Three-Field Potential for Soft-Wall Ads/QCD

Sean Bartz

University of Minnesota

In collaboration with
Joe Kapusta

U.S. DEPARTMENT OF ENERGY | Office of Science

AdS/CFT Correspondence

Duality between:

4D Conformal Field Theory ↔ 5D Gravity theory in AdS Space

Strongly Coupled CFT	Weakly coupled Gravity
Operators	Fields
Global Symmetries	Gauged Symmetries

AdS/QCD

- Study non-perturbative QCD
  - Hadron structure

Strongly Coupled QCD ⟷ Weakly coupled Gravity Dual in 5 Dimensions

- QCD not scale-invariant
  - Dilaton cutoff → Soft-wall Model

Full Action

- String Frame:

$$\mathcal{S}_{string} = \int d^5x\sqrt{-g}\left[e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - \frac{1}{2}\partial_M\chi\partial^M\chi - V(\Phi,\chi)\right) + e^{-\Phi}\mathcal{L}_{meson}\right]$$

$$\mathcal{L}_{meson} \equiv |DX|^2 + m_X^2|X|^2 - \kappa|X|^4 + \frac{1}{2g_5^2}(F_A^2 + F_V^2)$$

- Einstein Frame: $g_{MN}^{string} = e^{4\Phi/3}g_{MN}^{E}$ $\qquad \phi = \sqrt{\frac{8}{3}}\Phi$

$$\mathcal{S} = \int d^5x\sqrt{-g_E}\left[R_E - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{2}\partial_M\chi\partial^M\chi - V_E(\phi,\chi)\right]$$

Boundary Conditions

- Dilaton
  - IR: $\phi = \lambda z^2$
    → Slope of Meson Trajectory
- Chiral Field
  - UV: $\chi = \sigma z^3$
    → Chiral Condensate
  - IR: $\chi = \Gamma z$
    → Axial-Vector Mass Splitting

Background Equations

1
$$\sqrt{6}\phi''(z) - [\chi'(z)]^2 + \frac{2\sqrt{6}\phi'(z)}{z} = 0$$

2
$$3e^{2\phi(z)/\sqrt{6}}\frac{z^2}{L^2}\left[\frac{1}{\sqrt{6}}\phi''(z) - \frac{1}{2}[\phi'(z)]^2 - \frac{\sqrt{6}}{z}\phi'(z) - \frac{4}{z^2}\right] = V(\phi(z), \chi(z))$$

3
$$e^{2\phi(z)/\sqrt{6}}\frac{z^2}{L^2}\left[\chi''(z) - 3\chi'(z)\left(\phi'(z)/\sqrt{6} + \frac{1}{z}\right)\right] = \left.\frac{\partial V}{\partial \chi}\right|_{\phi=\phi(z),\chi=\chi(z)}$$

