Baryons in/and Lattice QCD

Chiral Dynamics 2012
Jefferson Laboratory,
Virginia, USA

André Walker-Loud
Lawrence Berkeley
National Laboratory

OUTLINE

- Baryons in lattice QCD
  - things I wish I had time to discuss
  - nucleon matrix elements $g_A,\ G_E(Q^2),\ G_M(Q^2)$
  - light quark mass dependence of the nucleon (baryons)
- Baryons and lattice QCD
  - electromagnetic self-energy of M_p - M_n
    and isovector nucleon magnetic polarizability

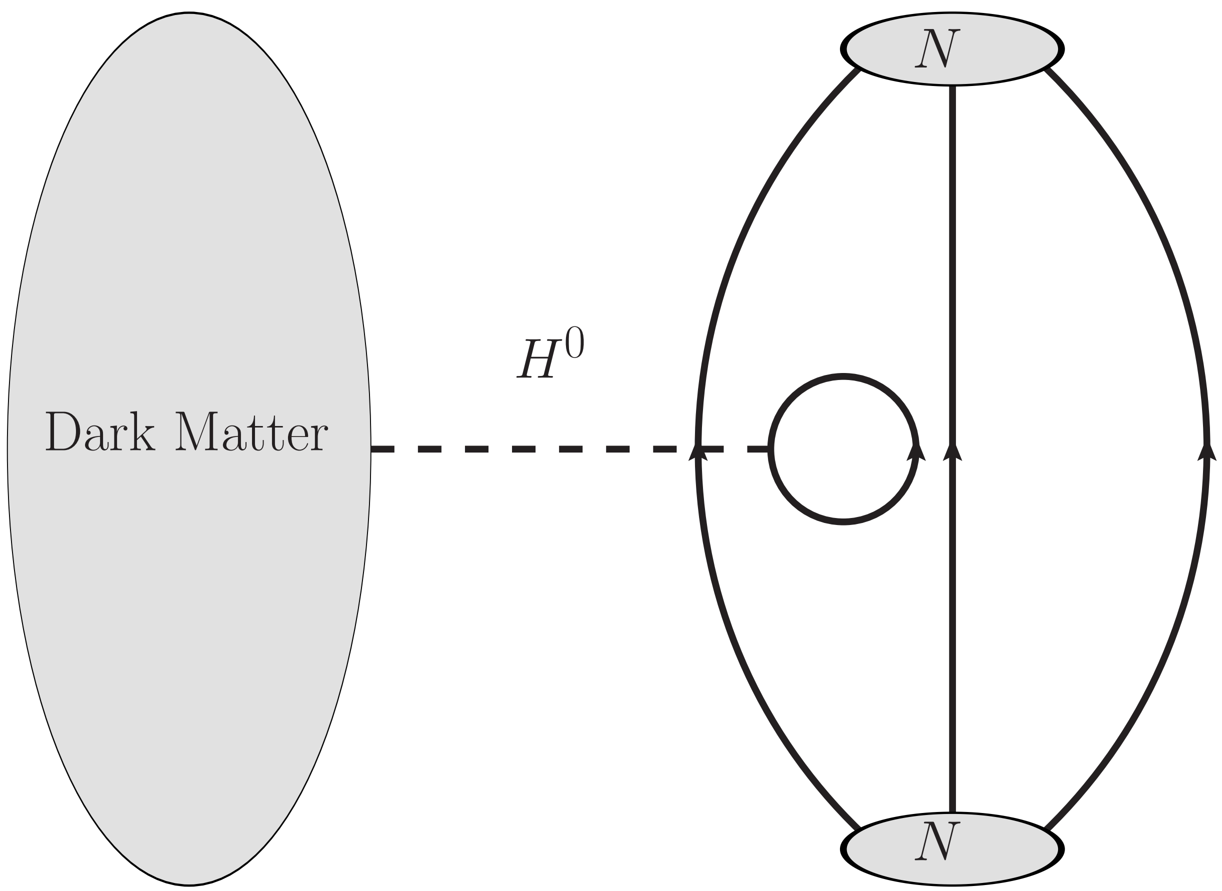
- electric polarizabilites and magnetic moments of the nucleon from lattice QCD

electromagnetic collaboration:
Will Detmold, Brian Tiburzi, AWL

see talk by Brian Tiburzi: "Lattice QCD methods for hadronic polarizabilities"
Monday, Hadron Structure and Meson Baryon Interactions

- $\langle N|\bar{q}_l q_l|N\rangle \qquad \langle N|\bar{s}s|N\rangle$

see talk by Jorge Martin Camalich:
"Baryon Chiral Perturbation Theory and Connection to Lattice QCD"



- see talk by Dru Renner:
  "Matrix elements from lattice QCD"
  Monday, Hadron Structure and Meson Baryon Interactions

Dru gave very nice talk, with cautionary summary I agree with 100%

for baryon matrix elements, lattice calculations currently lack sufficient study of systematic effects: finite volume, excited state contamination, continuum limit, ...

→ "Apparent conflicts with [experimental] measurements not justified"

→ "Apparent conflicts with χPT not compelling either"

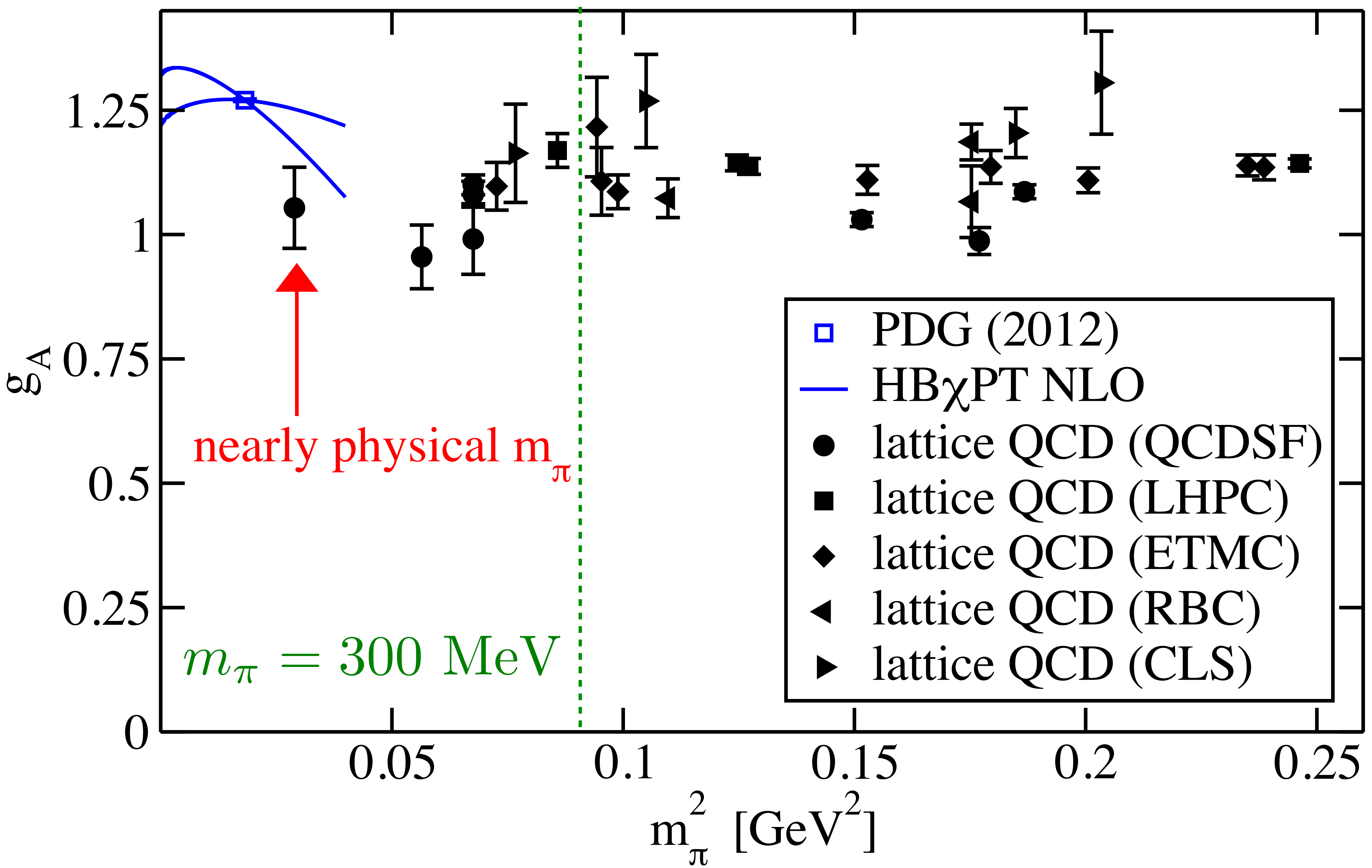
- see talk by Dru Renner:
  "Matrix elements from lattice QCD"
  Monday, Hadron Structure and Meson Baryon Interactions

If we don't take these cautions seriously, then we are forced to ask,

Is there something wrong with QCD?

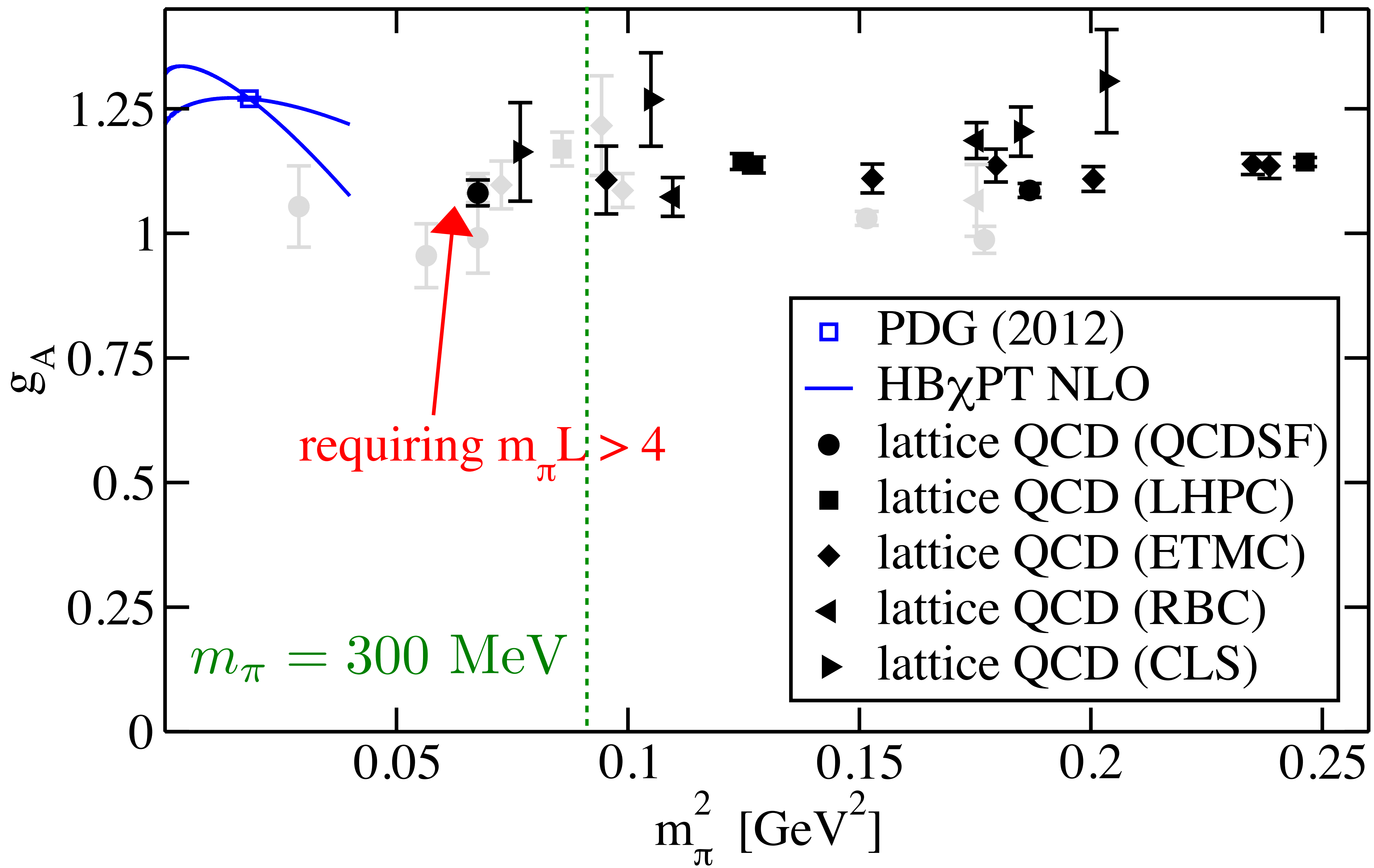
or

Is there something wrong with our lattice QCD calculations?



thanks to
Dru Renner

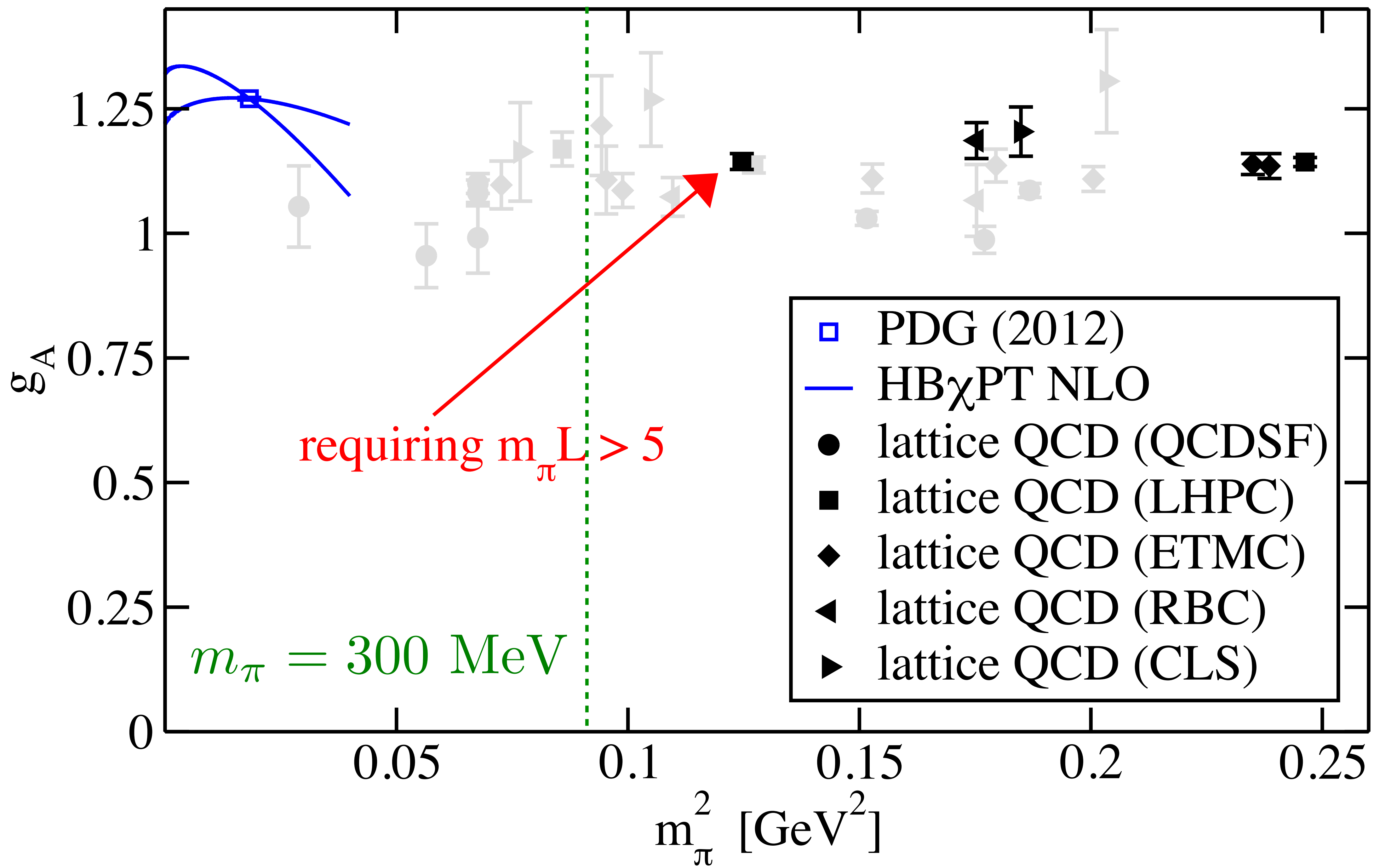
- no discernible pion mass dependence!
  must be large cancelations between different orders
  convergence is broken



