Lattice QCD Methods for Hadronic Polarizabilities

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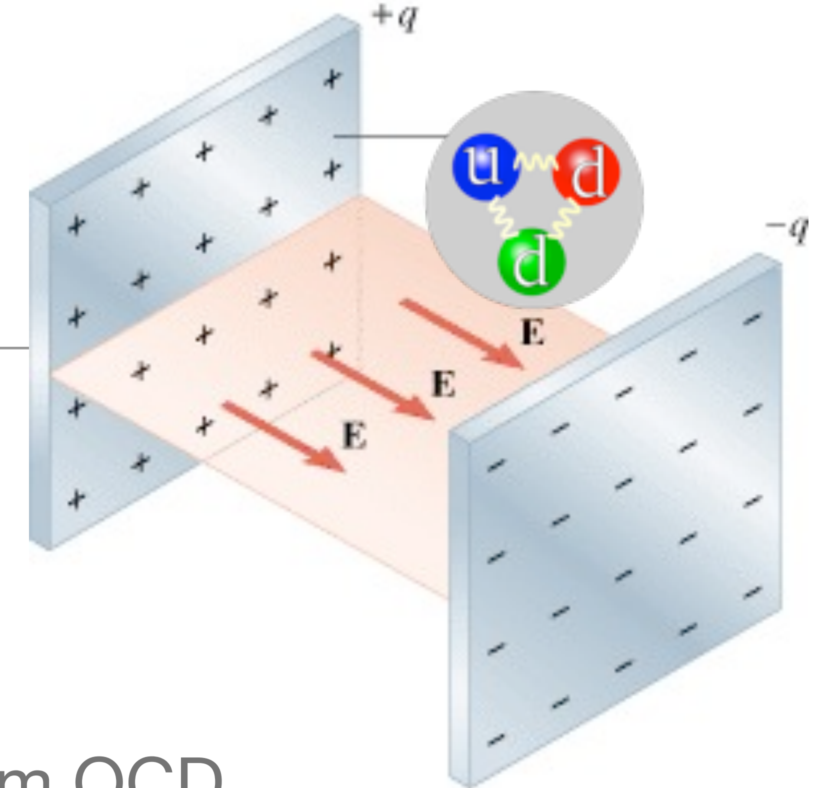
PSC CUNY

Overview



- Goal: determine electromagnetic polarizabilities from QCD

$$\alpha_E \quad \beta_M$$

- Confront: chiral perturbation theory predictions directly with QCD

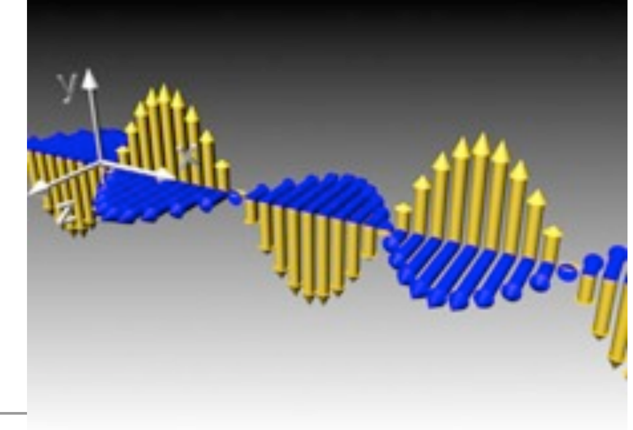
$$m_\pi$$

- Method: turn on external electromagnetic fields in lattice QCD

$$\vec{E} \quad \vec{B}$$

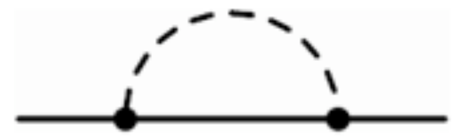
Based on work with: W. Detmold, A. Walker-Loud, S. Vayl

Electromagnetic Polarizabilities



- Stringent test of chiral perturbation theory description of low-energy QCD

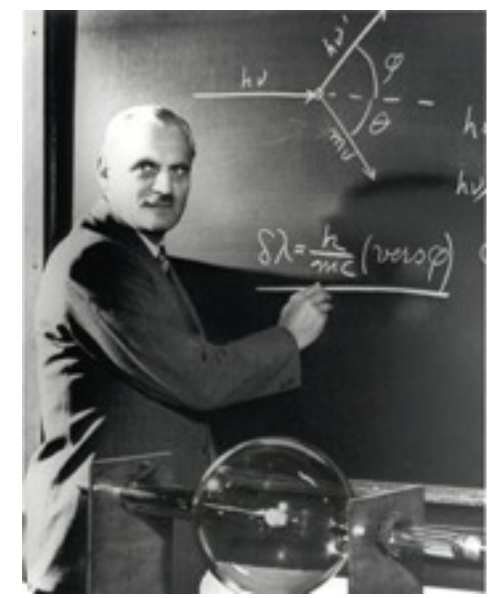
$$m_\pi^2 = 0 + \lambda\, m_q \langle \overline{\psi}\psi \rangle + \ldots$$



$$n \to \pi^- + p \to n$$

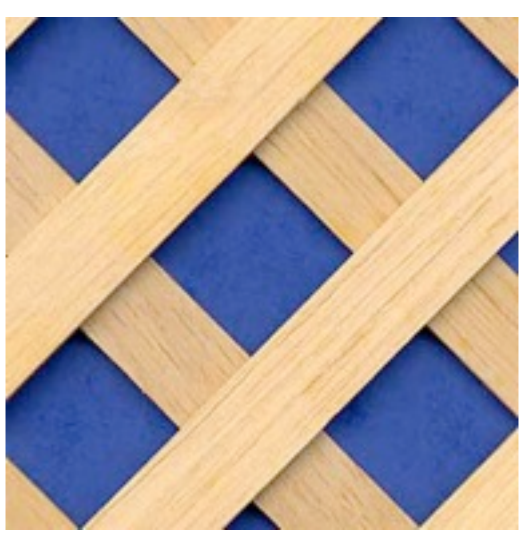
$$\alpha_E, \beta_M \sim \frac{1}{m_\pi}$$

- Experimentally accessible in Compton scattering.
  Caveats: low-energy expansion!, pions!, neutrons!



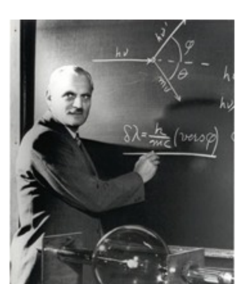
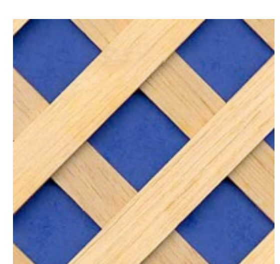
$$T^{\mu\nu}(k', k) = \int_{x,y} e^{ik\cdot y - ik'\cdot x} \langle H | T\left\{ J^\mu(x) J^\nu(y) \right\} | H \rangle$$

- Turn to Lattice QCD:
  first principles computations, stable pion, neutron "targets"



CHIRAL CORRECTIONS $m_\pi^2 \log m_\pi^2$

CHIRAL DOMINANCE $1/m_\pi$

 +  =?

