Hadron Structure in AdS/QCD

Valery Lyubovitskij

Institut für Theoretische Physik, Universität Tübingen
Kepler Center for Astro and Particle Physics, Germany

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

in collaboration with

Thomas Gutsche
Ivan Schmidt
Alfredo Vega

PRD 80 (2009) 055014
PRD 82 (2010) 074022
PRD 83 (2011) 036001
PRD 85 (2012) 076003
PRD 86 (2012) in print

Chiral Dynamics 2012, 7 August 2012, JLab, Newport News

Outline

- Introduction
  - Motivation
  - Classification of AdS/QCD models
  - Building bottom-up models
  - Highlights of main results in meson sector
- Baryons in soft-wall model
  - Basic blocks of effective action
  - EOM for the bulk profiles: mass spectra and wave functions
  - Inclusion of high Fock states
  - Consistency with large N_c
  - Inclusion photons
- Applications
  - Electromagnetic form factors
  - Generalized parton distributions
  - Roper resonance $N(1440)$

Introduction

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space

- HQCD models reproduce main features of QCD at low and high energies

- Motivation: AdS/CFT correspondence 1998 (Maldacena, Polyakov, Witten et al)

  Dynamics of the superstring theory in AdS$_{d+1}$ background is encoded in d conformal field theory living on the AdS boundary.

  More general: extra-dimensional (ED) theories including gravity are holographically equivalent to the gauge theories living on the boundary of ED space

- Symmetry arguments: Conformal group acting in boundary theory isomorphic to $SO(4,2)$ – the isometry group of AdS$_5$ space

Introduction

- AdS metric $ds^2 = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$ Poincaré form

  z is extra dimensional (holographic) coordinate; $z = 0$ is UV boundary

  UV asymptotics Klebanov, Witten

  $$\Phi(x,z)\Big|_{z\to 0} \to z^{d-\Delta}\left[\Phi_0(x) + O(z^2)\right] + z^{\Delta}\left[\Phi_{\rm ph}(x) + O(z^2)\right]$$

  $\Phi_0(x)$ is source of the CFT operator $\hat{\mathcal{O}}$,
  $\Phi_{\rm ph}(x) \sim \langle\hat{\mathcal{O}}\rangle$ is physical fluctuation

AdS/CFT dictionary

Gauge	Gravity
Operator $\hat{\mathcal{O}}$	Bulk field $\Phi(x,z)$
Δ — scaling dimension of $\hat{\mathcal{O}}$	m — mass of $\Phi(x,z)$
Source of $\hat{\mathcal{O}}$	Non-normalizable bulk profile near $z = 0$
$\langle\hat{\mathcal{O}}\rangle$	Normalizable bulk profile near $z = 0$

Introduction

- Top-down approaches Low-energy approximation of string theory trying to find a gravitational background with features similar to QCD (e.g. Sakai-Sugimoto model)
- Bottom-up approaches More phenomenological use the features of QCD to construct 5D dual theory including gravity on AdS space
- Towards to QCD:
  – Break conformal invariance and generate mass gap
  – Tower of normalized bulk fields (Kaluza-Klein modes) $\leftrightarrow$ Hadron wave functions
  – Spectrum of Kaluza-Klein modes $\leftrightarrow$ Hadrons spectrum
- Hard-wall:
  AdS geometry is cutted by two branes UV $(z = \epsilon \to 0)$ and IR $(z = z_{\rm IR})$
  Analogue of quark bag model, linear dependence on $J(L)$ of hadron masses
- Soft-wall:
  Soft cuttoff of AdS space by dilaton field $e^{-\varphi(z)}$
  Analytical solution of EOM, Regge behavior $M^2 \sim J(L)$

Introduction

Building HW/SW models

Erlich, Karch, Katz, Son, Stephanov, Da Rold, Pomarol, . . .

- Geometry: AdS space near $z = 0$ corresponds to QCD conformal in the UV

- Truncate AdS space by two branes:
  UV ($z = 0$) and IR ($z = z_{\rm IR}$) in HW or (dilaton) in SW

- 5D gauge groups

  Isospin: SU(2) isospin $\longrightarrow$ 5D SU(2) gauge group

  Vector mesons $\longrightarrow$ Vector KK gauge bosons

  Chiral symmetry: SU(2) $\times$ SU(2) $\longrightarrow$ SU(2) $\times$ SU(2) 5D gauge group

  Axial mesons $\longrightarrow$ Axial KK gauge bosons

  EBχS $\longrightarrow$ non-normalizable z-profile of scalar 5D field $S_0^{\rm non}(z) = \frac{\hat{m}}{2} z$

  SBχS $\longrightarrow$ normalizable z-profile of scalar 5D field $S_0^{\rm nor}(z) = \frac{\langle \bar{q} q \rangle}{2} z^3$

Introduction

- Scalar and Pseudoscalar fields $X(x,z) = \Big(S(x,z) + S_0(z)\Big)\, e^{\vec{\pi}(x,z)\vec{\tau}/F_\pi}$
  where $S_0(z) = S_0^{\rm non}(z) + S_0^{\rm nor}(z)$
- Pseudoscalar mesons are dual to the mixing of $\pi_i(x,z)$ and $\phi_i(x,z)$ – longitudinal component of axial field $A^i_\mu(x,z) = A^{i,\perp}_\mu(x,z) + \partial_\mu \phi^i(x,z)$
- Action

$$S = \int d^5 x e^{-\varphi(z)}\, \sqrt{g}\left\{ -\frac{1}{4g_5^2}(F_L^2 + F_R^2) - |DX|^2 + 3|X|^2 \right\}$$

where

$$\varphi(z) = \kappa^2 z^2\,, \quad g_5^2 = 12\pi^2/N_c\,,$$

$$V = \frac{1}{2}(A_L + A_R)\,, \quad A = \frac{1}{2}(A_L - A_R)\,,$$

$$F^{MN}_{L,R} = \partial^M A^N_{L,R} - \partial^N A^M_{L,R} - i[A^M_{L,R}, A^N_{L,R}]\,,$$

$$D^M X = \partial^M X - iA^M_L X + iXA^M_R$$

Introduction

- How it works: Mass spectrum and WFs of vector mesons
- Apply axial gauge $V_z = 0$ and derive EOM for the 4D-transverse component $V_\mu^\perp$

$$e^{\varphi(z)}\,\partial_z\left[\frac{e^{-\varphi(z)}}{z}\partial_z V_\mu^\perp(x,z)\right] - \partial^2 V_\mu^\perp(x,z) = 0$$

- KK expansion $V_\mu^\perp(x,z) = \sum_n V_{\mu,n}^\perp(x)\,v_n(z)$

$$e^{\varphi(z)}\,\partial_z\left[\frac{e^{-\varphi(z)}}{z}\partial_z v_n(z)\right] + M_{V_n}^2\, v_n(z) = 0$$

- After the substitution $v_n(z) = e^{\varphi(z)/2}\,\sqrt{z}\,\psi_n(z)$ derive Schrödinger EOM

$$-\partial_z^2\psi_n(z) + U(z)\psi_n(z) = M_{V_n}^2\,\psi_n(z)$$

where $U(z) = \kappa^4 z^2 + \frac{3}{4z^2}$ dilaton potential

- Analytical solutions

$$M_{V_n}^2 = 4\kappa^2(n+1)\,, \qquad \psi_n(z) = \sqrt{\frac{2n!}{(n+1)!}}\,\kappa^2\, z^{3/2}\, e^{-\kappa^2 z^2/2}\, L_n^1(\kappa^2 z^2)$$

Introduction

Main results in meson sector

- Reproduces concequences of chiral symmetry:

  Gell-Mann – Oakes – Renner relation $M_\pi^2\, F_\pi^2 \;=\; 2\hat{m}\,\langle\bar{q}q\rangle$

- Properties of correlators of vector and axial current:

  $$\Pi_V(Q^2) \to -\frac{N_c}{24\pi^2}\log Q^2 \quad \text{at} \;\; Q^2 \to \infty$$

  $$\Pi_A(Q^2) \to \frac{F_\pi^2}{Q^2} \quad \text{at} \;\; Q^2 \to 0$$

  VV (AA) Correlators are expanded as a sum over vector (axial) meson resonances

- Mass spectrum (Regge-like behavior)

- Form factors with correct power scaling, distribution functions, etc.

Baryons in soft-wall model

- Baryons in soft-wall model: Forkel–Beyer–Frederico, Brodsky–Teramond, Abidin–Carlson, Gutsche–Lyubovitskij–Schmidt–Vega, ...
- SW holographic approach for baryons with inclusion of high Fock states dual to bulk fermion fields of higher dimension.
- Objective: Application to nucleon form factors, GPDs, nucleon resonances (Roper)

Baryons in soft-wall model

- Bulk fermion fields
  $\Psi_+(x,z)$ and $\Psi_-(x,z)$ dual to $\mathcal{O}_R = (p_R, n_R)$ and $\mathcal{O}_L = (p_L, n_L)$
- Bulk fermion mass $\quad \pm m = \pm(\Delta - 3/2)$, where Δ - scaling dimension
- Scaling dimension $\equiv$ Twist-dimension $\quad \tau = N + L$,
  N - number of partons, $L = \max|L_z|$
- Action for the fermion field of twist τ

$$S_\tau = \int d^4x dz \sqrt{g}\, e^{-\varphi(z)} \sum_{i=+,-} \bar{\Psi}_{i,\tau}(x,z)\, \hat{\mathcal{D}}_i(z)\, \Psi_{i,\tau}(x,z)\,,$$

$$\hat{\mathcal{D}}_\pm(z) = \frac{i}{2}\Gamma^M \overleftrightarrow{\partial}_M \mp \frac{m + \varphi(z)}{R}$$

- dilaton $\quad \varphi(z) = \kappa^2 z^2$ (Regge behavior of hadron masses)
- metric $\quad g_{MN}(z) = \epsilon^a_M(z)\epsilon^b_N(z)\eta_{ab}, \quad g = |\mathrm{det} g_{MN}|$
- vielbein $\quad \epsilon^a_M(z) = e^{A(z)}\delta^a_M, \quad A(z) = \log(R/z)$ (conformal)
- interval $\quad ds^2 = g_{MN}dx^M dx^N = e^{2A(z)}(g_{\mu\nu}dx^\mu dx^\nu - dz^2)$

Baryons in soft-wall model

- P-transformations

$$U_P^{-1}\Psi_{\tau,\pm}(t,\vec{x},z)U_P = \pm\gamma^0\gamma^5\,\Psi_{\tau,\mp}(t,-\vec{x},z)$$

$$U_P^{-1}\bar{\Psi}_{\tau,\pm}(t,\vec{x},z)U_P = \pm\bar{\Psi}_{\tau,\mp}(t,-\vec{x},z)\,\gamma^0\gamma^5$$

$$\pm U_P^{-1}\,\bar{\Psi}_{\pm,\tau}(t,\vec{x},z)\,\Psi_{\pm,\tau}(t,\vec{x},z)\,U_P = \mp\bar{\Psi}_{\mp,\tau}(t,-\vec{x},z)\,\Psi_{\mp,\tau}(t,\vec{x},z)\,,$$

$$U_P^{-1}\,S_\tau^{\pm}\,U_P = S_\tau^{\mp}$$

- C-transformations

$$U_C^{-1}\Psi_\pm(x,z)U_C = \mp\,C\,\gamma^5\,\bar{\Psi}_\mp^T(x,z)$$

$$U_C^{-1}\bar{\Psi}_\pm(x,z)U_C \;=\; \pm\Psi_\mp^T(x,z)\gamma^5 C$$

$$\pm\,U_C^{-1}\bar{\Psi}_\pm(x,z)\Psi_\pm(x,z)U_C = \mp\,\bar{\Psi}_\mp(x,z)\Psi_\mp(x,z)$$

$$U_C^{-1}\,S_\tau^{\pm}\,U_C = S_\tau^{\mp}$$

Baryons in soft-wall model

- Redefinition $\Psi_{i,\tau}(x,z) = e^{\varphi(z)/2-2A(z)}\psi_{i,\tau}(x,z)$
- Expansion on left- and right-chirality components (eigenstates of γ^5)
  $\psi_{i,\tau}(x,z) = \psi^L_{i,\tau}(x,z) + \psi^R_{i,\tau}(x,z)$
- Kaluza-Klein expansion

$$\psi^{L/R}_{i,\tau}(x,z) = \frac{1}{\sqrt{2}} \sum_n \psi^{L/R}_n(x)\, f^{L/R}_{i,\tau,n}(z)\,,$$

