Matrix elements from lattice QCD

Dru Renner

JLab

reviewing work from many groups

presented at Chiral Dynamics 2012

Precision lattice QCD calculations

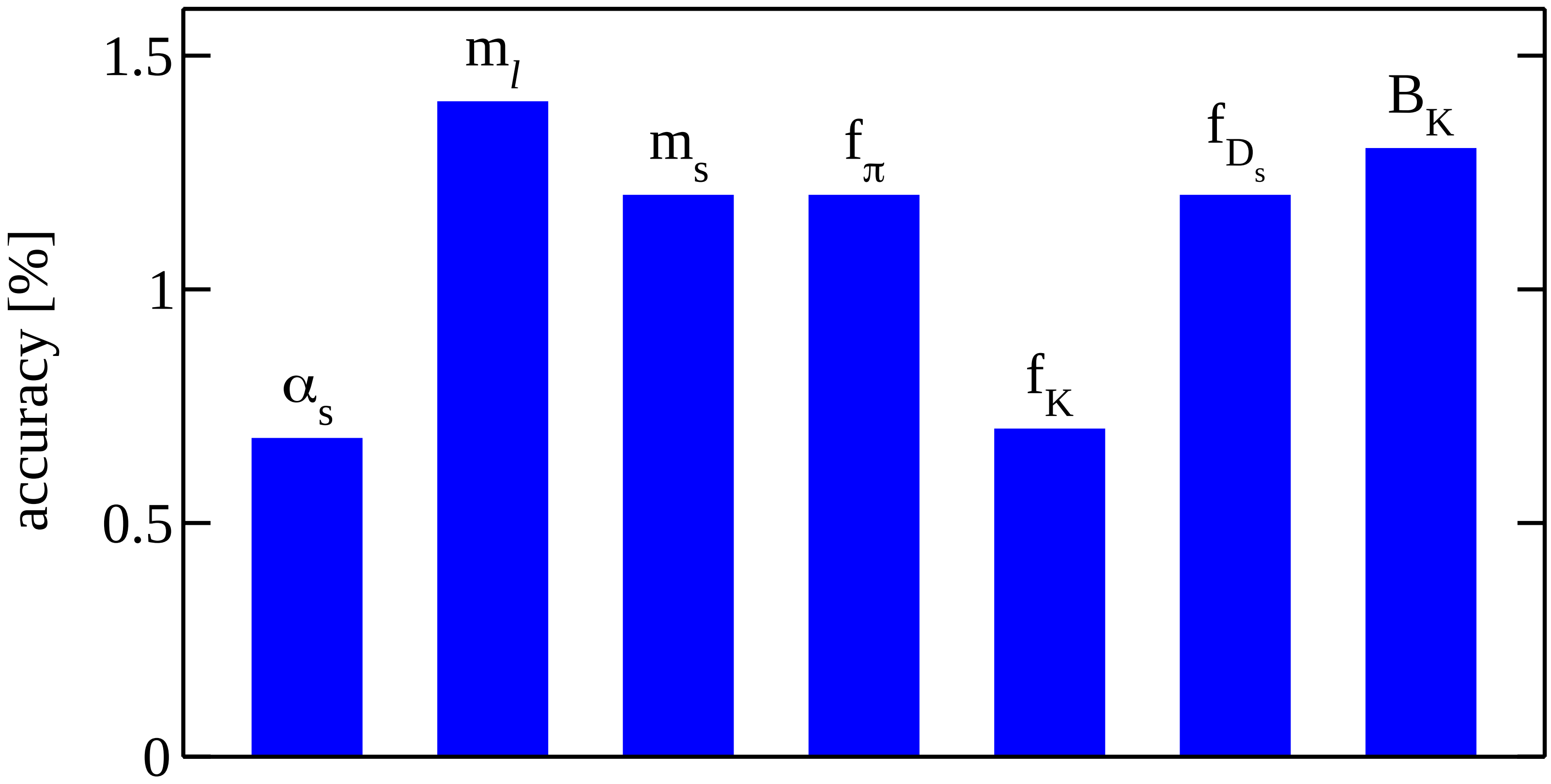
High-precision calculations of f_π,... in the last decade

Precision calculations of m_N,... in the last few years

Well-controlled pion matrix elements are possible now

Intense work on much harder nucleon matrix elements

Lattice QCD can do high-precision calculations

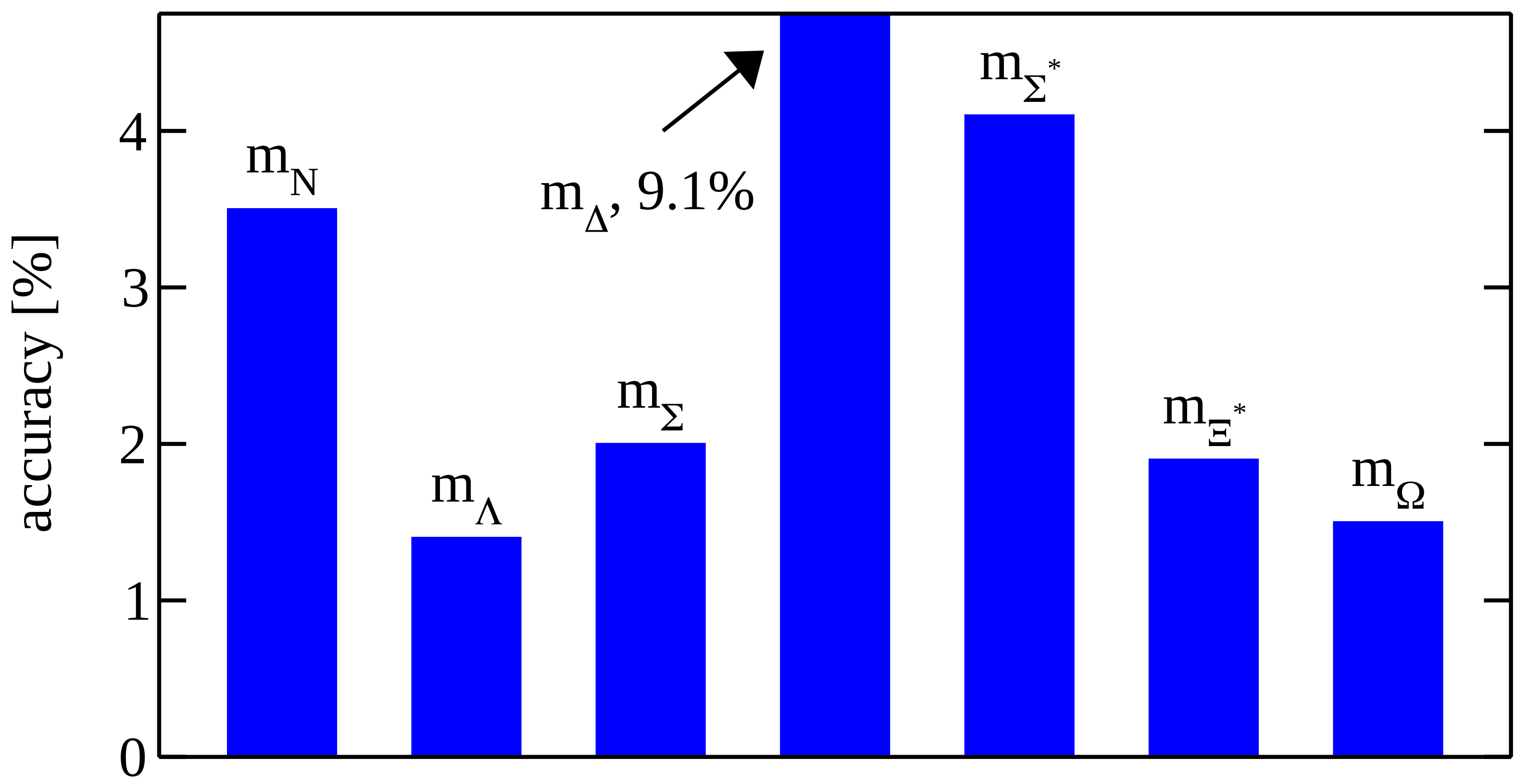


first precision calculations were completed in 2003

there is a notable absence of any baryonic properties

averages of results from arXiv:1110.0016 and http://latticeaverages.org/

Precision baryon physics is just recently feasible



first calculation in 2009 due to challenge of baryons

example of computational thresholds in lattice QCD

BMW collaboration, Science, arXiv:0906.3599, see ETMC and CP-PACS too

Pion form factor, simplest of the harder quantities

space-like form factor is calculable in Euclidean space

$$\langle \pi, p | V_\mu^{\text{em}} | \pi, p' \rangle = K_\mu F(Q^2) \qquad K_\mu = p_\mu + p'_\mu$$

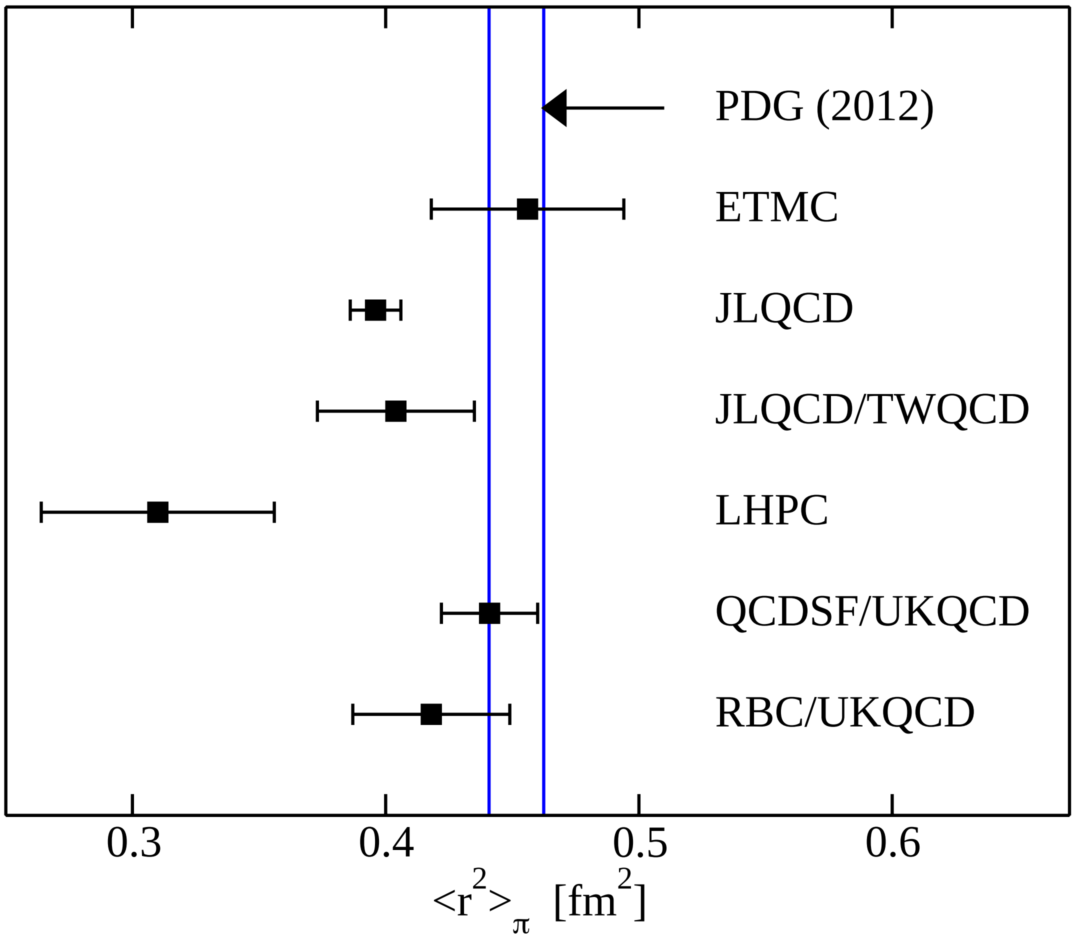
$\mathrm{F(0) = 1}$, so focus on the slope or the charge radius

