

Status of V_{us} and V_{ud} and implications for new physics

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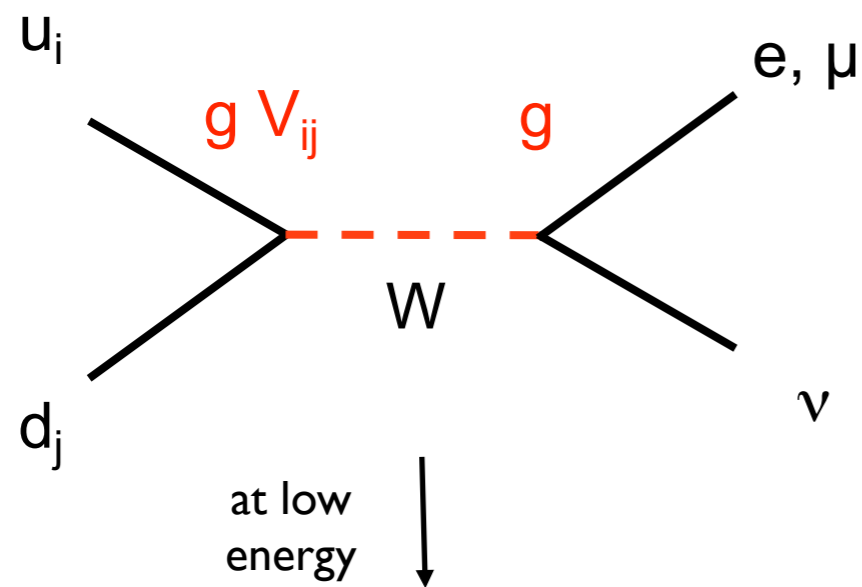


Outline

- Introduction: CC interactions and physics beyond the Standard Model
- V_{us} from K decays:
 - V_{us} from $Kl3$
 - V_{us}/V_{ud} from $K_{\mu 2}/\pi_{\mu 2}$
- Implications of Cabibbo universality tests for new physics
 - Model independent analysis
 - SUSY

CC interactions and BSM physics

- In the SM, W exchange \Rightarrow only V-A structure, universality relations



$$G_F \sim g^2 V_{ij} / M_W^2 \sim 1/v^2$$

Lepton universality

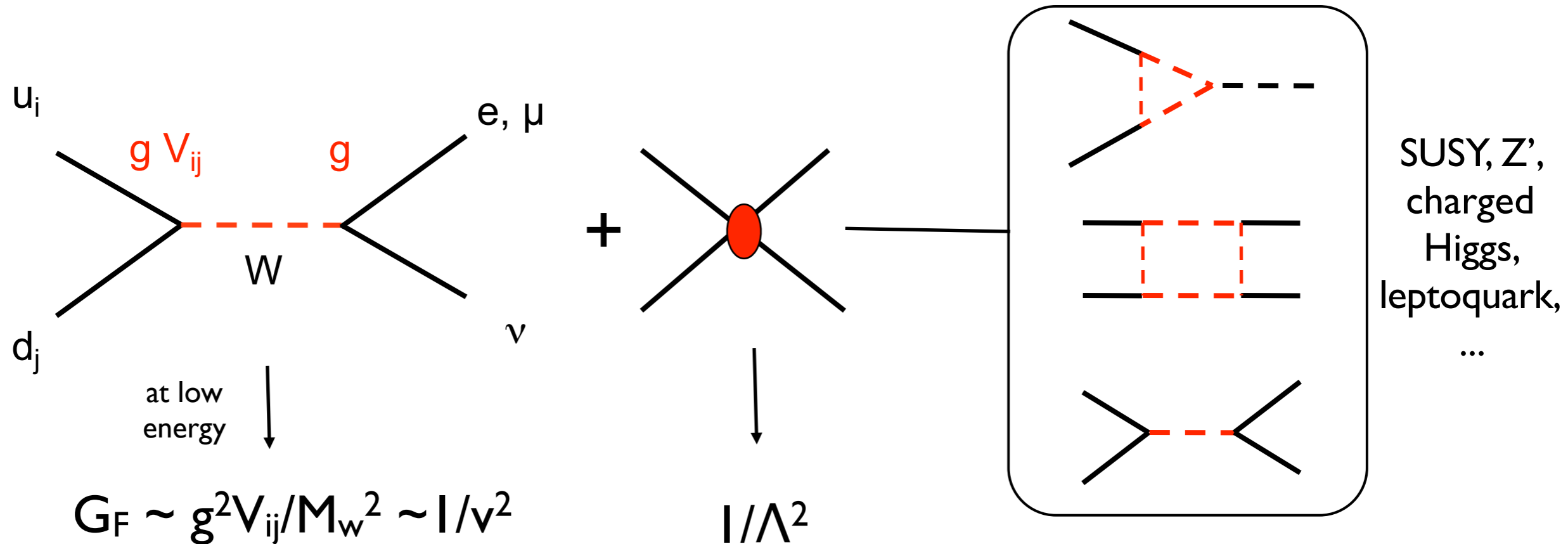
$$[G_F]_e / [G_F]_\mu = 1 + \Delta_{e/\mu}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |\cancel{V_{ub}}|^2 = 1 + \Delta_{\text{CKM}}$$

Cabibbo universality

CC interactions and BSM physics

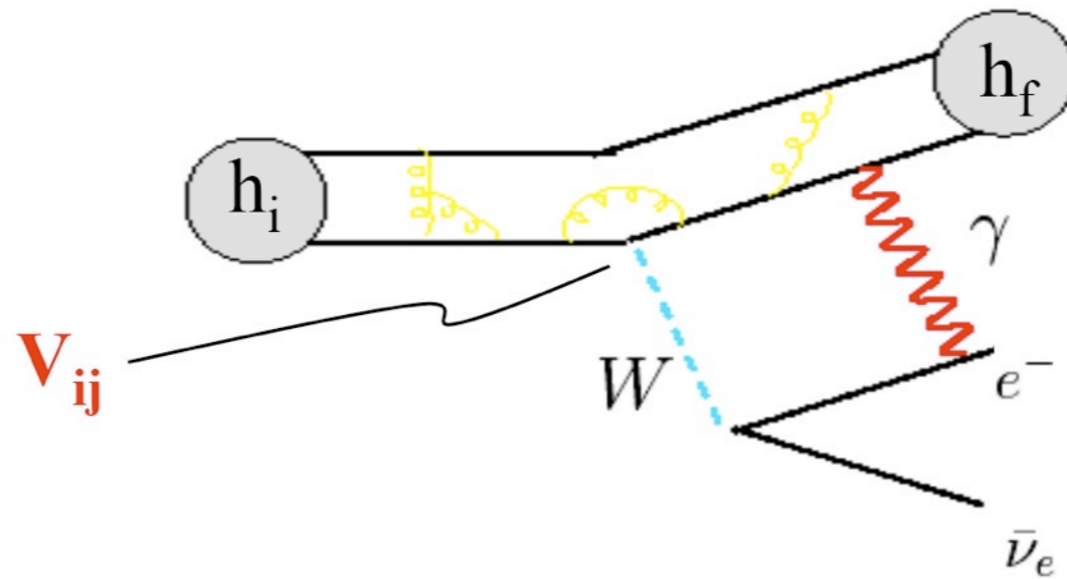
- In the SM, W exchange \Rightarrow only V-A structure, universality relations



- BSM: sensitive to tree-level and loop corrections from large class of models \rightarrow “broad band” probe of new physics, with BSM effects scaling as $\Delta_{CKM} \sim (v/\Lambda)^2$

Paths to V_{ud} and V_{us}

| | | | | |
|----------|--|--|---------------------------|---|
| V_{ud} | $0^+ \rightarrow 0^+$ ($\pi^\pm \rightarrow \pi^0 e \nu$) | $n \rightarrow p e \bar{\nu}$ | $\pi \rightarrow \mu \nu$ | $\tau \rightarrow h_{NS} \nu_\tau$ |
| V_{us} | $K \rightarrow \pi \ell \nu$ | $\Lambda \rightarrow p e \bar{\nu}, \dots$ | $K \rightarrow \mu \nu$ | $\tau \rightarrow h_S \nu_\tau$ (<i>inclusive</i>) |



Paths to V_{ud} and V_{us}

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- Theory golden modes: only V current contributes $\langle A(p_A) | \bar{q}^i \gamma_\mu q^j | B(p_B) \rangle$
 - Normalization known in SU(2) [SU(3)] symmetry limit
 - Corrections start at 2nd order in SU(2) [SU(3)] breaking

Ademollo-Gatto
Berhends-Sirlin

- Currently, most precise determinations of V_{ud} (0.02%) and V_{us} (0.5%)

Paths to V_{ud} and V_{us}

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- Both **V** and **A** currents contribute: need experimental input on $\langle A \rangle$ (e.g. β -asymmetry)
- Free of nuclear structure uncertainties
- Probe different combinations of BSM operators (compared to $0^+ \rightarrow 0^+$)

Paths to V_{ud} and V_{us}

| | | | | |
|----------|--|--|---------------------------|---|
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- Purely **A** transition: $\langle A \rangle \leftrightarrow$ decay constants (from Lattice QCD)
- Input on $F_K/F_\pi \Rightarrow$ information on ratio V_{ud}/V_{us}
- Probe different BSM operators than V-channels

Paths to V_{ud} and V_{us}

| | | | | |
|----------|--|--|---------------------------|---|
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- Use OPE to calculate inclusive BRs
- Information from exclusive modes, too
- See talk by E. Passemar

V_{us} from $K \rightarrow \pi l \nu$ decays

$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

$$\Gamma_{K_{\ell 3[\gamma]}} = C_K^2 \frac{G_\mu^2 S_{ew} M_K^5}{192\pi^3} \cdot |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \cdot I^{K\ell}(\lambda_i) \cdot [1 + 2 \Delta_{EM}^{K\ell} + 2 \Delta_{SU(2)}^K]$$

--- EXP input
--- TH input

- Experimental input:
 - FLAVIANET paper [1005.2323, Eur.Phys.J. C69 (2010) 399]
 - Updates: M. Moulson's talk at CIPANP 2012

V_{us} from $K \rightarrow \pi \ell \nu$ decays

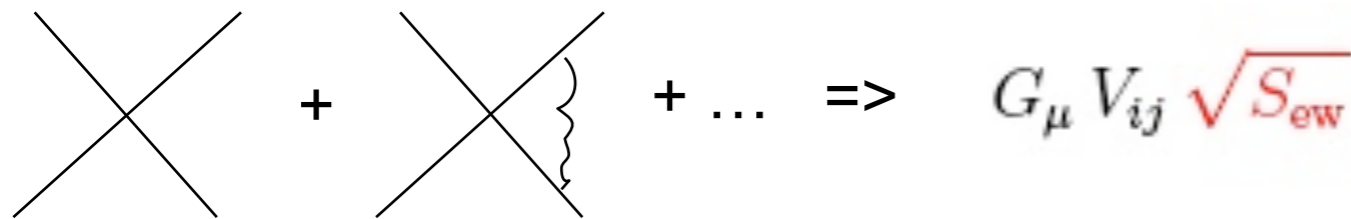
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Short distance
electroweak correction:

Sirlin '82

$$S_{ew} = 1 + \frac{2\alpha}{\pi} \left(1 + \frac{\alpha_s}{4\pi}\right) \log \frac{M_Z}{M_\rho} + O\left(\frac{\alpha\alpha_s}{\pi^2}\right) = 1.0232$$



$$\text{Tree} + \text{Loop} + \dots \Rightarrow G_\mu V_{ij} \sqrt{S_{ew}}$$

V_{us} from $K \rightarrow \pi \ell \nu$ decays

$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

$$\Gamma_{K\ell 3[\gamma]} = C_K^2 \frac{G_\mu^2 S_{ew} M_K^5}{192\pi^3} \cdot |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \cdot I^{K\ell}(\lambda_i) \cdot [1 + 2\Delta_{EM}^{K\ell} + 2\Delta_{SU(2)}^K]$$

$$t = (p_K - p_\pi)^2$$

1 in $SU(3)_V$ limit ($m_u = m_d = m_s$)

$$\langle \pi^-(p_\pi) | \bar{s} \gamma_\mu u | K^0(p_K) \rangle = f_+^{K^0\pi^-}(t) (p_K + p_\pi)_\mu + f_-^{K^0\pi^-}(t) (p_K - p_\pi)_\mu$$

- $f_+(0)$ is the key hadronic parameter
- Discuss effect of $SU(3)$ breaking later on

V_{us} from $K \rightarrow \pi \ell \nu$ decays

$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

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$$t = (p_K - p_\pi)^2$$

$$f_{+,0}(t) = f_+(0) \left(1 + \lambda_{+,0} \frac{t}{M_\pi^2} + \lambda''_{+,0} \frac{t^2}{M_\pi^4} + \dots \right)$$