- Relations between bulk profiles

$$\begin{aligned} f^R_{\tau,n}(z) &\equiv f^R_{+,\tau,n}(z) = -f^L_{-,\tau,n}(z)\,, \\ f^L_{\tau,n}(z) &\equiv f^L_{+,\tau,n}(z) = f^R_{-,\tau,n}(z)\,. \end{aligned}$$

- EOM

$$\left[-\partial_z^2 + \kappa^4 z^2 + 2\kappa^2\left(m \mp \frac{1}{2}\right) + \frac{m(m\pm 1)}{z^2}\right] f^{L/R}_{\tau,n}(z) = M^2_{n\tau}\, f^{L/R}_{\tau,n}(z),$$

Baryons in soft-wall model

- Solutions

$$f^L_{\tau,n}(z) \;=\; \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau)}}\,\kappa^\tau\; z^{\tau-1/2}\; e^{-\kappa^2 z^2/2}\; L_n^{\tau-1}(\kappa^2 z^2)\,,$$

$$f^R_{\tau,n}(z) \;=\; \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}}\,\kappa^{\tau-1}\; z^{\tau-3/2}\; e^{-\kappa^2 z^2/2}\; L_n^{\tau-2}(\kappa^2 z^2)$$

and

$$M^2_{n\tau} = 4\kappa^2\Big(n+\tau-1\Big)$$

with

$$\int\limits_0^\infty dz\, f^{L/R}_{\tau,n_1}(z)\, f^{L/R}_{\tau,n_2}(z) \;=\; \delta_{n_1 n_2}$$

Baryons in soft-wall model

- Inclusion of high Fock states

$$S = \sum_{\tau} c_\tau \, S_\tau$$

c_τ - set of free parameters

- Integration over z using normalizazion condition for $f^{L/R}$

$$S = \int d^4x \, \bar\psi_n(x) \Big[\underbrace{\sum_{\tau} c_\tau}_{=1} \, i \, \not\partial - \underbrace{\sum_{\tau} c_\tau M_{n\tau}}_{=M_n} \Big] \psi_n(x) \, .$$

Correct normalization of kinetic term of 4D spinor field

$$\sum_{\tau} c_\tau = 1 \, , \qquad \sum_{\tau} c_\tau M_{n\tau} = M_n \quad \text{(baryon mass)}$$

Baryons in soft-wall model

- Large N_c expansion $M_n = \sum_\tau c_\tau \, M_{n\tau} \sim \underbrace{\kappa}_{\sim\sqrt{N_c}} \cdot \underbrace{\sqrt{n+\tau-1}}_{\sim\sqrt{N_c}} \sim N_c$

- $\kappa \sim \sqrt{N_c}$ consistent with LFH (Brodsky-Teramond)
  $F_\pi = \frac{\sqrt{3}}{8}\kappa = 83 \text{ MeV} \sim \sqrt{N_c}$

- Dilaton can be identified with VEV of the scalar bulk field with dimension-2, which is holographically dual to the dimension-2 gluon operator A_μ^2.

- $\kappa^2 \sim N_c$ is related to the vacuum expectation value (VEV) $\langle \alpha_s A_\mu^2 \rangle \sim N_c$

- A_μ^2 has been discussed in the literature in detail:

  Celenza-Shakin; Chetyrkin-Narison-Zakharov; Gubarev-Stodolsky-Zakharov; Dorokhov-Broniowski; Arriola-Bowman-Broniowski

- Quadratic form of the dilaton profile is not unique

  See: Liu-Tseytlin model (top-down AdS/QCD approach)
  $e^{\varphi(z)} = 1 + qz^4$, where q related to dim-4 condensates $\langle \alpha_s G_{\mu\nu}^2 \rangle \sim N_c$ and $\langle \alpha_s G_{\mu\nu} \tilde{G}_{\mu\nu} \rangle \sim N_c$

Baryons in soft-wall model

- Holographic variable z (as conjugate to κ) scales as $z \sim \dfrac{1}{\sqrt{N_c}}$

- Physical interpretation:

  - Limit $N_c \to \infty$ means $z = 0$ or approaching to UV boundary

  baryons are bound states of infinitely number of N_c quarks

  - Limit $N_c \to 0$ means $z \to \infty$

  no bound states of quarks due to confinement imposed by the dilaton

  see also Brodsky, Huet PLB 417 (1998) 145

Electromagnetic stucture of nucleons

- Abidin-Carlson: First application of SW model (3q configurations)
- Coupling of bulk vector and fermion fields

$$\mathcal{L}_{\rm int}(x,z) = \sum_{i=+,-} \sum_{\tau} c_\tau \, \bar\Psi_{i,\tau}(x,z)\, \hat{\mathcal{V}}_i(x,z)\, \Psi_{i,\tau}(x,z)$$

$$\hat{\mathcal{V}}_\pm(x,z) = Q\,\Gamma^M V_M(x,z) \pm \frac{i}{4}\,\eta_V\,[\Gamma^M,\Gamma^N]\,V_{MN}(x,z) \pm g_V\,\tau_3\,\Gamma^M\, i\Gamma^z\, V_M(x,z)$$

$$\langle p'|J^\mu(0)|p\rangle \;=\; \bar u(p')\,\left[\gamma^\mu F_1^N(t) + \frac{i}{2m_N}\,\sigma^{\mu\nu}\,q_\nu F_2^N(t)\right] u(p)$$

$$\begin{aligned}
F_1^p(Q^2) &= C_1(Q^2) + g_V C_2(Q^2) + \eta_V^p C_3(Q^2)\\
F_2^p(Q^2) &= \eta_V^p C_4(Q^2)\\
F_1^n(Q^2) &= -g_V C_2(Q^2) + \eta_V^n C_3(Q^2)\\
F_2^n(Q^2) &= \eta_V^n C_4(Q^2)
\end{aligned}$$

Electromagnetic stucture of nucleons

- $$C_1(Q^2) = \frac{1}{2}\int_0^\infty dz\, V(Q,z)\, \sum_\tau c_\tau \left([f_\tau^L(z)]^2 + [f_\tau^R(z)]^2\right)$$
  $$= \sum_\tau c_\tau\, B(a+1,\tau)\, \left(\tau + \frac{a}{2}\right) \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$
- $$C_2(Q^2) = \frac{1}{2}\int_0^\infty dz\, V(Q,z)\, \sum_\tau c_\tau \left([f_\tau^R(z)]^2 - [f_\tau^L(z)]^2\right)$$
  $$= \frac{a}{2}\sum_\tau c_\tau\, B(a+1,\tau) \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$
- $$C_3(Q^2) = \frac{1}{2}\int_0^\infty dz z\, \partial_z V(Q,z)\, \sum_\tau c_\tau \left([f_\tau^L(z)]^2 - [f_\tau^R(z)]^2\right)$$
  $$= a\sum_\tau c_\tau\, B(a+1,\tau+1)\, \frac{a(\tau-1)-1}{\tau} \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$
- $$C_4(Q^2) = 2m_N \int_0^\infty dz z\, V(Q,z)\, \sum_\tau c_\tau\, f_\tau^L(z) f_\tau^R(z)$$
  $$= \frac{2m_N}{\kappa}\sum_\tau c_\tau\, (a+1+\tau)\, B(a+1,\tau+1)\, \sqrt{\tau-1} \sim \sum_\tau \frac{c_\tau}{a^\tau}$$
- $a = \frac{Q^2}{4\kappa^2}$, $\quad B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ is the Beta function.

Electromagnetic stucture of nucleons

- $V(Q,z)$ – propagator of trans. massless vector field (analogue of EM field)

$$V(Q,z) = \Gamma\left(1+\frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right)$$
$$= \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\dfrac{\kappa^2 z^2 x}{1-x}}$$

Radyushkin-Grigoryan (integral repr.)
Brodsky-Teramond (identify x with LC momentum fraction)

$$V(0,z) = 1\,, \qquad V(Q,0) = 1\,, \qquad V(Q,\infty) = 0\,.$$

At $Q^2 = 0$ functions $C_i(Q^2)$ are normalized as

$$C_1(0) = 1\,, \quad C_2(0) = C_3(0) = 0\,, \quad C_4(0) = \frac{2m_N}{\kappa} \sum_\tau c_\tau \sqrt{\tau - 1}$$

Electromagnetic stucture of nucleons

Choice of free parameters

- $\kappa = 383\ \mathrm{MeV}\,, \quad c_3 = 1.25\,, \quad c_4 = 0.16\,, \quad g_V = 0.3$
- c_5 is expressed through c_3 and c_4

$$c_5 = 1 - c_3 - c_4 = -0.41$$

- c_3, c_4 are constrained by the nucleon mass
- κ is fixed by the nucleon mass and nucleon electromagnetic radii
- g_V is fixed by fine tuning of the neutron electromagnetic radii
- Nonminimal couplings $\eta_V^{p,n}$ from nucleon magnetic moments

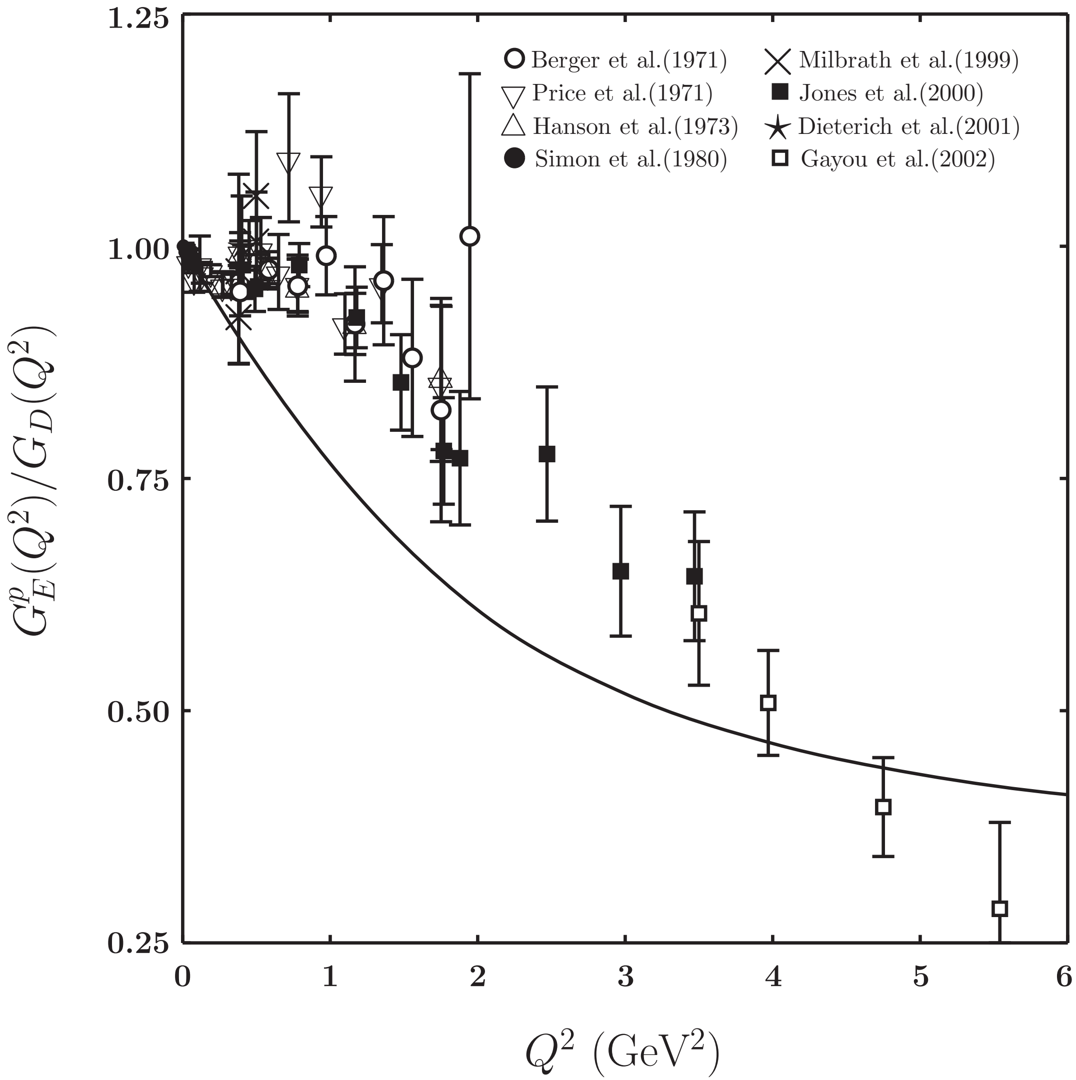
$$\eta_V^p = \frac{\kappa\,(\mu_p - 1)}{2m_N\,C_0} = 0.30\,, \quad \eta_V^n = \frac{\kappa\,\mu_n}{2m_N\,C_0} = -0.32\,, \quad C_0 = \sqrt{2}\,c_3 + \sqrt{3}\,c_4 + 2c_5$$

