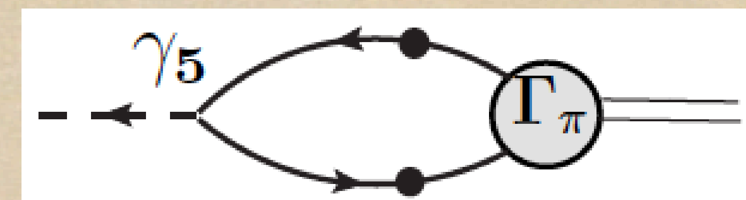


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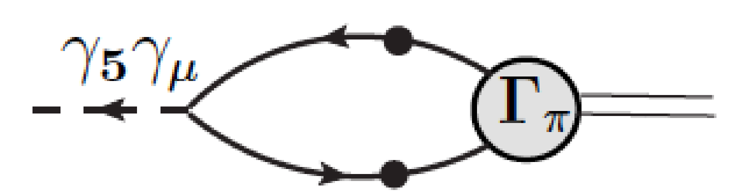


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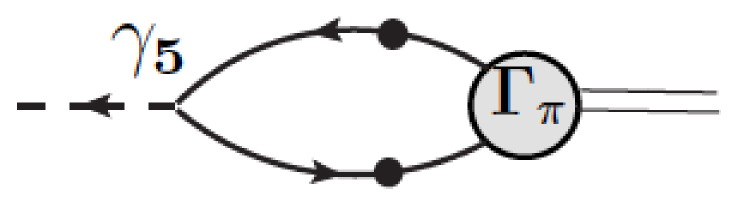


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- There is no reason to interpret either one as a mass scale that fills the vacuum
- But, does $\langle 0\,|\,\bar{q}\,\gamma_5\,q\,|\,\pi\rangle$ play a role in hadron physics? Yes, because........

New Perspectives on the Quark Condensate, S.J. Brodsky, C.D. Roberts, R.Shrock&P.C.Tandy, Phys. Rev, C82, 022201 (2010); + 3 other papers

pole residues of AV-WTI exactly give:

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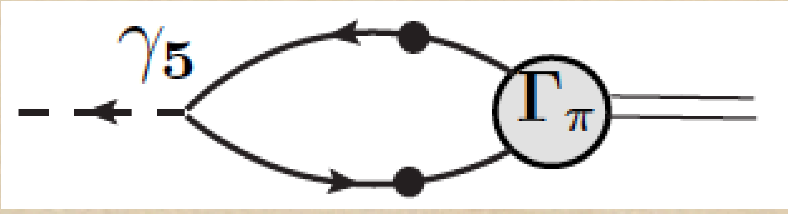


$$\rho_{ps}(\mu) = -\langle 0\,|\,\bar{q}\,\gamma_5\, q\,|\,ps\rangle$$

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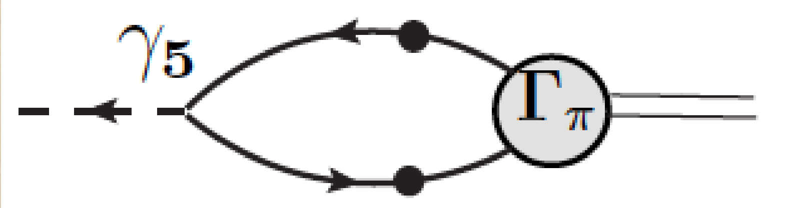


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Chiral limit

$$f_{ps}\, M_{ps}^2 = 2 m_q(\mu)\, \frac{\langle \bar{q}q \rangle_\mu}{f_{ps}} + \mathcal{O}(m_q^2)\ , \quad [\text{"GMOR"}, \text{today}]$$

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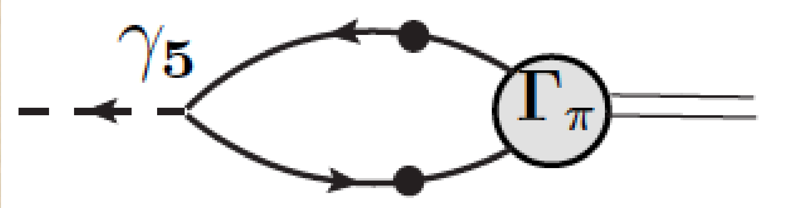


$$\rho_{\mathrm{ps}}(\mu) = -\langle \mathbf{0} \,|\, \bar{\mathbf{q}}\, \gamma_5\, \mathbf{q} \,|\, \mathbf{ps} \rangle$$

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In-pion condensate:

$$\langle \bar{q}q \rangle^\pi_\mu = \mathrm{i}\ \mathrm{f}_\pi\ \mathrm{tr_{cs}} \int^{\mathrm{E}}_{\mathrm{k}} \gamma_5\, \bar{\chi}_\pi(\mathbf{k}; -\mathbf{P}) \ = \ \mathrm{f}_\pi\, \langle 0|\, \bar{\mathbf{q}}(0)\, \gamma_5\, \mathbf{q}(0)\, |\pi(\mathbf{P})\rangle_\mu$$

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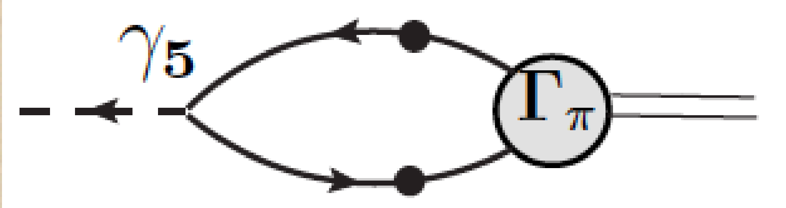


$$\rho_{\mathrm{ps}}(\mu) = -\langle \mathbf{0} \,|\, \bar{\mathbf{q}}\, \gamma_5\, \mathbf{q} \,|\, \mathbf{ps} \rangle$$

Chiral limit

$$\mathrm{f_{ps}\, M_{ps}^2 = 2m_q(\mu)\, \frac{\langle \bar{q}q \rangle_\mu}{f_{ps}} + \mathcal{O}(m_q^2)}\ , \quad [\text{"GMOR"}, \text{today}]$$

## In-pion condensate:

$$\langle \bar{\mathrm{q}}\mathrm{q} \rangle_\mu^\pi = \mathrm{i\ f_\pi\ tr_{cs}} \int_{\mathrm{k}}^{\mathrm{E}} \gamma_5\, \bar{\chi}_\pi(\mathrm{k}; -\mathrm{P}) \ = \ \mathrm{f_\pi}\ \langle 0 |\ \bar{\mathrm{q}}(0)\, \gamma_5\, \mathrm{q}(0)\ | \pi(\mathrm{P}) \rangle_\mu$$

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A Note of Caution:

Chiral Magnetism (or Magnetohadrochironics)

A. Casher and L. Susskind, Phys. Rev. D9 (1974) 436

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# Hadronic Charges $\langle\, \mathbf{H(P)} \,|\, \bar{\mathbf{q}}\, \mathcal{O}\, \mathbf{q} \,|\, \mathbf{H(P)} \,\rangle$

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**They are all properties internal to hadrons.**