$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

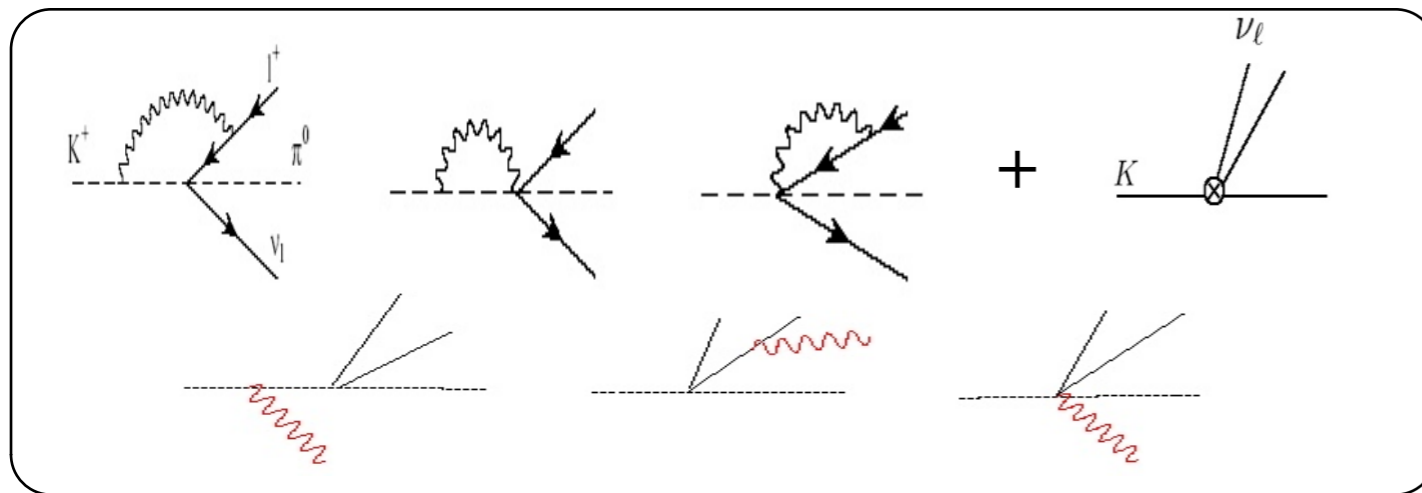
- Various parameterizations of the form factors exist (polynomial, pole, dispersive)
- Phase space integrals consistent within uncertainty

V_{us} from $K \rightarrow \pi \ell \nu$ decays

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Long-distance EM corrections:



| Δ^{EM} | |
|---------------|-------------------|
| K^0_{e3} | +0.50(11)% |
| K^+_{e3} | +0.05(13)% |
| $K^0_{\mu 3}$ | +0.70(11)% |
| $K^+_{\mu 3}$ | +0.01(13)% |

- ChPT to $O(e^2 p^2)$
- Use fully inclusive prescription for real photons
- Uncertainty estimate: LECs (100%) + higher chiral orders

V. Cirigliano, M. Giannotti, H. Neufeld:
JHEP 0811:006 (2008)

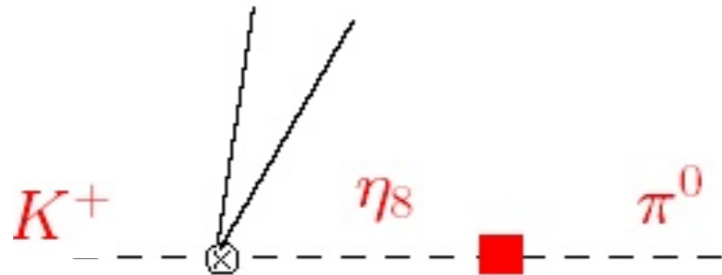
Anantharayan-Moussallam 2004,
Descotes-Jenon Moussallam 2005

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$$\Delta_{SU(2)}^K \equiv \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1$$



+ IB in 1-loop graphs + CT

V_{us} from $K \rightarrow \pi \ell \nu$ decays

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ChPT to $O(p^4)$ relates this to ratios of the light quark masses:

$$\Delta_{SU(2)}^K = \frac{3}{4} \frac{1}{Q^2} \left[\frac{m_K^2}{m_\pi^2} + \frac{\chi_{p^4}}{2} \left(1 + \frac{m_s}{\hat{m}} \right) \right]$$

$$Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

0.219
(calculable chiral corr.)

$$\hat{m} = \frac{m_u + m_d}{2}$$

V_{us} from $K \rightarrow \pi l \nu$ decays

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- Predict $\Delta_{SU(2)}$ from quark mass ratios:

$$m_s/\hat{m} = 24.7 \pm 1.1,$$

$$Q = 20.7 \pm 1.2$$

$$\Delta_{SU(2)} = (2.9 \pm 0.4) \%$$

A. Kastner, H. Neufeld, 2008

$$m_s/\hat{m} = 27.4 \pm 0.4$$

$$Q = 22.8 \pm 1.2$$

$$\Delta_{SU(2)} = (2.4 \pm 0.3) \%$$

FLAG 2010

V_{us} from $K \rightarrow \pi \ell \nu$ decays

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FLAG 2010

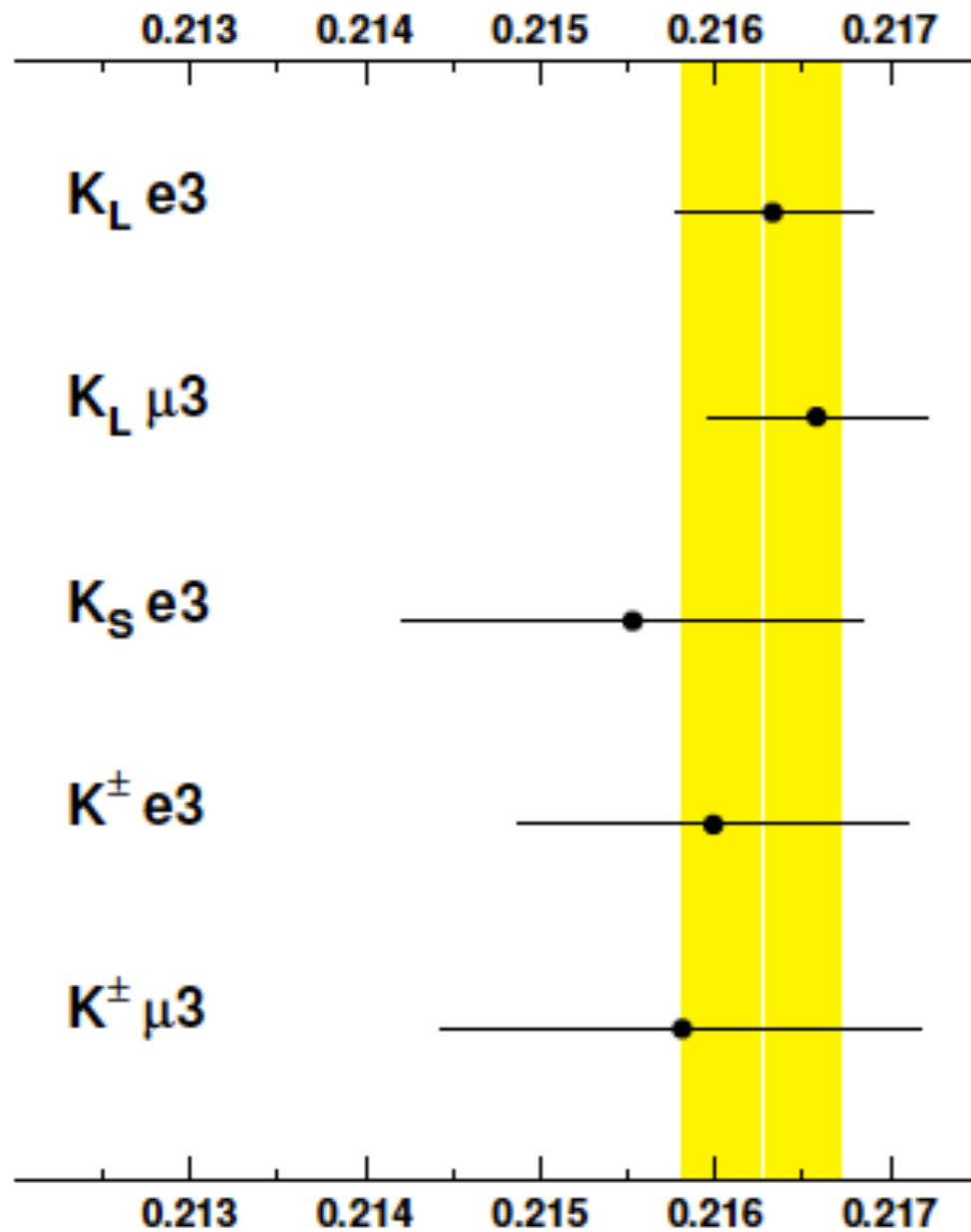
- Extract $\Delta_{SU(2)}$ from data + EM corr: constraint on quark masses

$$\Delta_{SU(2)}^K = \frac{\Gamma_{K_{\ell 3}^+}}{\Gamma_{K_{\ell 3}^0}} \cdot \frac{I^{K^0\ell}}{I^{K^+\ell}} \left(\frac{M_{K^0}}{M_{K^+}} \right)^5 - \frac{1}{2} - [\Delta_{EM}^{K^+\ell} - \Delta_{EM}^{K^0\ell}] \longrightarrow (2.7 \pm 0.4) \%$$

~ 0.15% from TH

V_{us} from $K \rightarrow \pi l \nu$ decays

- Experiment + rad. corr. (+ SU(2) corrections)



$$|V_{us}|f_+(0) = 0.2163 \pm 0.0005$$

0.25%

SU(3) breaking in $f_+(0)$

- CVC + Ademollo-Gatto theorem: $f_+^{K^0\pi^-}(0) - 1 = O((m_s - m_u)^2)$

$$f_+^{K^0\pi^-}(0) = 1 + \underbrace{f_{p^4}}_{O(m_q)} + \underbrace{f_{p^6}}_{O(m_q^2)} + \dots$$

chiral expansion

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chiral expansion

$$f_{p^4}$$

- One-loop graphs in EFT



- 1st order in m_q , 2nd order in $(m_s - m_u) \Rightarrow$

$$f_{p^4} \sim \frac{(m_s - m_u)^2}{m_s}$$

- No local operators, UV finite, free of uncertainty

$$f_{p^4} = -0.0227$$

SU(3) breaking in $f_+(0)$

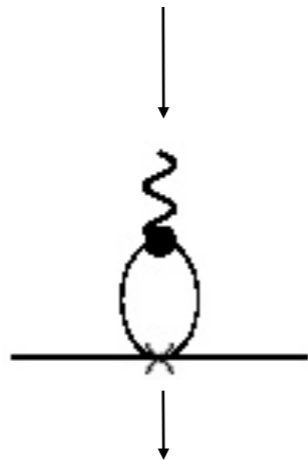
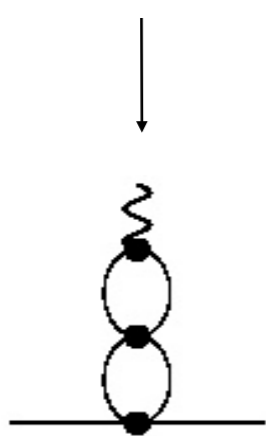
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chiral expansion

$$O(m_q) \quad O(m_q^2)$$

$$f_{p^6} = f_{p^6}^{2\text{-loops}}(\mu) + f_{p^6}^{L_i \times \text{loop}}(\mu) + f_{p^6}^{\text{tree}}(\mu)$$



$$8 \frac{(M_K^2 - M_\pi^2)^2}{F_\pi^2} \left[\frac{(L_5^r(M_\rho))^2}{F_\pi^2} - C_{12}^r(M_\rho) - C_{34}^r(M_\rho) \right]$$

LECs not fixed by chiral symmetry:
rely on quark model, large- N_c estimates, LQCD

