Recent Progress in Staggered Chiral Perturbation Theory

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1. Project: 1998 – Present
2. Staggered Fermion Formulation
   - Staggered Fermion Formulation
3. SChPT
   - Staggered Chiral Perturbation Theory
4. Appilcation
   - Pion Mass
   - Pion Decay Constants
   - B_K
   - $\pi - \pi$ Scattering Phase Shift
5. Summary and Conclusion

SWME Collaboration
1998 — Present

SWME Collaboration

- Seoul National University (SNU):
  Prof. Weonjong Lee
  Dr. Jon Bailey and Dr. Nigel Cundy (RA Prof.)
  11 graduate students.
- Brookhaven National Laboratory (BNL):
  Dr. Chulwoo Jung
  Dr. Hyung-Jin Kim (Postdoc)
- University of Washington, Seattle (UW):
  Prof. Stephen R. Sharpe.
- KISTI: Dr. Taegil Bae (Postdoc).
- University of Arizona, Tucson: Dr. Jongjeong Kim (Postdoc).

Lattice Gauge Theory Research Center (SNU)

- Center Leader: Prof. Weonjong Lee. (***)
- Research Assistant Prof.: Dr. Jon Bailey (***)
- Research Assitant Prof.: Dr. Nigel Cundy
- 11 graduate students (***)
- Secretary: Ms. Sora Park.
- more details on `http://lgt.snu.ac.kr/`.

Group Photo

Staggered Fermion Formulation

How to put quarks on the lattice ?

- Wilson Fermions:
  1. Clover Action: (**)
  2. Twisted mass fermions: (**)
  3. Domain Wall Fermions: (***)
  4. Overlap Fermions:
- Staggered Fermions;
  1. Asqtad action: (*)
  2. HYP staggered fermions: (***)
  3. $\overline{\text{Fat7}}$ staggered fermions:
  4. HISQ action: (***)

Cons and Pros for Staggered Fermions

- Advantages (Pros):
  1. Preserve part of exact chiral symmetries.
  2. Numerically cheapest on the lattice.
  3. No residual quark mass (no additive renormalization).
  4. Easy to improve with almost no extra cost.
  5. Staggered Chiral Perturbation Theory.

- Possess 4 degenerate tastes (pure lattice artifacts).

- Disadvantages (Cons):
  1. Born with taste symmetry breaking by construction.
  2. Theoretically more challenging to interprete the data.

Staggered Chiral Perturbation Theory

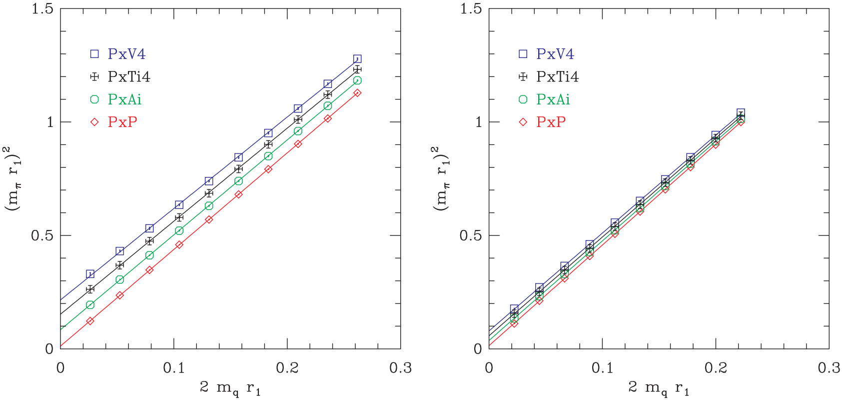
What is Staggered ChPT ?

- ChPT designed to analyze the data produced using staggered fermions.
- Dual expansion in powers of $p^2 \approx m_q$ and a^2.
- It incorporates all the taste symmetry breaking effects into the LECs order by order in a perturbative series.

