CHIRAL PERTURBATION THEORY AND MESONS

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Joaquim (Ximo) Prades

Dedicated to

Ximo Prades 1963-2010

Friend and collaborator

Symposium in his memory, 23 May 2011

http://www.ugr.es/∼fteorica/Ximo/

Joaquim (Ximo) Prades

We have worked together on

- $g-2$
- $\Delta I = 1/2$
- B_K, $\varepsilon'_K/\varepsilon_K$
- Quark models and ENJL
- electromagnetic effects, . . .
- and were working on rare kaon decays and $g-2$.

Other contributions

- m_s and V_{us} from τ-decays
- Quark-hadron duality
- Higgs
- *sigma*, meson-baryon
- ...

Outline

Exploring the consequences of
the chiral symmetry of QCD
and its spontaneous breaking
using effective field theory techniques

Derivation from QCD:
H. Leutwyler,
On The Foundations Of Chiral Perturbation Theory,
Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

For lectures, review articles: see
http://www.thep.lu.se/∼bijnens/chpt.html

Chiral Perturbation Theory

A general Effective Field Theory:

- Relevant degrees of freedom
- A powercounting principle (predictivity)
- Has a certain range of validity

Chiral Perturbation Theory:

- Degrees of freedom: Goldstone Bosons from spontaneous breaking of chiral symmetry
- Powercounting: Dimensional counting in momenta/masses
- Breakdown scale: Resonances, so about M_ρ.

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Chiral Symmetry

QCD: n_F light quarks: equal mass: interchange: $SU(n_F)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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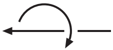
So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via

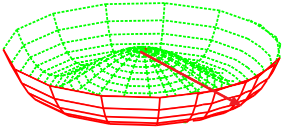
$v < c,\ m_q \neq 0 \Longrightarrow$

$v = c,\ m_q = 0 \not\Longrightarrow$



Goldstone Bosons

- $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
- $SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$
- 8 generators broken $\Longrightarrow$ 8 massless degrees of freedom and interaction vanishes at zero momentum
- Pictorially:

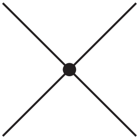


Need to pick a vacuum
$\langle \phi \rangle \neq 0$: Breaks symmetry
Massless mode along ridge

Goldstone Bosons

Power counting in momenta: Meson loops, Weinberg powercounting

rules

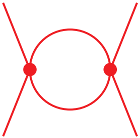


p^2

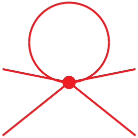
$1/p^2$

$\int d^4p$ p^4

one loop example



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$



$(p^2)\,(1/p^2)\,p^4 = p^4$

Chiral Pertubation Theories

- Which chiral symmetry: $SU(N_f)_L \times SU(N_f)_R$, for $N_f = 2, 3, \dots$ and extensions to (partially) quenched
- Or beyond QCD
- Space-time symmetry: Continuum or broken on the lattice: Wilson, staggered, mixed action
- Volume: Infinite, finite in space, finite T
- Which interactions to include beyond the strong one
- Which particles included as non Goldstone Bosons
- My general belief: if it involves soft pions (or soft K, η) some version of ChPT exists

Lagrangians: Lowest order

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\,\eta_8}{\sqrt{6}} \end{pmatrix}.$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger\rangle\}$,

$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$,
left and right external currents: $r(l)_\mu = v_\mu + (-) a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$ quark masses via scalar density: $s = \mathcal{M} + \cdots$

$\langle A \rangle = Tr_F\,(A)$

Lagrangians: NLO

$$\mathcal{L}_4 = L_1\langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2\langle D_\mu U^\dagger D_\nu U\rangle\langle D^\mu U^\dagger D^\nu U\rangle$$
$$+L_3\langle D^\mu U^\dagger D_\mu U D^\nu U^\dagger D_\nu U\rangle + L_4\langle D^\mu U^\dagger D_\mu U\rangle\langle \chi^\dagger U + \chi U^\dagger\rangle$$
$$+L_5\langle D^\mu U^\dagger D_\mu U(\chi^\dagger U + U^\dagger \chi)\rangle + L_6\langle \chi^\dagger U + \chi U^\dagger\rangle^2$$
$$+L_7\langle \chi^\dagger U - \chi U^\dagger\rangle^2 + L_8\langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger\rangle$$
$$-iL_9\langle F^R_{\mu\nu} D^\mu U D^\nu U^\dagger + F^L_{\mu\nu} D^\mu U^\dagger D^\nu U\rangle$$
$$+L_{10}\langle U^\dagger F^R_{\mu\nu} U F^{L\mu\nu}\rangle + H_1\langle F^R_{\mu\nu} F^{R\mu\nu} + F^L_{\mu\nu} F^{L\mu\nu}\rangle + H_2\langle \chi^\dagger \chi\rangle$$

L_i: Low-energy-constants (LECs)
H_i: Values depend on definition of currents/densities

These absorb the divergences of loop diagrams: $L_i \to L_i^r$
Renormalization: order by order in the powercounting

Lagrangians: Lagrangian structure

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

- All infinities known
- 3 flavour special case of 3+3 PQ: $\hat{L}_i^r, K_i^r \to L_i^r, C_i^r$
- Finite volume: no new LECs
- Other effects: (many) new LECs

Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars
- Chiral logarithms
- includes Isospin and the eightfold way ($SU(3)_V$)

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2}\log\frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu)\right] + \cdots$$

$M^2 = 2B\hat{m}$

$B \neq B_0$, $F \neq F_0$ (two versus three-flavour)

LECs and μ

$l_3^r(\mu)$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i}\, l_i^r(\mu) - \log\frac{M_\pi^2}{\mu^2}\,.$$

is independent of the scale μ.

For 3 and more flavours, some of the $\gamma_i = 0$: $L_i^r(\mu)$

Choice of μ :

- m_π, m_K: chiral logs vanish
- pick larger scale
- 1 GeV then $L_5^r(\mu) \approx 0$
  what about large N_c arguments????
- compromise: $\mu = m_\rho = 0.77$ GeV

Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exist
- Express orders in terms of physical masses and quantities (F_π, F_K)?
- Express orders in terms of lowest order masses?
- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in πK scattering
- Note: remaining μ dependence can occur at a given order
- Can make quite some difference in the expansion

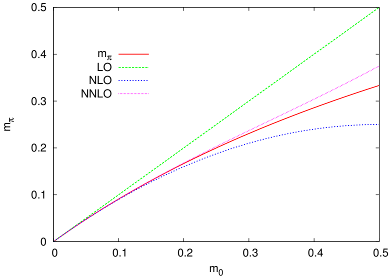
I prefer physical masses

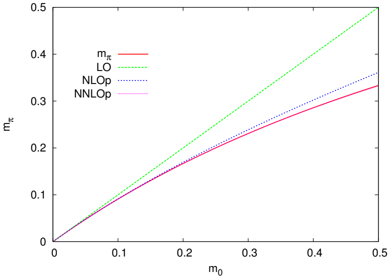
- Thresholds correct
- Chiral logs are from physical particles propagating

An example

$$m_\pi = \frac{m_0}{1 + a(m_0/f_0)} \qquad f_\pi = \frac{f_0}{1 + b(m_0/f_0)}$$

$$m_\pi = m_0 - a\frac{m_0^2}{f_0} + a^2\frac{m_0^3}{f_0^2} + \cdots \quad = m_0 - a\frac{m_\pi^2}{f_\pi} + a(b-a)\frac{m_\pi^3}{f_\pi^2} + \cdots$$





Example: $a = 1 \quad b = 0.5 \quad f_0 = 1$ convergence quite different

Two-loop calculations done

- Review paper on Two-Loops:

  JB, hep-ph/0604043 Prog. Part. Nucl. Phys. 58 (2007) 521
- $\eta \to 3\pi$

  JB, Ghorbani, JHEP 0711 (2007) 030 [arXiv:0709.0230]