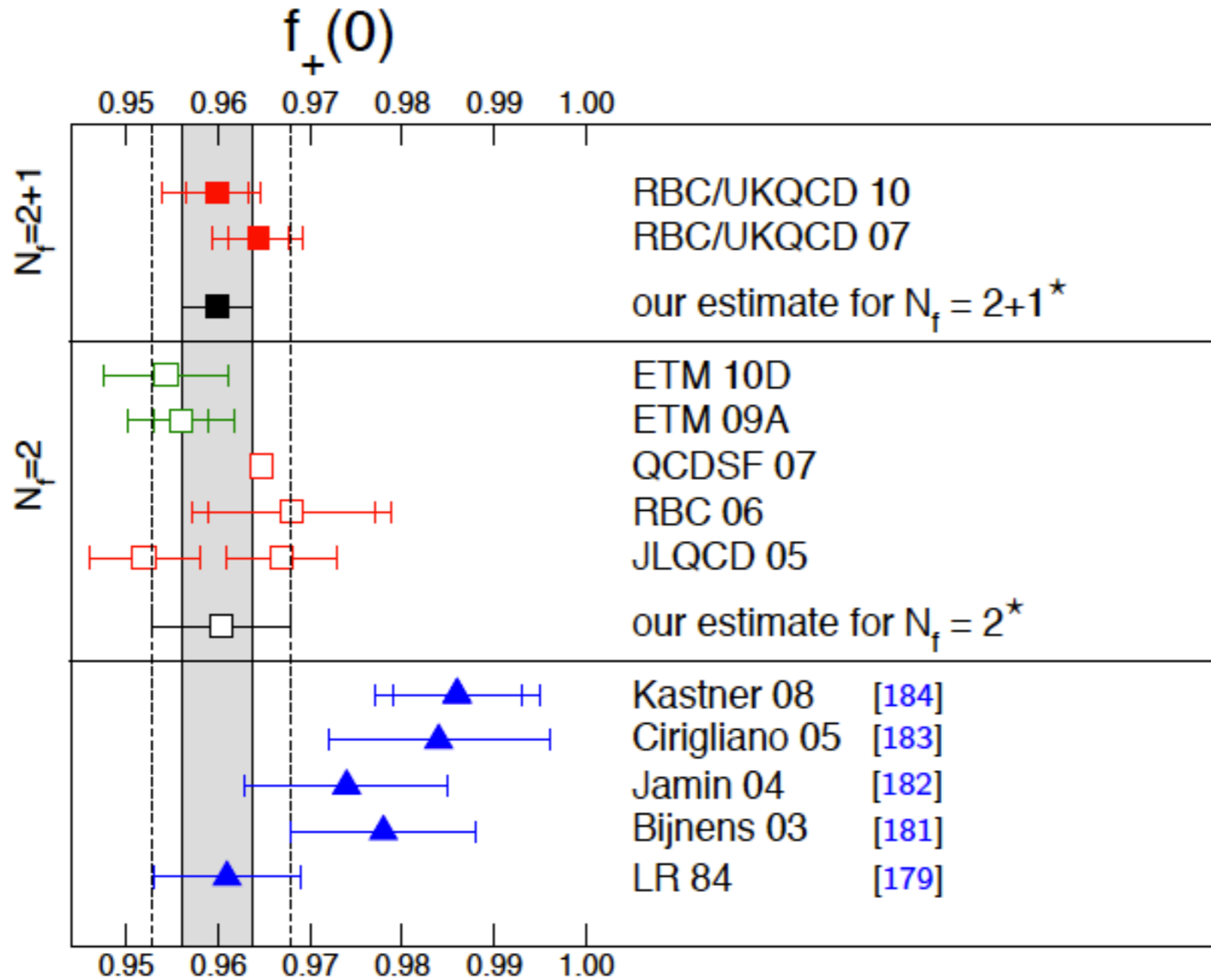
$$f_{p^6}^{L_i \times \text{loop}}(M_\rho) = -0.0020$$

$$f_{p^6}^{2\text{-loops}}(M_\rho) = 0.0113$$

Large and positive chiral loop contributions
@ $\mu = M_\rho$

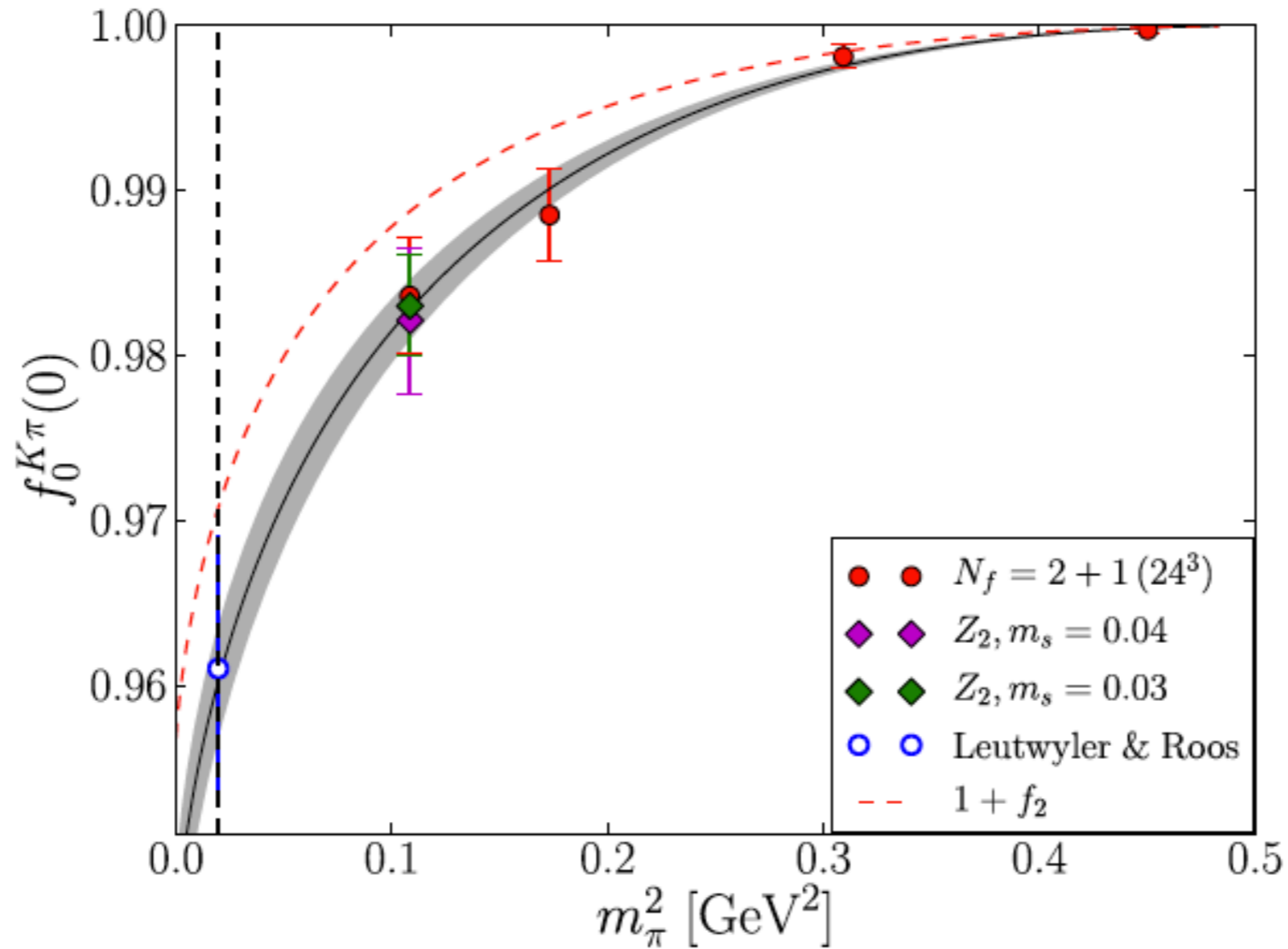
SU(3) breaking in $f_+(0)$

- Lattice QCD calculations vs ChPT + models:



SU(3) breaking in $f_+(0)$

- $N_f = 2+1$ Lattice QCD: RBC- UKQCD



RBC-UKQCD 10

$$f_+(0) = 0.959(5) \Rightarrow \boxed{V_{us} = 0.2255(13)} \quad 0.58\%$$

V_{us}/V_{ud} from $K \rightarrow \mu\nu$ / $\pi \rightarrow \mu\nu$

Marciano '04 , VC + H. Neufeld 2011

$$\frac{\Gamma_{K\ell 2(\gamma)}}{\Gamma_{\pi\ell 2(\gamma)}} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{F_K^2}{F_\pi^2} \frac{M_{K^\pm} (1 - m_\ell^2/M_{K^\pm}^2)^2}{M_{\pi^\pm} (1 - m_\ell^2/M_{\pi^\pm}^2)^2} (1 + \delta_{\text{EM}} + \delta_{\text{SU}(2)})$$

$$\frac{F_{K^\pm}^2}{F_{\pi^\pm}^2} = \frac{F_K^2}{F_\pi^2} (1 + \delta_{\text{SU}(2)})$$

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$$\frac{F_{K^\pm}^2}{F_{\pi^\pm}^2} = \frac{F_K^2}{F_\pi^2} (1 + \delta_{SU(2)})$$

- Extraction of V_{us}/V_{ud} requires:
 - F_K/F_π : lattice QCD in the isospin limit + isospin-breaking corrections (ChPT)
 - Long-distance radiative corrections: $\delta_{EM} = -0.0069(17)$
 - To $O(e^2p^2)$ in ChPT they are UV finite, no LECs.
Uncertainty due to higher order corrections

Knecht et al '99 , VC- Neufeld 2011

SU(2) breaking in F_K/F_π

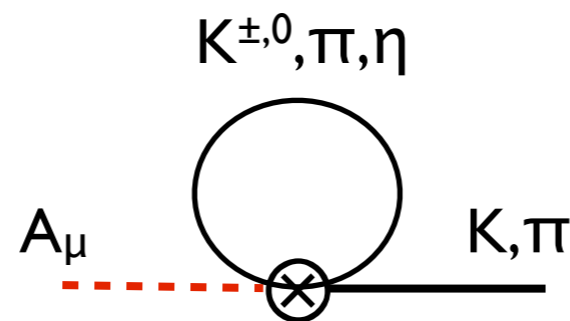
- SU(2) breaking to $O(p^4)$ in ChPT: $\frac{F_{K^\pm}^2}{F_{\pi^\pm}^2} = \frac{F_K^2}{F_\pi^2} (1 + \delta_{\text{SU}(2)})$

$$\delta_{\text{SU}(2)} = \sqrt{3} \varepsilon \left[-\frac{4}{3} (F_K/F_\pi - 1) + \frac{1}{3(4\pi)^2 F_0^2} \left(M_K^2 - M_\pi^2 - M_\pi^2 \ln \frac{M_K^2}{M_\pi^2} \right) \right]$$

Gasser-Leutwyler 1985 → VC-Neufeld 2011

Meson masses in the isospin limit

$$\varepsilon = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \hat{m}}$$



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$$\varepsilon = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \hat{m}}$$

$$\varepsilon = \frac{\sqrt{3}}{4R} = 0.0116(13), \quad F_K/F_\pi = 1.193(6)$$

FLAG 2010

$$\delta_{\text{SU}(2)} = -0.0043 (5)_{\text{input}} (11)_{\text{higher order}}$$

SU(2) breaking in F_K/F_π

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FLAG 2010

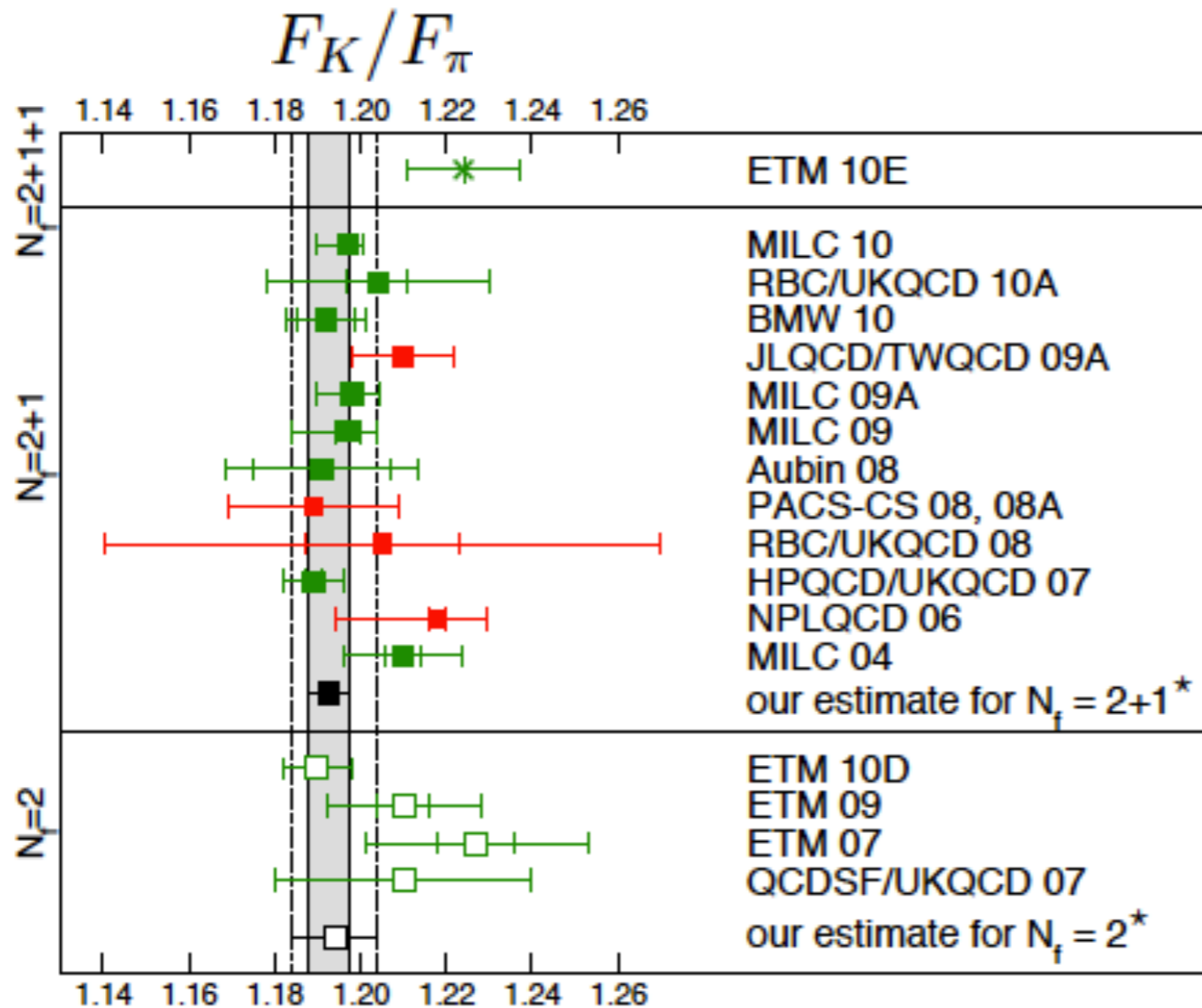
$$\delta_{\text{SU}(2)} = -0.0043 (5)_{\text{input}} (11)_{\text{higher order}}$$