Birth of Staggered ChPT

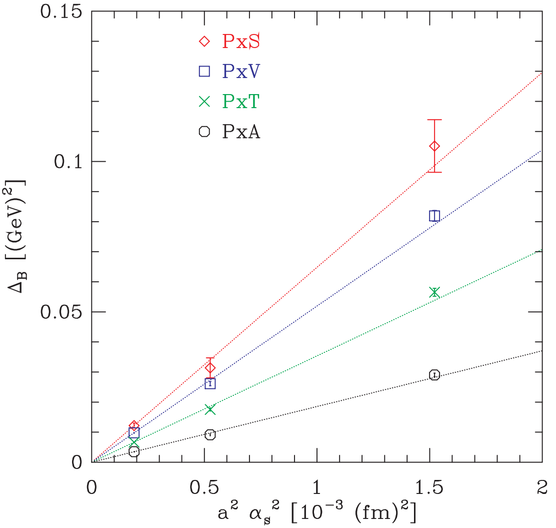
- At the leading order of $p^2 \approx m_q \approx m_\pi^2 \approx a^2$, we can prove that the pion spectrum respects $SO(4)$ taste symmetry out of the full SU(4) taste symmetry.
- Lee and Sharpe proved it for single flavor case (1999).
- Aubin and Bernard proved it for multiple flavor case (2003).
- Power counting rules are established through the numerical study on the lattice.

Splittings of Pion Multiplet Spectrum

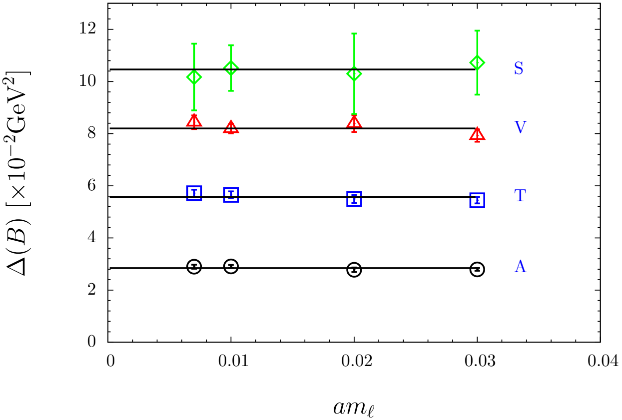


(1) Coarse lattice ($a = 0.12$fm) (2) Fine lattice ($a = 0.09$fm)

Scaling of the Splittings



Sea quark mass dependence of splittings



Staggered Chiral Perturbation Theory (SChPT)

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3. The systematic tool is the SChPT.

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3. The systematic tool is the SChPT.
4. Using the SChPT, we obtain the fitting functional form exactly order by order.

Staggered chiral perturbation theory

- **Power counting**

$$\mathcal{O}(a^2\Lambda_{\rm QCD}^2) \approx \mathcal{O}(p^2/\Lambda_\chi^2) \approx \mathcal{O}(m_\pi^2/\Lambda_\chi^2) \approx \mathcal{O}(m_q/\Lambda_{\rm QCD})$$

- **Lee & Sharpe Lagrangian for multiple flavors**

[Aubin and Bernard, 2003]

$$\mathcal{L}_{\rm LO} = \frac{f^2}{8}\mathrm{Tr}(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger) - \frac{1}{4}\mu f^2 \mathrm{Tr}(M\Sigma + M\Sigma^\dagger) + \frac{2m_0^2}{3}(U_I + D_I + S_I)^2 + a^2\mathcal{V}$$

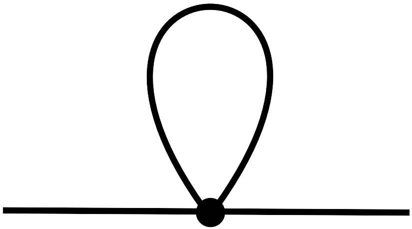
  - $M = \mathrm{diag}(m_u, m_d, m_s) \otimes \xi_I$
  - $\mathcal{V}$: taste symmetry breaking potential [Lee and Sharpe, 1999]

$$SO(4) \times SU(4)_T \xrightarrow{a\neq 0} SW_{4,\rm diag} \underset{p\ll\Lambda_\chi}{\subset} SO(4) \times SO(4)_T$$

