

*Chiral Dynamics, August 6 – 10, Jefferson Lab*

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Jefferson Lab

# Transversity parton distribution

# Nucleon landscape

Nucleon is a many body dynamical system of quarks and gluons

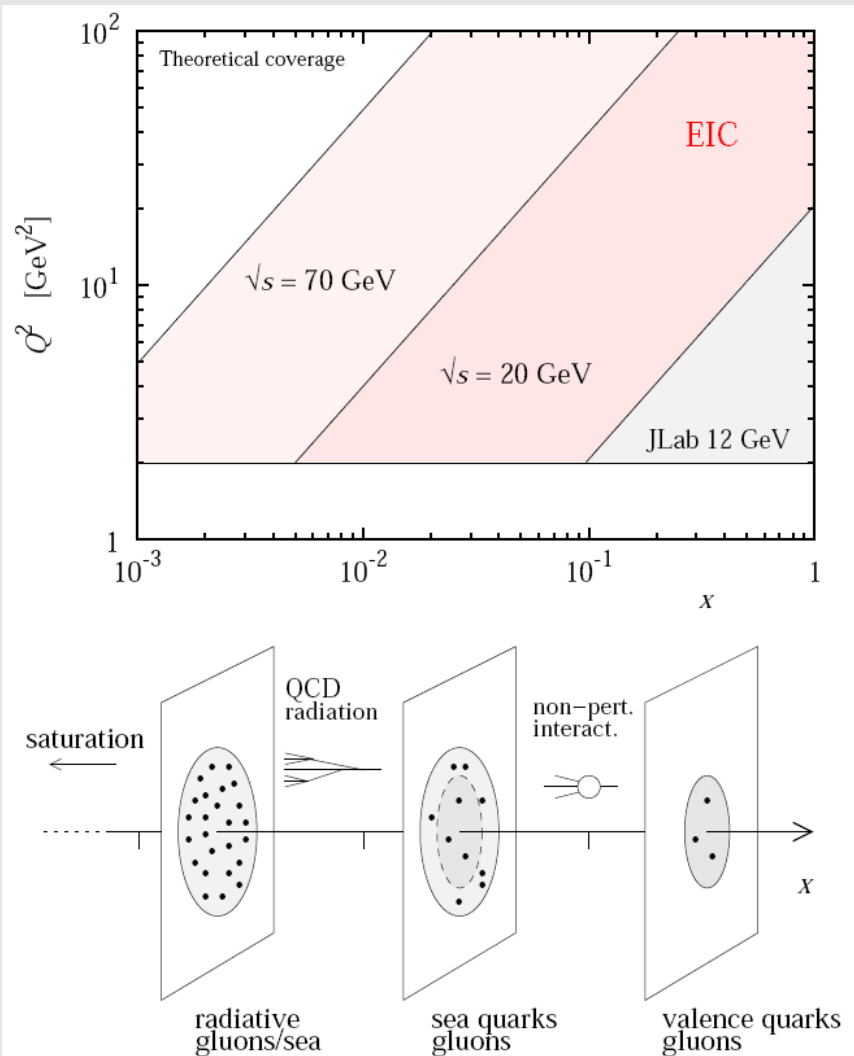
Changing  $x$  we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions and Transverse Momentum Dependent distributions

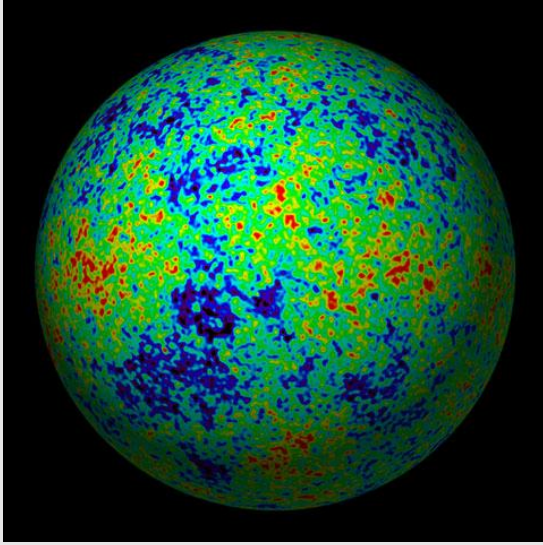
[See talk by Christian Weiss](#)

These distributions are also referred to as 3D (three-dimensional) distributions



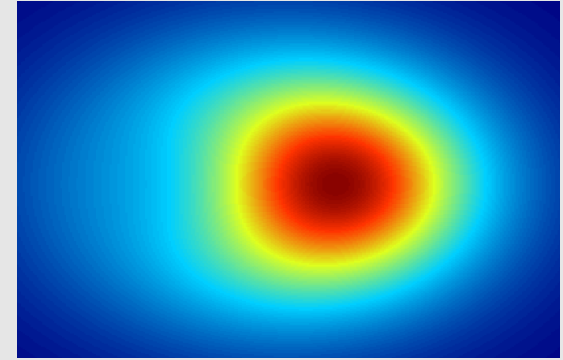
Plot courtesy of Christian Weiss

# Fundamental knowledge from 3D distributions



## **Cosmic Microwave Background**

is the source of information on history of our universe, inflation, distribution of matter, dark matter etc



## **3 Dimensional partonic picture**

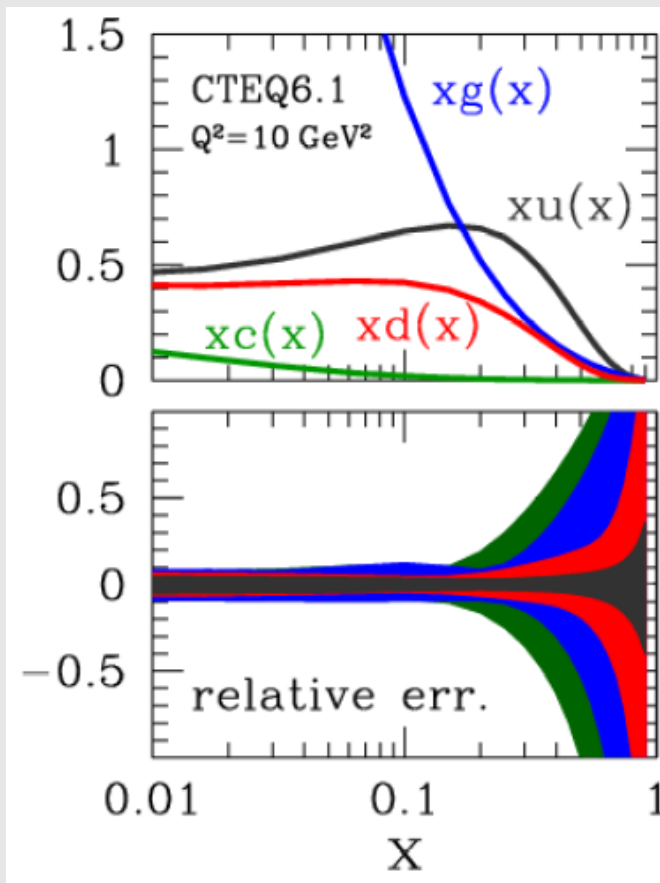
gives us insights on the dynamics of the confined system of quarks and gluons.

It also gives information on fundamental properties of the nucleon

Spin is one of these properties

# Hadron tomography

Conventional inclusive processes are sensitive to longitudinal momentum fraction of hadron momenta, they give no information on spatial or momentum 3D distribution of partons



Good knowledge of  
Parton  
Distribution  
Functions (PDFs)  
is acquired at HERA

However large-x behavior  
has still large uncertainties  
Data from JLab 12 will be  
important

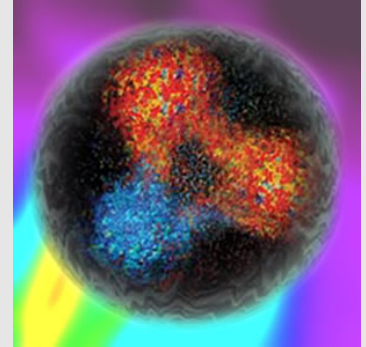
# Wigner distribution

Our goal is to understand 3 dimensional distributions of partons,  
How they move, where they are located inside a nucleon

Wigner distribution (1933) is a possibility

$$W(\mathbf{p}, \mathbf{r}) = \int d^3\eta e^{i\mathbf{p}\eta} \psi^*(\mathbf{r} + \eta/2) \psi(\mathbf{r} - \eta/2)$$

It gives both position and momenta

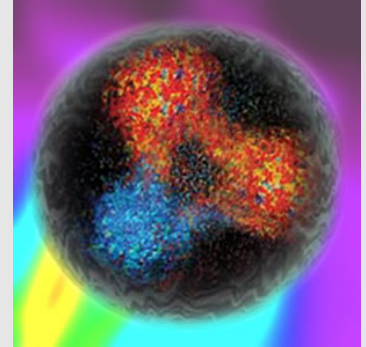


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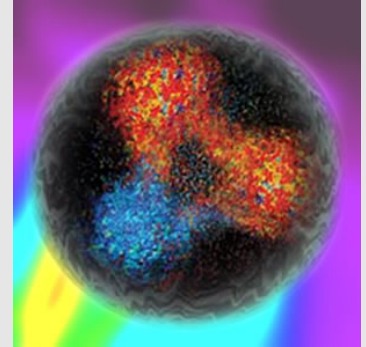
**Can it be measured?**

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It gives both position and momenta

**Can it be measured?**

**PROBABLY NOT!**

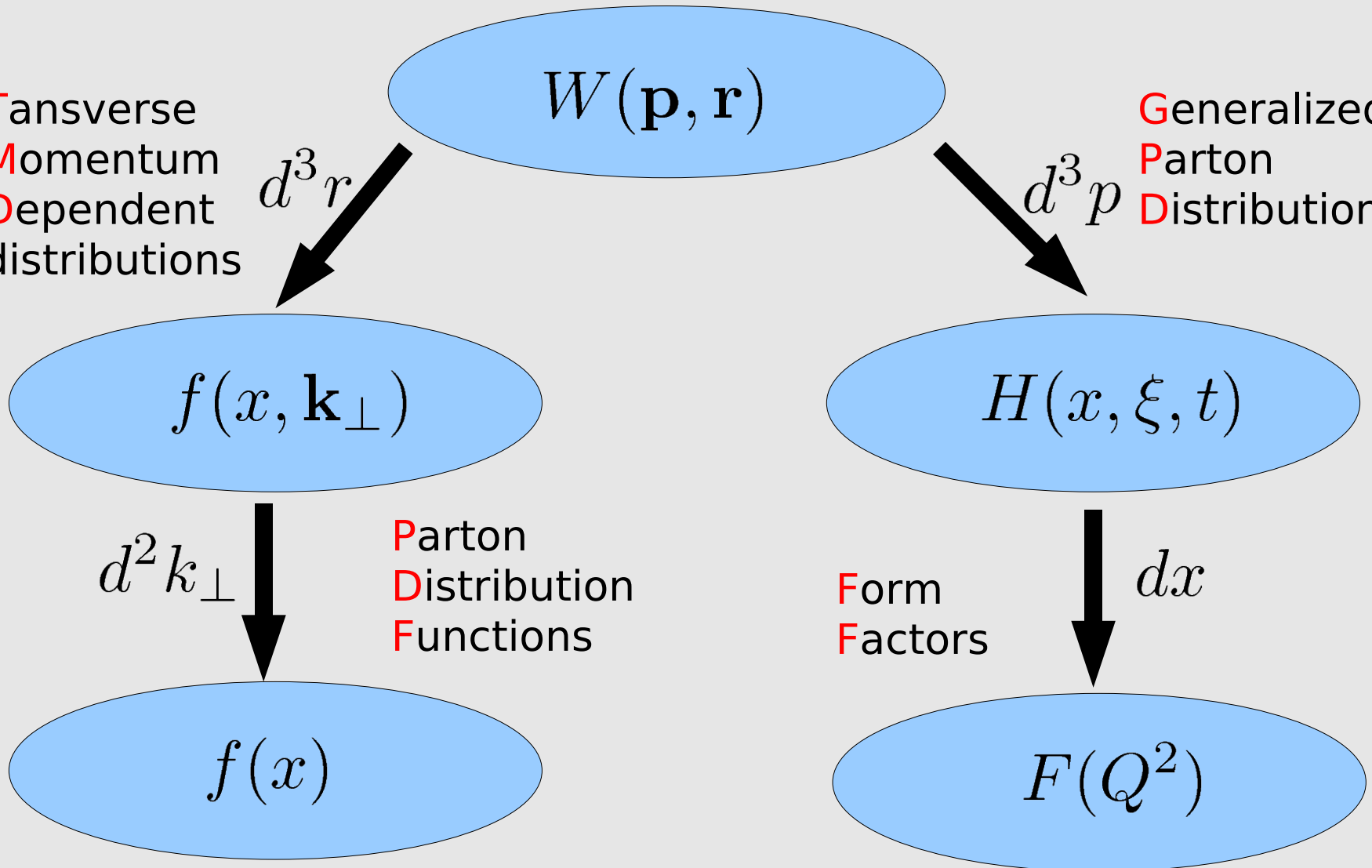
$$\Delta p \Delta r \geq \hbar/2$$

**No simultaneous knowledge on position  
and momenta**

# Wigner distribution

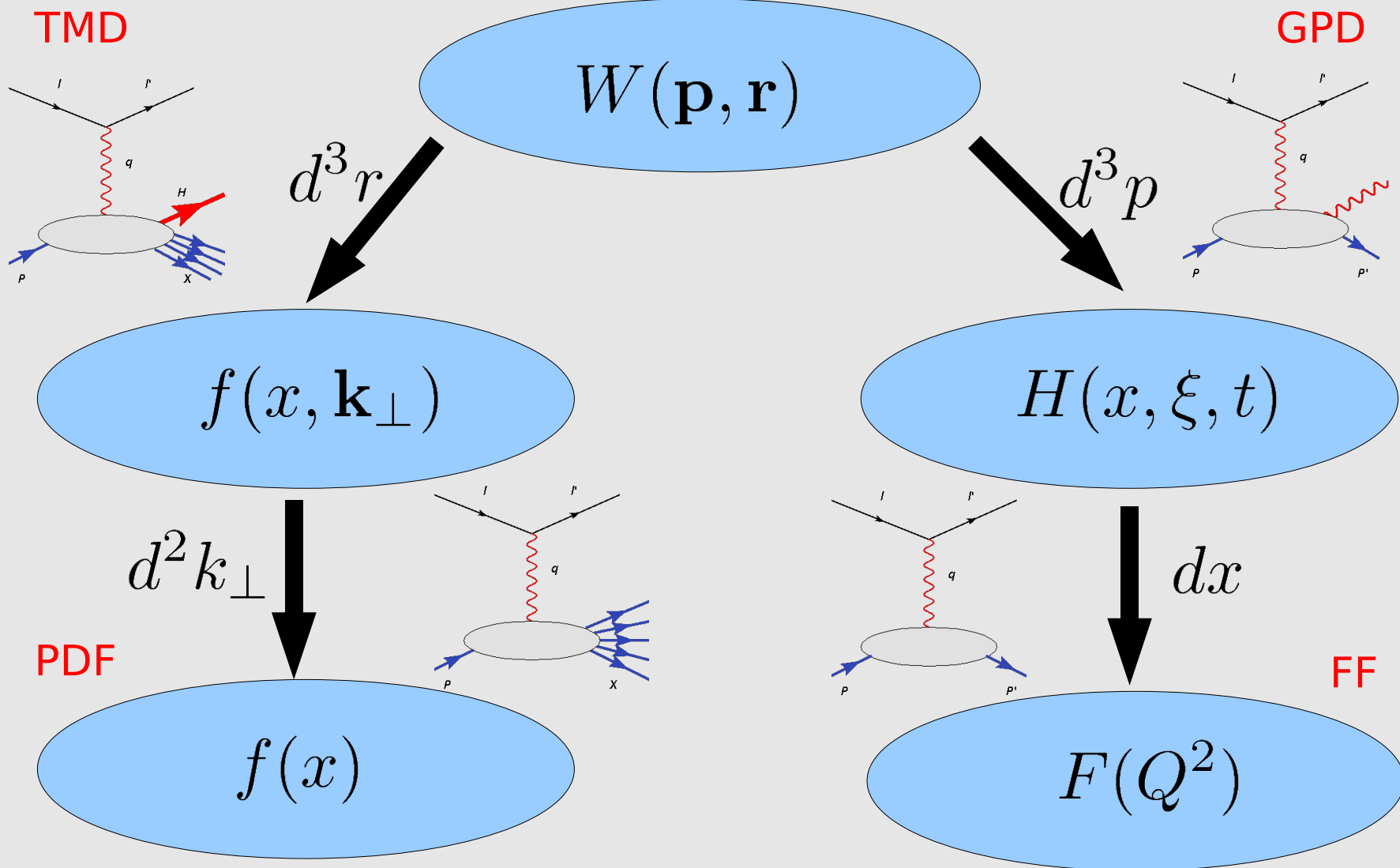
**T**ransverse  
**M**omentum  
**D**ependent  
distributions