thanks to Dru Renner

- apply rule of thumb cut $m_\pi L \geq 4$

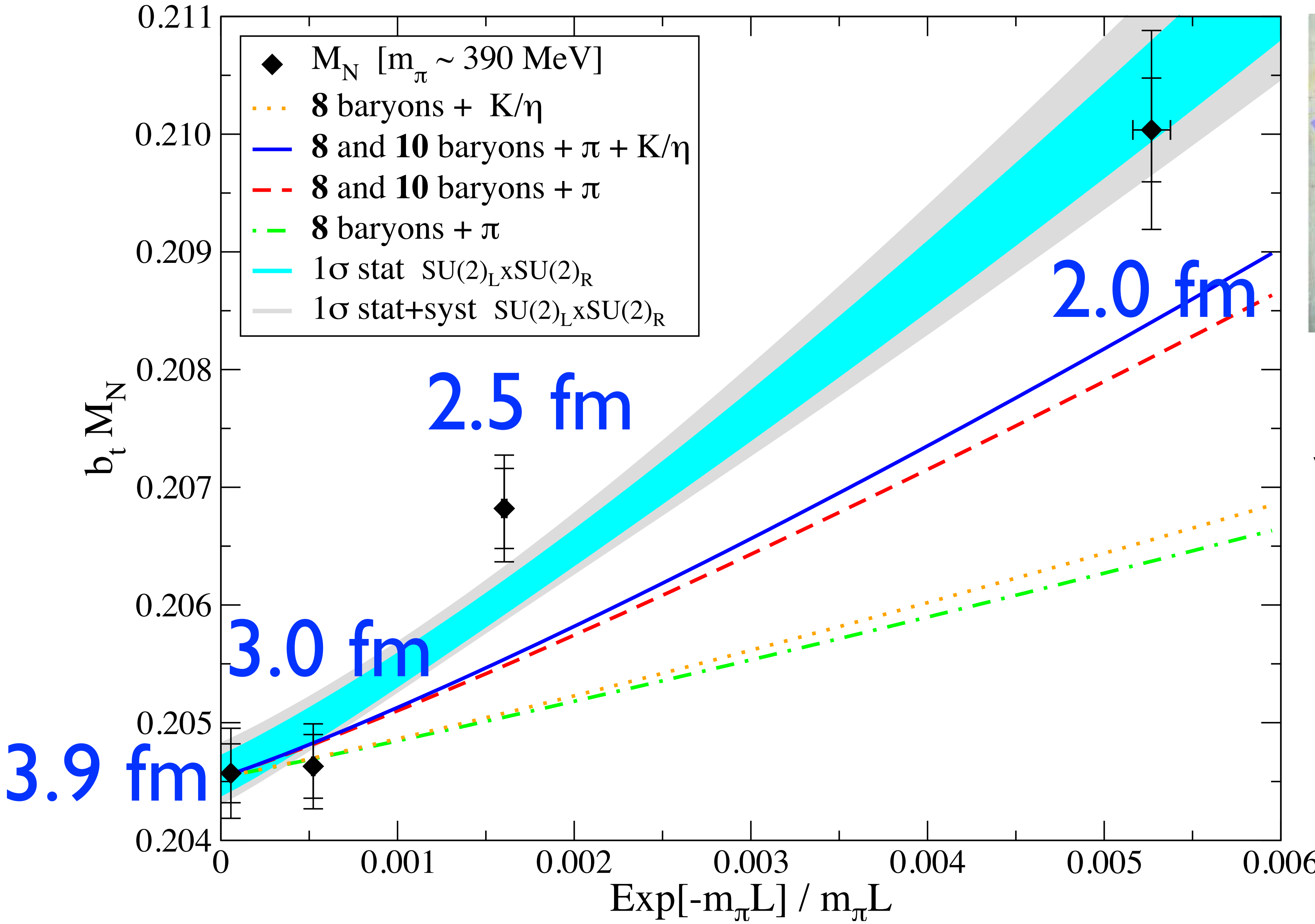
reasons to believe this may not be sufficient



thanks to
Dru Renner

- apply cut $m_\pi L \geq 5$

reasons to believe this may not be sufficient

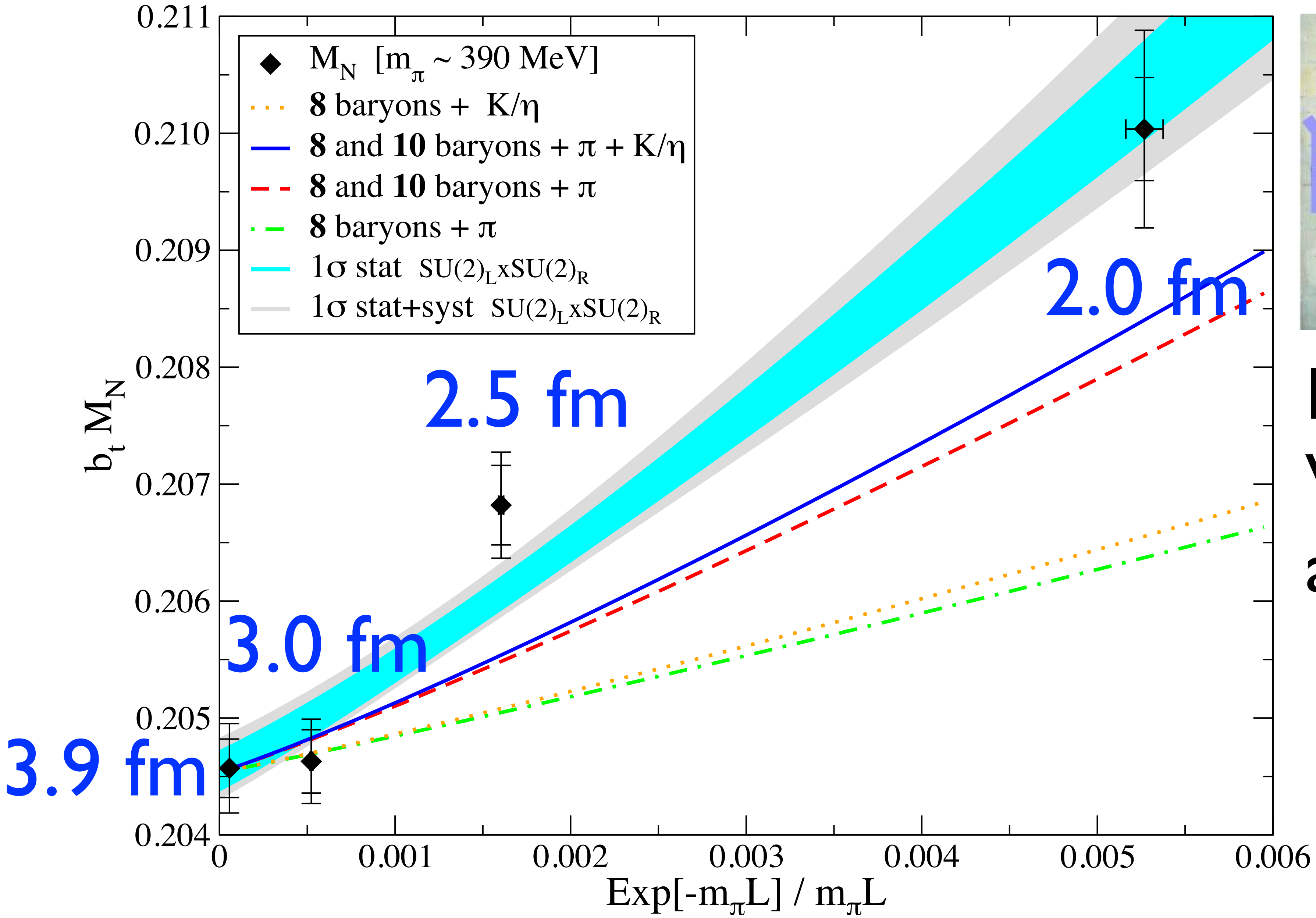


$m_\pi L =$ 7.7,5.8, 4.8 3.9

(AWL)

High statistics (IV): Volume Dependence arXiv:1104.4101

- rule of thumb which works well for pion/kaon, does not work as well baryons

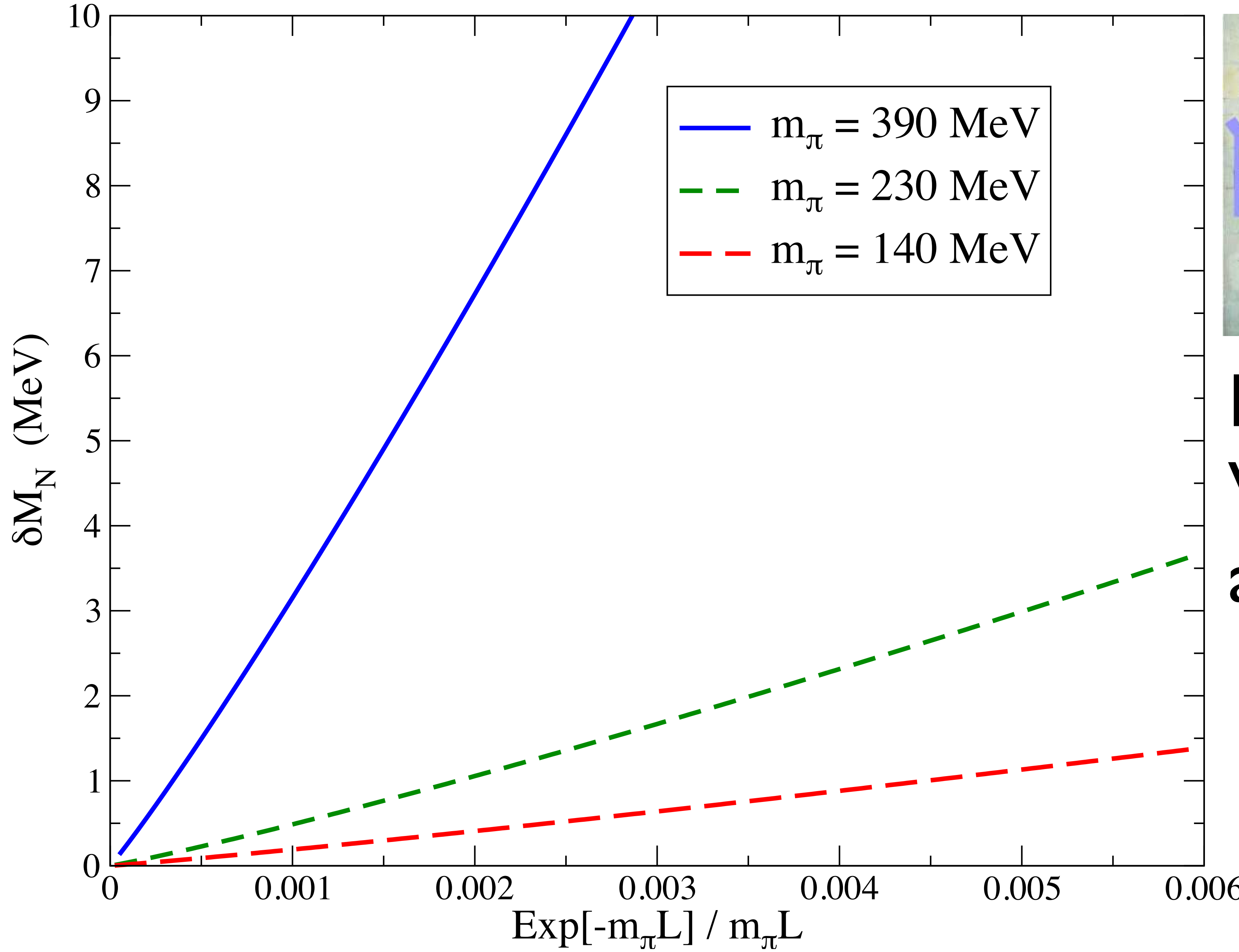


$m_\pi L =$ 7.7,5.8, 4.8 3.9

(AWL)

High statistics (IV): Volume Dependence arXiv:1104.4101

- quantities which are small mass splittings, or derivatives sigma terms from Feynman-Hellmann Theorem will be especially sensitive to these volume effects - see talk Jorge Martin Camalich



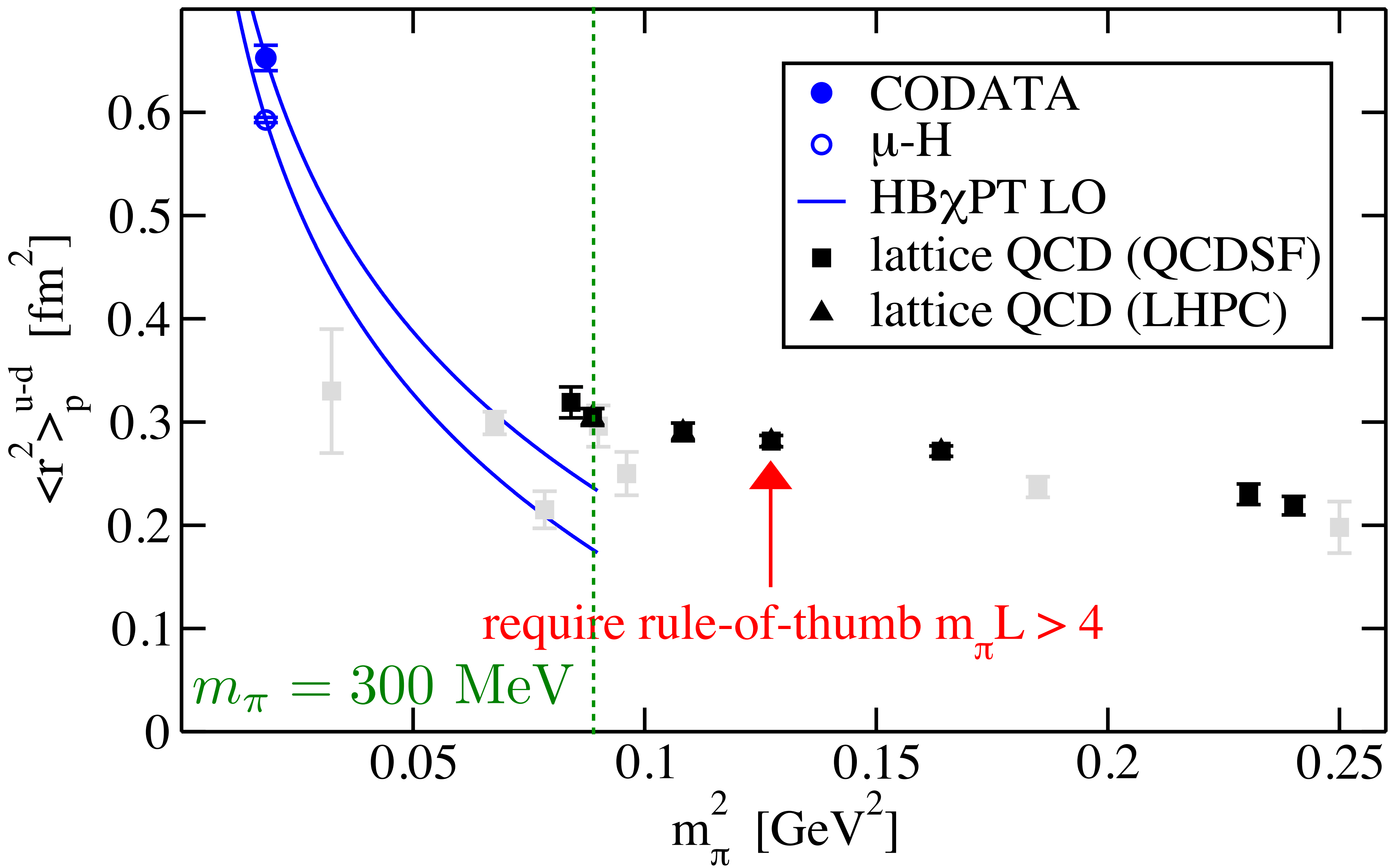
(AWL)

High statistics (IV): Volume Dependence arXiv:1104.4101

fixed $m_\pi L$ volume correction reduces as m_π^2 for nucleon mass

volume correction in SU(2)

