$1/N_c$ - ChPT in the one-Baryon sector

Álvaro Calle Cordón

Jefferson Lab

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In collaboration with José Goity

Outline

- $1/N_c$ - chiral perturbation theory ($SU(2)$ case)
- Baryon mass and axial current
- Application: lattice extrapolations
  - Nucleon and Delta masses
  - Nucleon axial charge g_A
- Summary

Large N_c limit + ChPT

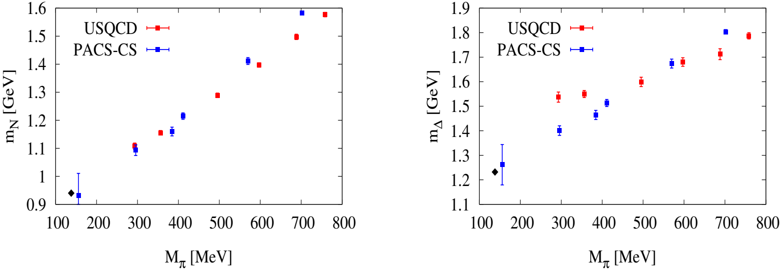
Previous works

- Dashen & Manohar. Baryon - pion couplings from large N_c QCD. PLB315 (1993) 425.
- Dashen & Manohar. $1/N_c$ corrections to the baryon axial currents in QCD. PLB315 (1993) 438.
- Luty & March-Russell. Baryons from quarks in the $1/N$ expansion. NPB426 (1994) 71.
- Luty & March-Russell. Baryon magnetic moments in a simultaneous expansion in 1/N and m(s). PRD51 (1995) 2332.
- Luty. Baryons with many colors and flavors. PRD51 (1995) 2322.
- Bedaque & Luty. Baryon masses at second order in large N chiral perturbation theory. PRD54 (1996) 2317.
- Jenkins. Chiral Lagrangian for baryons in the $1/N_c$ expansion. PRD53 (1996) 2625.
- Oh & Weise. Baryon masses in large N(c) chiral perturbation theory. EPJ A4 (1999) 363
- Flores-Mendieta et al. On the structure of large N(c) cancellations in baryon chiral perturbation theory. PRD62 (2000) 034001.
- Flores-Mendieta et al. Renormalization of the baryon axial vector current in large-N(c) chiral perturbation theory. PRD74 (2006) 094001.
- Flores-Mendieta. Baryon magnetic moments in large-N(c) chiral perturbation theory. PRD80 (2009) 094014.

Nucleon and Δ masses from lattice QCD

USQCD Collaboration, A. Walker-Loud et al., PRD79, 054502 (2009)

PACS-CS Collaboration, S. Aoki et al., PRD79, 034503 (2009)



The role of the Δ

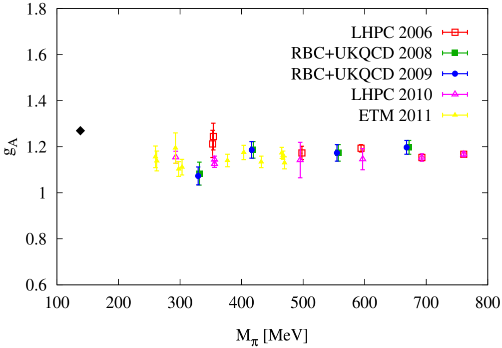
- Quark mass dependence in HBχPT:
  Octet: $m_B = m_0 + \delta m_B^{(1)} + \delta m_B^{(3/2)} + \delta m_B^{(2)} + \cdots$
  Decuplet: $m_T = m_0 + \Delta_0 + \delta m_T^{(1)} + \delta m_T^{(3/2)} + \delta m_T^{(2)} + \cdots$
  m_0: baryon mass in the chiral limit.
  Δ_0: decuplet-octet (delta-nucleon) mass splitting in the chiral limit.
  $\delta m_{B,T}^{(n)}$: corrections to the baryon mass scaling as $m_q^n \sim m_\pi^{2n}$
- large N_c limit: $\Delta = M_\Delta - M_N \to 0\,, N_c \to \infty$