Electromagnetic stucture of nucleons

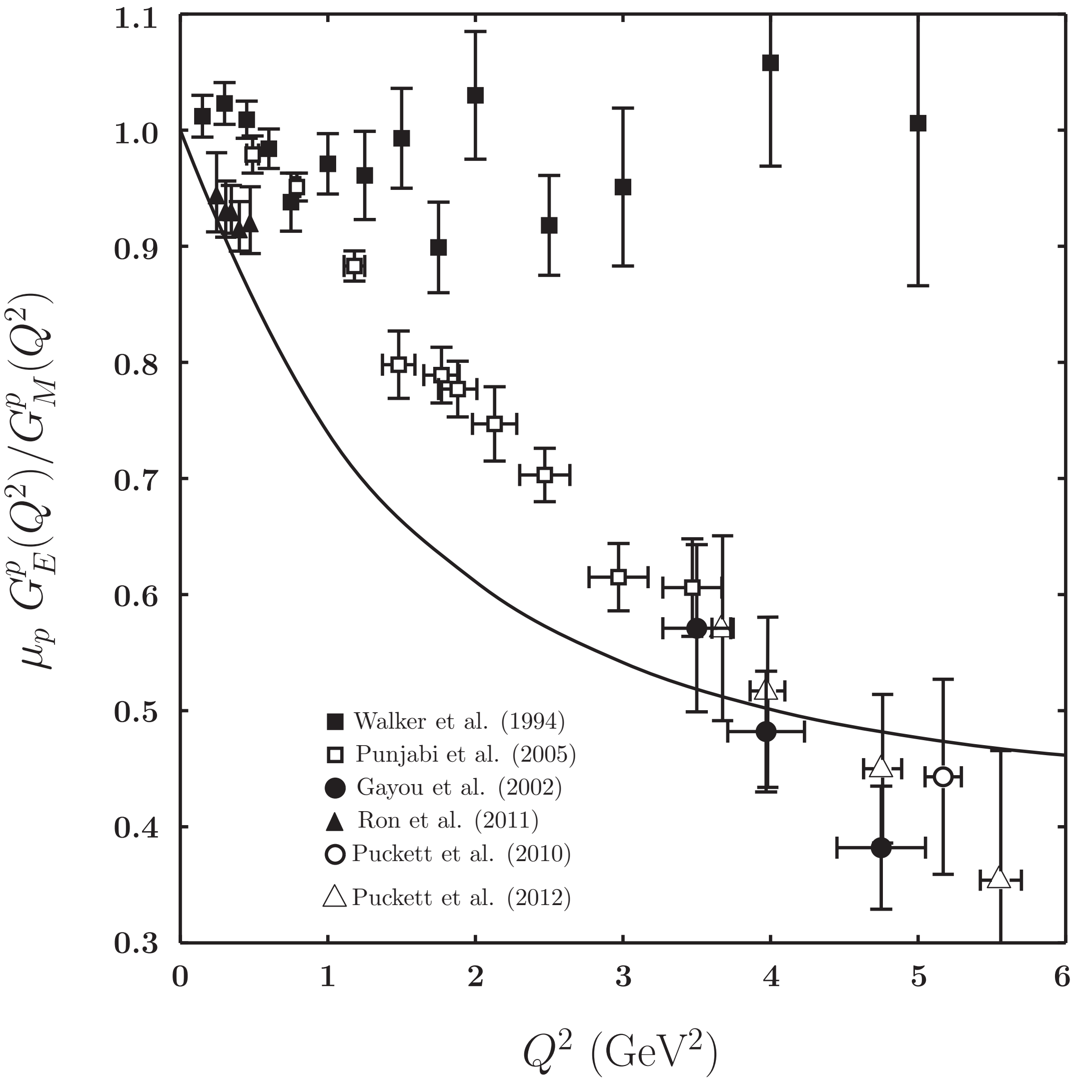
Mass and electromagnetic properties of nucleons

Quantity	Our results	Data
m_p (GeV)	0.93827	0.93827
μ_p (in n.m.)	2.793	2.793
μ_n (in n.m.)	-1.913	-1.913
r_E^p (fm)	0.840	0.8768 $\pm$ 0.0069
$\langle r_E^2 \rangle^n$ (fm^2)	-0.117	-0.1161 $\pm$ 0.0022
r_M^p (fm)	0.785	0.777 $\pm$ 0.013 $\pm$ 0.010
r_M^n (fm)	0.792	0.862$^{+0.009}_{-0.008}$
r_A (fm)	0.667	0.67$\pm$0.01

