Thermal Modifications of Quarkonium Spectral Functions from QCD Sum Rules with Maximum Entropy Method

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  arXiv: 1005.2459.
- **Charmonium spectra at finite temperature from QCD sum rules with the maximum entropy method.**
  P. Gubler, K. Morita, M.O., Phys. Rev. Lett. 107, 092003 (2011)
  arXiv:1104.4436.
- **A Bayesian analysis of the nucleon QCD sum rules.**
  K. Ohtani, P. Gubler, M.O., Eur. Phys. Jour. A47, 114 (2011)
  arXiv:1104.5577 and in preparation.
- **Thermal modification of bottomonium spectra from QCD sum rules with the maximum entropy method.**
  K. Suzuki, P. Gubler, K. Morita, M.O., arXiv:1204.1173

Spectral functions in QCD

- The spectrum of hadrons is a key to study the symmetry, structure and dynamics of hadrons in QCD. Recent discoveries of many new (exotic) states have brought new concepts, such as multi-quark states, molecular resonances, dynamically generated resonances.
- Hadron properties at finite temperature and/or at finite density (in hadronic medium) are crucial in mapping the QCD phase diagram.
- In these studies, the spectral functions of hadrons play key roles and should be computed from the first principle, QCD. Lattice QCD is obviously a strong method, while QCD sum rule is useful as a complimentary approach.

QCD Sum Rule

Two point function of composite operators

$$\Pi(p) \equiv i \int d^4x\, e^{ip\cdot x} \langle 0|T(J(x)J^\dagger(0)|0\rangle$$

(1) OPE (Operator Product Expansion)

$$\Pi(p_E^2) = \sum_n C_n(p_E^2)\langle 0|O_n(0)|0\rangle \qquad O_n \;:\; \text{Local operator}$$

at deep Euclid region $p_E^2 \equiv -p^2 \to \infty$ **where the QCD can be treated perturbatively.**

***C*$_\text{n}$ contains the hard QCD/perturbative contributions.**

Non-perturbative effects are taken into account as the matrix elements of the local operators, *O*$_\text{n}$, *i.e.*, QCD vacuum condensates.

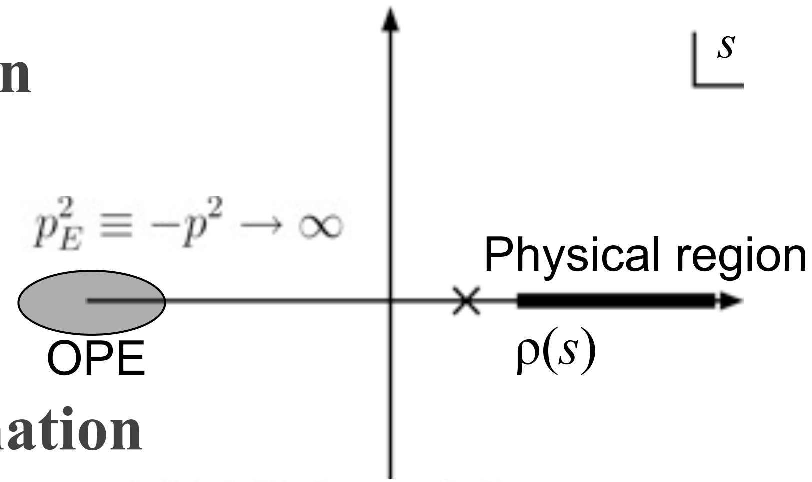
QCD Sum Rule

(2) Imaginary part of the $\Pi(p^2)$ gives the spectral function

$$\rho(p^2) \equiv \frac{1}{\pi}\mathrm{Im}\Pi(p^2) = \sum |\langle 0|J(0)|m\rangle|^2 \delta(p^2 - m^2)$$

(3) Causality: Dispersion relation

$$\Pi(p^2) = \int \frac{\rho(s)}{s - p^2 - i\varepsilon} ds$$



(4) To improve: Borel transformation

$$\mathcal{B}_{M^2}\Pi \equiv \tilde{\Pi}(M^2) = \lim_{p_E^2, n\to\infty, M^2 \equiv p_E^2/n = \text{finite}} \frac{(p_E^2)^{n+1}}{n!}\left(-\frac{d}{dp_E^2}\right)^n \Pi(p_E^2)$$

***M* is called the Borel mass, playing as a new variable.**

QCD Sum Rule

The Borel transform (1) eliminates unwanted subtraction terms, and (2) makes the OPE convergence better

Then we obtain the Borel sum rule

$$\boxed{\int e^{-s/M^2} \mathrm{Im}\, \Pi^{\mathrm{OPE}}(s)\, ds} = \int e^{-s/M^2} \rho(s)\, ds$$

Analytically calculable

to be obtained

(5) The conventional sum rule assumes a specific form of the spectral function, usually a pole plus continuum assumption,

$$\rho(s) = \lambda \delta(s - m^2) + \theta(s - s_0) \rho_{\mathrm{OPE}}(s)$$

and determine the mass (and the strength) of the pole by fitting the sum rule.

Maximum Entropy Method for Sum Rule

P. Gubler, M.O., Prog. Theor. Phys. 124 (2010) 995, arXiv:1005.2459

- A new approach using the Bayesian inference method allows us to obtain the spectral function **directly** from the sum rule without assuming its explicit form.
- The sum rule is reduced to a mathematical problem to invert the integral relation:

$$G(M) = \int_0^\infty d\omega\, K(M,\omega)\rho(\omega) \qquad K(M,\omega) = \frac{2\omega}{M^2}\, e^{-\omega^2/M^2}$$

$$\rho(\omega) \geq 0$$

- For the given *G*(*M*) by the OPE, the most probable ρ(ω) is extracted.

Maximum Entropy Method for Sum Rule

- **Bayes' Theorem**

$$P[\rho|G,I] = P[G|\rho,I]P[\rho|I]/P[G|I]$$

$P[A|B]$: **conditional probability of *A* given *B***

I : **general constraints for *ρ***

$P[G|\rho, I]$: **<u>likelihood function</u>**

$P[\rho|I]$: **<u>prior probability</u>**

- **Find the maximum of $P[\rho|G, I]$ to obtain the most probable spectral function. (Maximum Entropy Method)**
- **The same method was applied to obtain the spectral function from the lattice QCD data.**

M. Asakawa, T. Hatsuda, Y. Nakahara, Prog.Part.Nucl.Phys.46 (2001) 459-508.

Maximum Entropy Method for Sum Rule

Likelihood function

We assume the Gaussian distribution, similarly to the χ^2 fitting.

$$P[G|\rho, I] = Z_L^{-1} e^{-L[\rho]}$$

$$L[\rho] = \frac{1}{2(M_{\max} - M_{\min})} \int_{M_{\min}}^{M_{\max}} dM \, \frac{[G(M) - G_{\mathrm{OPE}}(M)]^2}{\sigma^2(M)}$$

$$G(M) = \int_0^\infty d\omega \, K(M,\omega)\rho(\omega)$$

Prior probability

$$P[\rho|I] = Z_s^{-1} e^{\alpha S[\rho]}$$

Shannon-Jaynes entropy

$$S[\rho] = \int_0^\infty d\omega [\rho(\omega) - m(\omega) - \rho(\omega) \log(\rho(\omega)/m(\omega))]$$

$m(\omega)$: ***default model, which maximizes the entropy if no information on G is given.***

Charmonium Spectrum at finite T

- Suppression of the J/ψ formation in Quark-Gluon Plasma matter

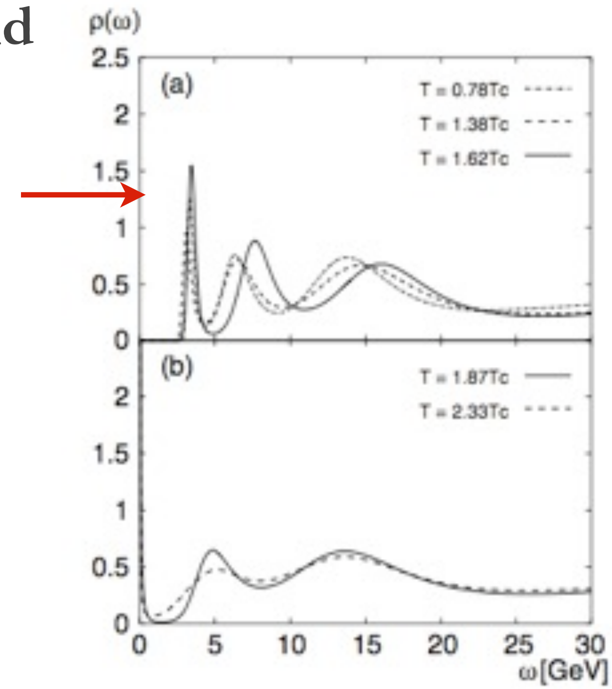
  T. Matsui and H. Satz, Phys. Lett. B 178, 416 (1986).

- During the last 10 years, a picture has emerged, from studies using quenched lattice QCD (and MEM), that the J/ψ survive above T_c.

  M. Asakawa and T. Hatsuda, PRL 92 012001 (2004).
  S. Datta et al, Phys. Rev. D69, 094507 (2004).
  T. Umeda et al, Eur. Phys. J. C39S1, 9 (2005).
  A. Jakovác et al, Phys. Rev. D75, 014506 (2007).

- But a new calculation gives different results.

  H.T. Ding et al., ArXiv:1204.4945 (2012).



MEM is most suitable for this problem.