**G**eneralized  
**P**arton  
**D**istributions



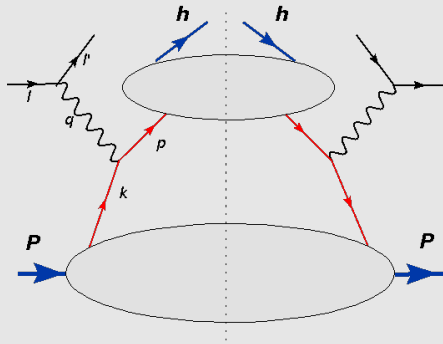


# Wigner distribution



# Transverse Momentum Dependent distributions

## SIDIS



If produced hadron has low transverse momentum

$$P_{hT} \sim \Lambda_{QCD} \ll Q$$

it will be sensitive to quark transverse momentum  $k_{\perp}$

$$\mathbf{l} + \mathbf{P} \rightarrow \mathbf{l}' + \mathbf{h} + \mathbf{X}$$

TMD factorization

Ji, Ma, Yuan (2002)

Collins (2011)

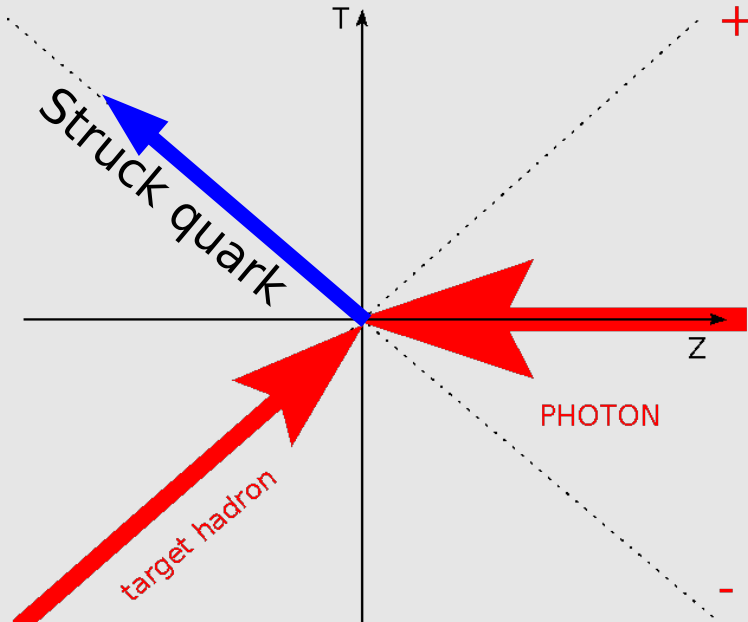
**GAUGE INVARIANT**

$$\Phi_{ij}(x, \mathbf{k}_{\perp}) = \int \frac{d\xi^{-}}{(2\pi)} \frac{d^2\xi_{\perp}}{(2\pi)^2} e^{ixP^{+}\xi^{-} - i\mathbf{k}_{\perp}\xi_{\perp}} \langle P, S_P | \bar{\psi}_j(0) \mathcal{U}(\mathbf{0}, \xi) \psi_i(\xi) | P, S_P \rangle$$

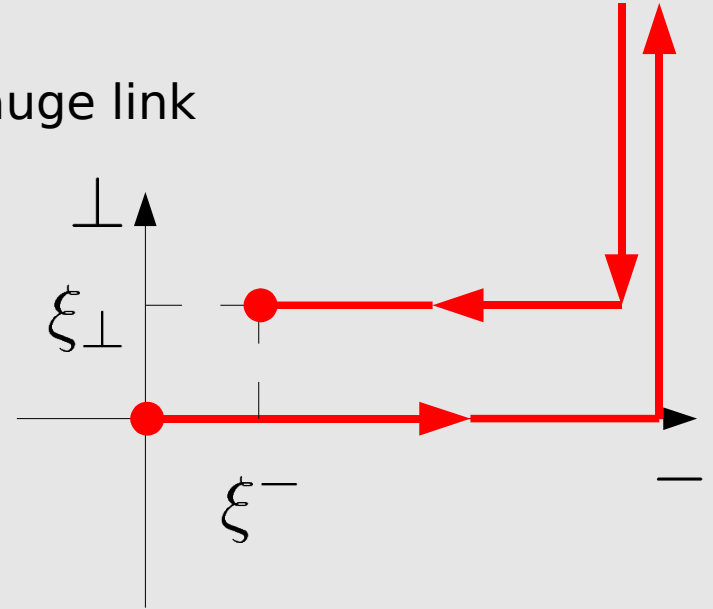
# Transverse Momentum Dependent distributions

$$\Phi_{ij}(x, \mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ixP^+\xi^- - i\mathbf{k}_\perp\xi_\perp} \langle P, S_P | \bar{\psi}_j(0) \mathcal{U}(\mathbf{0}, \xi) \psi_i(\xi) | P, S_P \rangle |_{\xi^+=0}$$

SIDIS in IMF:




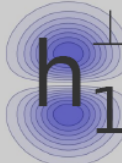


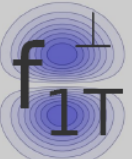

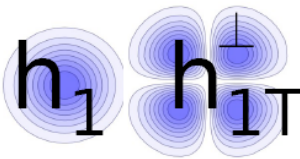
Gauge link



$$\mathcal{U}(a, b; n) = e^{-ig \int_a^b d\lambda n \cdot A_\alpha(\lambda n) t_\alpha}$$

Ensures gauge invariance of the distribution

# TMDs

$N \backslash q$	U	L	T
U			
L			
T			

8 functions in total (at leading Twist)

Each represents different aspects of partonic structure

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

# Twist-2 collinear PDFs

Quark-quark correlator can be decomposed by means of  
3 Parton Distributions Functions (PDF) in collinear (kt integrated) case

$$\Phi(x; P, S) = \frac{1}{2} \left\{ f_1(x) \not{P} + S_L g_1(x) \gamma_5 \not{P} + \frac{1}{2} h_1(x) \gamma_5 [\not{S}_T, \not{P}] \right\}$$

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Unpolarised PDF



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Unpolarised PDF



Helicity distribution



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Unpolarised PDF



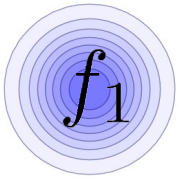
Helicity distribution



Transversity distribution



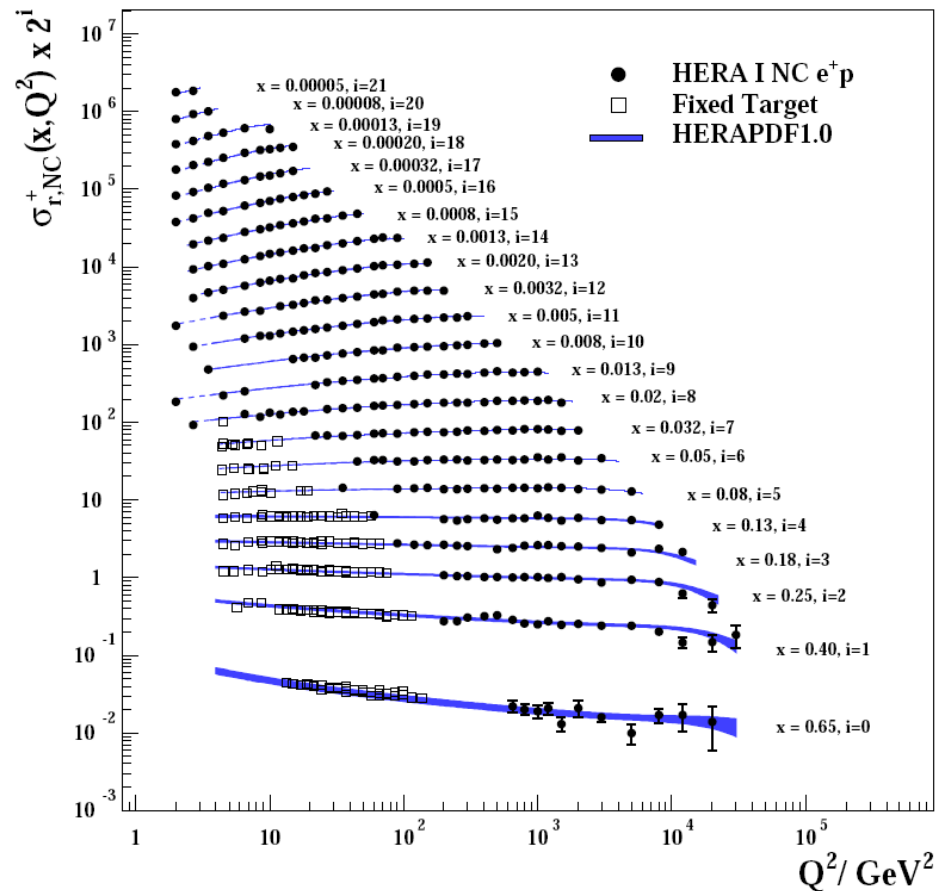


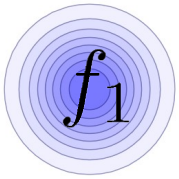


# Unpolarised PDFs

Good knowledge of unpolarised Parton Distribution Functions is acquired at HERA

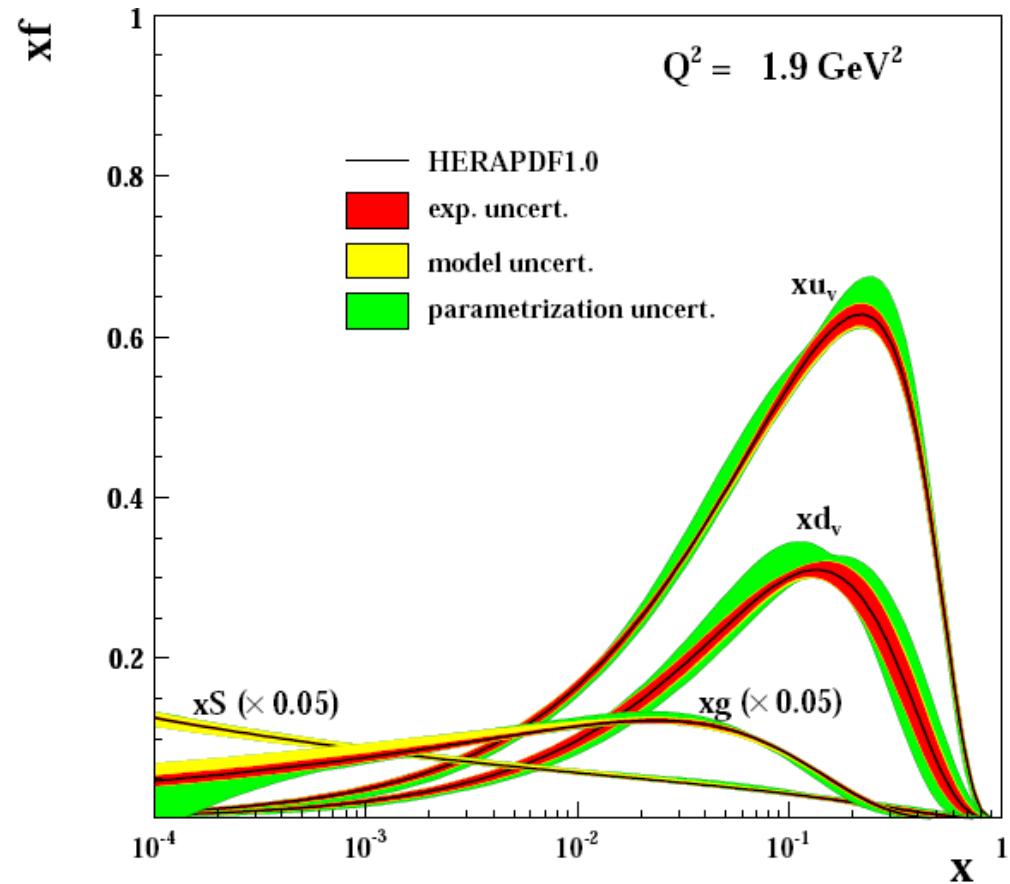
H1 and ZEUS





# Unpolarised PDFs

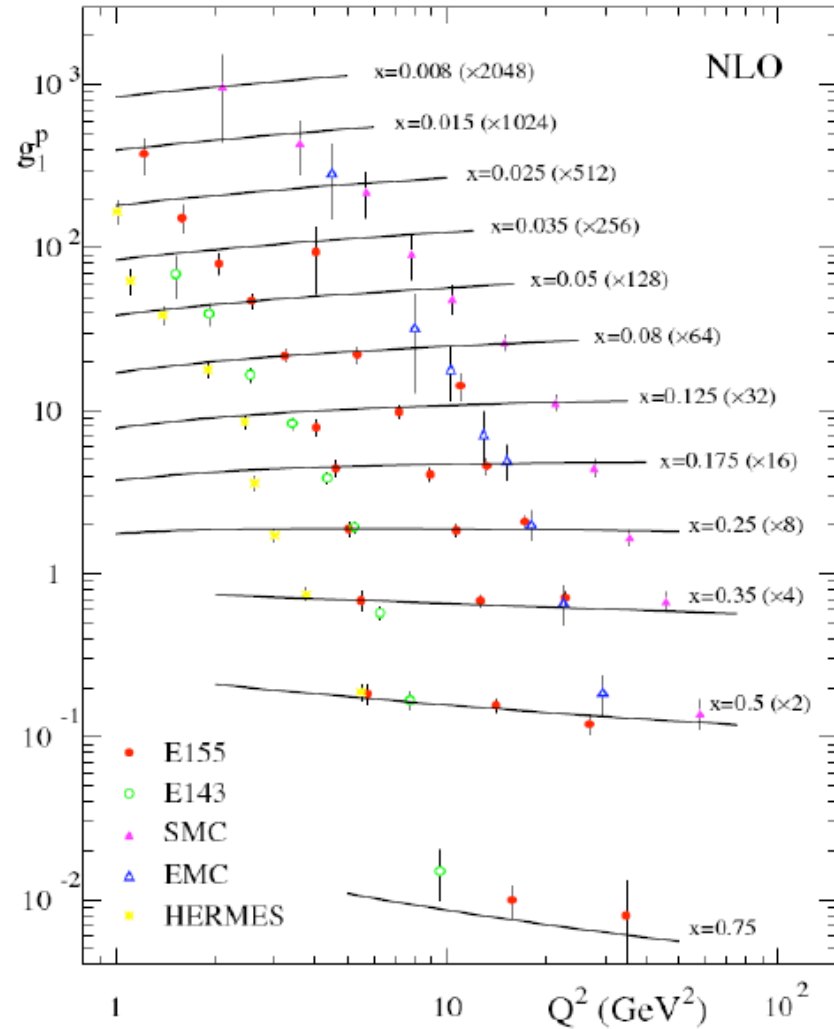
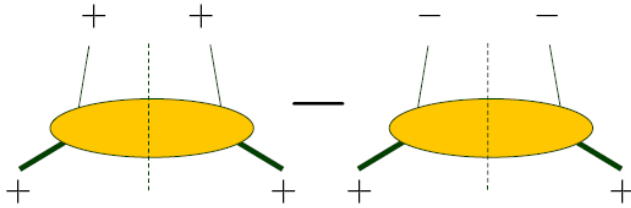
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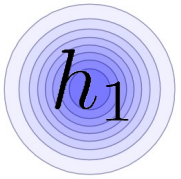