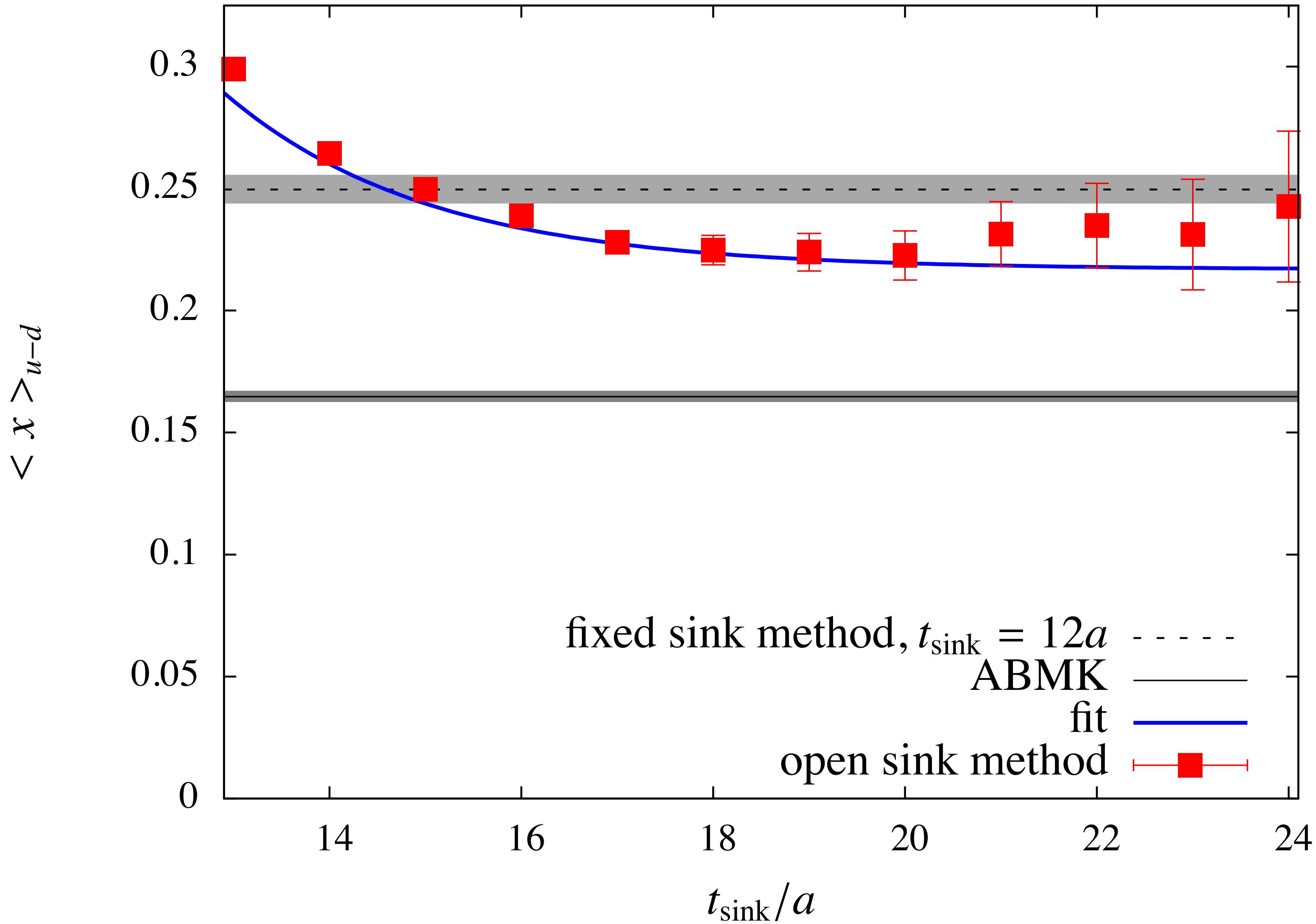
- $$\Delta M_N^{FV} = \frac{3\pi g_A^2}{(4\pi f_\pi)^2} \frac{m_\pi^3}{m_\pi L} \sum \mathbf{n} \neq 0 \frac{e^{-|\mathbf{n}| m_\pi L}}{|\mathbf{n}|}$$



thanks to Dru Renner

- similar issue for the elastic form factors, charge radii and anomalous magnetic moment

Lattice QCD Collaborations are actively exploring systematics

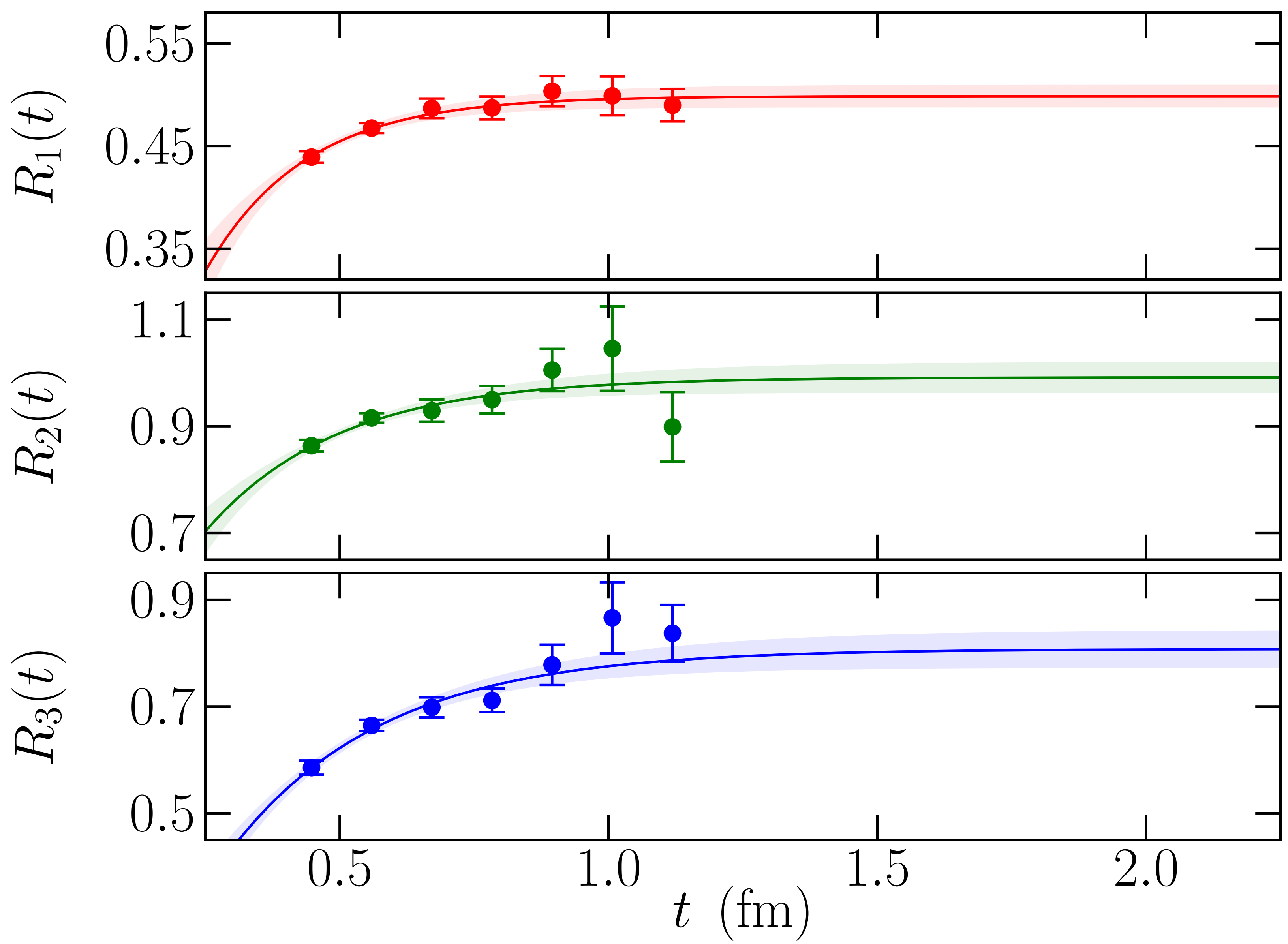


Dinter et al... Dru Renner:
arXiv:1108.1076

$m_\pi \simeq 380$ MeV

- gray band - traditional plateau method
- red values - "open sink" method - allows for more explicit study of excited state contamination

Lattice QCD Collaborations are actively exploring systematics



Detmold, Lin and Stefan Meinel: arXiv:1203.3378

$m_\pi \simeq 245$ MeV

Calculation of the axial charged of heavy (b) hadrons

See talk Wed. by Stefan Meinel

- construct matrix elements for many source-sink separations, allowing for robust study of excited state contamination

$$R_i(t) = (g_i)_{eff} - A_i e^{-\delta_i t}$$

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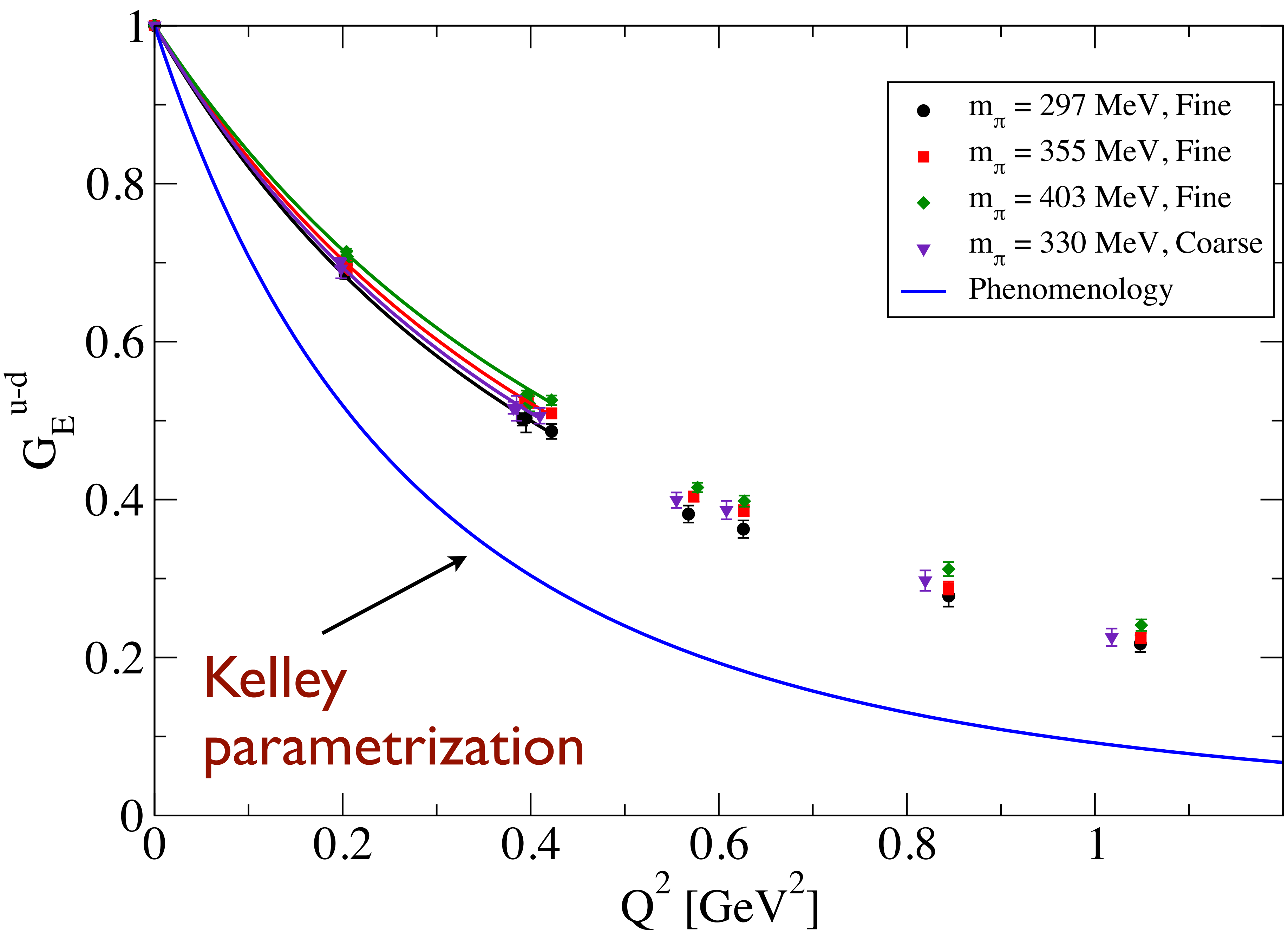


$$G_E = F_1 - \tau F_2$$

$$G_M = F_1 + F_2$$

Typical comparison of lattice QCD with experimental form factors: LHPC arXiv:0907.4194

Lattice QCD Collaborations are actively exploring systematics

Just Tuesday - I received new results from the LHP Collaboration

Michael Engelhardt
Jeremy Green
Stefan Krieg
John Negele
Andrew Pochinsky
Sergey Syritsyn

Results computed on BMWc ensemble isotropic clover Wilson with 2-level HEX-smeared gauge links