- SU(2) breaking from $N_f=2$ (ETM) lattice QCD

RM123 collaboration 2011

$$\delta_{\text{SU}(2)} = -0.0078 (6)_{\text{lattice}} (4)_{\text{Kaon EM mass splitting}}$$

F_K/F_π from lattice QCD

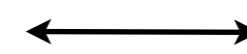


$$F_K/F_\pi = 1.193 \pm 0.006$$

FLAG 2010

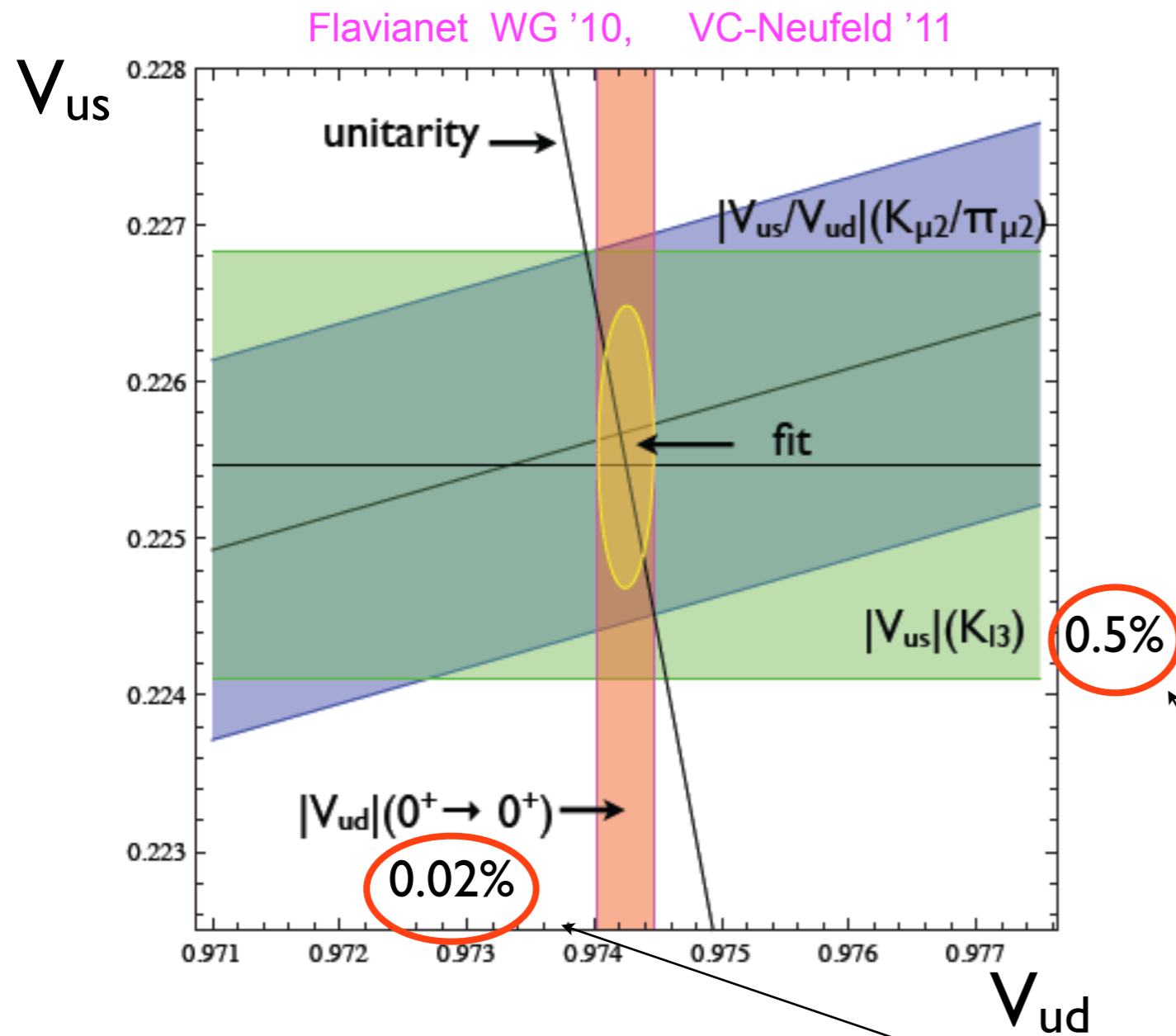
- Putting pieces together

$$V_{us}/V_{ud} = 0.2316(12)$$



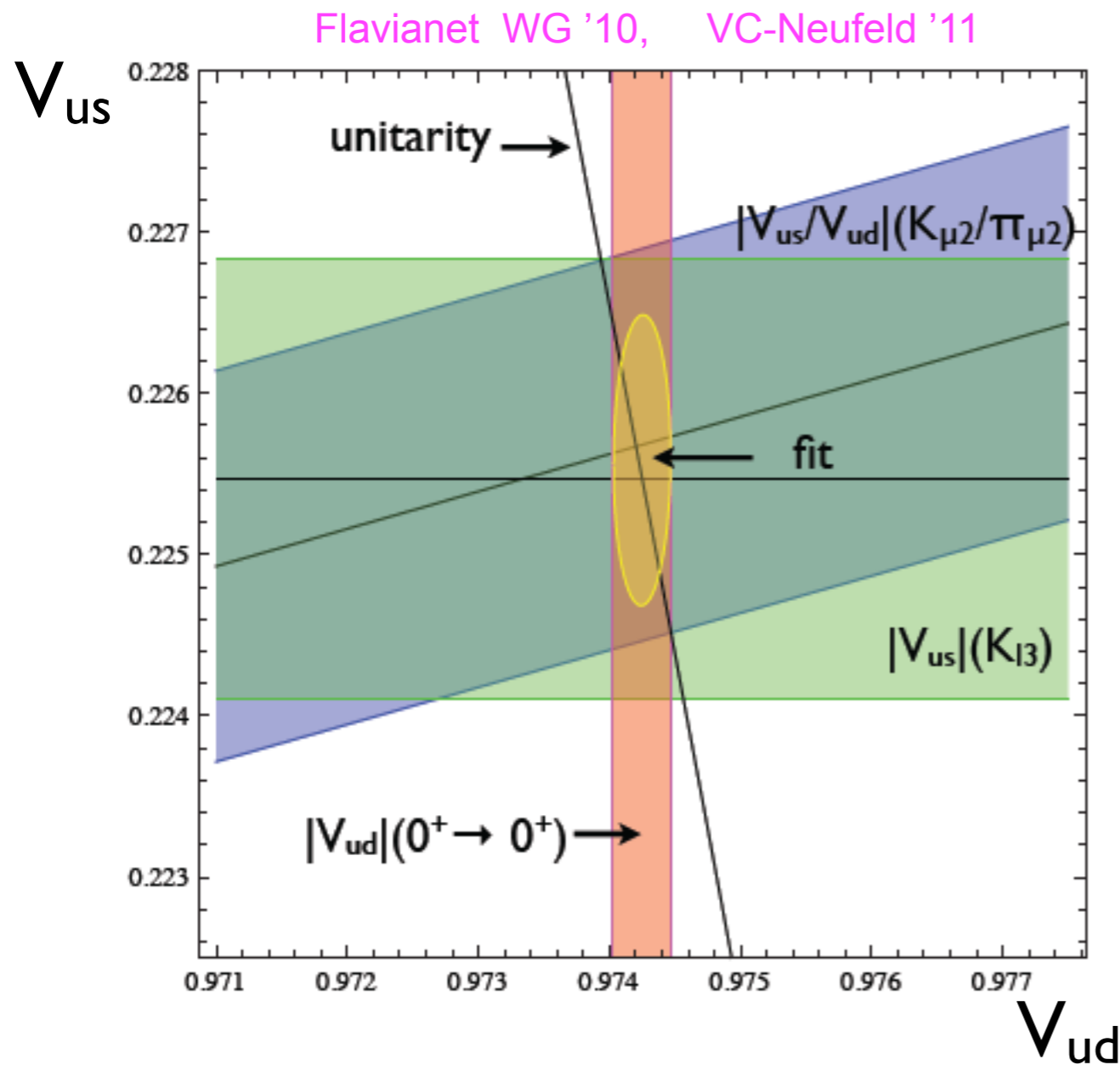
0.52%

Summary on V_{ud} and V_{us}



Theory dominated errors

Summary on V_{ud} and V_{us}



Fit result

$$V_{ud} = 0.97425 (22)$$

$$V_{us} = 0.2256 (9)$$



$$\Delta_{CKM} = (1 \pm 6) * 10^{-4}$$

Error equally shared between V_{ud} and V_{us}

- Great success of the SM: confirms large EW radiative corrections, would naively fit $M_Z = (90 \pm 7) \text{ GeV} !!$ [from S_{ew}]

Marciano

Implications for BSM physics

$$\frac{1 - |V_{uD}|^2}{1 - |U_{\mu N}|^2} \cdot \frac{1}{1 + BR_{\text{exotic}}^\mu} \cdot \frac{[G_F^{(\beta)}]^2}{[G_F^{(\mu)}]^2} = 1 + \Delta_{CKM}$$

Heavy fermion
mixing

Exotic
muon decays

Gauge
universality
violations

$$|V_{uD}| \leq 0.03$$

$$|U_{\mu N}| \leq 0.03$$

95% C.L.

$$BR_{\text{exotic}}^\mu < 0.001$$

95% C.L.