$$
\begin{aligned}
16\pi^2\, i\,\mathcal{I} &= \cdots + 4(\Delta^2 - M_\pi^2)^{3/2} J(\Delta, M_\pi) + \cdots \\
&\underset{M_\pi \gg \Delta}{\approx} \cdots + \frac{16}{3}\Delta^3 - 3\pi M_\pi \Delta^2 - 4M_\pi^2\Delta + 2\pi m^3 + \cdots
\end{aligned}
$$

$$
J(\Delta, M_\pi) = \begin{cases} \frac{\pi}{2} - \tanh^{-1}\left(\frac{\Delta}{\sqrt{\Delta^2 - M_\pi^2}}\right) & m < \Delta\,, \\ \frac{\pi}{2} - \tan^{-1}\left(\frac{\Delta}{\sqrt{M_\pi^2 - \Delta^2}}\right) & m > \Delta\,. \end{cases}
$$

Nucleon axial charge from lattice QCD

LHPC 2006: R. G. Edwards et al., PRL96, 052001 (2006)
LHPC 2010: J. D. Bratt et al., PRD82, 094502 (2010)
RBC+UKQCD 2008: T. Yamazaki et al., PRL100, 171602 (2008)
RBC+UKQCD 2009: T. Yamazaki et al., PRD79, 114505 (2009)
ETM 2011: C. Alexandrou et al., PRD83, 045010 (2011)
CM 2012: S Capitani et al., arXiv:1205.0180v1



The role of the Δ

PHYSICAL REVIEW D **75**, 014503 (2007)

Chiral extrapolation of g_A with explicit $\Delta(1232)$ degrees of freedom

M. Procura, B. U. Musch, T. R. Hemmert, and W. Weise
Physik-Department, Theoretische Physik, Technische Universität München, D-85747 Garching, Germany
(Received 18 October 2006; published 12 January 2007)

An updated and extended analysis of the quark-mass dependence of the nucleon's axial-vector coupling constant g_A is presented in comparison with state-of-the-art lattice QCD results. Special emphasis is placed on the role of the $\Delta(1232)$ isobar. It is pointed out that standard chiral perturbation theory of the pion-nucleon system at order p^4 fails to provide an interpolation between the lattice data and the physical point. In constrast, a version of chiral effective field theory with explicit inclusion of the $\Delta(1232)$ proves to be successful. Detailed error analysis and convergence tests are performed. Integrating out the $\Delta(1232)$ as an explicit degree of freedom introduces uncontrolled errors for pion masses $m_\pi \gtrsim 300$ MeV.

- The inclusion of the $\Delta(1232)$ at leading one-loop level is crucial in order to get a satisfactory description of the quark-mass dependence of g_A.
- Stable convergence pattern for pion masses below 300 MeV.
- Fine-tuning in the counterterms in order to get the rather flat dependence of g_A with m_π observed in the lattice.

$\Rightarrow$ Does this mean cancellations ?

$\Rightarrow$ We will understand why the $\Delta(1232)$ is so important !!!

Chiral Perturbation Theory & $1/N_c$ Expansion

Meson sector

- Unitary matrix of pion fields ($F_\pi = 92.4$)

$$U(x) = \exp\left(i \frac{\pi^a(x)\tau^a}{F_\pi} \right)$$

- The $\mathcal{O}(p^2)$ chiral Lagrangian reads ($\langle A \rangle \equiv \mathrm{Tr}(A)$),

$$\mathcal{L}_\pi^{(2)} = \frac{F_\pi^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi^\dagger U + \chi U^\dagger \rangle$$

with standard definitions:

$$U = u^2 \,, \ \nabla_\mu U \equiv \partial_\mu U - i r_\mu U + i U \ell_\mu$$
$$r_\mu = v_\mu + a_\mu \,, \ \ell_\mu = v_\mu + a_\mu \,, \ \chi = 2B(s + ip)$$

- External fields: vector (v_μ), axial (a_μ), pseudoscalar (p) and scalar (s).