$$m_\pi \simeq 150 \text{ MeV}$$

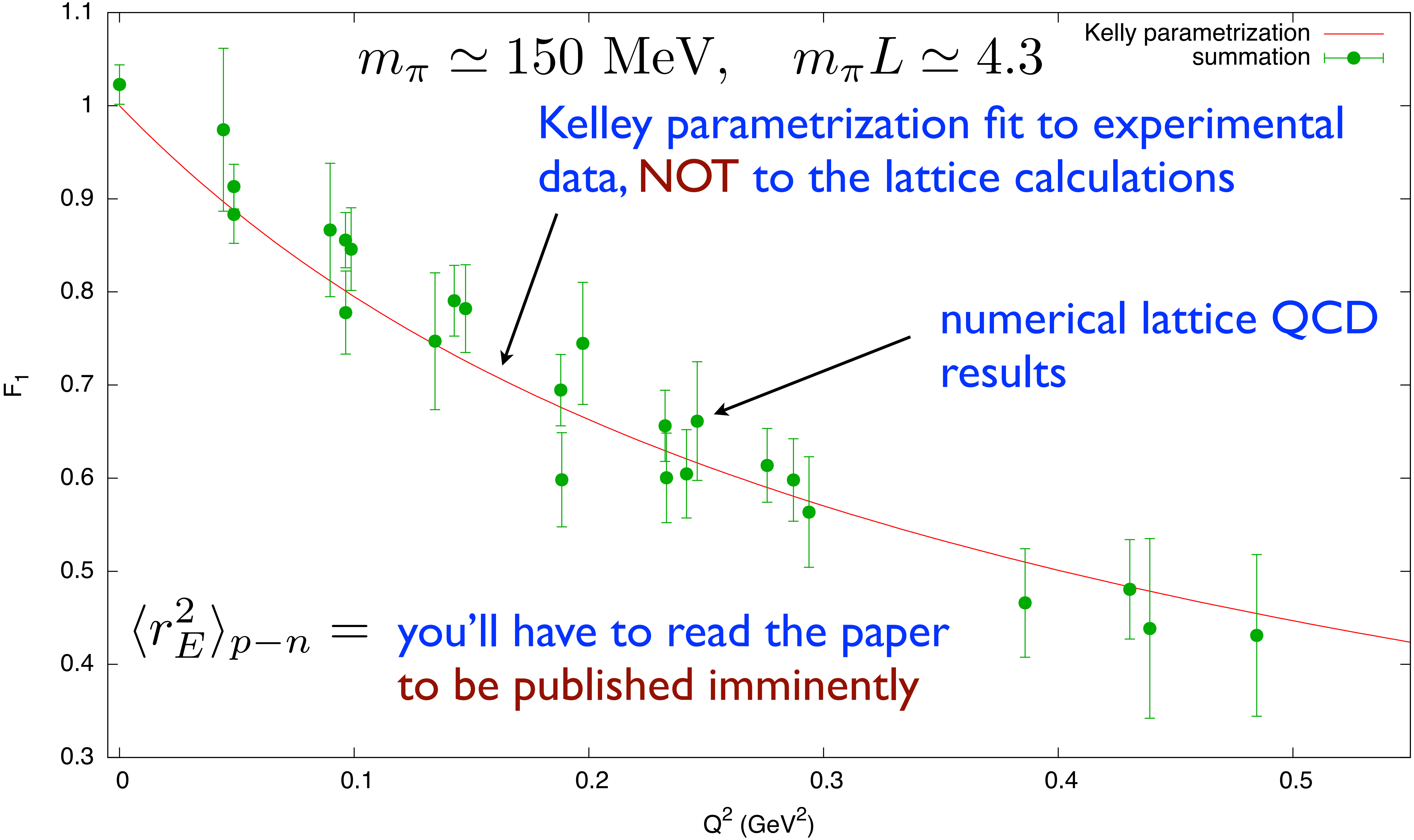
$$a \simeq 0.116 \text{ fm}$$

$$L \simeq 5.6 \text{ fm}$$

$$m_\pi L \simeq 4.3$$

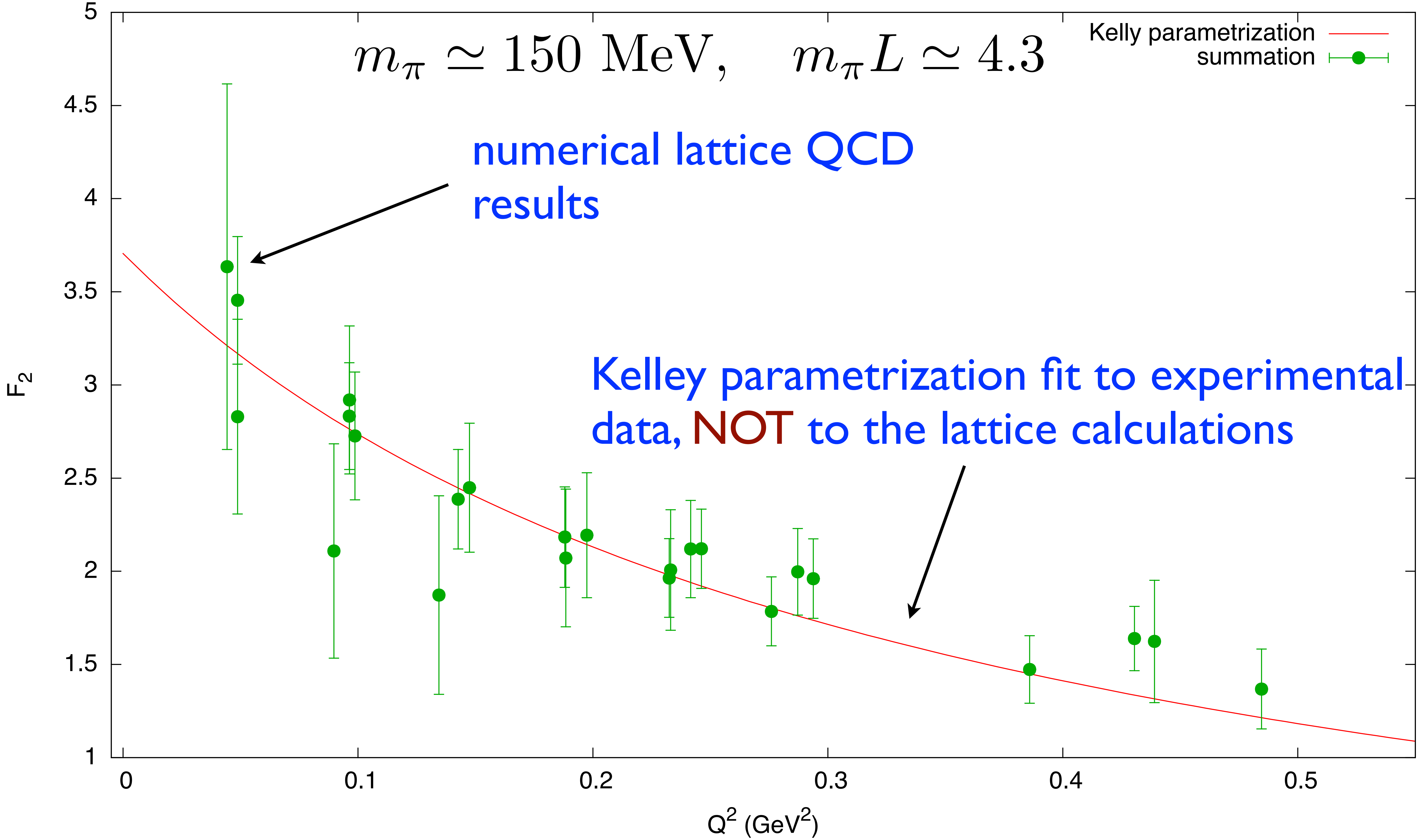
$$N_{src} = 7752$$

Lattice QCD Collaborations are actively exploring systematics



LHPC: Michael Engelhardt, Jeremy Green, Stefan Krieg, John Negele, Andrew Pochinsky, Sergey Syritsyn

Lattice QCD Collaborations are actively exploring systematics



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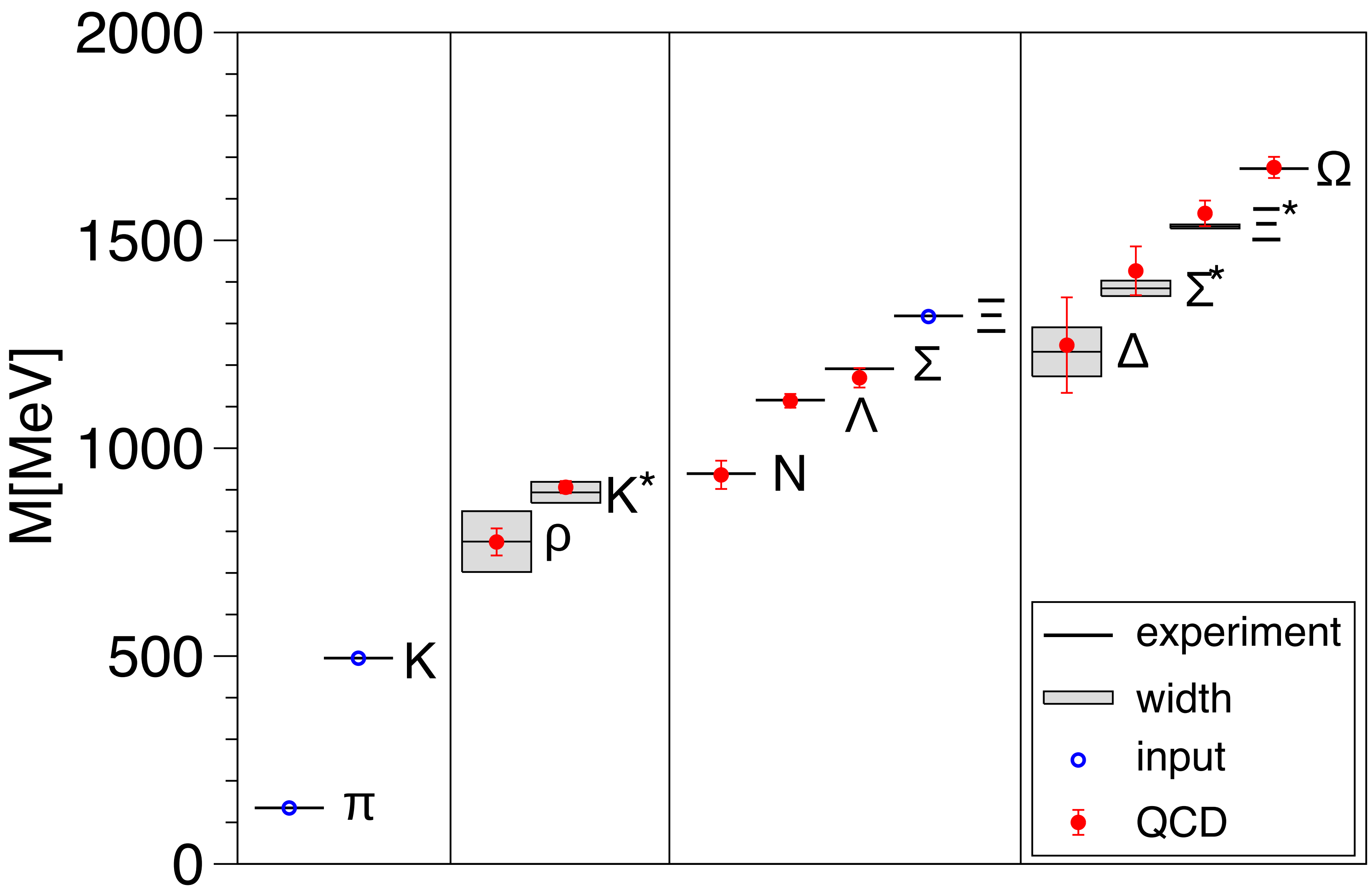
Baryons in lattice QCD: Conclusions I

- we believe lattice QCD, extrapolated to the continuum and infinite volume limits, and to the physical light quark masses, is the QCD of nature
  - while the ground state baryon spectrum has been nicely reproduced by lattice QCD calculations, nucleon matrix elements have proven to be significantly more challenging, alarming some even in the lattice community about the severity of the discrepancy - most notably with the nucleon axial charge, g_A
  - I share Dru Renner's opinion that after the 2008 lattice conference, too many groups fell into the trap of racing to the physical pion mass, without carefully checking their systematics.
  - sizes of physical volumes, pion masses and statistics which work for pion/kaon physics, heavy-quark physics, are typically not sufficient for computing properties of baryons
  - the latest LHPC results are the most significant thing to come from lattice QCD in this area since the onset of physical pion mass calculations
  - it remains for us to understand why their method works

Baryons in lattice QCD Light quark mass dependence of M_N



BMWc
Science 21 Nov 2008
Vol. 322 no. 5905 pp. 1224

$m_\pi \to m_l$

$m_K \to m_s$

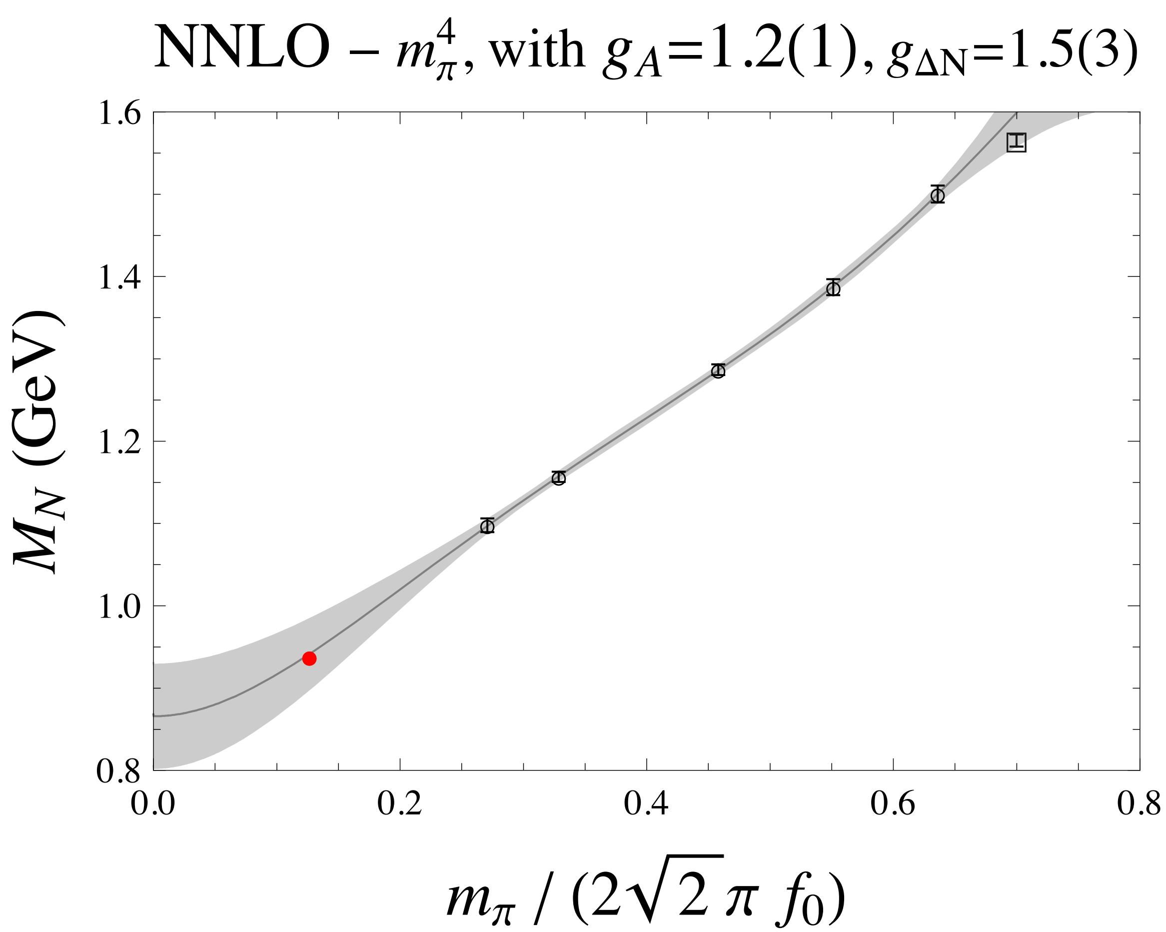
$m_\Xi \to \text{scale}$

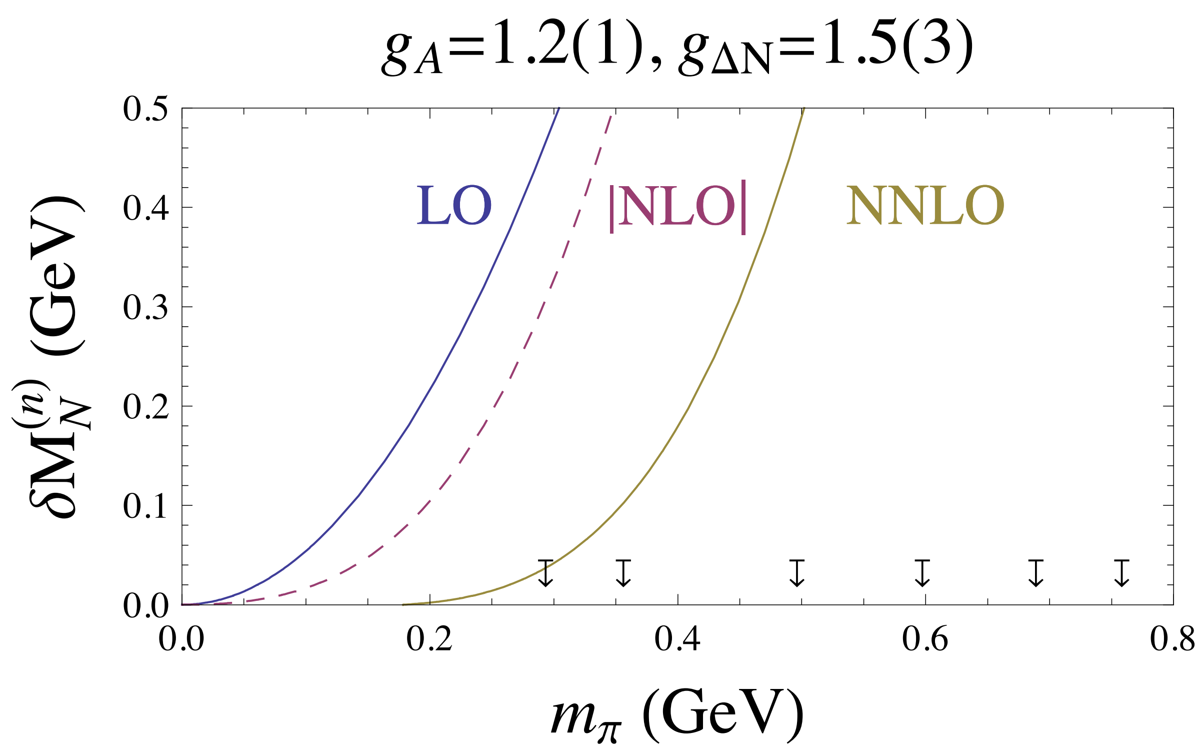
At the 2008 Lattice QCD Conference (Williamsburg), Budapest-Marseille-Wupertal collaboration (BMWc) surprised the community with calculations closer to the physical limit than the rest of us