$$\langle r^2 \rangle_\pi \equiv 6 \left. \frac{dF(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$$

no renormalization is required but $\mathrm{Q^2 \neq 0}$ is needed

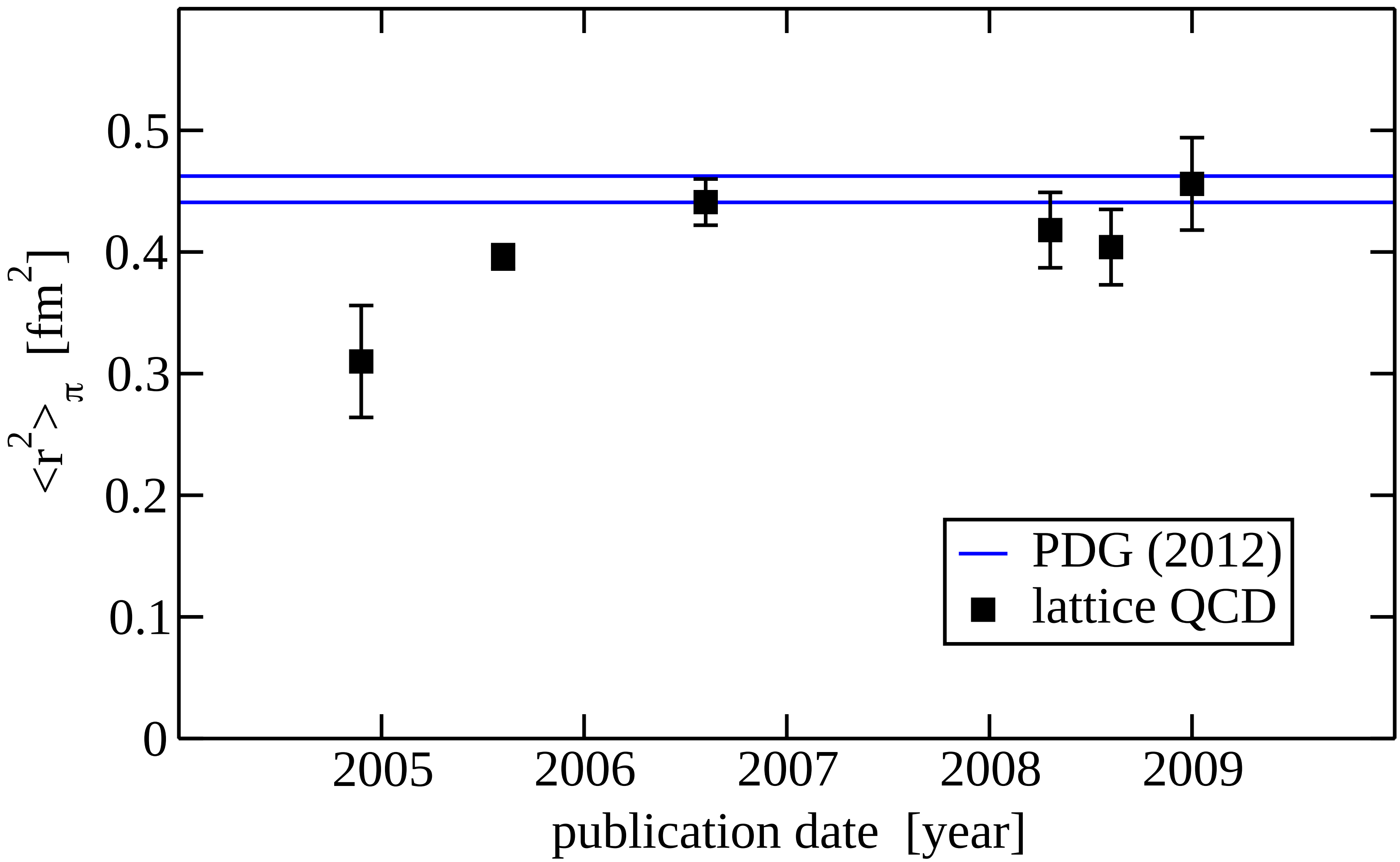
Precision calculations are essential

good example matrix element to illustrate the need ...



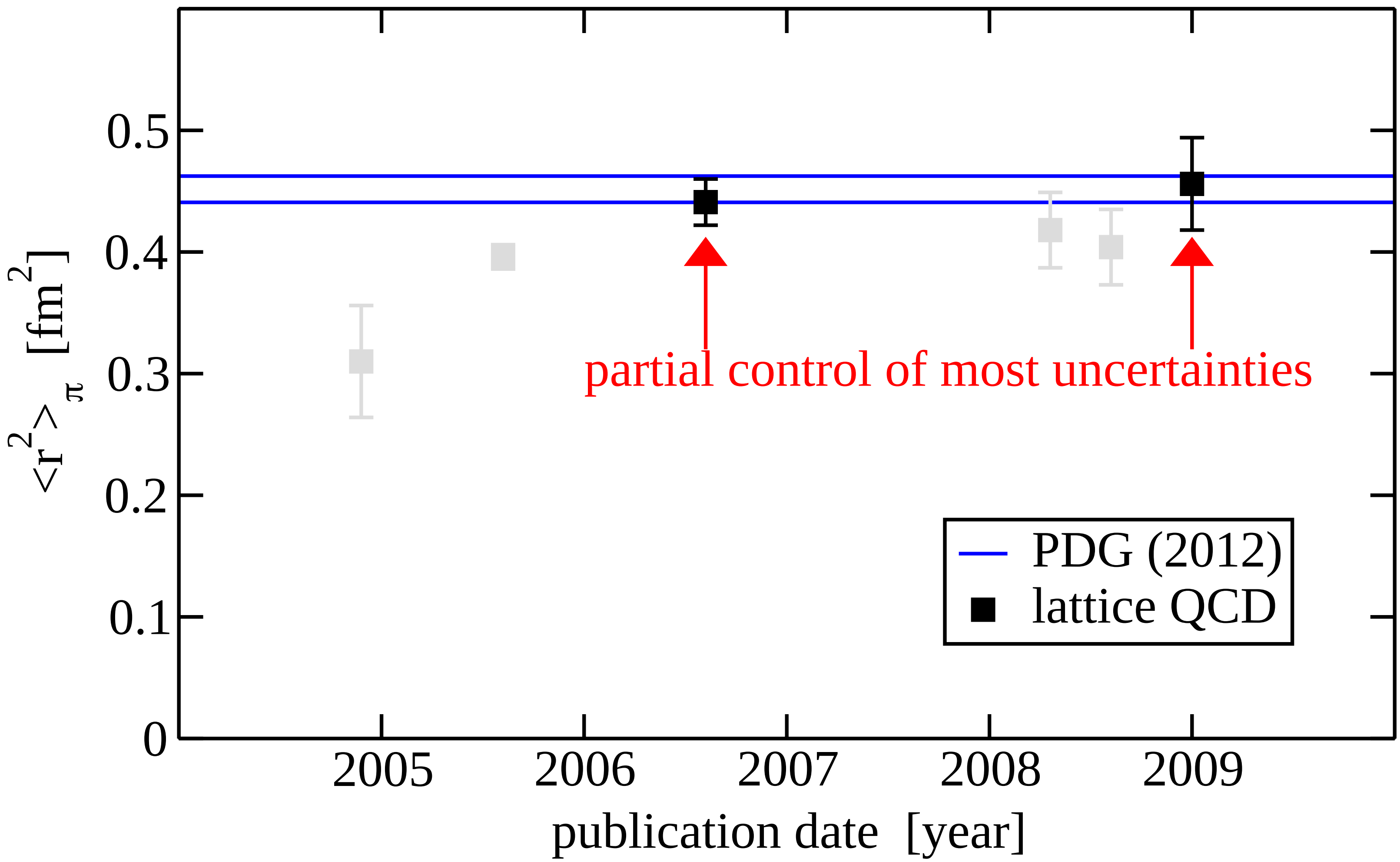
to carefully and precisely control all uncertainties

Pion charge radius



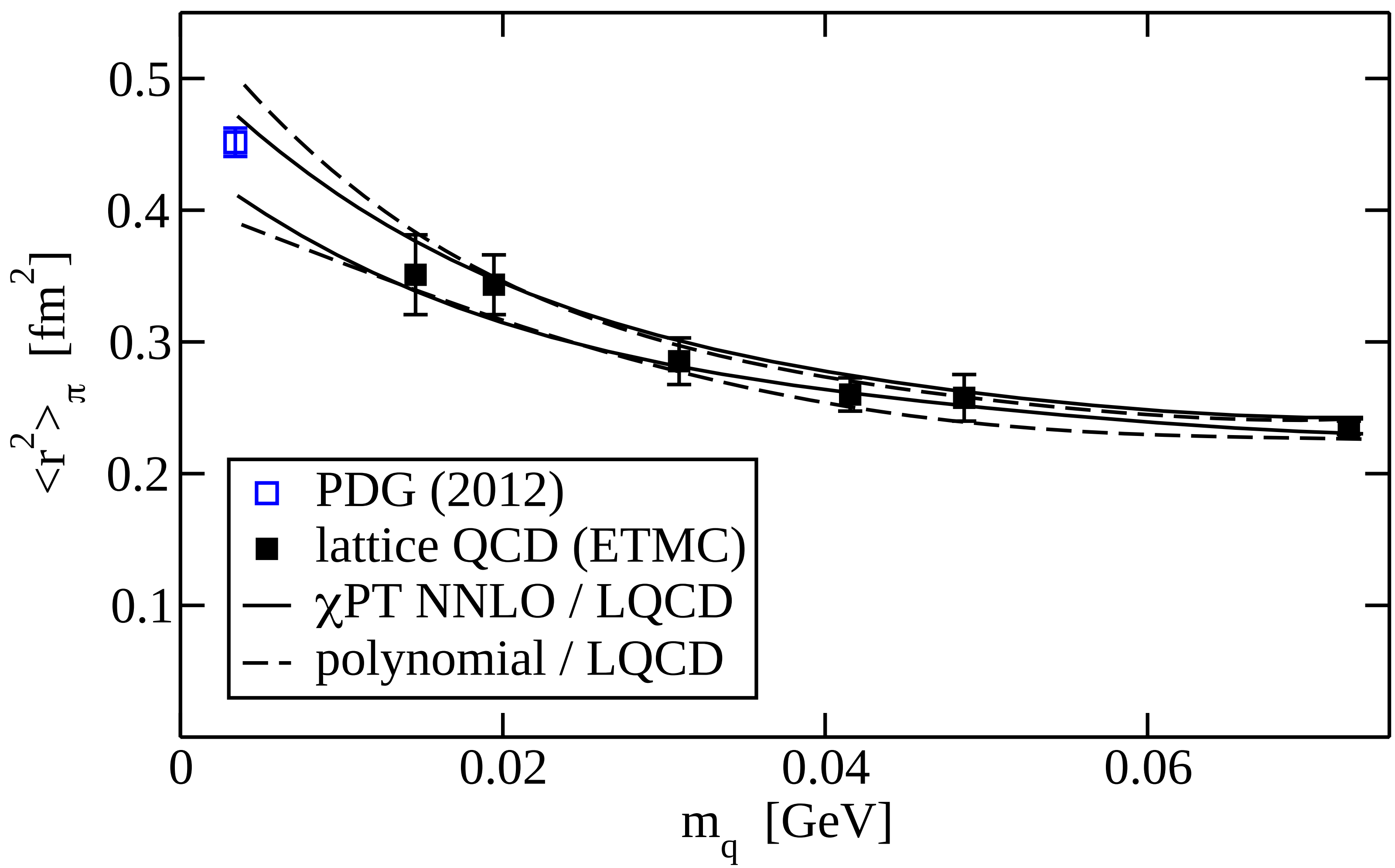
computational thresholds were not met in early results