  Plenary talk by Stefan Lanz
- $\pi^0 \to \gamma\gamma$

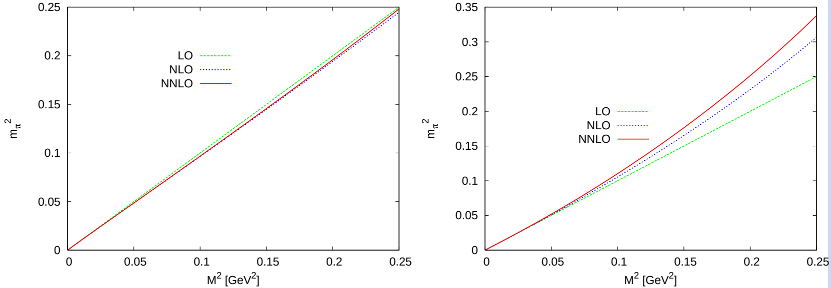
  Kampf, Moussallam, Phys.Rev. D79 (2009) 076005 [arXiv:0901.4688]
- $K_{\ell 3}$ isospin breaking due to $m_u - m_d$

  JB, Ghorbani, arXiv:0711.0148
- See also my talk in CD 2009

Two flavour LECs

- $\bar{l}_1$ to $\bar{l}_4$: ChPT at order p^6 and the Roy equation analysis in $\pi\pi$ and F_S Colangelo, Gasser and Leutwyler, *Nucl. Phys.* B 603 (2001) 125 [hep-ph/0103088] a related talk is G. Rios
- $\bar{l}_5$ and $\bar{l}_6$: from F_V and $\pi \to \ell\nu\gamma$ JB,(Colangelo,)Talavera and from $\Pi_V - \Pi_A$ González-Alonso, Pich, Prades
- $$\begin{aligned} \bar{l}_1 &= -0.4 \pm 0.6\,, & \bar{l}_2 &= 4.3 \pm 0.1\,,\\ \bar{l}_3 &= 2.9 \pm 2.4\,, & \bar{l}_4 &= 4.4 \pm 0.2\,,\\ \bar{l}_5 &= 12.24 \pm 0.21\,, & \bar{l}_6 - \bar{l}_5 &= 3.0 \pm 0.3\,,\\ \bar{l}_6 &= 16.0 \pm 0.5 \pm 0.7\,. \end{aligned}$$
- $l_7 \sim 5 \cdot 10^{-3}$ from π^0-η mixing Gasser, Leutwyler 1984
- Lattice: talks by Lellouch, Scholz, ...

A fitting caveat for chiral logs: m_π^2



Invisible $\bar{l}_3 = 2.9$ Visible $\bar{l}_3 = 2.9$

Three flavour LECs: uncertainties

- $m_K^2, m_\eta^2 \gg m_\pi^2$
- Contributions from p^6 Lagrangian are larger
- Reliance on estimates of the C_i much larger
- Typically: C_i^r: (terms with)
  kinematical dependence $\equiv$ measurable
  quark mass dependence $\equiv$ impossible (without lattice)
  100% correlated with L_i^r
- How suppressed are the $1/N_c$-suppressed terms?
- Are we really testing ChPT or just doing a phenomenological fit?

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Testing if ChPT works: relations

Yes: JB, Jemos, Eur.Phys.J. C64 (2009) 273-282 [arXiv:0906.3118]

Systematic search for relations between observables that do not depend on the C_i^r

Included:

- m_M^2 and F_M for π, K, η.
- 11 $\pi\pi$ threshold parameters
- 14 πK threshold parameters
- 6 $\eta \to 3\pi$ decay parameters,
- 10 observables in $K_{\ell 4}$
- 18 in the scalar formfactors
- 11 in the vectorformfactors
- Total: 76

We found 35 relations

Relations at NNLO: summary

- We did numerics for $\pi\pi$ (7), πK (5) and $K_{\ell 4}$ (1)
  13 relations
- $\pi\pi$: similar quality in two and three flavour ChPT
  The two involving a_3^- significantly did not work well
- πK: relation involving a_3^- not OK
  one more has very large NNLO corrections
- The relation with $K_{\ell 4}$ also did not work: related to that ChPT has trouble with curvature in $K_{\ell 4}$ talk by Stoffer
- Plot:
  - value of the loop part of the relation (C_i^r part $= 0$)
  - Normalization arbitrary
  - Large cancellations: sensitive to errors
  - Errors probably underestimated: correlations
- Conclusion: Three flavour ChPT "sort of" works

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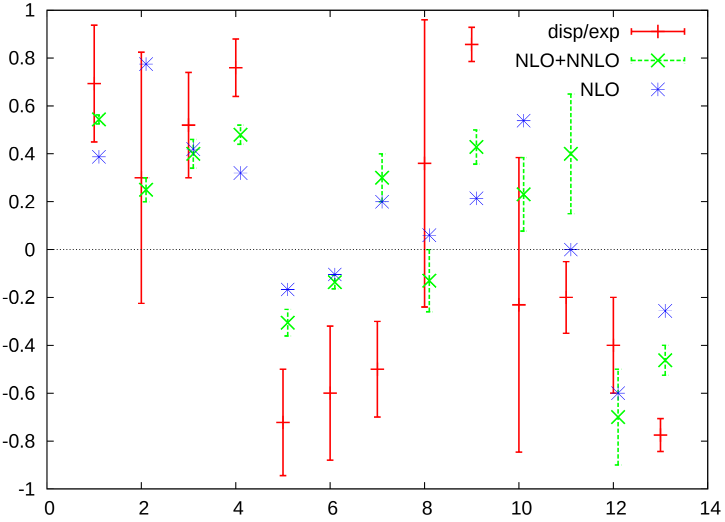


Fits: inputs

Main old determination of L_i^r: Amoros, JB Talavera 2001

$K_{\ell 4}$: $F(0)$, $G(0)$, λ_F, λ_G		E865 BNL $\Longrightarrow$ NA48
$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$		em with Dashen violation
F_{π^+}		$92.4 \Longrightarrow 92.2 \pm 0.05$ MeV
F_{K^+}/F_{π^+}	$1.22 \pm 0.01 \Longrightarrow 1.193 \pm 0.002 \pm 0.006 \pm 0.001$	
$m_s/\hat{m}$	24 (26) ($\Longrightarrow$ 27.8 Lattice)	L_4^r, L_6^r

Many more calculations done, especially $\pi\pi$ and F_S: include those as well

JB, Jemos, Nucl.Phys. B854 (2012) 631-665 [arXiv:1103.5945]

Main fit

	fit 10 iso	NA48/2	F_K/F_π	All $\star$	All
	old data			$\pi\pi$ πK $\langle r_S^2 \rangle$	$m_s/\hat{m}$
$10^3 L_1^r$	0.39 ± 0.12	0.88	0.87	0.89	0.88 ± 0.09
$10^3 L_2^r$	0.73 ± 0.12	0.79	0.80	0.63	0.61 ± 0.20
$10^3 L_3^r$	-2.34 ± 0.37	-3.11	-3.09	-3.06	-3.04 ± 0.43
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	0.60	0.75 ± 0.75
$10^3 L_5^r$	0.97 ± 0.11	0.91	0.73	0.58	0.58 ± 0.13
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	0.08	0.29 ± 0.85
$10^3 L_7^r$	-0.30 ± 0.15	-0.30	-0.26	-0.22	-0.11 ± 0.15
$10^3 L_8^r$	0.60 ± 0.20	0.59	0.49	0.40	0.18 ± 0.18
χ^2	0.26	0.01	0.01	1.20	1.28
dof	1	1	1	4	4

Main fit: variations

	All	$C_i^r \equiv 0$	All p^4	αC_i^r(CQMlike)
$10^3 L_1^r$	0.88 ± 0.09	0.65	1.12	0.66 ± 0.10
$10^3 L_2^r$	0.61 ± 0.20	0.11	1.23	0.24 ± 0.32
$10^3 L_3^r$	-3.04 ± 0.43	-1.47	-3.98	-1.80 ± 0.75
$10^3 L_4^r$	0.75 ± 0.75	0.80	1.50	0.77 ± 0.84
$10^3 L_5^r$	0.58 ± 0.13	0.68	1.21	0.83 ± 0.39
$10^3 L_6^r$	0.29 ± 0.85	0.29	1.17	0.32 ± 0.99
$10^3 L_7^r$	-0.11 ± 0.15	-0.14	-0.36	-0.15 ± 0.14
$10^3 L_8^r$	0.18 ± 0.18	0.19	0.62	0.27 ± 0.23
α	-	-	-	0.27 ± 0.47
χ^2	1.28	1.67	2.60	1.35
dof	4	4	4	3