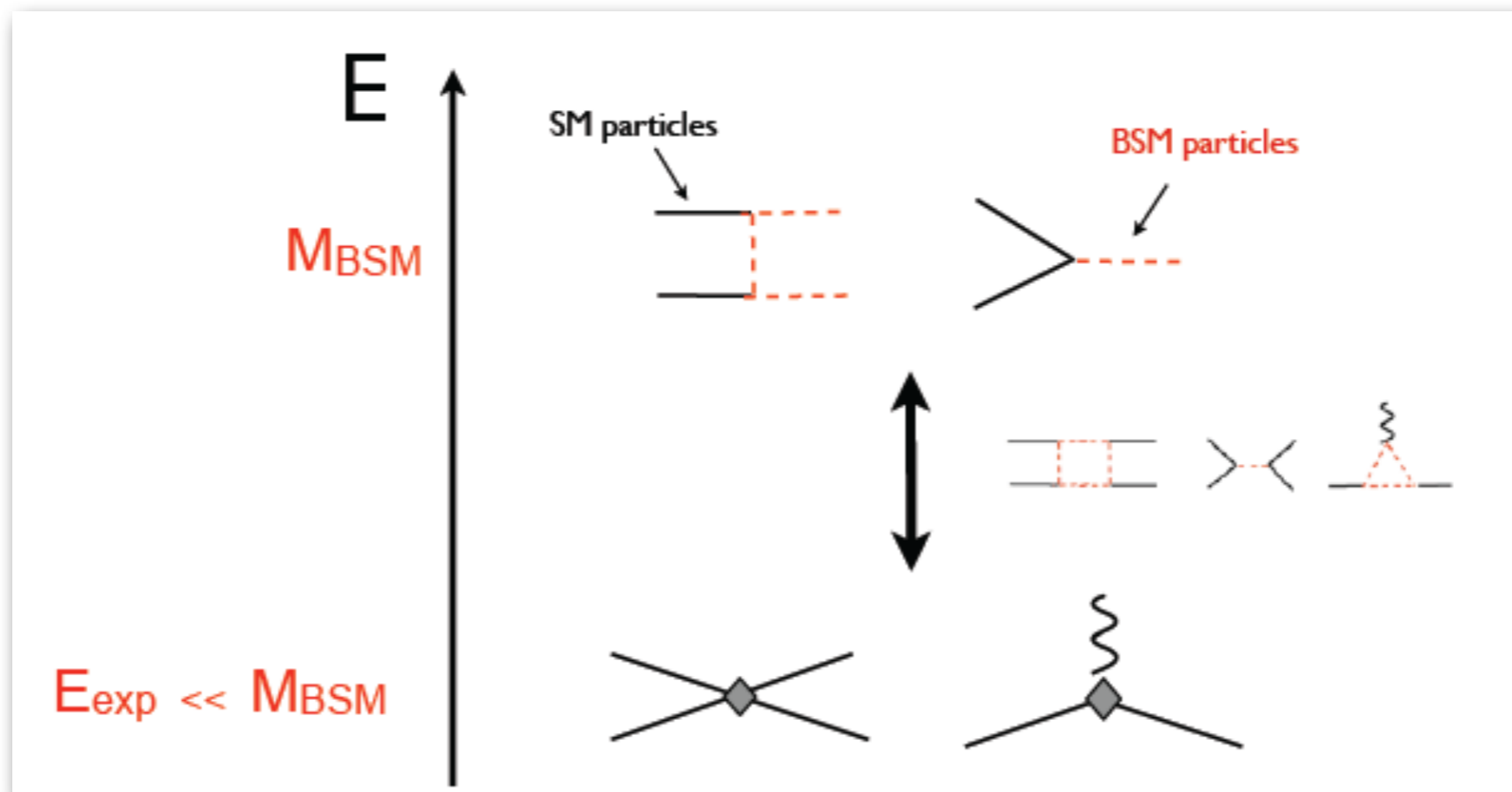
Stronger than direct limits

$$BR(\mu^+ \rightarrow e^+ \bar{\nu}_e \nu_\mu) < 0.012$$

Constraints on TeV
scale SM extensions

Model independent analysis

- Setup: integrate out heavy BSM degrees of freedom. Parameterize BSM interactions via $SU(2) \times U(1)$ gauge-invariant non-renormalizable operators built out of SM fields



Weinberg '79, Buchmuller-Wyler 1986, Han-Skiba 2004

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_X \frac{1}{\Lambda_X^2} O_X$$

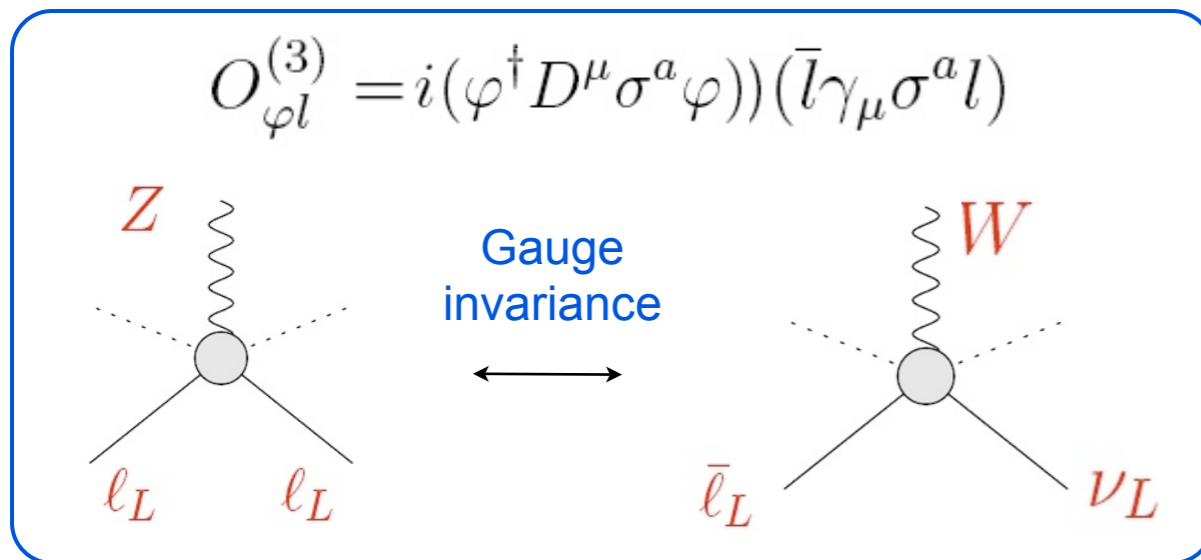
stop at dim=6

- Δ_{CKM} is sensitive to four operators**:

$$\Delta_{\text{CKM}} = 4 \left(\hat{\alpha}_{ll}^{(3)} - \hat{\alpha}_{lq}^{(3)} - \hat{\alpha}_{\varphi l}^{(3)} + \hat{\alpha}_{\varphi q}^{(3)} \right) \quad \hat{\alpha}_X = \frac{v^2}{\Lambda_X^2}$$

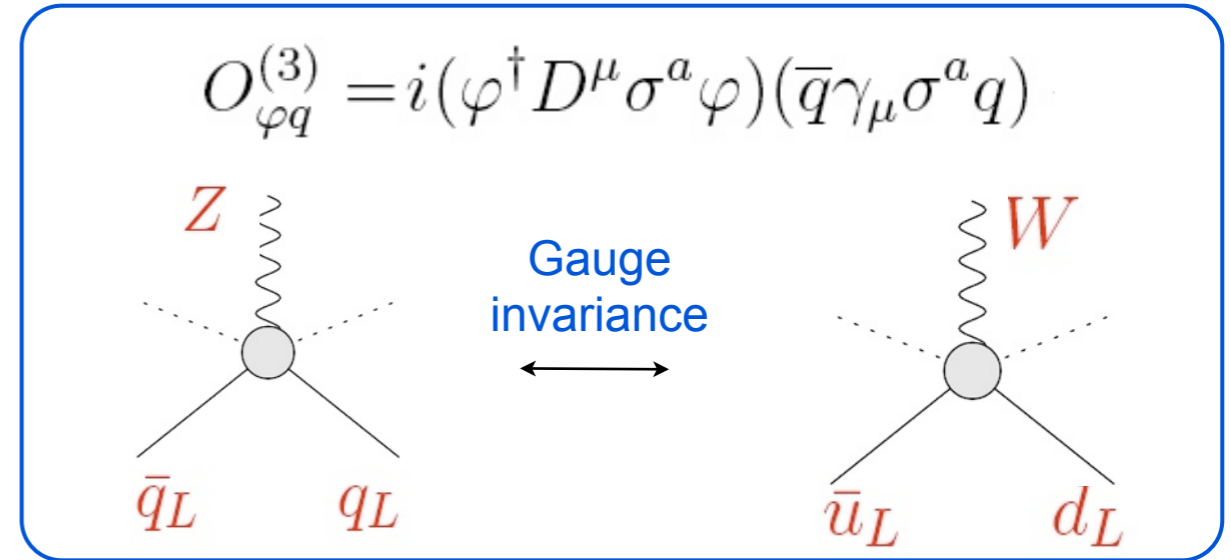
$v \sim 200 \text{ GeV}$

Vertex corrections



$$l^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$$

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$



$$q^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$$

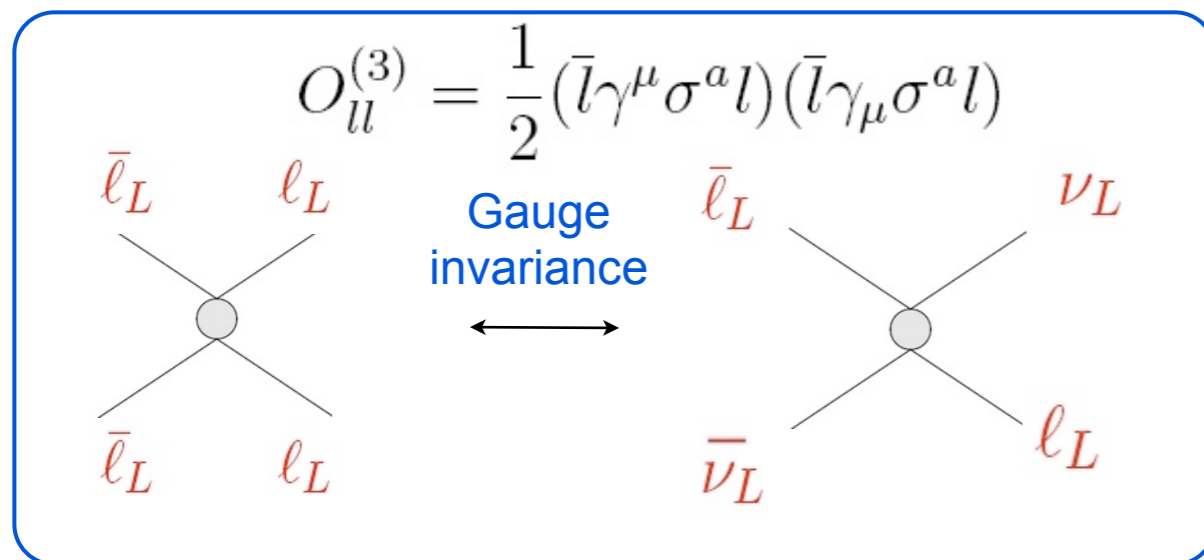
** Consider only $U(3)^5$ -invariant operators \Rightarrow no problems with FCNC.

- Δ_{CKM} is sensitive to four operators:

$$\Delta_{\text{CKM}} = 4 \left(\hat{\alpha}_{ll}^{(3)} - \hat{\alpha}_{lq}^{(3)} - \hat{\alpha}_{\varphi l}^{(3)} + \hat{\alpha}_{\varphi q}^{(3)} \right) \quad \hat{\alpha}_X = \frac{v^2}{\Lambda^2}$$

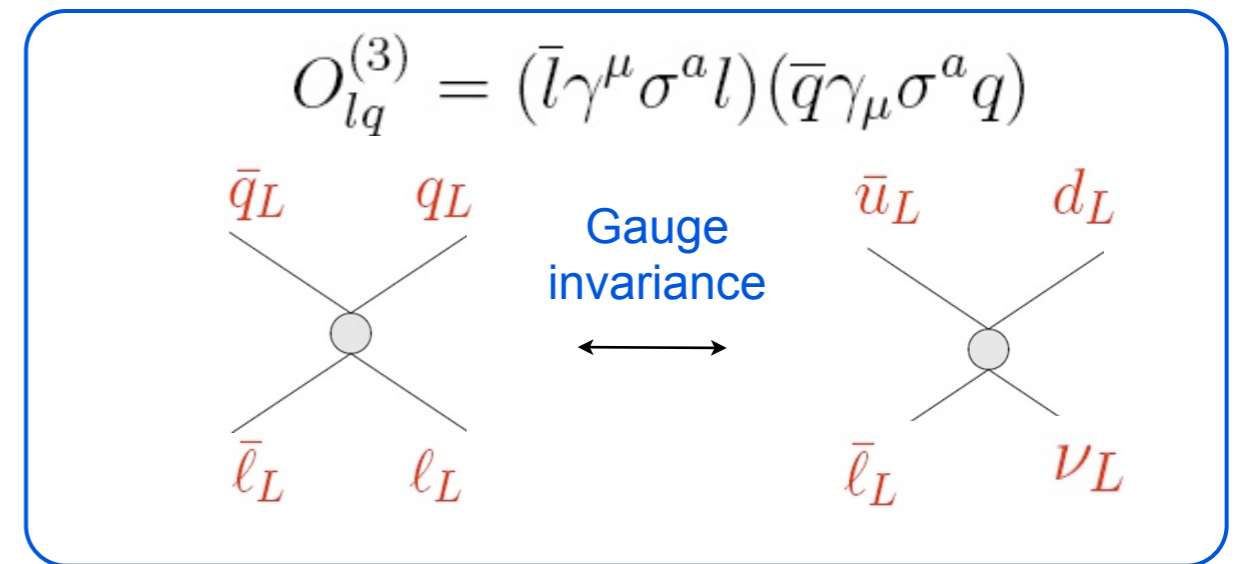
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4-fermion operators



$$l^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$$

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$



$$q^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$$

- Relevant operators affect other precision EW observables!