**Why use a different interpretation for the** $\mathcal{O} = 1$ **case?**

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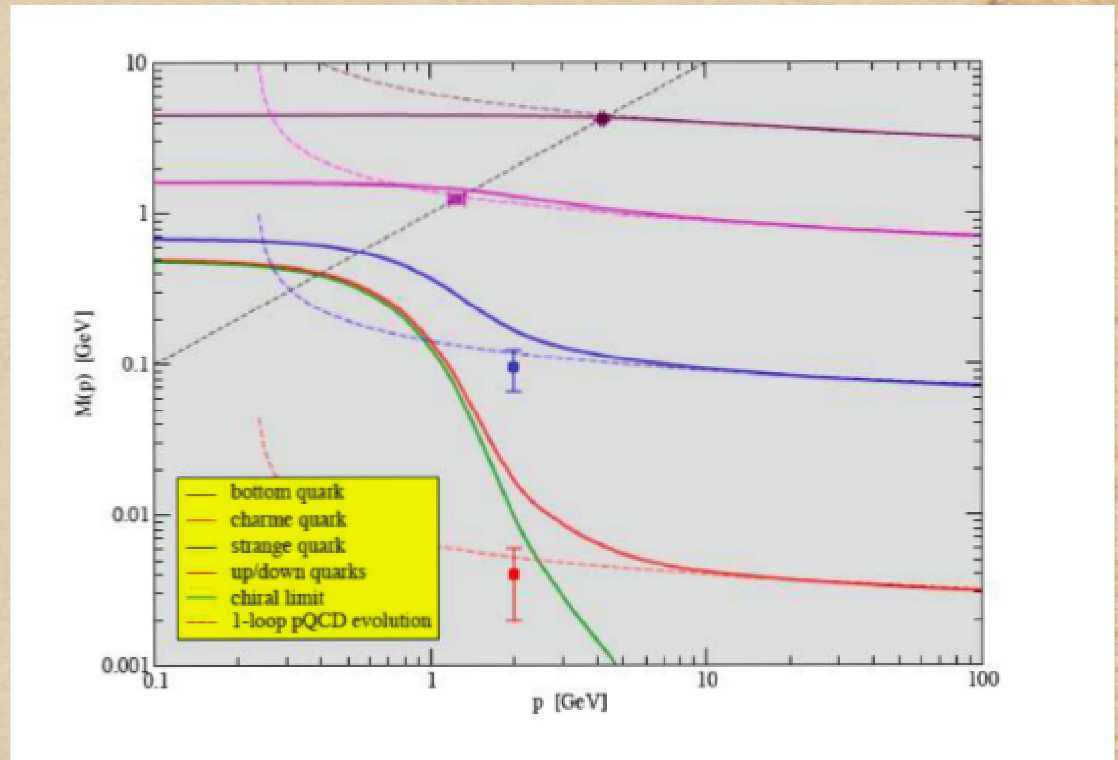

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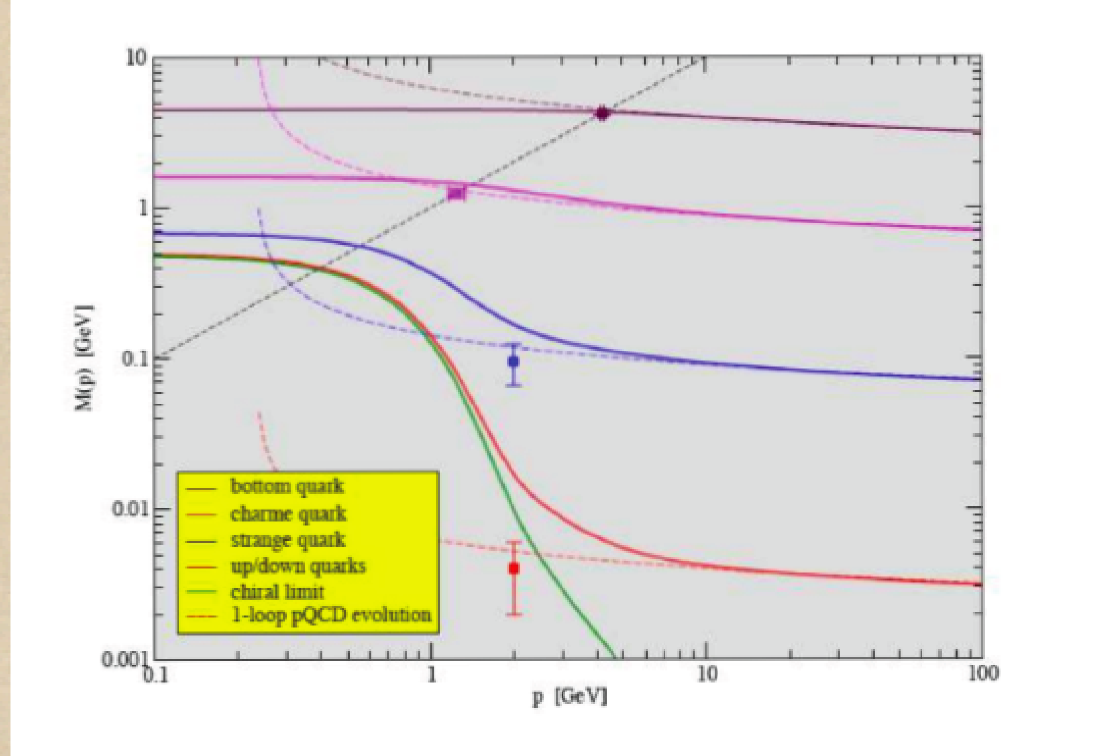


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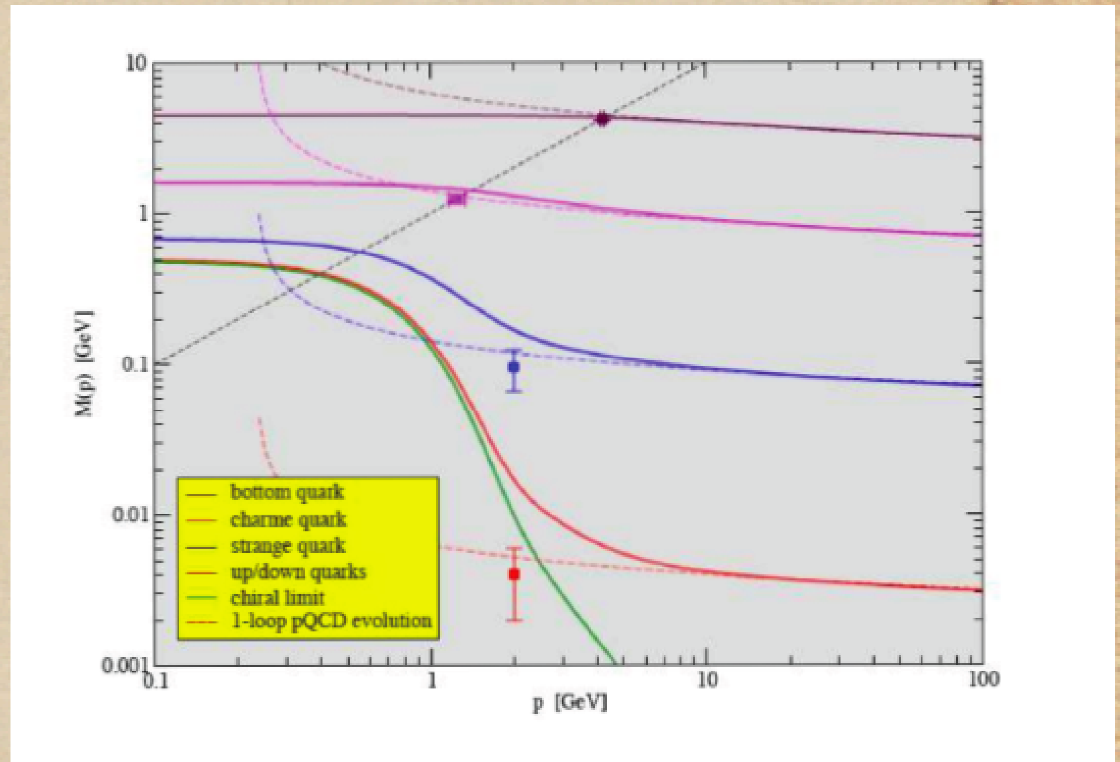