Pion charge radius



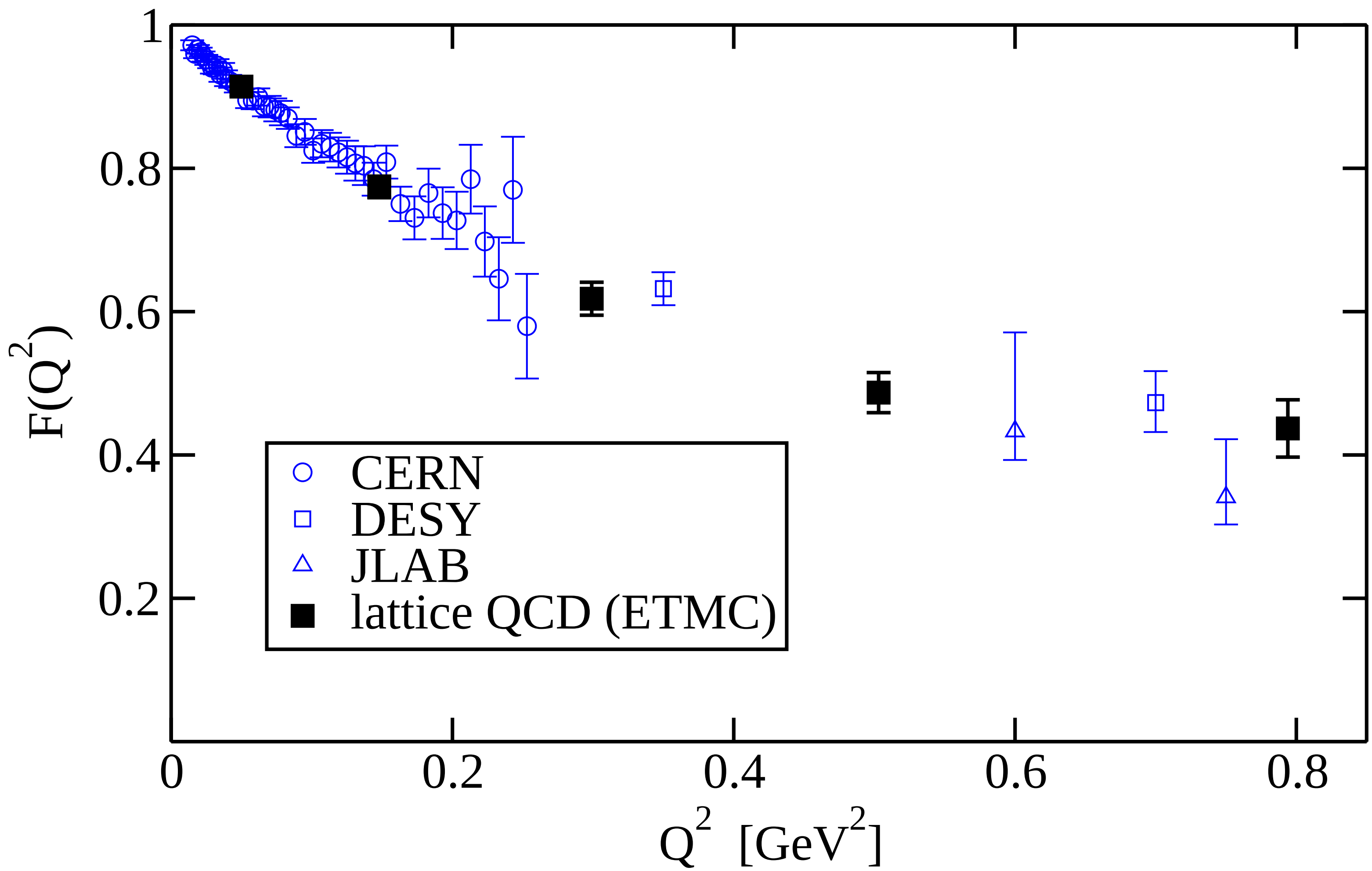
could now calculate pion charge radius to about 5%

Chiral extrapolation of pion charge radius



illustrates the χPT/LQCD extrapolation strategy

Pion form factor



lattice calculation is comparable to the measurements

Proton form factors are more challenging

proton leads to two vector form factors, F_1 and F_2

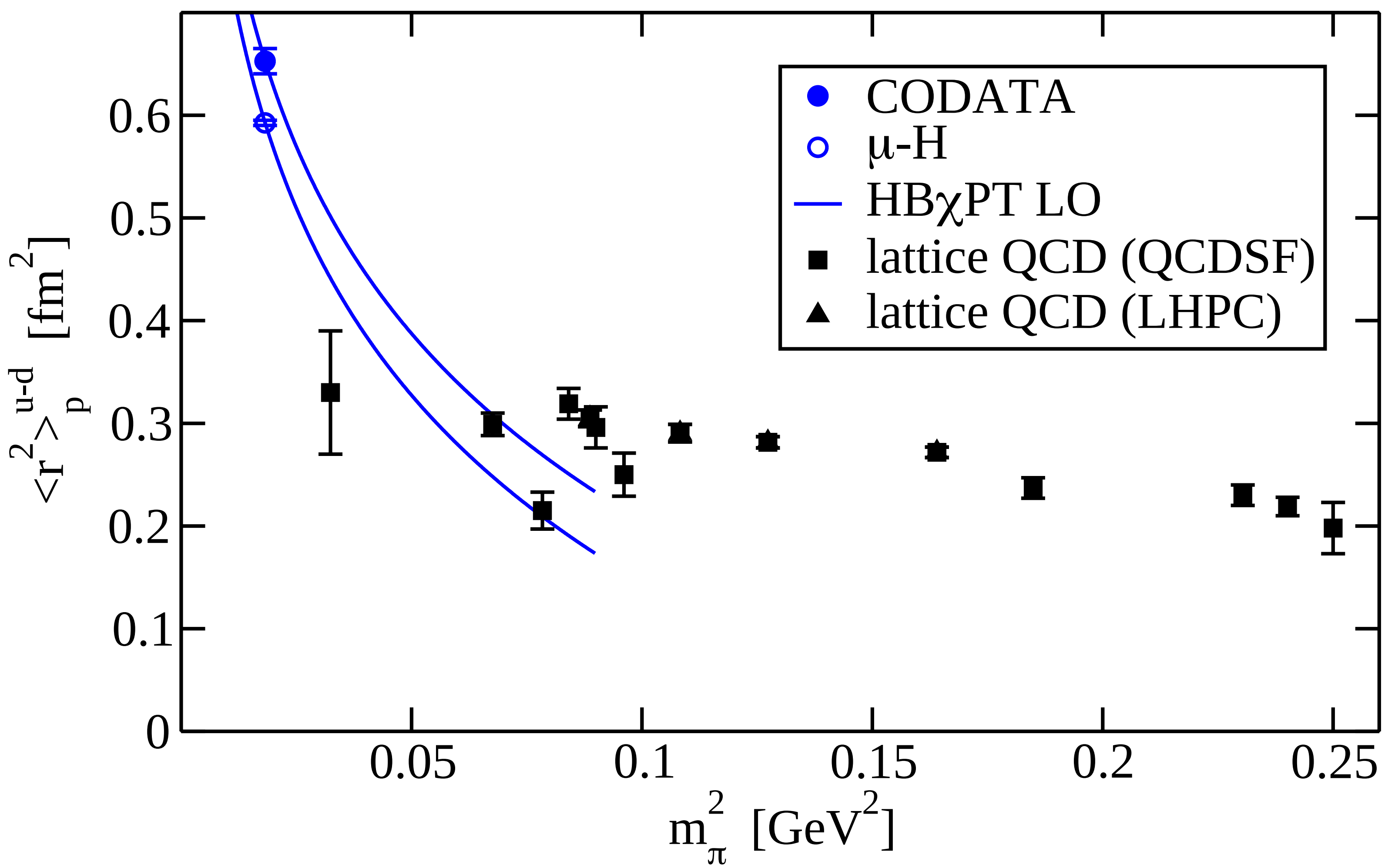
$$\langle N, p | V_\mu | N, p' \rangle = K_\mu^1 F_1(Q^2) + K_\mu^2 F_2(Q^2)$$

now, charge radius and anomalous magnetic moment

$$\langle r^2 \rangle_p \equiv 6 \left. \frac{dF_1(Q^2)}{dQ^2} \right|_{Q^2 \to 0} \qquad \kappa = \lim_{Q^2 \to 0} F_2(Q^2)$$