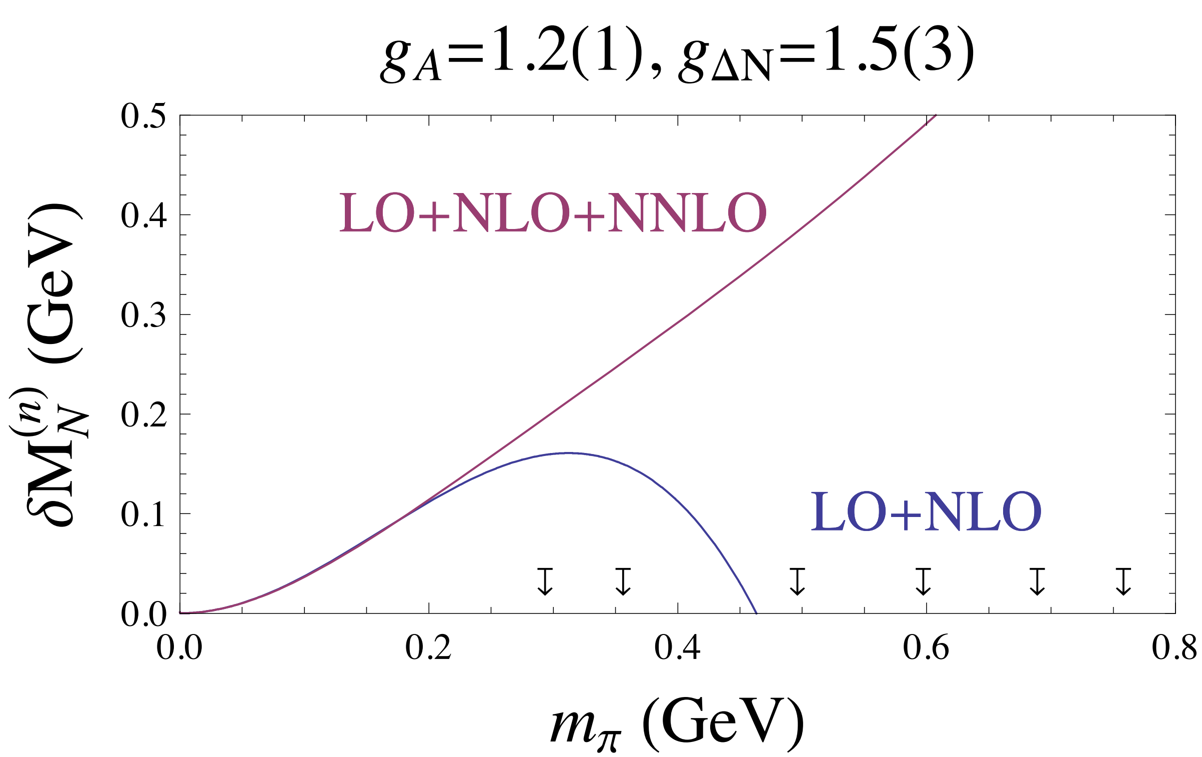
As Laurent Lellouch mentioned, this heralded the paradigm change in the relation between lattice QCD and effective field theory at least for simple quantities