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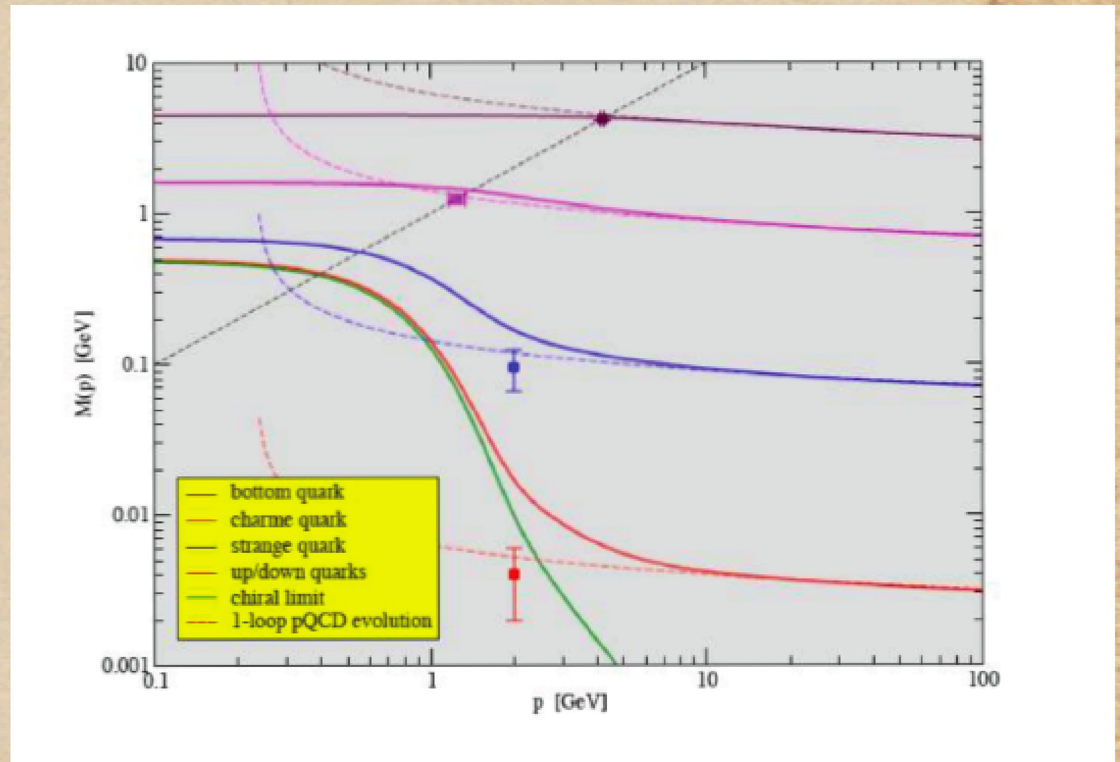
# Many Ways to Obtain the Chiral Quark Condensate

- $\langle \bar{\mathbf{q}}\mathbf{q} \rangle_\mu = -\mathbf{Z_4}(\mu, \mathbf{\Lambda})\ \mathrm{tr_{cd}} \int^{\mathbf{\Lambda}} \frac{\mathbf{d^4q}}{(\mathbf{2}\pi)^\mathbf{4}}\ \mathbf{S_0}(\mathbf{q}; \mu)$




- $M_0(p^2) \overset{\text{large}-p^2}{=} \frac{2\pi^2\gamma_m}{3} \frac{-\langle \bar{q}q \rangle^0}{p^2\left(\frac{1}{2}\ln\left[\frac{p^2}{\Lambda^2_{\mathrm{QCD}}}\right]\right)^{1-\gamma_m}}$

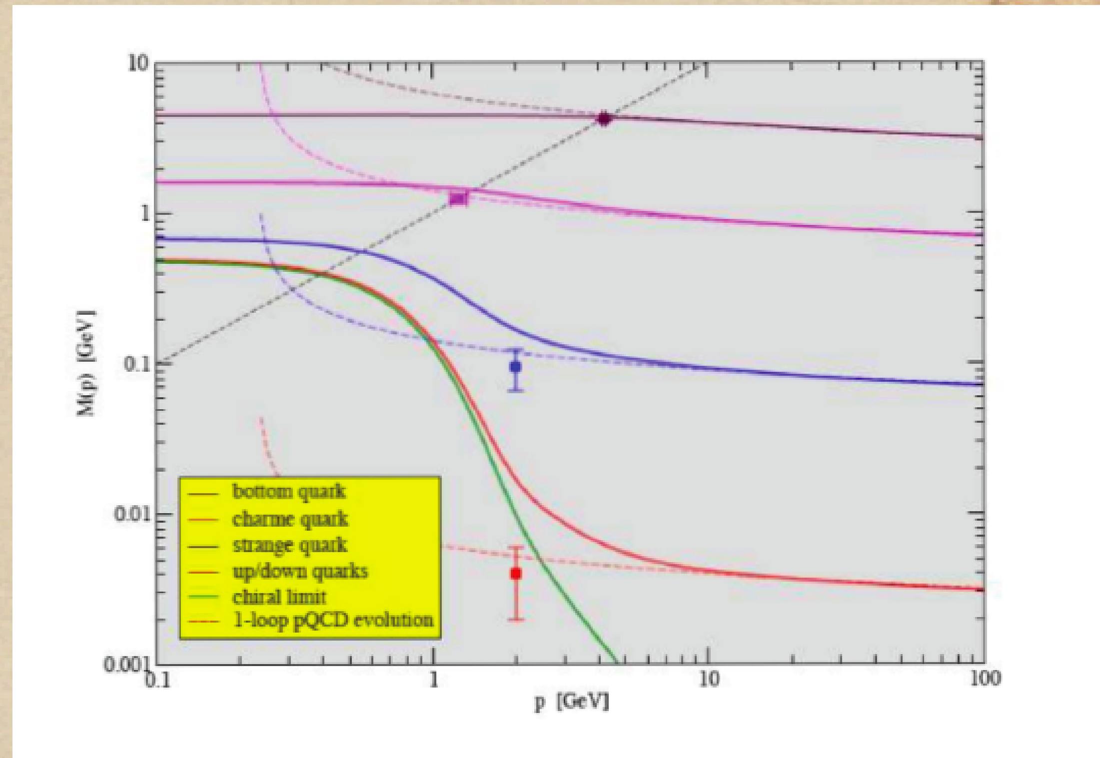
- $\mathbf{Via\ PS\ proj^n\ at\ r = 0\ of\ PS\ bound\ state\ wfn:}\ \langle \bar{\mathbf{q}}\mathbf{q} \rangle_\mu = \lim_{\hat{\mathbf{m}}\to\mathbf{0}} \mathbf{f}_\pi\ \langle \mathbf{0} | \bar{\mathbf{q}}\, \gamma_\mathbf{5}\, \mathbf{q} \,|\, \mathbf{ps} \rangle_\mu$

- $\mathbf{Via\ Casher-Banks\ formula:}\ \langle \bar{\mathbf{q}}\mathbf{q} \rangle_\mu = \lim_{\mathbf{m}\to\mathbf{0}} -\mathbf{2m} \int_\mathbf{0}^\infty \mathbf{d}\lambda\ \frac{\rho_{\not{D}}(\lambda)}{\lambda^\mathbf{2} + \mathbf{m^2}}$

- $\mathbf{Via\ scalar\ charge:}\ \langle \bar{\mathbf{q}}\mathbf{q} \rangle_\mu = \lim_{\mathbf{m_q}\to\mathbf{0}} \mathbf{f^2_M}\ \langle \mathbf{M} \,|\, \bar{\mathbf{q}}(\mathbf{0})\mathbf{q}(\mathbf{0}) \,|\, \mathbf{M} \rangle_\mu, \mathbf{M = PS,\ S\ Mesons}$

# Many Ways to Obtain the Chiral Quark Condensate

- $\langle \bar{q}q \rangle_\mu = -Z_4(\mu,\Lambda)\, \mathrm{tr_{cd}} \int^\Lambda \frac{d^4q}{(2\pi)^4}\, S_0(q;\mu)$




- $$M_0(p^2) \overset{\text{large}-p^2}{=} \frac{2\pi^2\gamma_m}{3} \frac{-\langle \bar{q}q \rangle^0}{p^2\left(\frac{1}{2}\ln\left[\frac{p^2}{\Lambda^2_{\mathrm{QCD}}}\right]\right)^{1-\gamma_m}}$$

- Via PS proj$^n$ at $r = 0$ of PS bound state wfn : $\langle \bar{q}q \rangle_\mu = \lim_{\hat{m}\to 0} f_\pi \langle 0 | \bar{q}\,\gamma_5\, q\, | ps \rangle_\mu$

- Via Casher – Banks formula : $\langle \bar{q}q \rangle_\mu = \lim_{m\to 0} -2m \int_0^\infty d\lambda \frac{\rho_{\not{D}}(\lambda)}{\lambda^2 + m^2}$

- Via scalar charge : $\langle \bar{q}q \rangle_\mu = \lim_{m_q\to 0} f_M^2 \langle M \,|\, \bar{q}(0)q(0) \,|\, M \rangle_\mu$, $M = $ PS, S Mesons

- Via PS or AV current – current correlators in lattice – QCD

# Condensate Summary

Condensates in QCD and the Cosmological Constant, S.J. Brodsky& R.Shrock, Proc. Nat. Acad. Sci., 108, 45 (2011).