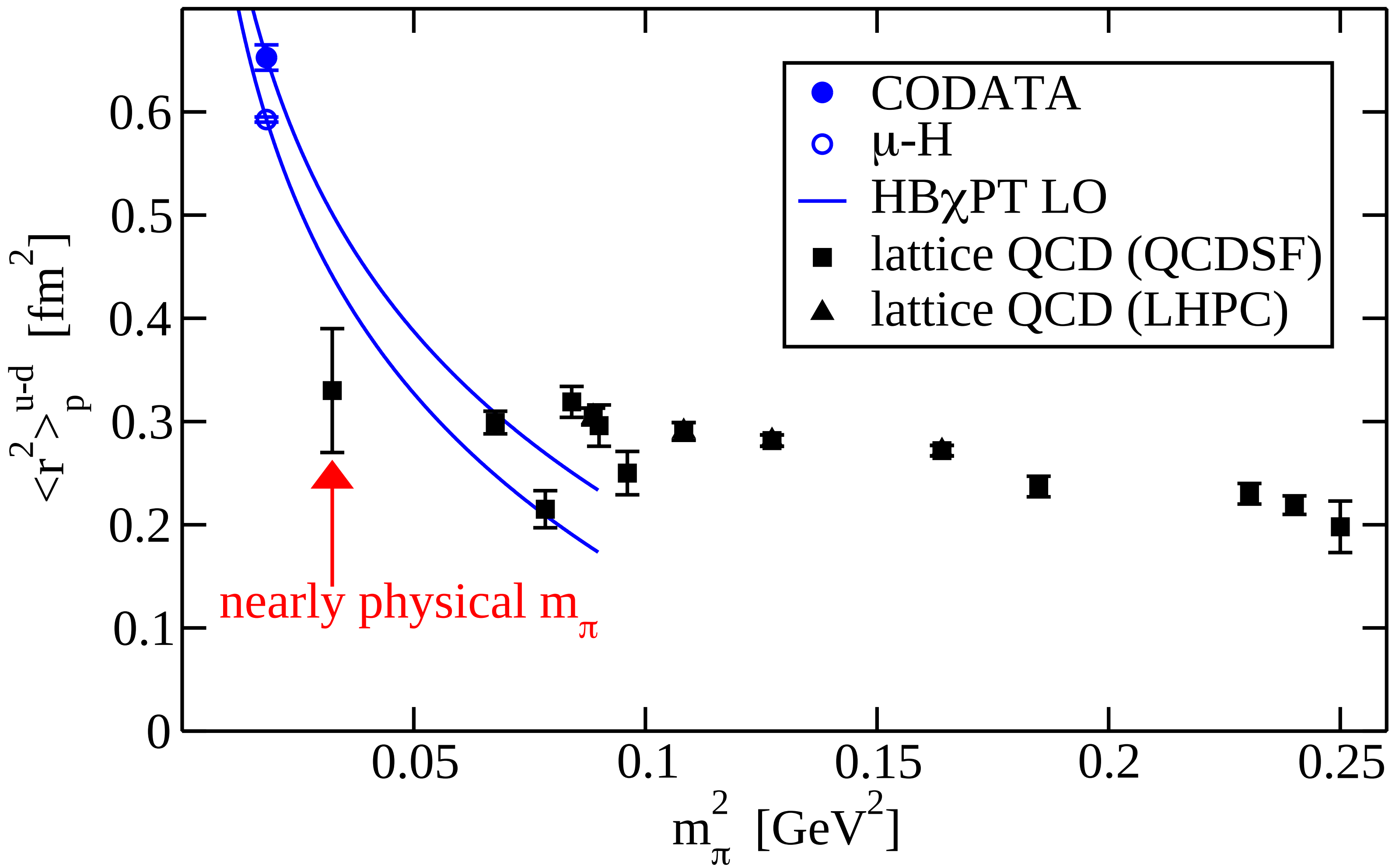
no renormalization is required but $\mathrm{Q}^2 \neq 0$ is needed

Proton charge radius



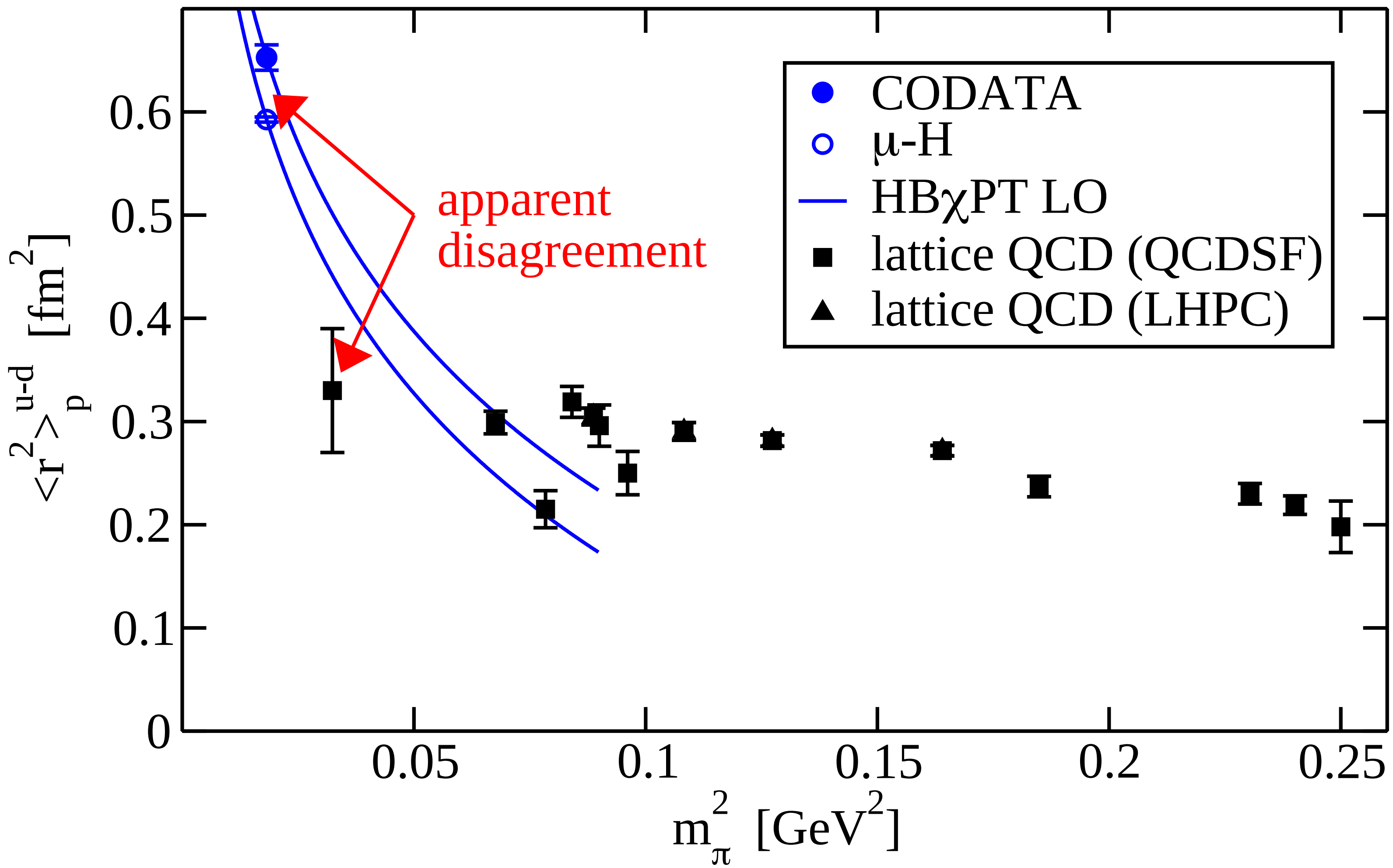
no reliable extrapolation, examine raw lattice results

Proton charge radius



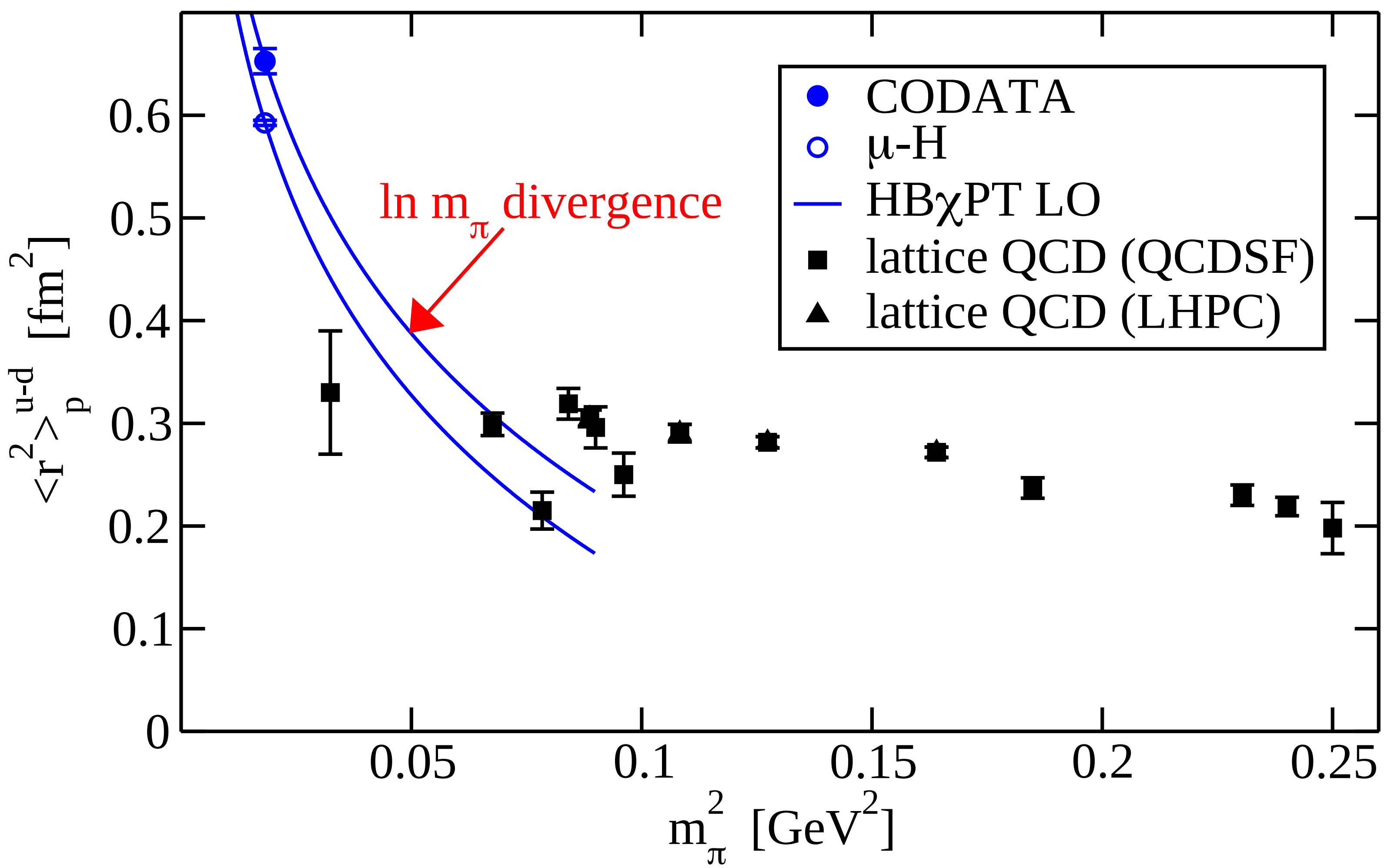
calculations nearly to the physical m_π a major triumph

