Chiral Perturbation Theory with a scalar field

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- χPT is the standard theory for computing observables with low–energy pions.
- However bad convergence has been observed in pion–pion scattering scalar I=0 channel (even at relatively low–momenta).
- Proposed solutions involve resumations of certain classes of diagrams, using unitaritzation techniques. eg Oller, Oset, Pelaez (98)
- This suggests the existence of a low–mass scalar isospin zero resonance, the sigma.
- In fact nowadays the sigma mass and decay width are known quite accurately.

$$\sqrt{s_\sigma} = 441^{+16}_{-8} - i272^{+9}_{-12.5}\text{MeV}$$

Caprini, Colangelo, Leutwyler (06), Garcia–Martin, Kaminski, Pelaez, Ruiz de Elvira (11)

- The sigma resonance has a low–mass similar to the kaon mass.
- Why not to include other low–mass states with different quantum numbers in χPT?

Aim

- Generalize the Chiral Lagrangian to include a light scalar with mass $m_S \ll \Lambda_\chi$
- The calculations can be systematically organized in powers of p^2/Λ_χ^2, $p \sim m_\pi \sim m_S$.
- Explore the physical consequences of such generalization.

Lagrangian χPT_S

$$\mathcal{L}_{\chi PT_S} = \mathcal{L}_{\chi PT} + \mathcal{L}_S + \mathcal{L}_{\pi S} + \mathcal{L}_{CT}$$

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$\mathcal{L}_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \ldots$ Gasser, Leutwyler (84) $\quad N_f = 2$

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}\left[D^\mu U^\dagger D_\mu U + \chi^\dagger U + U^\dagger \chi\right], \quad \chi = 2B_0\hat{m}\mathbf{1}, \quad m_\pi^2 = 2B_0\hat{m}, \quad \hat{m} = \frac{m_u + m_d}{2}$$

$$\begin{aligned}\mathcal{L}^{(4)} =& \frac{l_1}{4}\mathrm{Tr}\left[D^\mu U^\dagger D_\mu U\right]^2 + \frac{l_2}{4}\mathrm{Tr}\left[D^\mu U^\dagger D^\nu U\right]\mathrm{Tr}\left[D_\mu U^\dagger D_\nu U\right] + \\ &+ \frac{l_3}{16}\mathrm{Tr}\left[\chi^\dagger U + U^\dagger \chi\right]^2 + \frac{l_4}{4}\mathrm{Tr}\left[D^\mu U^\dagger D_\mu \chi + D^\mu U D_\mu \chi^\dagger\right] + \ldots\end{aligned}$$

$$U = e^{i\,2T\cdot\pi/F_0}, \quad \boldsymbol{\pi} \equiv \pi^\alpha \tau^\alpha = 2\pi^\alpha T^\alpha = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

- Standard $N_f = 2$ χPT Lagrangian.
- However! $F_0, B_0, l's$ are now parameters of a different theory.

$\mathcal{L}_S$

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mathring{m}_S^2 SS - \lambda_1 S - \frac{\lambda_3}{3!}S^3 - \frac{\lambda_4}{4!}S^4 + \cdots$$

- $\lambda_3 \sim \mathcal{O}(\Lambda_\chi)$ and $\lambda_4 \sim \mathcal{O}(1)$
- Strongly coupled !
- Scalar strongly coupled theories are believed to be trivial. Luscher (87), Frolich (82)
- We can take $\lambda_3 = \lambda_4 = 0$
- When scalar-pion interactions are introduced, small values are required to ensure renormalization.
- λ_1 is chosen to cancel scalar-vacuum mixing, zero at tree level, non-zero at higher orders.

$\mathcal{L}_{\pi S}$

$$\mathcal{L}_{\pi S} = \left(F_0 c_{1d} S + c_{2d} S^2 + \cdots\right) \mathrm{Tr}\left[D^\mu U^\dagger D_\mu U\right] + \\ + \left(F_0 c_{1m} S + c_{2m} S^2 + \cdots\right) \mathrm{Tr}\left[\chi^\dagger U + U^\dagger \chi\right]$$

- $c_{ij} \sim \mathcal{O}(1)$, $i = 1, 2$, $j = d, m$.
- χPT with S, linear terms Ecker, Gasser, Pich, de Rafael (89), Cirigliano, Ecker, Eidemuller, Kaiser, Pich, Portoles (06) and quadratic in Rosell, Ruiz–Femenia, Portoles (05)
- Pions and the scalar are dynamical in the same energy range!
- Operators not shown are suppressed by powers of $1/\Lambda_\chi$

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$\mathcal{L}_{CT}$

- New counterterms are required. Z_1 and Z_2 remormalize B_0, F_0.