At the 2008 Lattice QCD Conference,
something else unexpected happened

Baryons in lattice QCD   Light quark mass dependence of M_N







NNLO Heavy Baryon Fit

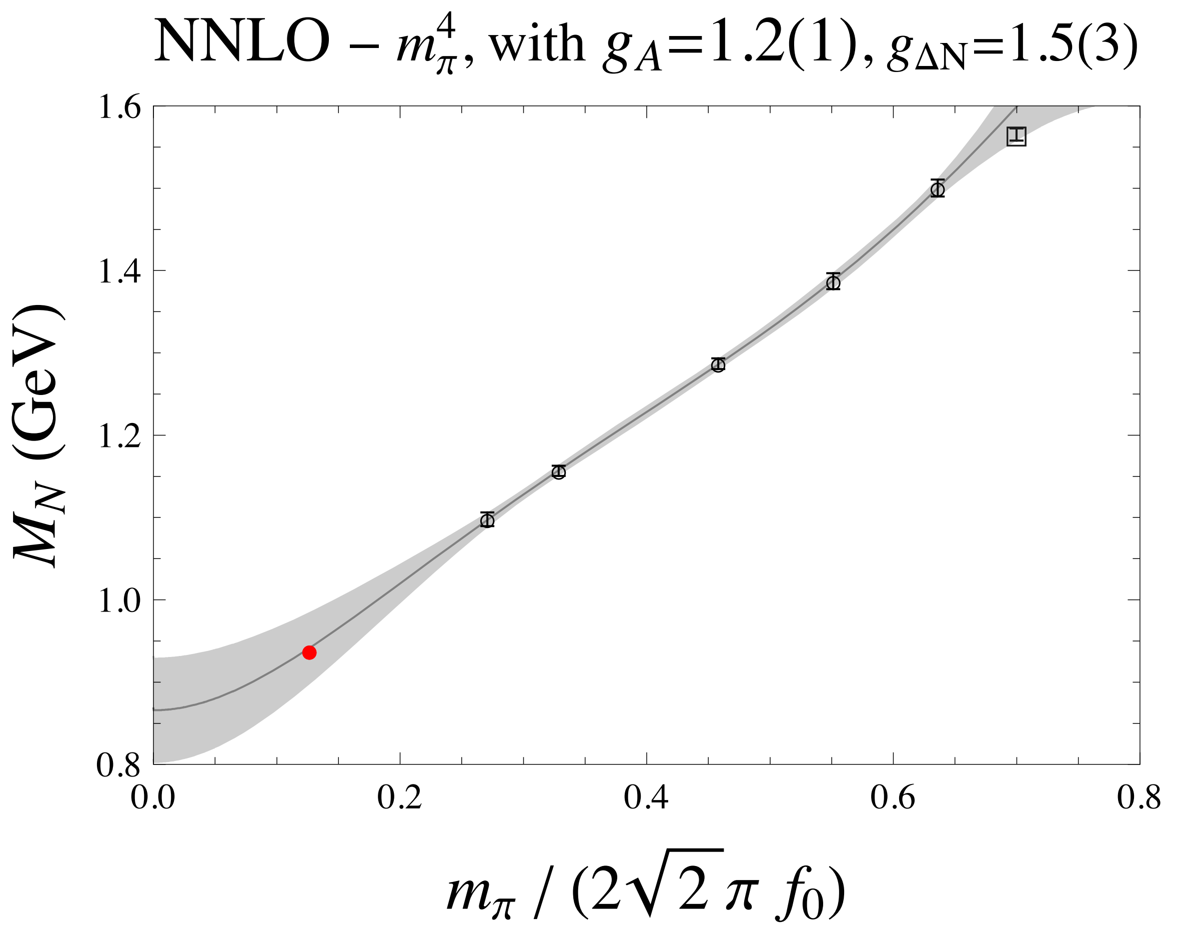
$$M_N = 954 \pm 42 \pm 20 \text{ MeV}$$

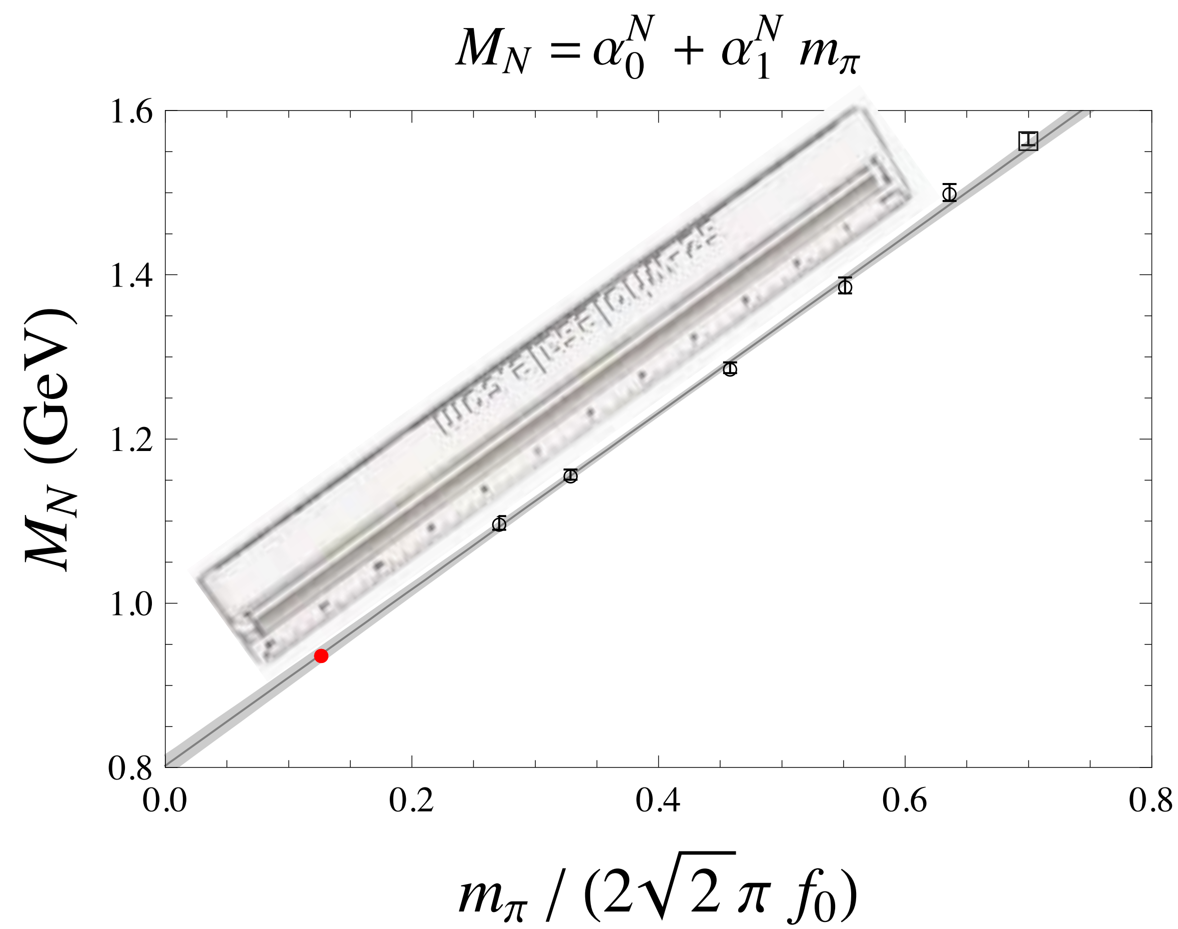
statistical

varying inputs

LHP Collaboration arXiv:0806.4549

Baryons in lattice QCD Light quark mass dependence of M_N





NNLO Heavy Baryon Fit

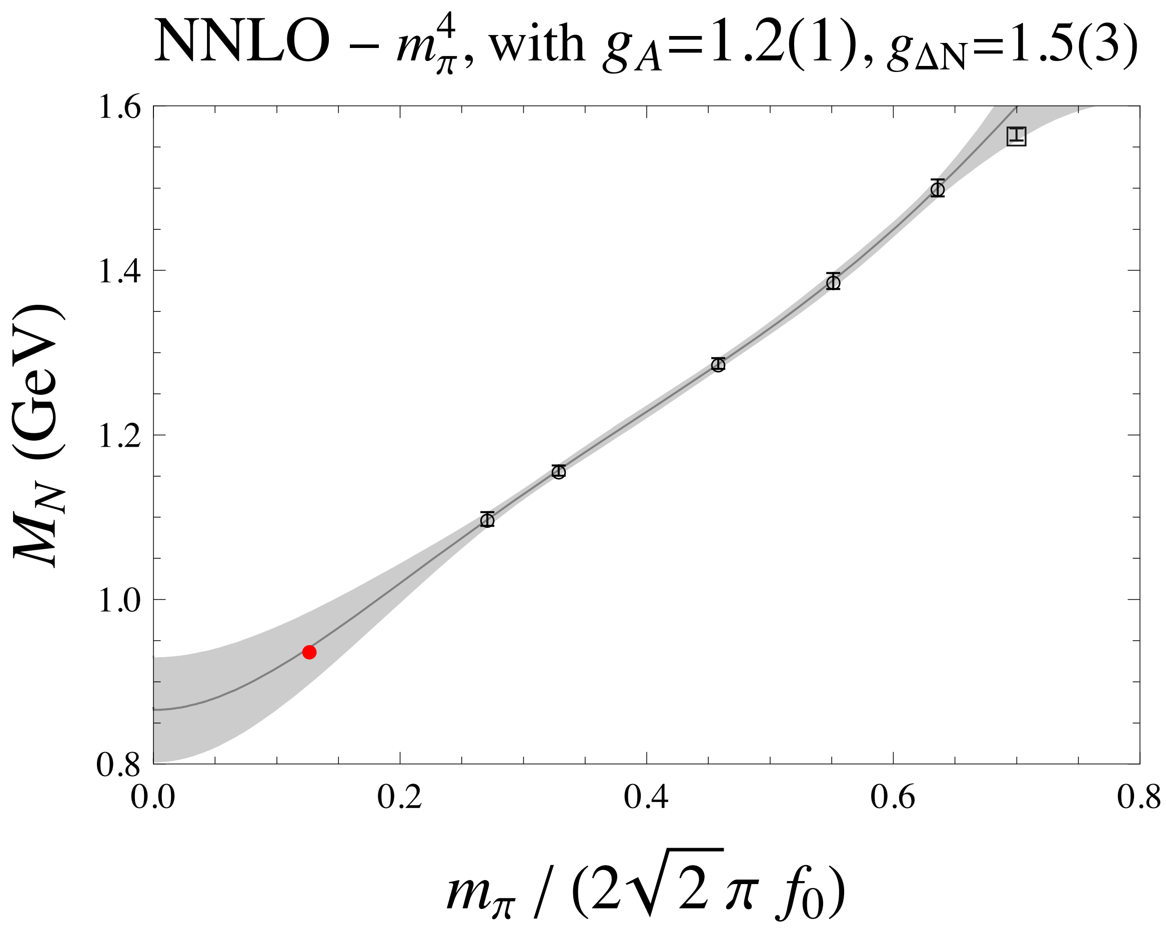
$$M_N = 954 \pm 42 \pm 20 \text{ MeV}$$

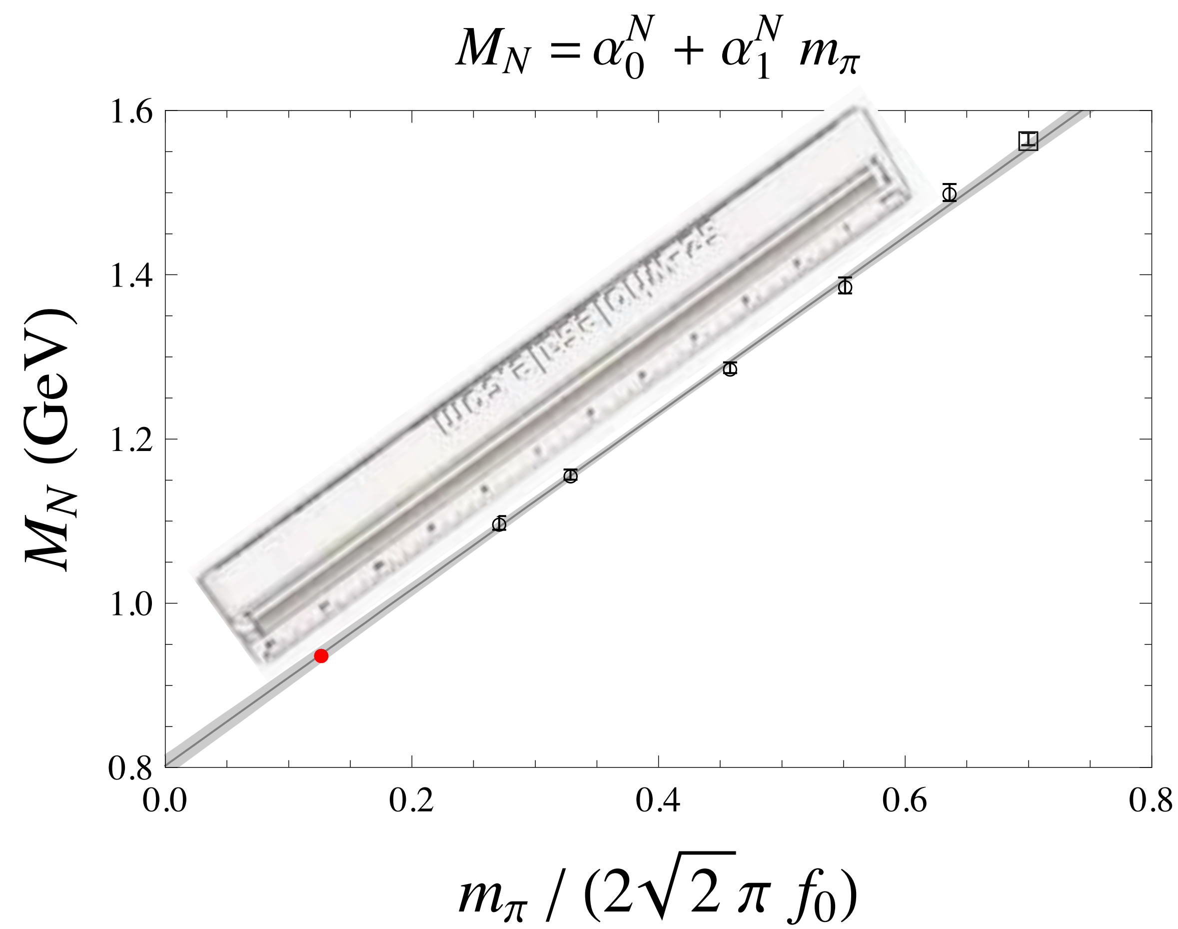
Ruler Approximation

$$M_N = \alpha_0^N + \alpha_1^N m_\pi$$

LHP Collaboration arXiv:0806.4549

Baryons in lattice QCD   Light quark mass dependence of M_N





NNLO Heavy Baryon Fit

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Ruler Approximation

$$M_N = \alpha_0^N + \alpha_1^N m_\pi$$
$$= 938 \pm 9 \text{ MeV}$$

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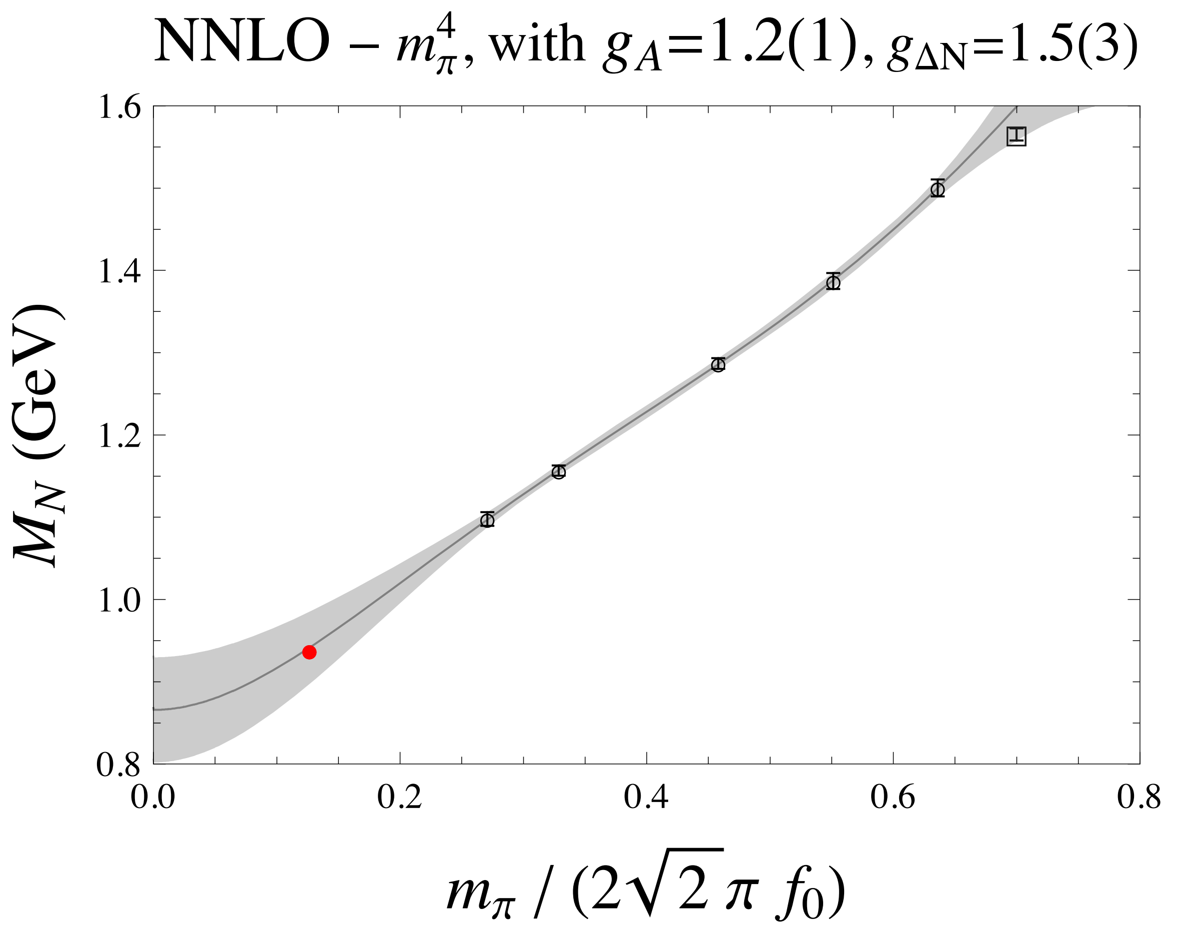
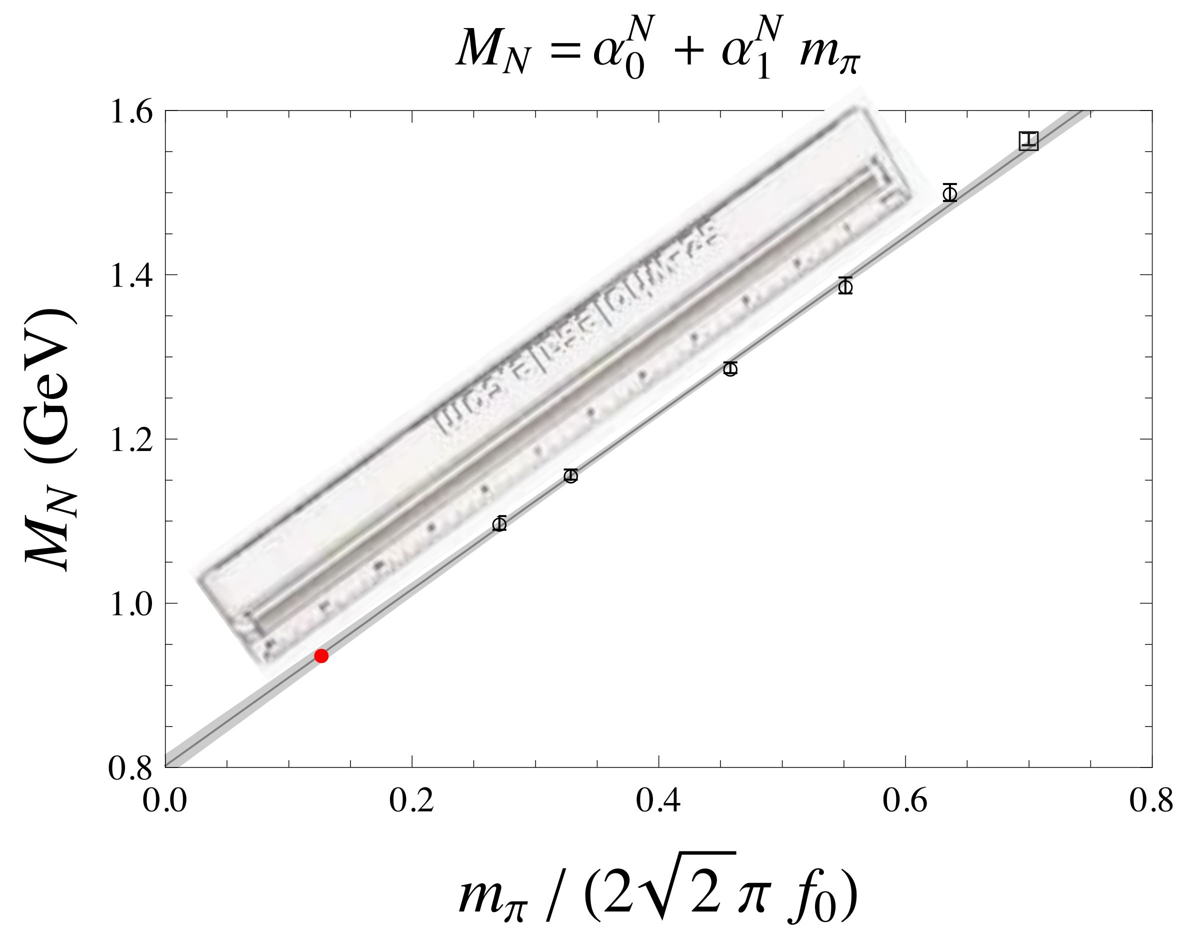
Light quark mass dependence of M_N





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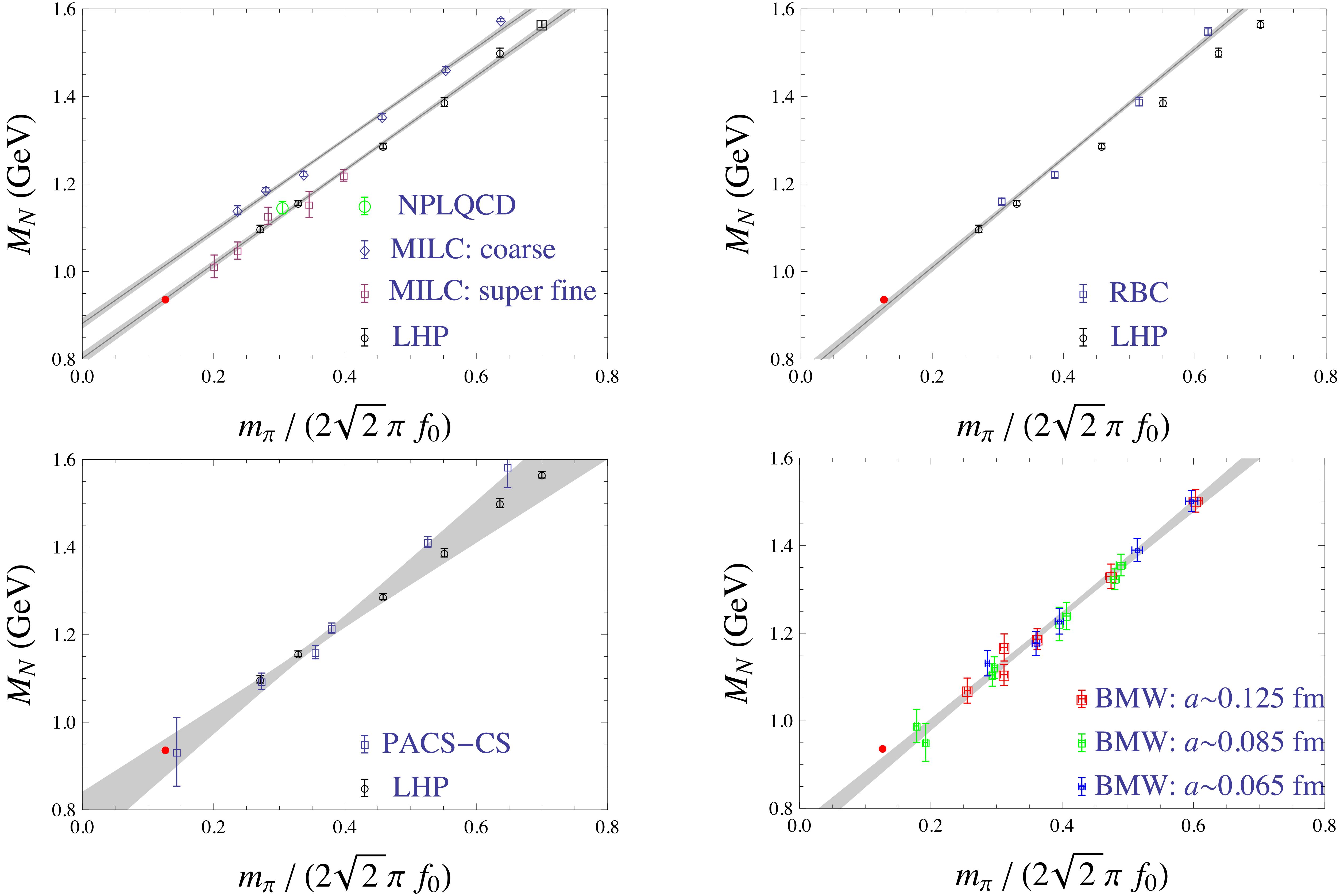
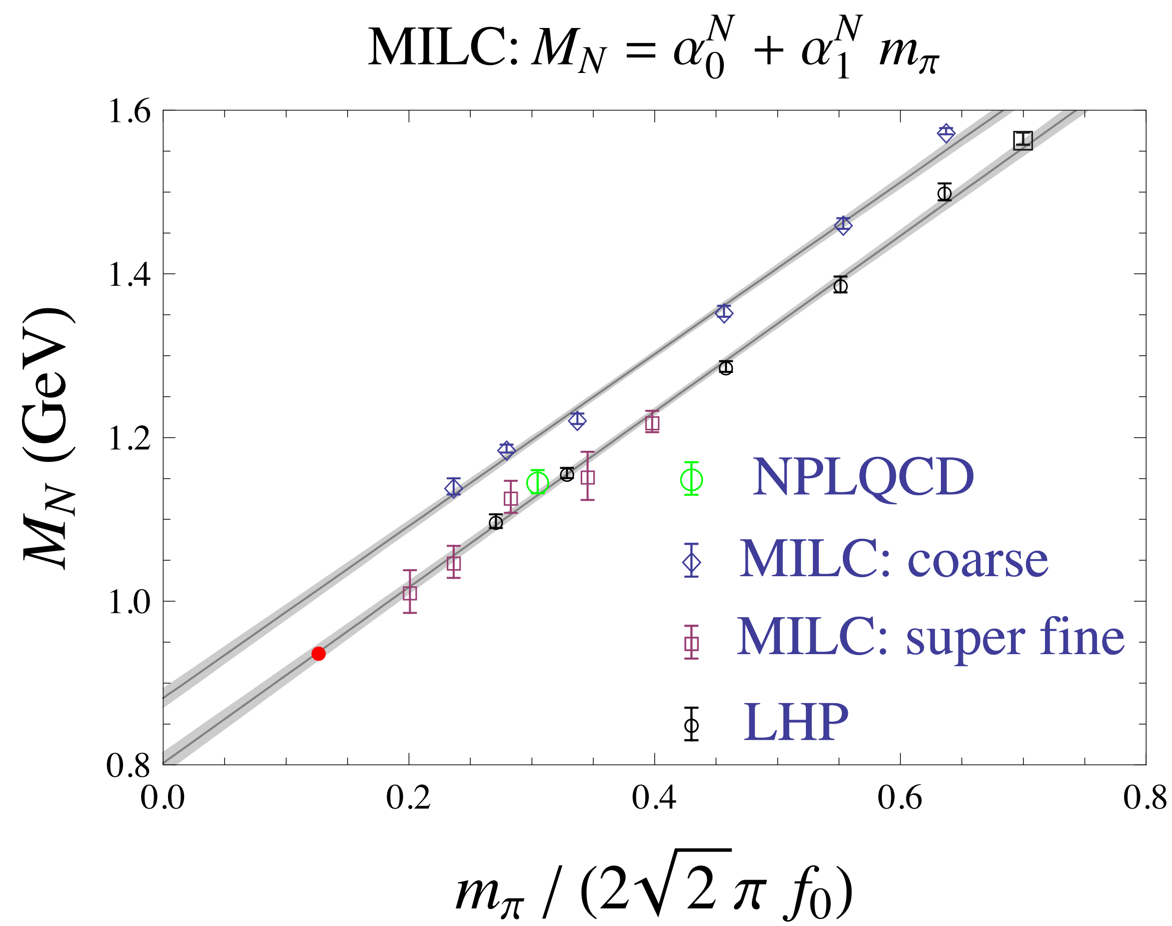
I am not advocating this as a good model for QCD!

LHP Collaboration arXiv:0806.4549

Baryons in lattice QCD   Light quark mass dependence of M_N

MILC: $M_N = \alpha_0^N + \alpha_1^N m_\pi$ Latt 2008, arXiv:0810.0663 $M_N = \alpha_0^N + \alpha_1^N m_\pi$





What does this teach us?

For these pion masses, there is a strong cancelation between LO, NLO and NNLO χPT contributions

perhaps should have been expected given poor convergence (but just not a straight line!!!)