Power-Law Fields

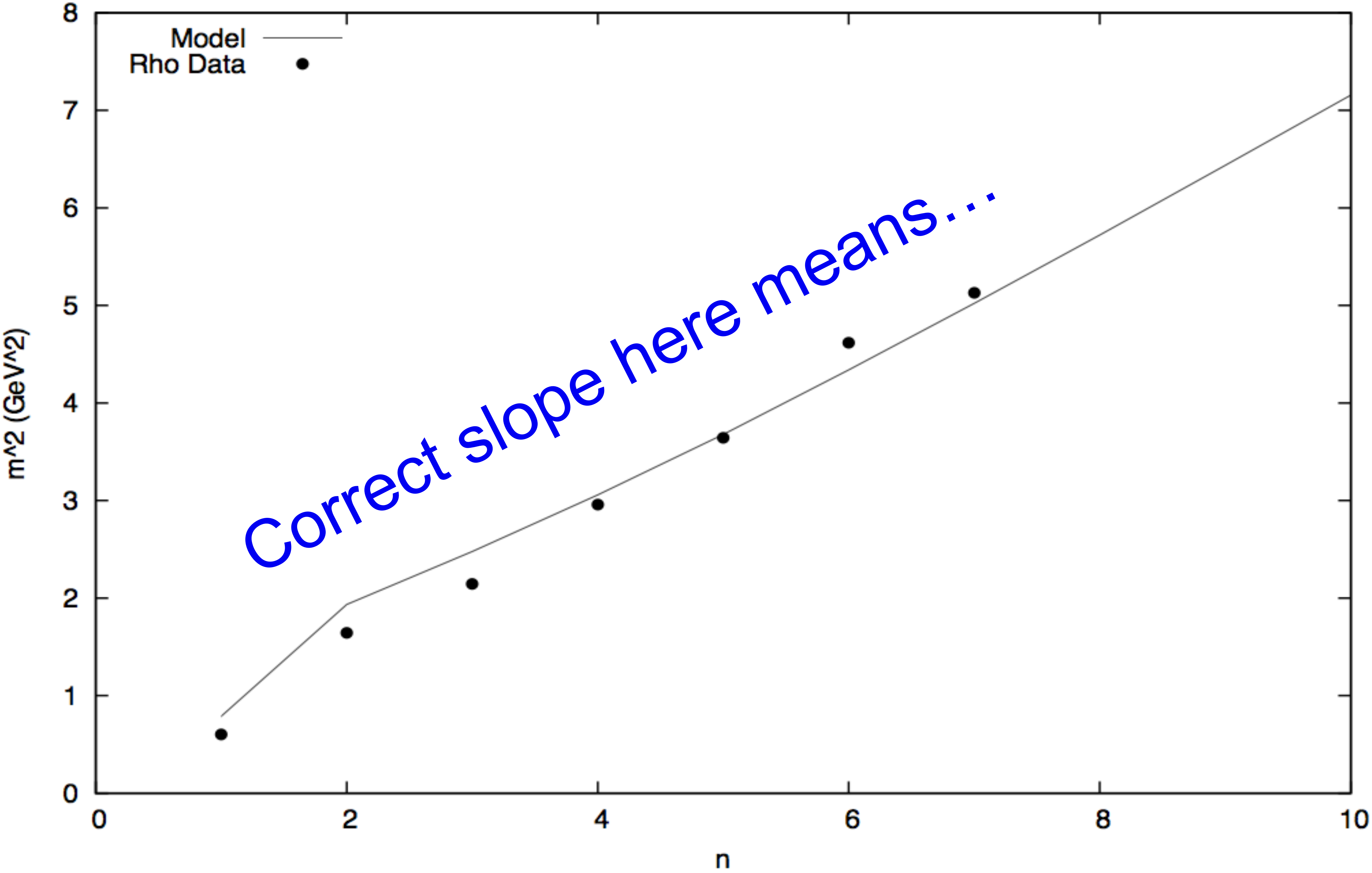
$$\chi = \chi_o z^n$$

1 $$\sqrt{6}\phi''(z) - [\chi'(z)]^2 + \frac{2\sqrt{6}\phi'(z)}{z} = 0$$

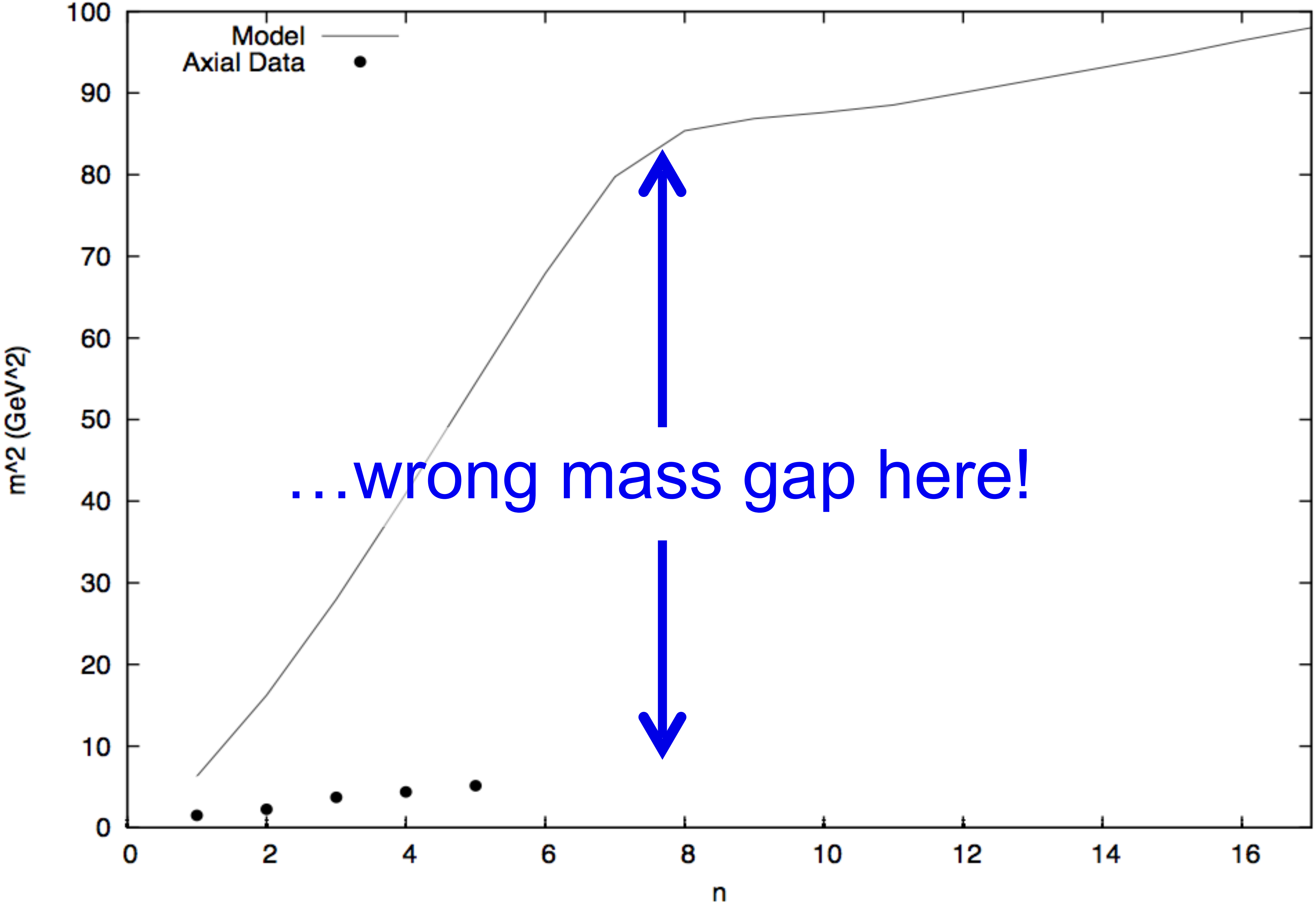
$$\phi = \frac{\sqrt{6}n}{12(2n+1)}\chi^2$$

So… λ and Γ are linked in IR! Problem?

2-Field Results – Vector



2-Field Results – Axial



3-Field Model

Add Glueball Field

1 $$\left(\frac{d\chi}{dz}\right)^2 + \boxed{\left(\frac{dG}{dz}\right)^2} = \frac{\sqrt{6}}{z^2}\frac{d}{dz}\left(z^2\phi'(z)\right)$$

2 $$\frac{1}{2}z^2 a\phi''(z) - \frac{3}{2}a[z\phi'(z)]^2 - 3za\phi'(z) = \tilde{V}\left(\phi(z), \chi(z), \boxed{G(z)}\right) + 12$$

3 $$\frac{z^2}{L^2}\left[\chi''(z) - 3\chi'(z)\left(a\phi'(z) + \frac{1}{z}\right)\right] = \frac{\partial\tilde{V}}{\partial\chi}$$

4 $$\boxed{\frac{z^2}{L^2}\left[G''(z) - 3G'(z)\left(a\phi'(z) + \frac{1}{z}\right)\right] = \frac{\partial\tilde{V}}{\partial G}}$$

Power-Law Potential

- Chiral and Glueball fields are power laws

$$\chi(z) = \chi_o z^n, \quad G(z) = g_o z^m$$

  - Insert into 1 to find dilaton
- Ansatz:

$$\tilde{V} = c_o + c_1\phi + \frac{1}{2}m_\chi^2\chi^2 + c_2\phi^2 + c_3 G^2 + c_4\chi^4 + c_5\phi\chi^2 + c_6 G^4 + c_7\phi G^2 + c_8 G^2\chi^2$$