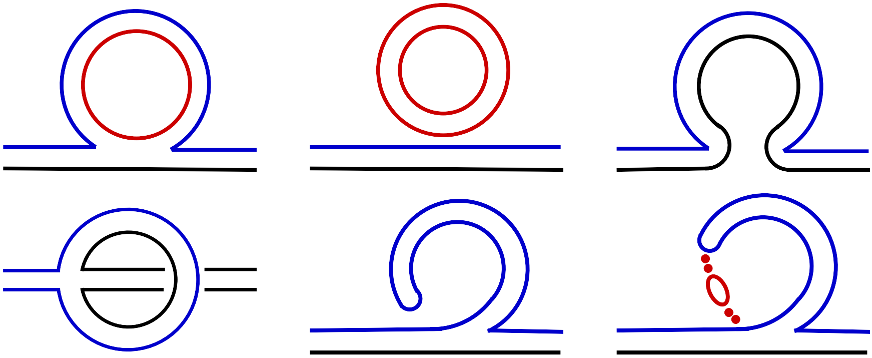
Application of SChPT

Pion Mass (Quark Mass)

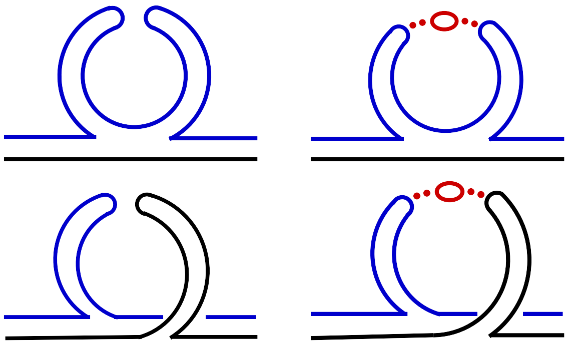
Pion Flow Diagrams



Quark Flow Diagrams (1)



Quark Flow Diagrams (2)



Results at NLO

- Pion self energy:

$$M_{\pi_F}^2 = m_{\pi_F}^2 + \Sigma(m_{\pi_F}^2) + \text{NNLO}$$
$$\Sigma(p^2) = \frac{1}{(4\pi f)^2}[\sigma_{\text{conn}}(p^2) + \sigma_{\text{disc}}(p^2)] + \sigma_{\text{anal}}(p^2)$$

- Connected Part:

$$\sigma_{\text{conn}} = a^2 \sum_B \left(\delta_{BF}^{\text{conn}} \ell(\pi_B^+) + \frac{\Delta_{BF}^{\text{conn}}}{48} [\ell(U_B) + 2\ell(\pi_B^+) + \ell(D_B)] \right)$$

- Disconnected Part:

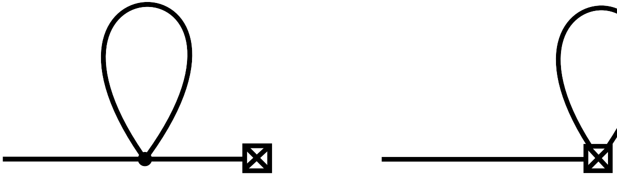
$$\sigma_{\text{disc}} = \frac{1}{12} \Big[2(-12X_5 + a^4(\Delta_{VF}^{\text{conn}} + \ldots)\delta'_V \left(R_{X\eta}^{\pi}(X_V)\tilde{\ell}(X_V) + \ldots \right) + \ldots$$

Progress History (Pion Mass)

- Goldstone pion sector at NLO : Aubin & Bernard (2003)
- Non-Goldstone pion sectors at NLO : Bailey & Kim & Lee (2012)
- Extension to mixed actions : underway by Yoon & Bailey & Lee (YBL) (2012)
  ※ Example of a mixed action :
  - valence quarks = HYP staggered fermions
  - sea quarks = asqtad staggered fermions
- Results have been used for the numerical study by MILC.
- We plan to apply the mixed action results to the data analysis.

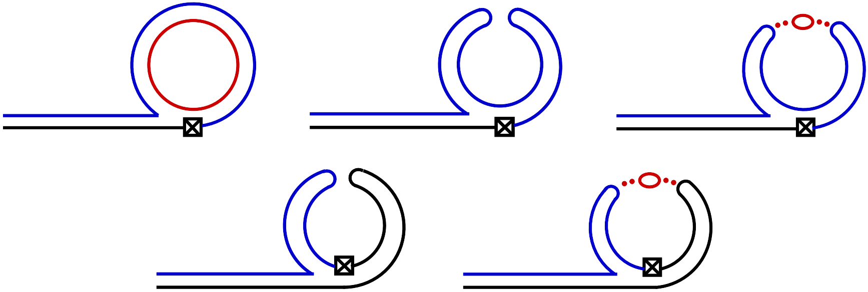
Pion Decay Constants

Pion Flow Diagrams



(13) Wavefunction Cont. (14) Current Cont.