Proton charge radius



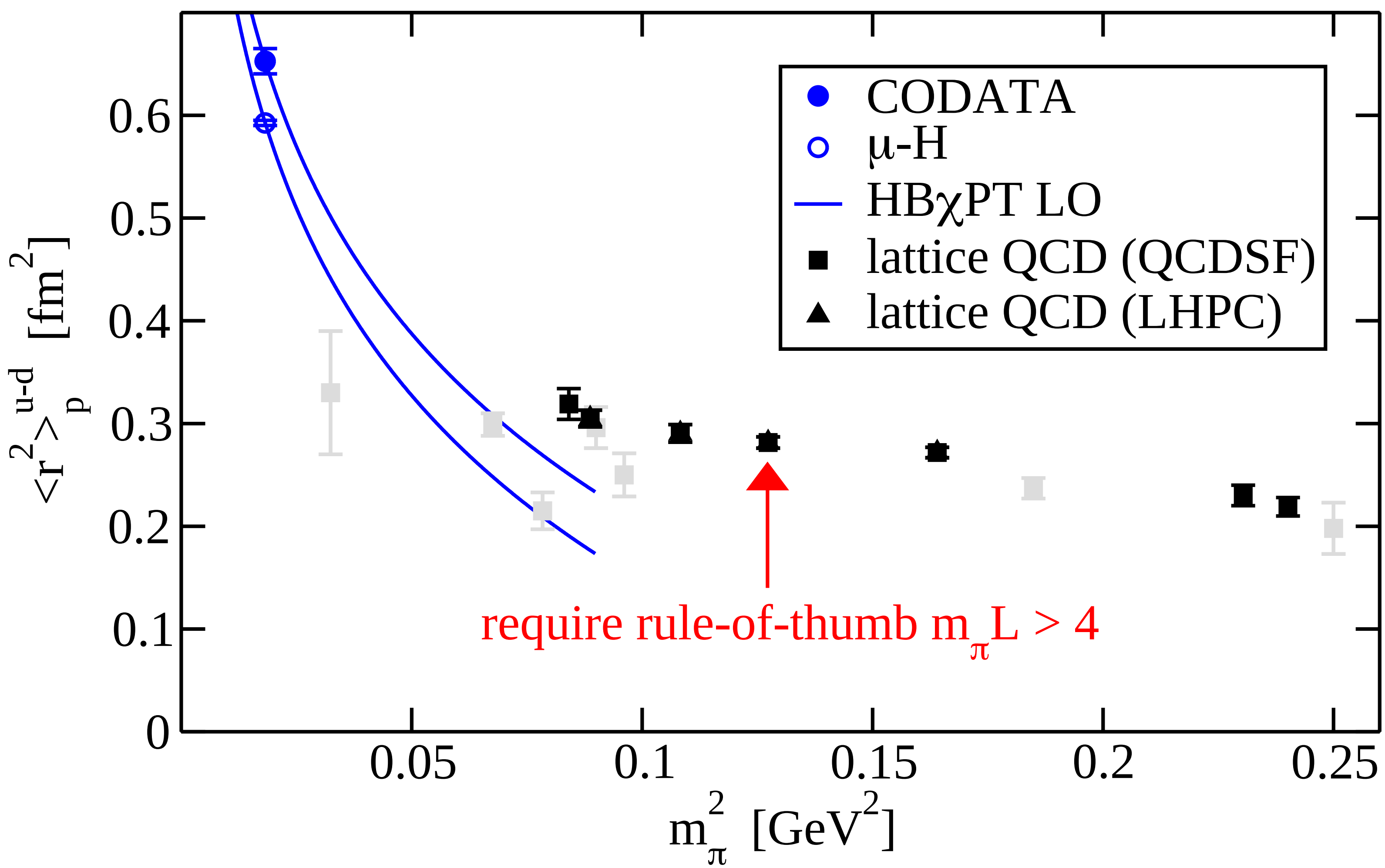
but well-controlled calculations needed to resolve this

Proton charge radius



divergence suggests possibly large finite-size effects

Proton charge radius



finite-size effects at small m_π may resolve this puzzle

Proton axial coupling should be easier

proton requires two axial form factors, g_A and g_P

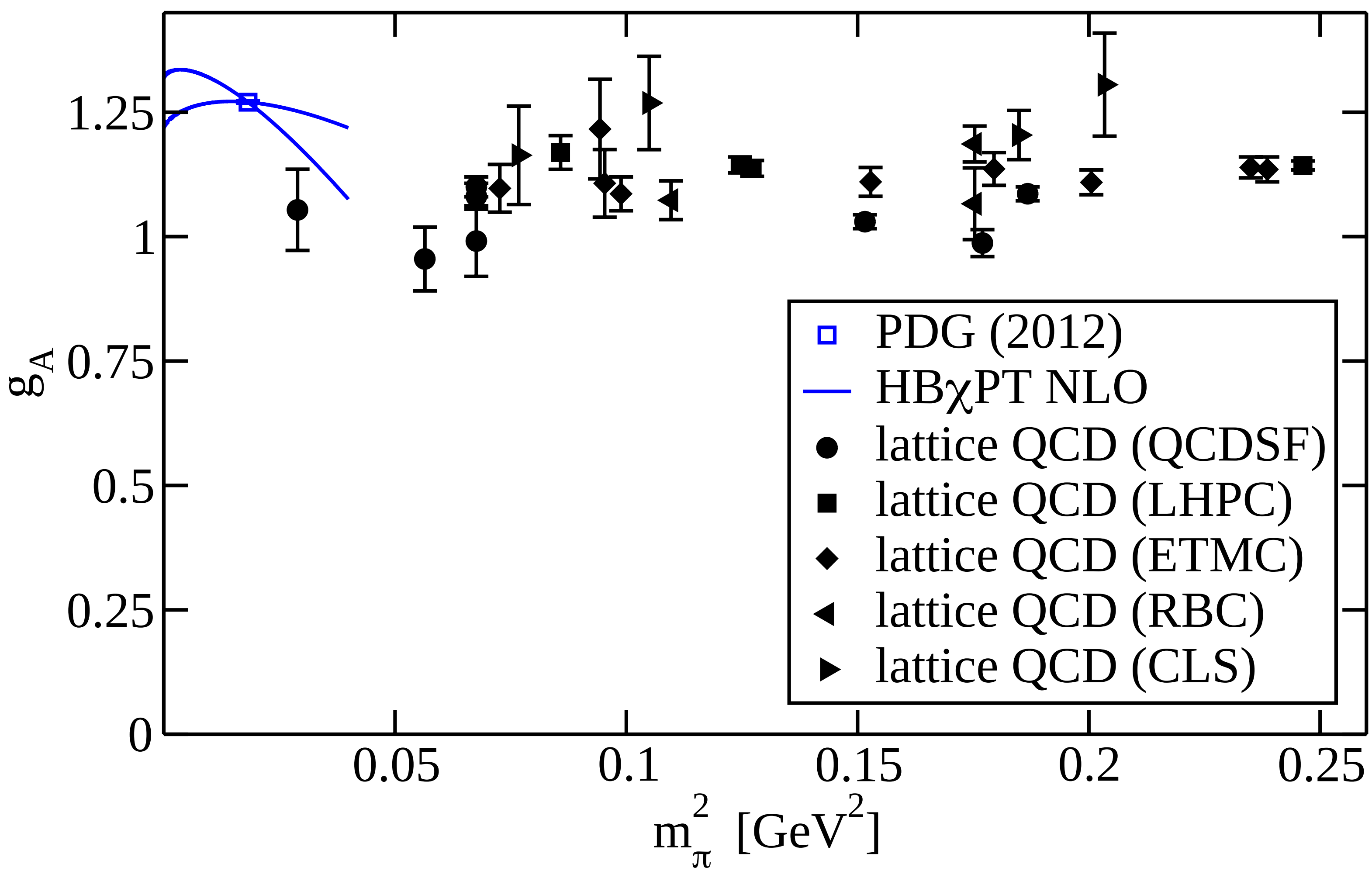
$$\langle N, p | A_\mu^{u-d} | N, p' \rangle = K_\mu^A g_A(Q^2) + K_\mu^P g_P(Q^2)$$

now both form factors have non-trivial forward limits

$$g_A = g_A(Q^2{=}0) \qquad g_P = \lim_{Q^2 \to 0} g_P(Q^2)$$

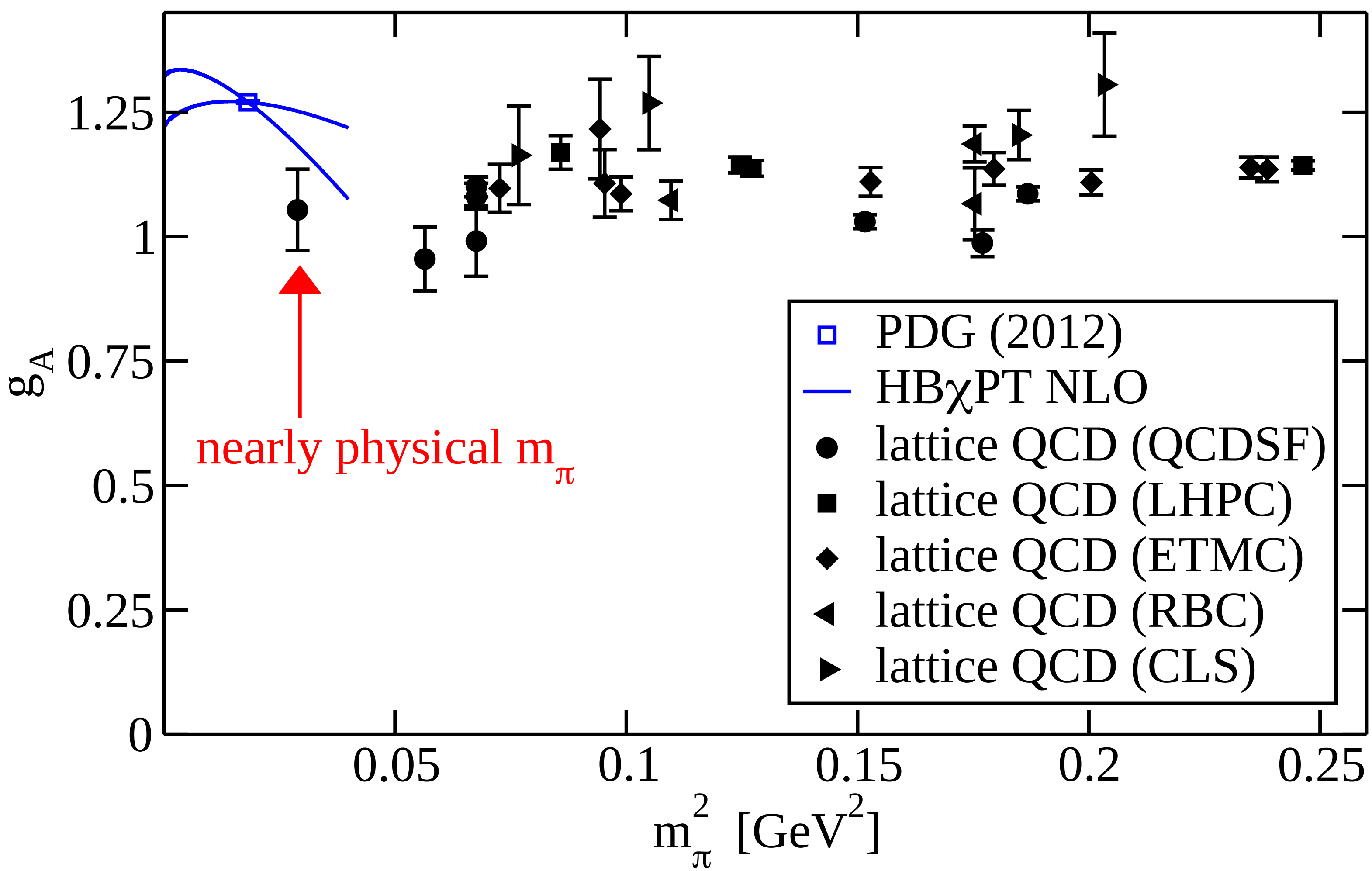
$Q^2 \neq 0$ is not needed for g_A, but a finite Z_A is required