- In this framework, we can assess the significance of Δ_{CKM} constraint vs other EW precision observables in a model-independent way

I) What is the range of Δ_{CKM} allowed by precision EW tests?

$$-9.5 \times 10^{-3} \leq \Delta_{\text{CKM}} \leq 0.1 \times 10^{-3} \quad 90\% \text{ C.L.}$$



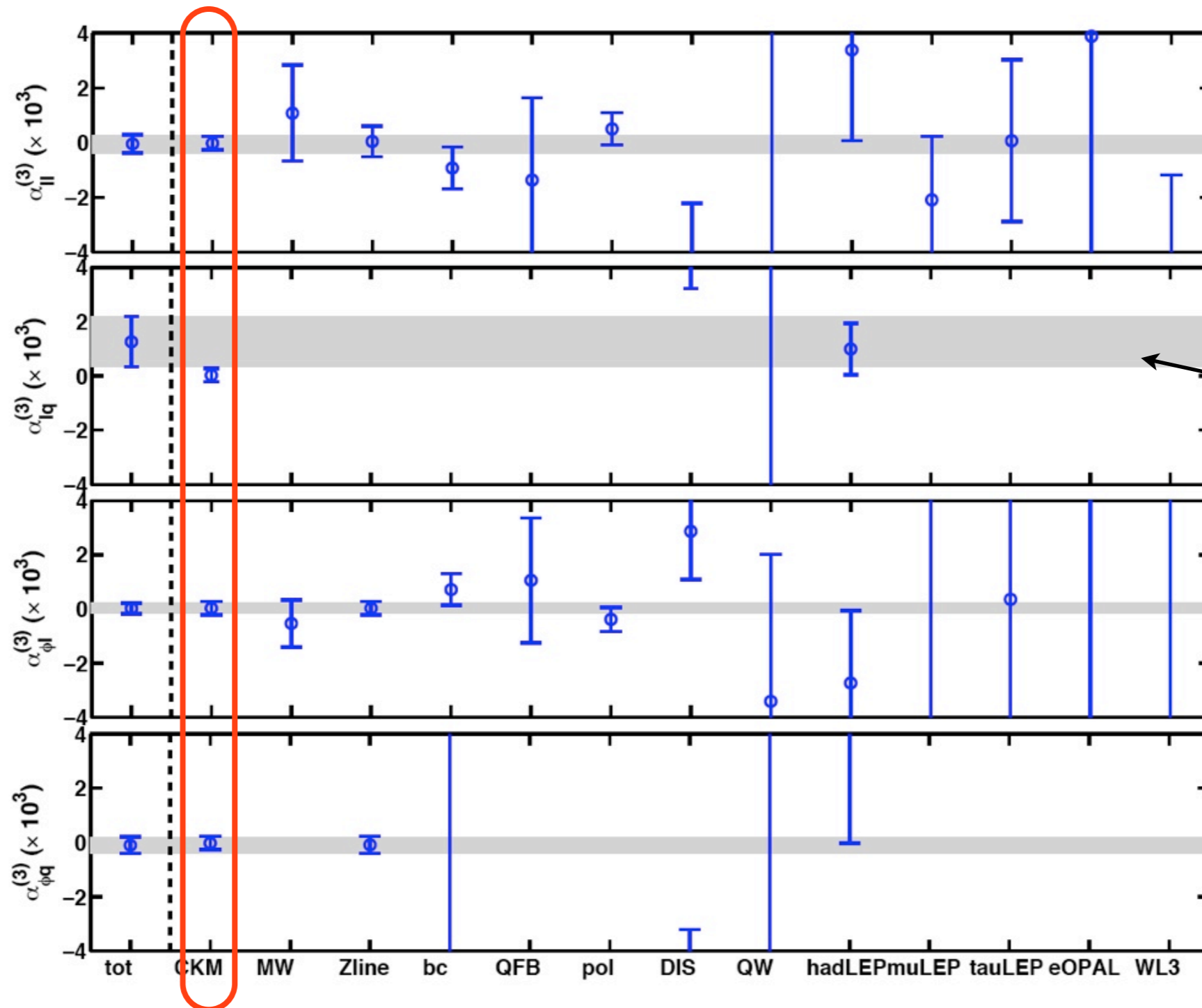
EW precision data alone would leave room for large Δ_{CKM} !

Direct constraint implies $|\Delta_{\text{CKM}}| \leq 1. \times 10^{-3}$ @ 90% CL

- In this framework, we can assess the significance of Δ_{CKM} constraint vs other EW precision observables in a model-independent way

2) What is the strength of Δ_{CKM} constraint? Same level or better than Z-pole observables (effective scale $\Lambda > 11 \text{ TeV @ 90\% CL}$)

$$\hat{\alpha}_X = \frac{v^2}{\Lambda_X^2}$$



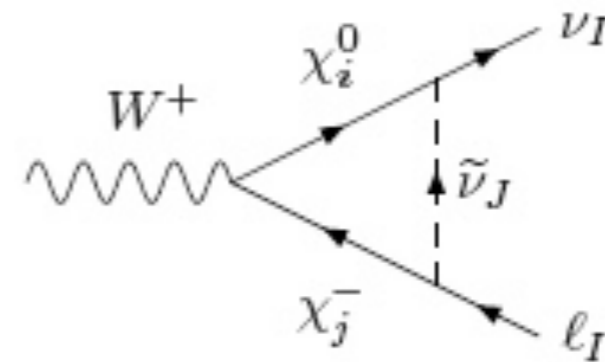
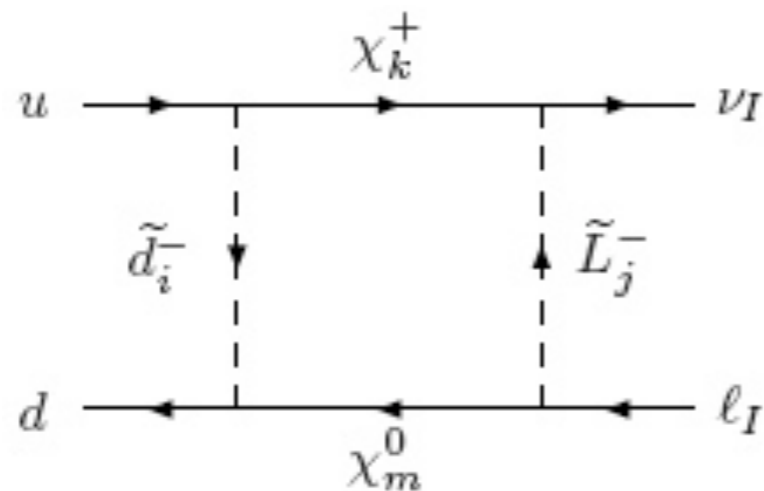
$O_{lq}^{(3)}$

Dramatic improvement over LEP2 and APV

Deviations as large as $\Delta_{\text{CKM}} \sim 0.01$ could be blamed on this operator

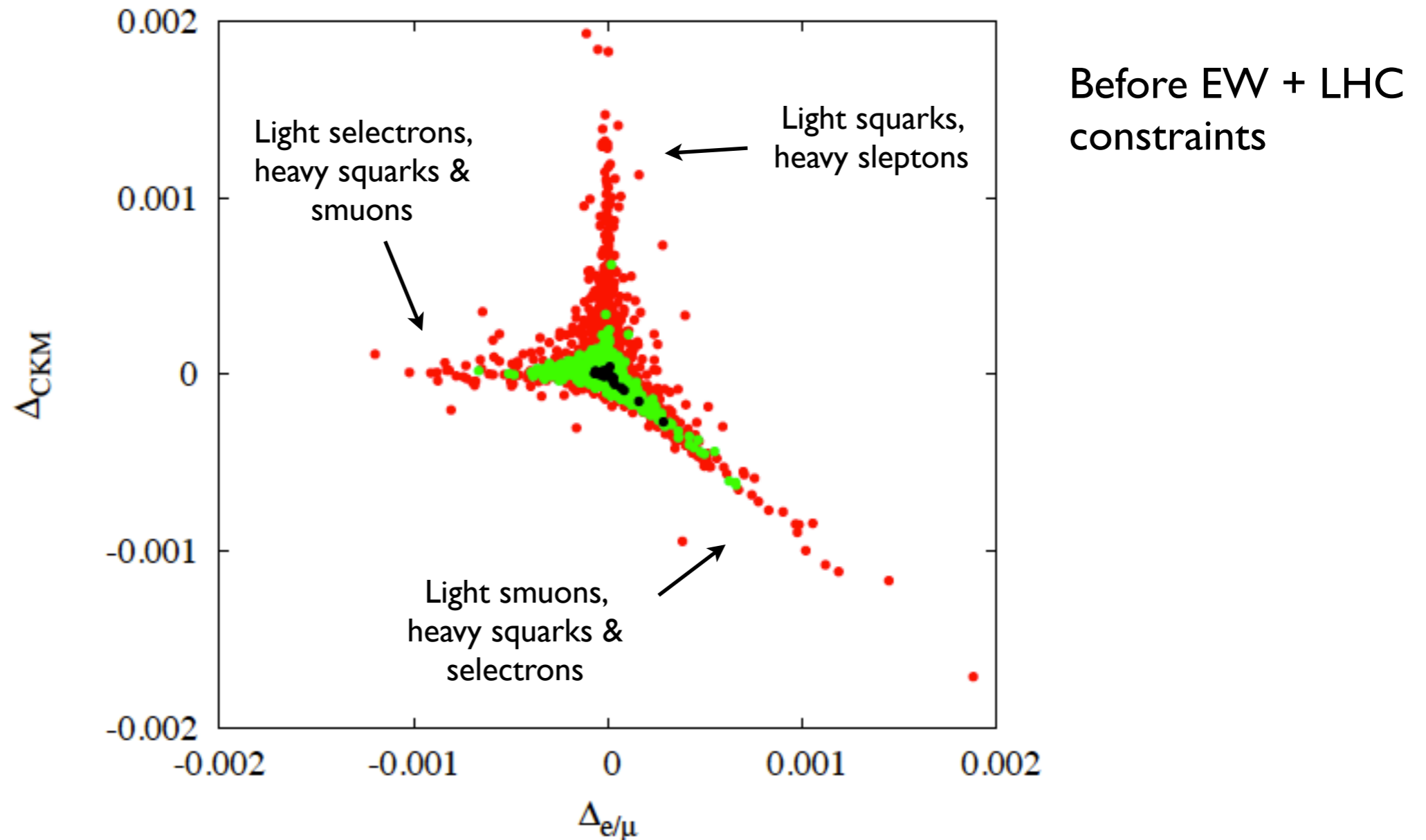
Universality and SUSY

- MSSM: box and vertex corrections induce non-universal corrections to the V-A CC operators
- S,P,T operators suppressed by insertions of Yukawa couplings



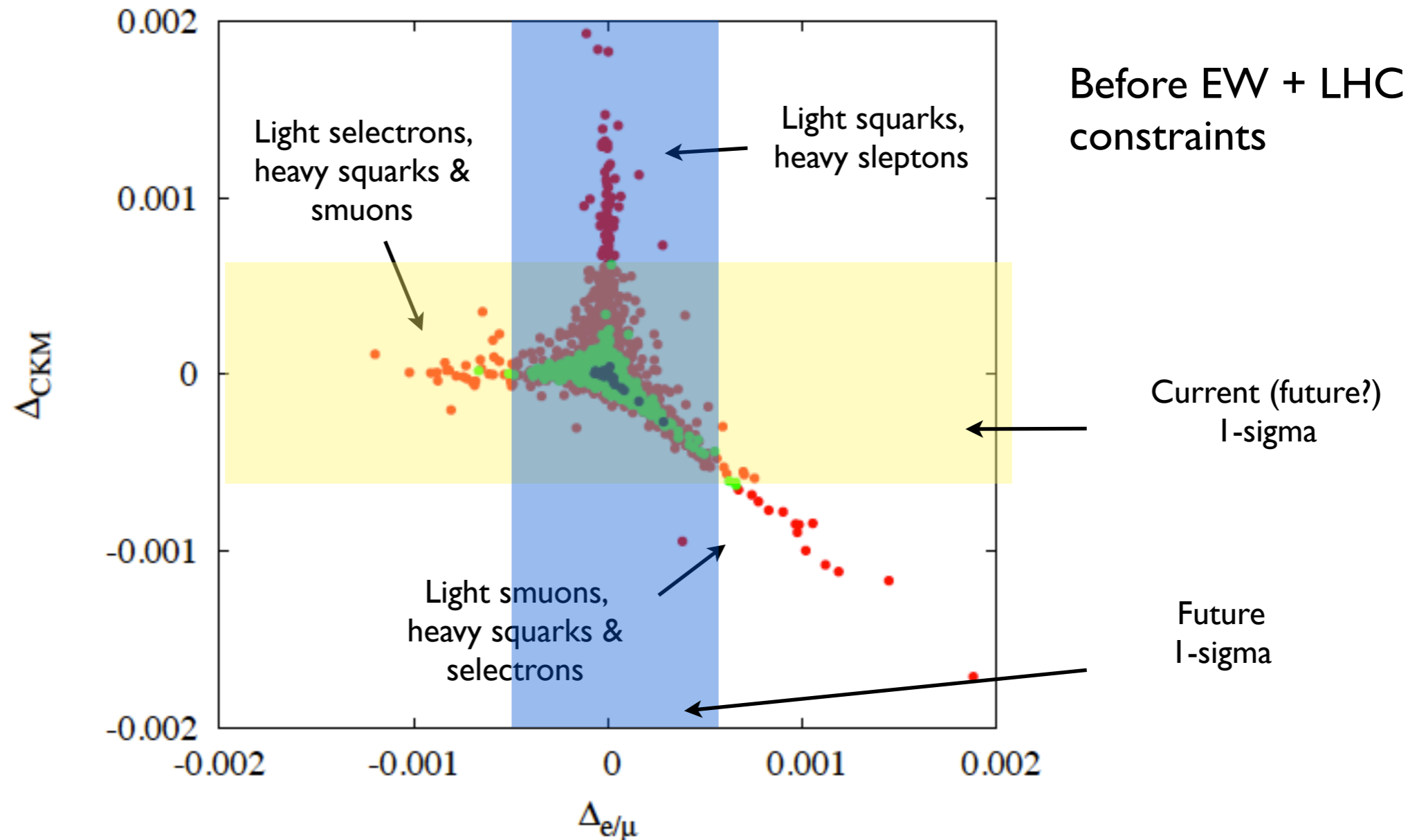
Barbieri et al '85
Hagiwara-Matsumoto-Yamada '95
Ramsey-Musolf Kurylov '01
Bauman, Erler, Ramsey-Musolf 2012