New Perspectives on the Quark Condensate, S.J. Brodsky, C.D. Roberts, R.Shrock&P.C.Tandy, Phys. Rev, C82, 022201 (2010).

Expanding the Concept of In-hadron Condensates, L.Chang, C.D. Roberts&P.C. Tandy, Phys. Rev. C85, 012201 (2012)

Confinement Contains Condensates, S.J. Brodsky, C.D. Roberts, R.Shrock&P.C.Tandy, Phys. Rev, C85, 065202 (2012).

# Condensate Summary

- Strong evidence exists that quark condensate is better thought of as a hadron property---explict ps & scalar meson matrix elements given
- Would solve the QCD vac energy problem for the Cosmological Constant
- Assumed confinement.  Consistent with the dynamically generated IR mass scale (max wavelength of confined fields in hadrons)
- Confinement suggests all the "vac condensates" of QCD Sum Rule fame are really inside hadrons
- A special thanks to my condensate of collaborators......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# Overall Summary

- Ladder-rainbow truncated DSE modeling of QCD for hadrons
- Seeking systematics from large to small scales, wherein hadrons are not describable as point fields
- Recent unified DSE treatment of light front quantities (PDFs, GPDs, DA) with other aspects of hadron structure: masses, decays, charge form factors, transition form factors.....
- Essentially one RL parameter apart from current q masses
- Can't seek precision---BSE kernel is not known in closed form
- With no free parameters, $u_\pi^v(x)$ and $u_K(x)/u_\pi(x)$ agree with Drell-Yan data
- Chiral loops have to be added after the "quark core" of hadrons is computed

# The End

Thank you !

# Confinement Implies all QCD Condensates are within Hadrons

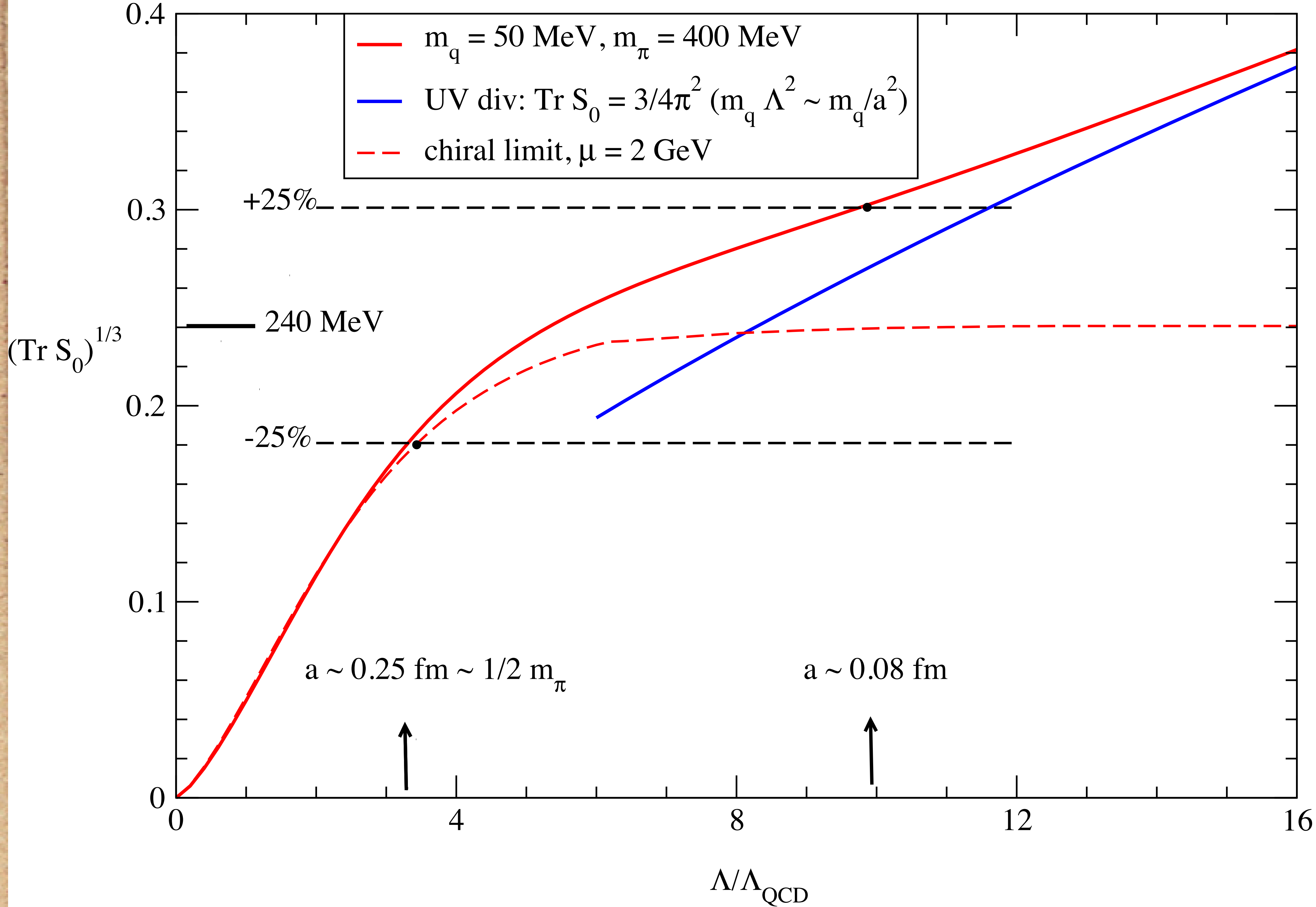
# Confinement Implies all QCD Condensates are within Hadrons

- Lattice-QCD and DSE modeling find that the dynamically generated IR masses of the gluon and u/d quarks are about 0.4-0.6 GeV
- Gives dynamical suppression of low momenta of these virtual fields in hadrons
- Gives suppression of wavelengths > 1-2 fm of .. .. .. .. ..
- Vacuum fluctuations? Casimir effect----interpretation under debate today:
- "No evidence for vacuum QCD fluctuations in absence of matter"---R. Jaffe, New Scientist, Feb 2012.
- Quark and gluon "propagators" are non-observable intermediate elements of theory to be used in construction of color singlet observables
- QCD Sum Rule approach: color singlet current-current correlators involve finite size matter distributions at one vertex; params are fixed by observable hadron data
- No virtual quark-gluon d.o.f. is needed more than a strong interaction distance from color singlet matter
- Lattice-QCD signal for quark condensate is pionic due to Goldstone/GT reln

# Doesn't the pion get fat and fill all space in the chiral limit?

# Doesn't the pion get fat and fill all space in the chiral limit?

- (So its in-pion condensate of quarks is spread throughout the vacuum?)
- Indeed the em charge radius of the pion does diverge in chiral limit due to virtual chiral meson loops
- But, it's due to the virtual tightly correlated PS qqbar pairs that fluctuate far from the pion's qqbar core
- There is no quark separated more than a strong interaction length from a qbar
- The in-hadron condensate is the qqbar-projected bound state wavefunction at zero separation---it is never in the vacuum
- The large distance fluctuations of virtual PS qqbar pairs carry their condensates inside them, the vacuum/void is left as is---empty