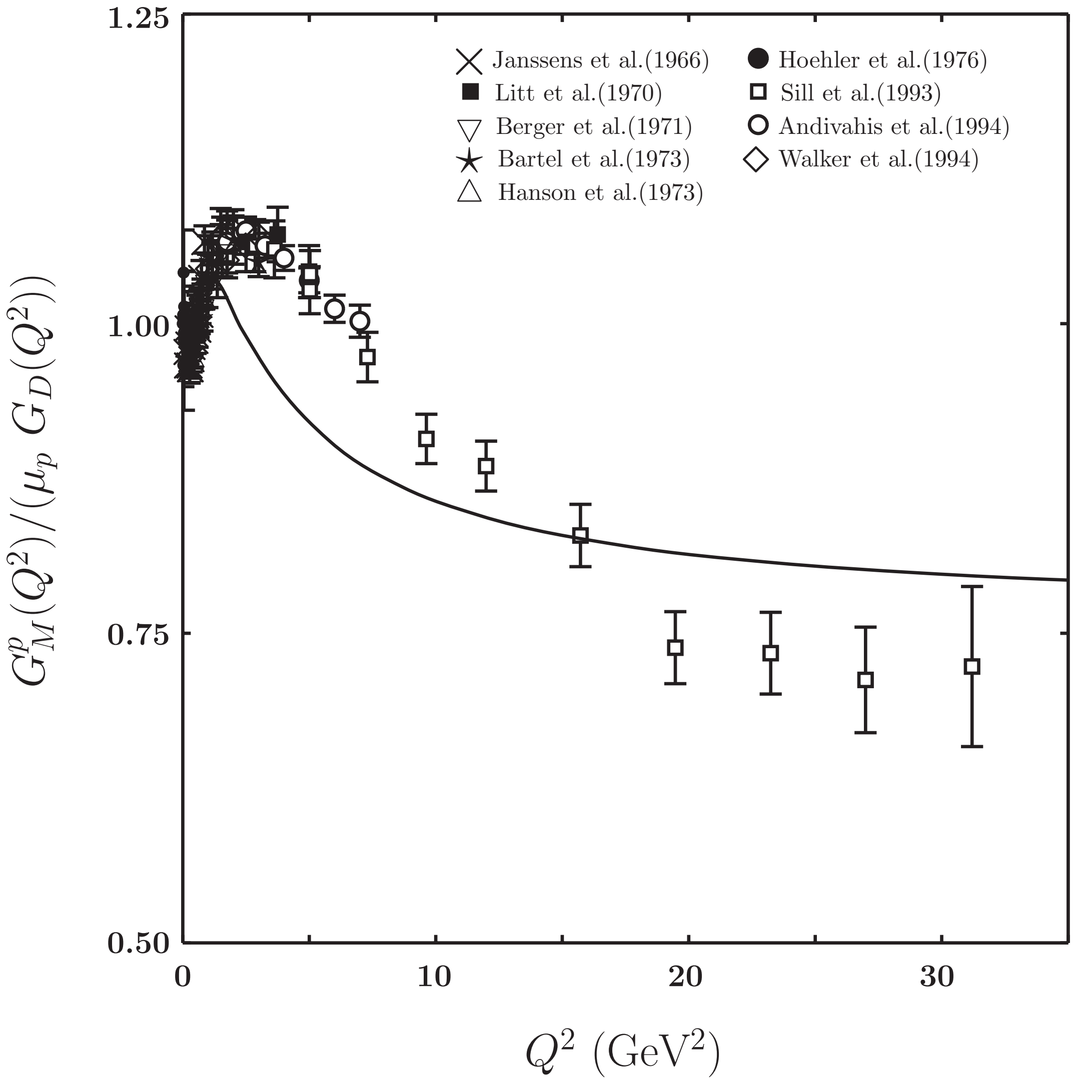
Electromagnetic stucture of nucleons



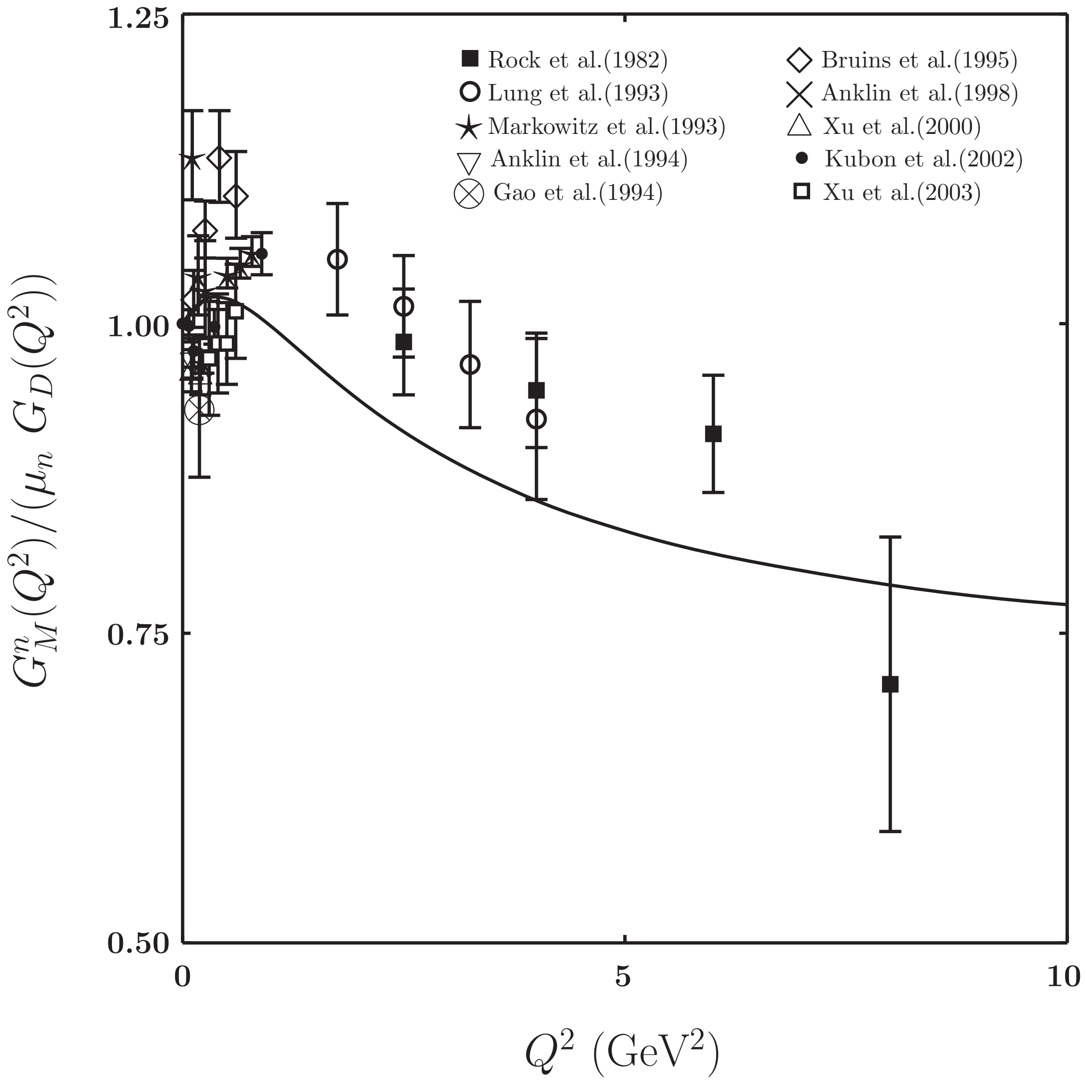
Electromagnetic stucture of nucleons



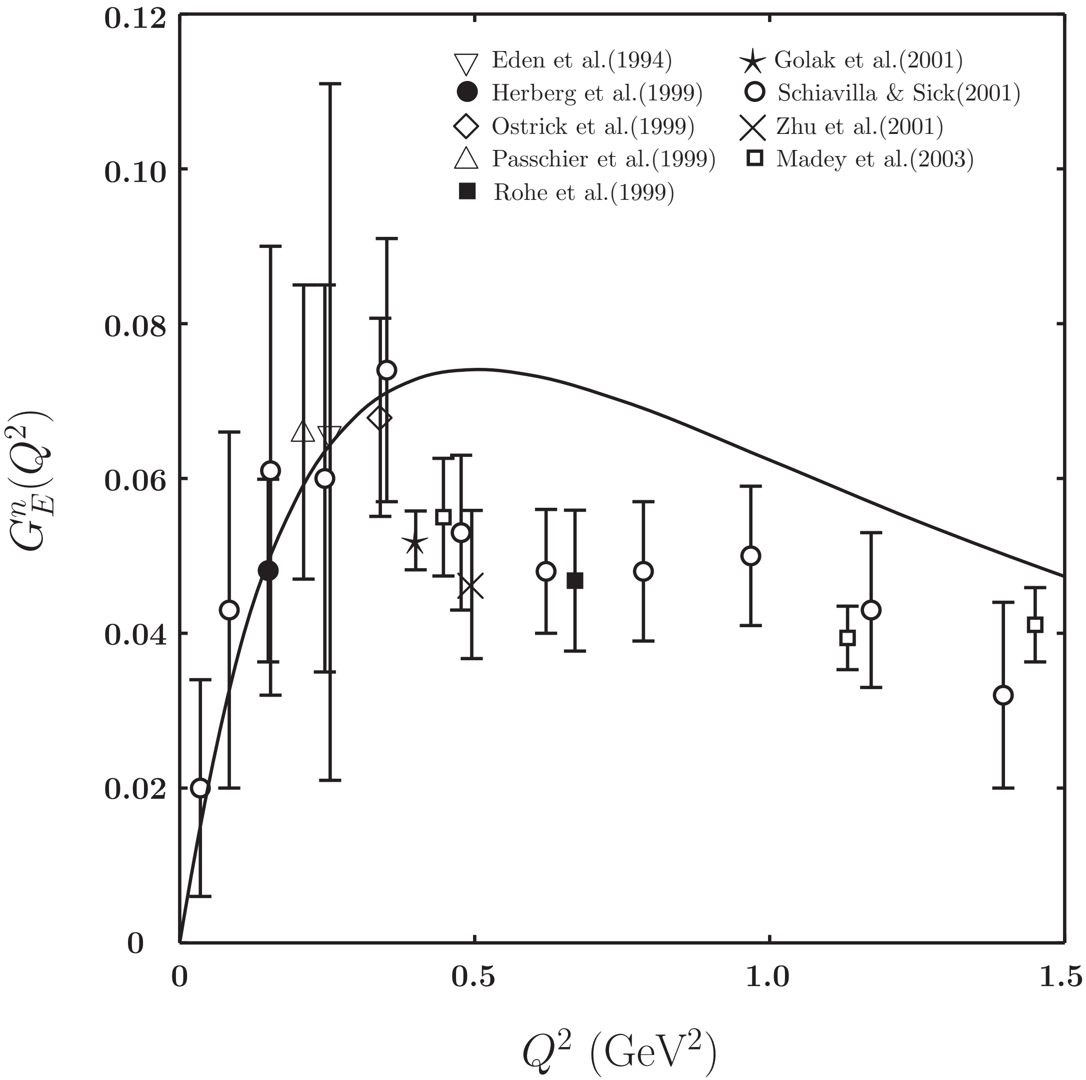
Electromagnetic stucture of nucleons



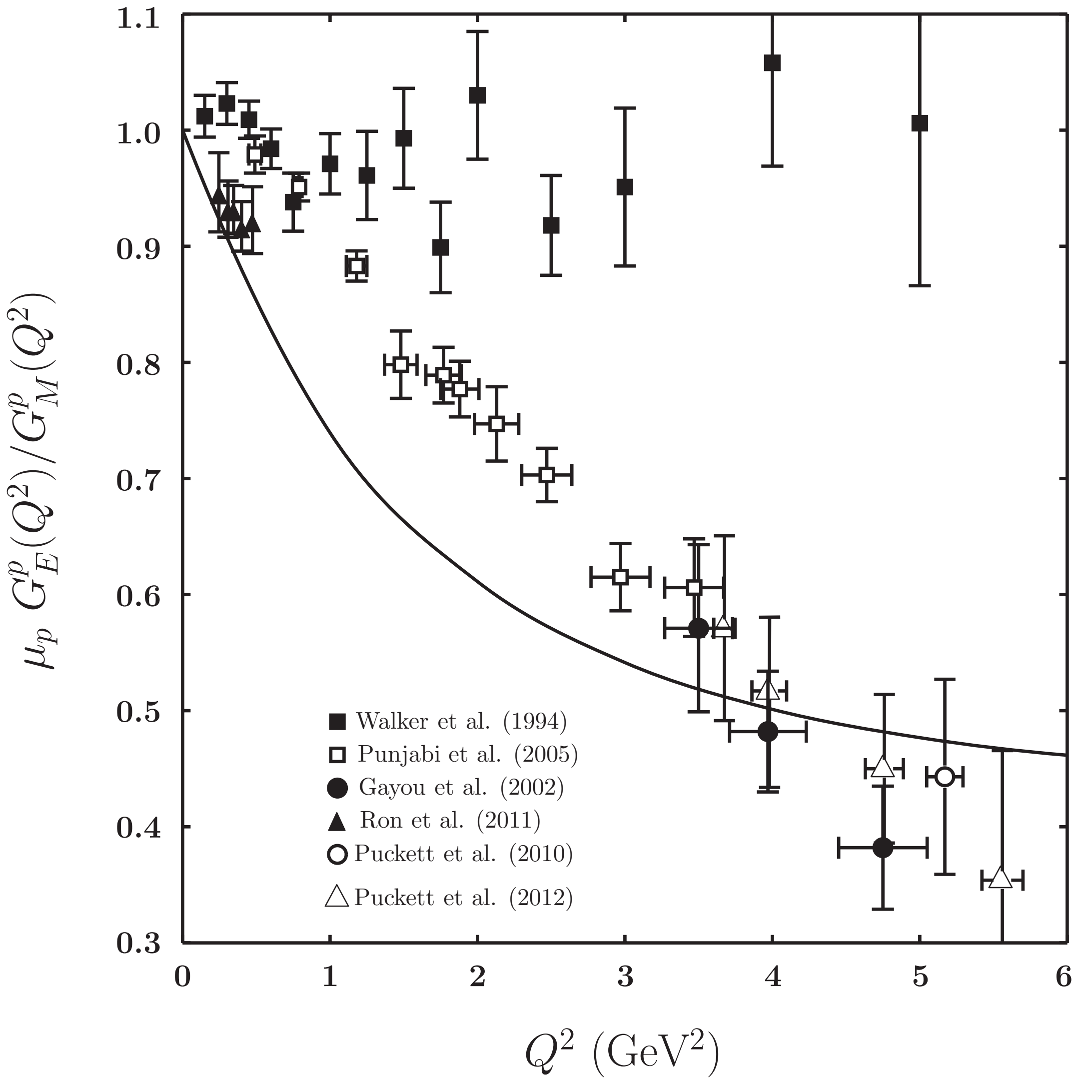
Electromagnetic stucture of nucleons



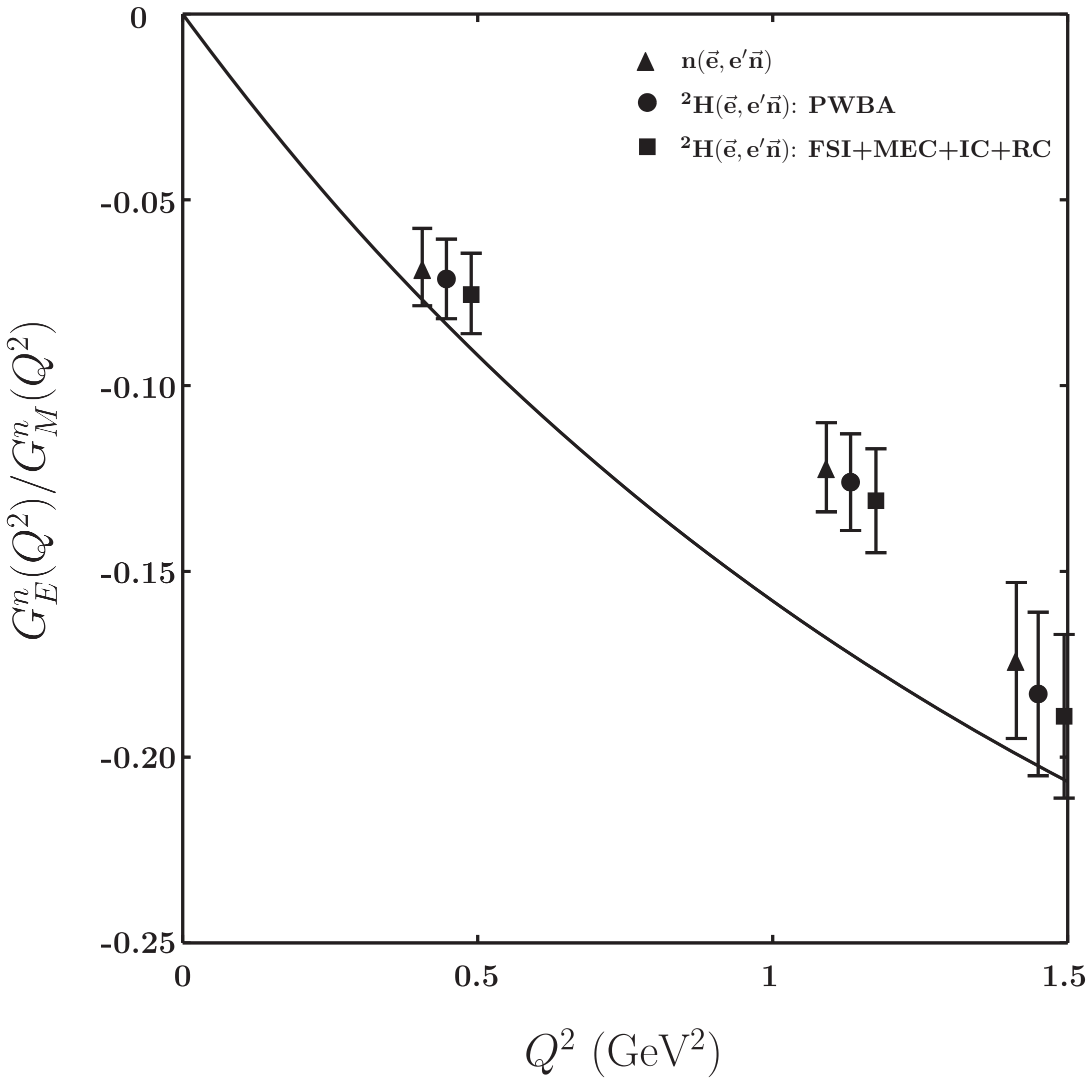
Electromagnetic stucture of nucleons



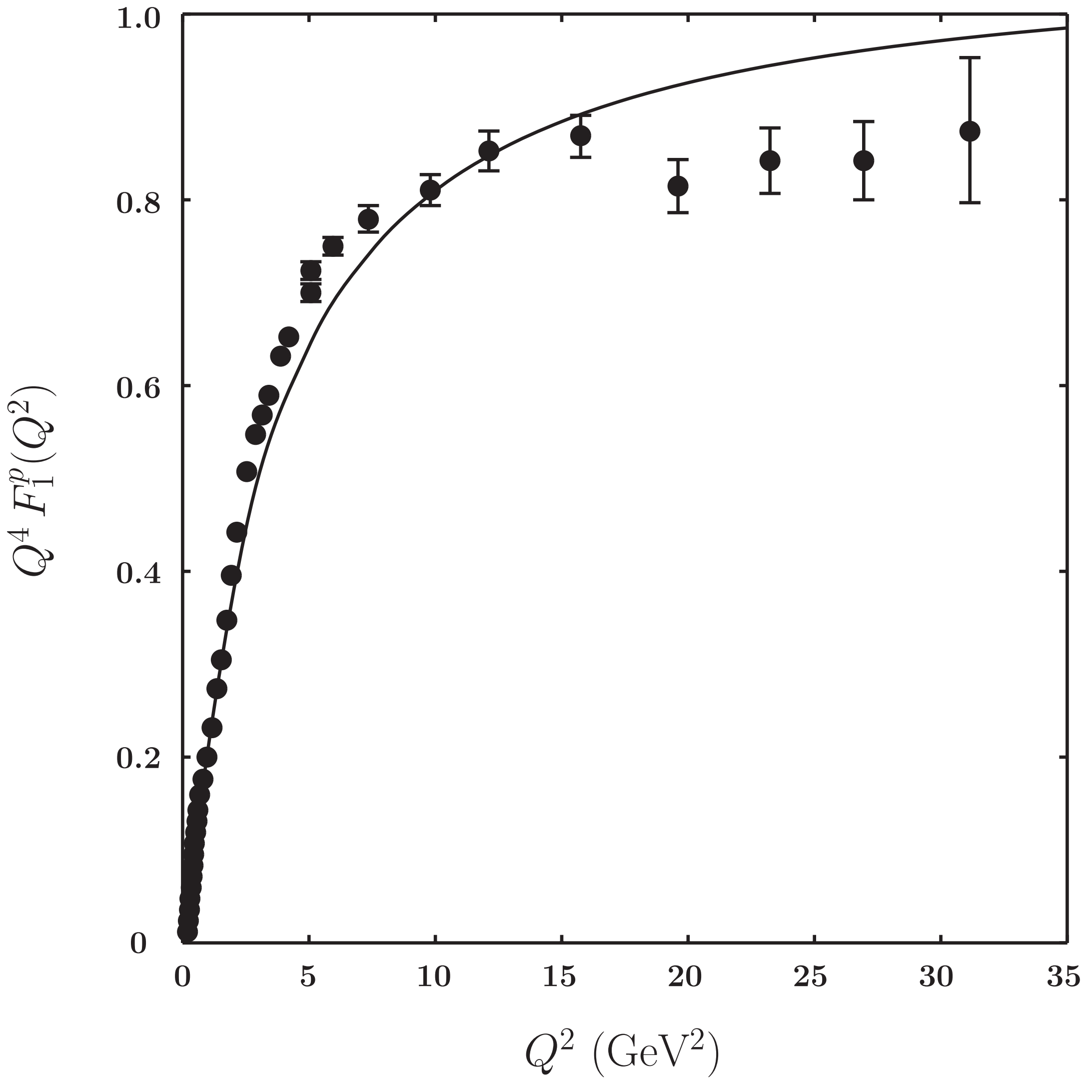
Electromagnetic stucture of nucleons



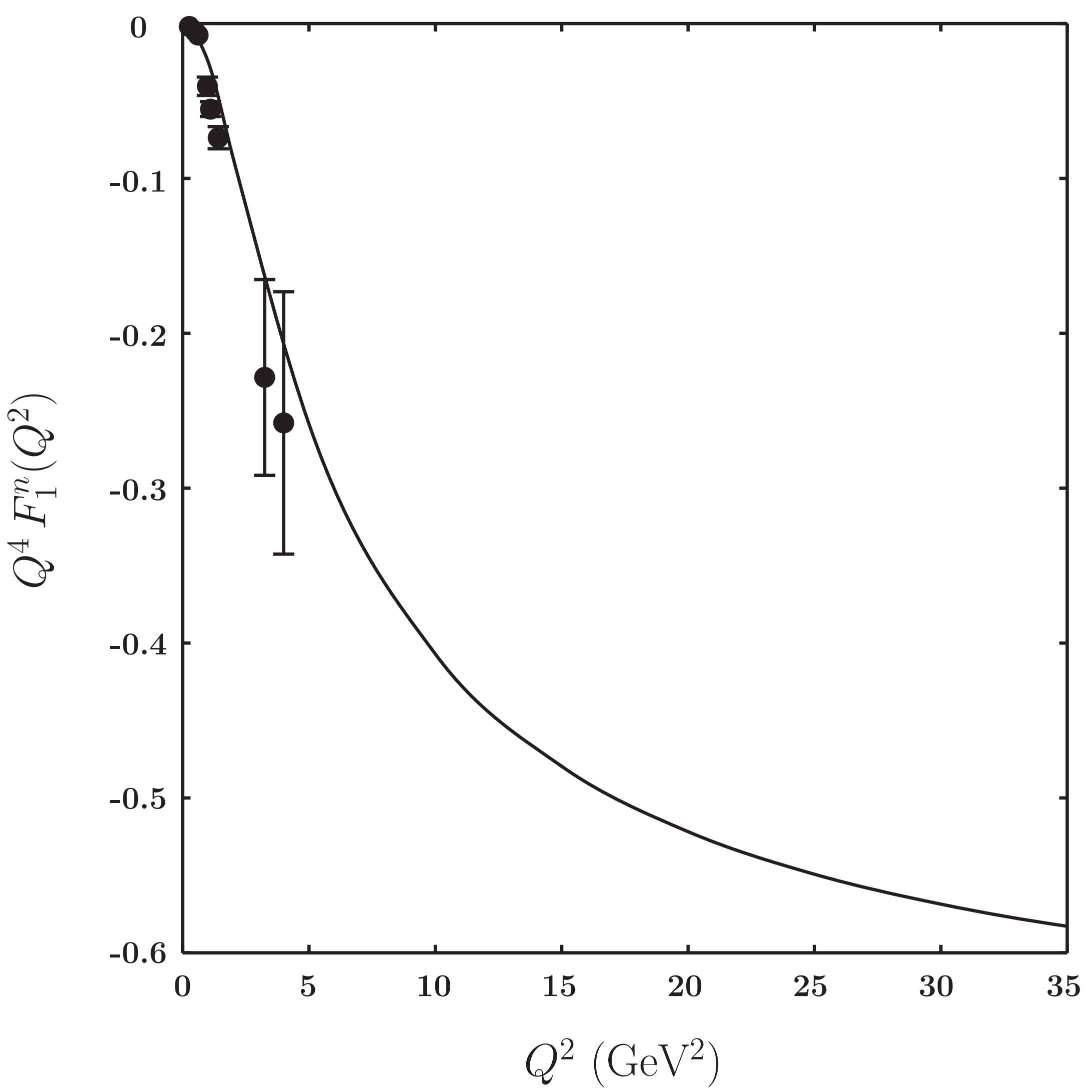
Electromagnetic stucture of nucleons



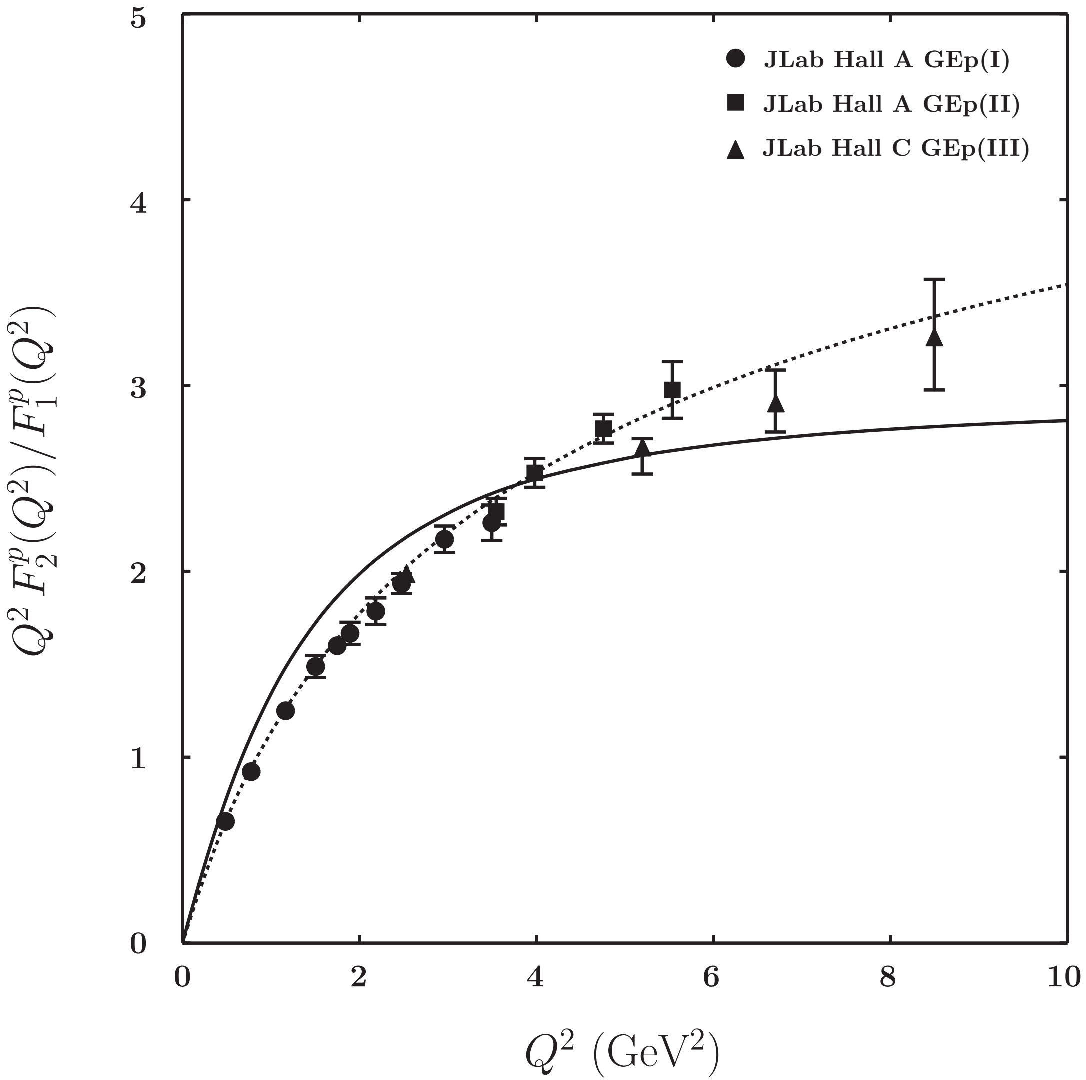
Electromagnetic stucture of nucleons



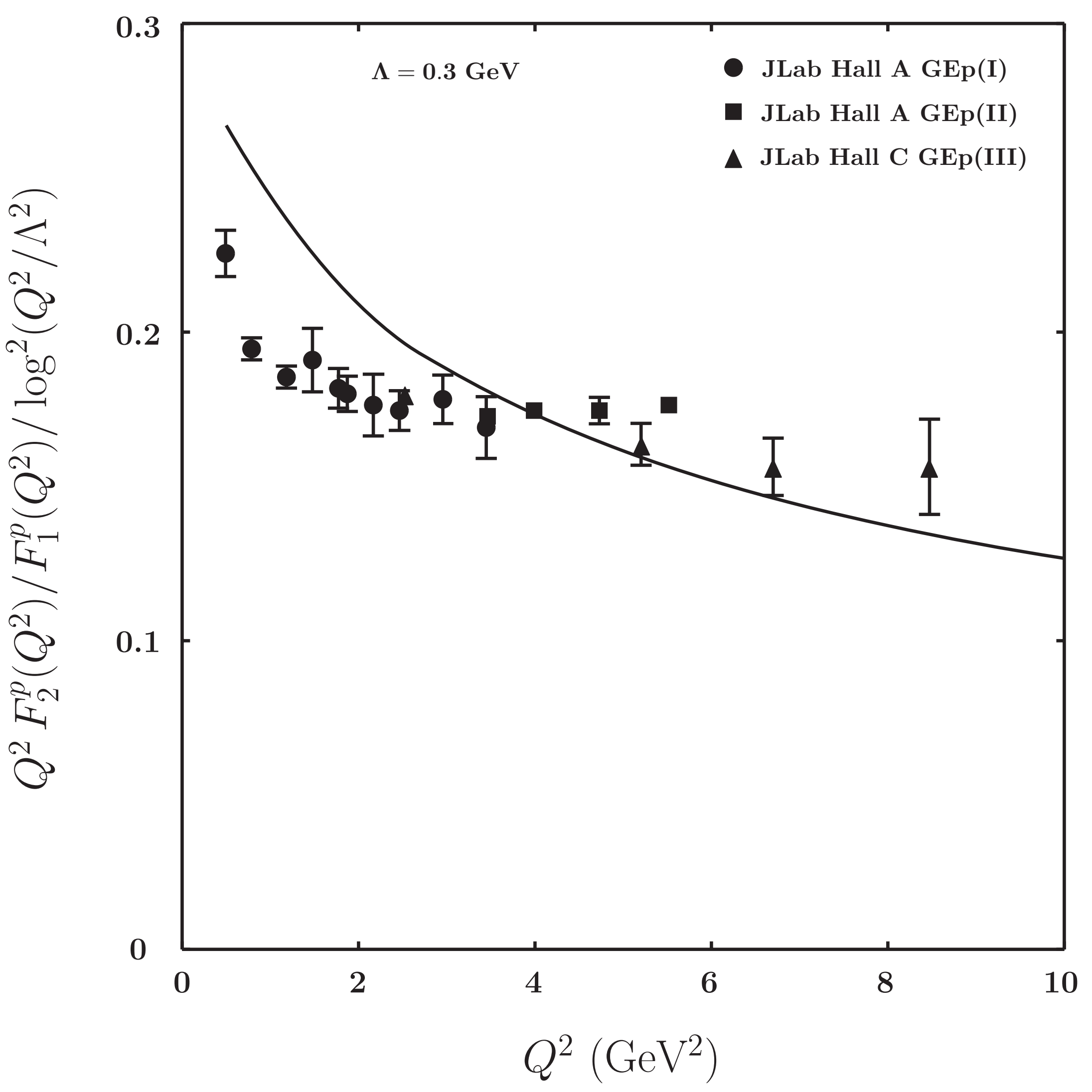
Electromagnetic stucture of nucleons



Electromagnetic stucture of nucleons



Electromagnetic stucture of nucleons



Nucleon GPDs

- Sum rules relating EM FF and GPDs Ji, Radyushkin