# Helicity distributions

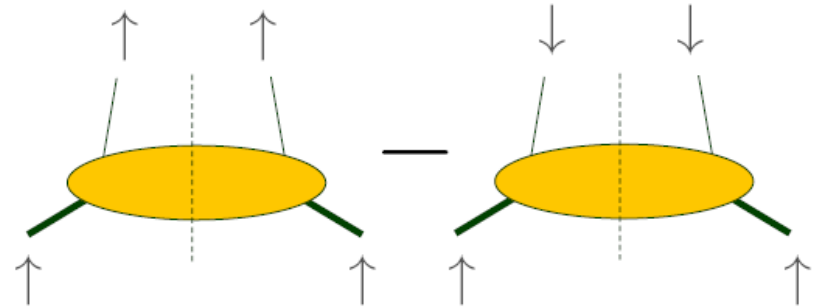
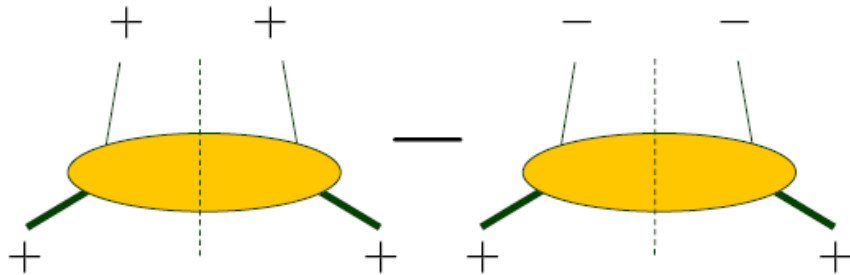
Helicity distributions  
are well known



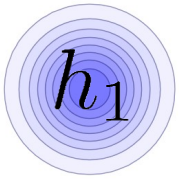


Helicity distribution

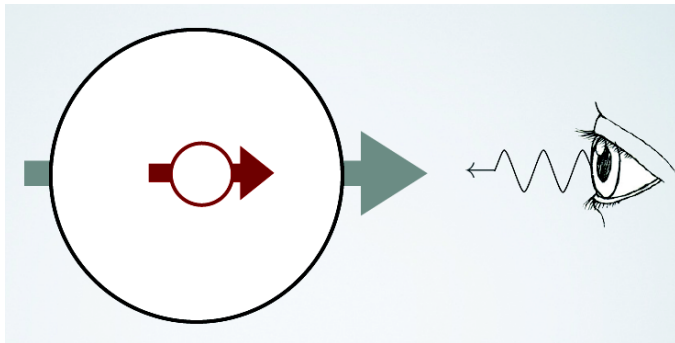
Transversity distribution



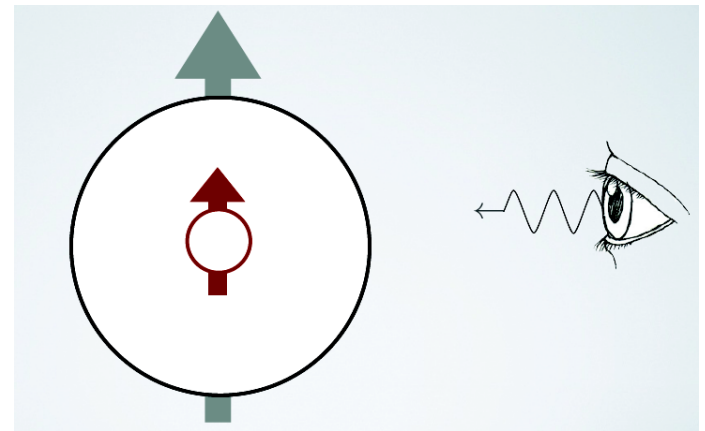
*Distribution of transversely polarised quarks inside transversely polarised nucleon*



## Helicity distribution

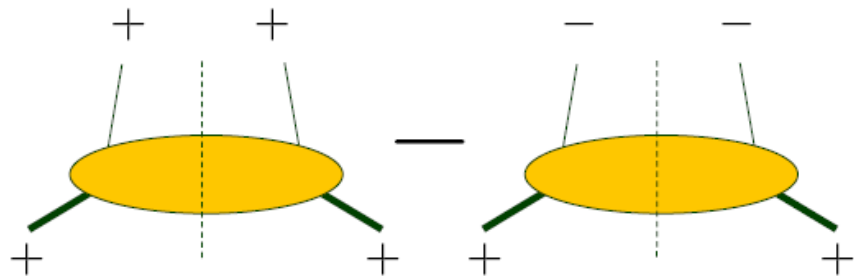


## Transversity distribution

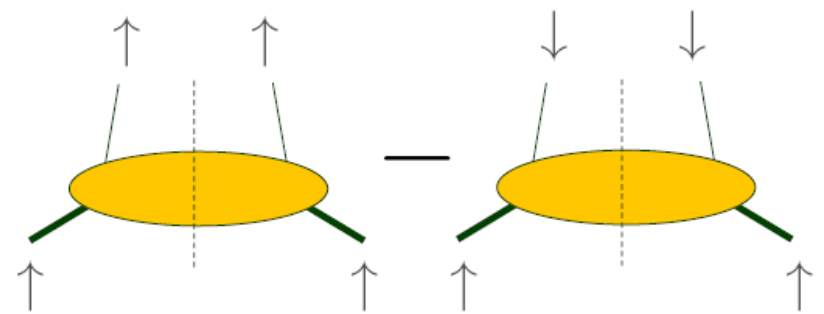


Boost and rotation do not commute  $\rightarrow$  helicity and transversity are different and difference a relativistic effect

## Helicity distribution



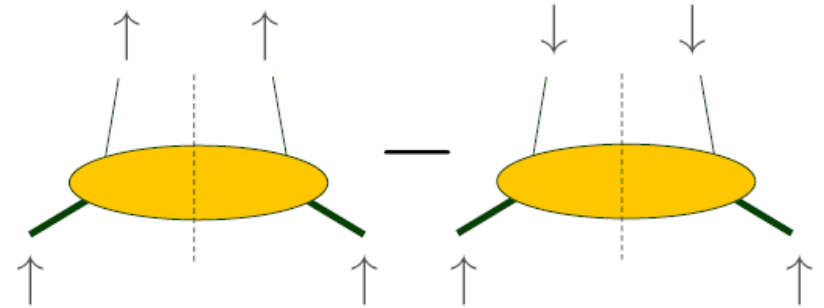
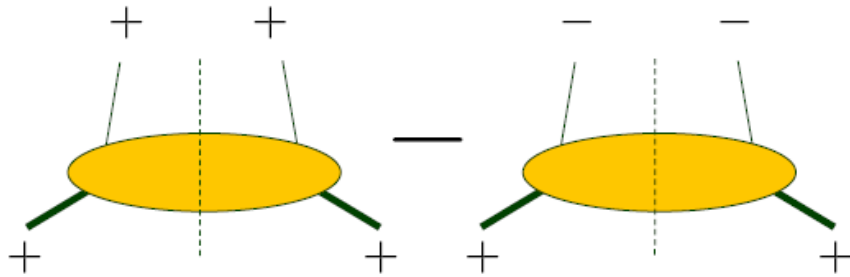
## Transversity distribution



*2005*: first data on transversity

## Helicity distribution

## Transversity distribution

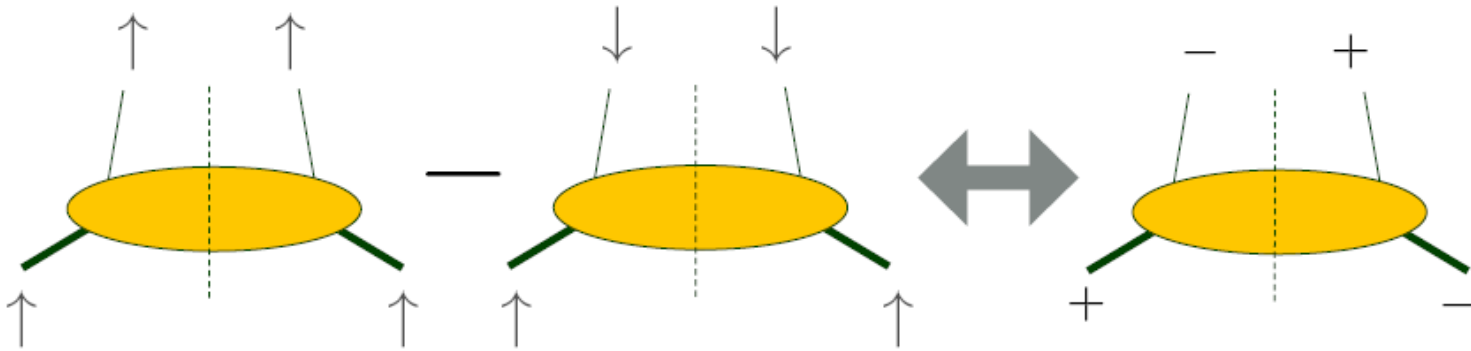


*2005:* first data on transversity

*2012:* hundred points from HERMES and COMPASS

# Why it is difficult to measure transversity distribution?

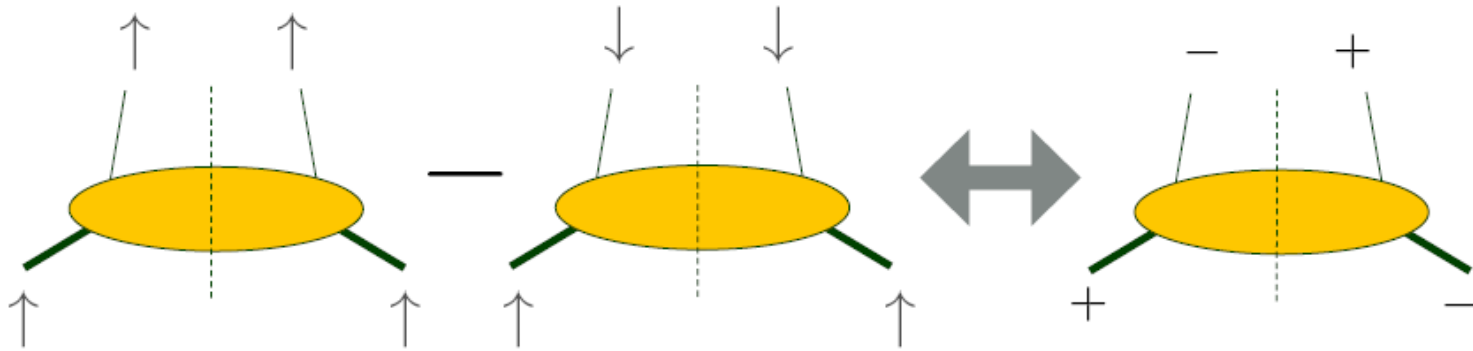
Transversity in helicity basis  $|\uparrow, \downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle)$





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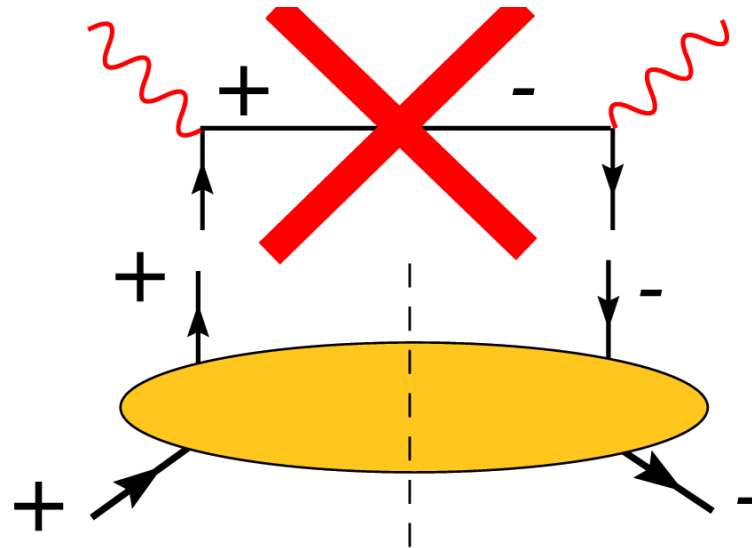
Transversity in helicity basis  $|\uparrow, \downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle)$



*Chiral Odd!*

# Transversity distribution

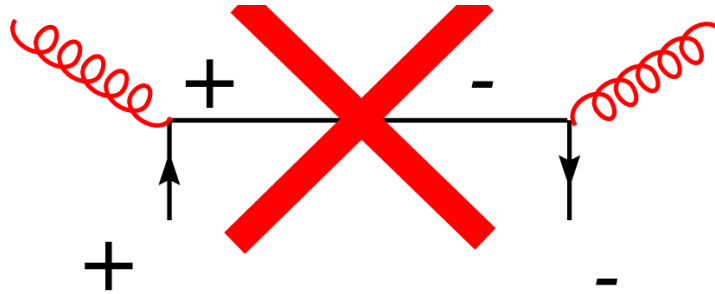
*Chiral Odd*: it cannot be measured in Deep Inelastic Scattering process