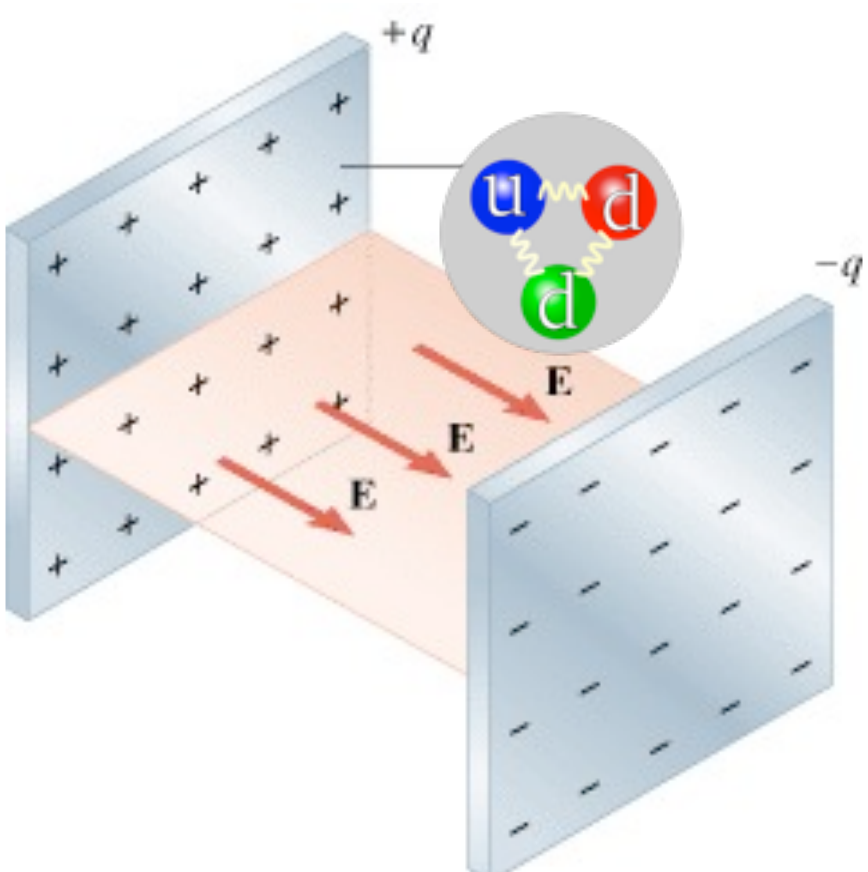
$$\psi(x+L) = \psi(x)$$

$$k = \frac{2\pi n}{L} \sim 400\,\mathtt{MeV}$$

Lattice QCD in External Fields

Couple classical electromagnetic fields to quarks and then study hadrons

$$D_\mu = \partial_\mu + ig\, G_\mu + iqA_\mu$$



Gauge links

$$U_\mu(x) = e^{igG_\mu(x)} \in SU(3)$$

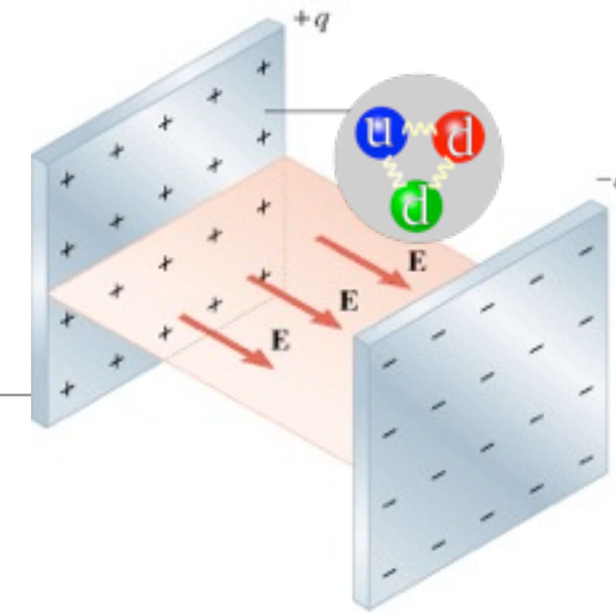
$$U_\mu^{\text{e.m.}}(x) = e^{iqA_\mu(x)} \in U(1)$$

In our exploratory studies:
U(1) field couples only to valence quarks

ChPT predicts the sea quark charge dependence of polarizabilities

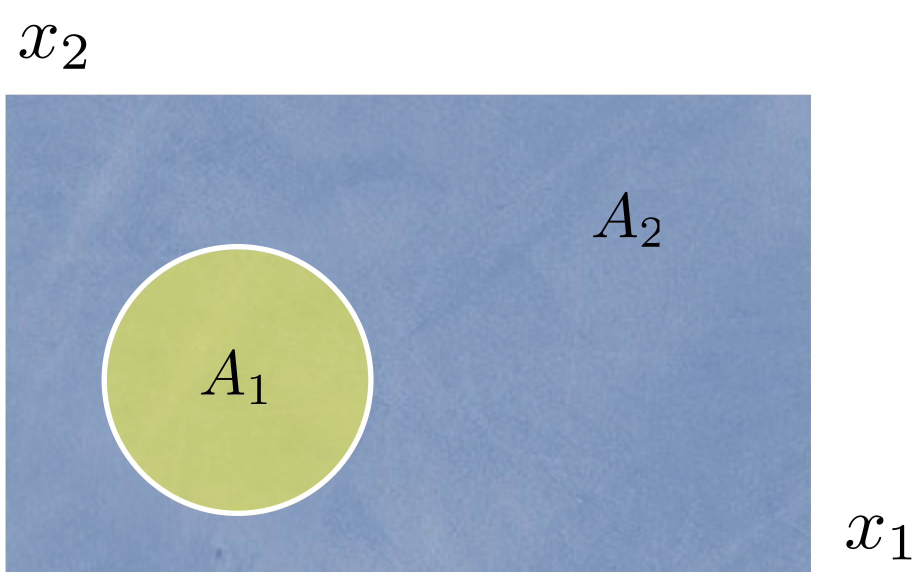
Detmold, Tiburzi, Walker-Loud, PRD 2006 Hu, Jiang, Tiburzi, PRD 2008

Lattice QCD in External Fields



Couple classical electromagnetic fields to quarks ...

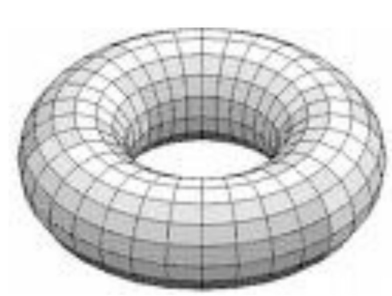
Magnetic field $\vec{B} = B\hat{x}_3$ Electric field $\vec{\mathcal{E}} = \mathcal{E}\hat{x}_3$



Gauge links

$$U_\mu^{\text{e.m.}}(x) = e^{iqA_\mu(x)} \in U(1)$$

Torus

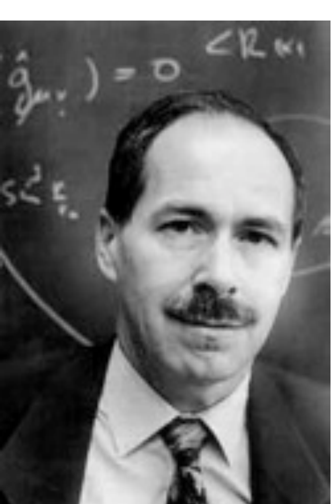


$$e^{iqBA_1} = e^{-iqBA_2}$$

$$A_1 + A_2 = L_1 L_2$$

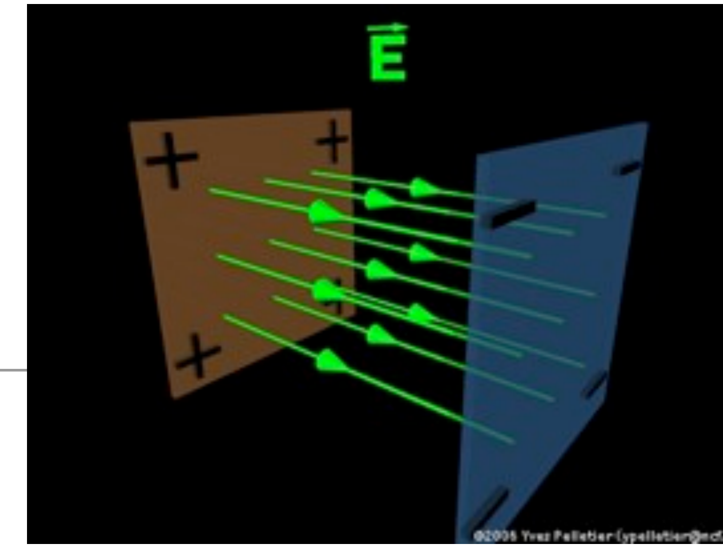
‘t Hooft quantization

$$q\,\mathcal{E} = \frac{2\pi n}{\beta L}$$



... and then study hadrons

Lattice QCD in External Fields



Method basics are basic

- Measure hadronic correlation functions in classical electromagnetic fields
- Study field strength dependence to determine parameters in effective action

E.g. neutral pion in electric field

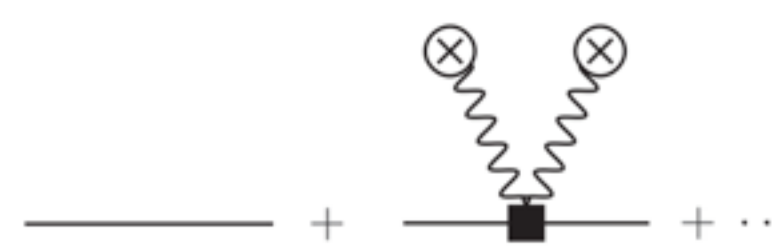
$$A_\mu = -\mathcal{E} x_4 \delta_{\mu 3}$$

Effective action $$\mathcal{L}(\vec{p} = 0, x_4) = \frac{1}{2}\pi^0 \left[-\partial_4 \partial_4 + m_\pi^2 + m_\pi \alpha_E \mathcal{E}^2\right] \pi^0$$

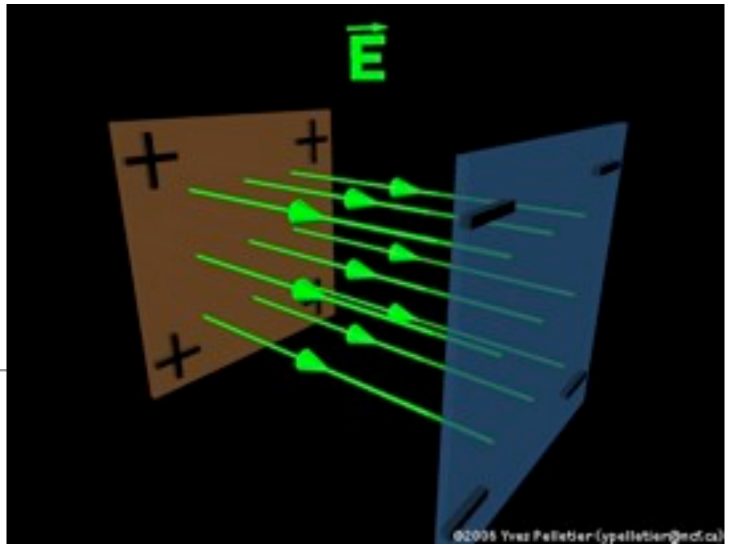
$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(\vec{0}, 0) \rangle = Z \exp(-E\tau)$$