Charmonium Sum Rules at T=0

The sum rule:

$$M(\nu) = \int_0^\infty e^{-\nu t} \rho(4m_c^2 t) dt \qquad \nu \equiv \frac{4m_c^2}{M^2}$$

$$M(\nu) = A(\nu)\left[1 + a(\nu)\alpha_s(\nu) + b(\nu)\frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{m_c^4} + d(\nu)\frac{\langle g^3 G^3 \rangle}{m_c^6}\right]$$

perturbative terms including α_s correction

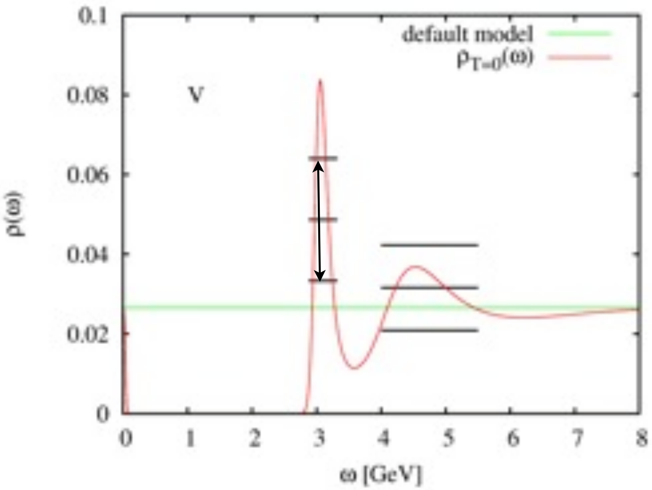
Non-perturbative corrections including condensates up to dim 6

M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979).
L.J. Reinders, H.R. Rubinstein and S. Yazaki, Nucl. Phys. B 186, 109 (1981).
R.A. Bertlmann, Nucl. Phys. B 204, 387 (1982).
J. Marrow, J. Parker and G. Shaw, Z. Phys. C 37, 103 (1987).

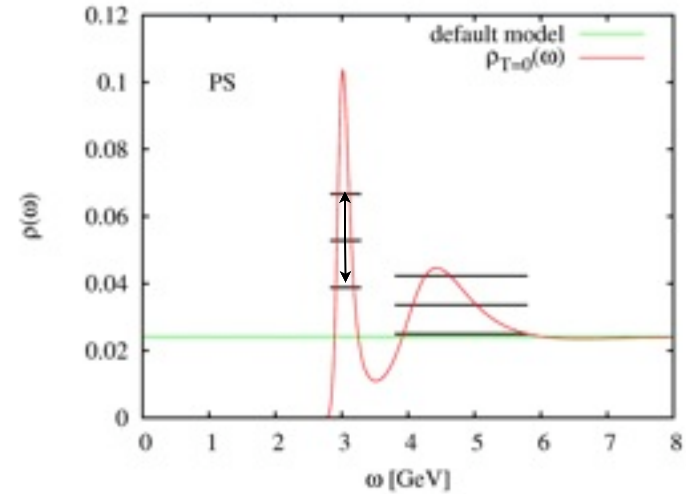
MEM Analysis at T=0

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \pm 0.0036 \text{ GeV}^4$$
$$m_c = 1.277 \pm 0.026 \text{ GeV}$$

S-wave

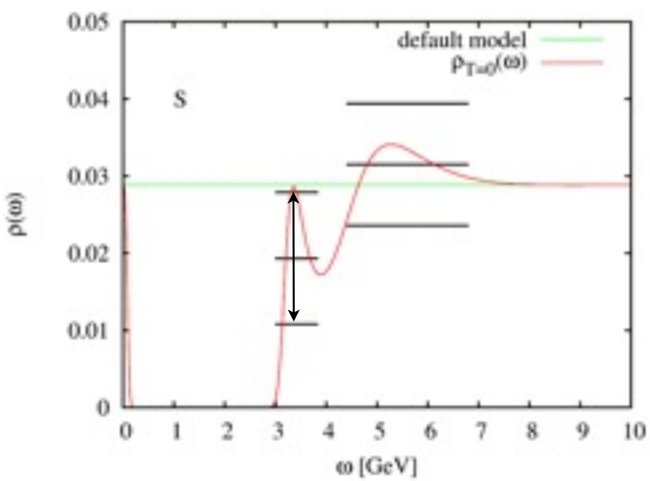


$m_{J/\psi}$=3.06 GeV (Exp: 3.10 GeV)

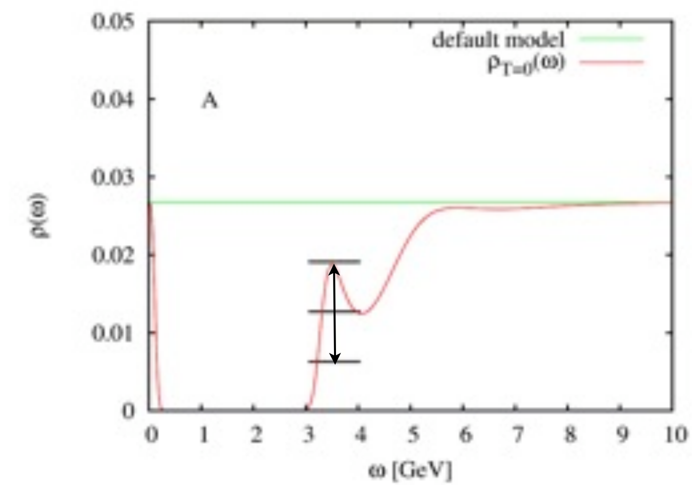


m_{η_c}=3.02 GeV (Exp: 2.98 GeV)

P-wave



$m_{\chi 0}$=3.36 GeV (Exp: 3.41 GeV)



$m_{\chi 1}$=3.50 GeV (Exp: 3.51 GeV)

Gubler, Morita, M.O., PRL 107, 092003 (2011)

Charmonium Spectrum at finite T

The Sum Rule approach has an advantage to Lattice QCD, where the finite T reduction of available data points makes the direct comparison of T=0 and T≠0 spectral functions difficult.

The QCD sum rule at finite T has been formulated in Hatsuda, Koike, Lee, Nucl. Phys. B 394, 221 (1993). The condensates bear the temperature dependences.

$$\mathcal{M}^J(\nu) = \pi e^{-\nu} A^J(\nu)[1 + \alpha_s(\nu) a^J(\nu) + b^J(\nu)\phi_b(T) + \boxed{c^J(\nu)\phi_c(T)}]$$

new term at finite T

$$\phi_b = \frac{4\pi^2}{9(4\boxed{m_h^2})^2} G_0(T) \qquad \nu = \frac{4m_h^2}{M^2}$$

$$\phi_c = \frac{4\pi^2}{3(4\boxed{m_h^2})^2} G_2(T)$$

Twist-2 condensate

$$(u^\alpha u^\beta - \frac{1}{4} g^{\alpha\beta}) G_2 = \left\langle \frac{\alpha_s}{\pi} G_\lambda^{a\alpha} G^{a\lambda\beta} \right\rangle$$

The Gluon condensate terms are suppressed by the heavy quark masses

T-dependence of the condensates

The energy-momentum tensor (of pure QCD)

$$T^{\alpha\beta} = -\mathcal{ST}(G^{a\alpha\lambda}G^{a\beta}_{\lambda}) + \frac{g^{\alpha\beta}}{4}\frac{\beta(g)}{2g}G^{a}_{\mu\nu}G^{a\mu\nu}$$

$$T^{\alpha\beta} = (\epsilon + p)(u^{\alpha}u^{\beta} - \frac{1}{4}g^{\alpha\beta}) + \frac{1}{4}g^{\alpha\beta}(\epsilon - 3p)$$

Matching the trace part and the Symmetric Traceless part

$$\langle\frac{\alpha_s}{\pi}G^2\rangle_T = \langle\frac{\alpha_s}{\pi}G^2\rangle_{\text{vac.}} - \frac{8}{11}(\boxed{\epsilon} - 3\boxed{p})$$

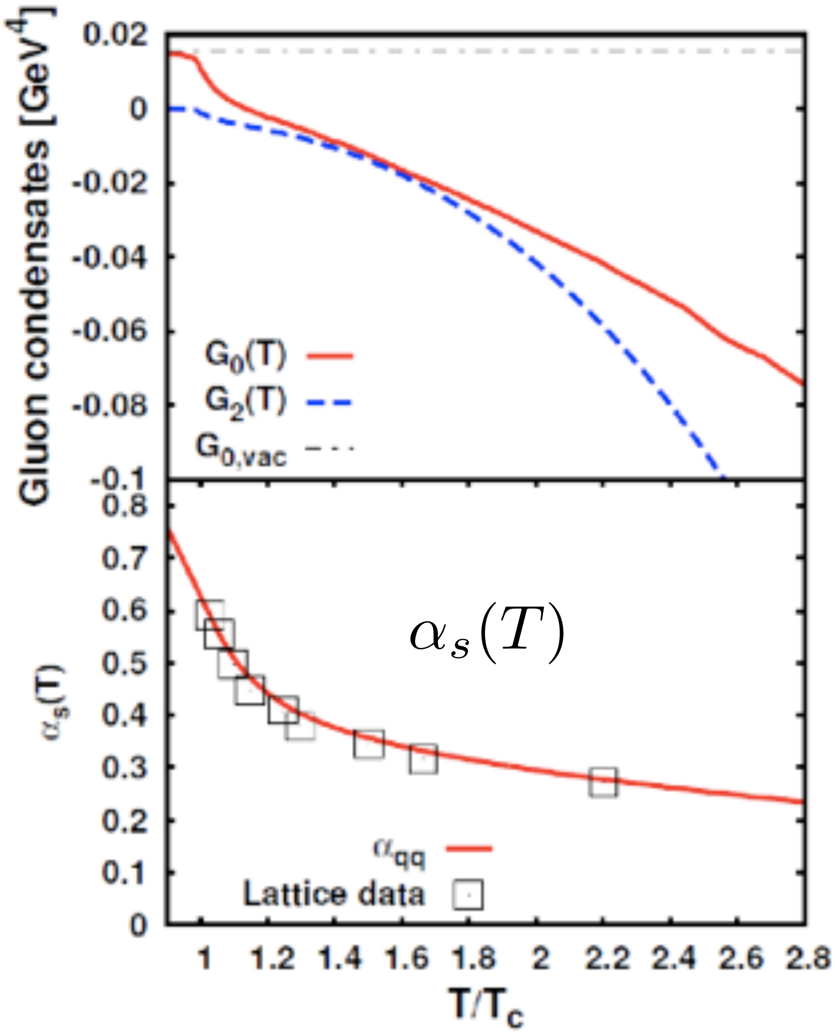
$$\langle\frac{\alpha_s}{\pi}G^2\rangle_{T,2} = -\frac{\boxed{\alpha_s(T)}}{\pi}(\boxed{\epsilon} + \boxed{p})$$

