Excited state contributions with two pions

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Motivation

Consider a two-point correlator for an operator P with zero spatial momentum

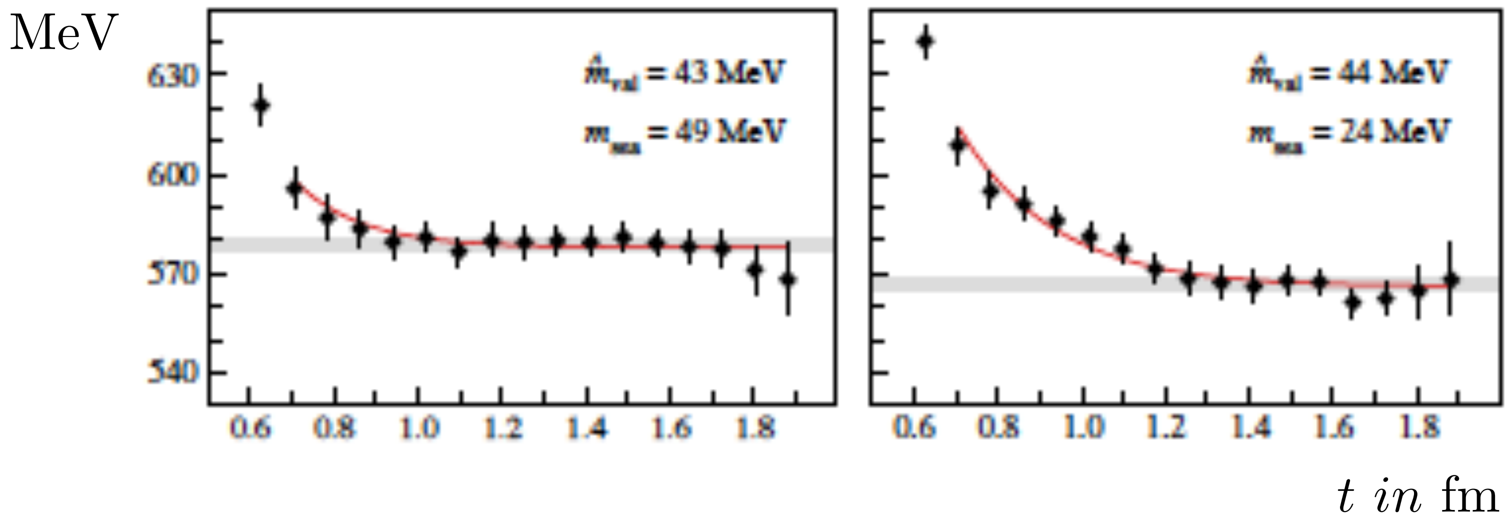
$$C(t) = \langle P(t) P^\dagger(0) \rangle = \sum_{n=0}^{\infty} C_n e^{-E_n t}$$

in a finite spatial volume L^3.
With the ground state a stable particle (e.g. π^+), excited states can correspond to single particle states (e.g. $\pi(1300)$) or multi-particle states (e.g. 3π).

⇒ Need to tell these apart for spectroscopy of excited states.

With light-enough quarks, the first excited state is the ground state plus two pions, with energy $E = E_0 + 2m_\pi$.

From Del Debbio et al., JHEP 0702 (2007) 056:



Effective mass plots of “valence-meson” mass, held fixed, with two different sea-quark masses (in partially quenched QCD, $m_{val} \neq m_{sea}$).

A good fit was obtained with (K stands for valence meson)

$$M_{eff}(t) = -\frac{d}{dt} \log C(t) = M_K \left(1 + c\, e^{-(M' - M_K)t}\right) + \dots$$

in which $M' = M_K + 2M_\pi$, with “π” made out of sea quarks.

Is this the correct interpretation of the second exponential?

- In a finite spatial volume, one expects that $c \sim 1/L^6$.
  Three-particle states contribution looks like

$$\frac{1}{L^6}\sum_{p,q,k}\delta_{p+q+k,0}\,\frac{1}{8E^\pi_p E^\pi_q E^K_k}\,|\langle 0|P(0)|\pi(p)\pi(q)K(k)\rangle|^2\,e^{-E_{tot}t}+\dots$$

  with lowest state having $p = q = k = 0$.

- In finite volume state with

$$E^\pi_p = E^\pi_q = M_\pi\sqrt{1+\left(\frac{2\pi}{M_\pi L}\right)^2}\ ,\quad p=-q=2\pi/L$$

  is much suppressed: for (typical) $M_\pi L \approx 4$, the square root ≈ 2 .