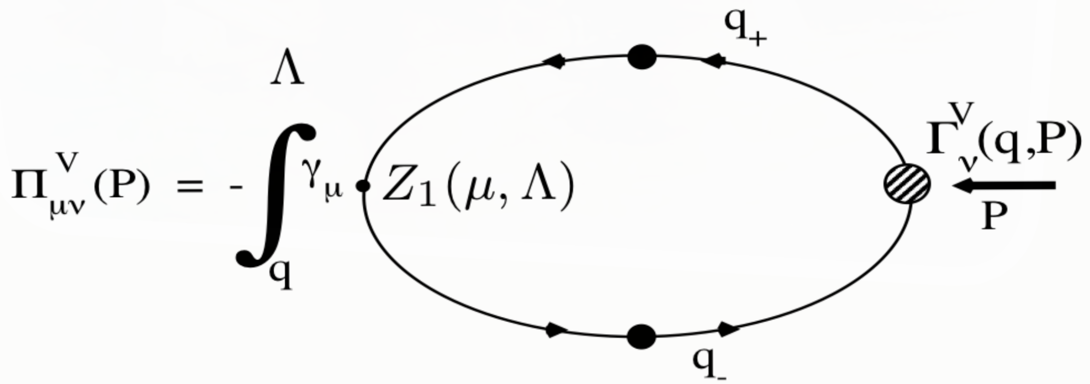

m_q = 50 MeV, m_π = 400 MeV
UV div: Tr S_0 = 3/4π^2 (m_q Λ^2 ~ m_q/a^2)
chiral limit, μ = 2 GeV
+25%
-25%
240 MeV
(Tr S_0)^1/3
a ~ 0.25 fm ~ 1/2 m_π
a ~ 0.08 fm
Λ/Λ_QCD
0.4
0.3
0.2
0.1
0
0
4
8
12
16

## The V-A Current Correlator

- $\Pi^V_{\mu\nu}(x) = \langle 0|\, T\, j_\mu(x) j^\dagger_\nu(0)\, |0\rangle$ , isovector currents $j_\mu = \bar{u}\gamma_\mu d, \quad j^5_\mu = \bar{u}\gamma_5\gamma_\mu d$

$$\Pi^V_{\mu\nu}(P) = (P^2\delta_{\mu\nu} - P_\mu P_\nu)\, \Pi^V(P^2)$$

$$\Pi^A_{\mu\nu}(P) = (P^2\delta_{\mu\nu} - P_\mu P_\nu)\, \Pi^A(P^2) + P_\mu P_\nu\, \Pi^L(P^2)$$




- $m_q = 0: \quad \Pi^V - \Pi^A = 0$ , to all orders in pQCD
- $\Pi^V - \Pi^A$ probes the scale for onset of non-perturbative phenomena in QCD

# Physics from the V-A correlator:

OPE:

$$\Pi^{V-A}(P^2) = \frac{32\pi\alpha_s\langle\bar{q}q\bar{q}q\rangle}{9\ P^6}\left\{1 + \frac{\alpha_s}{4\pi}\left[\frac{247}{4\pi} + \ln\left(\frac{\mu^2}{P^2}\right)\right]\right\} + \mathcal{O}\left(\frac{1}{P^8}\right)$$

| Model | $- < \bar{q}q >_{\mu=19}\ (GeV)^3$ | $< \bar{q}q\bar{q}q >_{\mu=19}\ (GeV)^6$ | $R(\mu = 19)$ |
|---|---|---|---|
| LR DSE | $(0.216)^3$ | $(0.235)^6$ | 1.65 |

Weinberg et al Sum Rules:

- I: $\frac{1}{4\pi^2}\int_0^\infty ds[\rho_v(s) - \rho_a(s)] = [P^2\,\Pi^{V-A}(P^2)]_{P^2\to 0} = -f_\pi^2$
- II: $P^2\,[P^2\,\Pi^{V-A}(P^2)]|_{P^2\to\infty} = 0$
- DGMLY: $\int_0^\infty dP^2\,[P^2\,\Pi^{V-A}(P^2)] = -\frac{4\pi f_\pi^2}{3\alpha}[m_{\pi^\pm}^2 - m_{\pi^0}^2]$

| Model | $f_\pi^2\,(GeV^2)$ | $f_\pi\,(MeV)$ | $f_\pi^{exp}/f_\pi^{num}$ | $\Delta m_\pi\,(MeV)$ | $(\Delta m_\pi)_{exp}$ |
|---|---|---|---|---|---|
| LR DSE | 0.0081 | 90.0 | 1.03 | 4.88 | $4.43 \pm 0.03$ |

*51* —T. Nguyen, PCT, in prep (2010)

# Build Kernel from Ampls of q-g Vertex

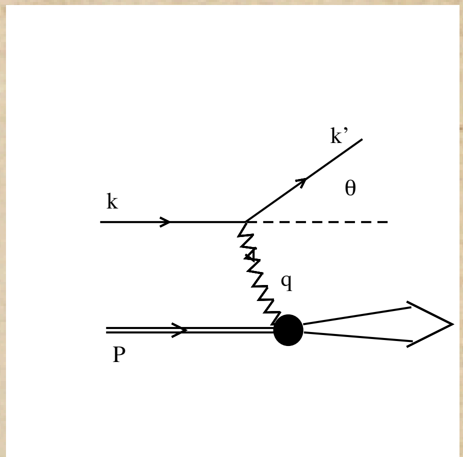
$$\Gamma_\mu^{BC}(p,q) = \frac{A(p^2)+A(q^2)}{2}\gamma_\mu + \frac{(p+q)_\mu}{p^2-q^2}\{[A(p^2)-A(q^2)]\frac{(\gamma\cdot p+\gamma\cdot q)}{2} - i[B(p^2)-B(q^2)]\}, \quad (2.$$

- Just as Ball-Chiu & Curtis-Pennington developed an Ansatz to obtain 3-pt vertex from 2-pt fns, so now get 4-pt fn from 3-pt
- Only longitudinal parts are controlled by symmetries of conserved & partially conserved currents
- There exist transverse WTIs [He Hanxin, Beijing], not yet employed
- Any important physics in the transverse parts?
- Not for the meson observables that LR already gets right
- Find Ansatz for DCSB-driven transverse parts of q-g vertex have strong influence on L > 0 bound states.....