T dependences are obtained from quench lattice QCD

K. Morita and S.H. Lee, Phys. Rev. Lett. 100, 022301 (2008).

K. Morita and S.H. Lee, Phys. Rev. C 77, 064904 (2008).

T-dependence of the condensates



The T dependences of ε, p and α_s are obtained from *quenched* lattice calculations:

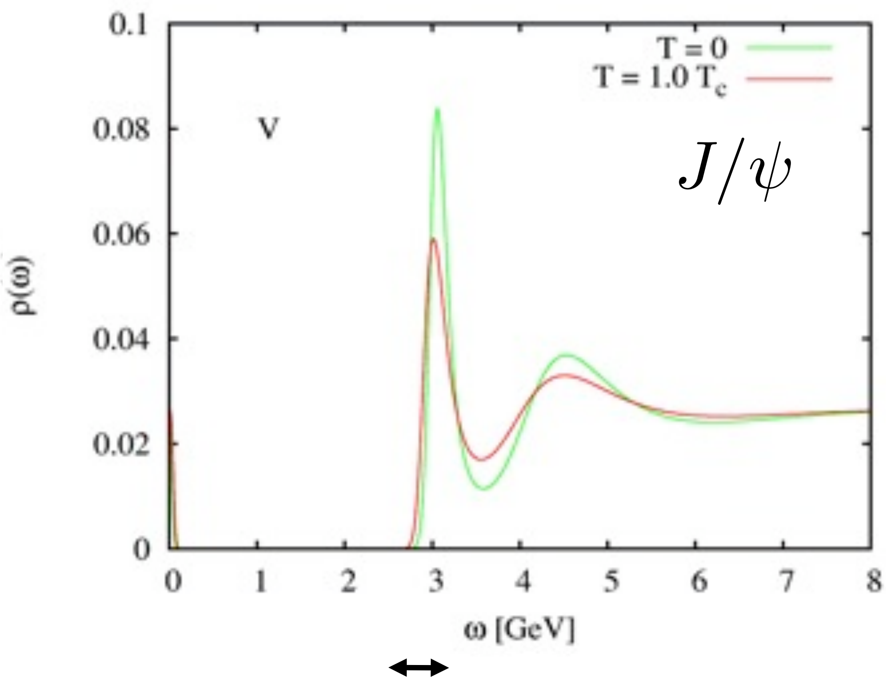
G. Boyd et al, Nucl. Phys. B 469, 419 (1996).
O. Kaczmarek et al, Phys. Rev. D 70, 074505 (2004).

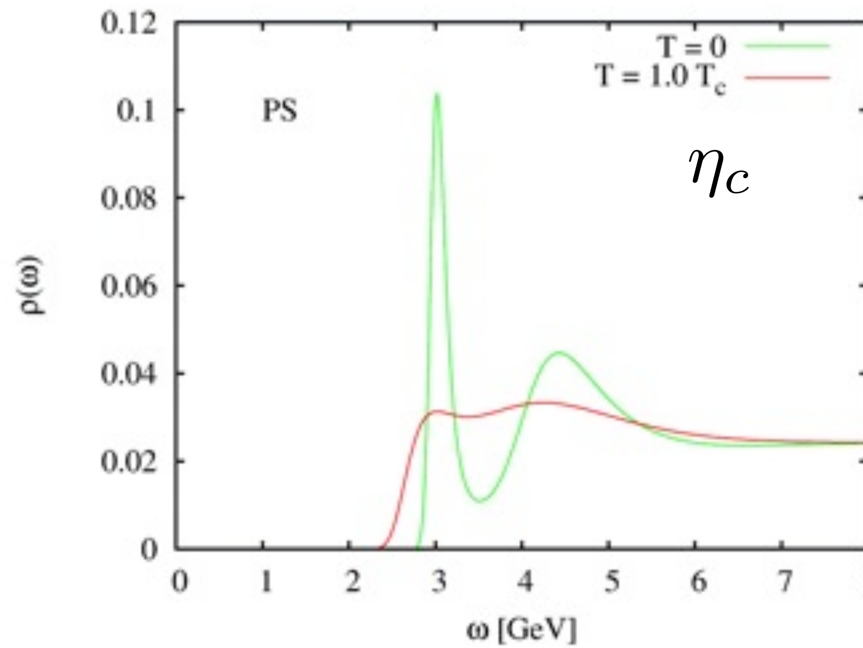
A sudden change of G_0(T) and G_2(T) above T_c is observed.