Leaving μ free, fits it to $\mu = 0.71 \pm 31$ GeV

Some results of this fit

Mass:

$$
\begin{aligned}
m_\pi^2|_{p^2} &= 1.035 & m_\pi^2|_{p^4} &= -0.084 & m_\pi^2|_{p^6} &= +0.049\,,\\
m_K^2|_{p^2} &= 1.106 & m_K^2|_{p^4} &= -0.181 & m_K^2|_{p^6} &= +0.075\,,\\
m_\eta^2|_{p^2} &= 1.186 & m_\eta^2|_{p^4} &= -0.224 & m_\eta^2|_{p^6} &= +0.038\,,
\end{aligned}
$$

Decay constants:

$$
\begin{aligned}
\left.\frac{F_\pi}{F_0}\right|_{p^4} &= 0.311 & \left.\frac{F_\pi}{F_0}\right|_{p^6} &= 0.108\\
\left.\frac{F_K}{F_0}\right|_{p^4} &= 0.441 & \left.\frac{F_K}{F_0}\right|_{p^6} &= 0.216\,,\\
\left.\frac{F_K}{F_\pi}\right|_{p^4} &= 0.129 & \left.\frac{F_K}{F_\pi}\right|_{p^6} &= 0.068\,.
\end{aligned}
$$

A problem with $\bar{l}_2$

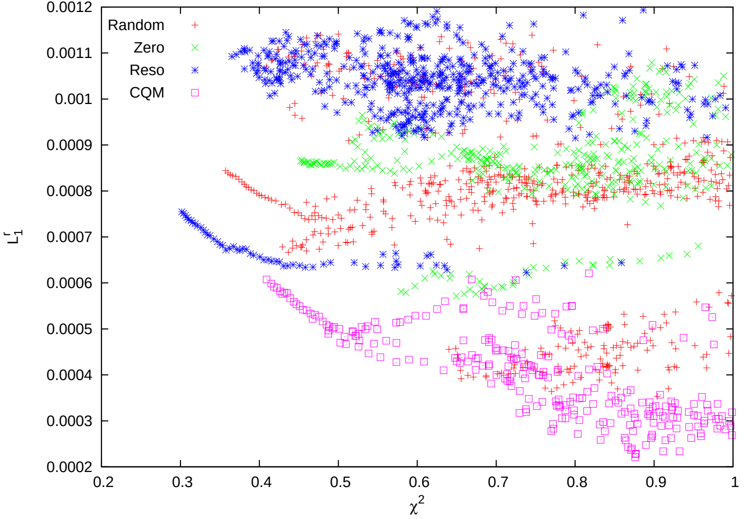
	fit 10 iso	All	"exp"
$\bar{\ell}_1$	$-0.6(0.5)$	$-0.1(1.1)$	-0.4 ± 0.6
$\bar{\ell}_2$	$5.7(4.9)$	$5.3(4.6)$	4.3 ± 0.1
$\bar{\ell}_3$	$1.3(2.9)$	$4.2(4.9)$	3.3 ± 0.7
$\bar{\ell}_4$	$4.0(4.1)$	$4.8(4.8)$	4.4 ± 0.4

- In brackets: p^4 relation between $\bar{l}_i$ and L_i^r
- $\bar{l}_2$ needs a $1/N_c$ suppressed C_i^r to work,: $2C_{13}^r - C_{11}^r$
- but then $\bar{l}_1$ gets off
- It goes to find "reasonable looking" C_i^r to get a fit but has several $1/N_c$ suppressed C_i^r nonzero
- Getting a low χ^2 is no problem with different L_r^i

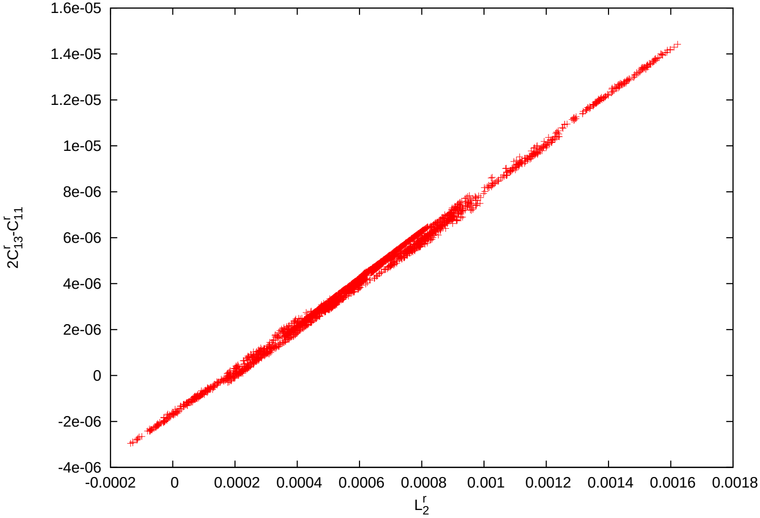
An example

- We start from choosing a starting set of C_i^r: 0, resonance or CQMlike
- Random start point usually bad fit
- Do a random walk in C_i^r space with steps size in $1/N_c$ suppressed directions 1/3 of leading in N_c directions
- refit L_r^i
- accept step with a Metropolis type acceptance on the χ^2
- Lots of fits with good χ^2

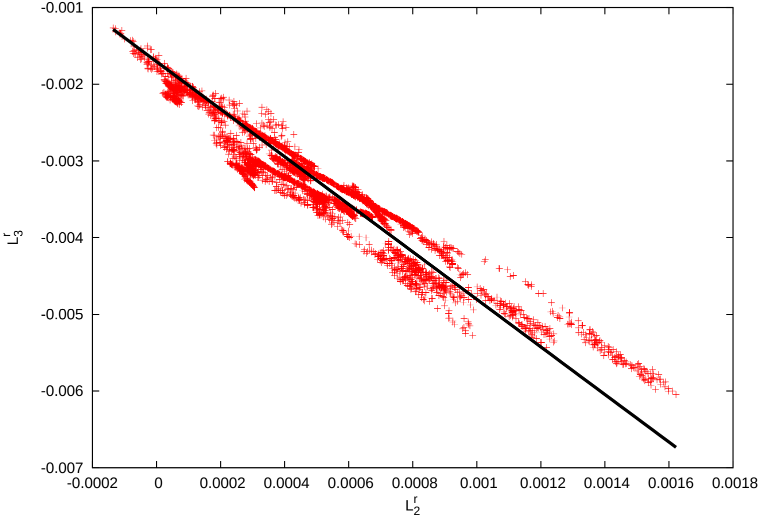
An example: value of L_1^r



An example: correlation L_2^r and $2C_{13}^r - C_{11}^r$ ($\bar{l}_2$)



An example: correlation L_2^r and L_3^r



Curve: $L_3^r = -3.1\,(L_2^r + 0.00055)$ (to guide the eye)

Need more information

- Need extra input for L_4^r, L_6^r
- Some preliminary fits using lattice have been done (by "continuum people")
  - G. Ecker, P. Masjuan, H. Neufeld, Phys.Lett. B692 (2010) 184-188, arXiv:1004.3422
  - V. Bernard, E. Passemar, JHEP 1004 (2010) 001 [arXiv:0912.3792]
- FLAG report Eur.Phys.J. C71 (2011) 1695 [arXiv:1011.4408]
- Lattice: many more talks here
- Reminder:
  - just ask me for the program for 2-loop (partially quenched) programs
  - Working on a C++ version of the programs
  - exists for isospin limit, expansion in physical masses but I never find the time to write the manual
- For now: fit ALL standard values especially for L_1^r, L_2^r, L_3^r.

Hard pion ChPT?

- In Meson ChPT: the powercounting is from all lines in Feynman diagrams having soft momenta
- thus powercounting = (naive) dimensional counting
- Baryon and Heavy Meson ChPT: $p, n, \ldots\ B, B^*$ or D, D^*
  - $p = M_B v + k$
  - Everything else soft
  - Works because baryon or b or c number conserved so the non soft line is continuous

  - Decay constant works: takes away all heavy momentum
  - General idea: M_p dependence can always be reabsorbed in LECs, is analytic in the other parts k.