Needs another chiral odd function to be measured

# Transversity distribution

*QCD Evolution*: no gluon contribution in the evolution



$h_1(x, Q^2)$  is suppressed at low  $x$

JLab 12 is an ideal place to measure transversity → as JLab explores high  $x$  region

# Transversity distribution

Bounds on transversity distribution: The Soffer Bound

$$|h_1(x)| \leq \frac{1}{2} (f_1(x) + g_1(x))$$

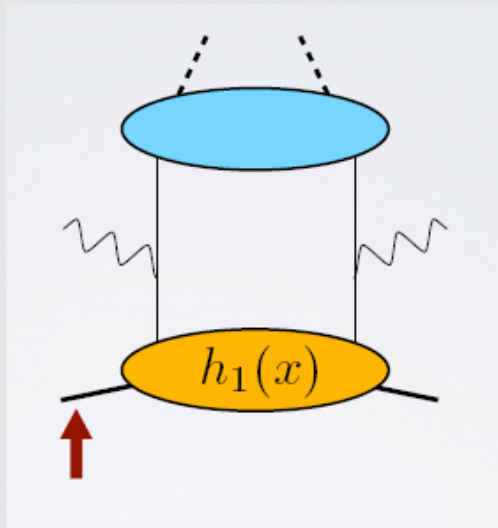
Valid at LO QCD, [Barone 97](#), [Bourelly et al 98](#)

Valid at NLO QCD, [Vogelsang 98](#)

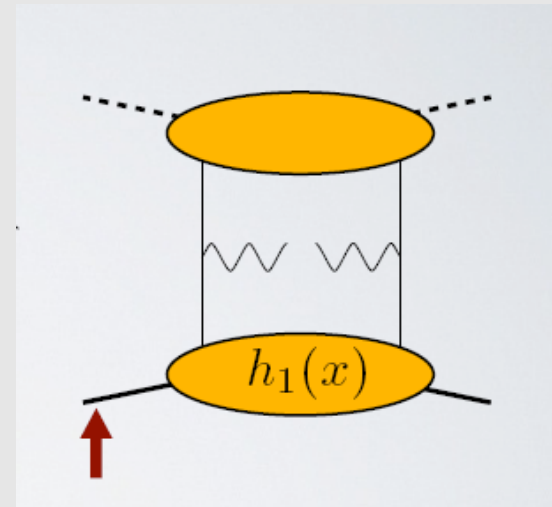
# Transversity how to measure?

Transversity needs another chiral odd function to be measured

## Semi Inclusive DIS (SIDIS)



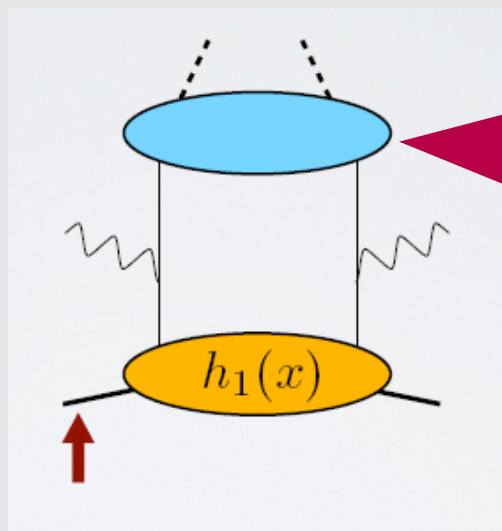
## Drell-Yan



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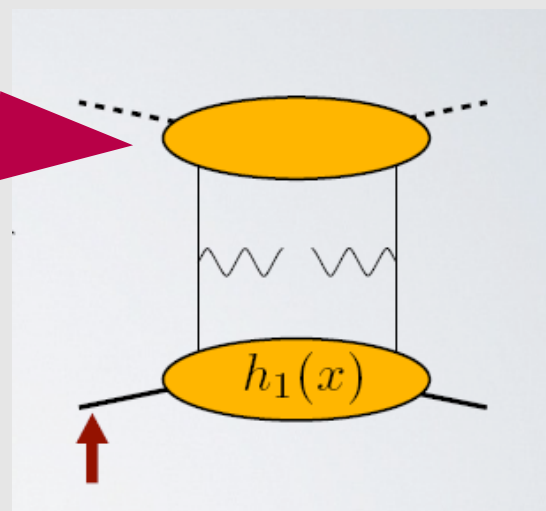
## Semi Inclusive DIS (SIDIS)



Collins fragmentation function

$$H_1^\perp(z, p_\perp)$$

## Drell-Yan



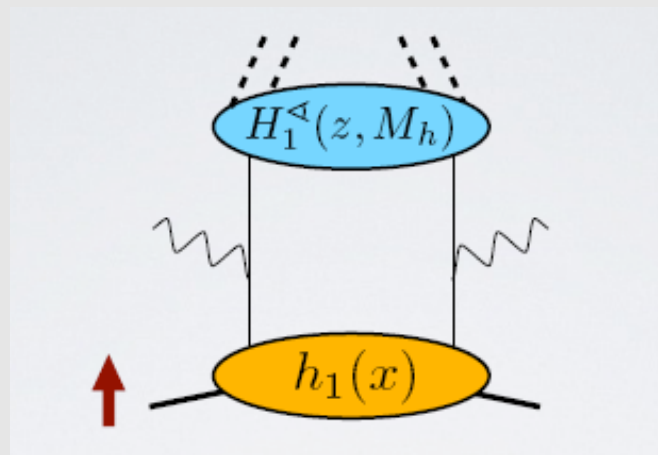
Transversity or Boer-Mulders function

$$h_1(x) \quad h_1^\perp(x, k_\perp)$$

# Transversity how to measure?

Another way to measure transversity is via dihadron fragmentation functions

## SIDIS

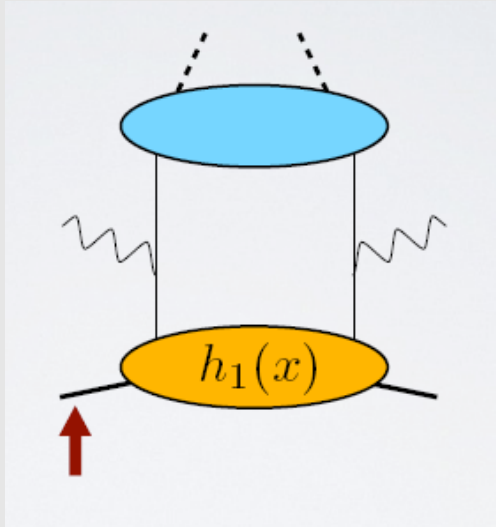


Dihadron fragmentation function

$$H_1^{\Delta}(z, M_h)$$

# Transversity from SIDIS

First extraction in 2007, [Anselmino et al 07](#)

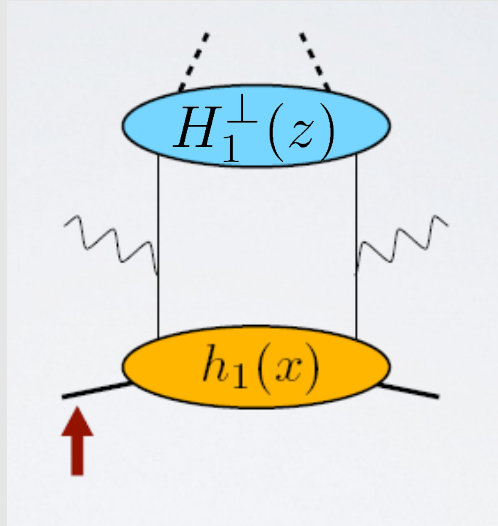


$$A_{UT}^{\sin(\Phi_h + \Phi_S)} \propto \frac{\sum e_q^2 h_1^q \otimes H_1^{\perp q}}{\sum e_q^2 f_1^q \otimes D_1^q}$$



# Transversity from SIDIS

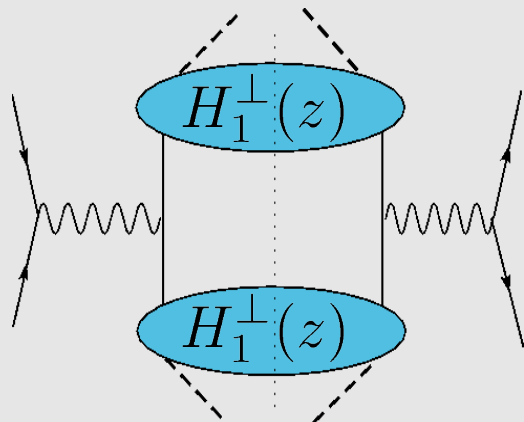
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$$A_{UT}^{\sin(\Phi_h + \Phi_S)} \propto \frac{\sum e_q^2 h_1^q \otimes H_1^{\perp q}}{\sum e_q^2 f_1^q \otimes D_1^q}$$

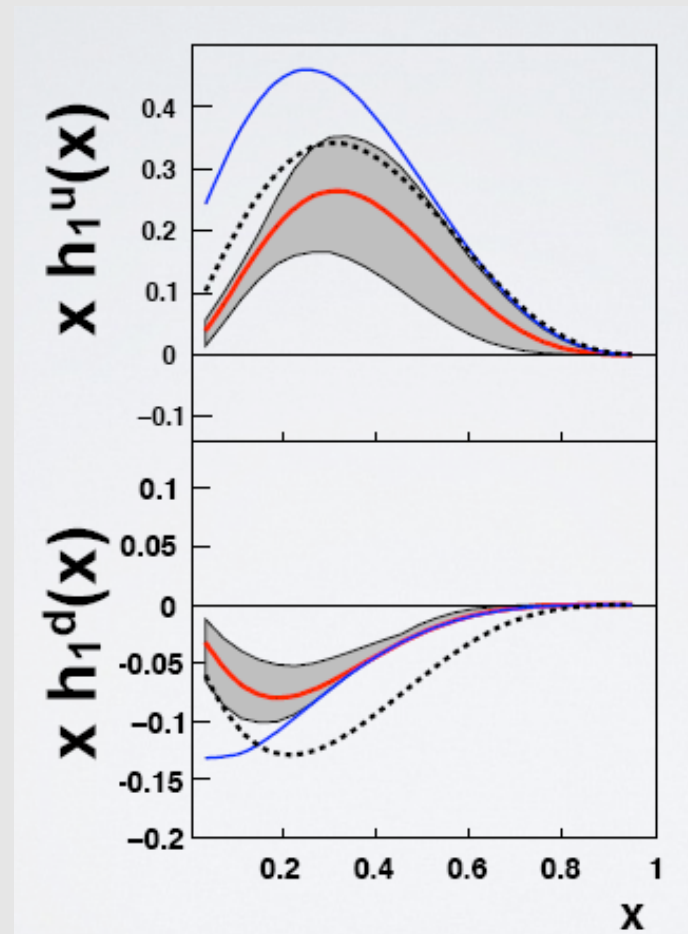
Two unknowns, transversity  $h_1(x)$   
Collins Fragmentation Function  $H_1^{\perp}(z)$

Fortunately information on  $H_1^{\perp}(z)$  is available from  $e^+e^-$



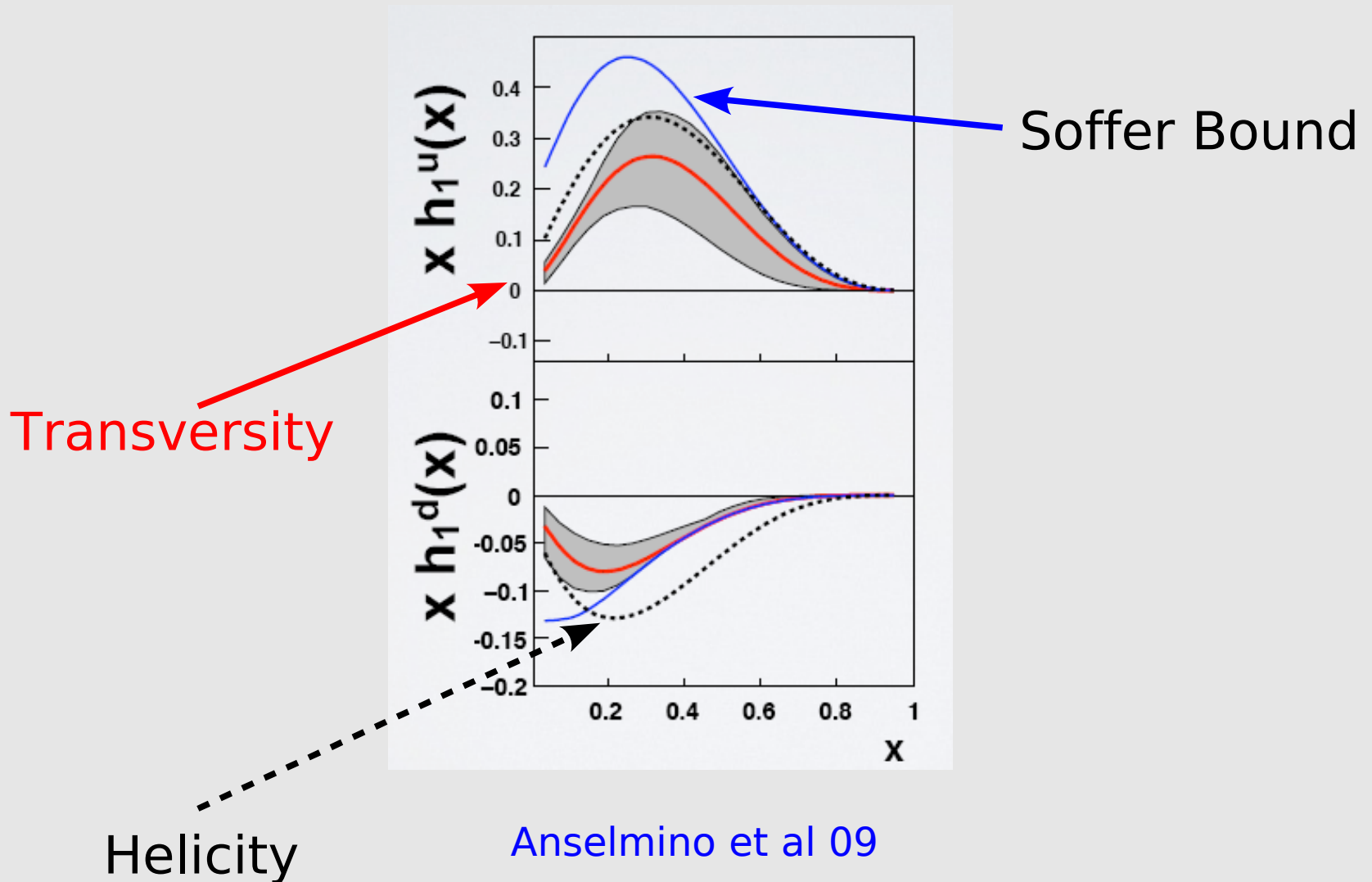
$$A_{e^+e^-} \propto \frac{\sum e_q^2 H_1^{\perp q} \otimes H_1^{\perp \bar{q}}}{\sum e_q^2 D_1^q \otimes D_1^{\bar{q}}}$$