K. Morita and S.H. Lee, Phys. Rev. D82, 054008 (2010).

Charmonium spectral function by MEM

MEM results at $T=T_c$



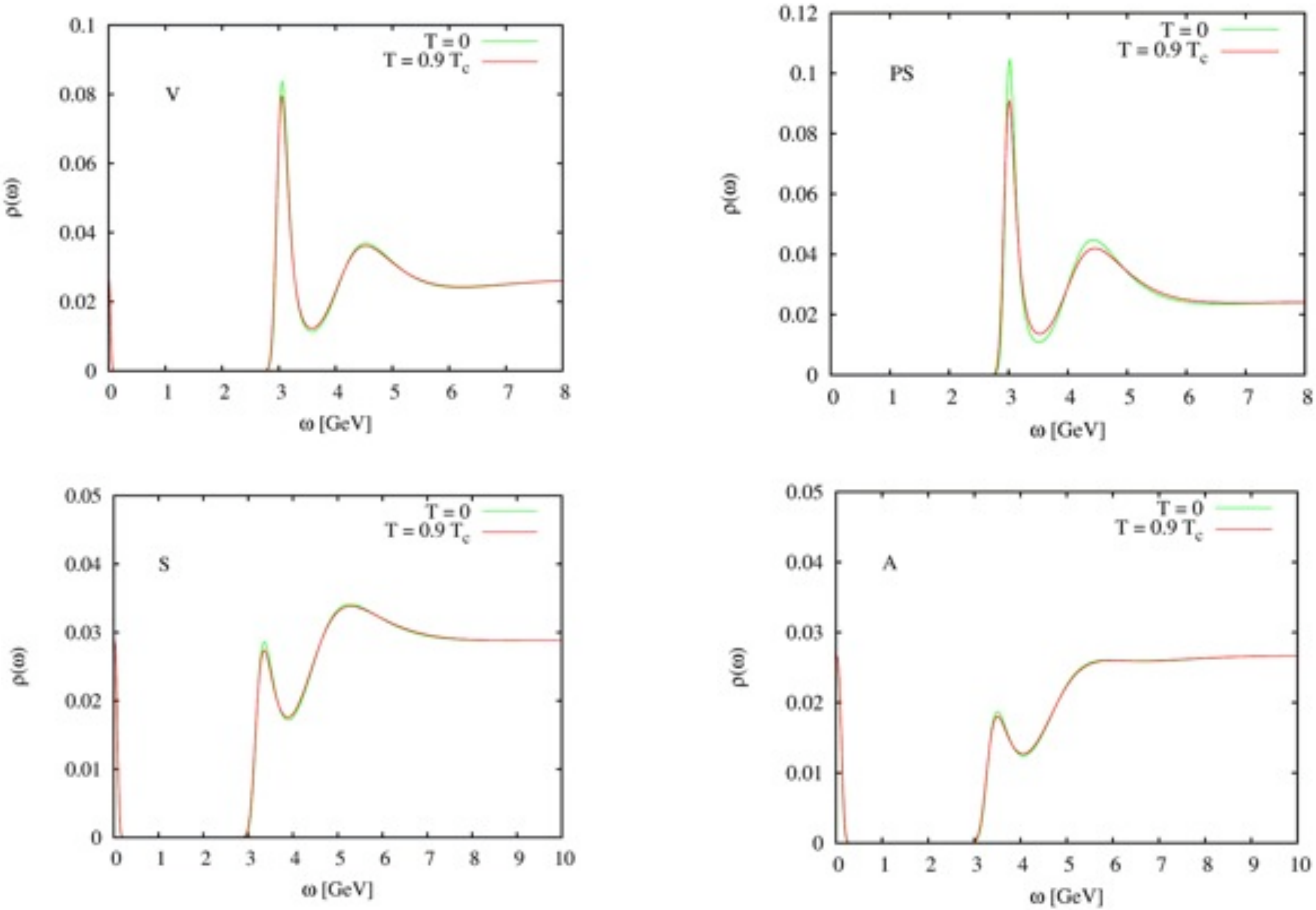


negative peak shift of ~50 MeV

almost disappearing

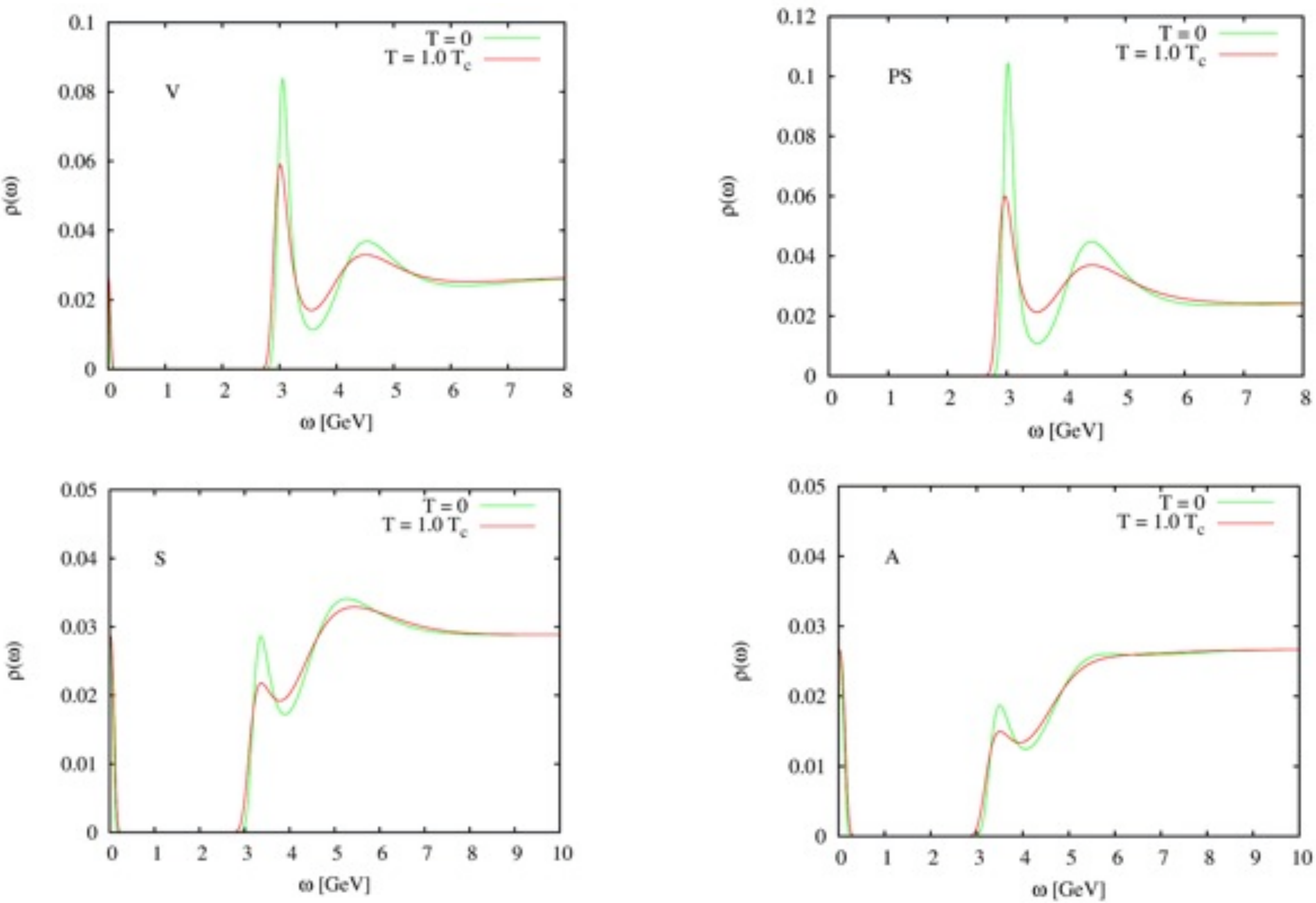
consistent with the previous sum rule analysis by Morita and Lee.

Charmonium spectral function by MEM



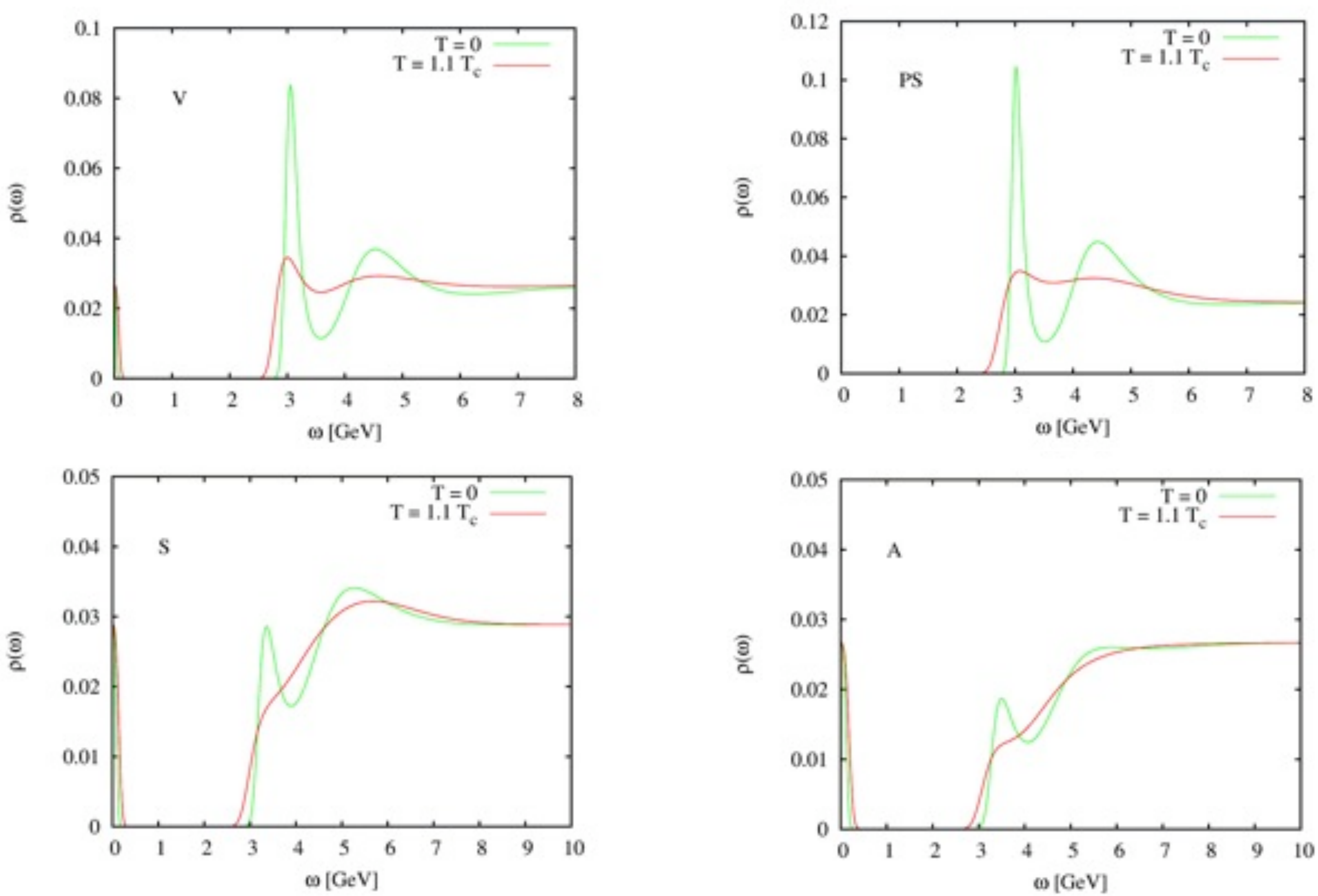
Gubler, Morita, M.O., PRL 107, 092003 (2011)