Universality and SUSY



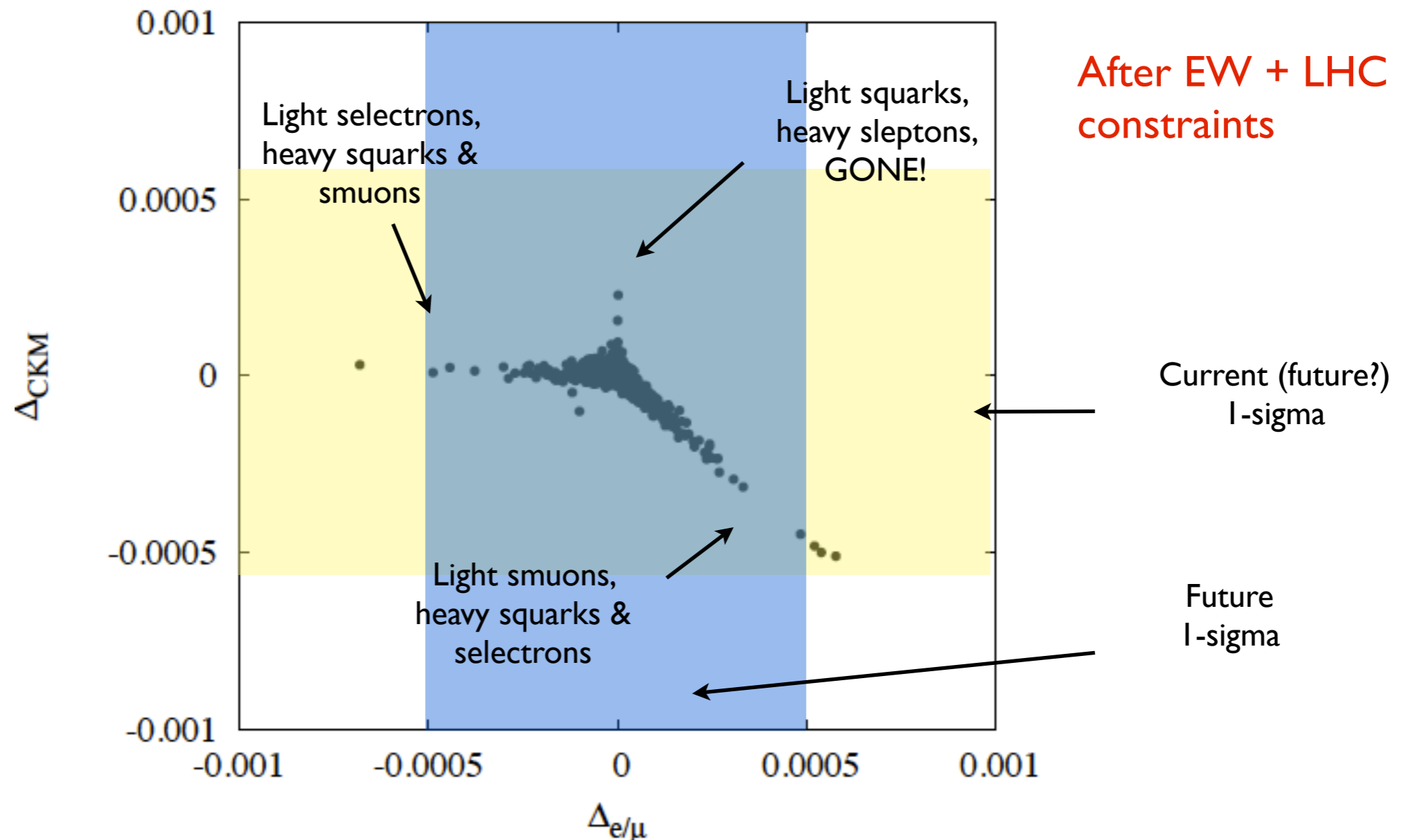
- Interesting correlation between Cabibbo universality and lepton universality: information on sfermion spectrum
- Essentially squark-slepton and selectron-smuon universality

Universality and SUSY



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Universality and SUSY



- Effects in the MSSM are small.
- Probing MSSM parameter space requires improved precision

Summary

- CC universality tests are a “broad band” probe of new physics
- Th. developments in extraction of V_{us} since Chiral 2009
 - Kl3: new lattice calculation of $f_+(0)$ [RBC-UKQCD]
 - Kl2: isospin breaking in F_K/F_π
- BSM physics reach: $\Delta_{CKM} = (1 \pm 6) * 10^{-4}$
 - Model independent EFT analysis: Cabibbo universality probes effective scale $\Lambda \sim 11 \text{ TeV}$
 - SUSY: probe squark, slepton spectrum. Effects of few 10^{-4}

Extra Slides

V_{ud} from $0^+ \rightarrow 0^+$ nuclear β decays

$$\frac{1}{t} = \frac{G_{\mu}^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} f(Q) (1 + RC) \longrightarrow ft (1 + RC) = \frac{2984.48(5) \text{ s}}{|V_{ud}|^2}$$

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$$(1 + RC) = (1 - \delta_C) (1 + \delta_R) (1 + \Delta_C)$$

$$\langle f | \tau_+ | i \rangle = \sqrt{2} (1 - \delta_C/2)$$

Coulomb distortion
of wave-functions

$$\delta_C \sim 0.5\%$$

Towner-Hardy
Ormand-Brown

Nucleus-dependent
rad. corr.

(Z, E^{\max} , nuclear structure)

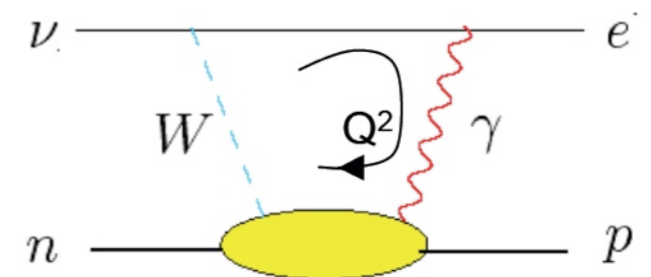
$$\delta_R \sim 1.5\%$$

Sirlin-Zucchini '86
Jaus-Rasche '87

Nucleus-independent
short distance rad. corr.

$$\Delta_R \sim 2.4\%$$

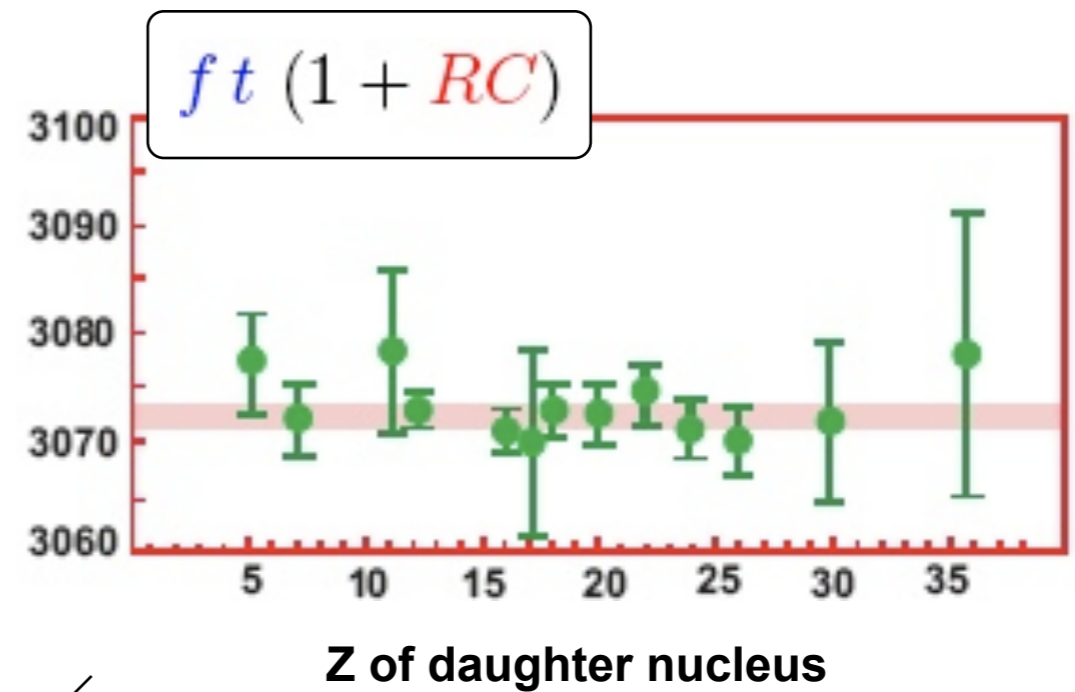
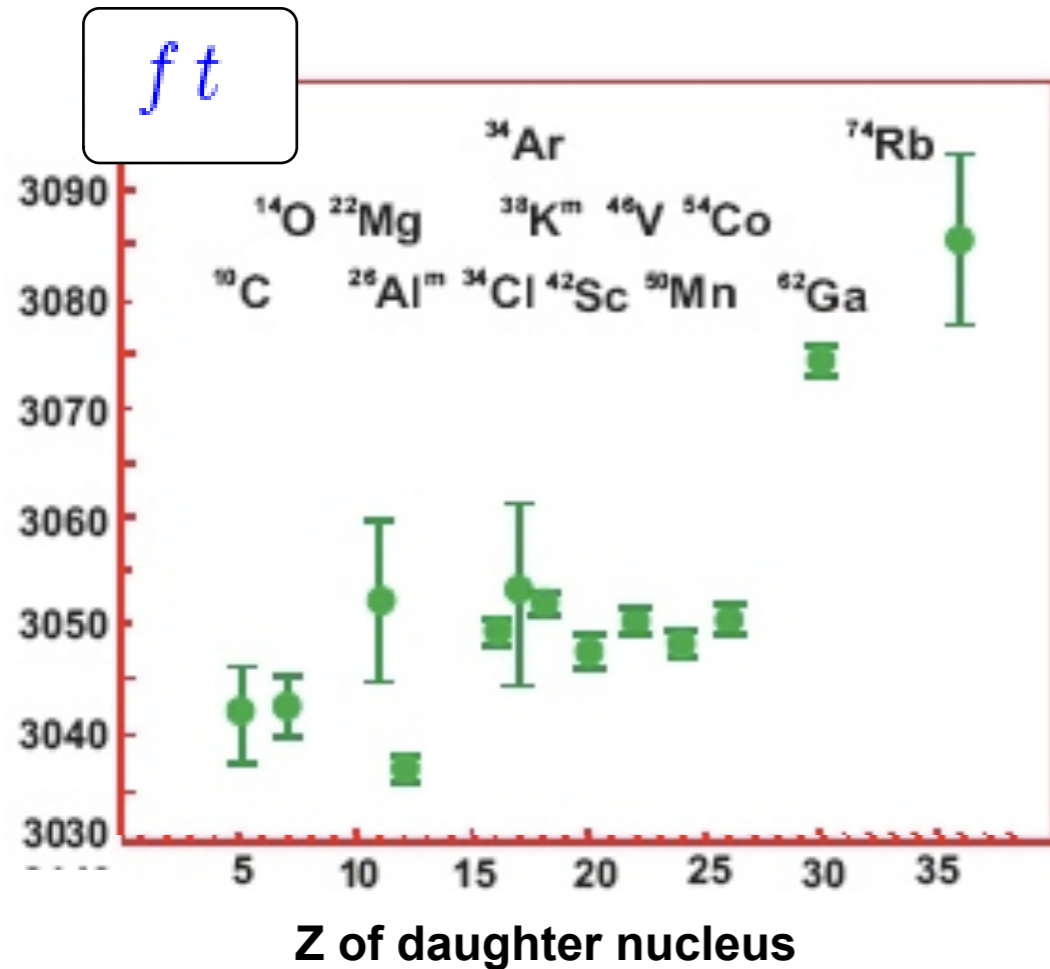
Marciano-Sirlin '06