# Transversity



Anselmino et al 09

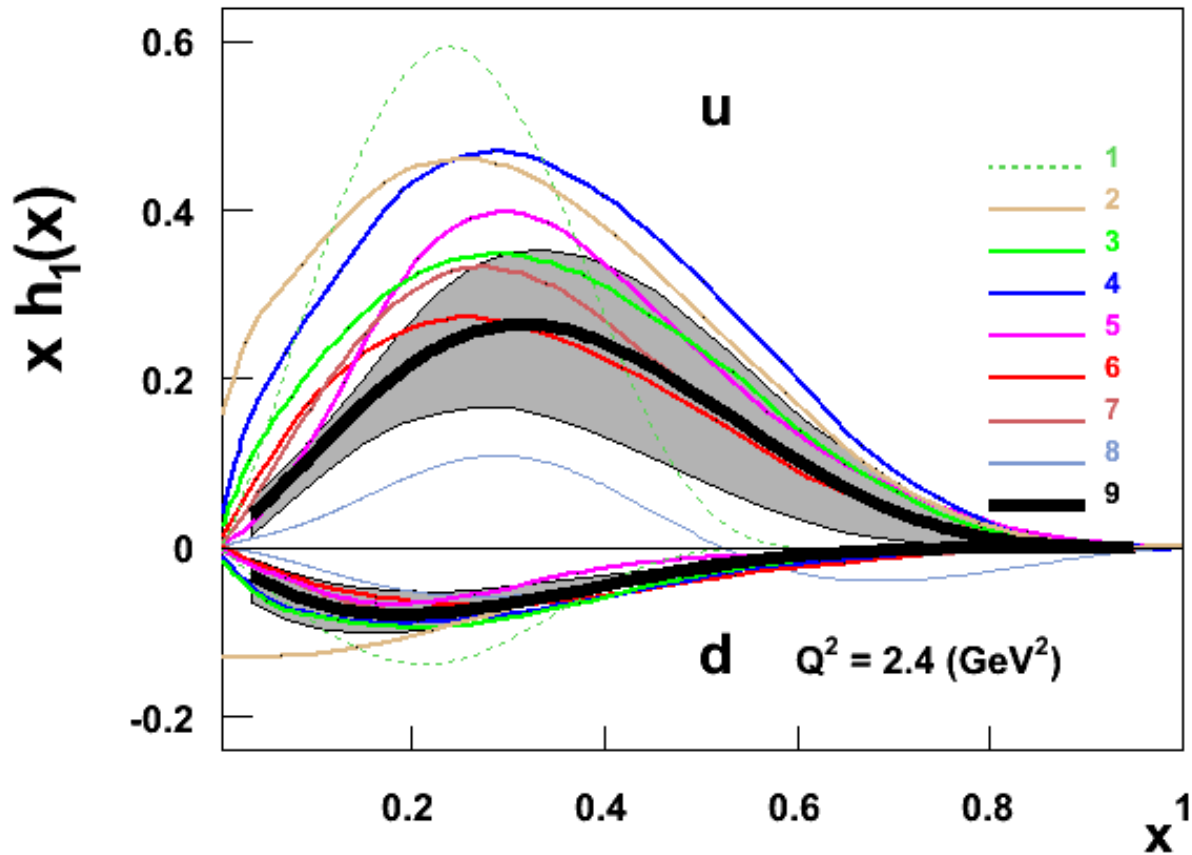
# Transversity



Anselmino et al 09

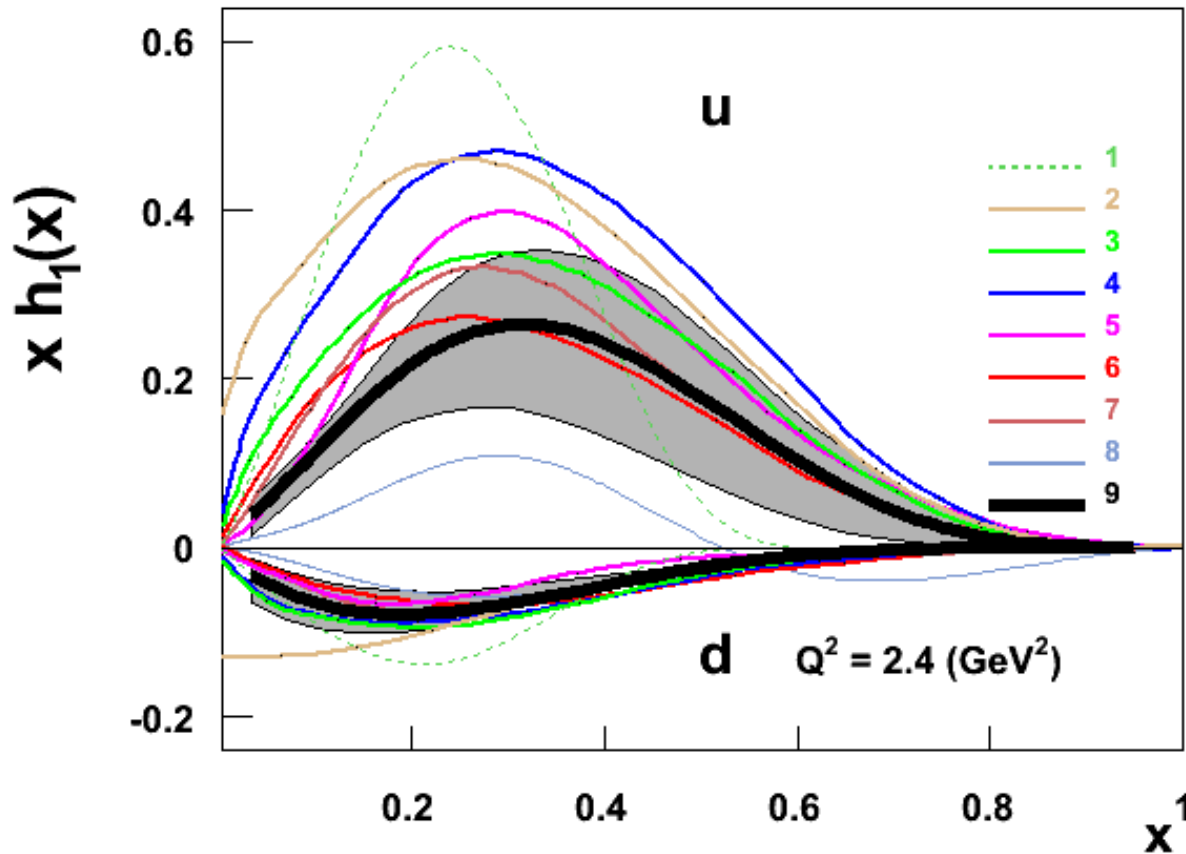
# Comparison with models

- 1 - Barone et al., (1997)
- 2 - Soffer et al., (2002)
- 3 - Korotkov et al., (2001)
- 4 - Schweitzer et al., (2001)
- 5 - Wakamatsu, (2007)
- 6 - Pasquini et al., (2005)
- 7 - Cloet et al., (2008)
- 8 - Bacchetta et al., (2008)
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# Comparison with models



Good agreement with models in sign and size of transversity.

High uncertainty especially in high-x region

# Tensor charge

Transversity is the only source of information on tensor charge

$$\delta q = \int_0^1 dx (h_1^q(x) - h_1^{\bar{q}}(x))$$

Fundamental quantity, as fundamental as vector or axial charges

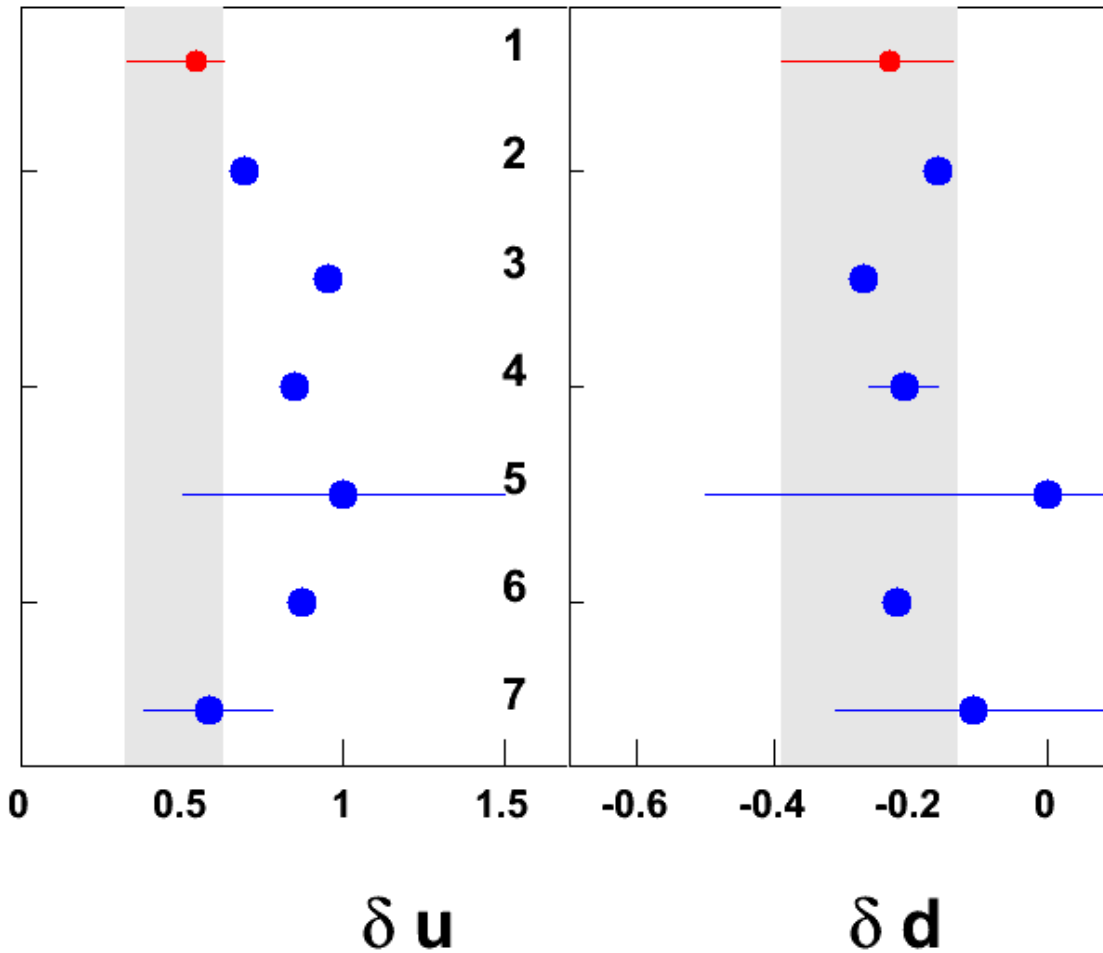
Caveat: no sum rules

# Tensor charge

- 1 - Anselmino et al., Nucl.Phys.Proc.Suppl. (2009)
- 2 - Cloet, Bentz and Thomas, Phys.Lett.B (2008)
- 3 - Wakamatsu, Phys.Lett.B (2007)
- 4 - Gockeler et al., Phys.Lett.B (2005)
- 5 - He and Ji, Phys. Rev. D (1995)
- 6 - Pasquini et al, Phys. Rev. D (2007)
- 7 - Gamberg and Goldstein, Phys. Rev. Lett. (2001)

$$\delta q = \int_0^1 dx (h_1^q(x) - h_1^{\bar{q}}(x))$$

$$\delta u = 0.54^{+0.09}_{-0.22}, \delta d = -0.23^{+0.09}_{-0.16}$$

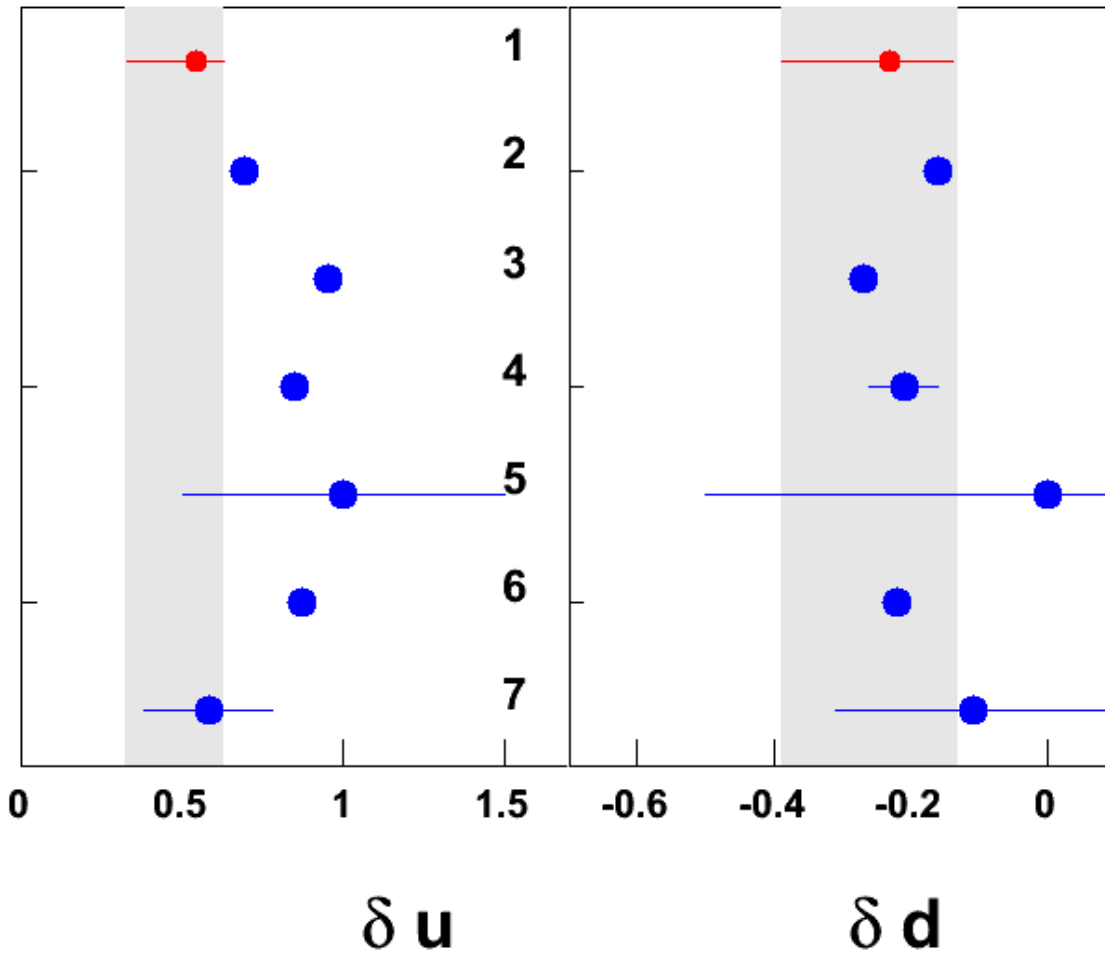


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$$\delta q = \int_0^1 dx (h_1^q(x) - h_1^{\bar{q}}(x))$$