$$E = m_\pi + \frac{1}{2}\alpha_E \mathcal{E}^2 + \ldots$$

Electric field strength dependence of energy yields polarizability



Lattice QCD in External Fields



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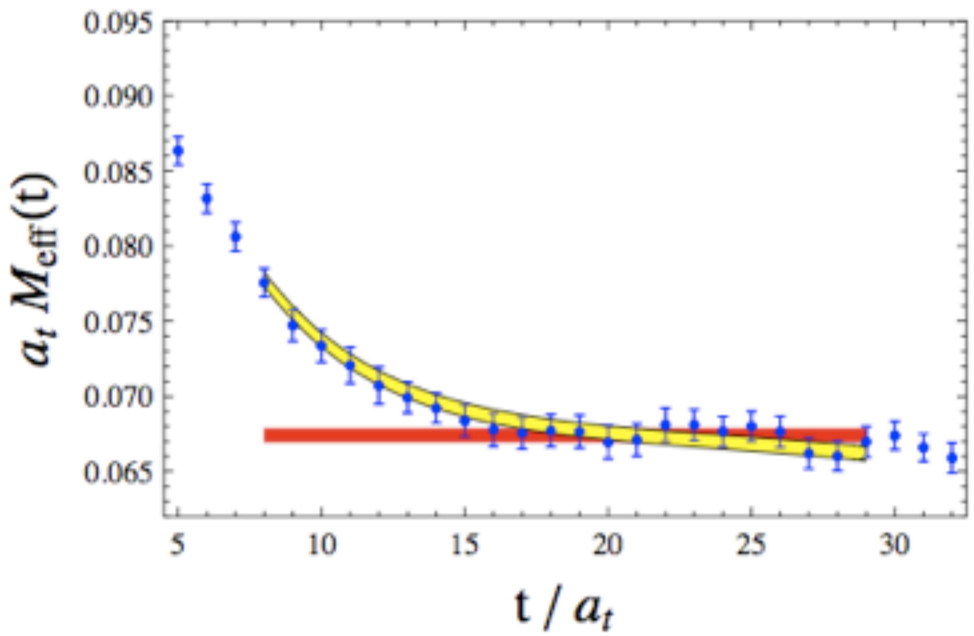
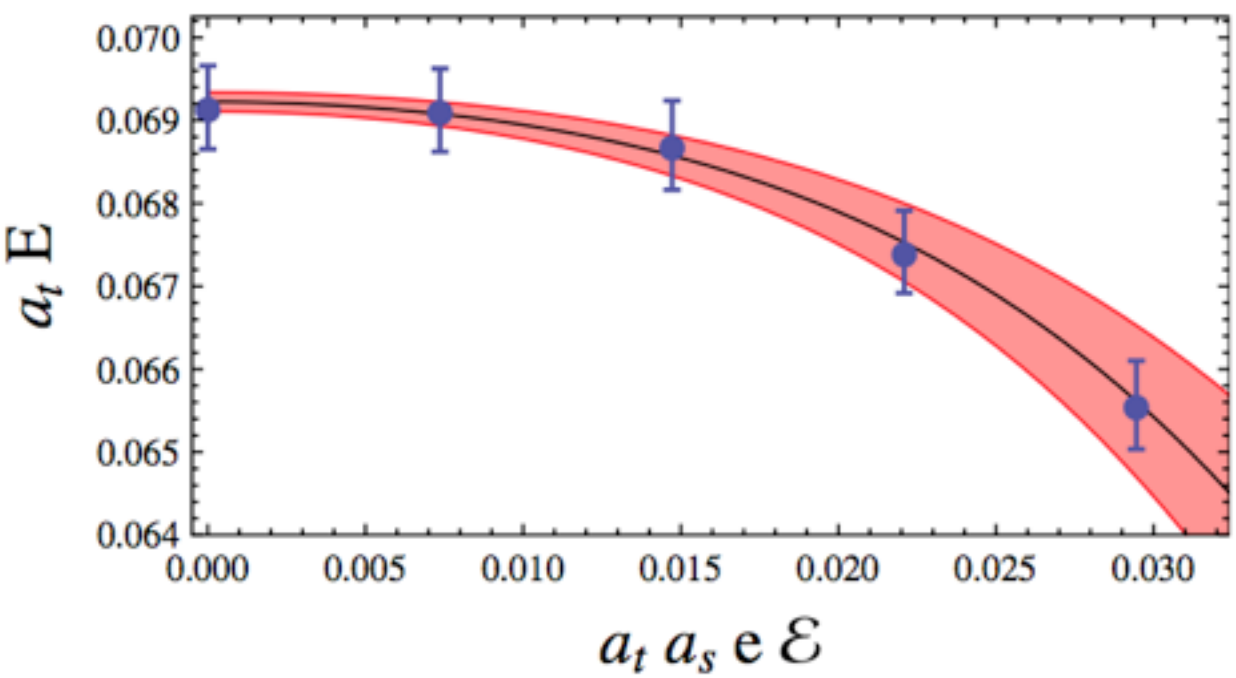


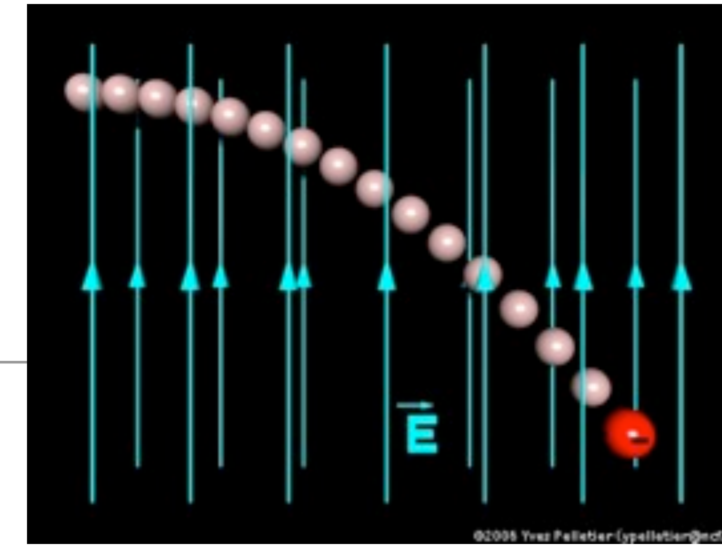


$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x},\tau)\chi^\dagger(\vec{0},0)\rangle = Z\exp(-E\tau)$$

Anisotropic clover lattices (HadSpec)

$20^3 \times 128 \qquad m_\pi = 390\,\texttt{MeV}$

Lattice QCD in External Fields



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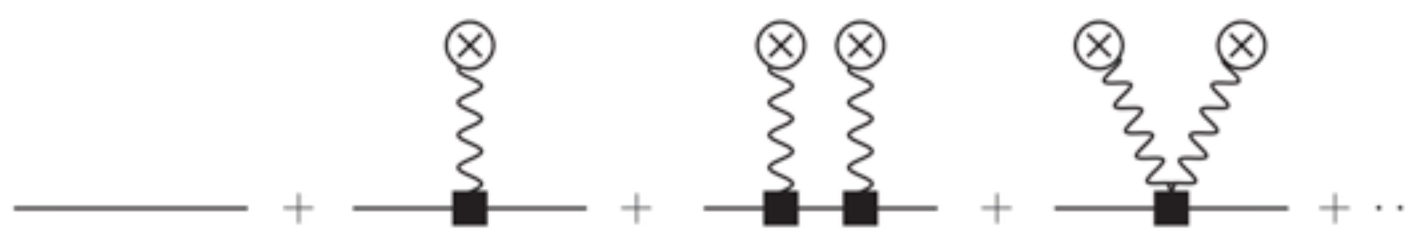

Electric field strength dependence of *rest energy* yields polarizability

$$G(\tau) = \langle \tau | \frac{1}{2\mathcal{H} + E^2} | 0 \rangle = \frac{1}{2}\int_0^\infty ds\, e^{-\frac{1}{2} s E^2} \langle \tau | e^{-s\mathcal{H}} | 0 \rangle$$

SCHWINGER, PR 1951
TIBURZI, NUPHA 2008

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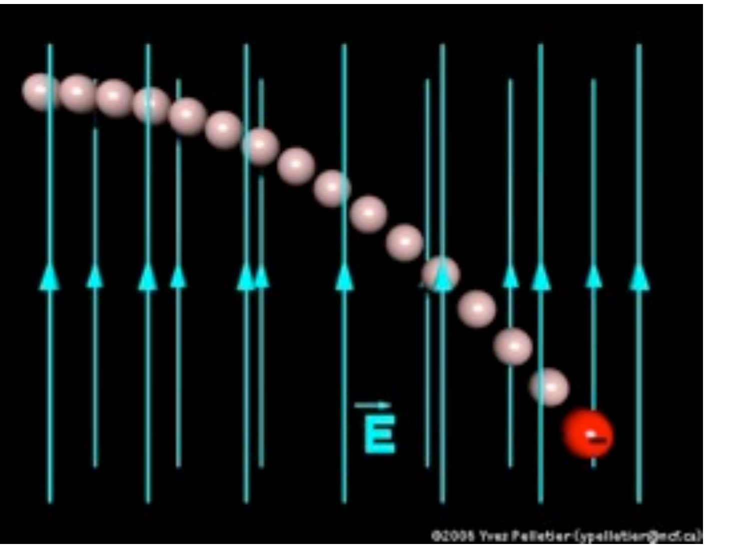