Quark Flow Diagrams (Current Contribution)



Results at NLO

- **Example:**
  - Pion decay constant for fully dynamical case ($xy = ud$)
  - SU(2) chiral perturbation theory ($m_u, m_d \ll m_s$)
  - 2+1 flavors ($m_u = m_d = m_\ell \neq m_s$)

$$
\begin{aligned}
f_{\pi_F} =& f \Bigg\{ 1 + \frac{1}{32\pi^2 f^2} \Bigg[-\frac{1}{4} \sum_B g_B \ell(\pi_B) \\
& \qquad\qquad + (4 - \Theta^{VF}) \Big\{ \ell(\pi_V) - \ell(\eta_V) \Big\} + (V \to A) \Bigg] \\
& \quad + L_4 \frac{16\mu}{f^2}(2m_\ell + m_s) + L_5 \frac{16\mu}{f^2} m_\ell + a^2 \mathcal{F}_F \Bigg\}
\end{aligned}
$$

Progress Report

- Goldstone Pion Sector: Aubin & Bernard (2003)
- Non-Goldstone Pion Sectors: Yoon & Bailey & Lee (2012)
- Extension to the mixed action: underway by YBL (2012)
- Results have been used for the numerical study by MILC.
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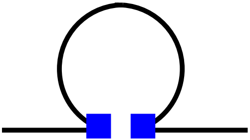
B_K (Indicrect CP Violation)

B_K definition in standard model

$$
\begin{aligned}
B_K &= \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle} \\
\hat{B}_K &= C(\mu) B_K(\mu), \\
C(\mu) &= \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]
\end{aligned}
$$

Pion Flow Diagrams for B_K



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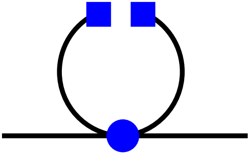

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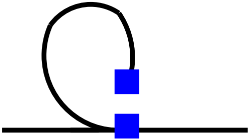

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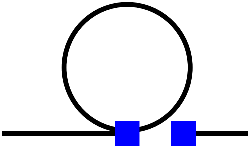
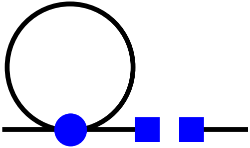


(23) X



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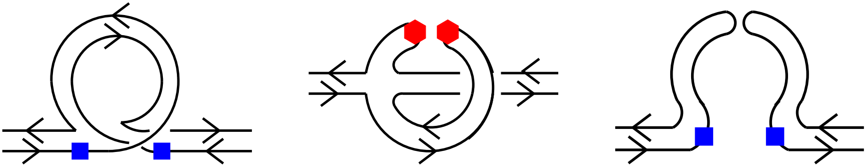
Quark Flow Diagrams for B_K



SU(2) Results at NLO

- B_K: $(m_u = m_d = m_\ell \ll m_s)$

$$B_K = d_1 Q_1 + d_2 \frac{X_P}{\Lambda_\chi^2} + d_3 \frac{L_P}{\Lambda_\chi^2} + \text{NNLO}$$

$$Q_1 = 1 + \frac{1}{32\pi^2 f^2}\left[(L_\text{I} - X_\text{I})\tilde{\ell}(X_\text{I}) + \ell(X_\text{I}) - 2\sum_\text{B} \tau^\text{B} \ell(X_\text{B})\right]$$

- $X_P = [m_\pi^{xx}(\xi_5)]^2$
- $L_P = [m_\pi^{\ell\ell}(\xi_5)]^2$

Progress History

- B_K at NLO : Sharpe & Van de Water (2006)
- Extension to the mixed action : Sharpe (2008)
- BSM operators at NLO : Bailey & Kim & Lee & Sharpe (2012)
- Application to the numerical study : SWME (2010 $\sim$ present)
- The SWME result of B_K is posted to FLAG officially (2012).

$$\begin{aligned}
\hat{B}_K &= 0.727 \pm 0.004(\text{stat}) \pm 0.038(\text{sys}) \\
\varepsilon_K &= (1.56 \pm 0.22) \times 10^{-3} \qquad (\text{Exclusive } V_{cb}) \\
&= (1.88 \pm 0.22) \times 10^{-3} \qquad (\text{Inclusive } V_{cb})
\end{aligned}$$

- We must reduce the errors of B_K and V_{cb} simultaneously.