$$\mathcal{L}_{CT} = Z_1 \mathring{m}_S^2 \mathrm{Tr}\left[\chi^\dagger U + \chi U^\dagger\right] + Z_2 \mathring{m}_S^2 \mathrm{Tr}\left[D_\mu U D^\mu U^\dagger\right] \\ + f_{2p} \Box S \Box S + d_{2m} \partial_\mu S \partial^\mu S \mathrm{Tr}\left[\chi^\dagger U + \chi U^\dagger\right] + b_{2m} S^2 \mathrm{Tr}\left[\chi^\dagger U + \chi U^\dagger\right]^2 \\ + a_{2m} S^2 \mathrm{Tr}\left[\chi^\dagger \chi\right] + e_{2m} S^2 \Re[\det(\chi)] \ldots .$$

Chiral Symmetry Constraints

Vacuum Configuration contribution, $U = \mathbf{1}$

$$\left(F_0 c_{1m} S + c_{2m} S^2 + \cdots\right) 8 B_0 \hat{m}$$

- Can be reshuffled into λ_1, m_S^2, $\cdots$

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$$\mathring{m}_S^2 \rightarrow m_S^2 = \mathring{m}_S^2 - 16 c_{2m} B_0 \hat{m}$$

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- Explicit Chiral Symmetry breaking introduces new contributions to the singlet–vacuum mixing.
- Excitations of the scalar field respect to the vacuum can be obtained by carrying out the shift:

$$S \to S + F_0 S_0 \qquad S_0 = c_{1m} \frac{8 B_0 \hat{m}}{m_S^2} - \frac{\lambda_1}{F_0 m_S^2}$$

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- In general after the shift the scalar field is not a singlet under Chiral Symmetry anymore.

- Specific parameter choices make the shift independent of the quark masses and restores Chiral Symmetry.

$$\lambda_1 = \frac{c_{1m}\mathring{m}_S^2 F_0}{2c_{2m}}\,.$$

- The Lagrangian resulting form such a field redefinition is equivalent to setting $c_{1m} = \lambda_1 = 0$.
- Equivalent to require the scalar field never to mix with the vacuum and to be always a singlet under Chiral Symmetry.

Renormalization

$$\ell_i := \ell_i^r + \gamma_i \lambda\,, \ (i=3,4) \quad Z_j := Z_j^r + \Gamma_j \lambda\,,$$
$$f_{2p} := f_{2p}^r + \Gamma_f \lambda\,, \quad d_{jm} := d_{jm}^r + \Delta_j \lambda\,, \ (j=1\,,2)$$

f.i.

$$\gamma_3 = -\frac{1}{2} + 32c_{2m}\left(c_{2m} - c_{2d}\right) - 8c_{1d}^2\left(1 - 4c_{2m}\right)\,, \quad \gamma_4 = 2 + 4c_{1d}^2\left(1 - 8c_{2m}\right) + 32c_{2d}c_{2m}$$

We have defined the scale independent quantities,

$$\ell_i^r = \frac{\gamma_i}{32\pi^2}\left[\bar{\ell}_i + \ln\left(\frac{m_\pi^2}{\Lambda^2}\right)\right] (i=3,4)\,, \quad Z_j^r = \frac{\Gamma_j}{32\pi^2}\left[\bar{Z}_j + \ln\left(\frac{m_S^2}{\Lambda^2}\right)\right] (j=1,2)\,,$$
$$f_{2p}^r = \frac{\Gamma_f}{32\pi^2}\left[\bar{f}_{2p} + \ln\left(\frac{m_\pi^2}{\Lambda^2}\right)\right]\,, \quad d_{im}^r = \frac{\Delta_i}{32\pi^2}\left[\bar{d}_{im} + \ln\left(\frac{m_\pi^2}{\Lambda^2}\right)\right]\,, (i=1,2)$$

$m^2_{\rm PS}$ and $F_{\rm PS}$ at NLO

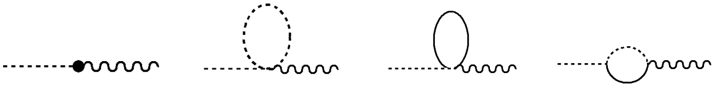
$m^2_{\rm PS}$

$$m^2_{\rm PS} = 2B_0\hat{m} + U_m + P_m + \mathcal{O}(p^6)\,,$$

$$U_m = -\frac{4c_{1d}^2}{F_0^2}\bar{J}(m_\pi^2, m_S^2; m_\pi^2)\left(m_S^2 - 2m_\pi^2\right)^2\,,$$

$$P_m = -\frac{m_\pi^4}{4\pi^2F_0^2}\left(\frac{\log\left(m_S^2/\Lambda^2\right) - \log\left(m_\pi^2/\Lambda^2\right)}{m_\pi^2 - m_S^2}\right)\left[c_{1d}^2 m_S^2 - 4c_{2m}\Gamma_1\left(m_\pi^2 - m_S^2\right)\right]$$

$$+\frac{m_\pi^4}{16\pi^2F_0^2}\bar{\ell}_3\gamma_3 + \frac{m_\pi^2\mathring{m}_S^2}{8\pi^2F_0^2}\bar{Z}_1\Gamma_1$$