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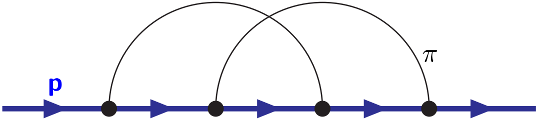
- thus powercounting = (naive) dimensional counting
- Baryon and Heavy Meson ChPT: $p, n, \dots\ B, B^*$ or D, D^*
  - $p = M_B v + k$
  - Everything else soft
  - Works because baryon or b or c number conserved so the non soft line is continuous



  - Decay constant works: takes away all heavy momentum
  - General idea: M_p dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon goes through
  - Applied to masses and πK scattering and decay constant Roessl, Allton et al., ...
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
  - Works like all the previous *heavy* ChPT
- Flynn-Sachrajda argued $K_{\ell 3}$ also for q^2 away from q^2_{max}.
- JB-Celis Argument generalizes to other processes with hard/fast pions and applied to $K \to \pi\pi$
- JB Jemos $B, D \to D, \pi, K, \eta$ vector formfactors, charmonium decays and a two-loop check
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Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle\pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

- Nothing prevents hard pions to be in the states α or β
- So by heavily using current algebra I should be able to get the light quark mass nonanalytic dependence

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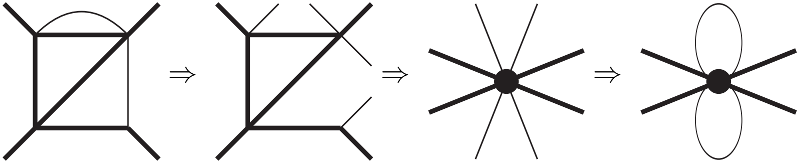
Hard pion ChPT?

Field Theory: a process at given external momenta

- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describably by an effective Lagrangian with coupling constants dependent on the external given momenta (Weinberg's folklore theorem)
- Lagrangian in hadron fields with all orders of derivatives



Hard pion ChPT?

- This effective Lagrangian as a Lagrangian in hadron fields but all possible orders of the momenta included: possibly an infinite number of terms
- If symmetries present, Lagrangian should respect them
- but my powercounting is gone
- In some cases we can argue that up to a certain order in the expansion in light masses, not momenta, matrix elements of higher order operators are reducible to those of lowest order.
- Lagrangian should be complete in *neighbourhood* of original process
- Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses
  Crucial part of the argument

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Hard Pion ChPT: A two-loop check

- Arguments work for the (2-flavour) pion vector and scalar formfactor JB-Jemos
- Therefore at any t the chiral log correction must go like the one-loop calculation.
- The one-loop log chiral log is with $t >> m_\pi^2$
- Predicts
$$F_V(t,M^2) = F_V(t,0)\left(1 - \frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$
$$F_S(t,M^2) = F_S(t,0)\left(1 - \frac{5}{2}\frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$
- $F_{V,S}(t,0)$ is now a coupling constant and can be complex

A two-loop check

- Two-loop ChPT is known and valid for $t, m_\pi^2 \ll \Lambda_\chi^2$
- expand in $t >> m_\pi^2$:

$$F_V(t,M^2) = F_V(t,0)\left(1 - \frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$
$$F_S(t,M^2) = F_S(t,0)\left(1 - \frac{5}{2}\frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

with

$$F_V(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(\frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6}\ln\frac{t}{\mu^2}\right)$$
$$F_S(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(1 + 16\pi^2 l_4^r + i\pi - \ln\frac{t}{\mu^2}\right)$$

- The needed coupling constants are complex
- Both calculations have two-loop diagrams with overlapping divergences
- The chiral logs should be valid for any t where a pointlike interaction is a valid approximation

$B, D \to \pi, K, \eta$

JB,Jemos

$$\langle P_f(p_f)\,|\overline{q}_i\gamma_\mu q_f|\,P_i(p_i)\rangle = (p_i+p_f)_\mu f_+(q^2) + (p_i-p_f)_\mu f_-(q^2)$$

$$f_{+B\to M}(t) = f^\chi_{+B\to M}(t) F_{B\to M}$$
$$f_{-B\to M}(t) = f^\chi_{-B\to M}(t) F_{B\to M}$$

- $F_{B\to M}$ is always the same for f_+, f_- and f_0
- This is not heavy quark symmetry: not valid at endpoint and valid also for $K \to \pi$.
- Not like Low's theorem, depends on more than just the external legs
- LEET: in this limit the two formfactors are related
  J. Charles et al, hep-ph/9812358

$B, D \to \pi, K, \eta$

$$
\begin{aligned}
F_{K\to\pi} &= 1 + \frac{3}{8F^2}\overline{A}(m_\pi^2) \qquad (2-\text{flavour})\\
F_{B\to\pi} &= 1 + \left(\frac{3}{8}+\frac{9}{8}g^2\right)\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4}+\frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24}+\frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},\\
F_{B\to K} &= 1 + \frac{9}{8}g^2\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{2}+\frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{6}+\frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},\\
F_{B\to\eta} &= 1 + \left(\frac{3}{8}+\frac{9}{8}g^2\right)\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4}+\frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24}+\frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},\\
F_{B_s\to K} &= 1 + \frac{3}{8}\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4}+\frac{3}{2}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24}+\frac{1}{2}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},\\
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\end{aligned}
$$

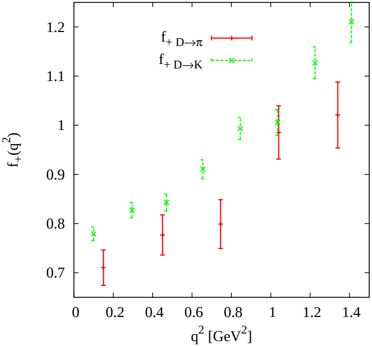
$F_{B_s\to\pi}$ vanishes due to the possible flavour quantum numbers.

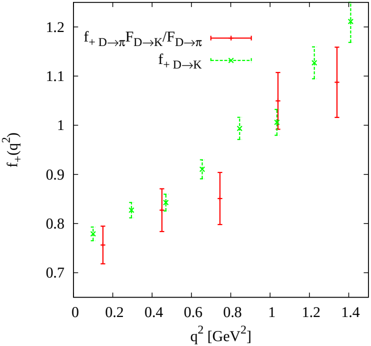
Note: $F_{B\to\pi} = F_{B\to\eta}$

$\overline{A}(M^2) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}$

Experimental check

CLEO data on $f_+(q^2)|V_{cq}|$ for $D \to \pi$ and $D \to K$ with $|V_{cd}| = 0.2253$, $|V_{cs}| = 0.9743$





$$f_{+D\to\pi} = f_{+D\to K} F_{D\to\pi}/F_{D\to K}$$

Applications to charmonium

- We look at decays $\chi_{c0}, \chi_{c2} \to \pi\pi, KK, \eta\eta$
- $J/\psi, \psi(nS), \chi_{c1}$ decays to the same final state break isospin or U-spin or V-spin, they thus proceed via electromagnetism or quark mass differences: more difficult.
- So construct a Lagrangian with a chiral singlet scalar and tensor field.
- $\mathcal{L}_{\chi_c} = E_1 F_0^2\,\chi_0\,\langle u^\mu u_\mu\rangle + E_2 F_0^2\,\chi_2^{\mu\nu}\,\langle u_\mu u_\nu\rangle\,.$
- No chiral logarithm corrections
- Expanding the energy-momentum tensor result Donoghue-Leutwyler at large q^2 agrees.
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Charmonium

- Phase space correction: $|\vec{p}_1| = \sqrt{m_\chi^2 - 4m_P^2}/2$.
- χ_{c0}:
  - $A \propto p_1 \cdot p_2 = (m_\chi^2 - 2m_P^2)/2$.
  - $\Longrightarrow$ $\boxed{G_0 = \sqrt{BR/|\vec{p}_1|}/(p_1.p_2)}$.
- χ_{c2}:
  - $A \propto T_\chi^{\mu\nu} p_{1\mu} p_{2\nu}$. (polarization tensor)
  - $|A|^2 \propto \frac{1}{5}\sum_{pol} T_\chi^{\mu\nu} p_{1\mu} p_{2\nu} T_\chi^{*\alpha\beta} p_{1\alpha} p_{2\beta} = \frac{1}{30}\left(m_\chi^2 - 4m_P^2\right)^2 \propto |\vec{p}_1|^4$.
  - $\Longrightarrow$ $\boxed{G_2 = \sqrt{BR/|\vec{p}_1|}/|\vec{p}_1|^2}$.
- $\times 2$ for $K_S^0 K_S^0$ to $K^0\overline{K^0}$, $\times 2/3$ for $\pi\pi$ to $\pi^+\pi^-$.