Charmonium spectral function by MEM



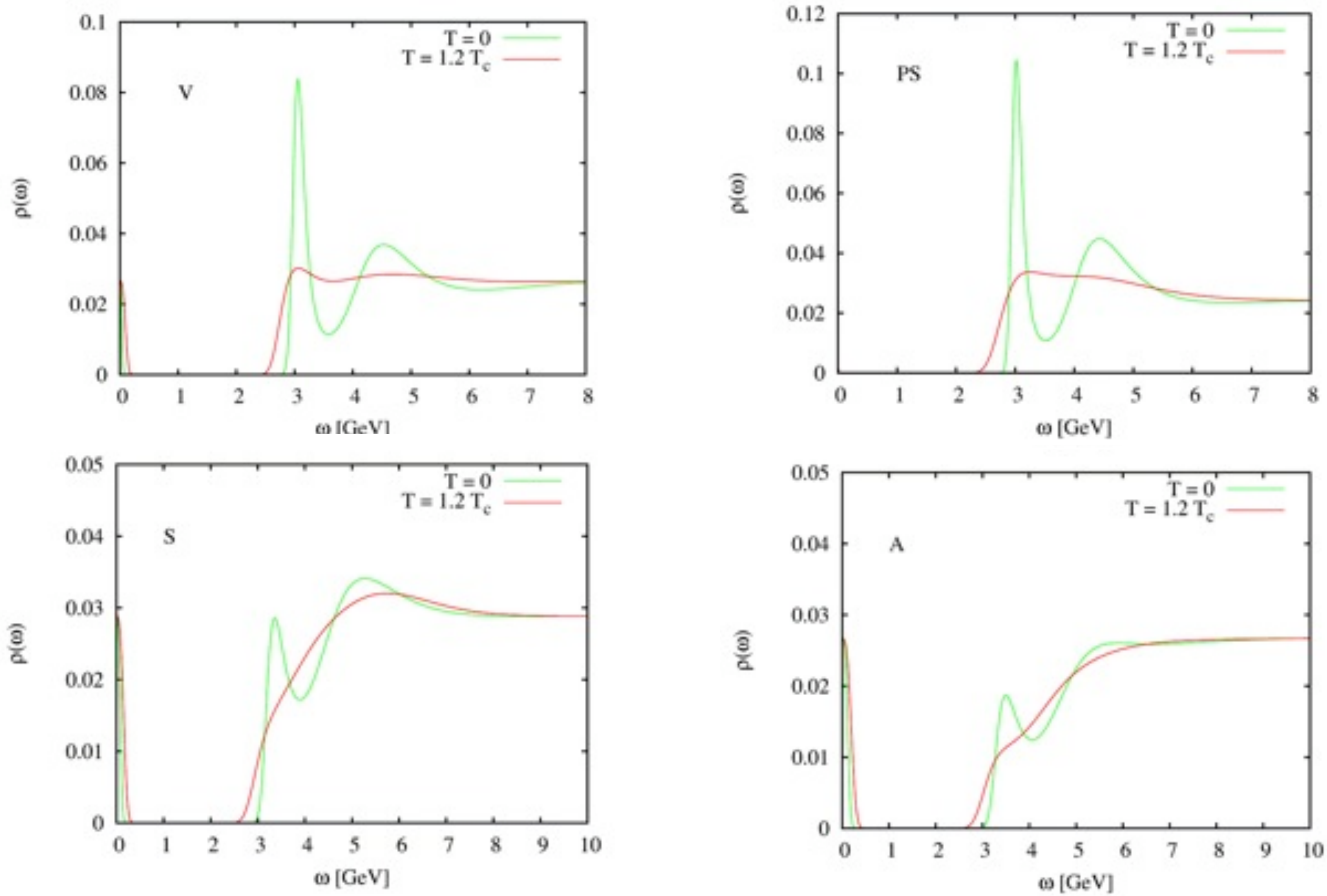
Gubler, Morita, M.O., PRL 107, 092003 (2011)

Charmonium spectral function by MEM



Gubler, Morita, M.O., PRL 107, 092003 (2011)

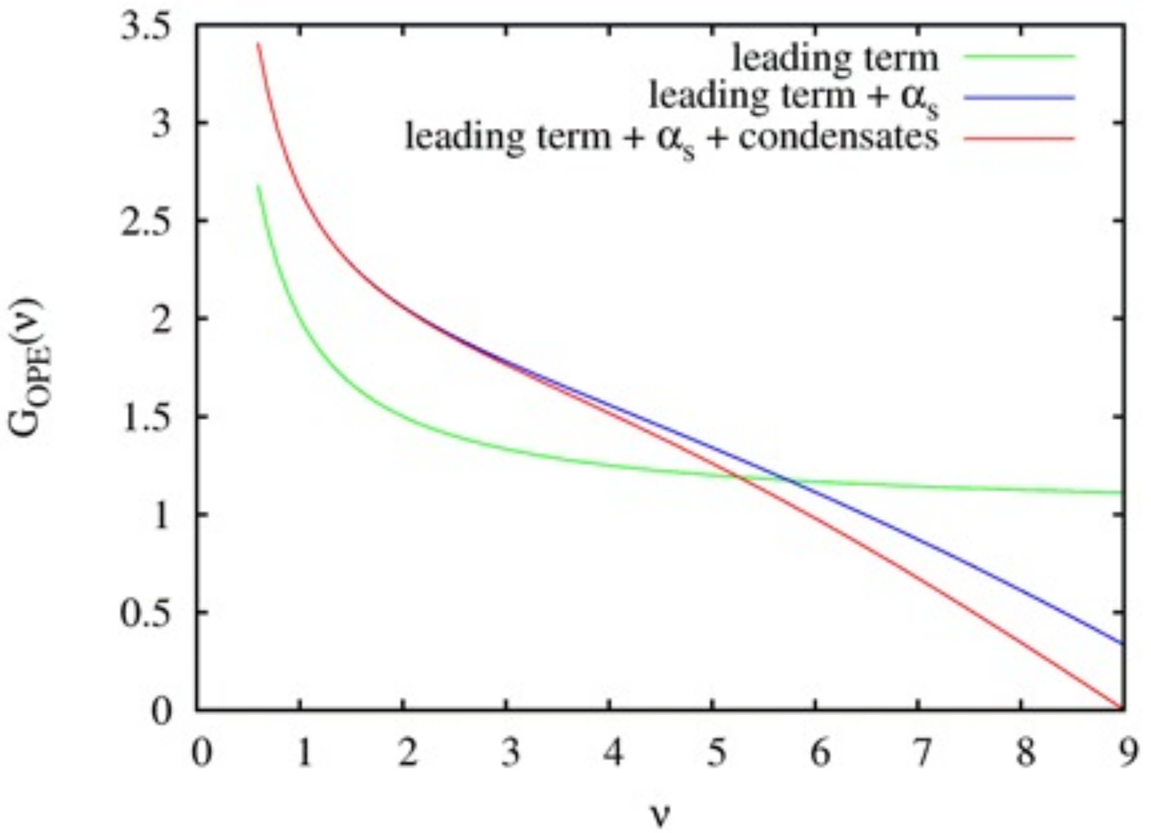
Charmonium spectral function by MEM



Gubler, Morita, M.O., PRL 107, 092003 (2011)

Charmonium spectral function by MEM

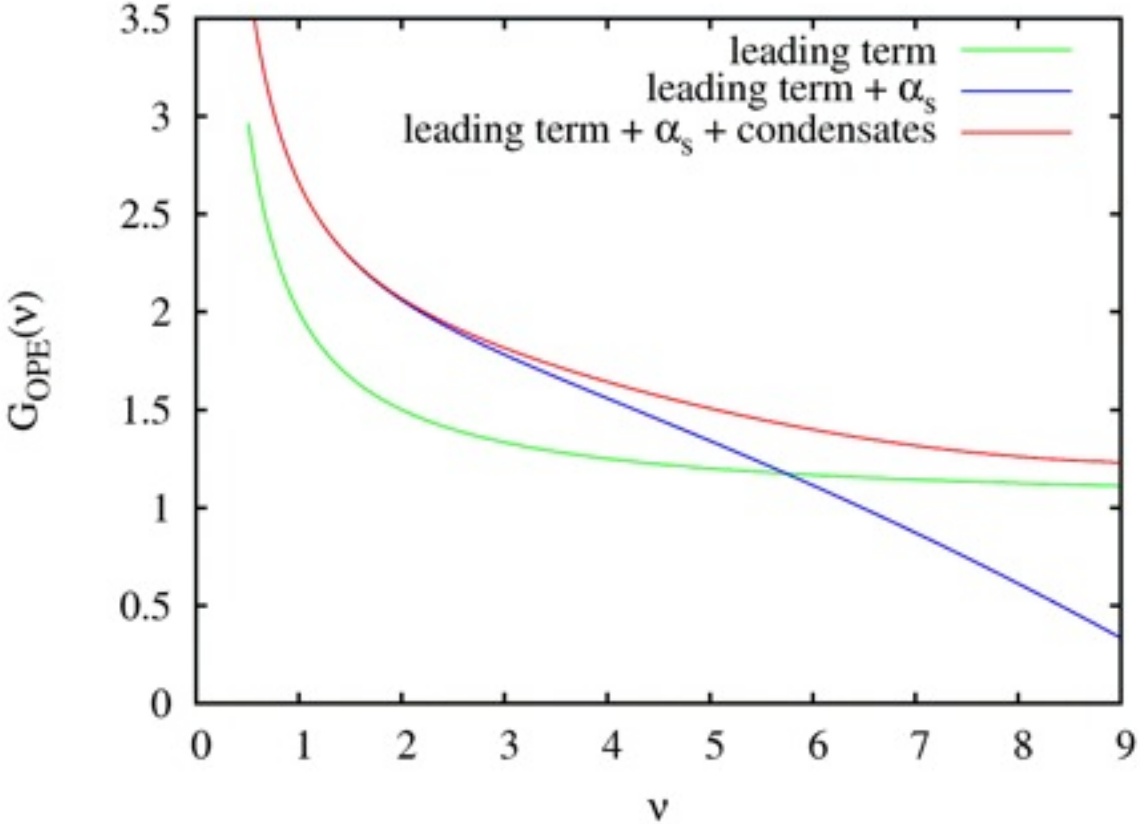
<u>The OPE data in the Vector channel at various T:</u>



T=0

Charmonium spectral function by MEM

The OPE data in the Vector channel at various T:



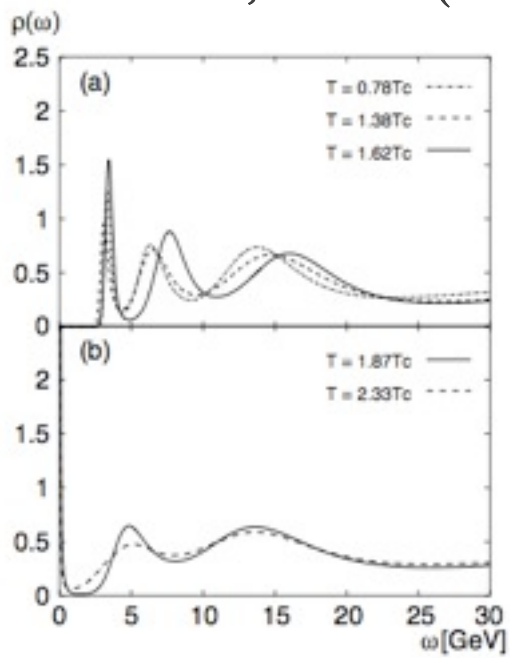
T=1.2 T_c

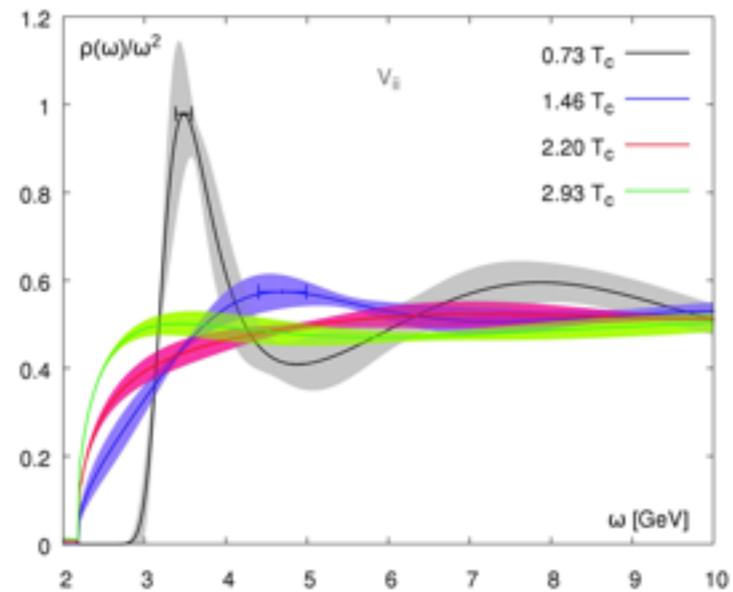
cancellation between α_s and condensate contributions

Charmonium spectrum at finite T

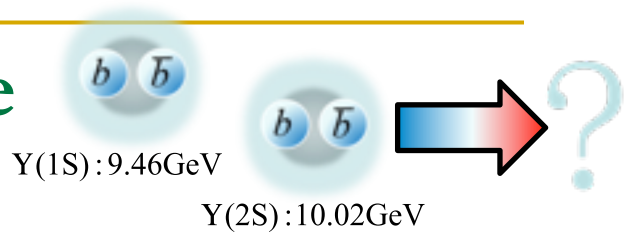
- **We have found that the sudden change of the gluon condensate (observed on the Quenched Lattice) induces a strong suppression of the charmonium peaks, especially the P-wave excited states.**
- **The J/ψ peak disappears quickly at $T > T_c$.**

Asakawa-Hatsuda, PRL 92 (2004) v.s. Ding et al., ArXiv:1204.4945 (2012).



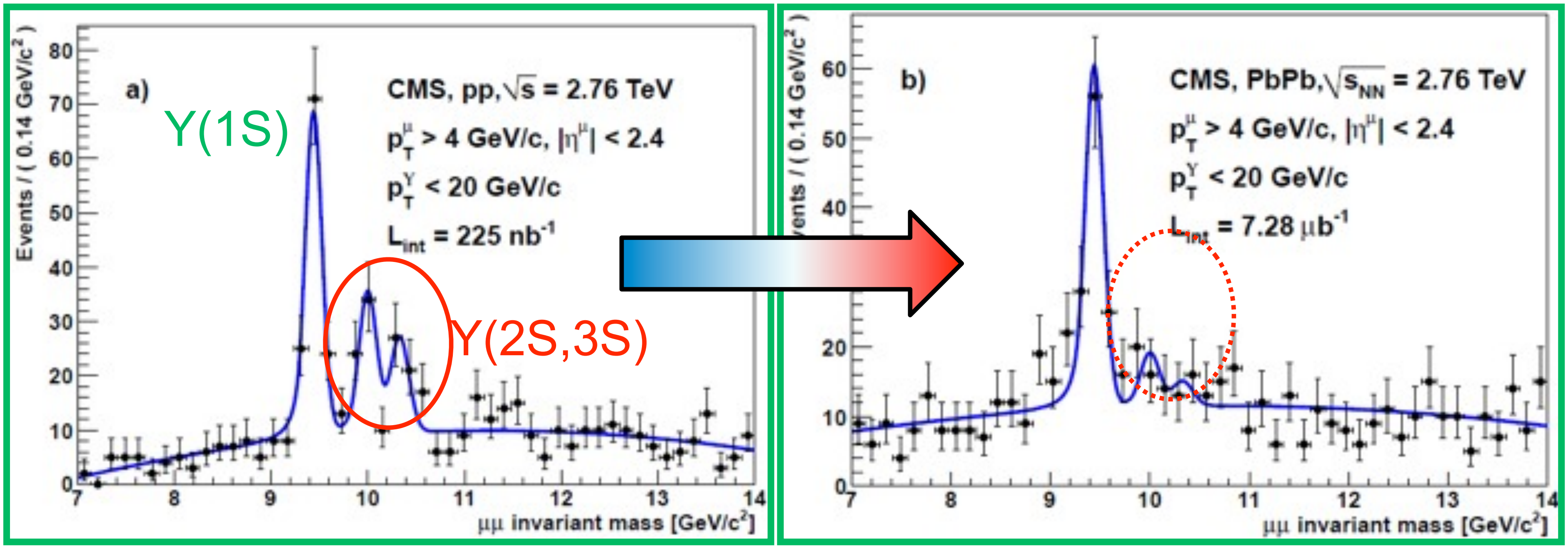


Bottomonia at finite temperature



Heavy ion collision at the LHC (CERN)

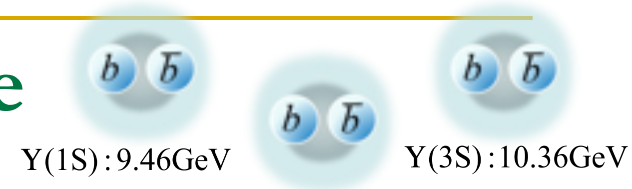
S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. Lett. **107**, 052302 (2011)



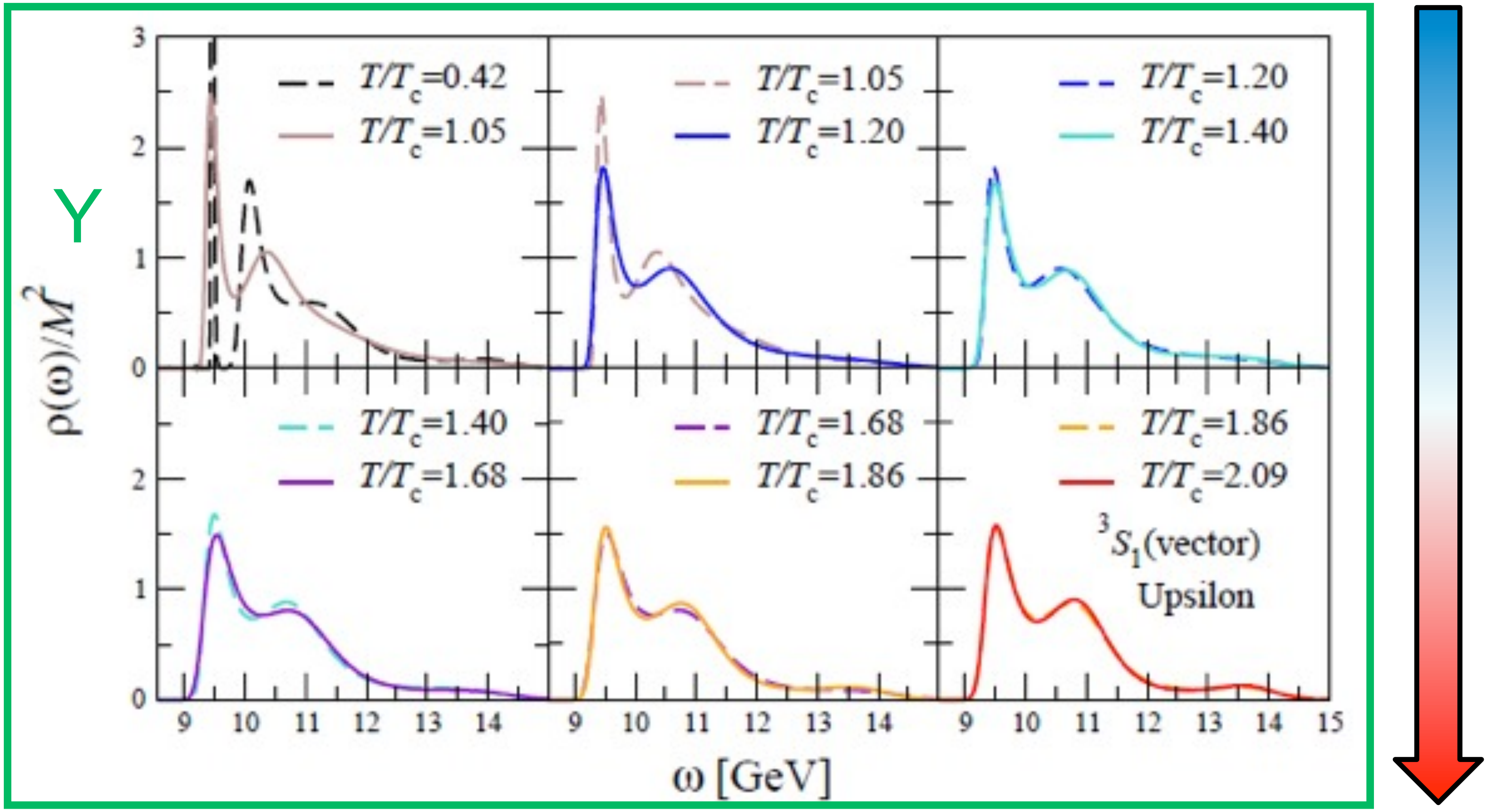
$\Upsilon(2S+3S)/\Upsilon(1S) = 0.78$ $\Upsilon(2S+3S)/\Upsilon(1S) = 0.24$

The data suggest that the excited states Υ(2S), Υ(3S) disappear at a lower temperature than ground state Υ(1S).