  - Insert into 2, 3, 4. Match powers to find c_i
  - Cosmological Constant $c_0 = -12$
  - Glueball mass term in IR
  - c_2 set by dilaton mass

Parametrization

- Redefine Parameters: $\alpha \sim \sigma, \quad \frac{\alpha}{\beta^2} = \Gamma, \quad A \sim G_o$
- Set Chiral, Glueball derivatives:

$$\chi' = \frac{\alpha}{\beta^2}(1 - e^{-\beta z})^2 \qquad G' = \frac{A}{B^3}(1 - e^{-Bz})^3$$

- Dilaton is complicated, but no special functions

Setting Parameters

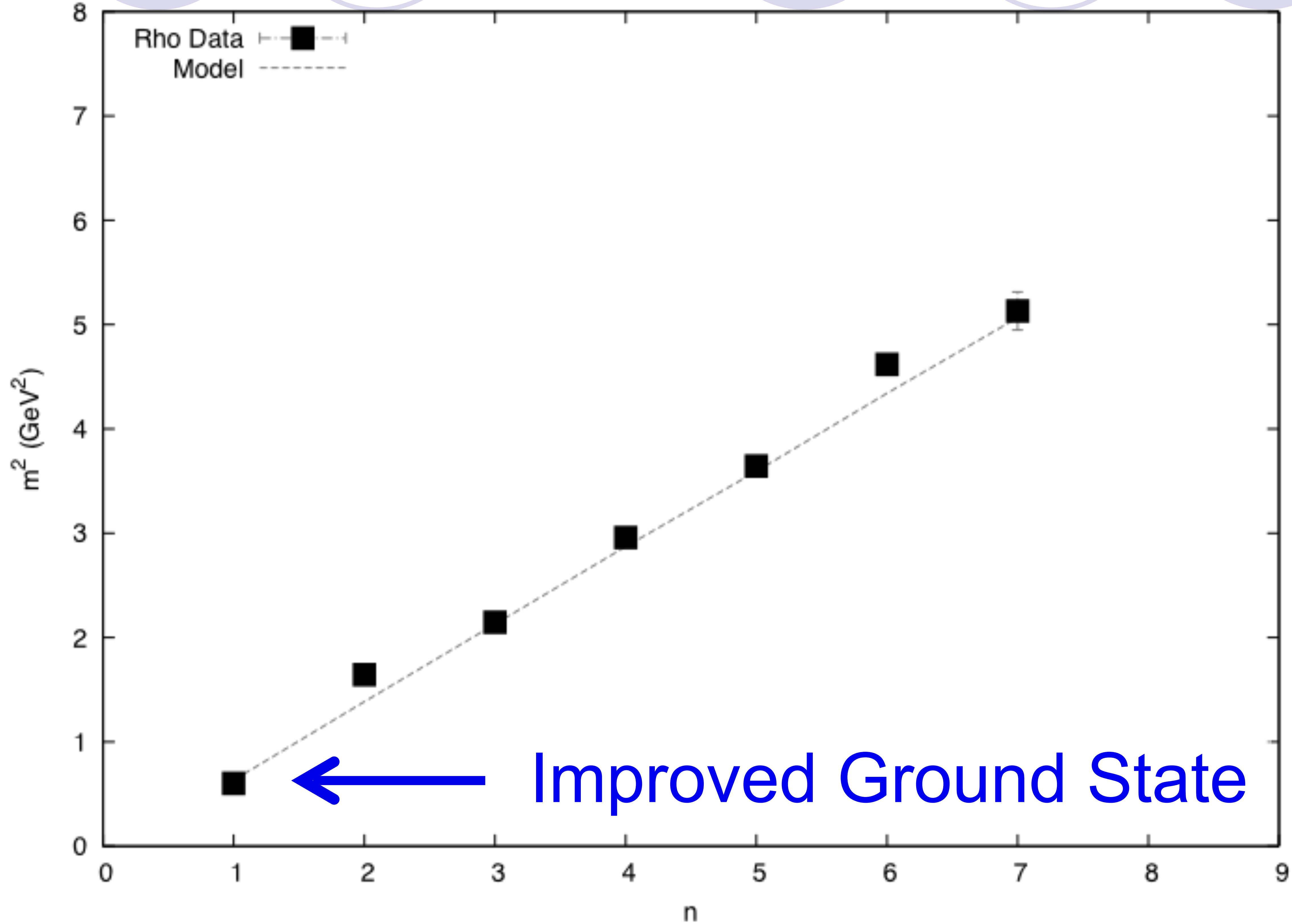
- Least-squares fit to meson spectra

$$\sigma = (0.375\,\mathrm{GeV})^3 \qquad G_o = (1.5\,\mathrm{GeV})^4$$

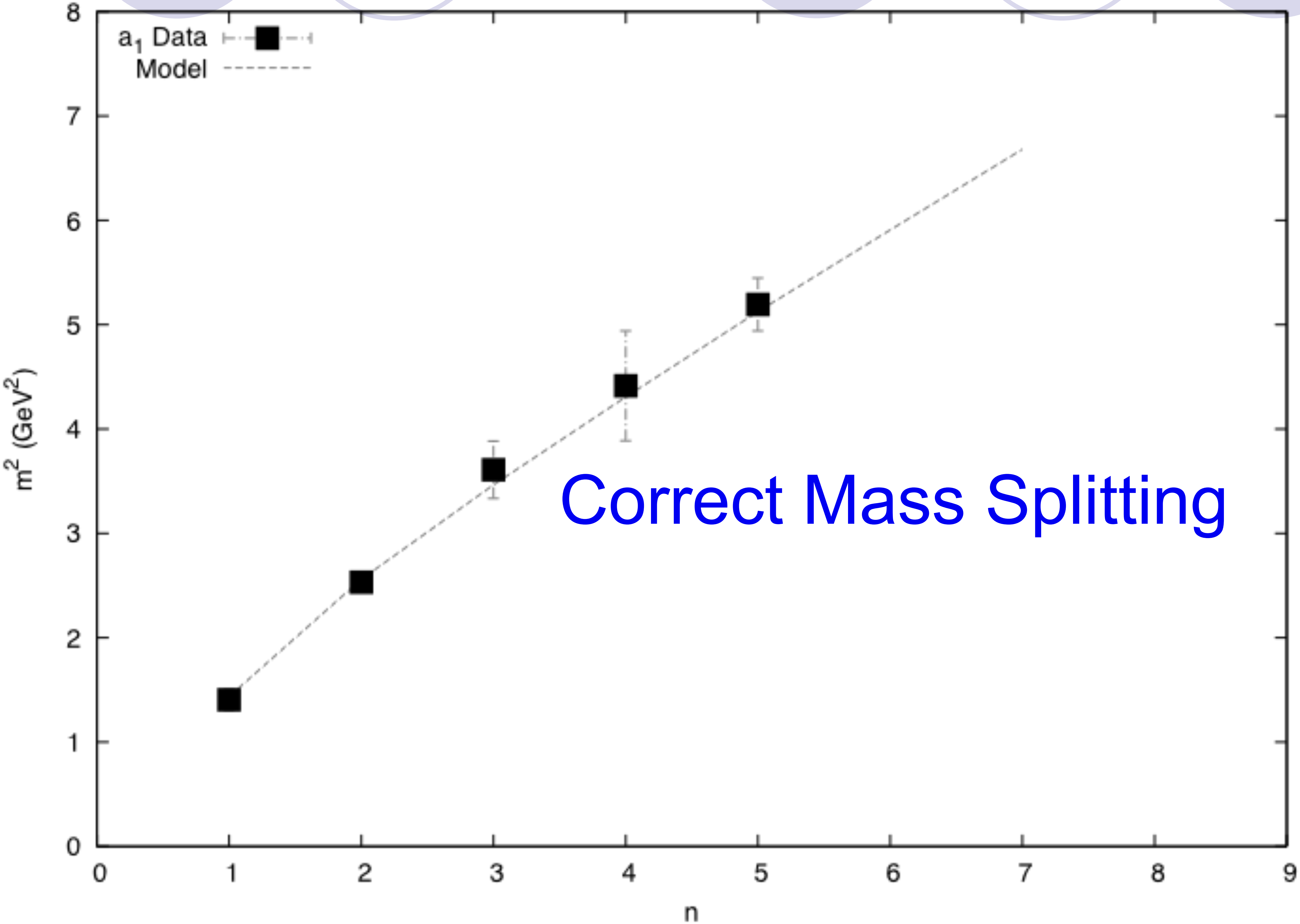
$$\lambda = (0.428\,\mathrm{GeV})^2 \qquad \kappa = -12.4$$