$$\begin{aligned} F_1^p(t) &= \int_0^1 dx \left(\frac{2}{3} H_v^u(x,t) - \frac{1}{3} H_v^d(x,t) \right) \\ F_1^n(t) &= \int_0^1 dx \left(\frac{2}{3} H_v^d(x,t) - \frac{1}{3} H_v^u(x,t) \right) \end{aligned}$$

$$\begin{aligned} F_2^p(t) &= \int_0^1 dx \left(\frac{2}{3} E_v^u(x,t) - \frac{1}{3} E_v^d(x,t) \right) \\ F_2^n(t) &= \int_0^1 dx \left(\frac{2}{3} E_v^d(x,t) - \frac{1}{3} E_v^u(x,t) \right) \end{aligned}$$

- Grigoryan-Radyushkin integral representation for bulk-to-boundary propagator

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\frac{x}{1-x}\kappa^2 z^2}$$

- LF mapping (Brodsky-Teramond): x is equivalent to LC momentum fraction

Nucleon GPDs

- GPDs $H_v^q(x,Q^2) = q(x)\,x^{\frac{Q^2}{4\kappa^2}}\,, \quad E_v^q(x,Q^2) = e^q(x)\,x^{\frac{Q^2}{4\kappa^2}}$
- Distribution functions $q(x)$ and $e^q(x)$

$$q(x) = \alpha^q\gamma_1(x) + \beta^q\gamma_2(x)\,, \quad e^q(x) = \beta^q\gamma_3(x)$$

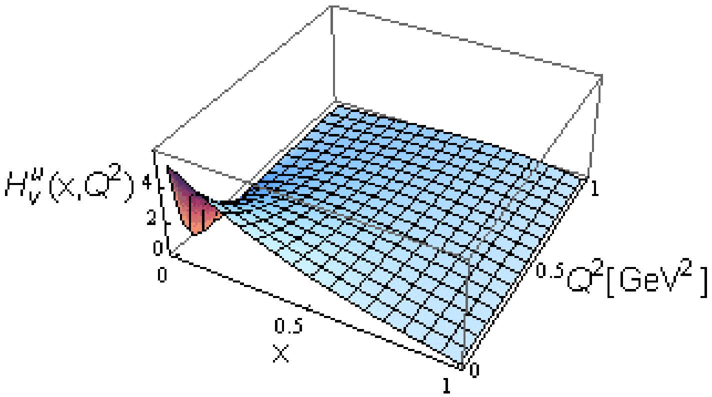
Flavor couplings α^q, β^q and functions $\gamma_i(x)$ are written as