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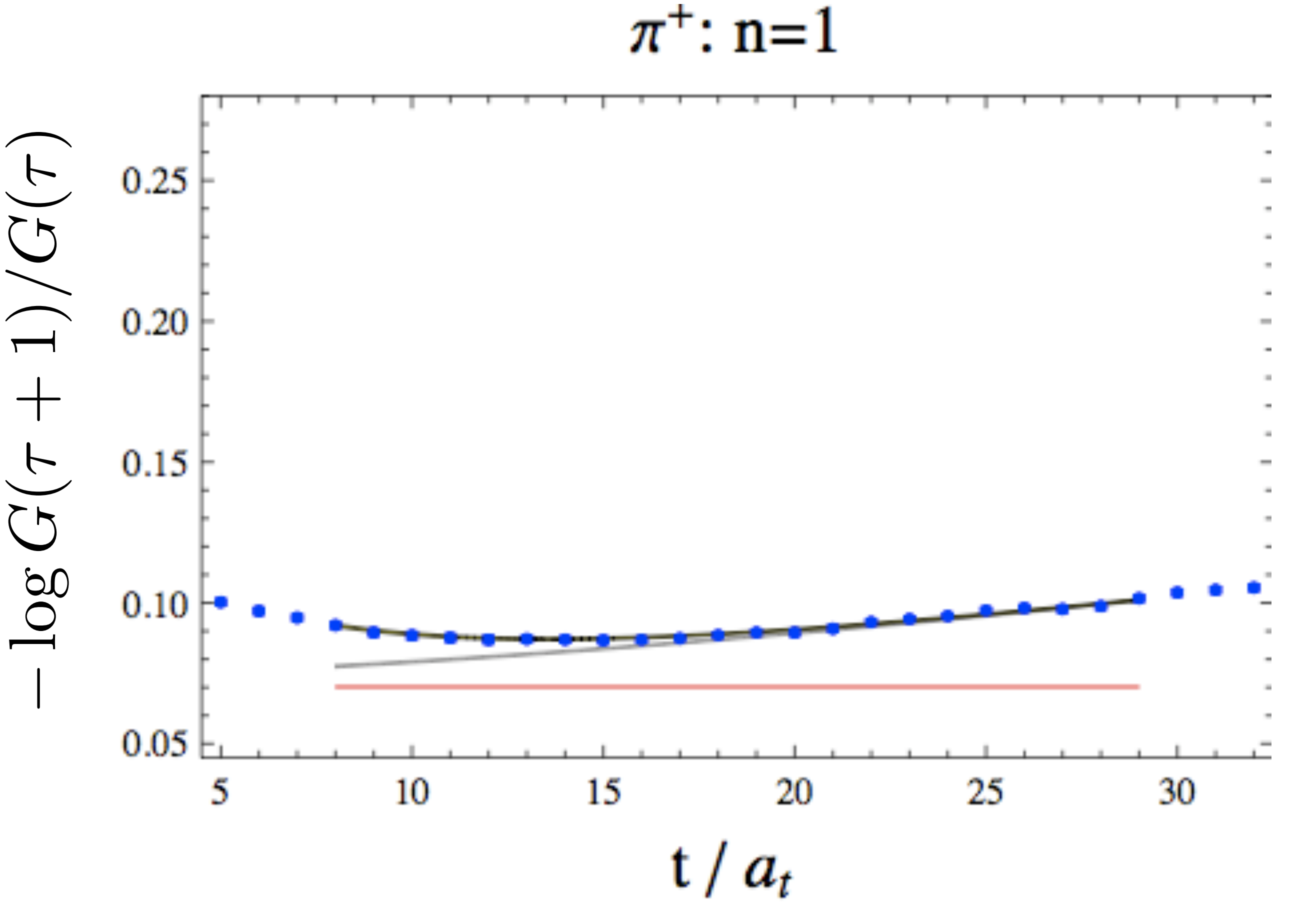

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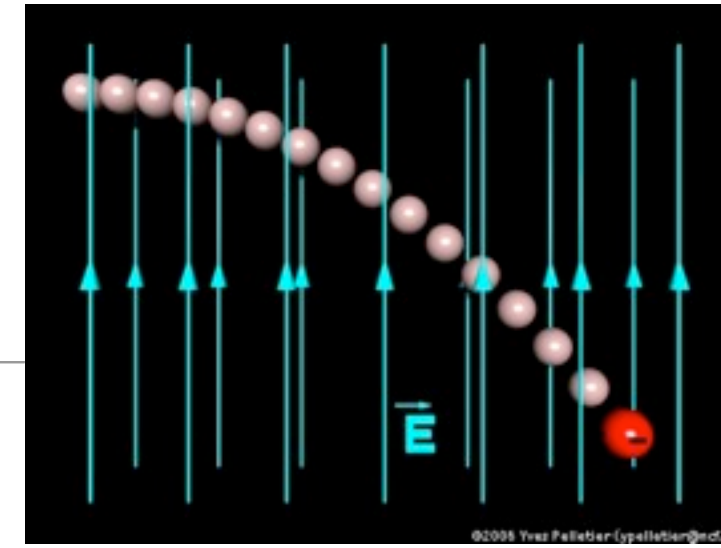

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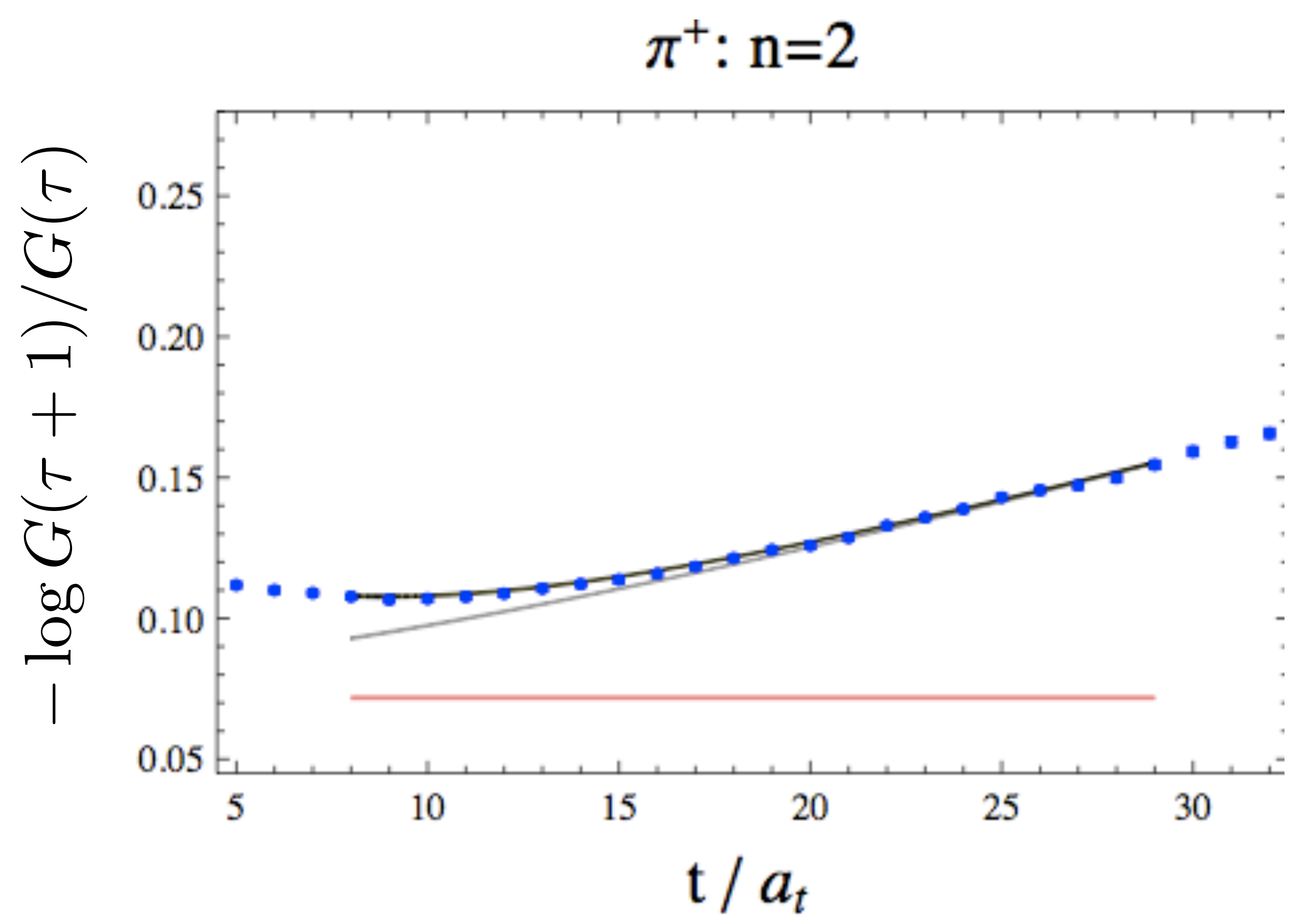