Charmonium

	χ_{c0}		χ_{c2}	
Mass Width	3414.75 $\pm$ 0.31 MeV 10.4 $\pm$ 0.6 MeV		3556.20 $\pm$ 0.09 MeV 1.97 $\pm$ 0.11 MeV	
Final state	10^3 BR	10^{10} $G_0[\mathrm{MeV}^{-5/2}]$	10^3 BR	10^{10} $G_2[\mathrm{MeV}^{-5/2}]$
$\pi\pi$	8.5 $\pm$ 0.4	3.15 $\pm$ 0.07	2.42 $\pm$ 0.13	3.04 $\pm$ 0.08
K^+K^-	6.06 $\pm$ 0.35	3.45 $\pm$ 0.10	1.09 $\pm$ 0.08	2.74 $\pm$ 0.10
$K_S^0 K_S^0$	3.15 $\pm$ 0.18	3.52 $\pm$ 0.10	0.58 $\pm$ 0.05	2.83 $\pm$ 0.12
$\eta\eta$	3.03 $\pm$ 0.21	2.48 $\pm$ 0.09	0.59 $\pm$ 0.05	2.06 $\pm$ 0.09
$\eta'\eta'$	2.02 $\pm$ 0.22	2.43 $\pm$ 0.13	$<$ 0.11	$<$ 1.2

Experimental results for $\chi_{c0}, \chi_{c2} \to PP$ and the factors corrected for the known m^2 effects.

- $\pi\pi$ and KK are good to 10% (Note: 20% for F_K/F_π)
- $\eta\eta$ OK

Summary of HPChPT

Why is this useful:

- Lattice works actually around the strange quark mass
- need only extrapolate in m_u and m_d.
- Applicable in momentum regimes where usual ChPT might not work
- Three flavour case useful for B, D, χ_c decays
- tells us something nontrivial about otherwise very difficult quantities

A cloud on the horizon?

- work by G.Colangelo, M. Procura, L. Rothen, R. Stucki, J. Tarrús Castellà
- talk by M. Procura this afternoon
- This info taken from talk by R. Stucki in QNP Paris
- Do a dispersive analysis of the pion form formfactor
- The two-body cut lives up to the prediction to all orders
- The four-body cut gives a contribution that does not live up to it

What now?

- The arguments for the general method are the same as for IR divergences, SCET,. . .
- $\Longrightarrow$ I believe something like HPChPT should exist
- The arguments for the proportionality to the lowest order are much weaker
  - Assumes each soft propagator has a free momentum
  - Lowest order is tricky: F_S
    $\langle \chi_+ \rangle$ and $\langle \chi_+ \rangle \langle u_\mu u^\mu \rangle$: different lowest order terms
- Some naive thoughts:
  - Does the ChPT dispersion need more subtraction constants?
  - The full calculation at 3 loops will be very difficult
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QCDlike and/or technicolor theories

- One can also have different symmetry breaking patterns from underlying fermions
- Three generic cases
  - $SU(N) \times SU(N)/SU(N)$
  - $SU(2N)/SO(2N)$
  - $SU(2N)/Sp(2N)$
- Many one-loop results existed especially for the first case (several discovered only after we published our work)
- Equal mass case pushed to two loops JB, Lu, 2009-11
- Related talks: Ruiz-Femena, Rosell, Buchoff

N_F fermions in a representation of the gauge group

- complex (QCD):
  - $q^T = (q_1\ q_2 \dots\ q_{N_F})$
  - Global $G = SU(N_F)_L \times SU(N_F)_R$
    $q_L \to g_L q_L$ and $g_R \to g_R q_R$
  - Vacuum condensate $\Sigma_{ij} = \langle \overline{q}_j q_i \rangle \propto \delta_{ij}$
  - $g_L = g_R$ then $\Sigma_{ij} \to \Sigma_{ij} \Longrightarrow$ conserved $H = SU(N_F)_V$:
- Real (e.g. adjoint): $\hat{q}^T = (q_{R1}\ \dots\ q_{RN_F}\ \tilde{q}_{R1}\ \dots\ \tilde{q}_{RN_F})$
  - $\tilde{q}_{Ri} \equiv C\overline{q}_{Li}^T$ goes under gauge group as q_{Ri}
  - some Goldstone bosons have baryonnumber
  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - $\langle \overline{q}_j q_i \rangle$ is really $\langle (\hat{q}_j)^T C \hat{q}_i \rangle \propto J_{Sij}$ $J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
  - Conserved if $g J_S g^T = J_S \Longrightarrow H = SO(2N_F)$

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- Pseudoreal (e.g. two-colours): $\hat{q}^T = (q_{R1}\ \ldots\ q_{RN_F}\ \tilde{q}_{R1}\ \ldots\ \tilde{q}_{RN_F})$
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  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - $\langle \overline{q}_j q_i \rangle$ is really $\epsilon_{\alpha\beta} \langle (\hat{q}_{\alpha j})^T C \hat{q}_{\beta i} \rangle \propto J_{Aij}$ $J_A = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$
  - Conserved if $g J_A g^T = J_A \Longrightarrow H = Sp(2N_F)$

Lagrangians

JB, Lu, arXiv:0910.5424: 3 cases similar with $u = \exp\left(\frac{i}{\sqrt{2}F}\phi^a X^a\right)$

But the matrices X^a are:

- Complex or $SU(N) \times SU(N)/SU(N)$:
  all $SU(N)$ generators
- Real or $SU(2N)/SO(2N)$:
  $SU(2N)$ generators with $X^a J_S = J_S X^{aT}$
- Pseudoreal or $SU(2N)/Sp(2N)$:
  $SU(2N)$ generators with $X^a J_A = J_A X^{aT}$
- Note that the latter are not the usual ways of parametrizing $SO(2N)$ and $Sp(2N)$ matrices

The main useful formulae

Calculating for equal mass case goes through using:

$$
\begin{aligned}
\text{Complex}: \quad & \langle X^a A X^a B\rangle = \langle A\rangle\langle B\rangle - \frac{1}{N_F}\langle AB\rangle\,,\\
& \langle X^a A\rangle\langle X^a B\rangle = \langle AB\rangle - \frac{1}{N_F}\langle A\rangle\langle B\rangle\,.\\
\text{Real}: \quad & \langle X^a A X^a B\rangle = \frac{1}{2}\langle A\rangle\langle B\rangle + \frac{1}{2}\left\langle A J_S B^T J_S\right\rangle - \frac{1}{2N_F}\langle AB\rangle\,,\\
& \langle X^a A\rangle\langle X^a B\rangle = \frac{1}{2}\langle AB\rangle + \frac{1}{2}\left\langle A J_S B^T J_S\right\rangle - \frac{1}{2N_F}\langle A\rangle\langle B\rangle\,.\\
\text{Pseudoreal}: \quad & \langle X^a A X^a B\rangle = \frac{1}{2}\langle A\rangle\langle B\rangle + \frac{1}{2}\left\langle A J_A B^T J_A\right\rangle - \frac{1}{2N_F}\langle AB\rangle\,,\\
& \langle X^a A\rangle\langle X^a B\rangle = \frac{1}{2}\langle AB\rangle - \frac{1}{2}\left\langle A J_A B^T J_A\right\rangle - \frac{1}{2N_F}\langle A\rangle\langle B\rangle
\end{aligned}
$$

So can do the calculations for all cases

Vacuum expectation value

All cases: $\langle \overline{q}q \rangle_{\rm LO} \equiv \sum_{i=1,N_F} \langle \overline{q}_{Ri} q_{Li} + \overline{q}_{Li} q_{Ri} \rangle_{\rm LO} = -N_F B_0 F^2$

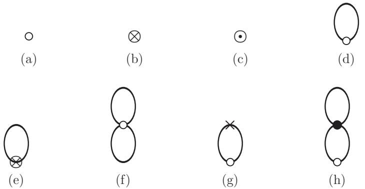
$M^2 = 2B_0\hat{m}$ and $\overline{A}(M^2) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}$.