$$\alpha^u = 2\,,\ \alpha^d = 1\,,\ \beta^u = 2\eta_p + \eta_n\,,\ \beta^d = \eta_p + 2\eta_n$$

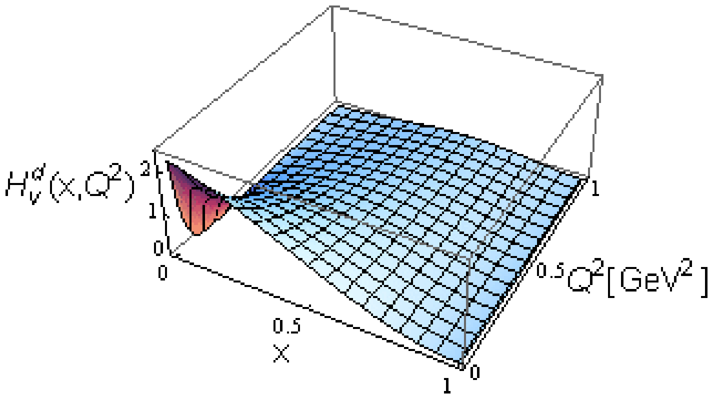
and

$$\begin{aligned}\gamma_1(x) &= \frac{1}{2}(5 - 8x + 3x^2)\\ \gamma_2(x) &= 1 - 10x + 21x^2 - 12x^3\\ \gamma_3(x) &= 24(1-x)^2\end{aligned}$$

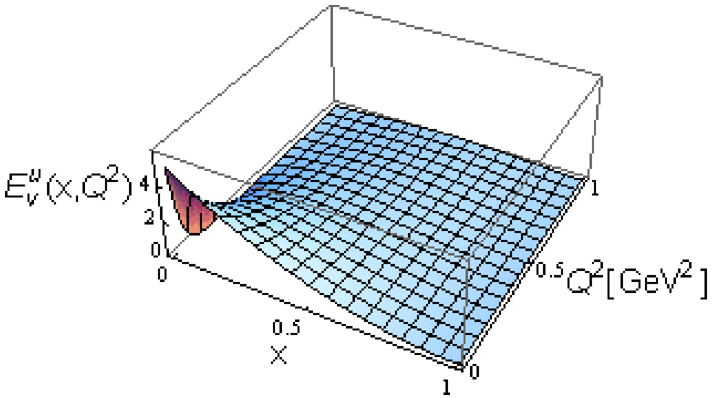
Nucleon GPDs



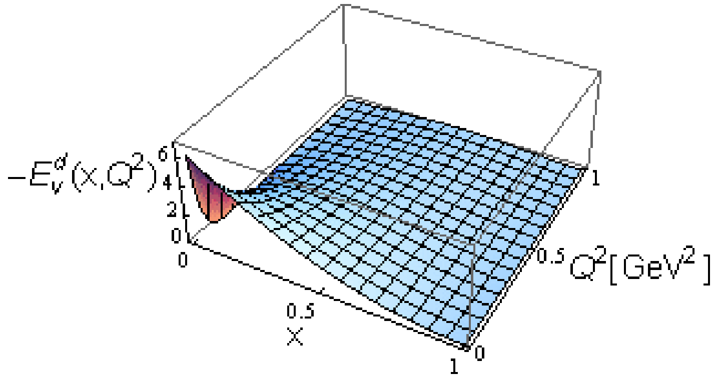
$H_v^u(x, Q^2)$



$H_v^d(x, Q^2)$



$E_v^u(x, Q^2)$



$E_v^d(x, Q^2)$

Nucleon GPDs

- Nucleon GPDs in impact space Burkardt, Miller, Diehl, Kroll et al

$$
\begin{aligned}
q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} H_q(x,\mathbf{k}_\perp^2) e^{-i\mathbf{b}_\perp \mathbf{k}_\perp} = q(x)\frac{\kappa^2}{\pi\log(1/x)} e^{-\frac{\mathbf{b}_\perp^2\kappa^2}{\log(1/x)}} \\
e^q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} E_q(x,\mathbf{k}_\perp^2) e^{-i\mathbf{b}_\perp \mathbf{k}_\perp} = e^q(x)\frac{\kappa^2}{\pi\log(1/x)} e^{-\frac{\mathbf{b}_\perp^2\kappa^2}{\log(1/x)}}
\end{aligned}
$$