Proton axial coupling



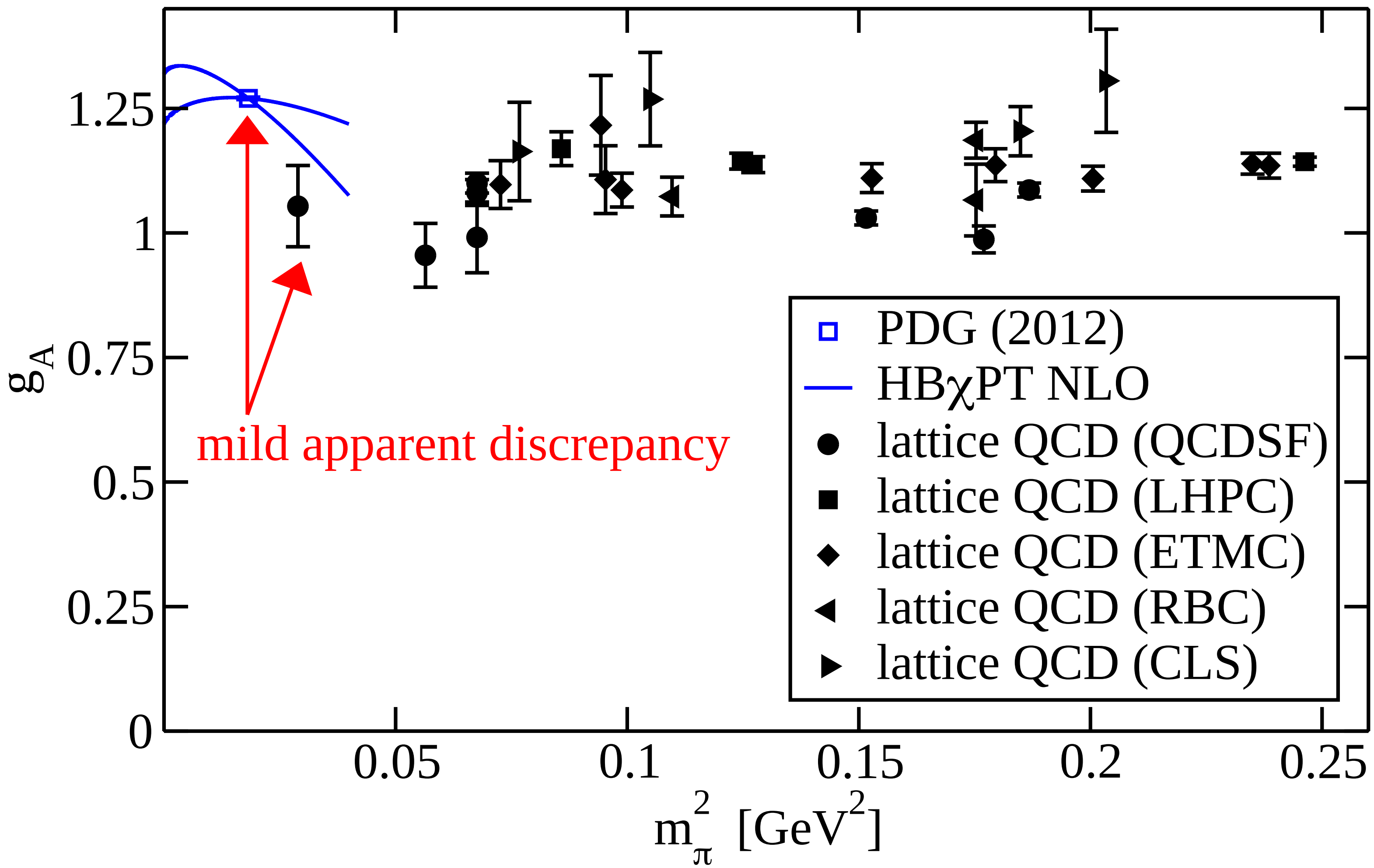
axial coupling has persistently been flat and too low

Proton axial coupling



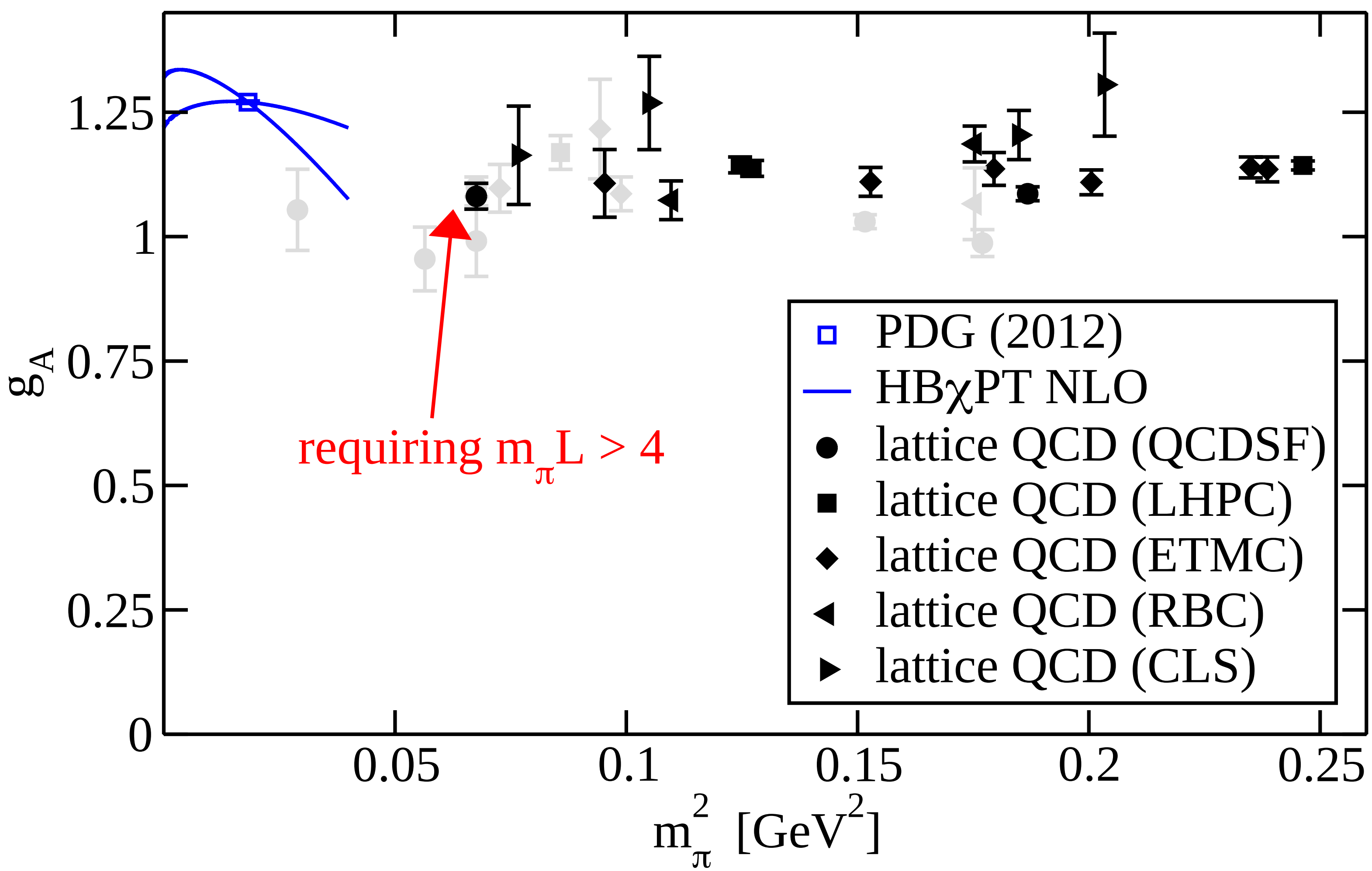
results approaching the physical point are still too low

Proton axial coupling



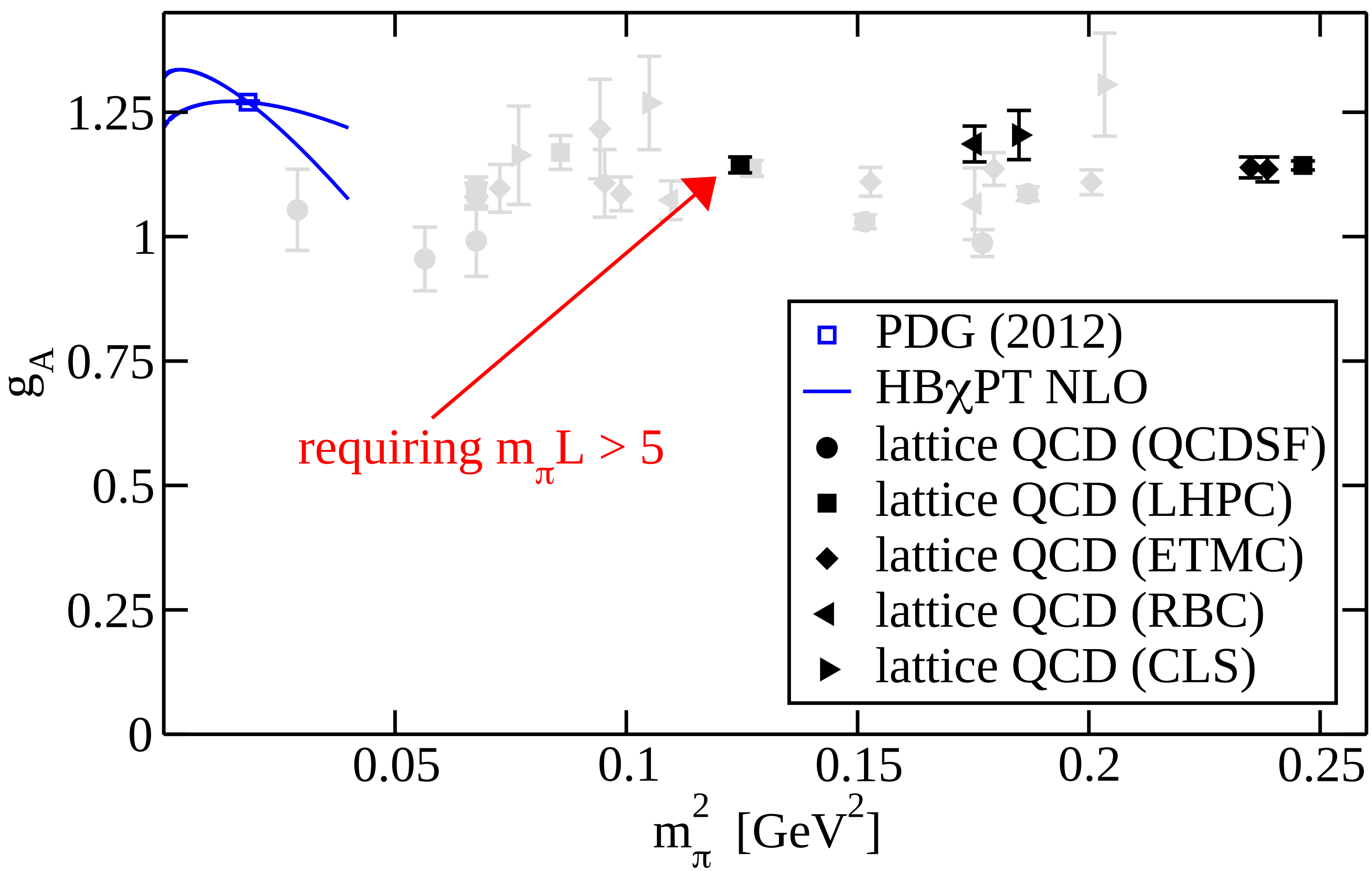
but apparent discrepancy is not too large to start with