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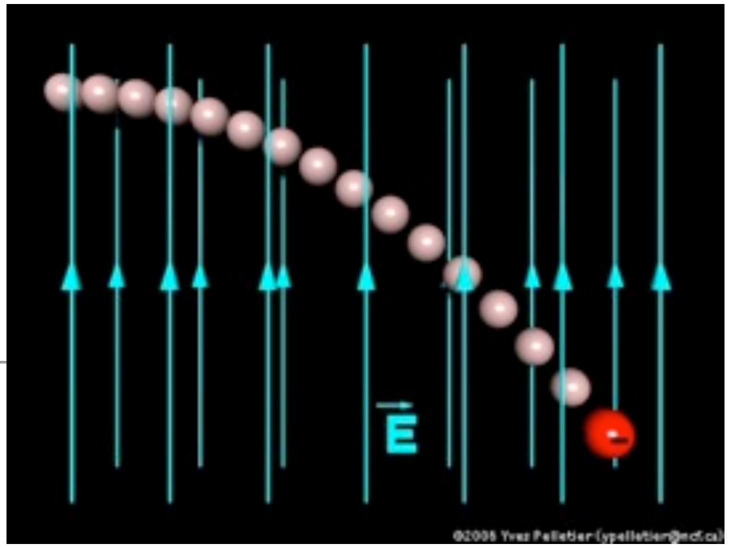

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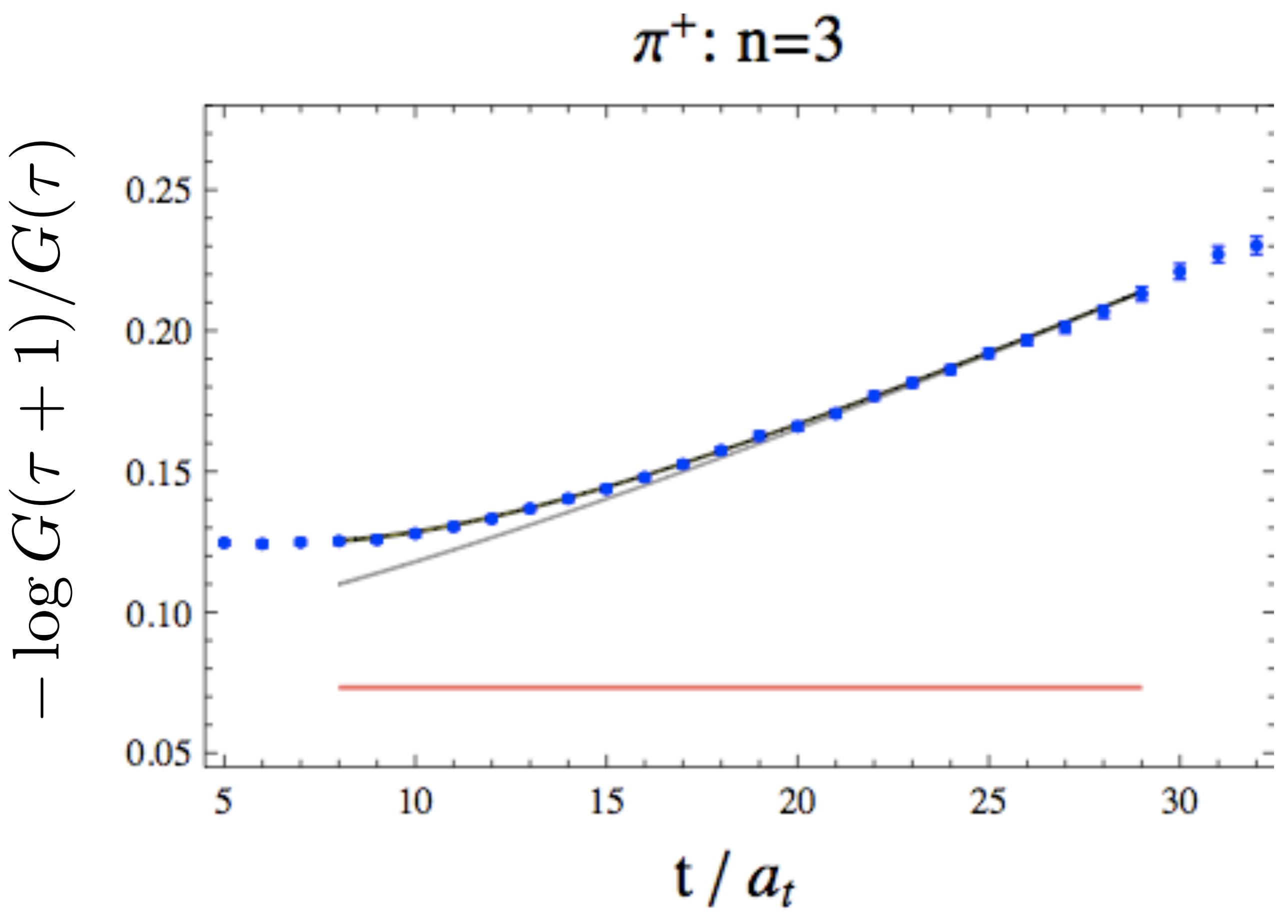

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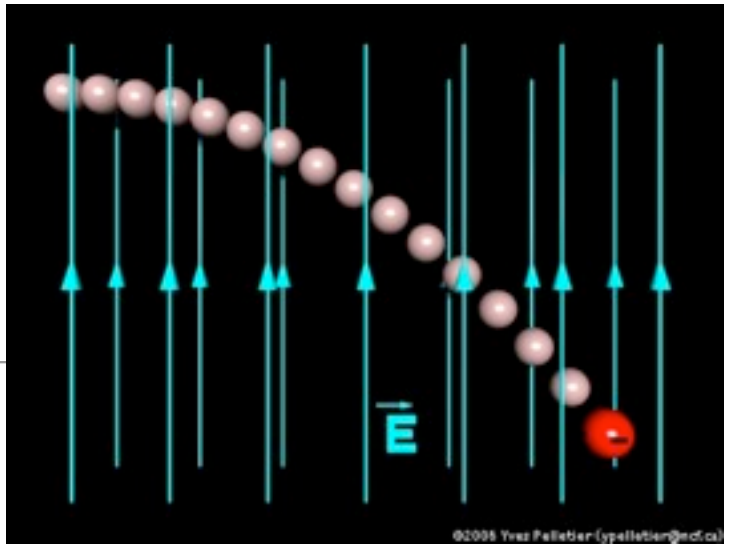

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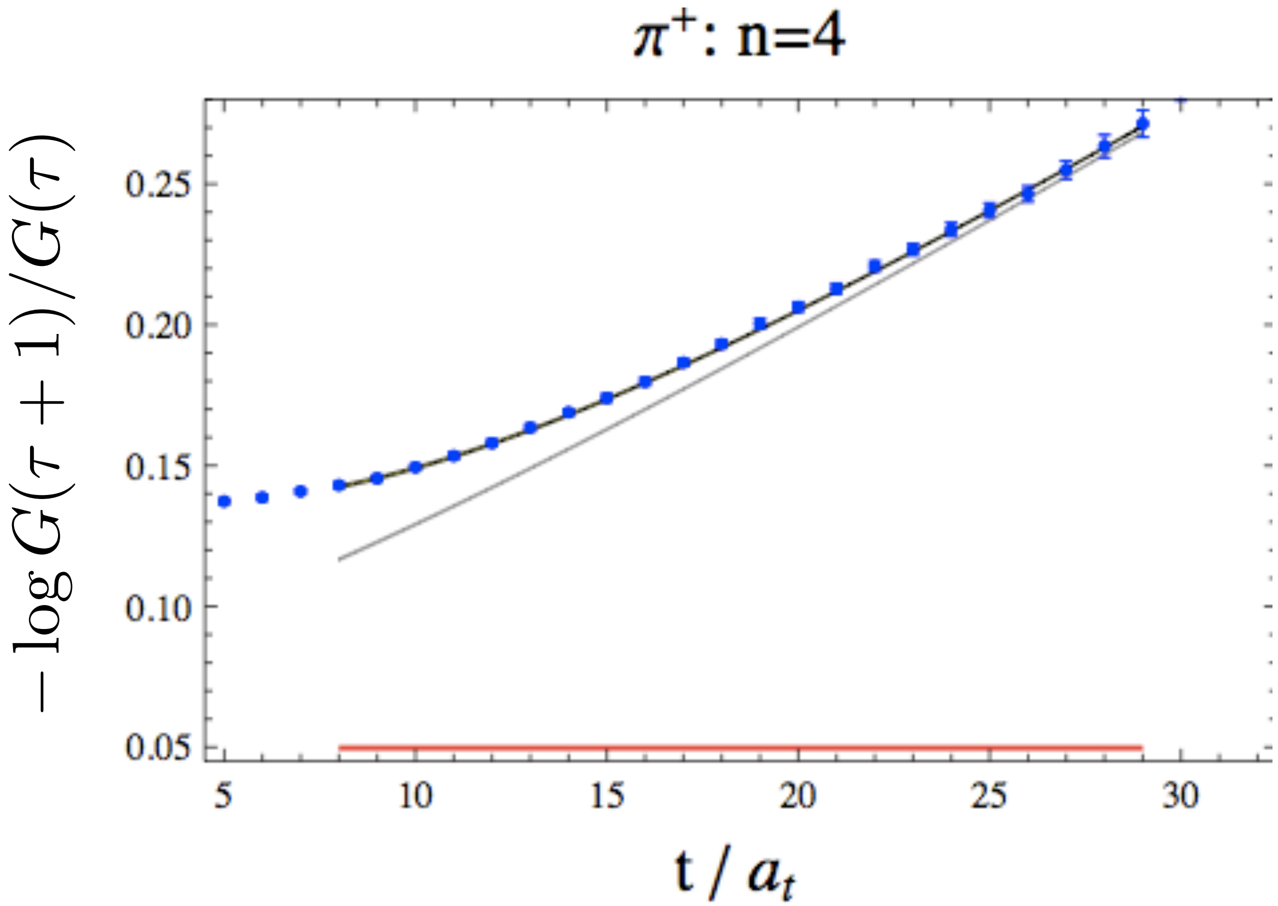