$\langle \overline{q}q \rangle = \langle \overline{q}q \rangle_{\rm LO} + \langle \overline{q}q \rangle_{\rm NLO} + \langle \overline{q}q \rangle_{\rm NNLO}$.

$$\langle \overline{q}q \rangle_{\rm NLO} = \langle \overline{q}q \rangle_{\rm LO}\left(a_V \frac{\overline{A}(M^2)}{F^2} + b_V \frac{M^2}{F^2}\right) ,$$

$$\langle \overline{q}q \rangle_{\rm NNLO} = \langle \overline{q}q \rangle_{\rm LO}\left(c_V \frac{\overline{A}(M^2)^2}{F^4} + \frac{M^2\overline{A}(M^2)}{F^4}\left(d_V + \frac{e_V}{16\pi^2}\right) + \frac{M^4}{F^4}\left(f_V + \frac{g_V}{16\pi^2}\right)\right) .$$

Diagrams:



Vacuum expectation value

	QCD	
a_V	$n - \frac{1}{n}$	
b_V	$16nL_6^r + 8L_8^r + 4H_2^r$	
c_V	$\frac{3}{2}\left(-1 + \frac{1}{n^2}\right)$	
d_V	$-24\left(n^2 - 1\right)\left(L_A + \frac{1}{n}L_B\right)$	$L_A = L_4^r - 2L_6^r$
e_V	$1 - \frac{1}{n^2}$	$L_B = L_5^r - 2L_8^r$
f_V	$48\left(K_{25}^r + nK_{26}^r + n^2K_{27}^r\right)$	
g_V	$8\left(n^2 - 1\right)\left(L_A + \frac{1}{n}L_B\right)$	
	Adjoint	2-colour
a_V	$n + \frac{1}{2} - \frac{1}{2n}$	$n - \frac{1}{2} - \frac{1}{2n}$
b_V	$32nL_6^r + 8L_8^r + 4H_2^r$	$32nL_6^r + 8L_8^r + 4H_2^r$
c_V	$\frac{3}{8}\left(-1 + \frac{1}{n^2} - \frac{2}{n} + 2n\right)$	$\frac{3}{8}\left(-1 + \frac{1}{n^2} + \frac{2}{n} - 2n\right)$
d_V	$-12\left(2n^2 + n - 1\right)\left(2L_A + \frac{1}{n}L_B\right)$	$-12\left(2n^2 - n - 1\right)\left(2L_A + \frac{1}{n}L_B\right)$
e_V	$\frac{1}{4}\left(1 - \frac{1}{n^2} + \frac{2}{n} - 2n\right)$	$\frac{1}{4}\left(1 - \frac{1}{n^2} - \frac{2}{n} + 2n\right)$
f_V	r_{VA}^r	r_{VT}^r
g_V	$4\left(2n^2 + n - 1\right)\left(2L_A + \frac{1}{n}L_B\right)$	$4\left(2n^2 - n - 1\right)\left(2L_A + \frac{1}{n}L_B\right)$

Note: relations in the large n limit.

$\phi\phi \to \phi\phi$

- $\pi\pi$ scattering
  - Amplitude in terms of $A(s,t,u)$

    $M_{\pi\pi}(s,t,u) = \delta^{ab}\delta^{cd}A(s,t,u) + \delta^{ac}\delta^{bd}A(t,u,s) + \delta^{ad}\delta^{bc}A(u,s,t)\,.$
  - Three intermediate states $I = 0, 1, 2$
- Our three cases
  - Two amplitudes needed $B(s,t,u)$ and $C(s,t,u)$

$$
\begin{aligned}
M(s,t,u) = {} & \left[\left\langle X^aX^bX^cX^d\right\rangle + \left\langle X^aX^dX^cX^b\right\rangle\right] B(s,t,u) \\
& + \left[\left\langle X^aX^cX^dX^b\right\rangle + \left\langle X^aX^bX^dX^c\right\rangle\right] B(t,u,s) \\
& + \left[\left\langle X^aX^dX^bX^c\right\rangle + \left\langle X^aX^cX^bX^d\right\rangle\right] B(u,s,t) \\
& + \delta^{ab}\delta^{cd}C(s,t,u) + \delta^{ac}\delta^{bd}C(t,u,s) + \delta^{ad}\delta^{bc}C(u,s,t)\,.
\end{aligned}
$$

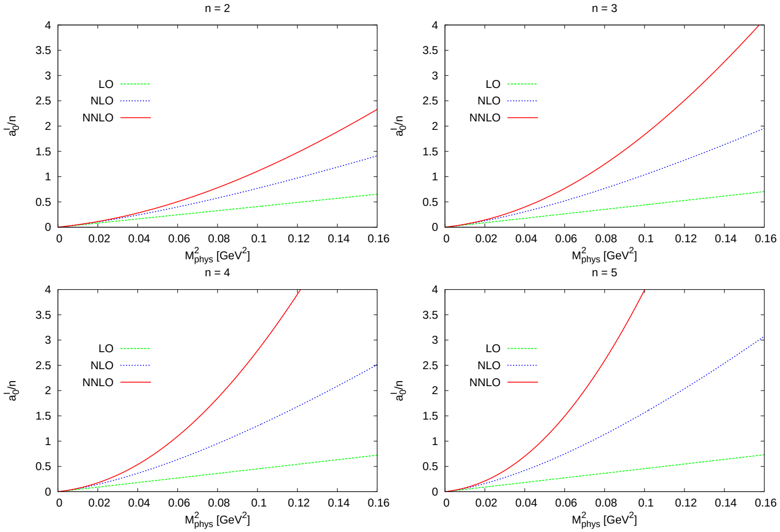
$B(s,t,u) = B(u,t,s) \qquad C(s,t,u) = C(s,u,t)\,.$
  - 7, 6 and 6 possible intermediate states
- All formulas similar length to $\pi\pi$ cases but there are so many of them

LUND UNIVERSITY

Chiral Perturbation Theory and Mesons

Johan Bijnens

$\phi\phi \to \phi\phi$: a_0^I/n



Conclusions for “Beyond QCD”

Calculations done:

- $M^2_{\rm phys}$
- $F_{\rm phys}$
- Meson-meson scattering
- Equal mass case: allows to get fully analytical result just as for 2-flavour ChPT
- Two-point functions relevant for S-parameter
  JB, Lu, arXiv:1102.0172
- Note: large N_F here not cactus but planar diagrams (in flavour lines)

To remember:

- Different symmetry patterns can appear for different gaugegroups and fermion representations
- Nonperturbative: lattice needs extrapolation formulae

Leading Logarithms

- More details: talk by Kampf this afternoon
- Take a quantity with a single scale: $F(M)$
- The dependence on the scale in field theory is typically logarithmic
- $L = \log(\mu/M)$
- $F = F_0 + F_1^1 L + F_0^1 + F_2^2 L^2 + F_1^2 L + F_0^2 + F_3^3 L^3 + \cdots$
- Leading Logarithms: The terms $F_m^m L^m$

The F_m^m can be more easily calculated than the full result

- $\mu\,(dF/d\mu) \equiv 0$
- Ultraviolet divergences in Quantum Field Theory are always local

Renormalizable theories

- Loop expansion $\equiv \alpha$ expansion
- $F = \alpha + f_1^1\alpha^2 L + f_0^1\alpha^2 + f_2^2\alpha^3 L^2 + f_1^2\alpha^3 L + f_0^2\alpha^3 + f_3^3\alpha^4 L^3 + \cdots$
- f_i^j are pure numbers
- $\mu\frac{d\alpha}{d\mu} = \beta_0\alpha^2 + \beta_1\alpha^3 + \cdots$
- $\mu\frac{dF}{d\mu} = 0 \Longrightarrow \boxed{\beta_0 = -f_1^1 = f_2^2 = -f_3^3 = \cdots}$
- Relies on the α the same in all orders
- In effective field theories: different Lagrangian at each order
- The recursive argument does not work

Renormalizable theories

- Loop expansion $\equiv$ α expansion
- $F = \alpha + f_1^1\alpha^2 L + f_0^1\alpha^2 + f_2^2\alpha^3 L^2 + f_1^2\alpha^3 L + f_0^2\alpha^3 + f_3^3\alpha^4 L^3 + \cdots$
- f_i^j are pure numbers
- $\mu\dfrac{d\alpha}{d\mu} = \beta_0\alpha^2 + \beta_1\alpha^3 + \cdots$
- $\mu\dfrac{dF}{d\mu} = 0 \Longrightarrow \boxed{\beta_0 = -f_1^1 = f_2^2 = -f_3^3 = \cdots}$
- Relies on the α the same in all orders
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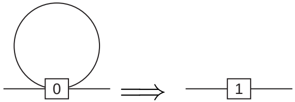
Renormalizable theories