Proton axial coupling



rule-of-thumb $m_\pi L > 4$ is known to be insufficient here

Proton axial coupling



it is known that $m_\pi L > 6$ may even be necessary here

Proton parton distributions

GPDs related to generalized form factors $\mathrm{A_{ni}}$, $\mathrm{B_{ni}}$, $\mathrm{C_n}$

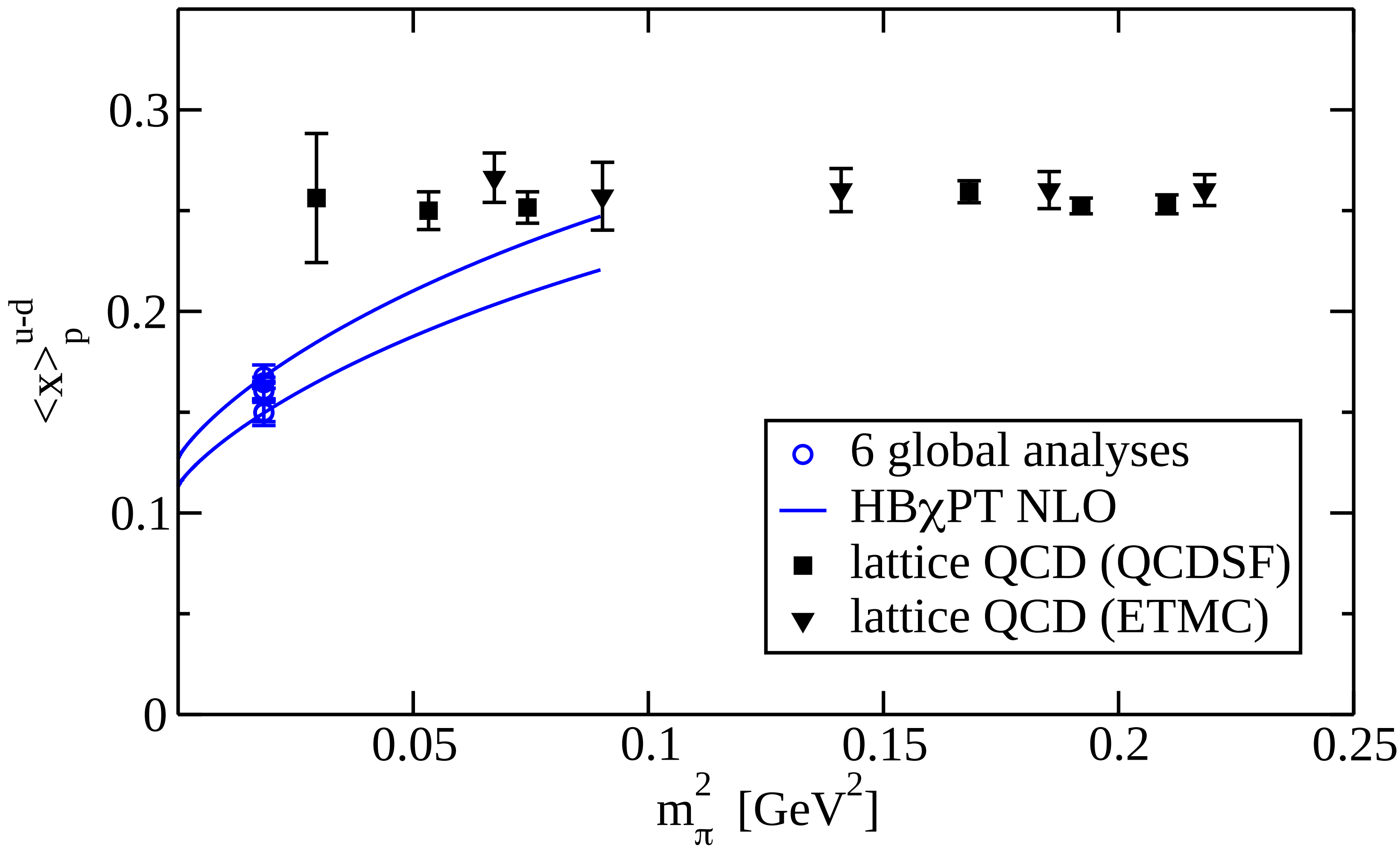
$$\langle N,p|O_{\mu\nu}|N,k\rangle = K^A_{\mu\nu}A_{20}(Q^2) + K^B_{\mu\nu}B_{20}(Q^2) + K^C_{\mu\nu}C_2(Q^2)$$

like $\mathrm{F_1}$ or $\mathrm{g_A}$, only $\mathrm{A_{n0}}$ has an accessible forward limit

$$\int_{-1}^{1} dx\, x\, q(x) = \langle x \rangle = A_{20}(Q^2{=}0)$$

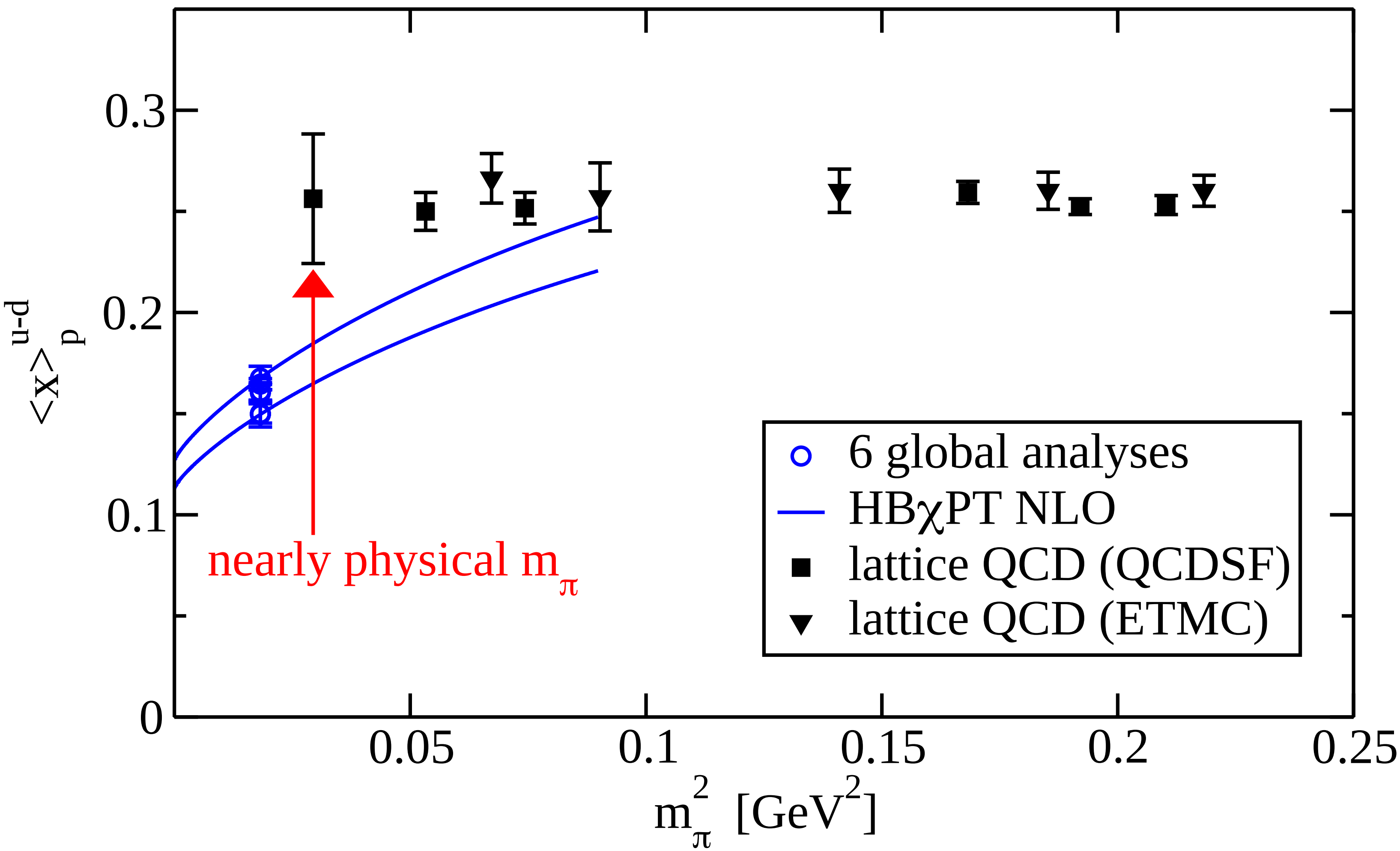
scale-dep. renormalization is needed but $\mathrm{Q^2} \neq 0$ is not