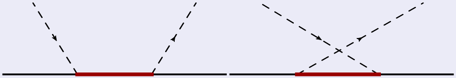
Baryon sector: Spin-Flavor algebra in the large N_c limit

Gervais & Sakita (1984), Dashen & Manohar (1993)

From Witten's counting rules:

Pion-nucleon scattering amplitude $\sim \mathcal{O}(1)$ ... but pion-nucleon vertex $\sim \mathcal{O}(\sqrt{N_c})$

Cancellation between diagrams: consistency conditions



$$\propto \frac{N_c^2 g_A^2}{F_\pi^2}\left[X^{ia}, X^{jb}\right] = O(1)$$

Spin-Flavor $SU(2N_f)$ symmetry

Ground state baryon $=$ spin-flavor symmetric multiplet **B** of $SU(2N_f)$ with S=I

$$\left[S^i, T^a\right] = 0$$

$$\left[S^i, S^j\right] = i\epsilon^{ijk}S^k \, , \, \left[T^a, T^b\right] = i\epsilon^{abc}T^c$$

$$\left[S^i, G^{ja}\right] = i\epsilon^{ijk}G^{ka} \, , \, \left[T^a, G^{ib}\right] = i\epsilon^{abc}G^{ic}$$

$$\left[G^{ia}, G^{jb}\right] = \frac{i}{2N_F}\delta^{ab}\epsilon^{ijk}S^k + \frac{1}{4}\delta^{ij}f^{abc}T^c + \frac{i}{2}\epsilon^{ijk}d^{abc}G^{kc} \, .$$

$$J = I = \frac{1}{2}, \frac{3}{2}, \cdots, \frac{N_c}{2}$$

$$\mathbf{B} = \begin{pmatrix} N \\ \Delta_{3/2} \\ \vdots \\ \Delta_{N_c/2} \end{pmatrix} \rightarrow \mathbf{B} = \begin{pmatrix} N \\ \Delta \end{pmatrix}$$

Chiral Perturbation Theory & $1/N_c$ Expansion

Baryon sector

- $\mathcal{O}(p)$ HB chiral lagrangian

$$\mathcal{L}^{(1)}_{\pi B} = i\,\mathbf{B}^\dagger D_0 \mathbf{B} + g_A\,\mathbf{B}^\dagger u_i^a G^{ia}\mathbf{B} + \mathbf{B}^\dagger \delta m_S \mathbf{B}\,,$$

$$\begin{aligned} D_\mu \mathbf{B} &= \partial_\mu\,\mathbf{B} - i\,\Gamma^a_\mu\; T^a \mathbf{B}\,, \\ \Gamma_\mu &= \frac{1}{2}\left[u\,(\partial_\mu - i r_\mu)\,u^\dagger + u^\dagger\,(\partial_\mu - i\ell_\mu)\,u\right]\,, \\ u_\mu &= i\left[u^\dagger\,(\partial_\mu - i r_\mu)\,u - u\,(\partial_\mu - i\ell_\mu)\,u^\dagger\right]\,. \end{aligned}$$

  and $\Gamma^a_\mu \equiv \frac{1}{2}\,\langle \tau^a\,\Gamma_\mu \rangle$, $u^a_\mu \equiv \frac{1}{2}\,\langle \tau^a\,u_\mu \rangle$, $\delta m_S = \frac{C_{HF}}{N_c}\,\vec{S}^2$,

- All terms have same chiral order, they do have different $1/N_c$ orders.
  The Weinberg-Tomozawa $\sim 1/F_\pi^2$ and overall $\mathcal{O}(p/N_c)$
  The πB coupling $\sim 1/F_\pi$ and overall $\mathcal{O}(p\sqrt{N_c})$
- Key scalings with N_c: $F_\pi = \mathcal{O}(\sqrt{N_c})$, $g_A = \mathcal{O}(N_c^0)$, $m_B = \mathcal{O}(N_c)$, $M_\pi = \mathcal{O}(N_c^0)$.
- Higher order lagrangians (under construction):
  General form $B^\dagger\; O_\chi \otimes \mathcal{G}\;\mathbf{B}$ & EOM
  $O_\chi =$ tensor $\{u_\mu\,, D_\mu\,, \chi_\pm\}$, and $\mathcal{G} =$ tensor $\{1, S^i\,, T^a\,, G^{ia}\}$

Chiral Perturbation Theory & $1/N_c$ Expansion

A new combined power counting: the ξ-expansion

$$\mathcal{M} = i\frac{g_A^2}{F_\pi^2} k_1^i k_2^j \sum_n G^{jb}\mathcal{P}_n G^{ia} \frac{1}{p^0 + k_1^0 - \delta m_n} + \left(a \to b\,, i \to j\,, p^0 \to p'^0\,, k_1^0 \to -k_1^0\right)$$

The HB propagator can only be expanded when p^0, p'^0, $\delta m_n \ll k_1^0$, k_2^0.