$$\Gamma = 0.25\,\mathrm{GeV}$$

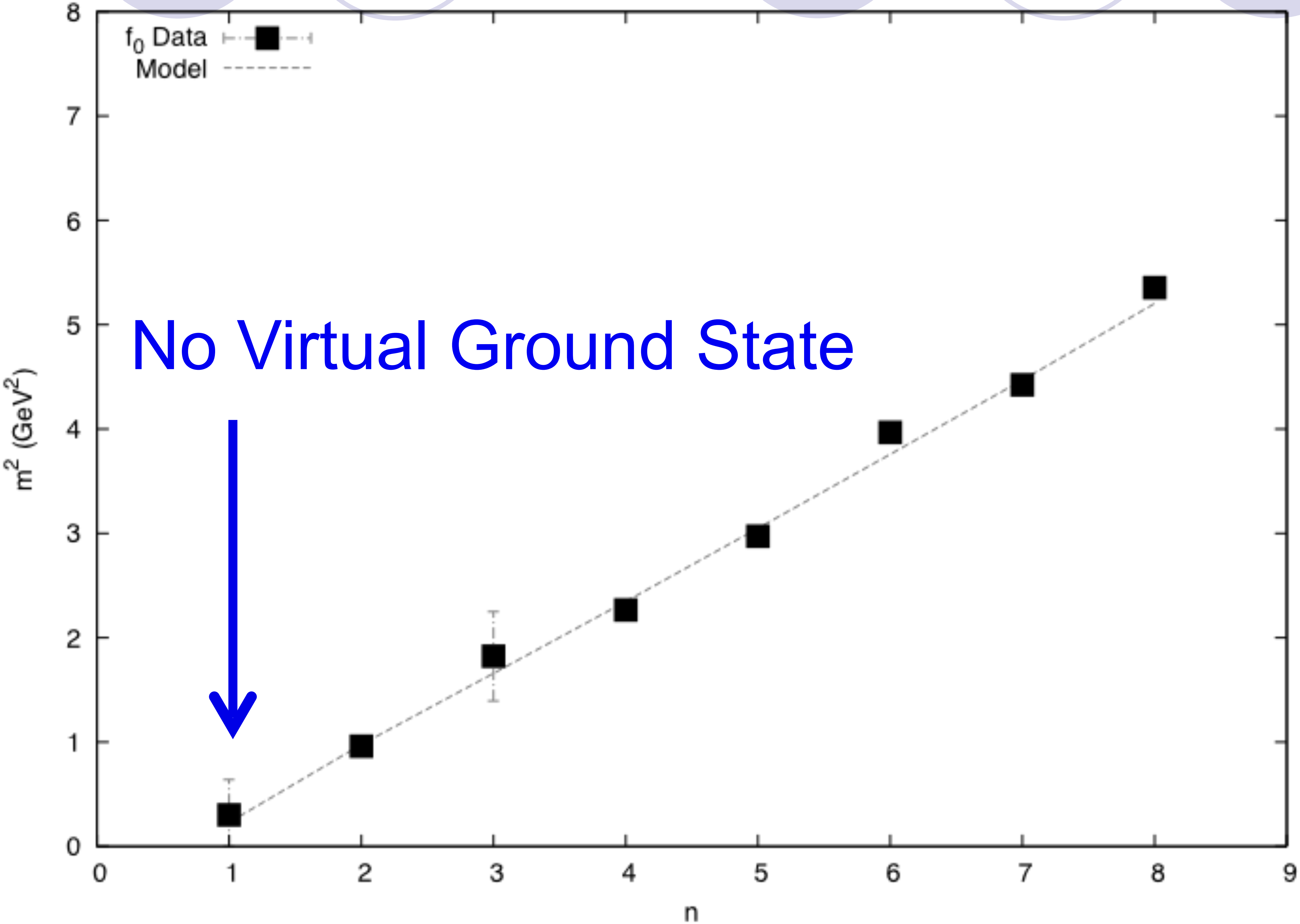
Vector Results



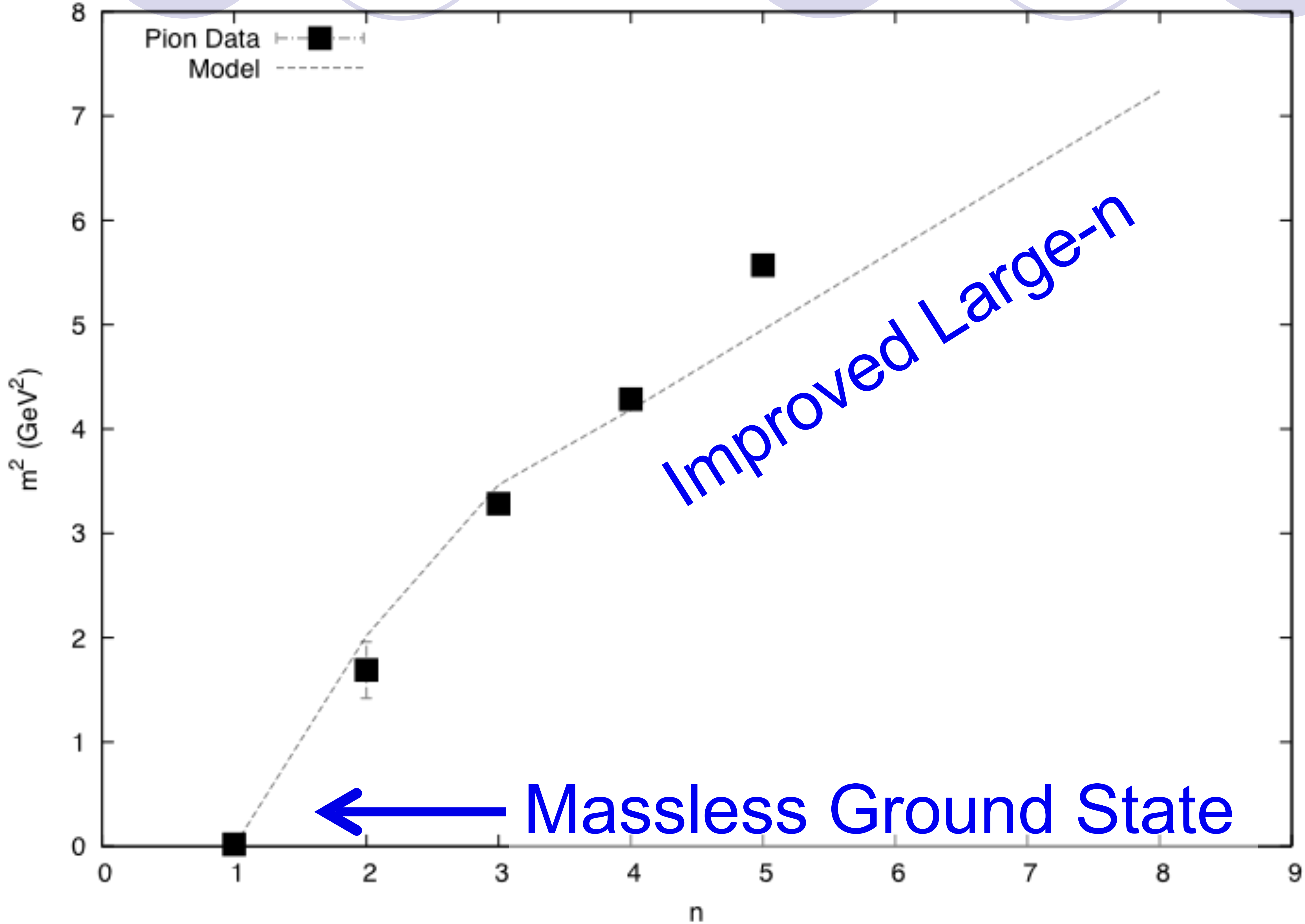
Axial Results



Scalar Results



Pion Results



Next Steps

- Construct Potential using Parametrization
- Use Potential for Finite Temperature

Summary

- Two-field model gives wrong meson spectrum
- Three-field model allows flexibility
- Can parametrization come from potential?

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