Proton momentum fraction



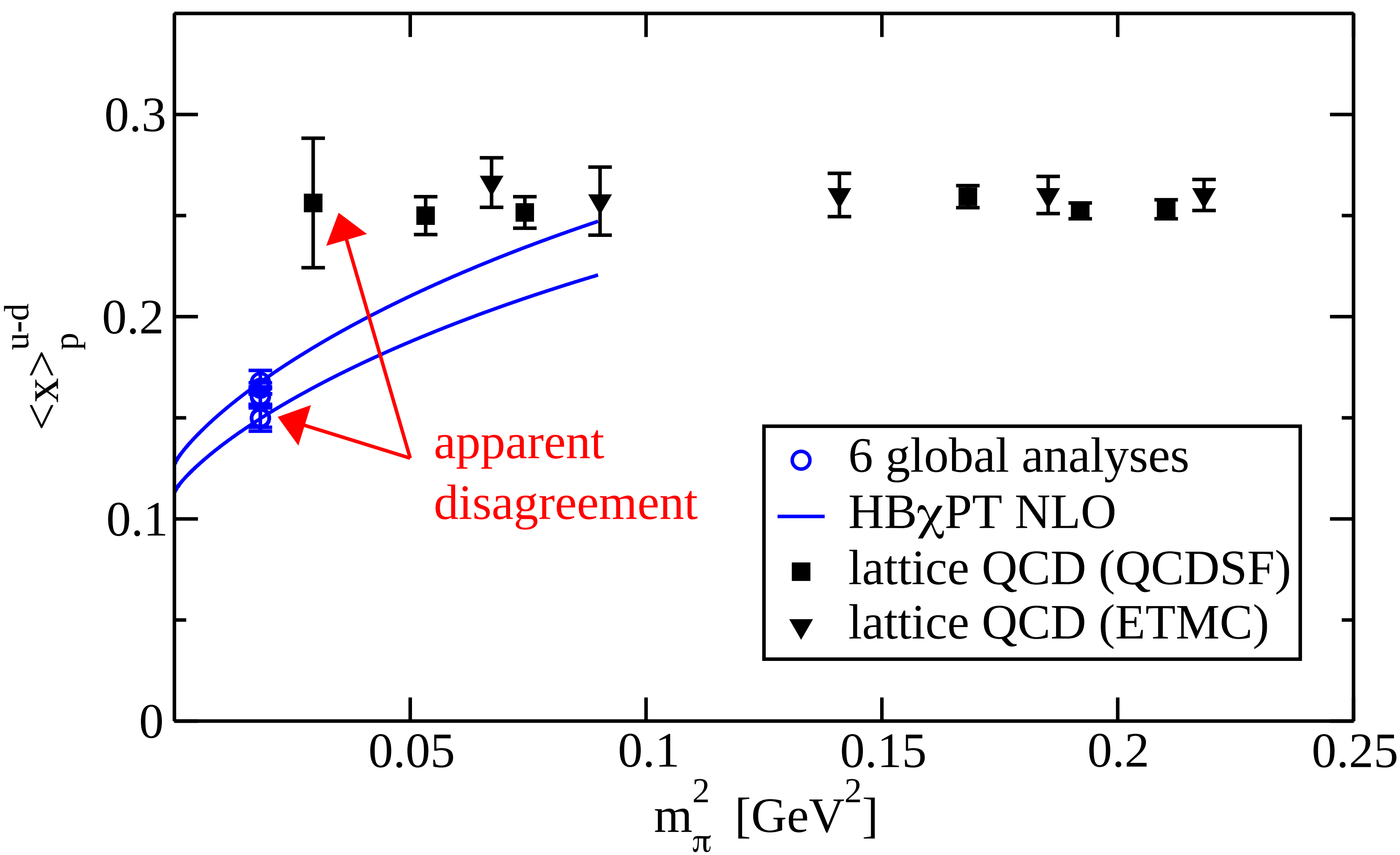
long standing trend for $\langle x \rangle^{u-d}$ to be quite flat in m_π

Proton momentum fraction



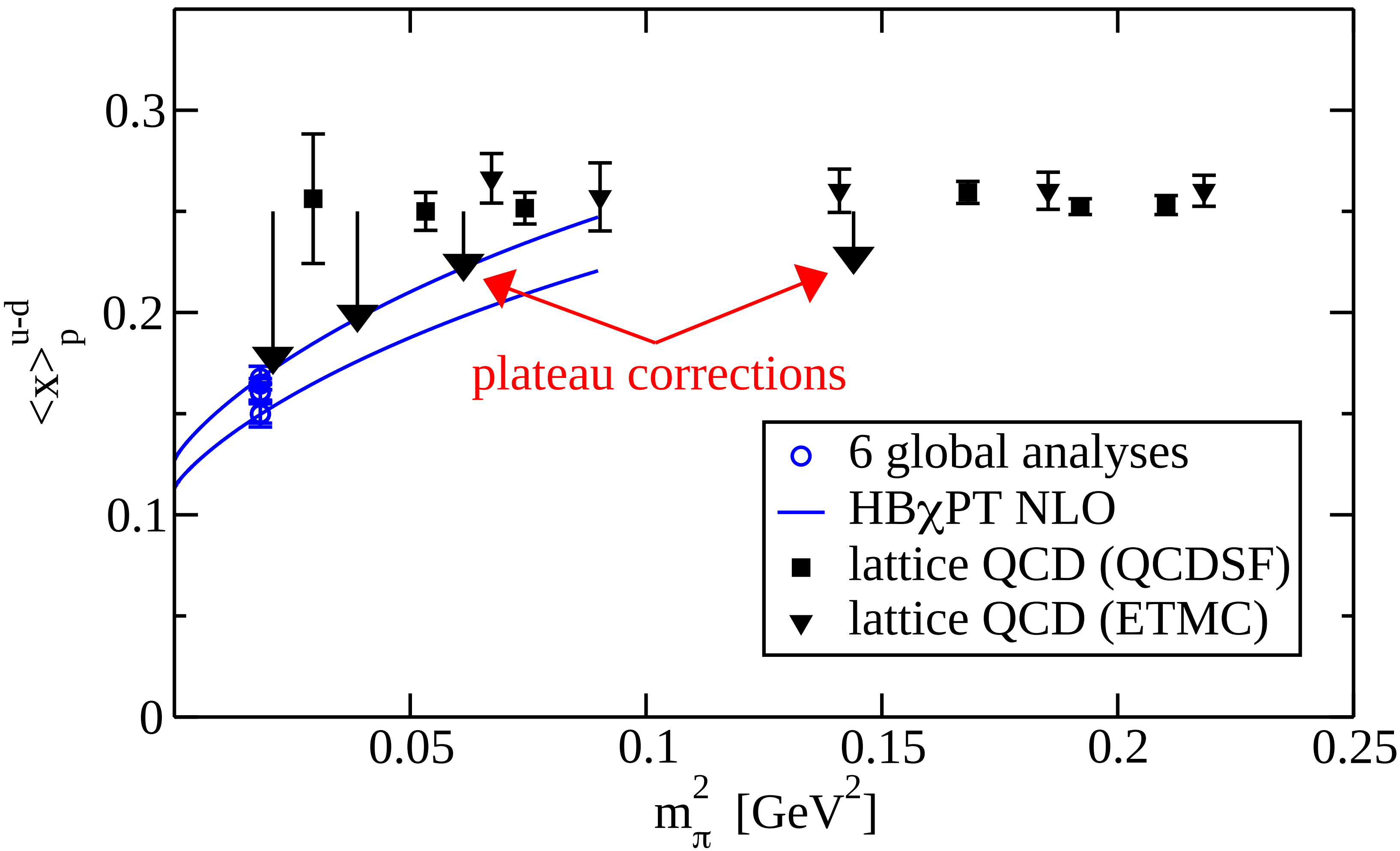
will return shortly to this result close to physical limit

Proton momentum fraction



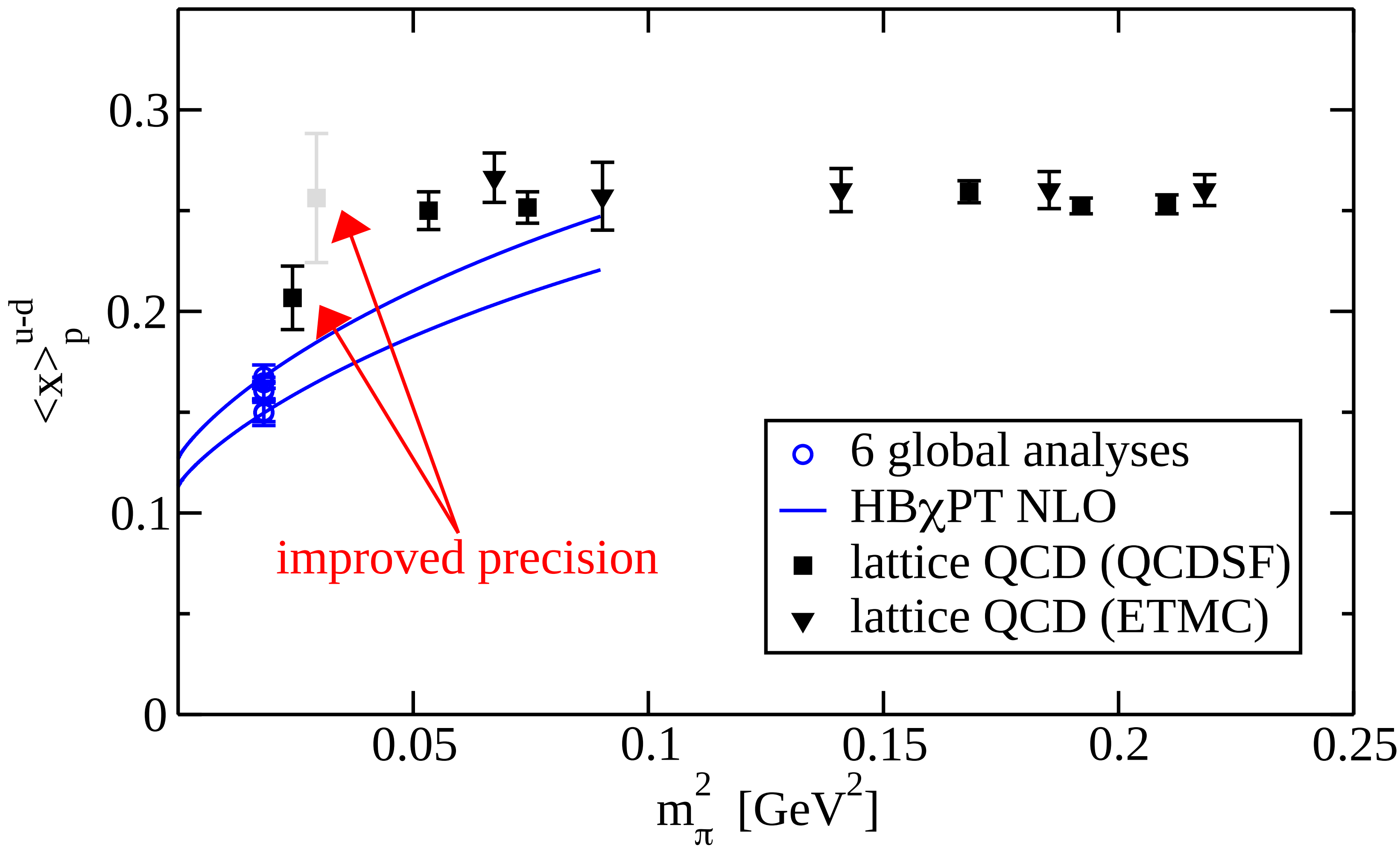
again, well-controlled calculations needed here too

Proton momentum fraction



corrections by LHPC/ETMC may resolve this puzzle

Proton momentum fraction



possible curvature but lightest point has $m_\pi L = 2.7$

Summary

Well-controlled calculations for the pion are feasible

Intense progress for the nucleon is being made

Apparent conflicts with measurements not justified

Apparent conflicts with χPT not compelling either