Precision is not enough to discriminate among different calculations



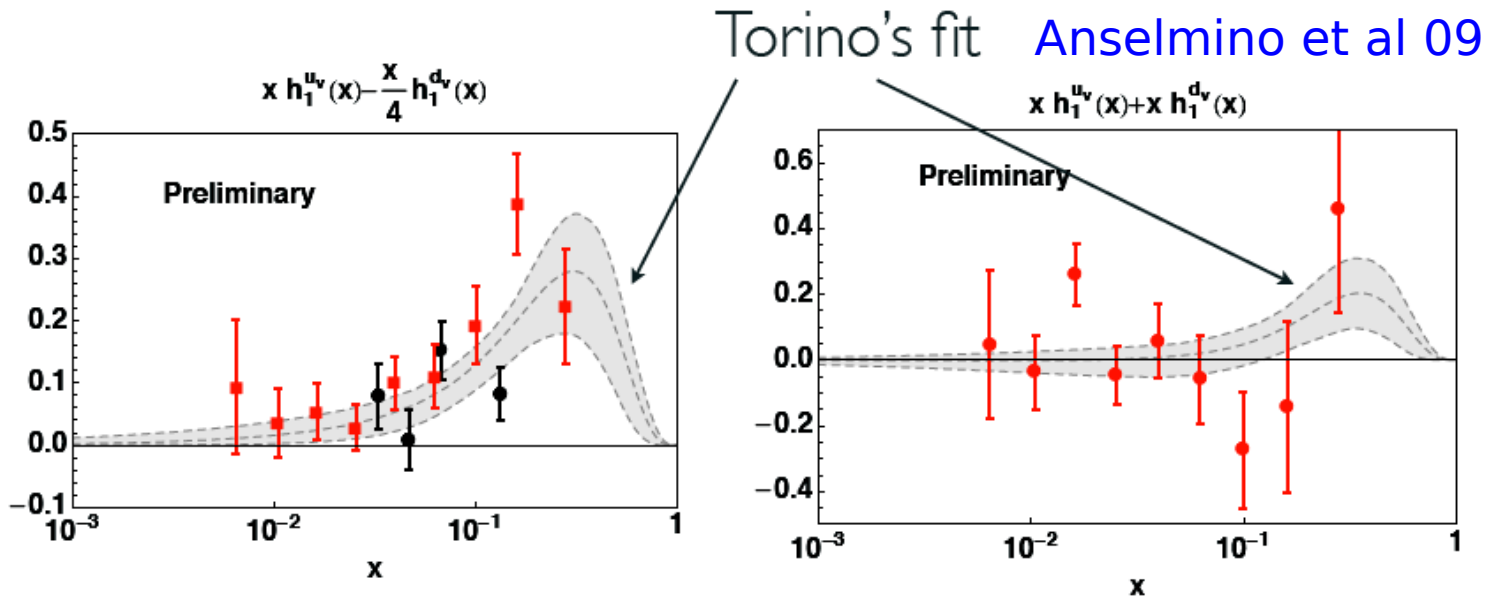
# Tensor vs axial charges

$$\Delta q = \int_0^1 dx (g_1^q(x) + g_1^{\bar{q}}(x)) \quad \delta q = \int_0^1 dx (h_1^q(x) - h_1^{\bar{q}}(x))$$

	Axial, DSSV	Tensor, Anselmino
u	0.82	0.54
d	-0.45	-0.23
s	-0.11	0
sum	0.26	0.39

$$Q = 1, 0.9(\text{GeV})$$

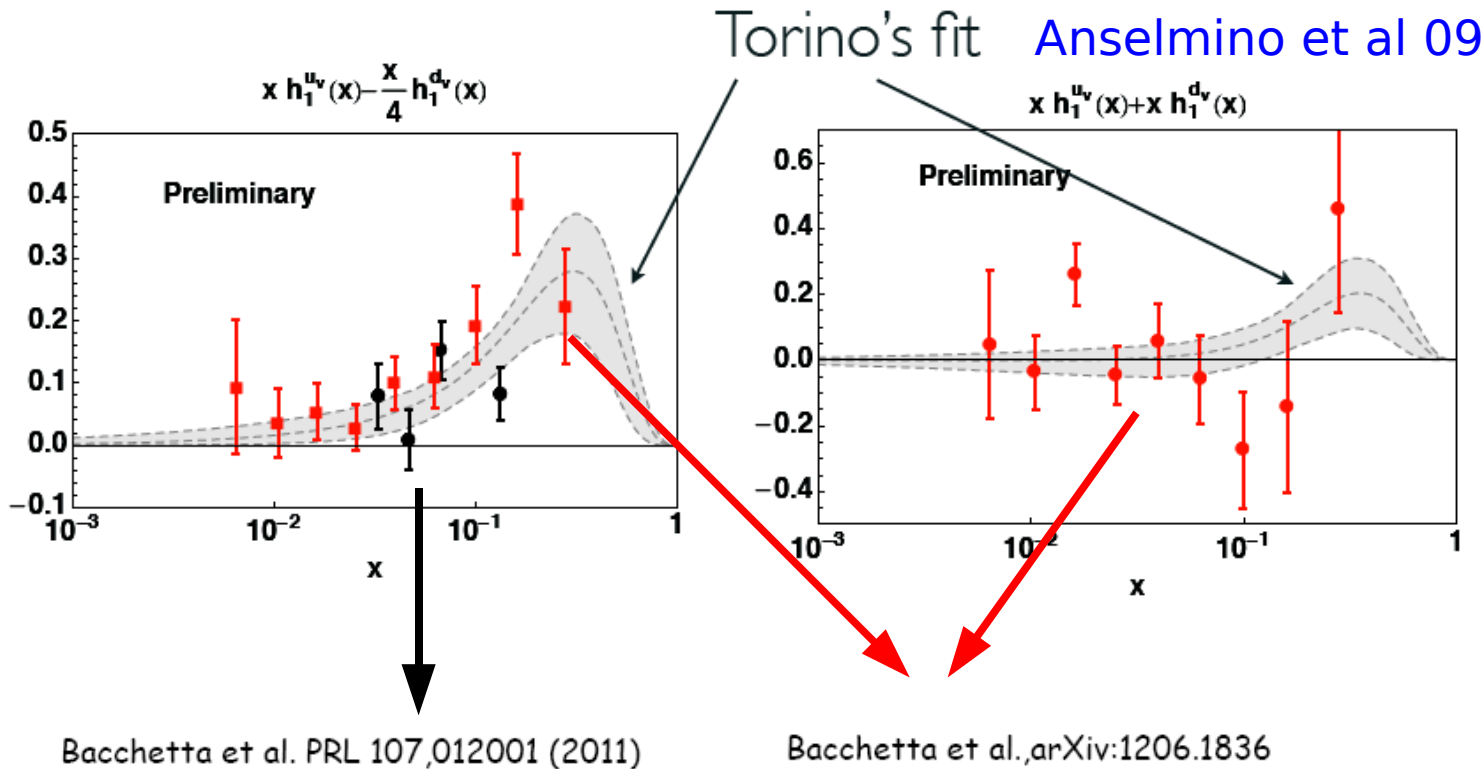
# Transversity from dihadron fragmentation



Bacchetta et al. PRL 107,012001 (2011)

Bacchetta et al., arXiv:1206.1836

# Transversity from dihadron fragmentation



Good qualitative agreement of two methods of extraction

What do we expect from  
JLab 12?

# 12 GeV approved experiments

	Hall A	Hall B	Hall C	Hall D	Total
The Hadron Spectra as Probes of QCD (GluEx & heavy baryon and meson spectroscopy)		1		1	2
The Transverse Structure of the Hadrons (elastic and transition form factors)	4	3	2		9
The Longitudinal Structure of the Hadrons (Unpolarized and polarized parton distributions)	2	2	5		9
The 3D Structure of the Hadrons (GPDs and TMDs)	5	10	3		18
Hadrons and Cold Nuclear Matter	3	2	6		11
Low-Energy Tests of the Standard Model and Fundamental Symmetries	2			1	3
<b>Total</b>	<b>16</b>	<b>18</b>	<b>16</b>	<b>2</b>	<b>52</b>

See talk by Bob McKeown

# 12 GeV approved experiments

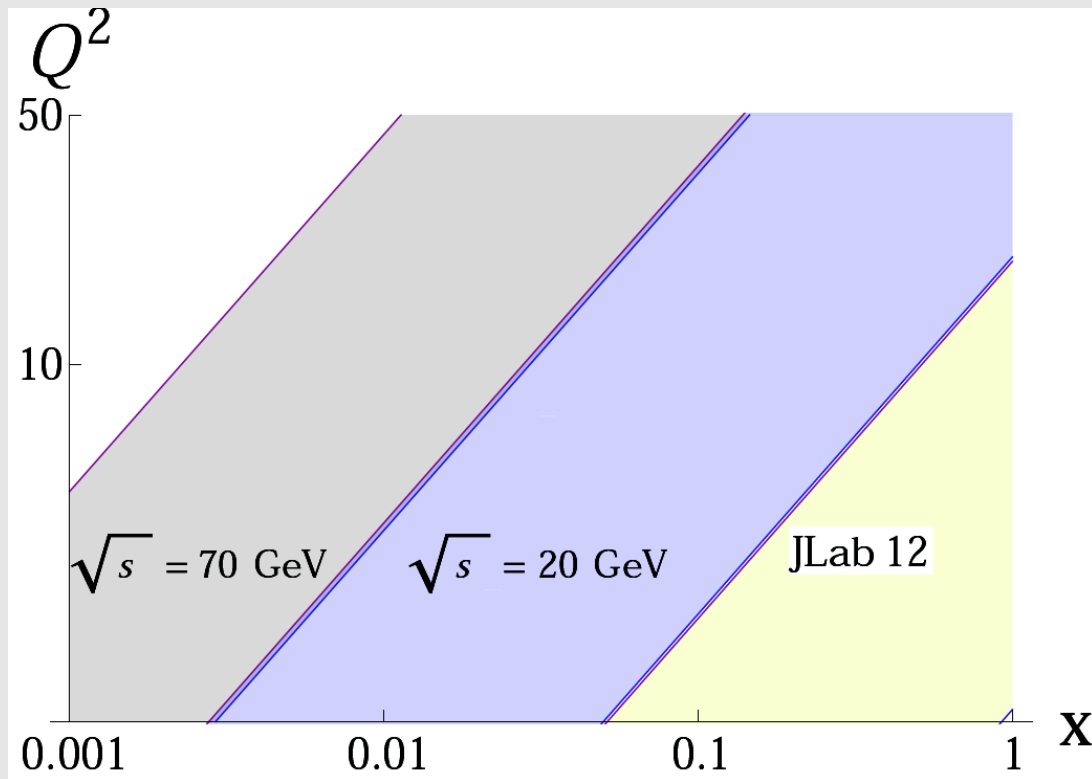
	Hall A	Hall B	Hall C	Hall D	Total
The Hadron Spectra as Probes of QCD (GluEx & heavy baryon and meson spectroscopy)		1		1	2
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**>30% is dedicated to 3D structure!**

See talk by Bob McKeown

# Kinematics

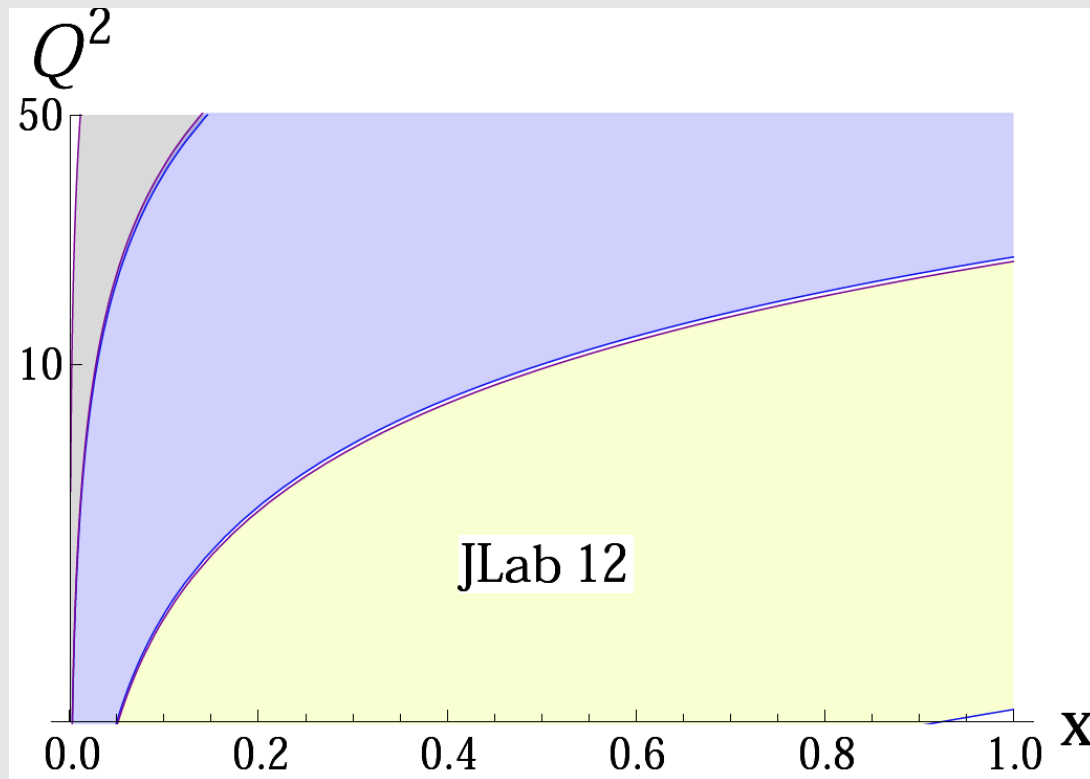
Kinematics  $Q^2 \simeq sxy$



JLab 12 and future  
Electron Ion Collider  
are complimentary

# Kinematics

Kinematics  $Q^2 \simeq sxy$

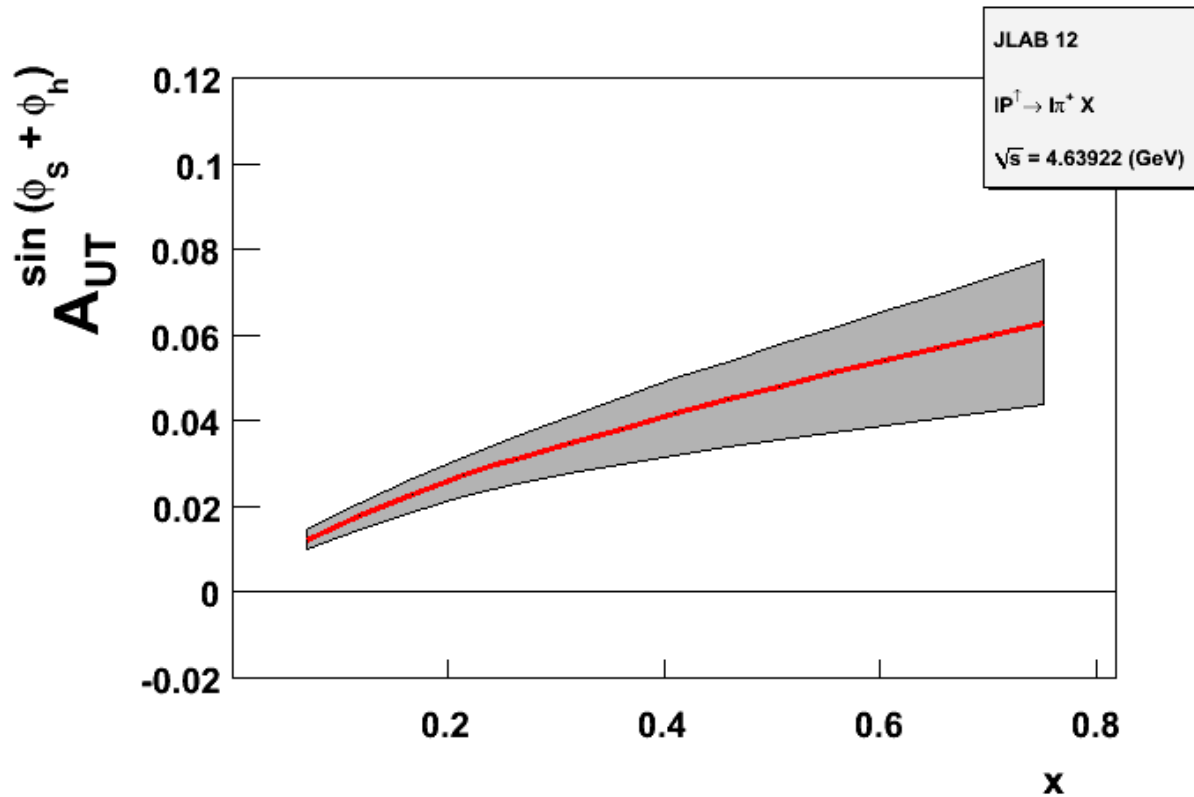


JLab 12 and future  
Electron Ion Collider  
are complimentary

JLab and EIC are going to  
provide fine 4D binning  
of the data.