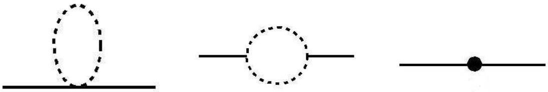
$F_{\rm PS}$

$$F_{\rm PS} = F_0\left(1 + U_F + P_F + \mathcal{O}(p^6)\right) .$$

$$U_F = \frac{2c_{1d}^2}{F_0^2 m_\pi^2}\bar{J}(m_\pi^2, m_S^2; m_\pi^2)\frac{2m_\pi^2 - m_S^2}{4m_\pi^2 - m_S^2}\left(14m_\pi^4 - 15m_\pi^2 m_S^2 + 3m_S^4\right) ,$$

$$P_F = \frac{c_{1d}^2}{8\pi F_0^2}\frac{\left(m_S^2 - 2m_\pi^2\right)^2}{4m_\pi^2 - m_S^2} - \frac{m_\pi^2}{4\pi^2 F_0^2}\left(\frac{\log\left(m_S^2/\Lambda^2\right) - \log\left(m_\pi^2/\Lambda^2\right)}{m_\pi^2 - m_S^2}\right)\Bigg[c_{1d}^2\frac{\left(m_S^2 - 2m_\pi^2\right)^2}{\left(4m_\pi^2 - m_S^2\right)}$$
$$+4c_{2m}\Gamma_2\left(m_\pi^2 - m_S^2\right)\Bigg] + \frac{m_\pi^2}{32\pi^2 F_0^2}\gamma_4\bar{\ell}_4 + \frac{\mathring{m}_S^2}{8\pi^2 F_0^2}\bar{Z}_2\Gamma_2$$

Γ and $m^2_{S,\mathrm{NLO}}$



$m^2_{S,\mathrm{NLO}}$

$$m^2_{S,\,\mathrm{NLO}} = m_S^2 - \frac{3c_{1d}^2 m_S^4 \bar{f}_{2p}}{8\pi^2 F_0^2} - \frac{3m_\pi^4 \bar{d}_{1m}}{4\pi^2 F_0^2}\left(c_{2m} - c_{2d} + 6c_{1d}^2\right) + \frac{9c_{1d}^2 m_S^2 m_\pi^2 \bar{d}_{2m}}{4\pi^2 F_0^2} - \frac{6c_{1d}^2}{F_0^2}\bar{J}(m_\pi^2, m_\pi^2; m_S^2)\left(m_S^2 - 2m_\pi^2\right)^2$$

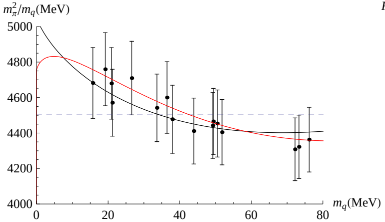
Γ_σ

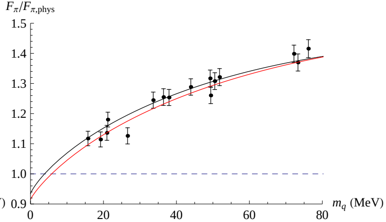
$$\frac{\Gamma_\sigma}{2} = \frac{3c_{1d}^2}{8\pi F_0^2 m_S}\sqrt{1 - \frac{4m_\pi^2}{m_S^2}}\left[m_S^2 - 2m_\pi^2\right]^2$$

Taking $m_{S,\,\mathrm{CCL}} = 441.2\,\mathrm{MeV}$, $\Gamma_{\mathrm{CCL}}/2 = 272$, from Caprini, Colangelo, Leutwyler (06) and standard values for m_π^2 and F_π we obtain $c_{1d}^2 = 0.457$.