$\pi - \pi$ Scattering

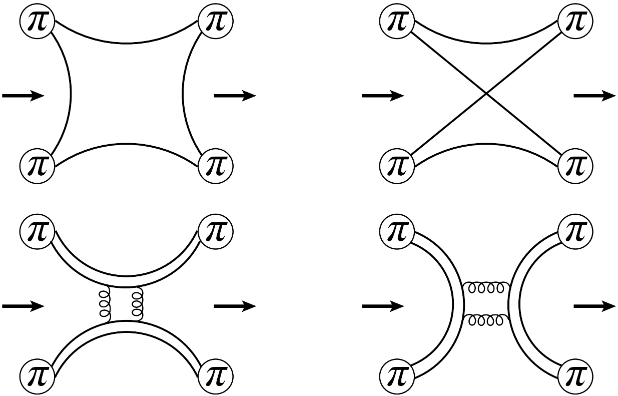
$\pi - \pi$ Scattering and S–matrix

- Five channels of two pion states in staggered fermion formulation:

$$\pi(P) - \pi(P), \quad \pi(A) - \pi(A), \quad \pi(T) - \pi(T),$$
$$\pi(V) - \pi(V), \quad \pi(S) - \pi(S),$$

- The trouble is that their energy eigenvalues are non-degenerate.
- Recently, Hansen & Sharpe make it possible to study multi-channel scattering problem by modifying the Luscher formula.
- Now, it is possible to study the $N = 5$ multi-channel $\pi - \pi$ scattering problem on the lattice using staggered fermions.

Quark Flow Diagrams for $\pi - \pi$ Scattering



Unitarity Violation by Rooting Technique

- If the SU(4) taste symmetry is exactly conserved, then rooting cannot make a trouble of unitarity violation.
- However, if the SU(4) taste symmetry is broken, then rooting makes a unitarity violation.
- The staggered fermion formulation has taste symmetry breaking by construction.
- Hence, the rooting triggers the unitarity violation for staggered fermions.
- As a consequence, there are two kinds of unitarity violation on the lattice using staggered fermions: one from partially quenched QCD and the other from the rooting.

Rooting Technique

- Fermion Determinant of Staggered Fermions:

$$\int [d\psi][d\bar{\psi}] \exp[\int \bar{\psi}(D + m_1)\psi] = \det(D + m_1)$$

- Here, the Dirac operator $(D + m_1)$ contains 4 copies of degenerate tastes.

- In order to reduce the number of tastes to one, we use the rooting technique in the numerical study.

$$\det(D + m_1) \longrightarrow [\det(D + m_1)]^{\frac{1}{4}}$$

- However, if the SU(4) taste symmetry is broken, then the rooting causes a unitarity violation since sea quarks and valence quarks have different Dirac operators.

How to get around the trouble: SChPT

- SChPT can, in principle, trace the rooting part and the unitarity violation terms.
- Hence, we fit the numerical data to the functional form suggested by SChPT.
- Then, we can remove the unitarity violating terms by hand.
- Then, the remaining part will be unitary, which corresponds to the S-matrix defined by Hansen & Sharpe.
- The SChPT calculation is underway by Yoon & Bailey & Lee.

Summary

Summary of Current Status in SChPT

physics	Goldstone	Non-Goldstone	mixed	numerical
m_π^2	$\bigcirc$	$\bigcirc$	$\triangle$	$\bigcirc$
f_π	$\bigcirc$	$\bigcirc$	$\triangle$	$\bigcirc$
B_K	$\bigcirc$	$\times$	$\bigcirc$	$\bigcirc$
BSM op	$\bigcirc$	$\times$	$\bigcirc$	$\triangle$
$\pi - \pi$	$\triangle$		$\times$	$\triangle$
$K \to \pi\pi$	$\times$		$\times$	$\times$
V_{cb}	$\bigcirc$		$\times$	$\bigcirc$

Sincere apologies
for omitting some topics

Thank you very much !!!