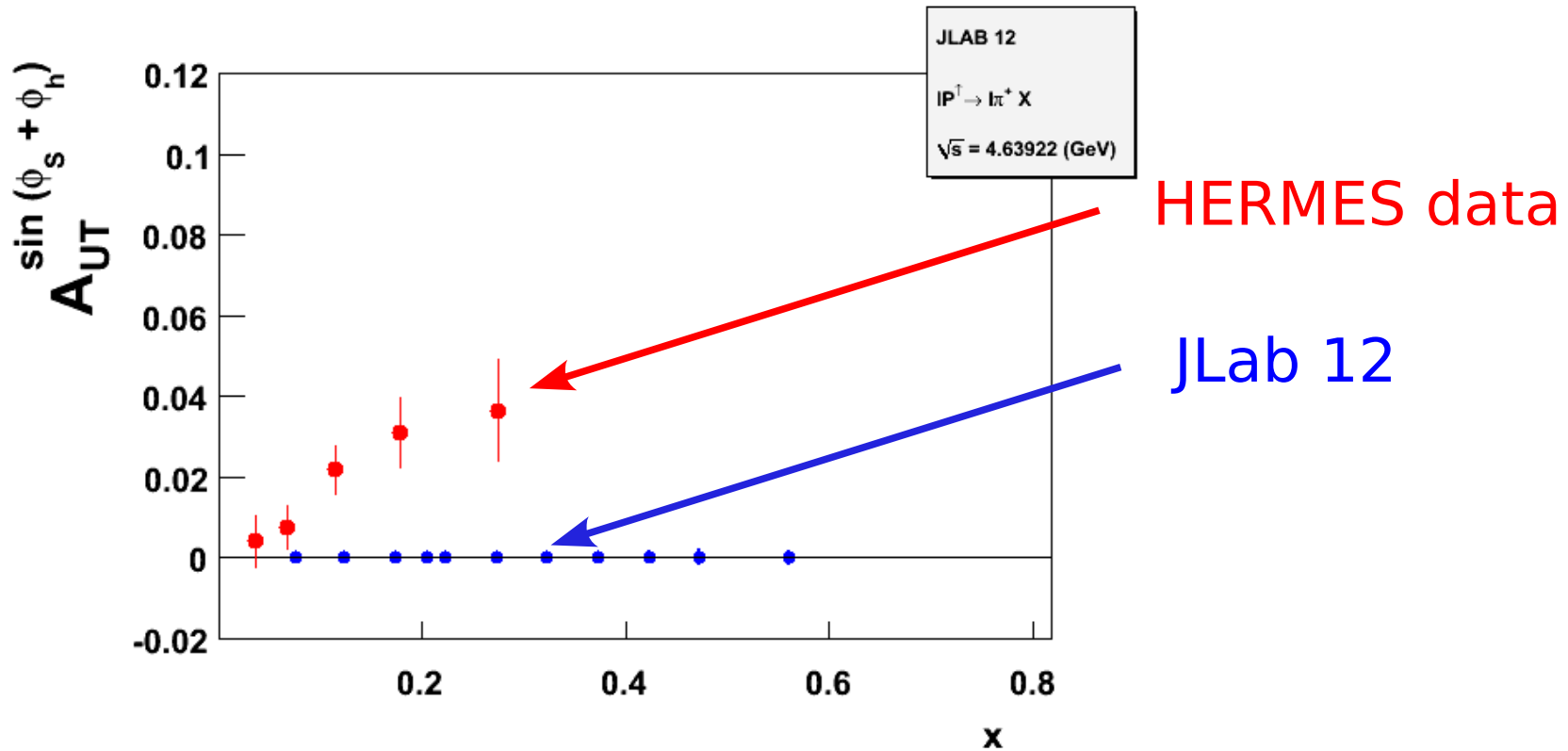


# What do we expect at JLab?



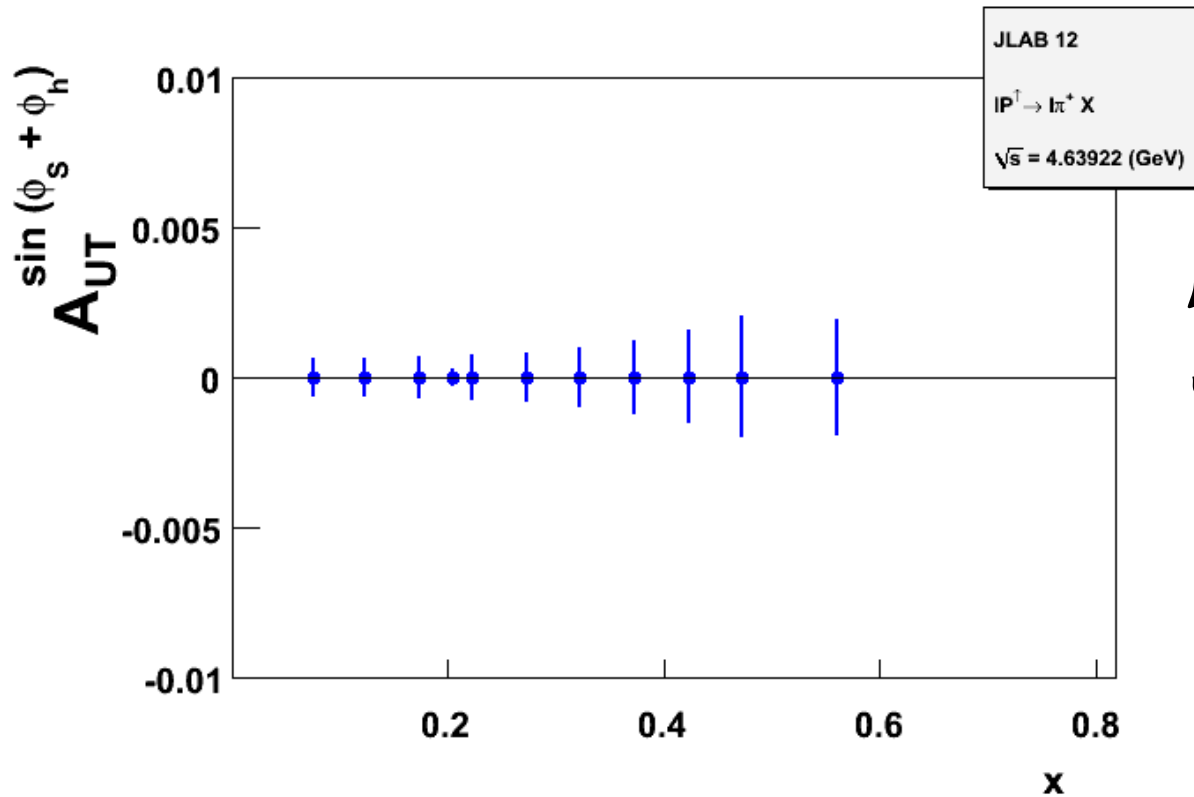
Prediction for JLab 12 kinematics based on [Anselmino et al 09](#)

# What do we expect at JLab?



Estimates of experimental error for JLab 12

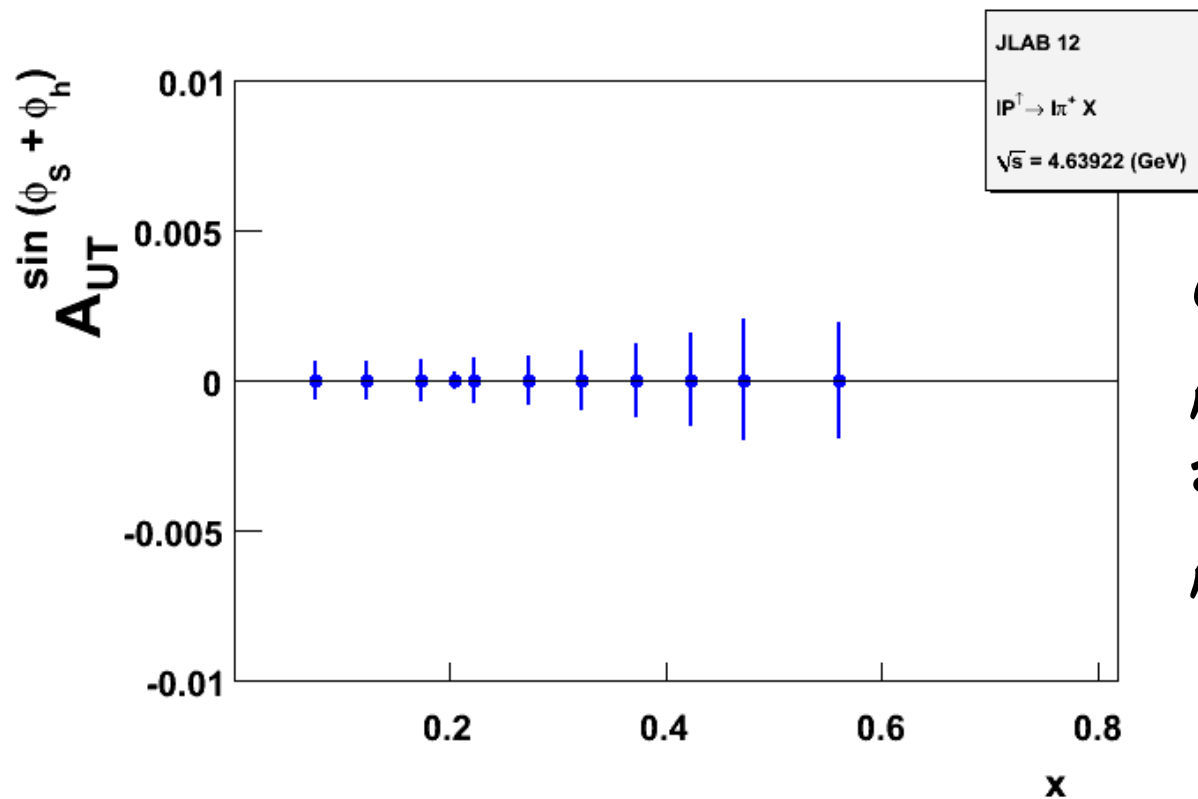
# What do we expect at JLab?



Less than 0.25 %  
error!