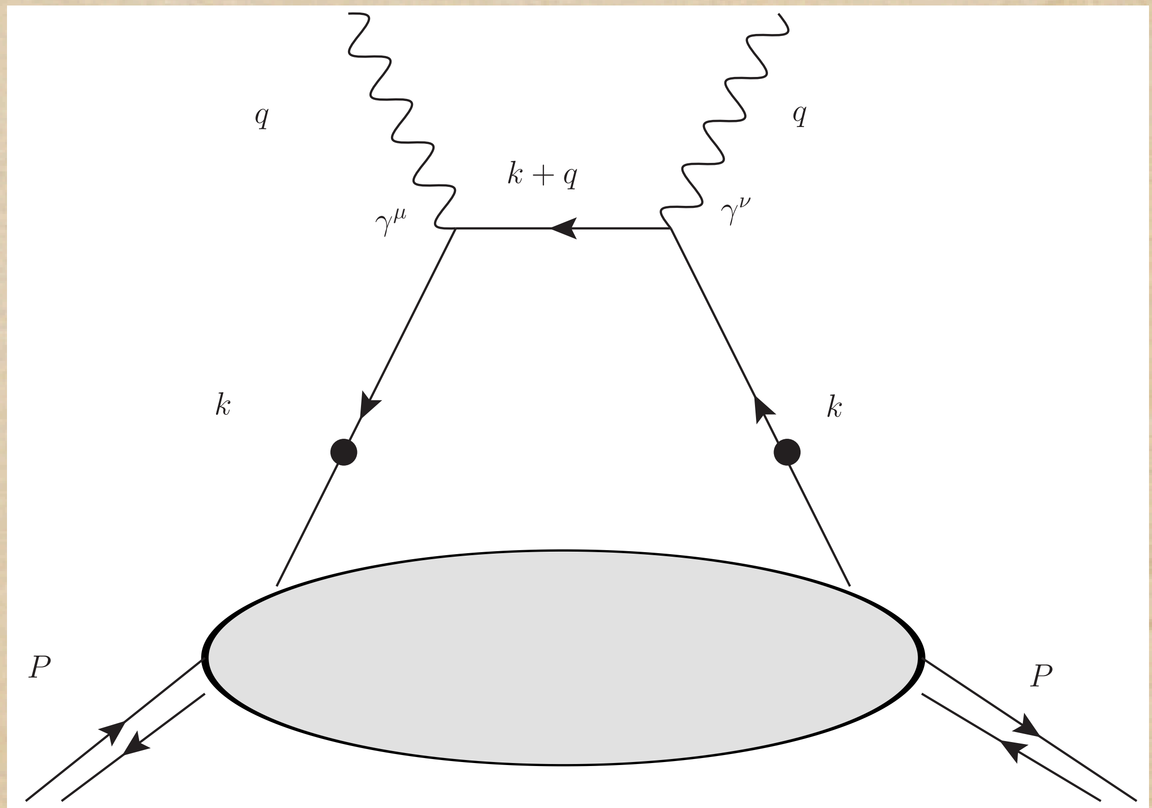
# DSE Approach to QCD's Parton Distributions

- Unify DSE treatment of PDFs with other aspects of hadron structure: masses, decays, charge form factors, transition form factors.....
- PDFs have their own blend of hard and soft, perturbative and non-perturbative, aspects of QCD.
- E.g. $\langle \mathbf{x^m} \rangle : \ \mathbf{small\ m \sim F_\pi(Q^2 \approx 0),\ large\ m \sim\ uv\ structure\ of\ bound\ state}$
- Can a DSE approach to PDFs compete with a lattice-QCD approach ? Eg, how difficult is it to calculate a lot of moments, enough to reproduce the distribution?

# Deep Inelastic Lepton Scattering




- PDFs: $u_\pi(x), \quad u_K(x), \quad s_K(x)$
- Drell-Yan data exists
- Pion and Kaon/Pion Ratio
- Employ LR DSE model
- Bjorken limit fixes quark $\mathbf{k}^+$
- Covariant formulation, explicitly integrate: $\int \mathbf{dk^-} \; \mathbf{\Gamma(k^2, k \cdot P)}$
- Evolve from model scale via LO DGLAP



# DIS in Bjorken Kinematic Limit

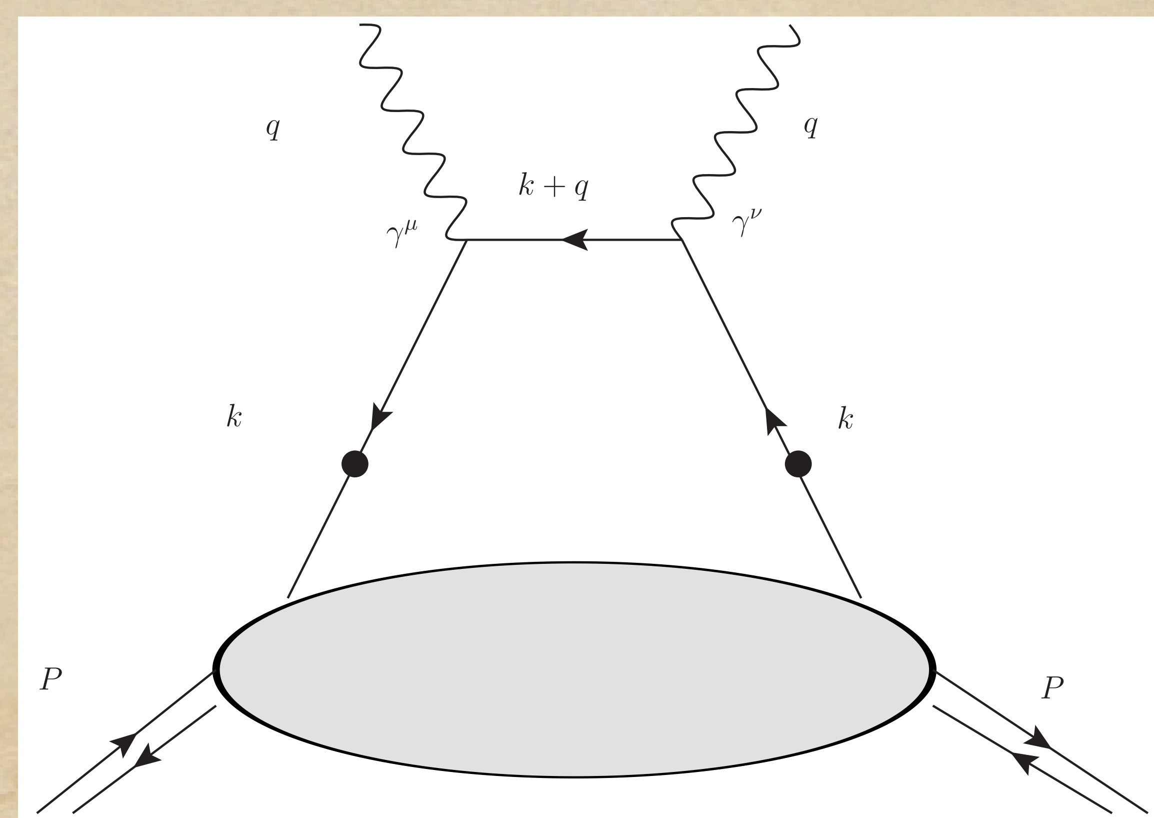
$$\mathbf{T}^{\mu\nu} = \mathbf{i} \int \mathbf{d^4z \;\; e^{iq\cdot z}} \;\; \langle \pi(\mathbf{P}) \,|\, \mathbf{T\, J^{\mu}_{H}(z)\, J^{\nu}_{H}(0)} \,|\, \pi(\mathbf{P}) \rangle_{\mathbf{c}}$$

Bjorken limit seeks out dominant singularity : $\mathbf{z^2 = 0^+}$

$$\mathbf{q \cdot n \equiv q^+ = -x\,P \cdot n} \quad \Rightarrow \quad \mathbf{z \cdot p \equiv z^- \sim \frac{1}{M_\pi x}}$$

$$\mathbf{q \cdot p \equiv q^- = 2\nu} \quad \Rightarrow \quad \mathbf{z \cdot n \equiv z^+ \sim 0}$$

$$\mathbf{S(k+q)} \;\to\; \frac{\gamma \cdot \mathbf{n}}{\mathbf{2(k \cdot n - xP \cdot n) + i\epsilon}}$$




DIS is hard & fast,
confinement is soft & slow

$$W^{\mu\nu} \propto \{T^{\mu\nu}(\epsilon) - T^{\mu\nu}(-\epsilon)\} \Rightarrow \text{Euclidean model elements can be continued}$$