- For light-enough pions, we can calculate c in ChPT, and check!
  Here we do this in LO ChPT with three light flavors.
  (Note: partial quenching not relevant, as long as $m_{sea} < m_{val}$.)

ChPT results (leading order):

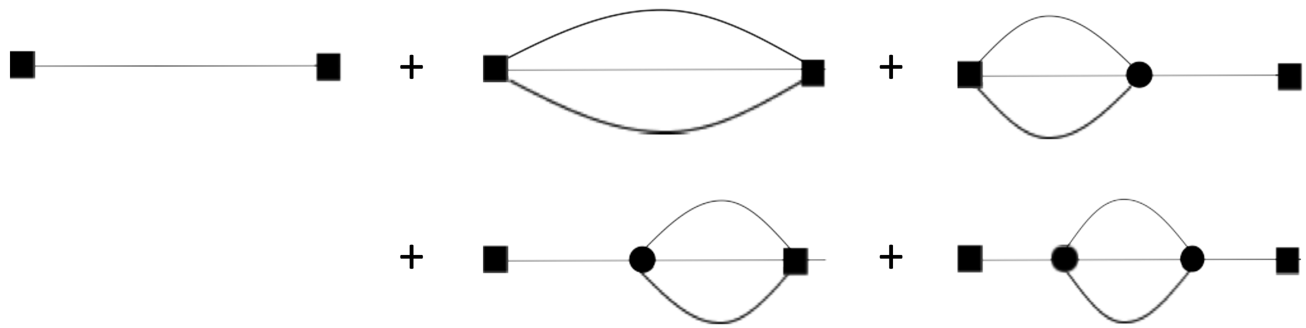
Take $P = \overline{d}\gamma_5 u$ then

$$C(t) = -\frac{f^2 B^2}{2M_\pi}\, e^{-M_\pi t}\left[1 + \frac{45}{512(fL)^4(M_\pi L)^2}\; e^{-2M_\pi t}\right]$$

Note that this contains the leading 3-particle state only, with two additional pions at rest; there are other states with non-zero momentum particles, additional kaon pairs, etc.

At this order $f = f_\pi = f_K$ (normalization: $f_\pi = 93$ MeV),
B is leading-order Gasser-Leutwyler constant.

Diagrams:



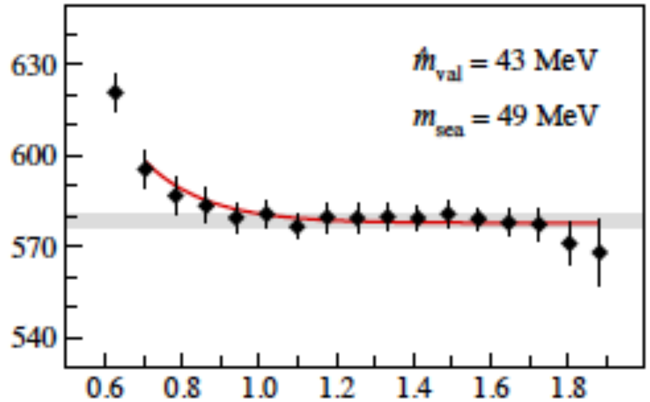
(squares denote the operator P, circles denote strong vertices)

Power counting: $m_\pi \sim m_K \sim p \sim 1/L \sim \pi_a$

With this power counting all other diagrams are higher order.

Back to Del Debbio et al.:

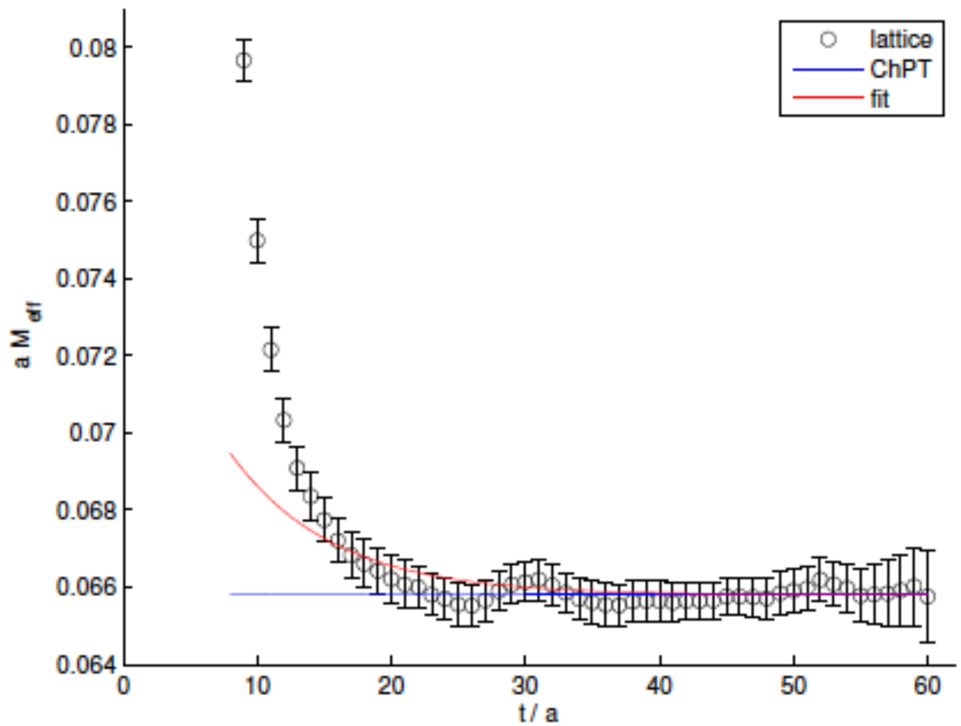
From JHEP 0702 (2007) 056, 082: $fL \approx 1.0$, $M_\pi L \approx 5.9$