Estimates of experimental error for JLab 12

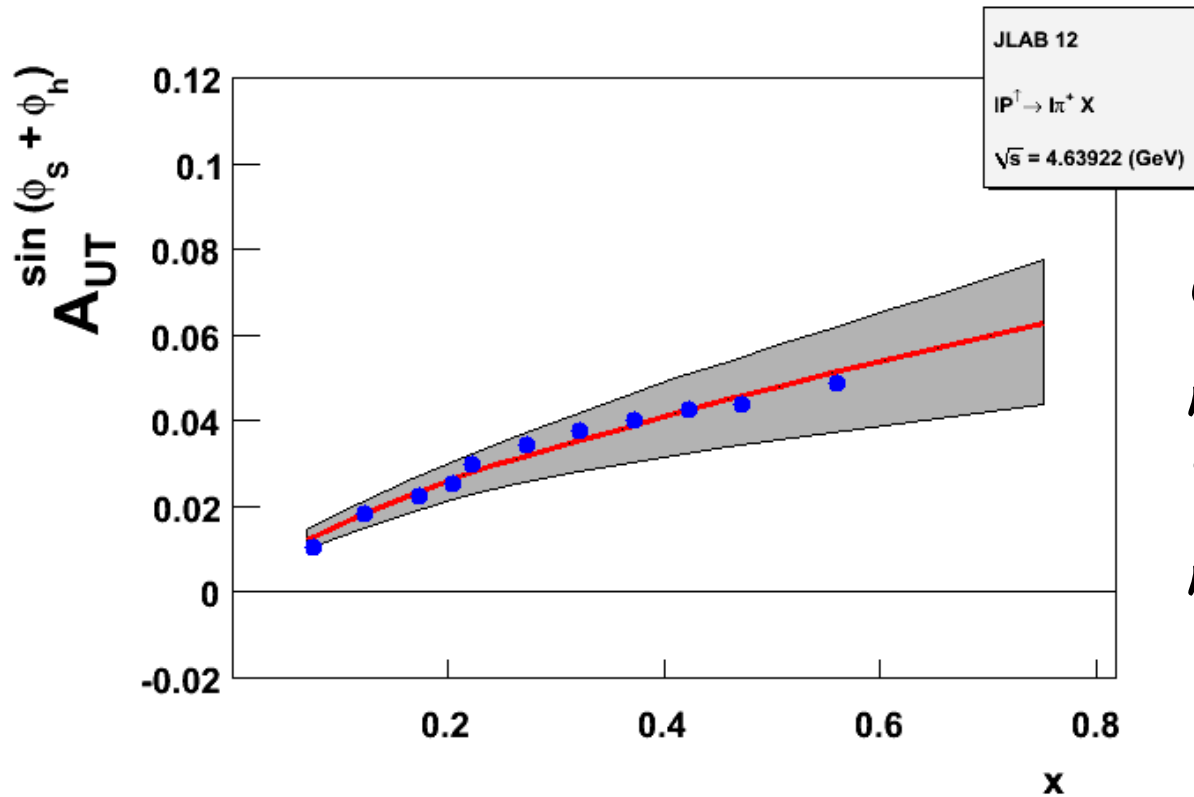
# Generate pseudo-data



*Generate  
pseudodata  
around our  
predictions*

Estimates of experimental error for JLab 12

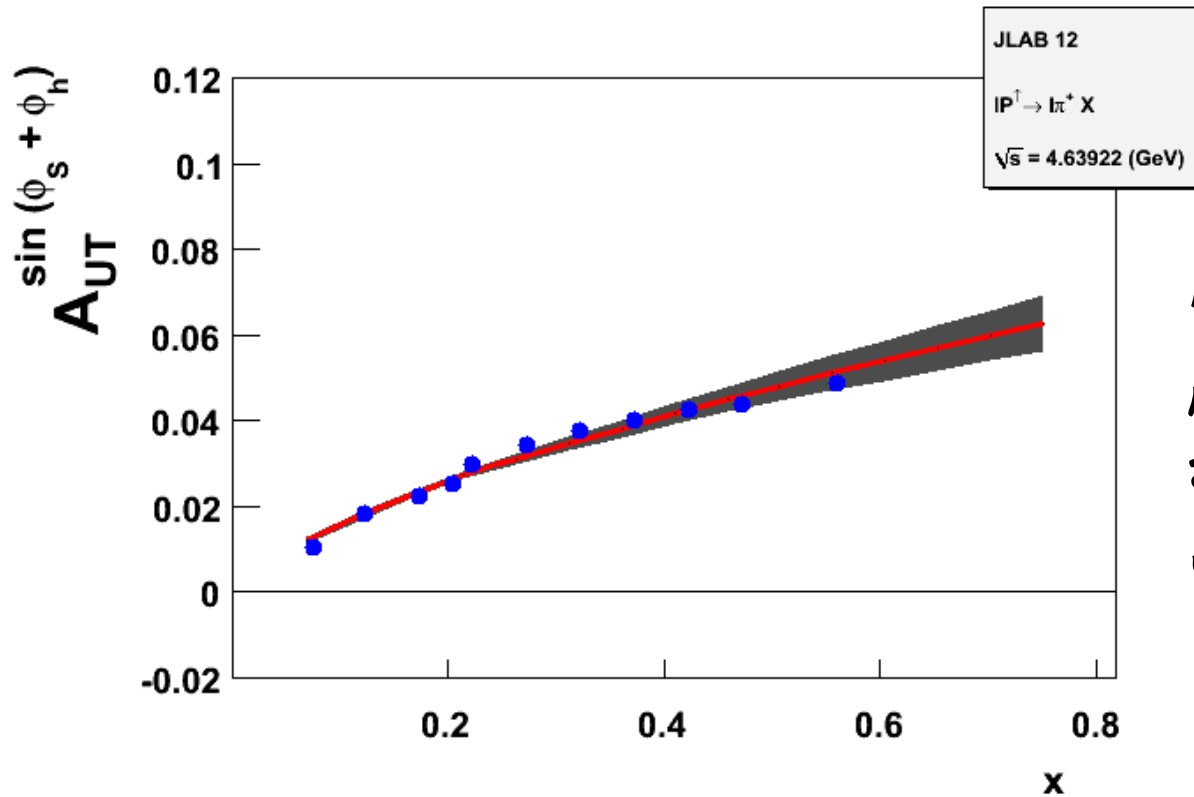
# Generate pseudo-data



*Generate  
pseudodata  
 $1-\sigma$  around our  
predictions*

Based on [Anselmino et al 09](#)

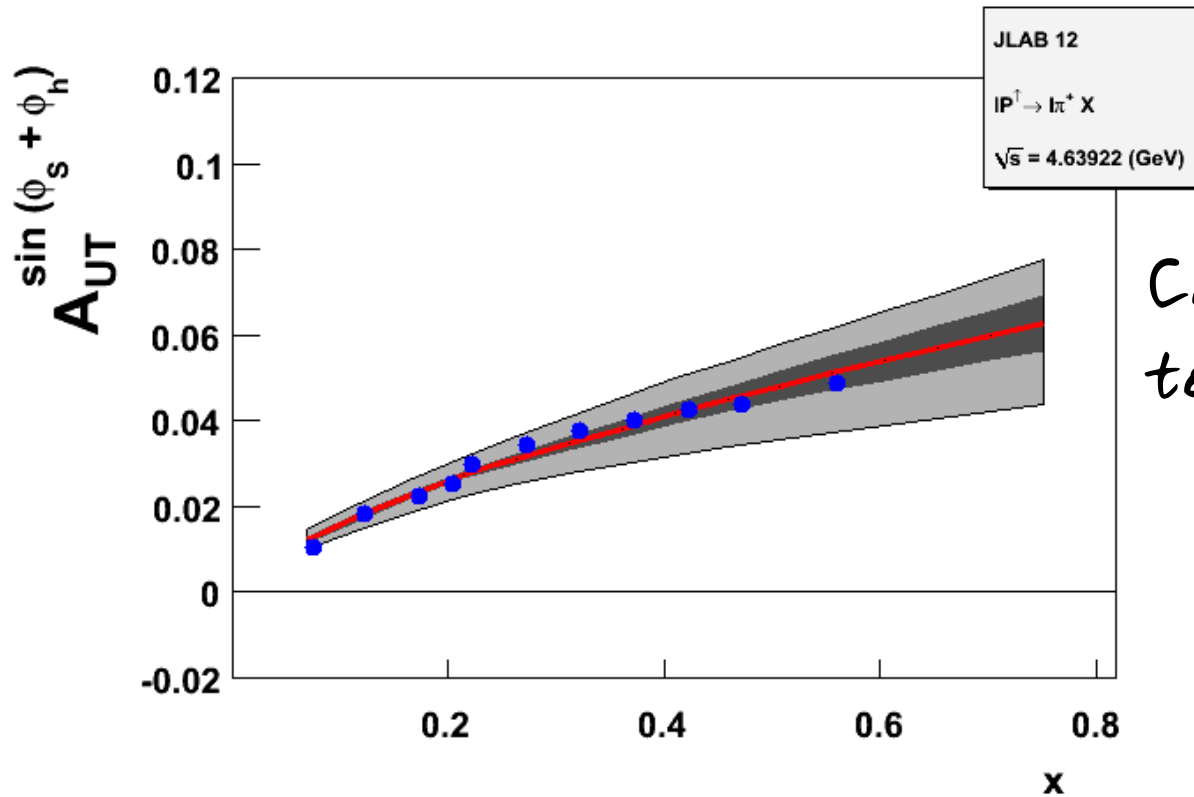
# Fit the pseudo-data



Fit the  
pseudodata  
and estimate the  
error

Based on [Anselmino et al 09](#)

# Generate pseudo-data



*Compare the error to the existing one*

Based on [Anselmino et al 09](#)

# Tensor charge

## 1 - JLab 12

2 - Anselmino et al., Nucl.Phys.Proc.Suppl. (2009)

3 - Cloet, Bentz and Thomas, Phys.Lett.B (2008)

4 - Wakamatsu, Phys.Lett.B (2007)

5 - Gockeler et al., Phys.Lett.B (2005)

6 - He and Ji, Phys. Rev. D (1995)

7 - Pasquini et al, Phys. Rev. D (2007)

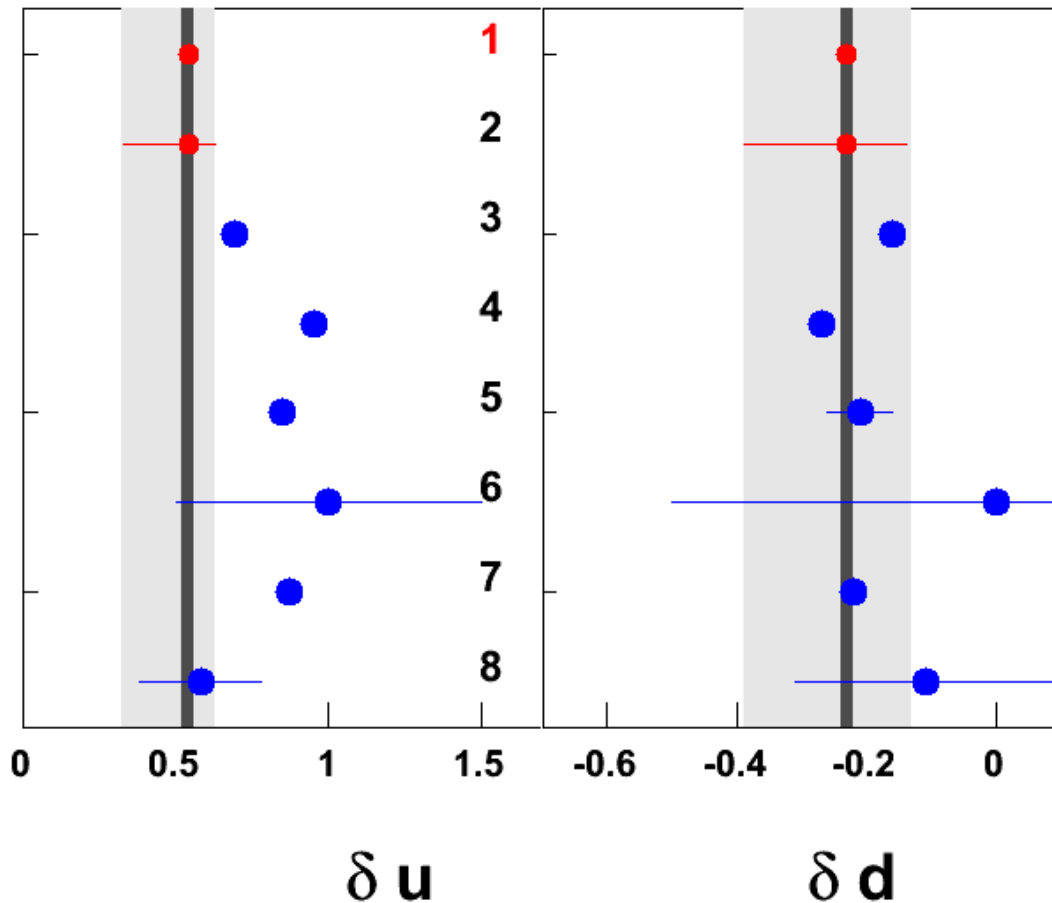
8 - Gamberg and Goldstein, Phys. Rev. Lett. (2001)

$$\delta q = \int_0^1 dx ( h_1^q(x) - h_1^{\bar{q}}(x) )$$

$$\delta u = 0.54^{+0.09}_{-0.22}, \delta d = -0.23^{+0.09}_{-0.16}$$

## JLab 12 Proton and He<sup>3</sup> targets

$$\delta u = 0.54^{+0.02}_{-0.02}, \delta d = -0.23^{+0.01}_{-0.01}$$





# Tensor charge

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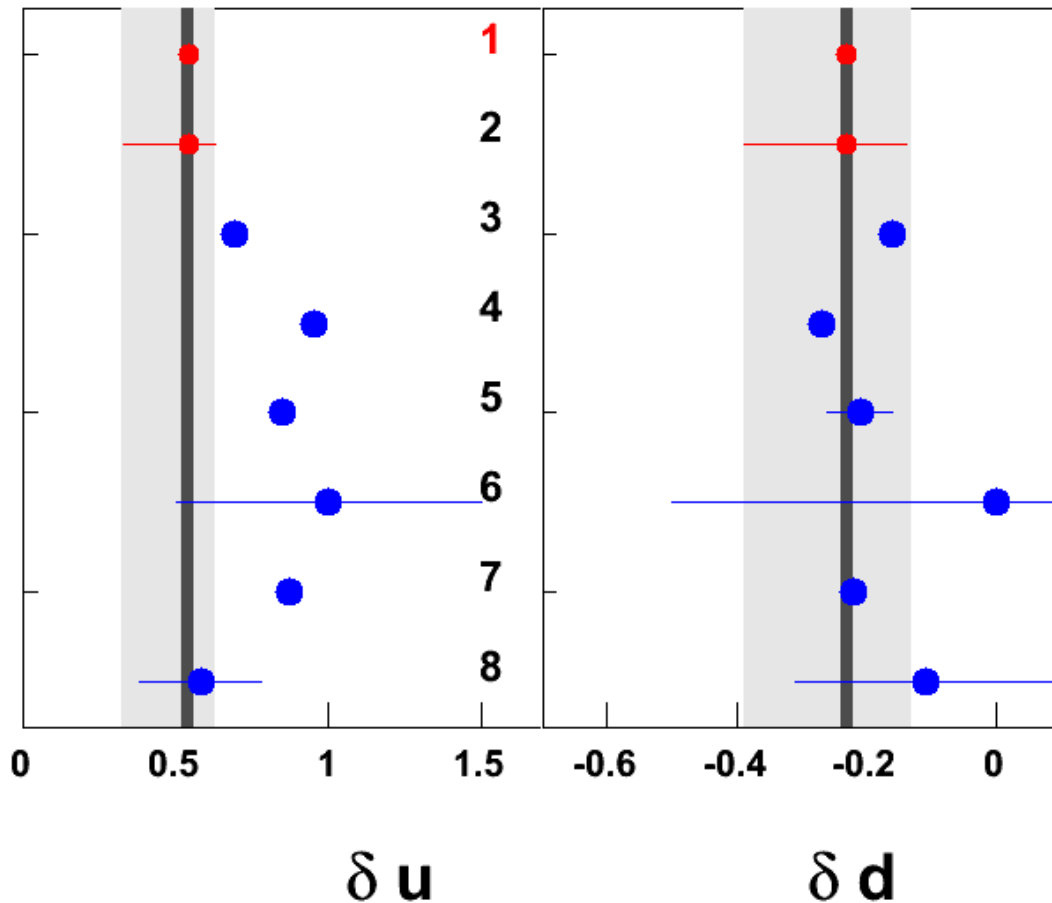
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## JLab 12 Proton and He<sup>3</sup> targets

$$\delta u = 0.54^{+0.02}_{-0.02}, \delta d = -0.23^{+0.01}_{-0.01}$$



*Enough precision  
to discriminate  
among models!*

# CONCLUSIONS

~~CONCLUSIONS~~

Message

# ~~CONCLUSIONS~~

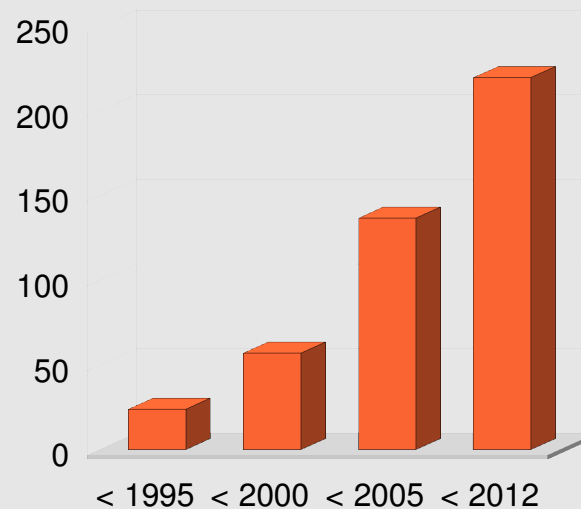
## Message

- Transversity has clear operator structure

$$\langle \bar{q} \gamma^+ \gamma^1 \gamma_5 q \rangle$$

- Transversity is equally important part of the nucleon structure as the other two leading twist distributions

- Title “transversity”  
in the literature



- JLab 12 promises unprecedented accuracy  $\pm 5\%$