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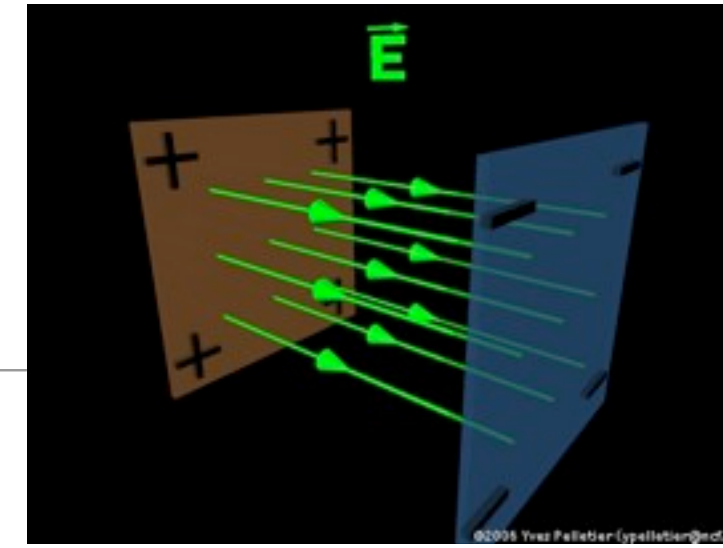
Lattice QCD in External Fields



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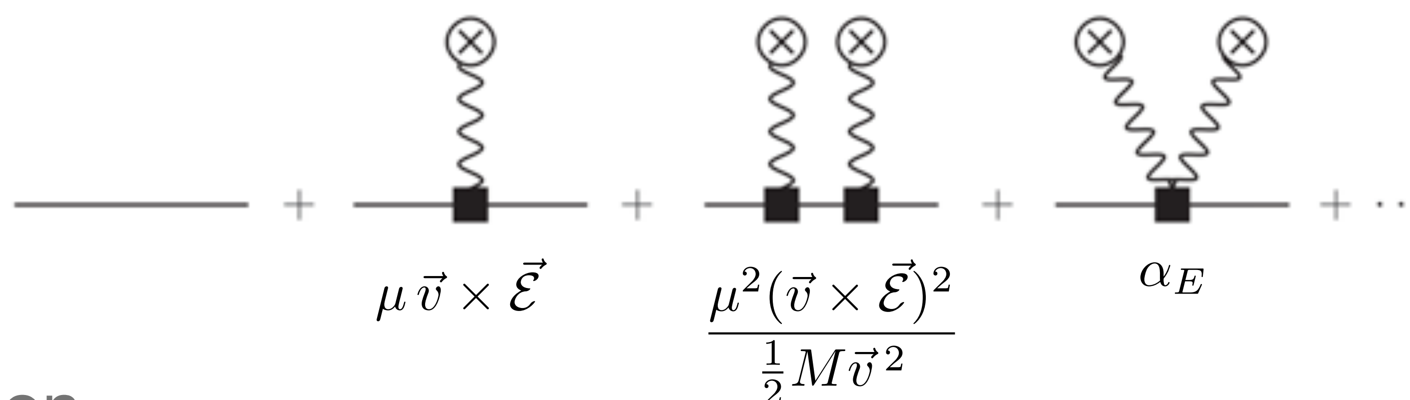
E.g. neutron in electric field

Detmold, Tiburzi, Walker-Loud, PRD 2010

Effective action $$\mathcal{L} = N^\dagger \left[\gamma_\mu \partial_\mu + E - \frac{\mu}{4M_N} \sigma_{\mu\nu} F_{\mu\nu} \right] N$$

$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(\vec{0}, 0) \rangle$$

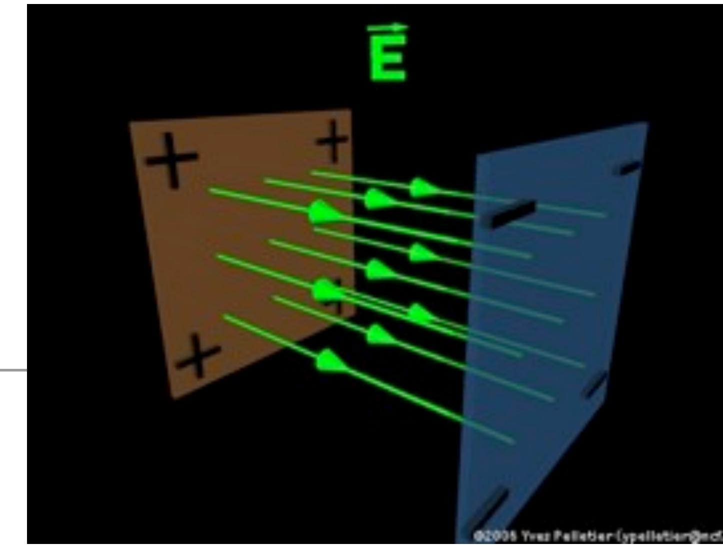


Unpolarized correlation function

$$G(\tau) = Z e^{-E_{\text{eff}}\, \tau}$$

$$E_{\text{eff}} = M + \frac{1}{2} \mathcal{E}^2 \left(\alpha_E - \frac{\mu^2}{4M^3} \right) + \ldots$$

Lattice QCD in External Fields



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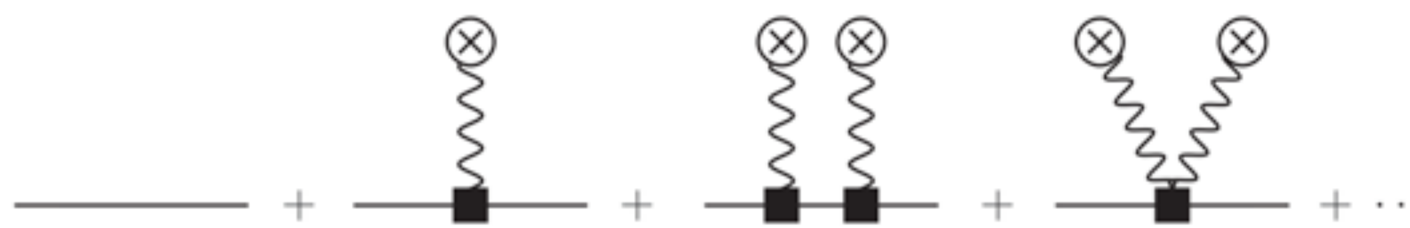
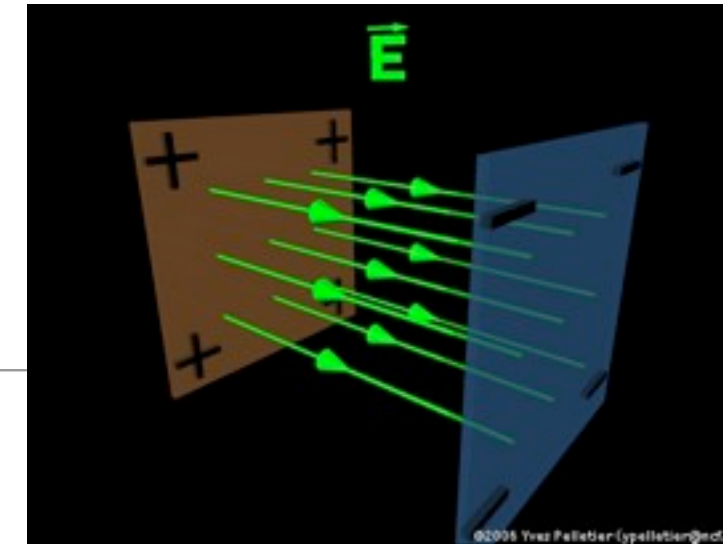


Magnetic Field: spin projected correlators $$G_\pm(\tau) = \mathrm{Tr}\left[\Sigma_\pm G(\tau)\right]$$

$$\sigma_{\mu\nu} F_{\mu\nu} = \vec{S} \cdot \vec{B}$$ $$\Sigma_\pm = \frac{1}{2}(1 \pm S_3)$$