- Loop expansion $\equiv \alpha$ expansion
- $F =$
  $\alpha + f_1^1 \alpha^2 L + f_0^1 \alpha^2 + f_2^2 \alpha^3 L^2 + f_1^2 \alpha^3 L + f_0^2 \alpha^3 + f_3^3 \alpha^4 L^3 + \cdots$
- f_i^j are pure numbers
- $\mu \dfrac{d\alpha}{d\mu} = \beta_0 \alpha^2 + \beta_1 \alpha^3 + \cdots$
- $\mu \dfrac{dF}{d\mu} = 0 \Longrightarrow \boxed{\beta_0 = -f_1^1 = f_2^2 = -f_3^3 = \cdots}$
- Relies on the α the same in all orders
- In effective field theories: different Lagrangian at each order
- The recursive argument does not work

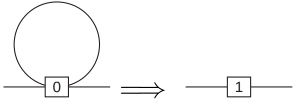
Weinberg's argument

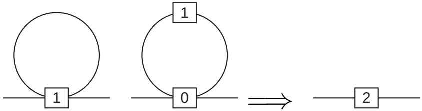
- Weinberg, Physica A96 (1979) 327
- Two-loop leading logarithms can be calculated using only one-loop: Weinberg consistency conditions
- Proof at all orders:
  - using β-functions: Büchler, Colangelo, hep-ph/0309049
  - Proof with diagrams: JB, Carloni, arXiv:0909.5086
- Proof relies on
  - μ: dimensional regularization scale
  - $d = 4 - w$
  - at n-loop order ($\hbar^n$) must cancel:
    - $1/w^n$, $\log \mu/w^{n-1}, \ldots, \log^{n-1} \mu/w$
    - This allows for relations between diagrams
    - All needed for $\log^n \mu$ coefficient can be calculated from one-loop diagrams

Mass to $\hbar^2$

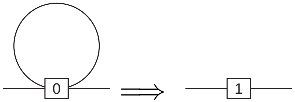
- $\hbar^1$: 

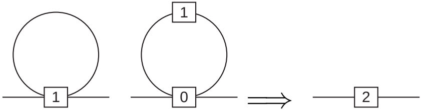
Mass to $\hbar^2$

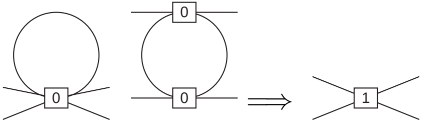
- $\hbar^1$: 

- $\hbar^2$: 

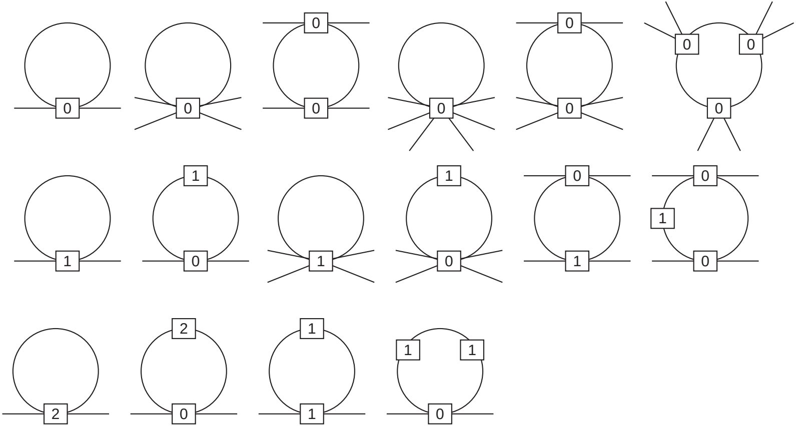
Mass to $\hbar^2$

- $\hbar^1$: 

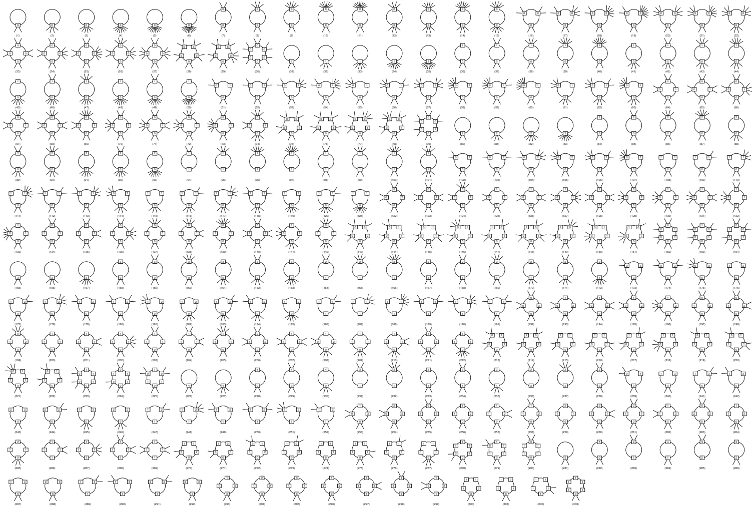
- $\hbar^2$: 

- but also needs $\hbar^1$: 

Mass to order $\hbar^3$



Mass to order $\hbar^6$



General

- Calculate the divergence
- rewrite it in terms of a local Lagrangian
  - Luckily: symmetry kept: we know result will be symmetrical, hence do not need to explicitly rewrite the Lagrangians in a nice form
  - Luckily: we do not need to go to a minimal Lagrangian
  - So everything can be computerized
- We keep all terms to have all 1PI (one particle irreducible) diagrams finite

Massive $O(N)$ sigma model

- $O(N+1)/O(N)$ nonlinear sigma model
- $\mathcal{L}_{n\sigma} = \frac{F^2}{2}\partial_\mu\Phi^T\partial^\mu\Phi + F^2\chi^T\Phi\,.$
  - Φ is a real $N+1$ vector; $\Phi \to O\Phi$; $\Phi^T\Phi = 1$.
  - Vacuum expectation value $\langle\Phi^T\rangle = (1\ 0\ldots0)$
  - Explicit symmetry breaking: $\chi^T = \left(M^2\ 0\ldots0\right)$
  - Both spontaneous and explicit symmetry breaking
  - N-vector ϕ
- N (pseudo-)Nambu-Goldstone Bosons
- $N = 3$ is two-flavour Chiral Perturbation Theory

Massive $O(N)$ sigma model: Φ vs ϕ

- $$\Phi_1 = \begin{pmatrix} \sqrt{1-\frac{\phi^T\phi}{F^2}} \\ \frac{\phi^1}{F} \\ \vdots \\ \frac{\phi^N}{F} \end{pmatrix} = \begin{pmatrix} \sqrt{1-\frac{\phi^T\phi}{F^2}} \\ \frac{\phi}{F} \end{pmatrix}$$ Gasser, Leutwyler

- $$\Phi_2 = \frac{1}{\sqrt{1+\frac{\phi^T\phi}{F^2}}}\begin{pmatrix} 1 \\ \frac{\phi}{F} \end{pmatrix} \qquad \Phi_3 = \begin{pmatrix} 1-\frac{1}{2}\frac{\phi^T\phi}{F^2} \\ \sqrt{1-\frac{1}{4}\frac{\phi^T\phi}{F^2}}\,\frac{\phi}{F} \end{pmatrix}$$ only mass term

- $$\Phi_4 = \begin{pmatrix} \cos\sqrt{\frac{\phi^T\phi}{F^2}} \\ \sin\sqrt{\frac{\phi^T\phi}{F^2}}\,\frac{\phi}{\sqrt{\phi^T\phi}} \end{pmatrix} \qquad \Phi_5 = \frac{1}{1+\frac{\phi^T\phi}{4F^2}}\begin{pmatrix} 1-\frac{\phi^T\phi}{4F^2} \\ \frac{\phi}{F} \end{pmatrix}$$ CCWZ, Weinberg

Massive $O(N)$ sigma model: Checks

Need (many) checks:

- use the five different parametrizations
- compare with known results:

$$M^2_{phys} = M^2 \left(1 - \frac{1}{2} L_M + \frac{17}{8} L_M^2 + \cdots \right) ,$$

$$L_M = \frac{M^2}{16\pi^2 F^2} \log \frac{\mu^2}{\mathcal{M}^2}$$

  Usual choice $\mathcal{M} = M$.