1. *Strict* large-N_c limit (we can expand the HB propagator)

$$\frac{1}{N_c} \sim \delta m_n \ll k^0 \sim p\,, \quad \text{i.e.}\,, \quad \frac{1}{N_c} \sim \mathcal{O}(p^2)$$

   It has been known as large-N_c-ChPT

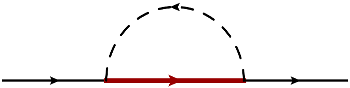
   Luty'95, Jenkins'96, Oh&Weise'99, Flores − Mendieta'00 . . .

2. *Soft* limit (we cannot expand the HB propagator)

$$\frac{1}{N_c} \sim \delta m_n \sim k^0 \sim p\,, \quad \text{i.e.}\,, \quad \frac{1}{N_c} \sim \mathcal{O}(p)$$

   Our expansion parameters is $\xi \sim 1/N_c \sim m_\pi$. Real life $M_\pi \lesssim m_\Delta - m_N$

Baryon self energy



$$
\begin{aligned}
\Delta_n &= \delta m_n - p^0 \,, \\
p^0 &= \delta m_{in} + \mathfrak{p}^0 \,, \\
\delta m &= \frac{C_{HF}}{N_c} \vec{S}^2 \,.
\end{aligned}
$$

$$
\begin{aligned}
\delta\Sigma_{(1)} &= i \frac{g_A^2}{F_\pi^2} \frac{1}{d-1} \sum_n G^{ia} \mathcal{P}_n G^{ia} \, I_{(1)}(n, p^0, M_\pi) \,, \\
I_{(1)}(n, p^0, M_\pi) &= \int \frac{d^d k}{(2\pi)^d} \frac{\vec{k}^2}{k^2 - M_\pi^2 + i\epsilon} \frac{1}{k^0 - \Delta_n + i\epsilon} \,, \\
&= -\frac{i}{16\pi^2} \Big\{ \Delta_n (2\Delta_n^2 - 3M_\pi^2) \left(\frac{1}{\epsilon} - \gamma + \log(4\pi) - \log\left(\frac{M_\pi^2}{\mu^2}\right)\right) \\
&\quad - 4(\Delta_n^2 - M_\pi^2)^{3/2} \tanh^{-1}\left(\frac{\Delta_n}{\sqrt{\Delta_n^2 - M_\pi^2}} \right) \\
&\quad - 2\pi (M_\pi^2 - \Delta_n^2)^{3/2} - \Delta_n (5M_\pi^2 - 4\Delta_n^2) \Big\} \,,
\end{aligned}
$$

$$
\delta m_{(1)} = \delta\Sigma_{(1)}\Big|_{\mathfrak{p}^0 \to 0} \,, \qquad \delta Z_{(1)} = \frac{\partial \delta\Sigma_{(1)}}{\partial \mathfrak{p}^0}\Bigg|_{\mathfrak{p}^0 \to 0}
$$

Baryon mass and WF renormalization

WF renormalization CT structure

- Finite + UV pieces:

$$\delta Z_{(1)} = \delta Z_{(1)}^{Finite} + \delta Z_{(1)}^{UV}$$

- Counter-terms structure

	1	$\vec{S}^2$	$\vec{S}^4$
M_π^0	$-\frac{3g_A^2 C_{HF}^2}{64\pi^2 F_\pi^2}\frac{(N_c+4)}{N_c^2}$ $\sim \mathcal{O}(p^0/N_c) \sim \mathcal{O}(\xi)$	$-\frac{g_A^2 C_{HF}^2}{32\pi^2 F_\pi^2}\frac{(N_c+2)(N_c+6)}{N_c^2}$ $\sim \mathcal{O}(p^0/N_c) \sim \mathcal{O}(\xi)$	$\frac{g_A^2 C_{HF}^2}{8\pi^2 F_\pi^2}\frac{1}{N_c^2}$ $\sim \mathcal{O}(p^0/N_c^3) \sim \mathcal{O}(\xi^3)$
M_π^2	$\frac{3g_A^2}{512\pi^2 F_\pi^2} N_c(N_c+4)$ $\sim \mathcal{O}(p^2 N_c) \sim \mathcal{O}(\xi)$	$-\frac{g_A^2}{64\pi^2 F_\pi^2}\frac{1}{N_c}$ $\sim \mathcal{O}(p^2/N_c) \sim \mathcal{O}(\xi^3)$	

$$\delta Z_{(1)}^{CT} = \frac{\omega_1}{N_c} + \frac{\omega_2}{N_c}\vec{S}^2 + \frac{\omega_3}{N_c^3}\vec{S}^4 + z_1 N_c M_\pi^2 + \frac{z_2}{N_c}\vec{S}^2 M_\pi^2$$

Baryon mass and WF renormalization

Baryon mass CT structure

- Finite + UV pieces:

$$\delta m_{(1)} = \delta m_{(1)}^{Finite} + \delta m_{(1)}^{UV}$$

- Counter-terms structure

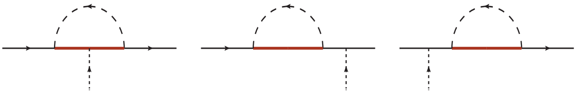
	$\mathbf{1}$	$\vec{S}^2$	$\vec{S}^4$
M_π^0	$\frac{g_A^2 C_{HF}^3}{16\pi^2 F_\pi^2}\frac{(N_c+4)}{N_c^2}$ $\sim \mathcal{O}(p^0/N_c^2) \sim \mathcal{O}(\xi^2)$	$\frac{g_A^2 C_{HF}^3}{48\pi^2 F_\pi^2}\frac{5N_c(N_c+4)-24}{N_c^3}$ $\sim \mathcal{O}(p^0/N_c^2) \sim \mathcal{O}(\xi^2)$	$\frac{-7g_A^2 C_{HF}^3}{12\pi^2 F_\pi^2}\frac{1}{N_c^3}$ $\sim \mathcal{O}(p^0/N_c^4) \sim \mathcal{O}(\xi^4)$
M_π^2	$\frac{-3g_A^2 C_{HF}}{128\pi^2 F_\pi^2}(N_c+4)$ $\sim \mathcal{O}(p^2/N_c^0) \sim \mathcal{O}(\xi^2)$	$\frac{5g_A^2 C_{HF}}{32\pi^2 F_\pi^2}\frac{1}{N_c}$ $\sim \mathcal{O}(p^2/N_c^2) \sim \mathcal{O}(\xi^4)$	

$$\delta m_{(1)}^{CT} = \frac{\bar{m}_0}{N_c} + \frac{\mu_0}{N_c^2}\vec{S}^2 + \mu_1 M_\pi^2 + \frac{\mu_2}{N_c}\vec{S}^2 M_\pi^2 + \frac{\mu_3}{N_c^3}\vec{S}^4$$

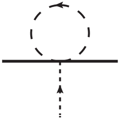
Baryon mass

$$\delta m_B(S) = N_c m_0 + \frac{C_{HF}}{N_c}S(S+1) + c_1 N_c M_\pi^2 + \delta m_{(1)}(S)/(1-\delta Z_{(1)}(S))$$