Differing energies $$E_\pm = M \left(1 \pm \frac{\mu B}{2M^2} \right)$$

Lattice QCD in External Fields



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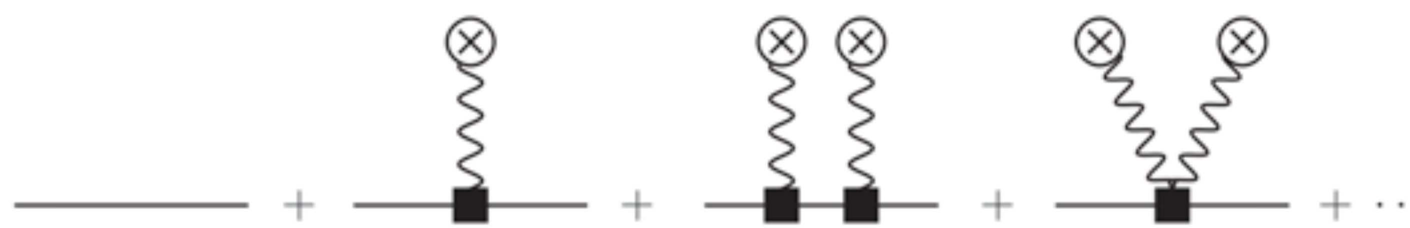
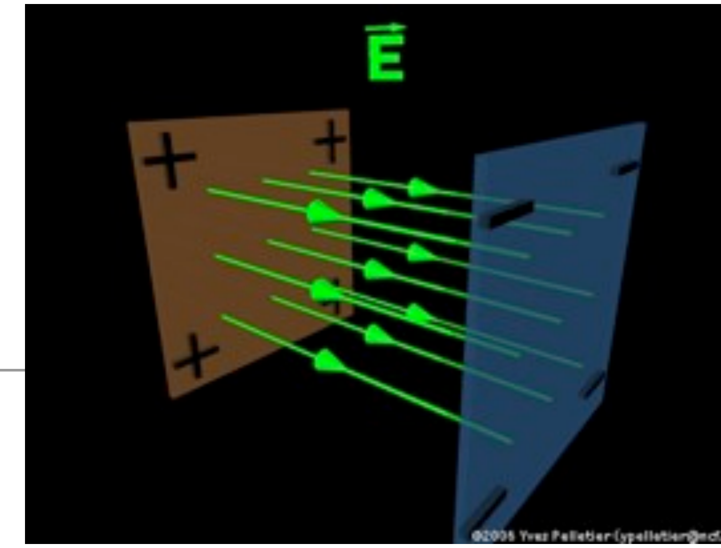


Electric Field: boost projected correlators $G_\pm(\tau) = \mathrm{Tr}\left[\mathcal{P}_\pm G(\tau)\right]$

$\sigma_{\mu\nu} F_{\mu\nu} = \vec{K} \cdot \vec{\mathcal{E}}$ $\mathcal{P}_\pm = \frac{1}{2}(1 \pm K_3)$ Differing amplitudes $Z_\pm = Z\left(1 \pm \frac{\mu\mathcal{E}}{2M^2}\right)$

Lattice QCD in External Fields



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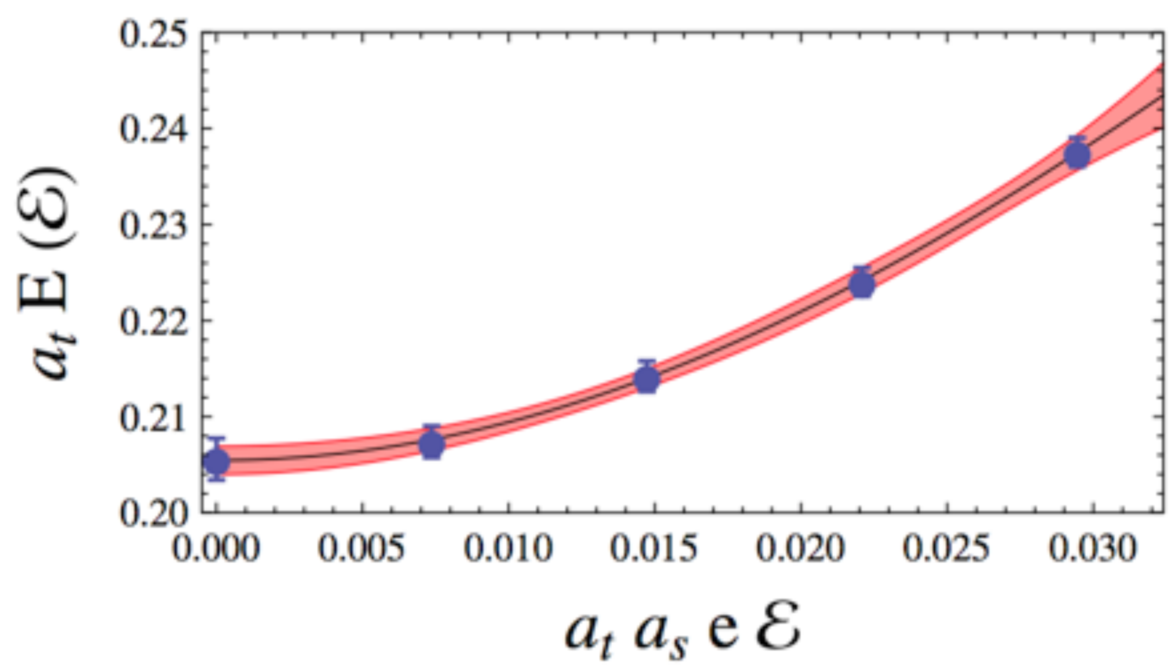
E.g. neutron in electric field

DETMOLD, TIBURZI, WALKER-LOUD, PRD 2010



$$E = M + \frac{1}{2}\mathcal{E}^2 \alpha_E + \dots$$

$$\mathrm{Tr}[\mathcal{P}_\pm G(\tau)] = Z\left(1 \pm \frac{\mu\mathcal{E}}{2M_N^2}\right)\exp(-\tau E_{\text{eff}})$$

Simultaneous fit to boost projected correlators

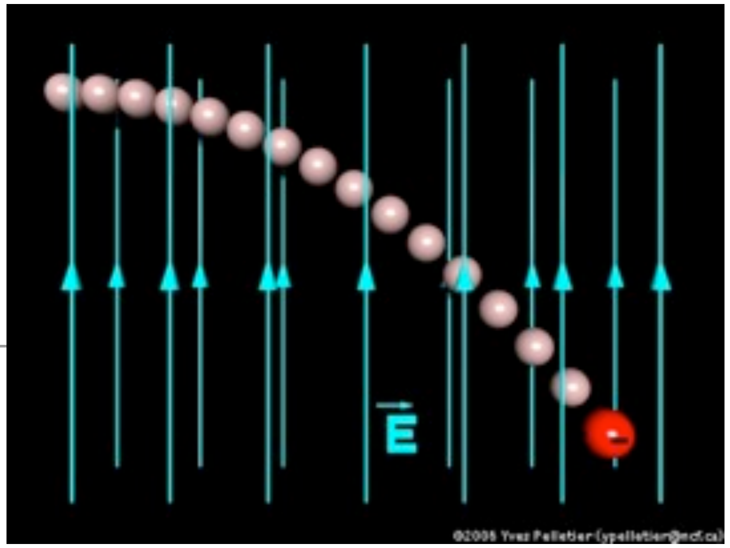
$$\mu_n = -1.6(1)\,[\mu_N]$$
$$(\mu_n)_{\text{exp}} = -1.9\,[\mu_N]$$

$$\alpha_E^n = 3(1) \times 10^{-4}\,\mathtt{fm}^3$$
$$(\alpha_E^n)_{\text{exp}} = 11(2) \times 10^{-4}\,\mathtt{fm}^3$$

Our results have known sources of systematic error

Not E_{eff} which includes *Born term*

Proton in Background Electric Field



Charge and anomalous magnetic moment

$$G_{\pm}(\tau) = Z\left(1 \pm \frac{\kappa \mathcal{E}}{2M^2}\right)\langle \tau | \frac{1}{2\mathcal{H} + E^2 \pm Q\mathcal{E}} | 0 \rangle$$

Boost projection

- First results for proton

$$\alpha_E^p = 2(1) \times 10^{-4}\,\texttt{fm}^3$$
$$(\alpha_E^p)_{\text{exp}} = 12(1) \times 10^{-4}\,\texttt{fm}^3$$

Relativistic contribution from magnetic moment

$$\mu_p = 2.6(1)\,[\mu_N]$$
$$(\mu_p)_{\text{exp}} = 2.8\,[\mu_N]$$

Known sources of systematic error:
pion mass, lattice spacing, lattice volume, ...

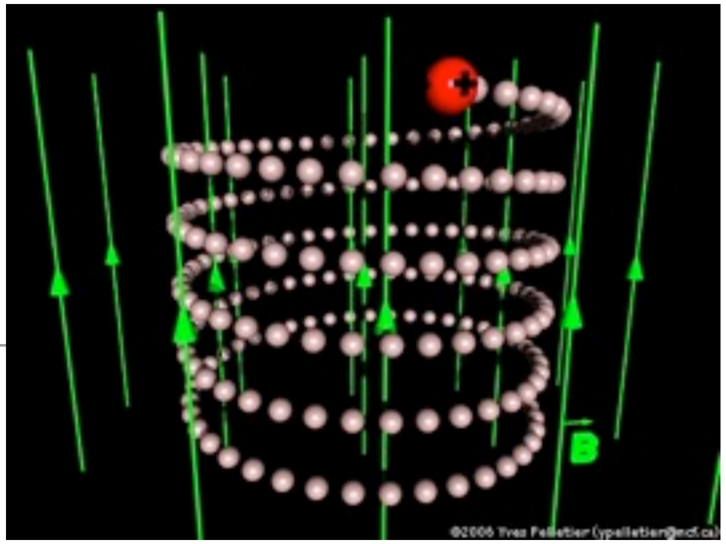
$$\alpha_E \sim \frac{1}{m_\pi} \qquad \mu \sim \mu_0 + m_\pi$$

Can test ChPT directly

Known solution:

DETMOLD, TIBURZI, WALKER-LOUD, PRD 2010

Particles in Magnetic Fields



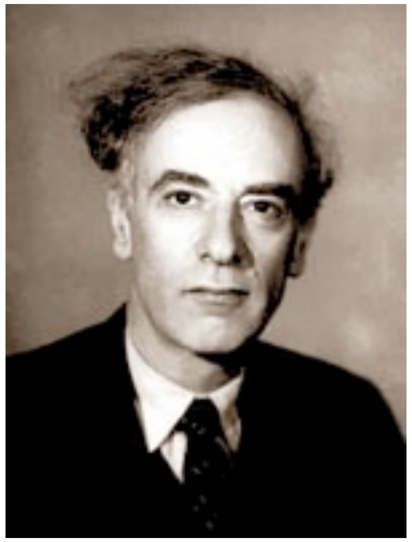
- Lattice QCD + external magnetic fields $\mu \quad \beta_M$

- Quantization condition restrictive $qB = \dfrac{2\pi n}{L^2}$
  Ideally $QB \ll m^2$

Pile up!

$$\Delta E_n / m^2 = QB/m^2$$

- Charged particles: Landau levels $E_n = QB(n + \frac{1}{2})$

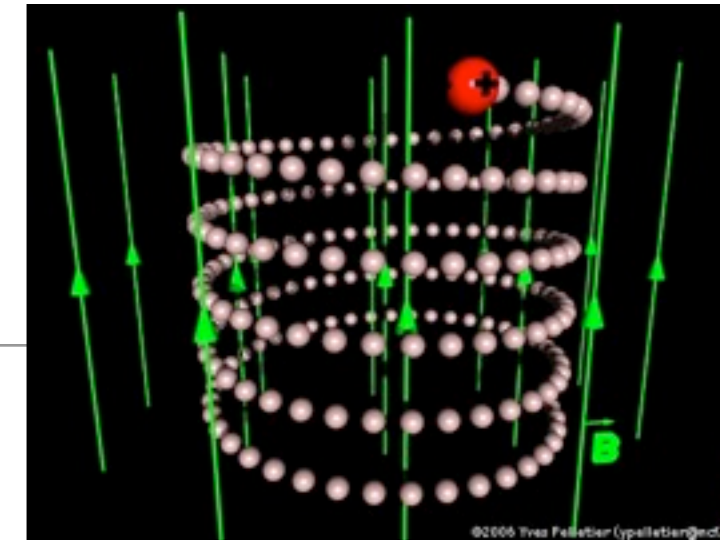


- Lattice two-point correlation function $A_\mu = -Bx_2\delta_{\mu 1}$

$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(\vec{0}, 0) \rangle$$

$$= Z_0 e^{-E_0\tau} + Z_1 e^{-E_1\tau} + Z_2 e^{-E_2\tau} + \cdots$$

Particles in Magnetic Fields



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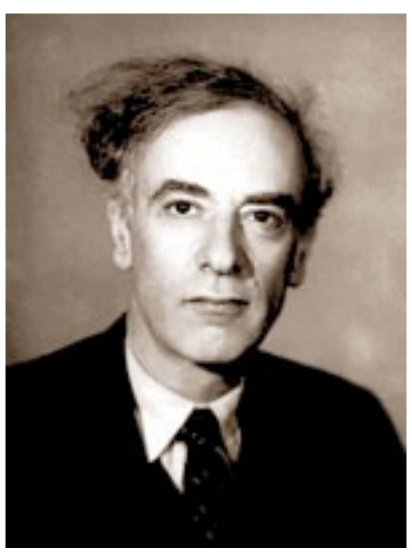
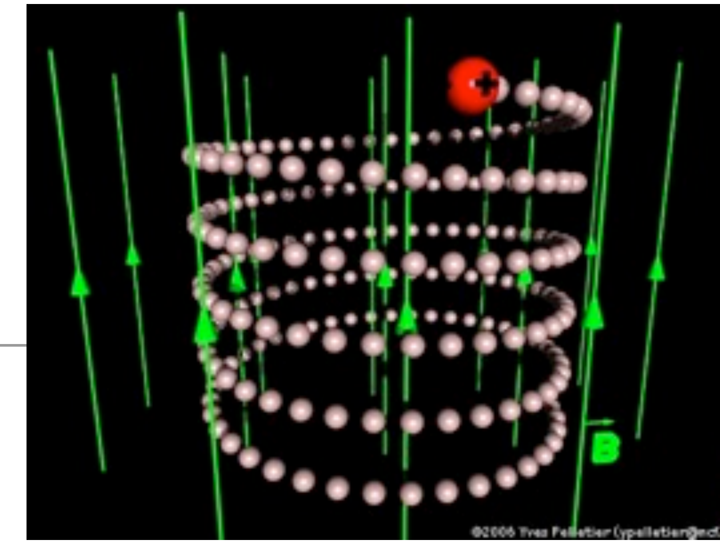


- Lattice two-point correlation function $A_\mu = -Bx_2\delta_{\mu 1}$

$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x},\tau)\chi^\dagger(\vec{0},0)\rangle \; \times \psi^*_{\vec{p}=0}(\vec{x})$$

$$= Z_0 e^{-E_0\tau} + Z_1 e^{-E_1\tau} + Z_2 e^{-E_2\tau} + \cdots$$

Particles in Magnetic Fields



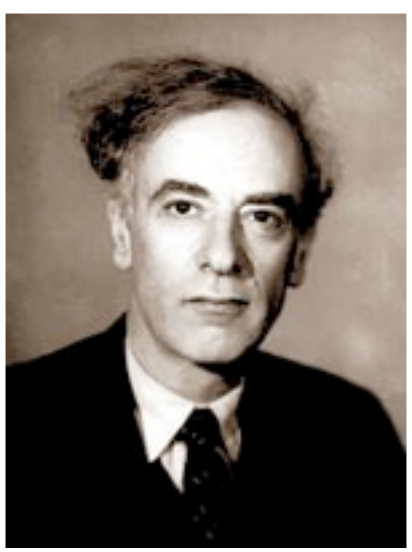
- Lattice QCD + external magnetic fields $\mu \quad \beta_M$

- Quantization condition restrictive $qB = \dfrac{2\pi n}{L^2}$
  Ideally $QB \ll m^2$

Pile up!

$$\Delta E_n / m^2 = QB/m^2$$

- Charged particles: Landau levels $E_n = m + QB(n + \frac{1}{2})$



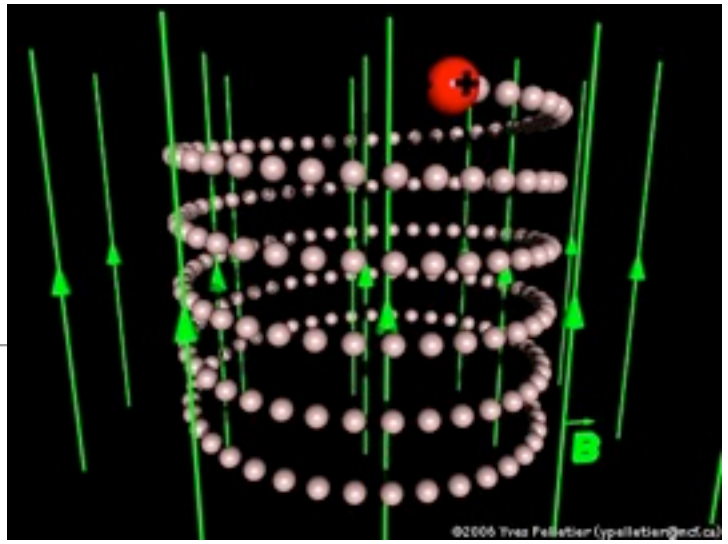
- Lattice two-point correlation function $A_\mu = -Bx_2\delta_{\mu 1}$

$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x},\tau)\chi^\dagger(\vec{0},0)\rangle \times \psi^*_{n=0}(x_2)$$

À LA SCHWINGER

$$= Z_0 e^{-E_0\tau} \qquad + \cdots$$

Particles in Magnetic Fields

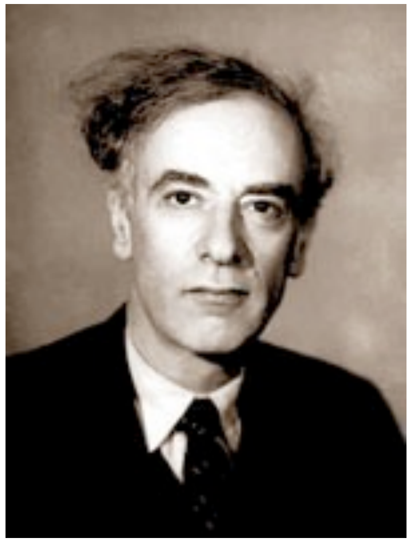


- Need to solve for lowest lattice Landau level

Discretization can perturb about continuum limit

$\Delta\psi_{n=0}$ negligible $$E_0^2 = M^2 + QB - \left(\frac{a^2 Q^2}{8} + M\beta_M\right) B^2 + \cdots$$

Finite Volume add series of magnetic periodic images to $\psi_{n=0}(x_2)$



- Lattice two-point correlation function $A_\mu = -Bx_2\delta_{\mu 1}$

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À la Schwinger

$$= Z_0 e^{-E_0\tau} + \cdots$$

Method remains to be tested in actual lattice calculations

Into Focus

Spontaneous chiral symmetry breaking has fundamental consequences for the low-energy structure of hadrons. Confirmation from both experiment and lattice will be a milestone.

- Electromagnetic polarizabilities probe low-energy hadron structure at a fundamental level
- Experimental confrontation with chiral dynamics motivates study from first principles
- External field method allows lattice determination; progress made, much work to be done