# Valence $u_\pi(x)$ from DSE-BSE solutions

- Valence quarks, handbag diagram

**Nguyen, Bashir, Roberts, PCT, arXiv : 1102.2448 (2011).**

- Data: Conway et al, PRD39, 92 (1989). $M_{l\bar{l}} = 4.05$ GeV

- Prev DSE (phen): Hecht et al, PRC63, 025213 (2001),

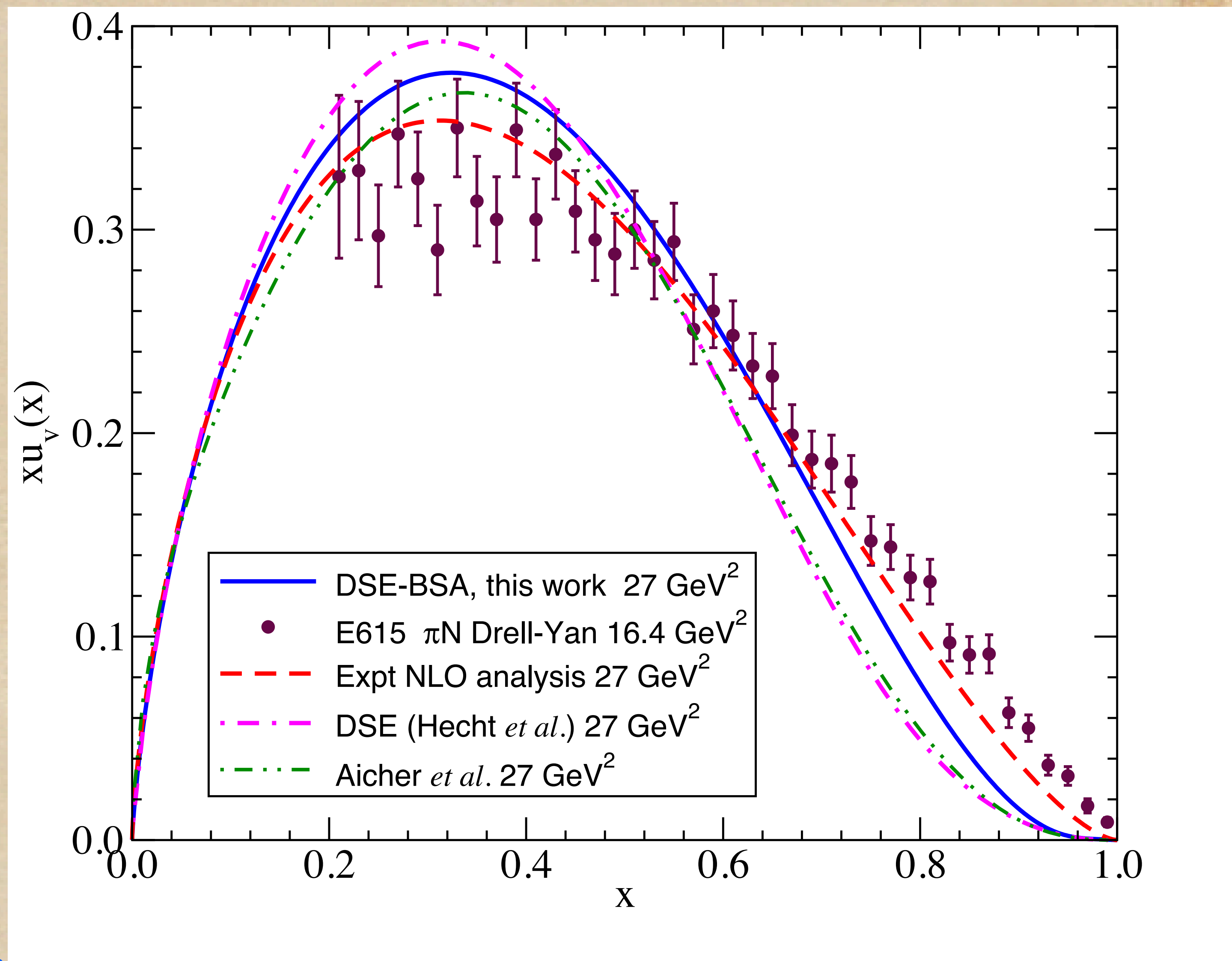
$$\Gamma_\pi(k^2, k\cdot P = 0) \sim i\gamma_5 B_0(k^2)/f_\pi^0 + \cdots$$

$S_{phen}(k)$

- Large x behavior: $(1-x)^{2.08}$

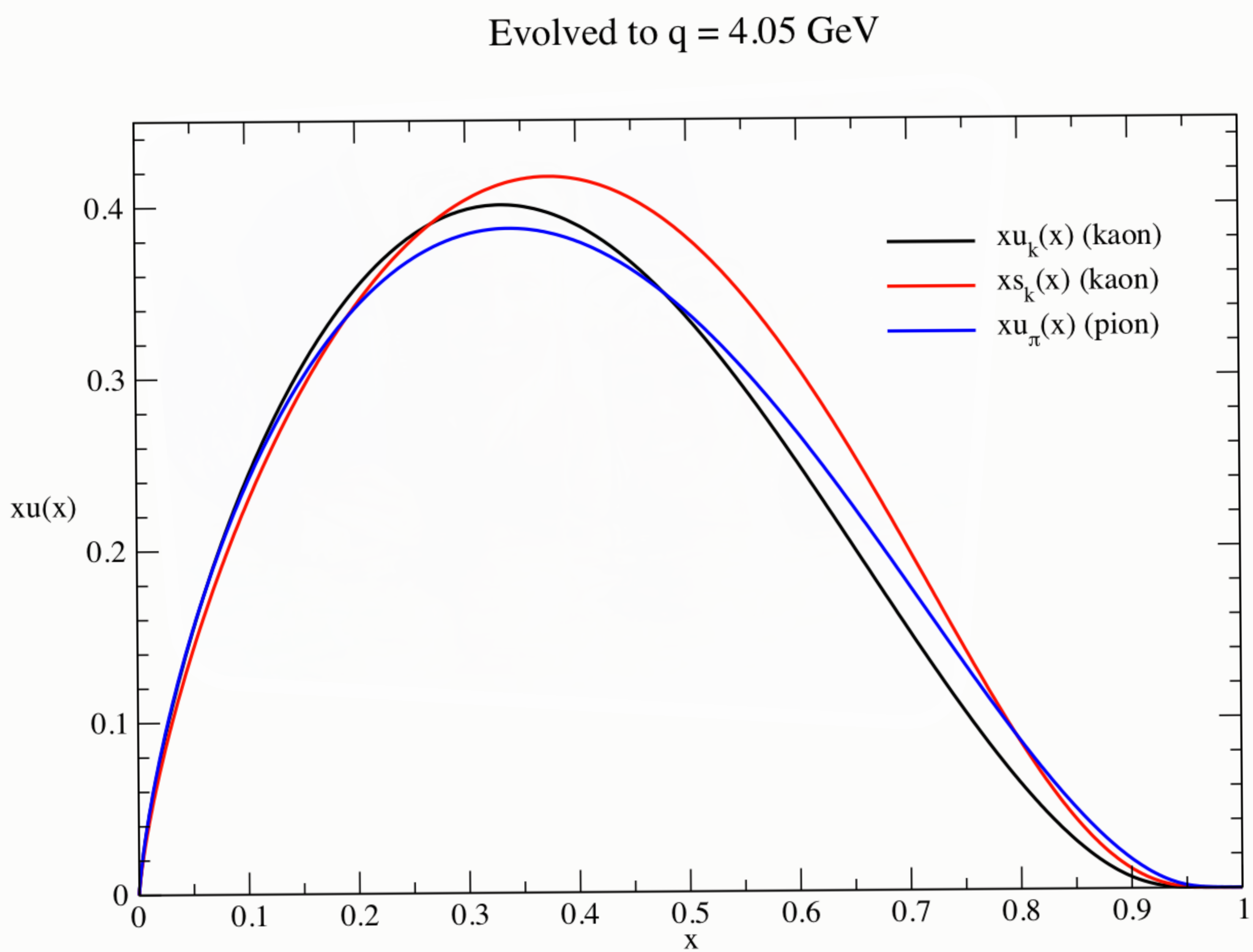
- T. Nguyen, PhD 2010, KSU,

- Wijesooriya, Reimer&Holt, PRC72, 065203 (2005)




Momentum Sum Rule: $\langle x \rangle_{Q_0^2} = 0.76$

# *Quark Distributions in $\pi$ and $K$*




- Environmental depn of $u(x)$ in accordance with effective quark mass
- $u(x), \quad s(x)$ difference in $K$ in accordance with effective quark mass

