Model independent form factor relations at large N_c

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based on T.D. Cohen, V. Krejčiřík, Phys. Rev. C **85** 035205 (2012)

introduction

- Quantum chromodynamics — theory of strong interaction
  - gauge theory of quarks and gluons based on $SU(N_c = 3)$ symmetry

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  - expansion around non-interacting theory
  - corrections in the powers of coupling constant

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- Practical problem — QCD is strongly coupled at low energies
  - conventional perturbative expansion is not applicable
  - expansion around non-interacting theory
  - corrections in the powers of coupling constant
- Some useful approaches
  - expansion around large-N_c limit
  - expansion around massless-quark (chiral) limit

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  - number of colors N_c is a hidden free parameter of QCD
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- Promising idea — develop models of QCD in these two limits
  - even though these limits do not completely describe the real world, they are believed to capture many of its (at least qualitative) details.
  - systematic procedure how to include corrections in the powers of m_π or $1/N_c$

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- Double limit is not uniform and ordering of limits does matter (for certain observables).

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  - large N_c — encoded in the very core of the models, in the semiclassical treatment
  - chiral — imposed later as a constraint on the dynamic of meson fields

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  - large N_c — encoded in the very core of the models ($N_c \to \infty$ taken first)
  - looks totally different (if nothing else they are formulated in five dimensions)

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  - the need for new model-independent relation

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- New model-independent relation
  - use position-space electric and magnetic form factors (Fourier transforms of standard momentum-space ones[4])
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- Not all models on the market satisfy it
  - something is wrong with Sakai-Sugimoto (”top-down”) model
  - the underlying reason for this appears to be due to a failure of the flat-space instanton approximation[6]

(5) A. Cherman, T.D. Cohen, M. Nielsen, Phys. Rev. Lett. **103** (2009) 022001.

(6) A. Cherman, T.Ishii, arXiv:1109.4665v2[hep-th] (2011).

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- The relation is proved in the large N_c chiral perturbation theory

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- Features of large N_c χPT
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  - the form of pion-baryon-baryon$'$ vertex is determined by the large N_c consistency relations

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- Feynman rules for vertices
  - photon-two pions : $\epsilon_{a3b} A_\mu \, (p_a^\mu + p_b^\mu)$
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    - matrices $\tau^{(BB')}$ ($\sigma^{(BB')}$) act in isospin (spin) space, they are a generalization of Pauli matrices, which appear in the pion-nucleon-nucleon vertex

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- Coupling matrices

$$\tau_1^{(NN)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2^{(NN)} = i \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \tau_3^{(NN)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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    - non-relativistic propagators for baryons

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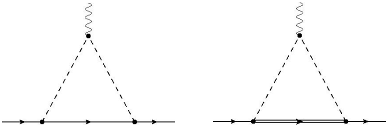

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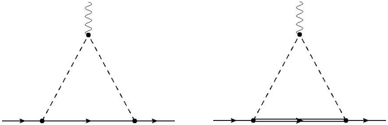

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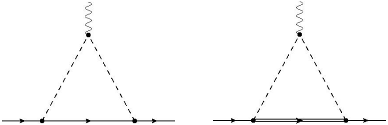

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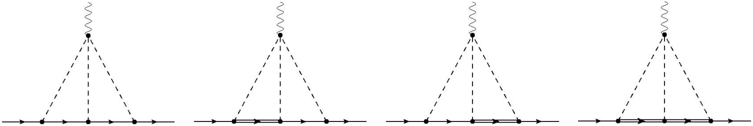

  - three pion loop with nucleons and deltas in the intermediate states
  - totally four diagrams to be taken into account

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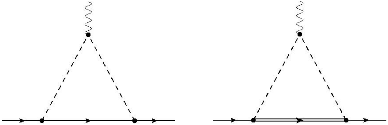
$$\lim_{r\to\infty} \widetilde{G}_M^{I=0} = \frac{3}{2^9\pi^5}\,\frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3\frac{\Delta}{r^7} \qquad \lim_{r\to\infty} \widetilde{G}_M^{I=1} = \frac{1}{2^5\pi^2}\left(\frac{g_A}{f_\pi}\right)^2\frac{1}{r^4}$$

- Model-independent relation $\lim_{r\to\infty} \frac{r^2\,\widetilde{G}_E^{I=0}\,\widetilde{G}_E^{I=1}}{\widetilde{G}_M^{I=0}\,\widetilde{G}_M^{I=1}} = 18$ holds!
  - as advertised, all low-energy constants canceled

comment about the presence of Δs

- For isovector form factors



- matrix structure for iso-space (same for the spin space matrices σ):

$$\tau_a^{(NN)}\tau_b^{(NN)} = \delta_{ab}\,\mathbf{I}_\tau + i\,\epsilon_{abc}\,\tau_c \quad , \qquad \tau_a^{(\Delta N)}\tau_b^{(N\Delta)} = -\sqrt{2}\,\delta_{ab}\,\mathbf{I}_\tau + \frac{i}{\sqrt{2}}\epsilon_{abc}\,\tau_c$$

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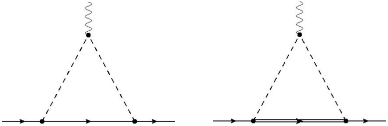

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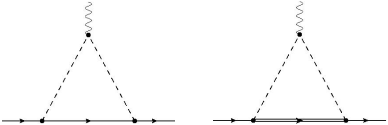

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- Isoscalar-vector ($\widetilde{G}_M^{I=0}$) and isovector-scalar ($\widetilde{G}_E^{I=1}$) channels, the amplitudes subtract exactly in the leading order in $1/N_c$ (where $\Delta = 0$)
- For isoscalar-scalar ($\widetilde{G}_E^{I=0}$) and isovector-vector ($\widetilde{G}_M^{I=1}$) channels, Δ in the intermediate state only leads to a multiplicative factor

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limit $m_\pi \to 0$

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- $\lim\limits_{N_c \to \infty} \lim\limits_{r \to \infty} \lim\limits_{m_\pi \to 0} \frac{r^2 \, \widetilde{G}_E^{I=0} \, \widetilde{G}_E^{I=1}}{\widetilde{G}_M^{I=0} \, \widetilde{G}_M^{I=1}} = 9$

conclusion

- The relation $\lim_{r\to\infty} \frac{r^2\, \widetilde{G}_E^{I=0}\, \widetilde{G}_E^{I=1}}{\widetilde{G}_M^{I=0}\, \widetilde{G}_M^{I=1}} = 18$ was proven in large N_c χPT
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- It may serve as an honest model-independent constrain on baryon models based on large N_c and chiral physics
  - it was shown to hold for:
    - Skyrme model(5)
    - "bottom-up" holographic model(5)
    - "top-down" holographic model (if treated properly)(6)

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