Vertex correction



$\sim \mathcal{O}(p^3 N_c^{3/2})$



$\sim \mathcal{O}(p^3 N_c^{1/2})$

$$
\begin{aligned}
\delta\Gamma_{(1)} &= -i\left\{(1) + \frac{1}{2}((2)+(3))_{no-pole} + (4)\right\} \\
(1) &= i\left(\frac{g_A}{F_\pi}\right)^3 \frac{q^i}{d-1}\sum_{n,n'} G^{jb}\mathcal{P}_{n'}G^{ia}\mathcal{P}_n G^{jb}\,\frac{\left(I_{(1)}(n,p^0,M_\pi) - I_{(1)}(n',p'^0,M_\pi)\right)}{p^0 - p'^0 - \delta m_n + \delta m_{n'}}\,, \\
(2)+(3) &= -\frac{g_A}{F_\pi}q^i\left\{G^{ia}\delta Z_{(1)} + \delta Z_{(1)}G^{ia} + \cdots\right\} + pole - terms\,, \\
(4) &= i\frac{g_A}{3F_\pi^3}\,q_i\,\Delta(M_\pi)G^{ia}
\end{aligned}
$$

$$
(1) + \frac{1}{2}((2)+(3))_{no-pole}\Big|_{N_c\to\infty} = \frac{i}{2}\left(\frac{g_A}{F_\pi}\right)^3 \frac{q^i}{d-1}\underbrace{[[G^{jb},G^{ia}],G^{jb}]}_{1/N_c^2}\frac{\partial}{\partial p^0}I_{(1)}(p^0,M_\pi) \sim \mathcal{O}(p^3 N_c^{1/2})
$$

Axial current (CT structure)

- Finite + UV pieces:

$$\delta A_{(1)} = \delta A^{Finite}_{(1)} + \delta A^{UV}_{(1)}$$

- Counter-terms structure

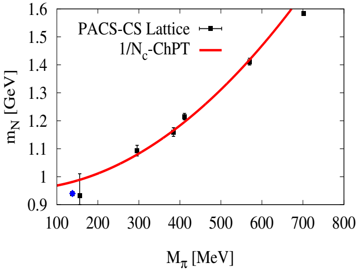
	M_π^0	M_π^2
G^{ia}	$\frac{g_A^3 C_{HF}^2}{32\pi^2 F_\pi^2} \frac{2(2-N_c(N_c+4))}{3N_c^2}$ $\sim \mathcal{O}(p^0/N_c) \sim \mathcal{O}(\xi)$	$\frac{g_A(2+g_A^2)}{32\pi^2 F_\pi^2}$ $\sim \mathcal{O}(p^2/N_c) \sim \mathcal{O}(\xi^3)$
$\left\{G^{ia}, \hat{S}^2\right\}$	$\frac{g_A^3 C_{HF}^2}{8\pi^2 F_\pi^2 N_c^2} \frac{1}{3} \left\{G^{ia}, \hat{S}^2\right\}$ $\sim \mathcal{O}(p^0/N_c^2) \sim \mathcal{O}(\xi^2)$	
$\left[\hat{S}^2, \left[\hat{S}^2, G^{ia}\right]\right]$	$-\frac{g_A^3 C_{HF}^2}{32\pi^2 F_\pi^2 N_c^2} \frac{7}{3} \left[\hat{S}^2, \left[\hat{S}^2, G^{ia}\right]\right]$ $\sim \mathcal{O}(p^0/N_c^2) \sim \mathcal{O}(\xi^2)$	

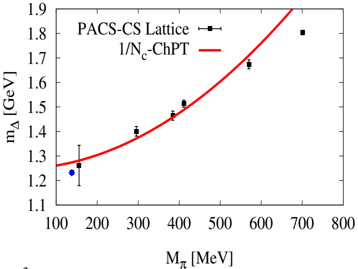
$$\delta g_A^{CT} = \frac{C_0^A}{N_c} + C_1^A M_\pi^2 + \frac{C_2^A}{N_c}(\vec{S}^2 + \vec{S}'^2) + \frac{C_3^A}{N_c^3}(\vec{S}^2 - \vec{S}'^2)^2$$

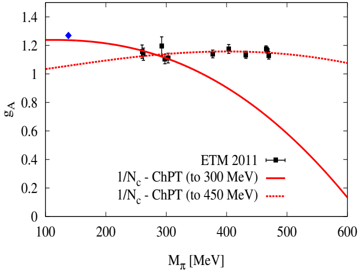
Axial charge

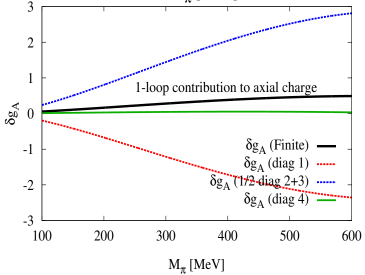
$$g_A(S) = g_A^0 + \delta g_A^{Finite}(S) + \delta g_A^{CT}(S)$$