For these parameter values $c \approx 5 \times 10^{-3}$, much to small to explain the curve, for which c would have to be more than two orders of magnitude larger.

Note that $M_\pi \approx 620$ MeV .
Similar conclusion for the plot with lower $M_\pi \approx 420$ MeV .

At a lighter sea-pion mass ($M_\pi = 270$ MeV), the ALPHA collaboration (Fritzsch et al., arXiv:1205.5380) produced new data for pion correlator:

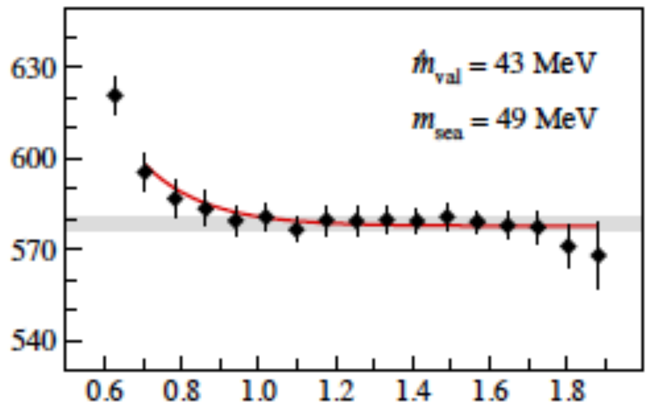


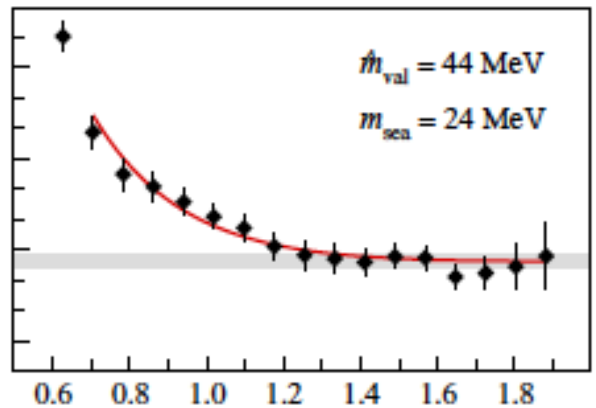
Here $c = 1.6 \times 10^{-3}$, again too small to explain the data (blue curve). For a fit (red curve) we need again two orders of magnitude more.

Try with single-particle excited state: $M_{eff} = M\left(1 + A\, e^{-(M'-M)t}\right)$

(Note: speculation!)

with M' the mass of the equivalent of the $\pi(1300)$.





$M' = 1.67$ GeV $\qquad$ $M' = 1.45$ GeV

$A = 1.8$ $\qquad$ $A = 2.2$

If we parameterize $M' = M'_0 + b\, M_\pi^2$ and $A = A_0 + a\, M_\pi^2$,

then $M'(M_\pi = 140\ \mathrm{MeV}) = 1.27\ \mathrm{GeV},\ b = 1.1\ \mathrm{GeV}^{-1},\ a = -1.9\ \mathrm{GeV}^{-2}$.

Conclusions

- In a given channel, the ground-state energy and the energy of a state with two extra pions are related by chiral symmetry, hence can be calculated in ChPT.

- In a finite spatial volume, the state with two extra pions at rest is suppressed by the square of the volume. Suppression is of order $(f_\pi L)^{-4}(M_\pi L)^{-2}$ times a numerically small factor $\sim 1/10$ (even smaller for the axial current). Can be ignored in analysis of current lattice data.

- We considered states created by the pseudo-scalar density and axial current, but observations should generalize to other channels .