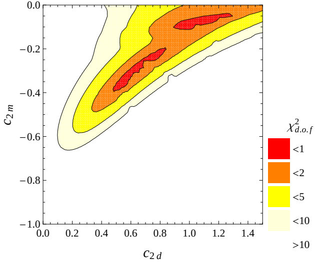
Matching $m_{\rm PS}^2$ and $F_{\rm PS}$ with lattice data

Lattice data from ETM Collaboration, R.Baron *et al.*





LO (dashed line), NLO χPT (solid black line) and χPT_S (solid red line)



$\chi^2_{d.o.f} = \frac{16.7}{26}$ $\quad B_0 = 1680.5\,\text{MeV}$

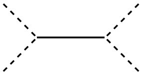
$\ell_3^r = -1.12 \times 10^{-3}$ $\quad \ell_4^r = 6.94 \times 10^{-3}$

$c_{2d} = 1.21$ $\quad c_{2m} = -0.083$

$F_0 = 101.2\,\text{MeV}$ $\quad \mathring{m}_S^2 = 426\,\text{MeV}$

Pion–Pion Scattering Lengths a_0^0 , a_0^2

a_0^0 , a_0^2



$$a_0^0 = \frac{7m_\pi^2}{32\pi F_0^2} - \frac{3m_\pi^4}{2\pi F_0^2}\frac{c_{1d}^2}{4m_\pi^2 - m_S^2} + \frac{m_\pi^4}{\pi m_S^2 F_0^2} c_{1d}^2 \,,$$

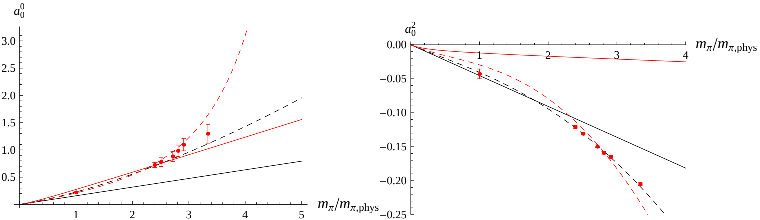
$$a_0^2 = -\frac{m_\pi^2}{16\pi F_0^2} + \frac{m_\pi^4}{\pi m_S^2 F_0^2} c_{1d}^2 \,.$$

Decoupling limit

Large tree level finite contribution to the parameter $\bar{l}_1$ of χPT,

$$\bar{l}_1^S = 192\pi^2 \frac{F_0^2 c_{1d}^2}{\mathring{m}_S^2} \sim 38$$

Lattice data from Z. Fu (12)



Dashed line χPT$_S$ LO, red solid line χPT$_S$ LO+l_1 term, black solid line χPT LO, black dashed line χPT NLO. The parameters are constrained so Γ and m_S^2 always coincide with the ones given in Caprini, Colangelo, Leutwyler (06).

	a_0^0	a_0^2
Exp.(stat)(syst)	0.2210(47)(40)	−0.0429(44)(28) Batley (10)
Beyond NLO χPT	0.220 ± 0.005	-0.0444 ± 0.0010 Colangelo (01)
χPT, LO	0.159	−0.0454
χPT, NLO	0.228	−0.0405
χPT$_S$, LO	0.275	−0.0121
χPT$_S$, LO+ℓ_1	0.210	−0.0296

Extracting m_S^2, Γ_σ from lattice data

$$c_{2m} = -0.228 \quad , \quad \bar{\ell}_1 = -10.9 \quad , \quad c_{1d}^2 = 0.304 \quad , \quad \mathring{m}_S = 483\text{MeV} \quad , \quad \chi^2_{d.o.f} = 13.7\,,$$

which translates to

$$m_S = 486\text{MeV} \quad , \quad \frac{\Gamma_\sigma}{2} = 236\text{MeV} \quad , \quad a_0^0 = 0.177 \quad , \quad a_0^2 = -0.0361\,,$$

Compared with the Caprini, Colangelo, Leutwyler (06)

$$m_S = 441^{+16}_{-8}\,, \quad \frac{\Gamma_\sigma}{2} = 272^{+9}_{-12.5}\text{MeV}$$

Garcia–Martin, Kaminski, Pelaez, Ruiz de Elvira (11)

$$m_S = 457^{+14}_{-13}\,, \quad \frac{\Gamma_\sigma}{2} = 279^{+11}_{-7}\text{MeV}$$

Conclusions χPT_S

- We constructed a consistent Chiral effective theory including a light scalar.
- The theory has the same degrees–of–freedom as the linear sigma model, but: Greater parameter flexibility due chiral symmetry breaking at Λ_χ, exept c_{1m}. Predictions can be improved systematically.
- Good description of lattice data for m_π^2 , F_π.
- Good description of lattice data for a_0^0 but worse than χPT for a_0^2.
- Allows to extract the sigma resonance parameters from chiral extrapolations of lattice data.

Thank you for your attention