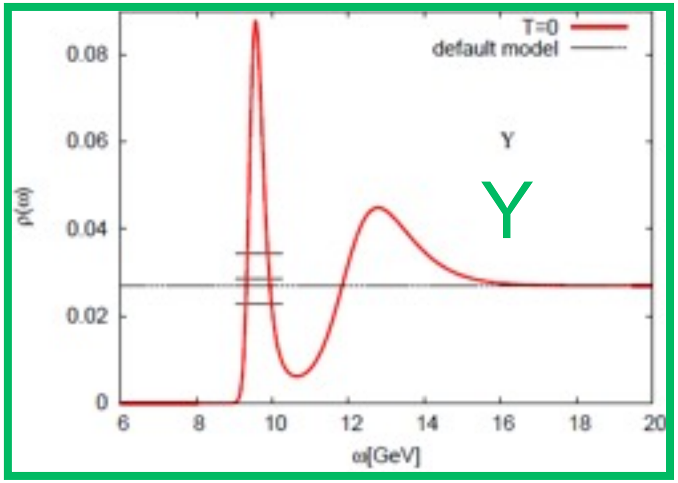
Bottomonia at finite temperature



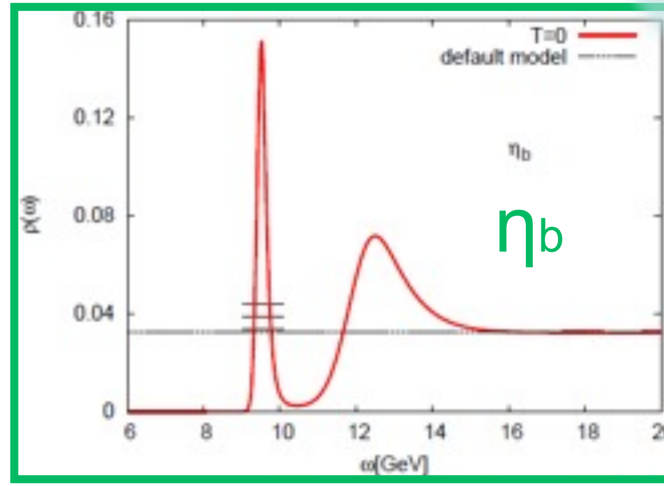
Lattice QCD + NRQCD with MEM   G. Aarts et al., JHEP 1111 (2011) 103



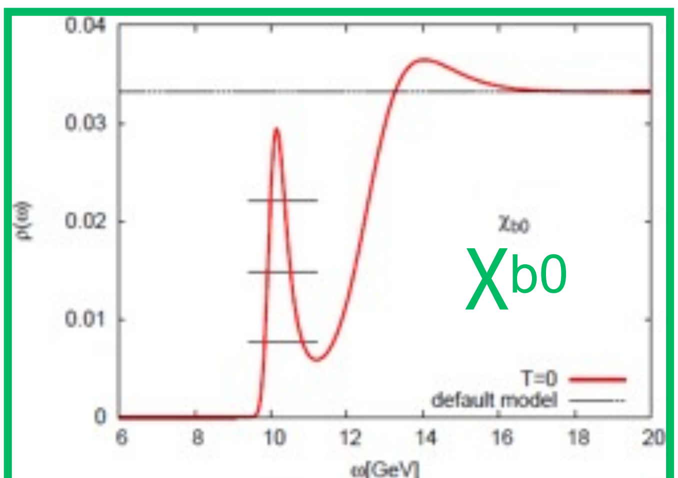
Bottomonia at zero temperature



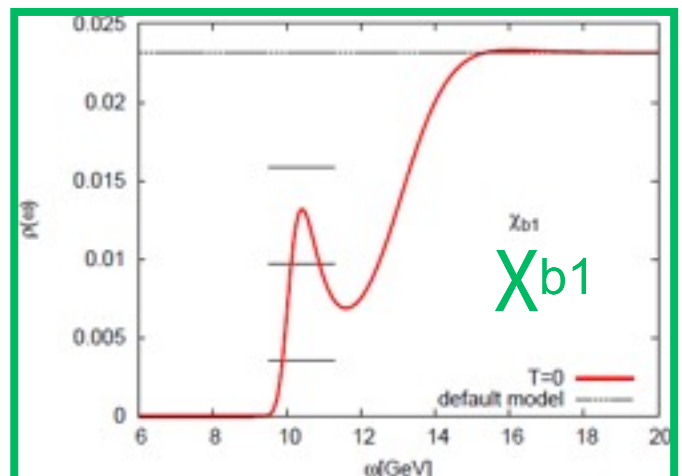
Mass=9.63GeV (exp. 9.46GeV)



Mass=9.55GeV (exp. 9.39GeV)

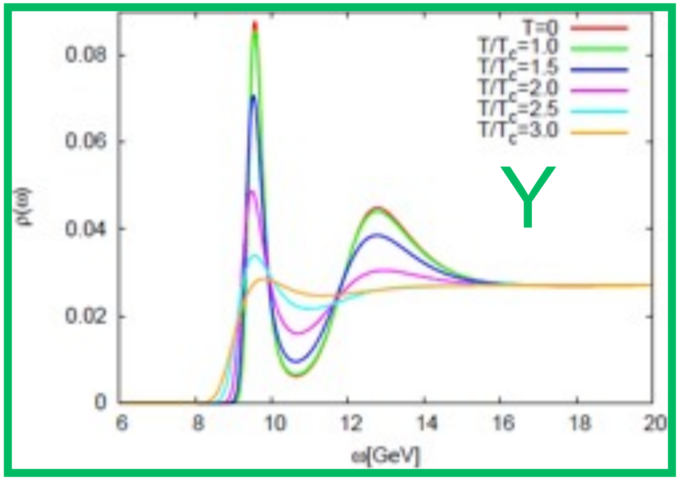


Mass=10.18GeV (exp. 9.86GeV)

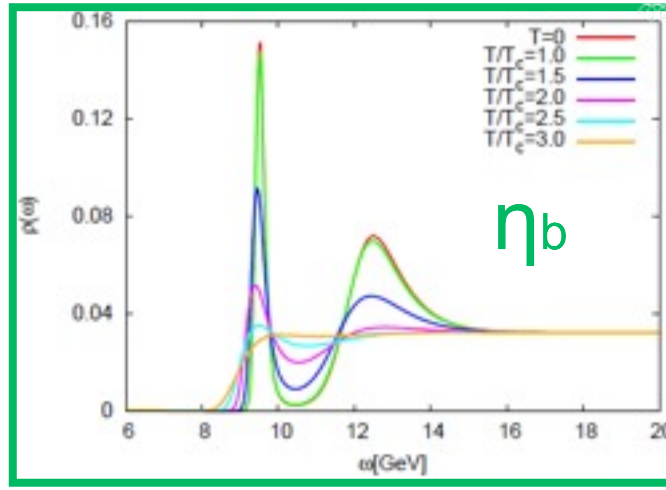


Mass=10.44GeV (exp. 9.89GeV)