Chiral extrapolations (results to $\mathcal{O}(\xi^4)$)







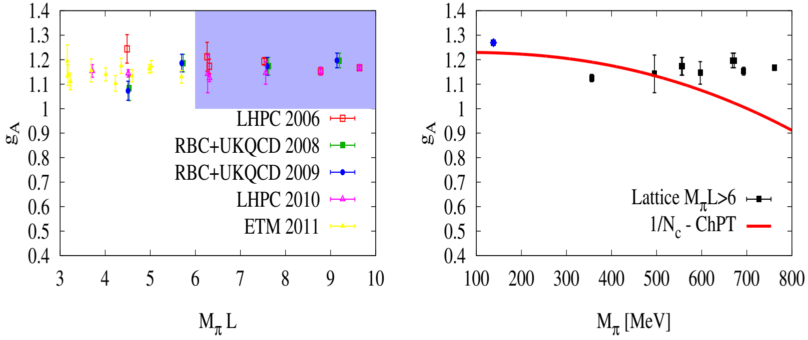


Summary

- Chiral Symmetry and Large N_c are fundamental features of QCD
- Although chiral and large N_c limits do not commute, one can developed a combined power counting implementing them simultaneously.
- In the large N_c limit, the N and Δ are degenerated. A consistent way of formulating $\Delta - \chi$EFT is by imposing large N_c constraints in the chiral lagrangians.
- We have analyzed baryon masses in $1/N_c$-ChPT for $N_f = 2$, and applied our results to perform chiral extrapolations of lattice data.
- We fit the Nucleon, Delta and the axial charge simultaneously.
- A strong cancellation between diagrams occurs in the case of the axial charge giving rise to a mild dependence with the quark mass.
- A systematic analysis of finite size and volume effects is worth to consider.

MORE STUFF

Finite volume effect in gA



Needs checking and some more work !!!

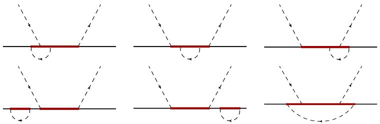
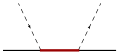
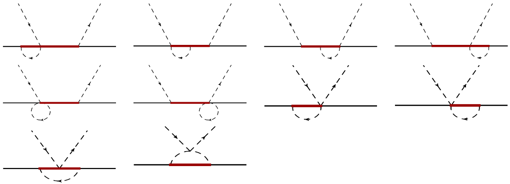
Chiral extrapolations (fits)

	g_A^0	m_0 [MeV]	C_{HF} [MeV]	c_1 [GeV^{-1}]	μ_2 [MeV^{-1}]	z_1 [MeV^{-2}]	C_1^A [GeV^{-2}]	M_N [MeV]	M_Δ [MeV]	g_A	χ^2/DOF
LHPC10	0.94 (2)	293 (4)	537 (8)	0.47 (15)	50.7 (5)	0.068978 (2)	-0.8 (2)	~ 981	~ 1274	~ 1.0	3
UKQCD09	0.85 (5)	293 (4)	689 (5)	0.47 (17)	24.9 (3)	0.20966 (1)	-0.5 (3)	~ 981	~ 1273	~ 1.0	3.5
ETM11	0.96 (3)	293 (4)	529 (8)	0.47 (17)	48.3 (4)	0.68024 (16)	-1.0 (3)	~ 981	~ 1274	~ 1.0	3

Chiral Perturbation Theory & $1/N_c$ Expansion

πN scattering `[ACC & Goity (in progress)]`

πN Feynman diagrams up to one-loop. Crossed diagrams are not shown.

	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$
$\mathcal{O}(N_c^2)$			
$\mathcal{O}(N_c)$			
$\mathcal{O}(N_c^0)$			
$\mathcal{O}(1/N_c)$