- Parton charge and magnetization densities in transverse impact space

$$
\begin{aligned}
\rho_E^N(\mathbf{b}_\perp) &= \sum_q e_q^N \int_0^1 dx q(x,\mathbf{b}_\perp) = \frac{\kappa^2}{\pi}\sum_q e_q^N \int_0^1 \frac{dx}{\log(1/x)} q(x) e^{-\frac{\mathbf{b}_\perp^2\kappa^2}{\log(1/x)}} \\
\rho_M^N(\mathbf{b}_\perp) &= \sum_q e_q^N \int_0^1 dx e^q(x,\mathbf{b}_\perp) = \frac{\kappa^2}{\pi}\sum_q e_q^N \int_0^1 \frac{dx}{\log(1/x)} e^q(x) e^{-\frac{\mathbf{b}_\perp^2\kappa^2}{\log(1/x)}}
\end{aligned}
$$

where $e_u^p = e_d^n = 2/3$ and $e_u^n = e_d^p = -1/3$

Nucleon GPDs

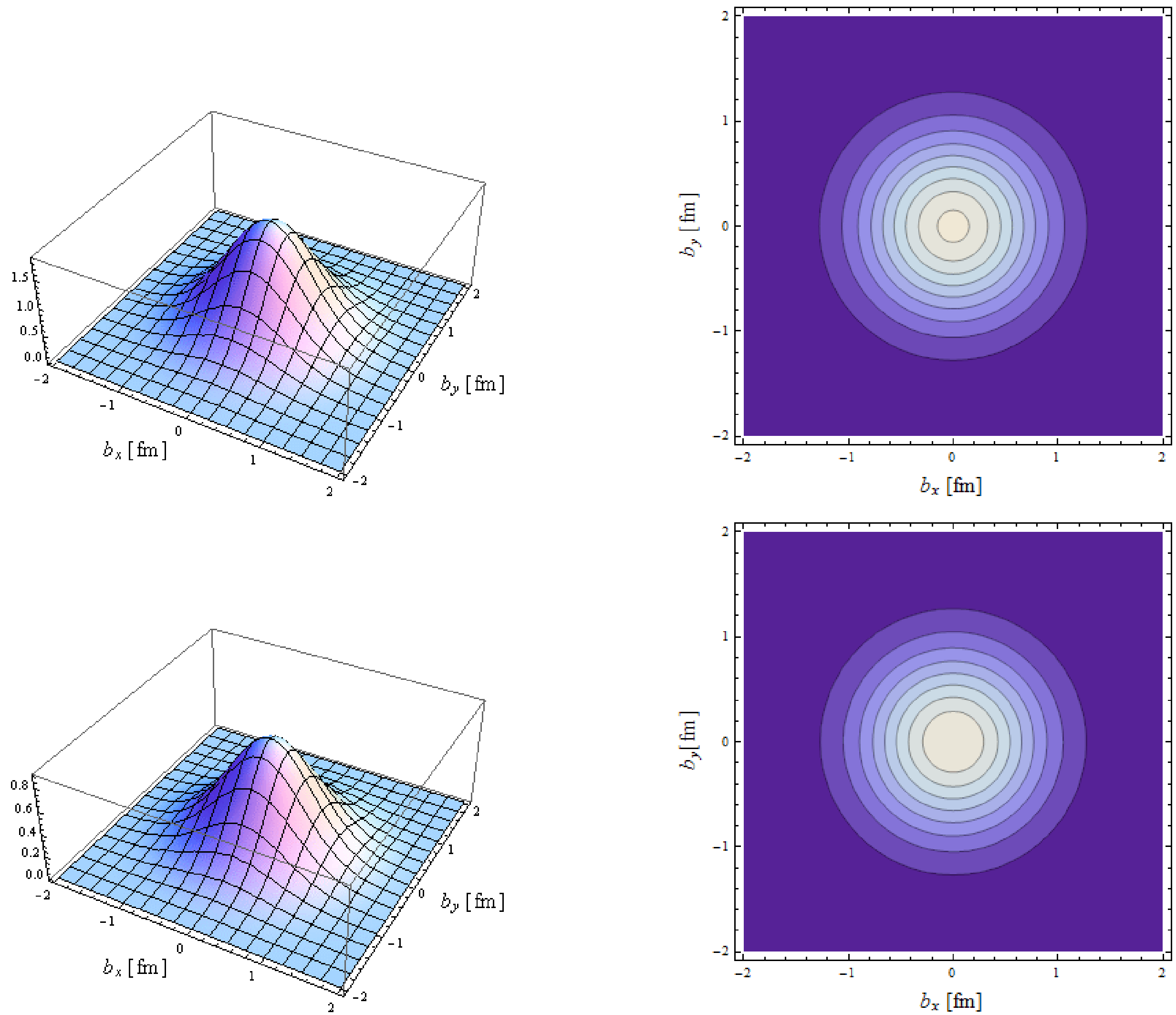
- Transverse width of impact parameter dependent GPD

$$\langle R_\perp^2(x)\rangle_q = \frac{\int d^2\mathbf{b}_\perp \mathbf{b}_\perp^2\, q(x,\mathbf{b}_\perp)}{\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)} = -4\frac{\partial \log H_v^q(x,Q^2)}{\partial Q^2}\Big|_{Q^2=0} = \frac{\log(1/x)}{\kappa^2}$$

- Transverse rms radius

$$\langle R_\perp^2\rangle_q = \frac{\int d^2\mathbf{b}_\perp \mathbf{b}_\perp^2 \int_0^1 dx q(x,\mathbf{b}_\perp)}{\int d^2\mathbf{b}_\perp \int_0^1 dx q(x,\mathbf{b}_\perp)} = \frac{1}{\kappa^2}\left(\frac{5}{3}+\frac{\beta^q}{12\alpha^q}\right) \simeq 0.527\ \mathrm{fm}^2$$

Nucleon GPDs



Plots for $q(x, \mathbf{b}_\perp)$ for $x = 0.1$: $u(x, \mathbf{b}_\perp)$ - upper pannels, $d(x, \mathbf{b}_\perp)$ - lower pannels

Roper resonance $N(1440)$

- Put $n = 1$ and get solutions dual to Roper:

$$M_{\mathcal{R}} \simeq 1440 \text{ MeV}$$

- $N \to R + \gamma$ transition

$$M^\mu = \bar{u}_{\mathcal{R}} \left[\gamma_\perp^\mu F_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{M_{\mathcal{R}}} F_2(q^2) \right] u_N , \quad \gamma_\perp^\mu = \gamma^\mu - q^\mu \frac{\not{q}}{q^2}$$

- Helicity amplitudes

$$H_{\pm\frac{1}{2}0} = \sqrt{\frac{Q_-}{Q^2}} \left(F_1 M_+ - F_2 \frac{Q^2}{M_{\mathcal{R}}} \right)$$

$$H_{\pm\frac{1}{2}\pm 1} = -\sqrt{2Q_-} \left(F_1 + F_2 \frac{M_+}{M_{\mathcal{R}}} \right)$$

- Alternative set [Weber, Capstick, Copley et al]

$$A_{1/2} = -b\, H_{\frac{1}{2}1} , \quad S_{1/2} = b \frac{|\mathbf{p}|}{\sqrt{Q^2}} H_{\frac{1}{2}0} ,$$

$$Q_\pm = M_\pm^2 + Q^2 , \quad M_\pm = M_{\mathcal{R}} \pm M_N , \quad b = \sqrt{\frac{\pi\alpha}{2E M_{\mathcal{R}} M_N}}$$

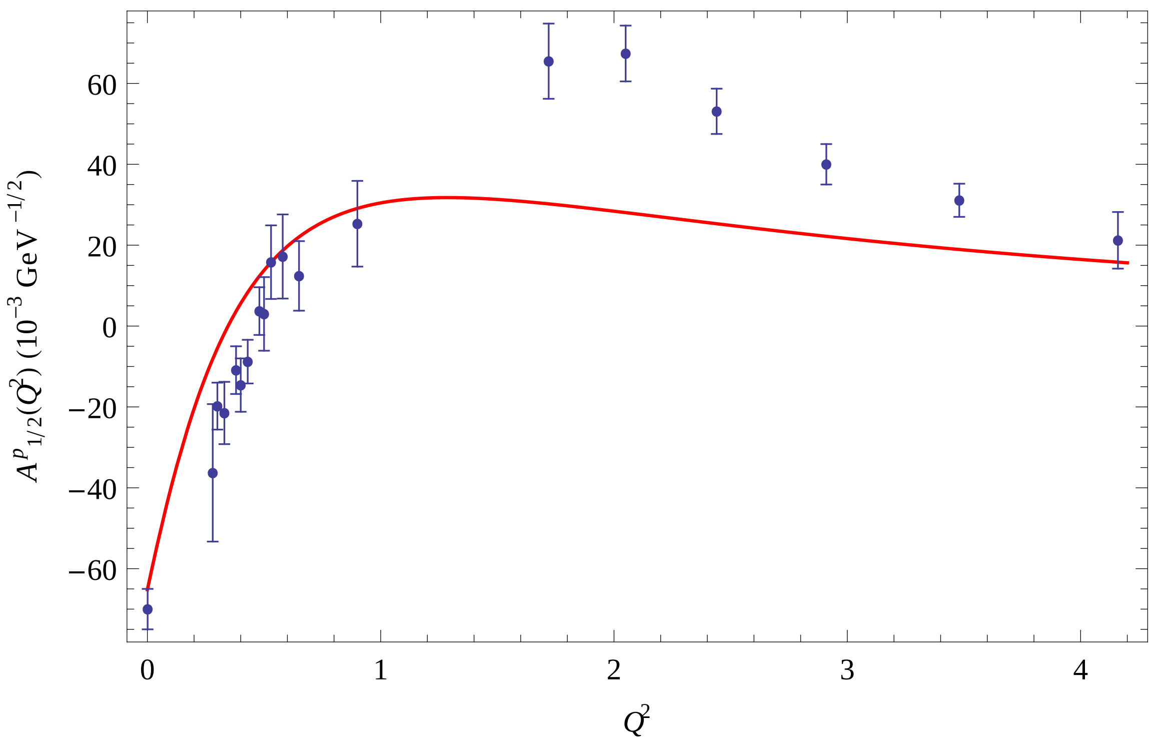
Roper resonance $N(1440)$

Helicity amplitudes $A^N_{1/2}(0)\,,\ S^N_{1/2}(0)$

Quantity	Our results	Data
$A^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.065	-0.065 $\pm$ 0.004
$A^n_{1/2}(0)$ (GeV$^{-1/2}$)	0.040	0.040 $\pm$ 0.010
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	0.040	
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.040	

Roper resonance $N(1440)$

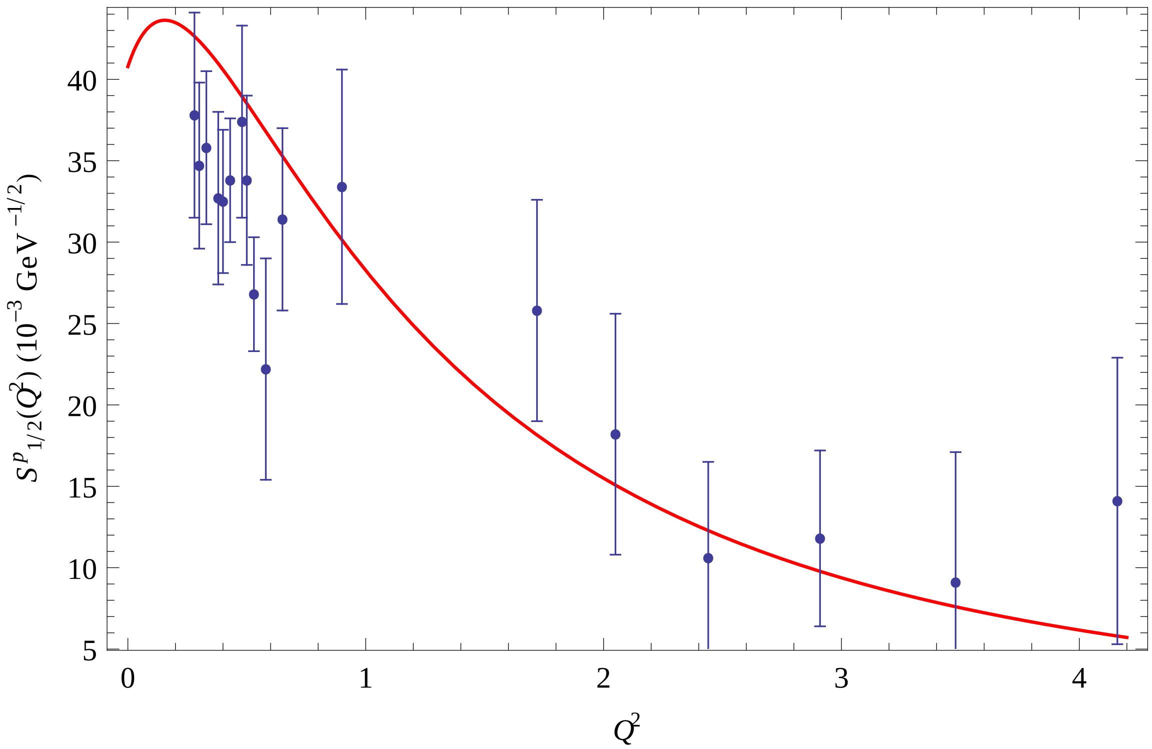
Helicity amplitude $A^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, 1205.3948 [nucl-ex]