What if we consider the octet and decuplet in the three flavor theory?

$$M_N = M_0 + \alpha_N^\pi m_\pi^2 + \alpha_N^K m_K^2 - \frac{1}{16\pi^2 f^2}\Big[3\pi(D+F)^2 m_\pi^3 + \frac{\pi}{3}(D-3F)^2 m_\eta^3 + \frac{2\pi}{3}(5D^2 - 6DF + 9F^2)m_K^3 + \frac{8}{3}\mathcal{F}(m_\pi,\Delta,\mu) + \frac{2}{3}\mathcal{F}(m_K,\Delta,\mu)\Big]$$

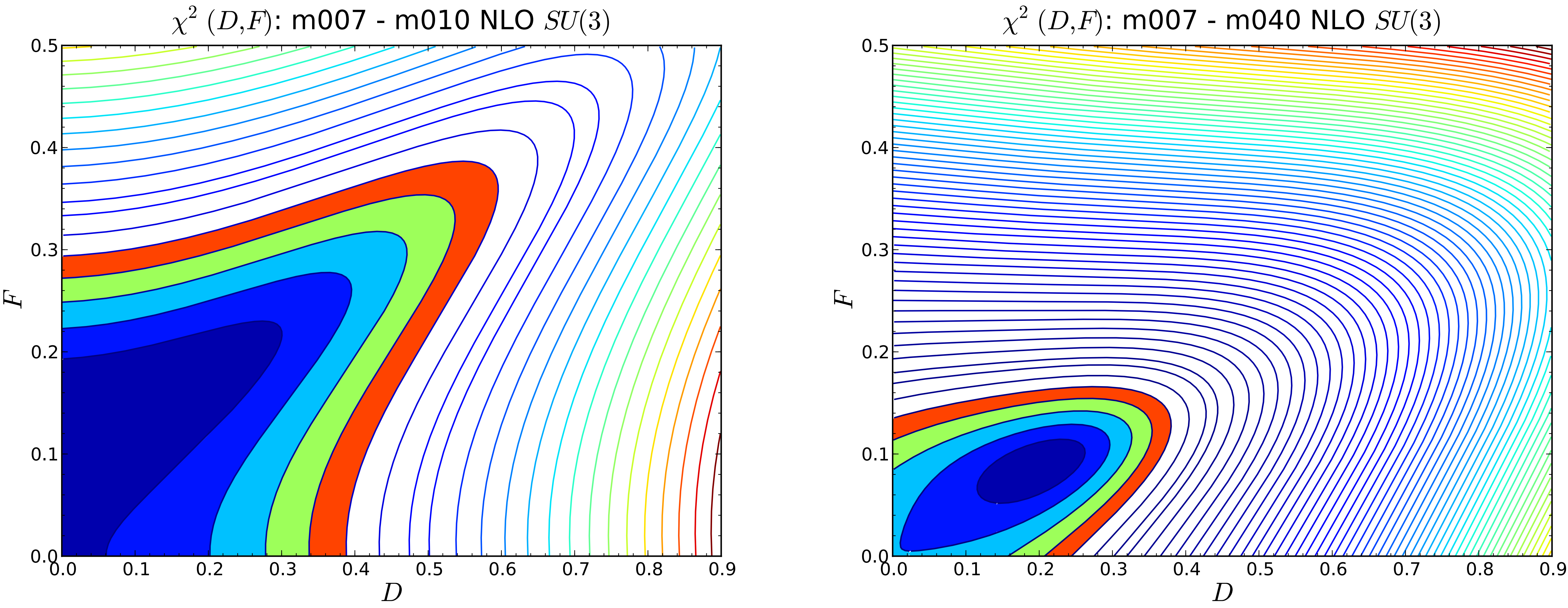
Possible convergence is significantly challenged (fails) by kaon and eta loops

LHP Collaboration arXiv:0806.4549

PACS-CS Collaboration arXiv:0905.0962

Baryons in lattice QCD   Light quark mass dependence of M_N

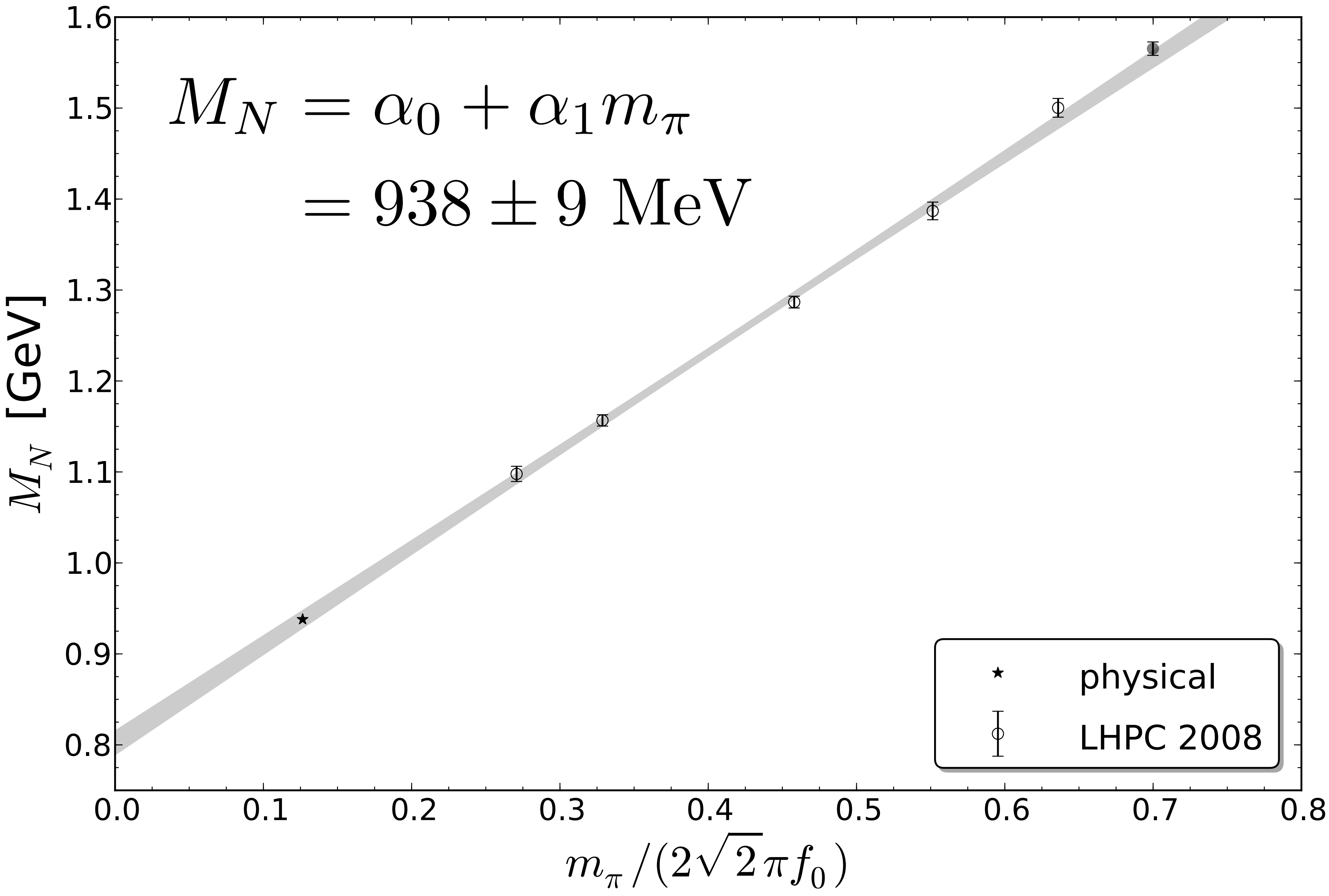
figures: Jenkins, Manohar, Negele and AWL arXiv:0907.0529



NLO SU(3) chiral fits to spectrum are not consistent with phenomenological values of D, F

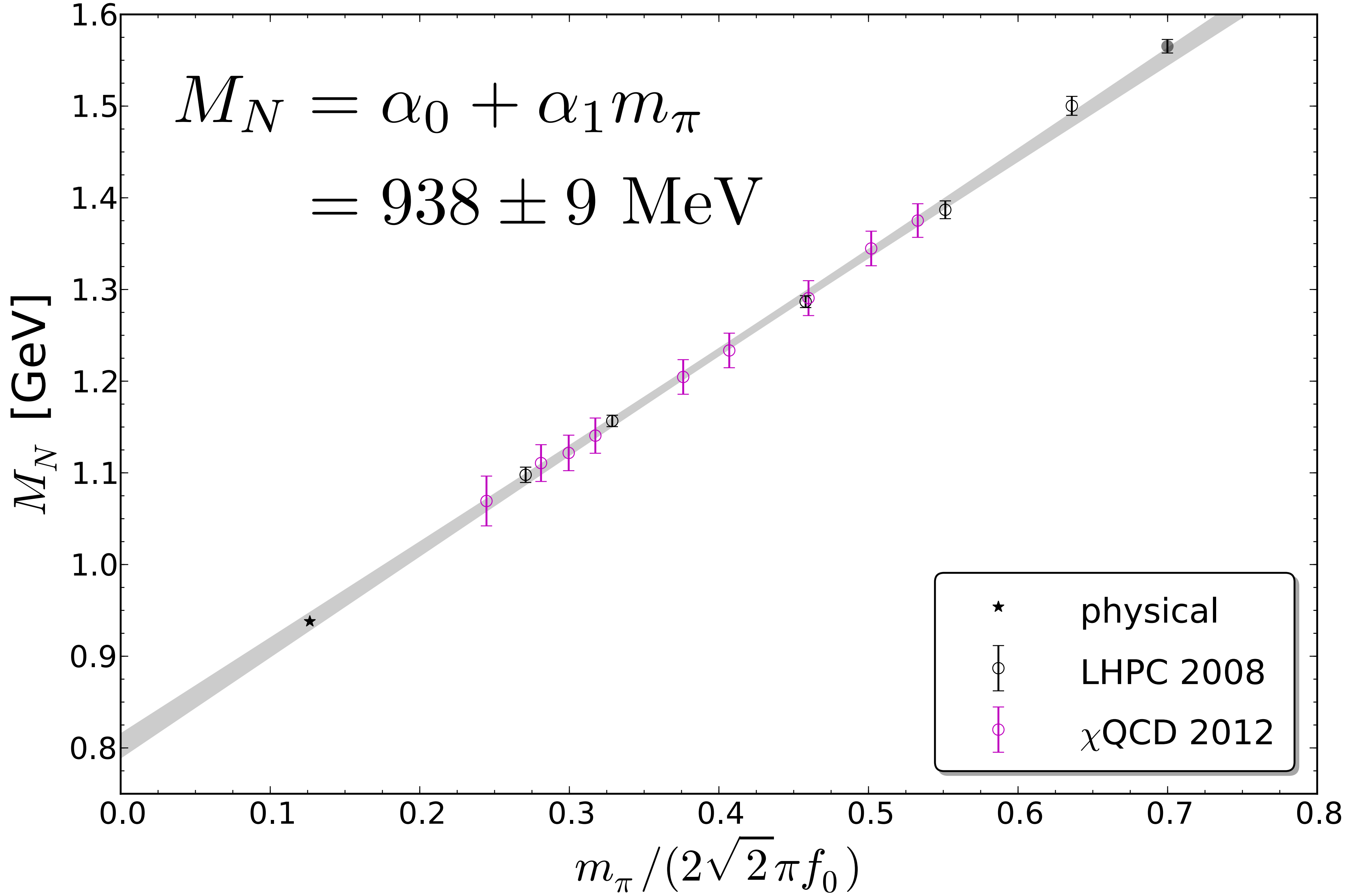
$$D \sim 0.75, \quad F \sim 0.50$$

What is the status now (2012)?



Physical point NOT included in fit

What is the status now (2012)?



χQCD Collaboration uses Overlap Valence fermions on Domain-Wall (RBC-UKQCD) sea fermions

Light quark mass dependence of M_N

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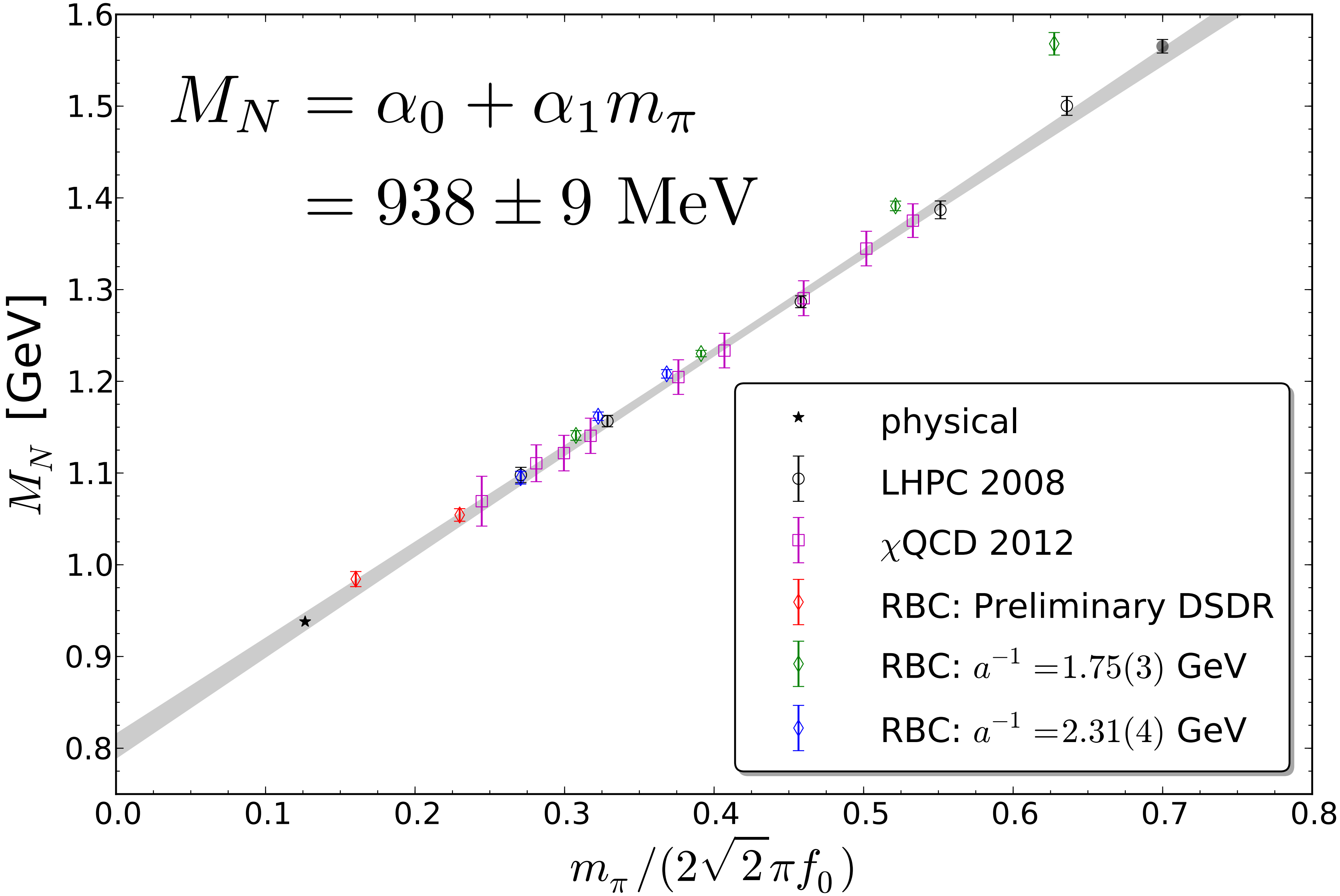
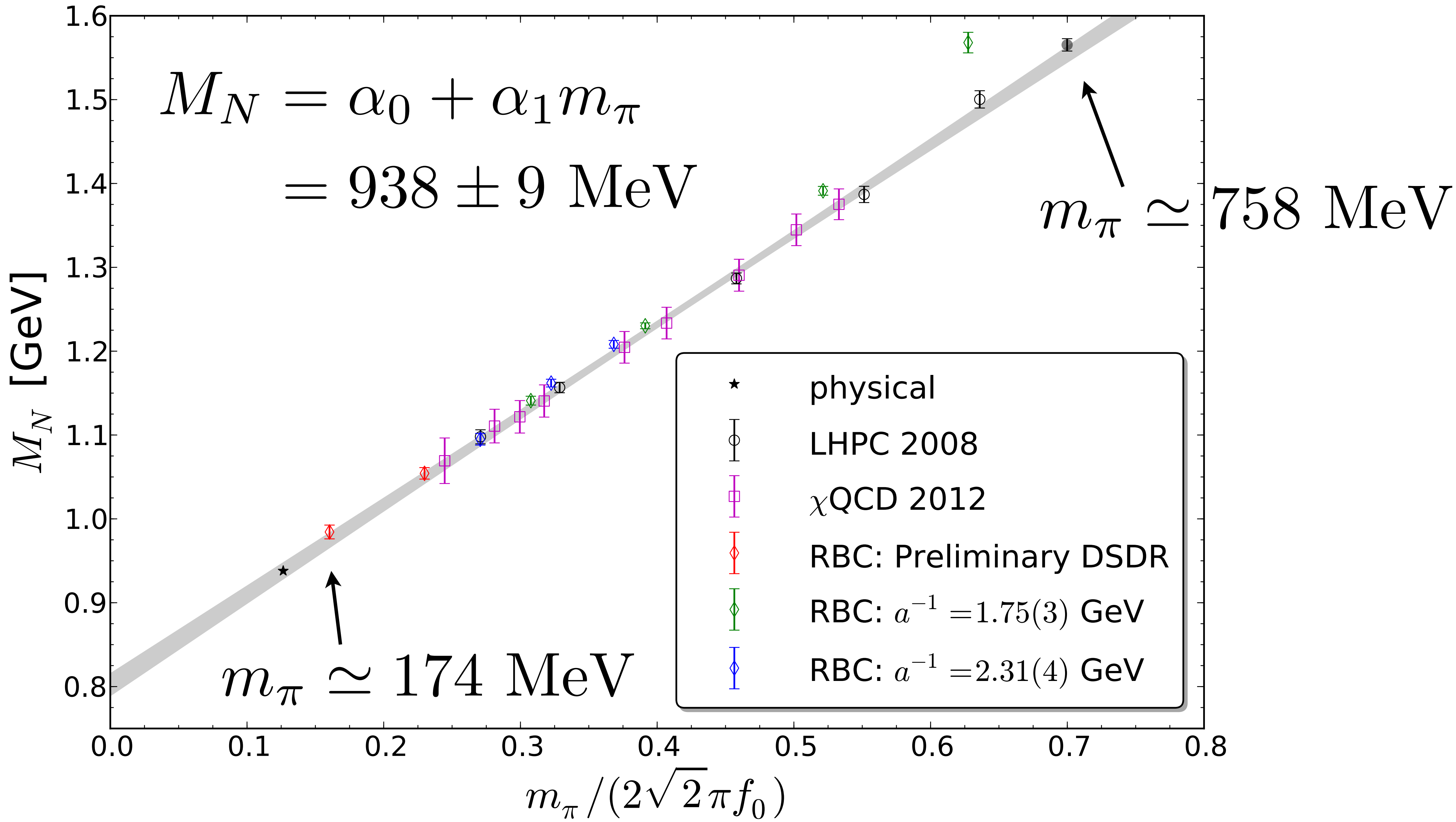


RBC-UKQCD Collaboration uses Domain-Wall valence and sea fermions

Light quark mass dependence of M_N

What is the status now (2012)?