- large N (but known results only for massless case)
  Coleman, Jackiw, Politzer 1974
- large N massive later found partly in appendix of Kivel, Polyakov, Vladimirov on distribution functions.

Results

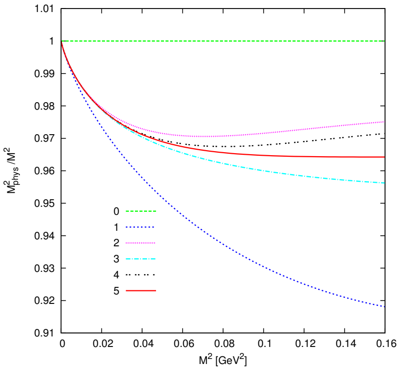
- $M^2_{\text{phys}} = M^2(1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \ldots)$

  $L_M = \frac{M^2}{16\pi^2 F^2} \log \frac{\mu^2}{M^2}$

i	a_i, $N = 3$	a_i for general N
1	$-\frac{1}{2}$	$1 - \frac{N}{2}$
2	$\frac{17}{8}$	$\frac{7}{4} - \frac{7N}{4} + \frac{5\ N^2}{8}$
3	$-\frac{103}{24}$	$\frac{37}{12} - \frac{113N}{24} + \frac{15\ N^2}{4} - N^3$
4	$\frac{24367}{1152}$	$\frac{839}{144} - \frac{1601\ N}{144} + \frac{695\ N^2}{48} - \frac{135\ N^3}{16} + \frac{231\ N^4}{128}$
5	$-\frac{8821}{144}$	$\frac{33661}{2400} - \frac{1151407\ N}{43200} + \frac{197587\ N^2}{4320} - \frac{12709\ N^3}{300} + \frac{6271\ N^4}{320} - \frac{7\ N^5}{2}$

- $F_{\text{phys}}, \langle \bar{q}_i q_i \rangle$ as well done
- Anyone recognize any funny functions?
- Many more and larger tables in the papers

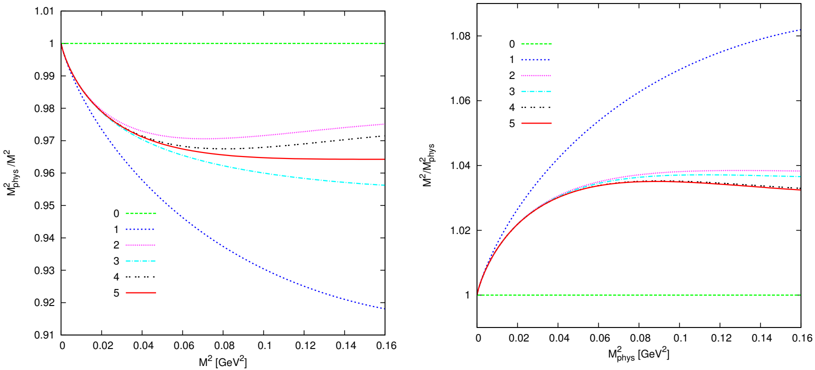
Numerical results (inspired from large N)



Left: $\frac{M^2_{\rm phys}}{M^2} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \cdots$

F $= 90$ MeV, $\mu = 0.77$ GeV

Numerical results (inspired from large N)



Left: $\frac{M^2_{\text{phys}}}{M^2} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \cdots$

Right: $\frac{M^2}{M^2_{\text{phys}}} = 1 + d_1 L_{M_{\text{phys}}} + d_2 L^2_{M_{\text{phys}}} + d_3 L^3_{M_{\text{phys}}} + \cdots$

F $= 90$ MeV, $\mu = 0.77$ GeV

Anomaly for $O(4)/O(3)$

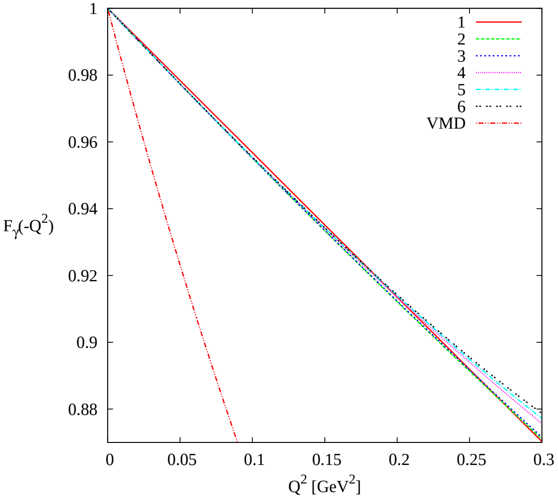
JB, Kampf, Lanz, arXiv:1201.2608

- $$\mathcal{L}_{WZW} = -\frac{N_c}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\Big\{\epsilon^{abc}\left(\frac{1}{3}\Phi^0\partial_\mu\Phi^a\partial_\nu\Phi^b\partial_\rho\Phi^c - \partial_\mu\Phi^0\partial_\nu\Phi^a\partial_\rho\Phi^b\Phi^c\right)v^0_\sigma + (\partial_\mu\Phi^0\Phi^a - \Phi^0\partial_\mu\Phi^a)v^a_\nu\partial_\rho v^0_\sigma + \frac{1}{2}\epsilon^{abc}\Phi^0\Phi^a v^b_\mu v^c_\nu\partial_\rho v^0_\sigma\Big\}.$$
- $A(\pi^0 \to \gamma(k_1)\gamma(k_2)) = \epsilon_{\mu\nu\alpha\beta}\,\varepsilon_1^{*\mu}(k_1)\varepsilon_2^{*\nu}(k_2)\,k_1^\alpha k_2^\beta\,F_{\pi\gamma\gamma}(k_1^2,k_2^2)$
- $F_{\pi\gamma\gamma}(k_1^2,k_2^2) = \dfrac{e^2}{4\pi^2 F_\pi}\hat{F}F_\gamma(k_1^2)F_\gamma(k_2^2)F_{\gamma\gamma}(k_1^2,k_2^2)$
- $\hat{F}$: on-shell photon; $F_\gamma(k^2)$: formfactor; $F_{\gamma\gamma}$ nonfactorizable part

Anomaly for $O(4)/O(3)$

- Done to six-loops
- $\hat{F} = 1 + 0 - 0.000372 + 0.000088 + 0.000036 + 0.000009 + 0.0000002 + \dots$
- Really good convergence
- $F_{\gamma\gamma}$ only starts at three-loop order (could have been two)
- $F_{\gamma\gamma}$ in the chiral limit only starts at four-loops.
- The leading logarithms thus predict this part to be fairly small.
- $F_{\gamma}(k^2)$: plot

Anomaly for $O(4)/O(3)$



Leading logs small, converge fast

Other results

- JB,Carloni, arXiv:1008.3499
  - massive case: $\pi\pi$, F_V and F_S to 4-loop order
  - large N for these cases also for massive $O(N)$.
  - done using bubble resummations or recursion eqation which can be solved analytically
- JB, Kampf, Lanz, arXiv:1201.2608
  - Mass, F_π, F_V to six loops
  - Anomaly: $\gamma^* 3\pi$ (five) and $\pi^0\gamma^*\gamma^*$ (six loops)
  - large N not relevant in this case
- JB, Kampf, Lanz, in preparation
  - $SU(N) \times SU(N)/SU(N)$

Other results

- Bissegger, Fuhrer, hep-ph/0612096 Dispersive methods, massless Π_S to five loops
- Kivel, Polyakov, Vladimirov, 0809.3236, 0904.3008, 1004.2197, 1012.4205
  - In the massless case tadpoles vanish
  - $\Longrightarrow$ number of external legs needed does not grow
  - All 4-meson vertices via Legendre polynomials
  - can do divergence of all one-loop diagrams analytically
  - algebraic (but quadratic) recursion relations
  - massless $\pi\pi$, F_V and F_S to arbitrarily high order
  - large N agrees with Coleman, Wess, Zumino
  - large N is not a good approximation

Conclusions Leading Logs

- Several quantities in massive $O(N)$ LL known to high loop order
- Large N in massive $O(N)$ model solved
- Had hoped: recognize the series also for general N
- Limited essentially by CPU time and size of intermediate files
- Some first studies on convergence etc.
- $\pi\pi$, F_V and F_S to four-loop order (F_V higher)
- The technique can be generalized to other models/theories
  - $SU(N) \times SU(N)/SU(N)$: under way
  - One nucleon sector: planned/hoped

Conclusions