Roper resonance $N(1440)$

Helicity amplitude $S^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, 1205.3948 [nucl-ex]

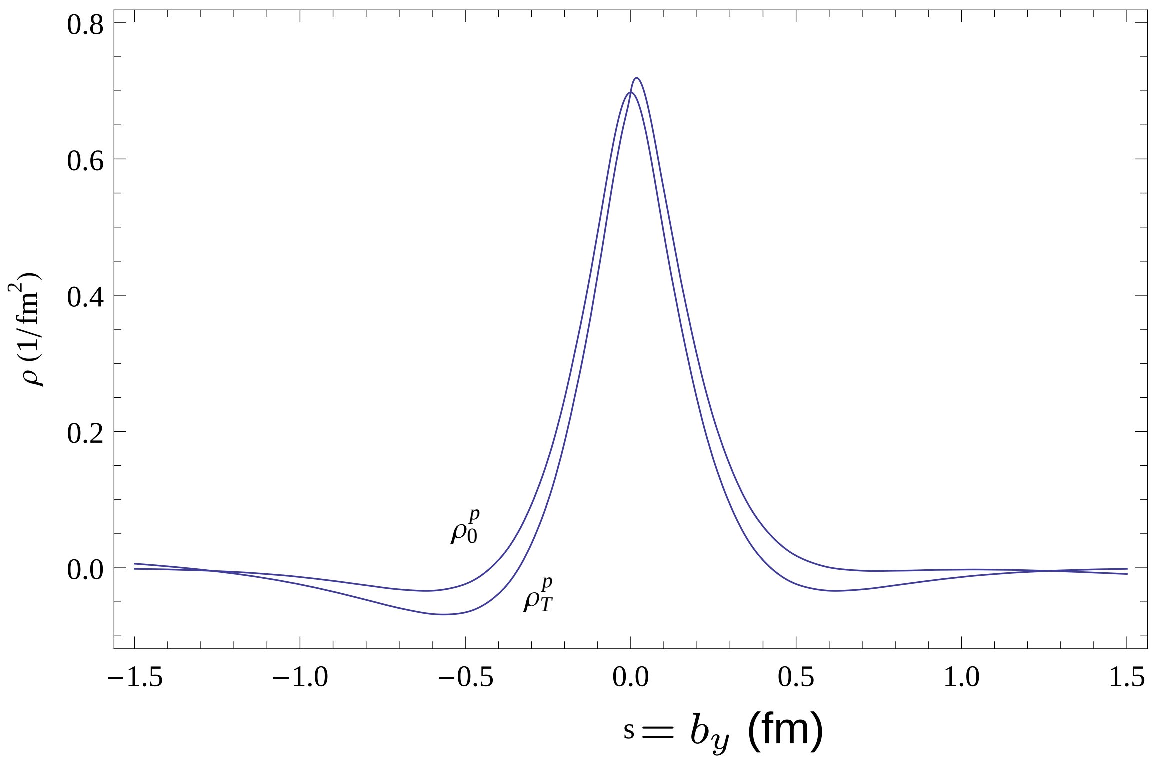
Roper resonance $N(1440)$

Tiator-Vanderhaeghen:

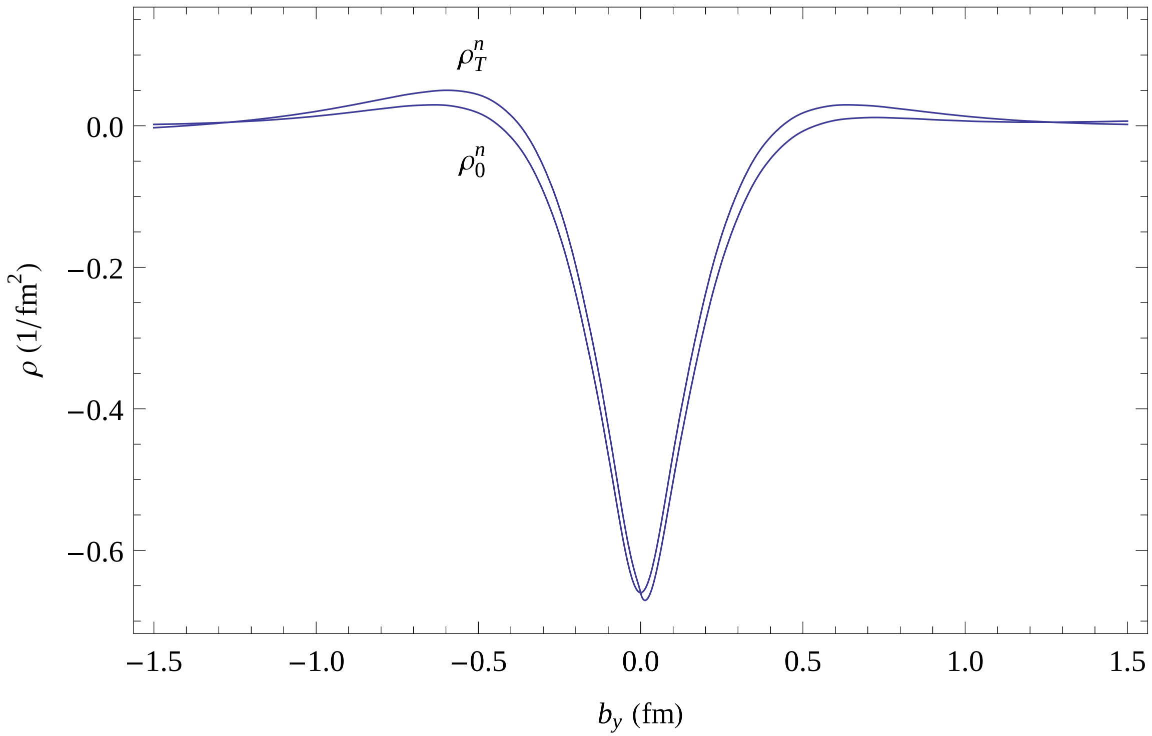
Quark transition charge density in the transverse plane

$$\rho_0(\vec{b}_\perp) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{q}_\perp \vec{b}_\perp}\ \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda \rangle \qquad [\,\text{unpol.}\,]$$

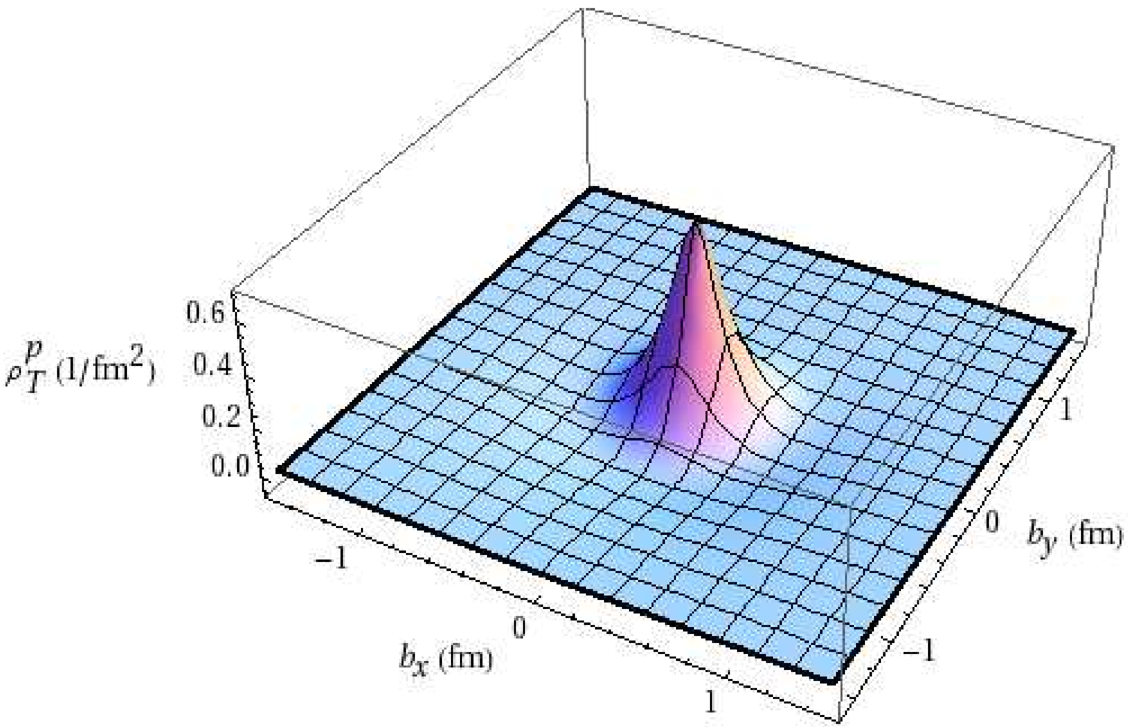
$$\rho_T(\vec{b}_\perp) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{q}_\perp \vec{b}_\perp}\ \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, s_\perp \rangle \qquad [\,\text{T} - \text{pol.}\,]$$



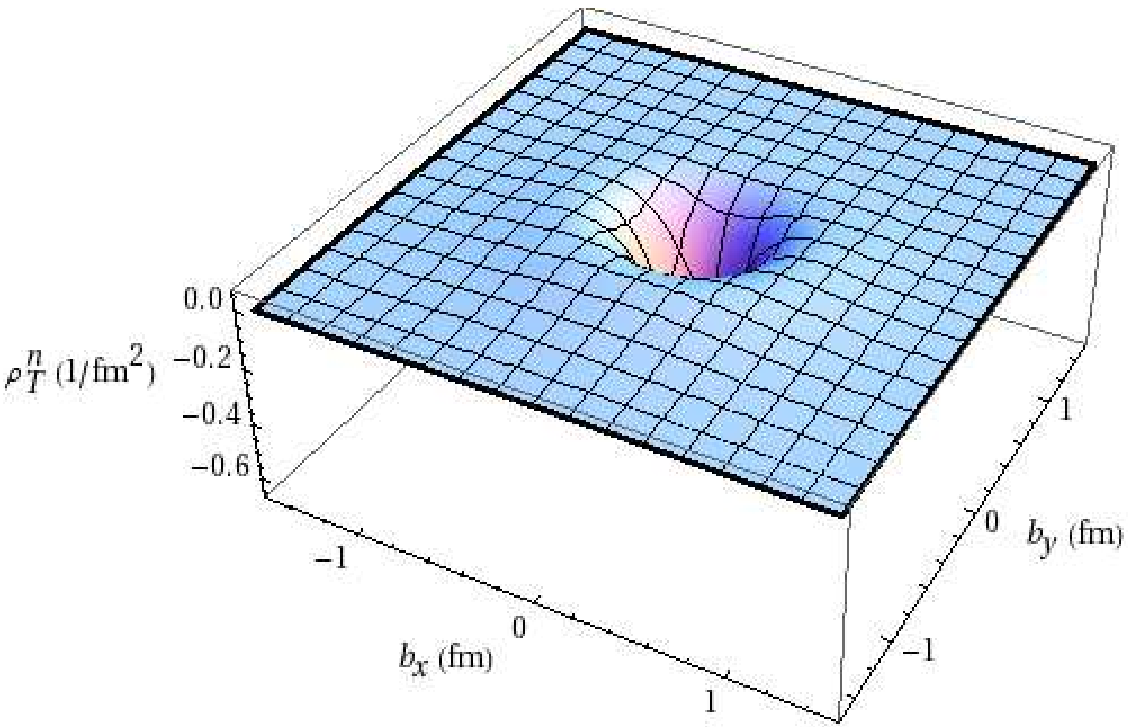
Roper resonance $N(1440)$



Roper resonance $N(1440)$



Roper resonance $N(1440)$



Summary

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- HQCD models reproduce main features of QCD at low and high energies
- We develop a soft–wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances
- Mesons, baryons and exotic states from unified point view and including high Fock states
- Future work: nucleon TMDs, DVCS