V_{ud} from $0^+ \rightarrow 0^+$ nuclear β decays

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Towner-Hardy, Sirlin-Zucchini, Marciano-Sirlin



$$V_{ud} = 0.97425 (22)$$

Towner-Hardy 2009

Isospin symmetry limit

- $m_u = m_d$, $e = 0$
- Choice for the values of isospin symmetric meson masses

$$M_\pi^2 = M_{\pi^0}^2$$

$$M_K^2 = \frac{1}{2} (M_{K^\pm}^2 + M_{K^0}^2 - M_{\pi^\pm}^2 + M_{\pi^0}^2)$$

Subtraction of higher order EM effects produces tiny changes in $\delta_{\text{SU}(2)}$

- To leading order:

$$M_\pi^2 = 2B_0\hat{m}, \quad M_K^2 = B_0(m_s + \hat{m}).$$

$$\hat{m} = \frac{1}{2}(m_u + m_d)$$

F_K, F_π : one loop results

$$F_{\pi^\pm} = F_0 \left\{ 1 + \frac{4}{F_0^2} \left[L_4^r(\mu)(M_\pi^2 + 2M_K^2) + L_5^r(\mu)M_\pi^2 \right] - \frac{1}{2(4\pi)^2 F_0^2} \left[2M_\pi^2 \ln \frac{M_\pi^2}{\mu^2} + M_K^2 \ln \frac{M_K^2}{\mu^2} \right] \right\}, \quad \varepsilon = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \hat{m}}$$

$$F_{K^\pm} = F_0 \left\{ 1 + \frac{4}{F_0^2} \left[L_4^r(\mu)(M_\pi^2 + 2M_K^2) + L_5^r(\mu)M_K^2 \right] - \frac{1}{8(4\pi)^2 F_0^2} \left[3M_\pi^2 \ln \frac{M_\pi^2}{\mu^2} + 6M_K^2 \ln \frac{M_K^2}{\mu^2} + 3M_\eta^2 \ln \frac{M_\eta^2}{\mu^2} \right] - \frac{8\sqrt{3}\varepsilon}{3F_0^2} L_5^r(\mu)(M_K^2 - M_\pi^2) - \frac{\sqrt{3}\varepsilon}{4(4\pi)^2 F_0^2} \left[M_\pi^2 \ln \frac{M_\pi^2}{\mu^2} - M_\eta^2 \ln \frac{M_\eta^2}{\mu^2} - \frac{2}{3}(M_K^2 - M_\pi^2) \left(\ln \frac{M_K^2}{\mu^2} + 1 \right) \right] \right\}.$$



$$\frac{F_{K^\pm}^2}{F_{\pi^\pm}^2} = \frac{F_K^2}{F_\pi^2} (1 + \delta_{\text{SU}(2)})$$

$$\delta_{\text{SU}(2)} = -\frac{16\sqrt{3}\varepsilon}{3F_0^2} L_5^r(\mu)(M_K^2 - M_\pi^2) - \frac{\sqrt{3}\varepsilon}{2(4\pi)^2 F_0^2} \left[M_\pi^2 \ln \frac{M_\pi^2}{\mu^2} - M_\eta^2 \ln \frac{M_\eta^2}{\mu^2} - \frac{2}{3}(M_K^2 - M_\pi^2) \left(\ln \frac{M_K^2}{\mu^2} + 1 \right) \right]$$

Neutron decay and V_{ud}

$$V_{ud} = \left[\frac{4908.7(1.9) \text{ s}}{\tau_n (1 + 3g_A^2)} \right]^{1/2}$$

\leftarrow Theory input (rad. corr.) under control
 Czarnecki, Marciano, Sirlin 2004
 \leftarrow $(1-2\epsilon_R)*g_A$

- Experimental input not settled yet
- Competitive V_{ud} extraction requires:

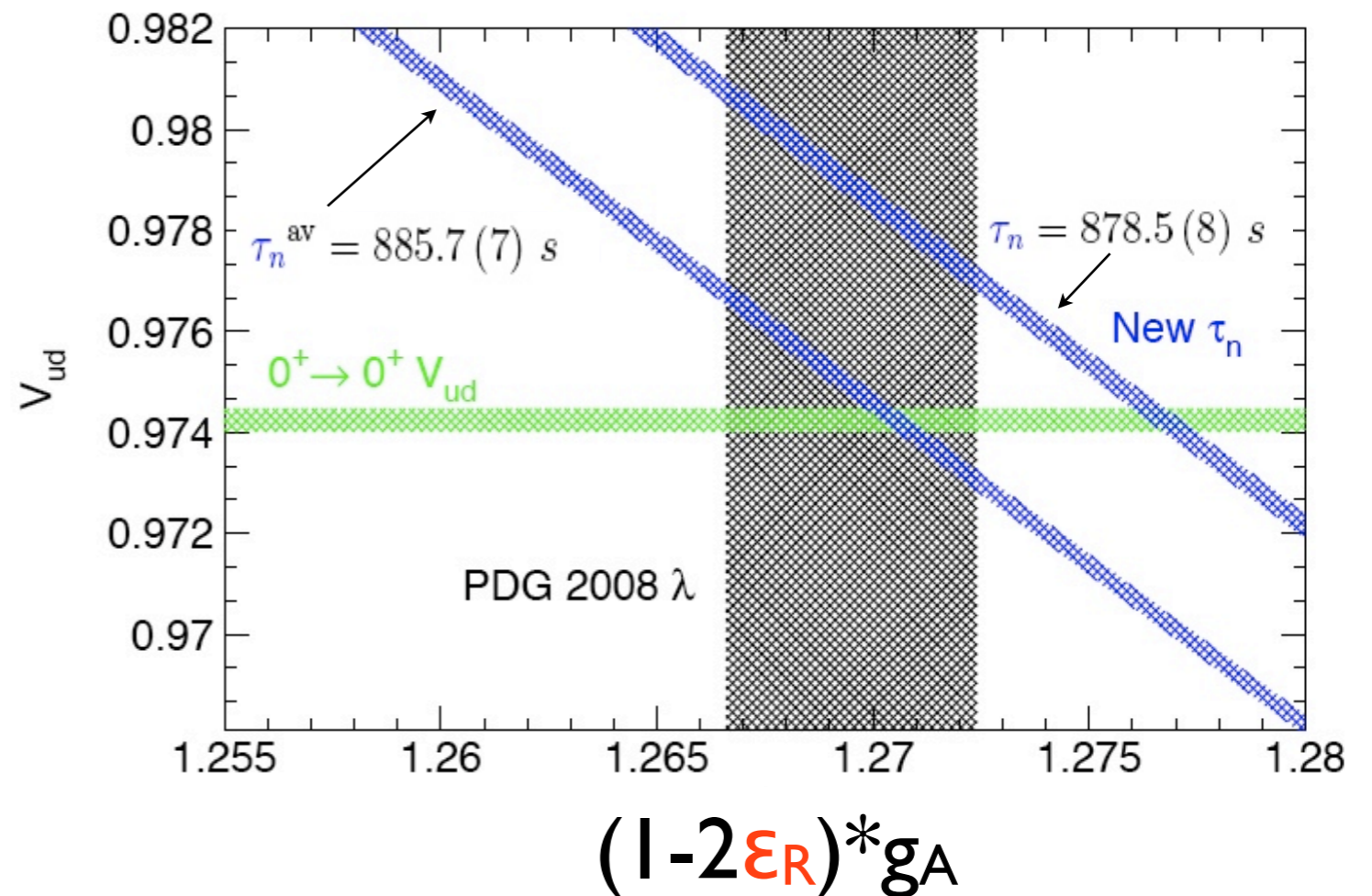
$$\delta g_A / g_A \sim 0.025\%$$

$$(\delta A / A \sim 0.1\%)$$

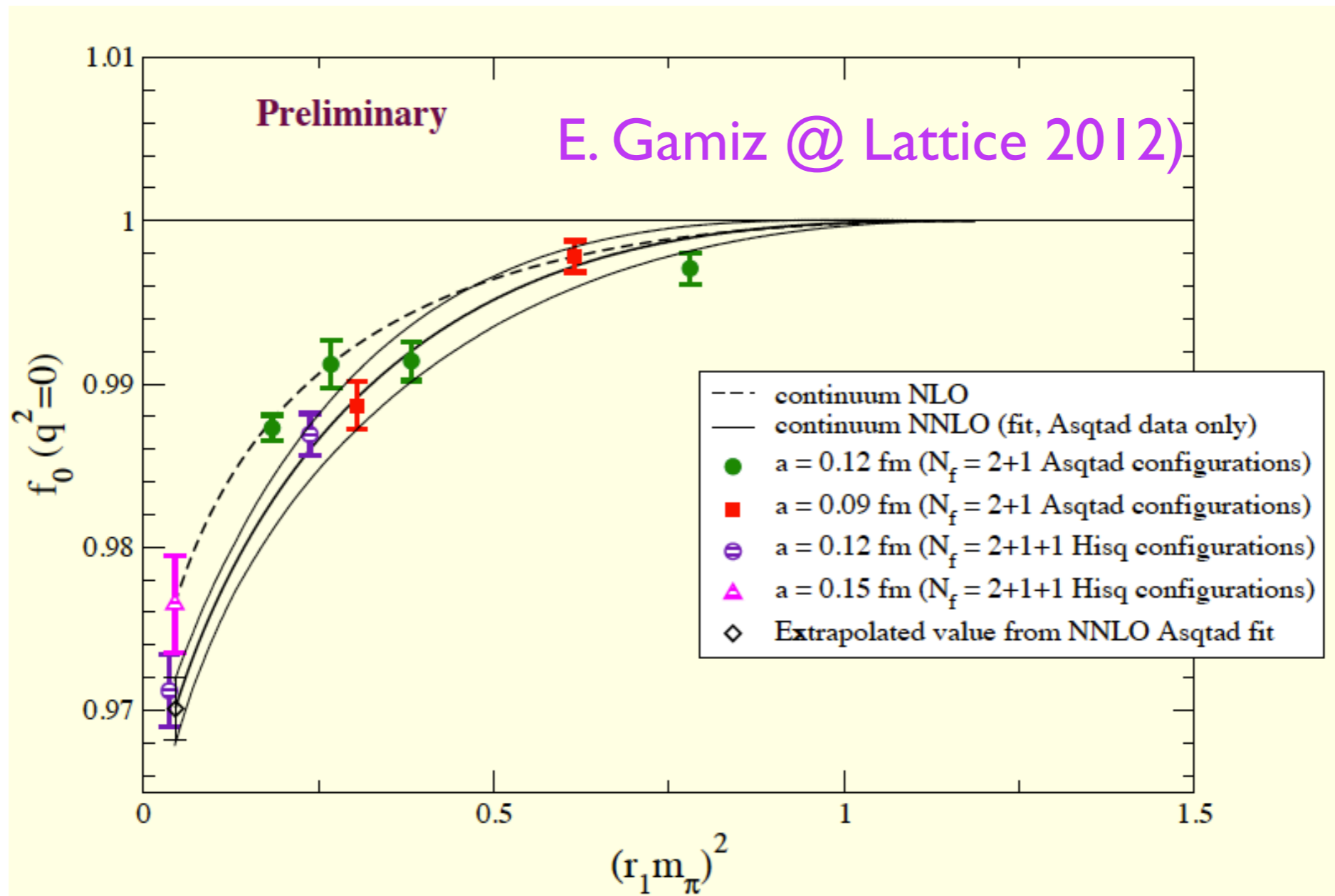
$$\delta \tau_n \sim 0.35 \text{ s}$$

$$\delta \tau_n / \tau_n \sim 0.04 \%$$

Brad Plaster, CKM 2008



- $N_f = 2+1$ Lattice QCD: MILC preliminary



- Best fit central value $f_+(0) = 0.97$, error estimate 0.35-0.5% [no volume extrapolation yet]