# Large x Estimate of $\frac{u_K(x)}{u_\pi(x)}$

- Approximations : $\Gamma_{K/\pi}(q^2) \sim \gamma_5 \, N_{K/\pi} \, / \, (q^2 + \Lambda^2_{K/\pi})$ $\qquad S(k) \sim 1/(i \, \not{k} + M_q)$

$$\Rightarrow \qquad u_K(x) = N \int_0^\infty d\hat{\mu} \, \frac{\frac{a}{1-x} + b + \hat{\mu}}{[\frac{a}{1-x} + c + \hat{\mu}]^2} \, (\frac{a}{1-x} + d + \hat{\mu})^{-2} \quad , \qquad a = xM_s^2$$

$$\frac{a}{1-x} >> \text{ any other mass scale} \Rightarrow \quad u_{K/\pi}(x) \ \propto \ N^2_{K/\pi} \, \frac{(1-x)^2}{M^4_{spect}}$$

In a covariant and properly regularized formulation, $(1-x)^2$ is due totally to the divergence of the relative momentum argument of both $\Gamma_{K/\pi}(q^2)$ ie 1 – gluon exchange binding in pQCD

$$\frac{u_K(1)}{u_\pi(1)} \sim \frac{f_\pi^2}{f_K^2} \, (\frac{M_u}{M_s})^4 \sim 0.3 \qquad\qquad \text{cf } \ 0.35 \text{ from DSE – BSE}$$

# Feynman Integral Method/Representation

- For triangle diagram, need all momentum integral variables to appear in denominators that are powers of quadratic forms, with necessary finite powers in numerator
- [Could apply to BSE eqn]---see 1960-70s---Perturbation Integral Repn, and Nakanashi Repn of BSE amplitudes
- Here we use it as a convenient fit/representation of existing numerical solns of DSE for q propagator, and of BSE for meson BSE ampls.
- Momentum integrals done analytically, remaining Feyn parameter integrations done numerically
- Only singularities in the resulting physical quantity come from true contour pinches demanded by unitarity and physical thresholds from open hadronic decays
- Accommodates confining propagators via complex conjugate location of spectral properties--non-positive spectral densities
- Trust in essential content of QFT: analyticity, unitarity, principal mass scales, causality.....etc.

# A Note of Caution: Casher & Susskind (1974)

Chiral Magnetism (or Magnetohadrochironics)

A. Casher and L. Susskind, Phys. Rev. D9 (1974) 436

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- Authors argue that DCSB can be realized as a property of hadrons
- No need for a non-trivial vac exterior to the measurable d.o.f
- Compatible with light-front field theory with its trivial vacuum
- Infinite # d.o.f. is the essential element for DCSB
- Brodsky and Shrock picked up this theme & advocate max wavelength for quarks and gluons (relative to matter)
- Brodsky and Shrock advocate LF-QCD gives cosmological const = 0

# Condensates of Confined Fields

PHYSICAL REVIEW C **82**, 022201(R) (2010)

## New perspectives on the quark condensate

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$$\lim_{\hat{\mathbf{m}}\to\mathbf{0}} \mathbf{f}_\pi \, \langle \mathbf{0} | \, \bar{\mathbf{q}} \, \gamma_\mathbf{5} \, \mathbf{q} \, | \pi \rangle_\mu = -\mathbf{Z_4}(\mu, \mathbf{\Lambda}) \, \mathrm{tr}_{\mathrm{cd}} \int^{\mathbf{\Lambda}} \frac{\mathbf{d^4q}}{(\mathbf{2}\pi)^\mathbf{4}} \, \mathbf{S_0}(\mathbf{q}; \mu) = \langle \, \bar{\mathbf{q}}\mathbf{q} \, \rangle_\mu$$

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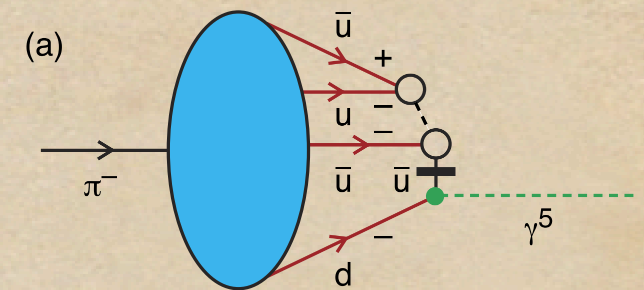
$$\lim_{\hat{m}\to 0} \mathbf{f}_\pi \, \langle \mathbf{0} | \, \bar{\mathbf{q}} \, \gamma_5 \, \mathbf{q} \, | \pi \rangle_\mu = -\mathbf{Z}_4(\mu, \Lambda) \, \mathrm{tr}_{\mathrm{cd}} \int^\Lambda \frac{\mathbf{d}^4\mathbf{q}}{(2\pi)^4} \, \mathbf{S}_0(\mathbf{q}; \mu) = \langle \, \bar{\mathbf{q}}\mathbf{q} \, \rangle_\mu$$

- So-called vacuum chiral quark condensate is really a property of the Goldstone boson BSE wavefunction
- Its a constant mass scale that does not leak outside of its containers (hadrons): An in-hadron condensate.
- Above relation is dictated by DCSB: $\mathbf{GT_q}: \;\; \Gamma_\pi(\mathbf{k}^2; 0) = \mathbf{i}\gamma_5 \, \tau \, \frac{\frac{1}{4}\mathrm{tr}\, \mathbf{S}_0^{-1}(\mathbf{k})}{\mathbf{f}_\pi^0} + \cdots$
- (1-body problem and 2-body problem coincide)
- Removes the 46 orders of magnitude in QCD's vacuum energy over-estimate of cosmological constant

# DCSB in Pion in LF Field Theory & its Trivial Vacuum

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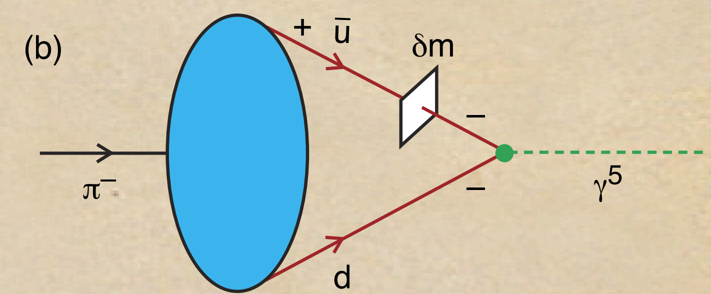

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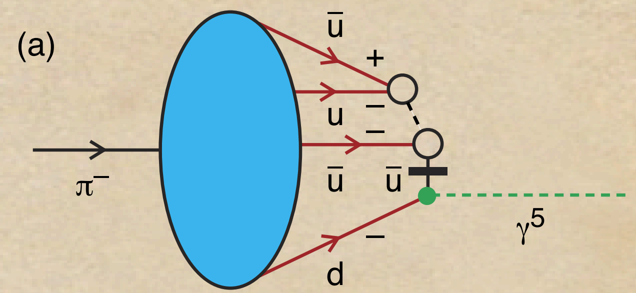


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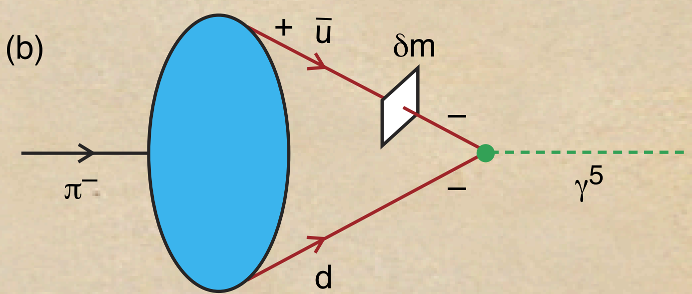


- Higher Fock state components & the LF instantaneous interaction can combine to simulate the required helicity non-conservation
- Effect would look like a dynamically generated mass function
- Infinite # d.o.f. is the essential element for DCSB
- Does this in fact happen? Under investigation.