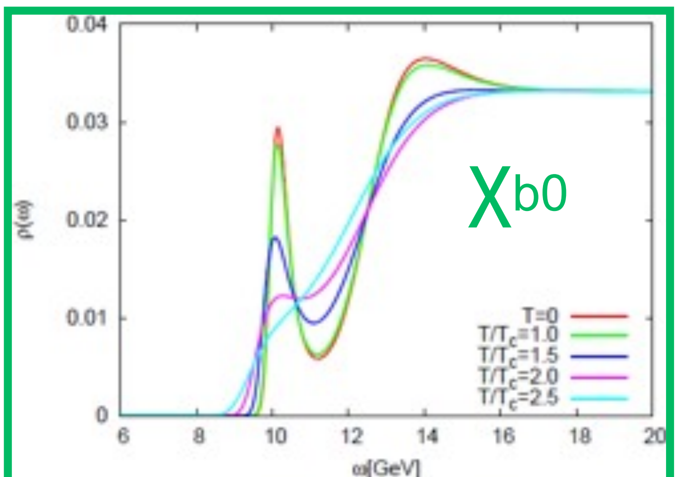
Bottomonia at finite temperature



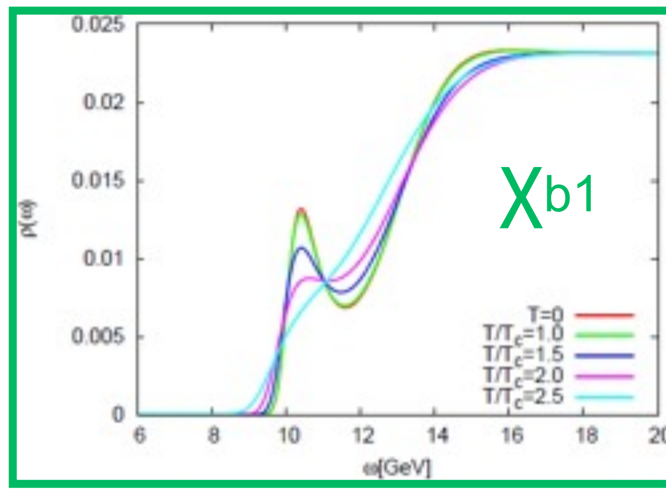
disappear at T >2.3T$_c$



disappear at T >2.1T$_c$



disappear at T=1.3-2.5T$_c$

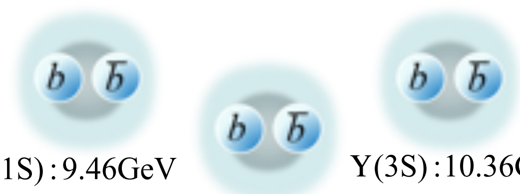


disappear at T < 2.5T$_c$

Bottomonia at finite temperature

- The bb^{bar} mesons survive at higher temperatures than the charmonia, because the gluon condensate terms of the sum rule are suppressed by $1/m_b^2$ factor.
- The obtained Upsilon spectral function is found to contain contributions of the Υ(1s), Υ(2s) and Υ(3s) states. We cannot separate individual states in our method.

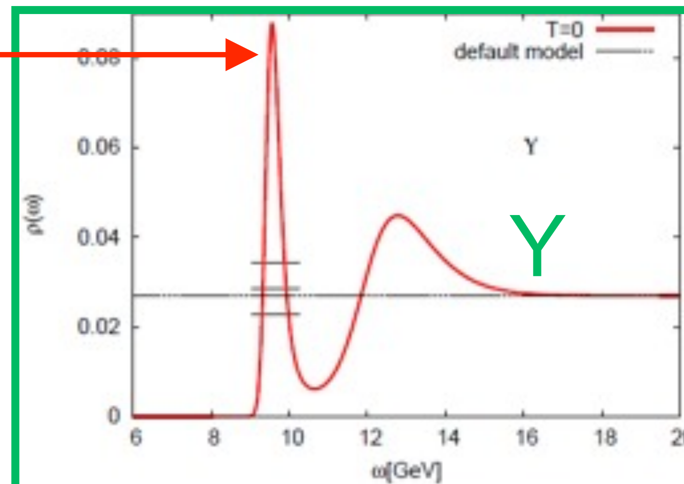
Υ(1S), Υ(2S) and Υ(3S)



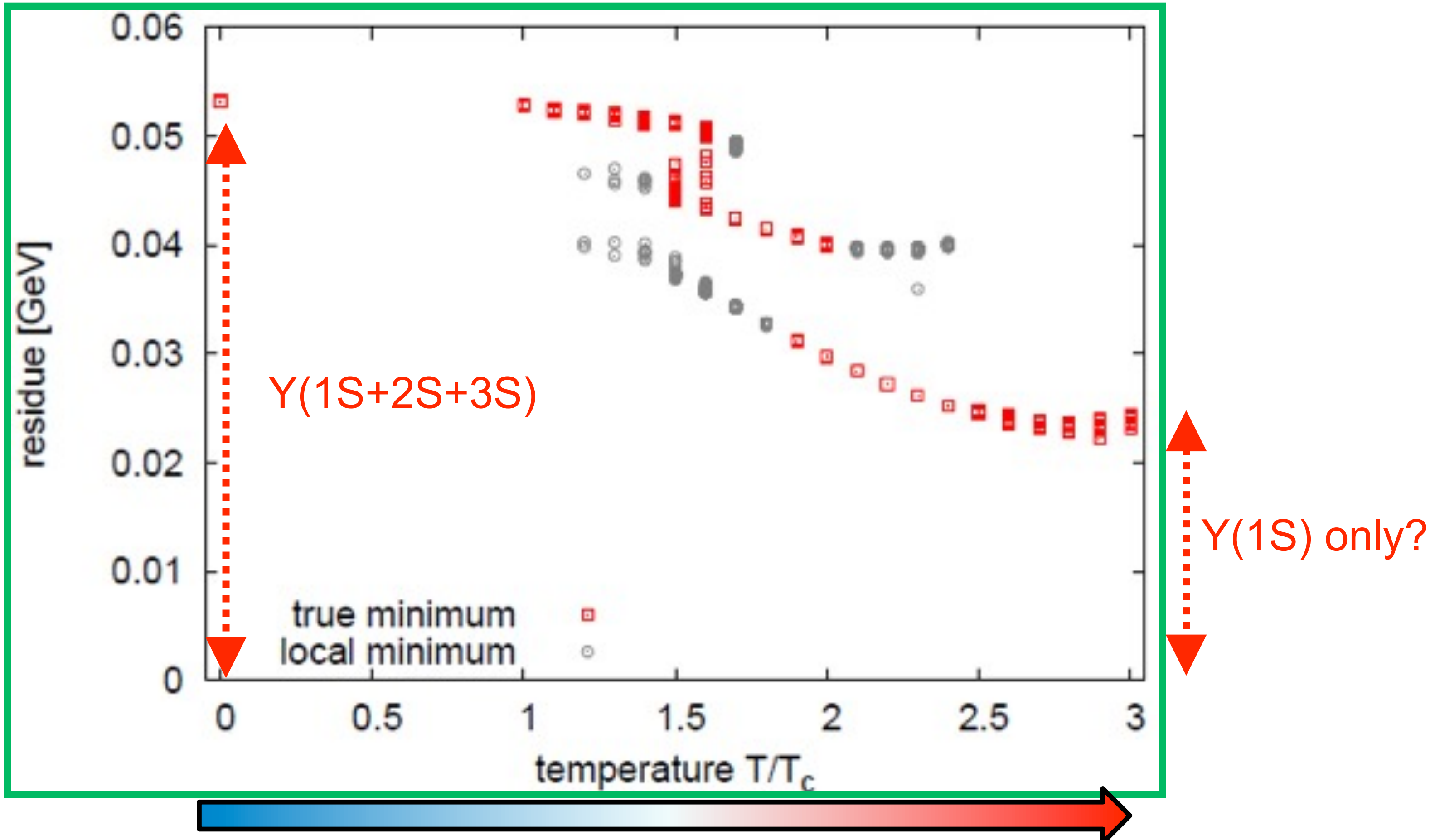
Υ(1S) : 9.46GeV

Υ(2S) : 10.02GeV

Υ(3S) : 10.36GeV



Bottomonia at finite temperature



The residue of the Υ peak decreases as T increases, which suggests that the excited states (2S, 3S) melt away at 1.5-2.0T_c, while the ground state(1S) survives further.

Conclusion

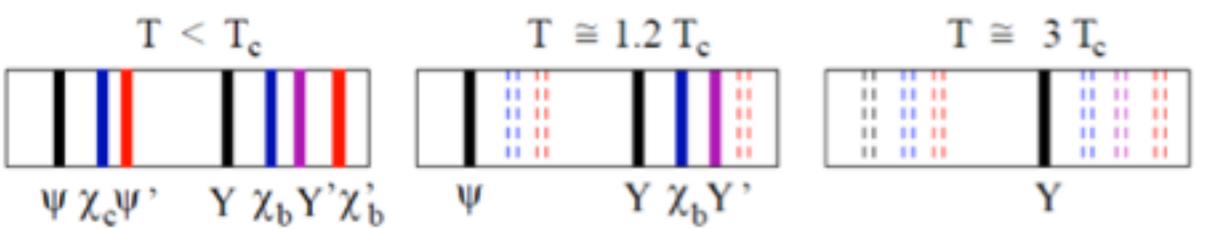
- **We have formulated a novel method for the analysis of the QCD sum rules using the Maximum Entropy Method (MEM). The spectral functions can be extracted directly from the sum rule without parametrizing them in a functional form such as pole+continuum.**
- **We have applied the new method to the analysis of the finite temperature quarkonium spectrum and found that the quarkonium peaks indeed disappear due to the rapid change of the gluon condensates at above T_c.**
- **The peaks of quarkonia disappear at temperatures,**

J/Ψ	η_c	χ_{c0}	χ_{c1}
$1.2\ T_c$	$1.1\text{-}1.2\ T_c$	$1.0\text{-}1.1\ T_c$	$1.0\text{-}1.1\ T_c$
Υ	η_b	χ_{b0}	χ_{b1}
$> 2.3\ T_c$	$> 2.1\ T_c$	$1.3\text{-}2.5\ T_c$	$< 2.5\ T_c$

Conclusion

- The bb^{bar} mesons survive at higher temperatures than the charmonia, because they are less sensitive to the changes of the gluon condensates.
- The results are consistent with the picture in which the bottomonium excited states Υ(2S,3S) melt away at lower temperatures than the ground state Υ(1S).

The lighter quarkonia melt at a lower T, while the heavier ones melt at a higher T → *Thermometer for the QGP*



by H. Satz