Taking this seriously yields

$$\sigma_{\pi N} = 67 \pm 4 \text{ MeV}$$

I am not advocating this as a good model for QCD!

Large N_c and SU(3) Chiral Perturbation Theory

What can we do?

- Consider 2-flavor expansion for hyperons

Beane, Bedaque, Parreno and Savage nucl-th/0311027

Tiburzi and AWL arXiv:0808.0482

Jiang and Tiburzi arXiv:0905.0857

Mai, Bruns, Kubis and Meissner arXiv:0905.2810

Jiang, Tiburzi and AWL arXiv:0911.4721

Jiang and Tiburzi arXiv:0912.2077

Large N_c and SU(3) Chiral Perturbation Theory

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- Read the literature and apply an old idea to our new problem

  combine the constraints of large N_c and SU(3) symmetries

Large N_c and SU(3) Chiral Perturbation Theory

Combined large N_c and SU(3) symmetries

't Hooft 1974
Witten 1979
Coleman 1979
Dashen, Jenkins, Manohar 1993

...

see talks at this conference by

Alvaro Calle Cardon: "$1/N_c$ Chiral Perturbation Theory in the one-Baryon Sector"

Vojtech Krejcirik: "Model-independent form factor relations at large N_c"

Mathias Lutz: "Strangeness in the baryon ground states"

Large N$_c$ and SU(3) Chiral Perturbation Theory

- theory is placed on solid theoretical foundation

$$\lim_{N_c \to \infty} M_B = \infty$$

controlled expansion in $1/N_c$ (at least formally)

- inclusion of spin 3/2 dof well defined field theoretically

$$M_\Delta - M_N \propto \frac{1}{N_c}$$

- naturally explains smallness of baryon octet GMO relation

$$N_c m_s^{3/2} \propto \text{flavor-}\mathbf{1}$$

$$m_s^{3/2} \propto \text{flavor-}\mathbf{8}$$

$$m_s^{3/2}/N_c \propto \text{flavor-}\mathbf{27}$$ leading correction to GMO

Large N_c and SU(3) Chiral Perturbation Theory

- gives you "smarter" observables to measure/calculate

eg: Spectrum $$M = M^{1,0} + M^{8,0} + M^{27,0} + M^{64,0}$$

$$M^{1,0} = c^{1,0}_{(0)} N_c \mathbf{1} + c^{1,0}_{(2)} \frac{1}{N_c} J^2$$

$$M^{8,0} = c^{8,0}_{(1)} T^8 + c^{8,0}_{(2)} \frac{1}{N_c} \{J^i, G^{i8}\} + c^{8,0}_{(3)} \frac{1}{N_c^2} \{J^2, T^8\}$$

$$M^{27,0} = c^{27,0}_{(2)} \frac{1}{N_c} \{T^8, T^8\} + c^{27,0}_{(3)} \frac{1}{N_c^2} \{T^8, \{J^i, G^{i8}\}\}$$

$$M^{64,0} = c^{64,0}_{(3)} \frac{1}{N_c^2} \{T^8, \{T^8, T^8\}\}$$

$J^i = q^\dagger (J^i \otimes \mathbf{1}) q$ one-body spin operator

$T^a = q^\dagger (\mathbf{1} \otimes T^a) q$ one-body flavor operator

$G^{ia} = q^\dagger (J^i \otimes T^a) q$ one-body spin-flavor operator

Large N_c and SU(3) Chiral Perturbation Theory

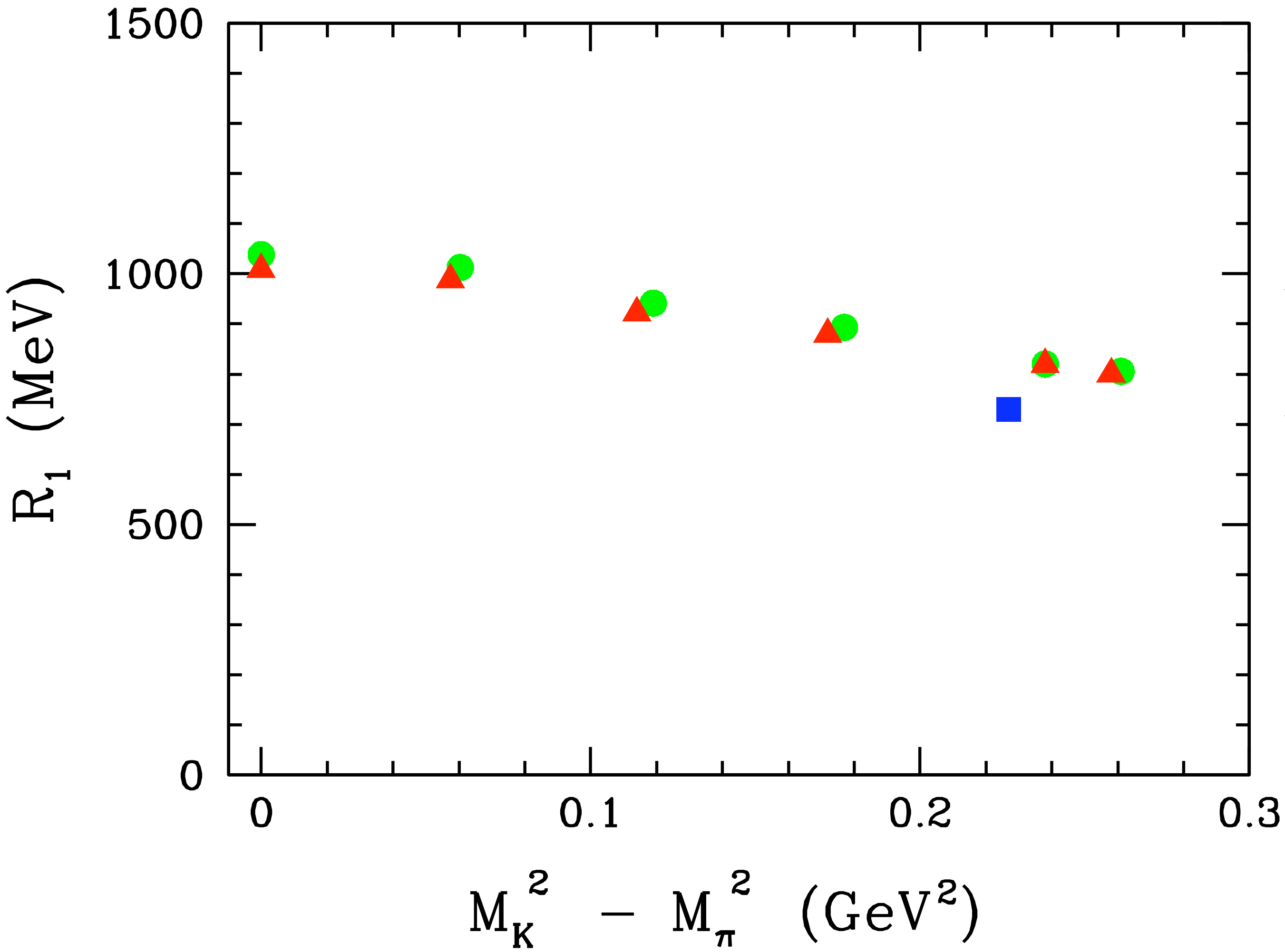
Jenkins and Lebed hep-ph/9502227

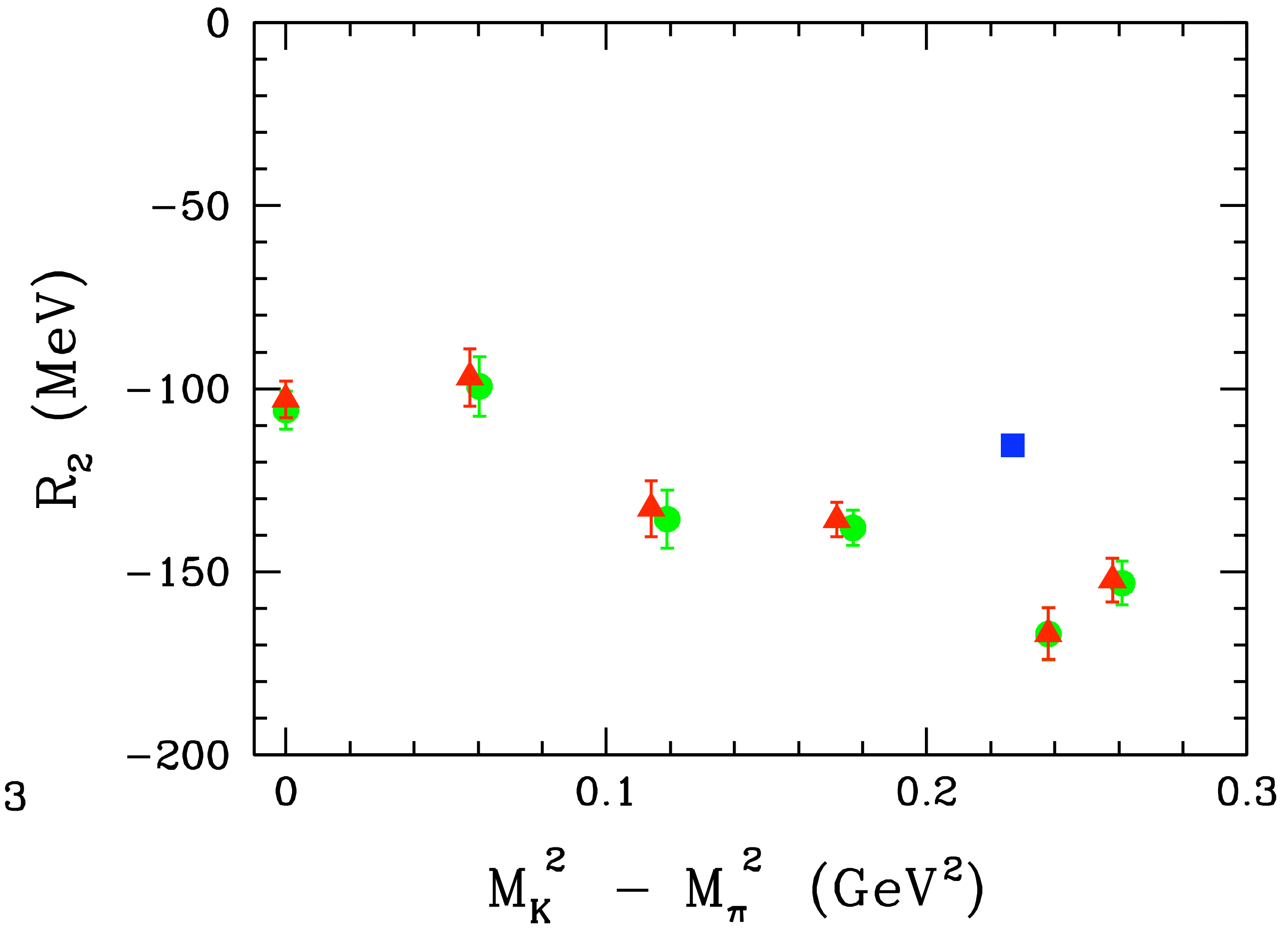
Label	Operator	Coefficient	Mass Combination	$1/N_c$	$SU(3)$
M_1	$\mathbb{1}$	$160\, N_c\, c^{1,0}_{(0)}$	$25(2N+\Lambda+3\Sigma+2\Xi)-4(4\Delta+3\Sigma^*+2\Xi^*+\Omega)$	N_c	1
M_2	J^2	$-120\,\frac{1}{N_c}\, c^{1,0}_{(2)}$	$5(2N+\Lambda+3\Sigma+2\Xi)-4(4\Delta+3\Sigma^*+2\Xi^*+\Omega)$	$1/N_c$	1
M_3	T^8	$20\sqrt{3}\,\epsilon\, c^{8,0}_{(1)}$	$5(6N+\Lambda-3\Sigma-4\Xi)-2(2\Delta-\Xi^*-\Omega)$	1	ϵ
M_4	$\{J^i, G^{i8}\}$	$-5\sqrt{3}\,\frac{1}{N_c}\,\epsilon\, c^{8,0}_{(2)}$	$N+\Lambda-3\Sigma+\Xi$	$1/N_c$	ϵ
M_5	$\{J^2, T^8\}$	$30\sqrt{3}\,\frac{1}{N_c^2}\,\epsilon\, c^{8,0}_{(3)}$	$(-2N+3\Lambda-9\Sigma+8\Xi)+2(2\Delta-\Xi^*-\Omega)$	$1/N_c^2$	ϵ
M_6	$\{T^8, T^8\}$	$126\,\frac{1}{N_c}\,\epsilon^2\, c^{27,0}_{(2)}$	$35(2N-3\Lambda-\Sigma+2\Xi)-4(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)$	$1/N_c$	ϵ^2
M_7	$\{T^8, J^iG^{i8}\}$	$-63\,\frac{1}{N_c^2}\,\epsilon^2\, c^{27,0}_{(3)}$	$7(2N-3\Lambda-\Sigma+2\Xi)-2(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)$	$1/N_c^2$	ϵ^2
M_8	$\{T^8,\{T^8,T^8\}\}$	$9\sqrt{3}\,\frac{1}{N_c^2}\,\epsilon^3\, c^{64,0}_{(3)}$	$\Delta-3\Sigma^*+3\Xi^*-\Omega$	$1/N_c^2$	ϵ^3
M_A			$(\Sigma^*-\Sigma)-(\Xi^*-\Xi)$	$1/N_c^2$	$-$
M_B			$\frac{1}{3}(\Sigma+2\Sigma^*)-\Lambda-\frac{2}{3}(\Delta-N)$	$1/N_c^2$	$-$
M_C			$-\frac{1}{4}(2N-3\Lambda-\Sigma+2\Xi)+\frac{1}{4}(\Delta-\Sigma^*-\Xi^*+\Omega)$	$1/N_c^2$	$-$
M_D			$-\frac{1}{2}(\Delta-3\Sigma^*+3\Xi^*-\Omega)$	$1/N_c^2$	$-$

$$R \equiv \frac{\sum_i c_i M_i}{\sum_i |c_i|}$$

$$\epsilon \propto m_s - m_l$$

Large N_c and SU(3) Chiral Perturbation Theory



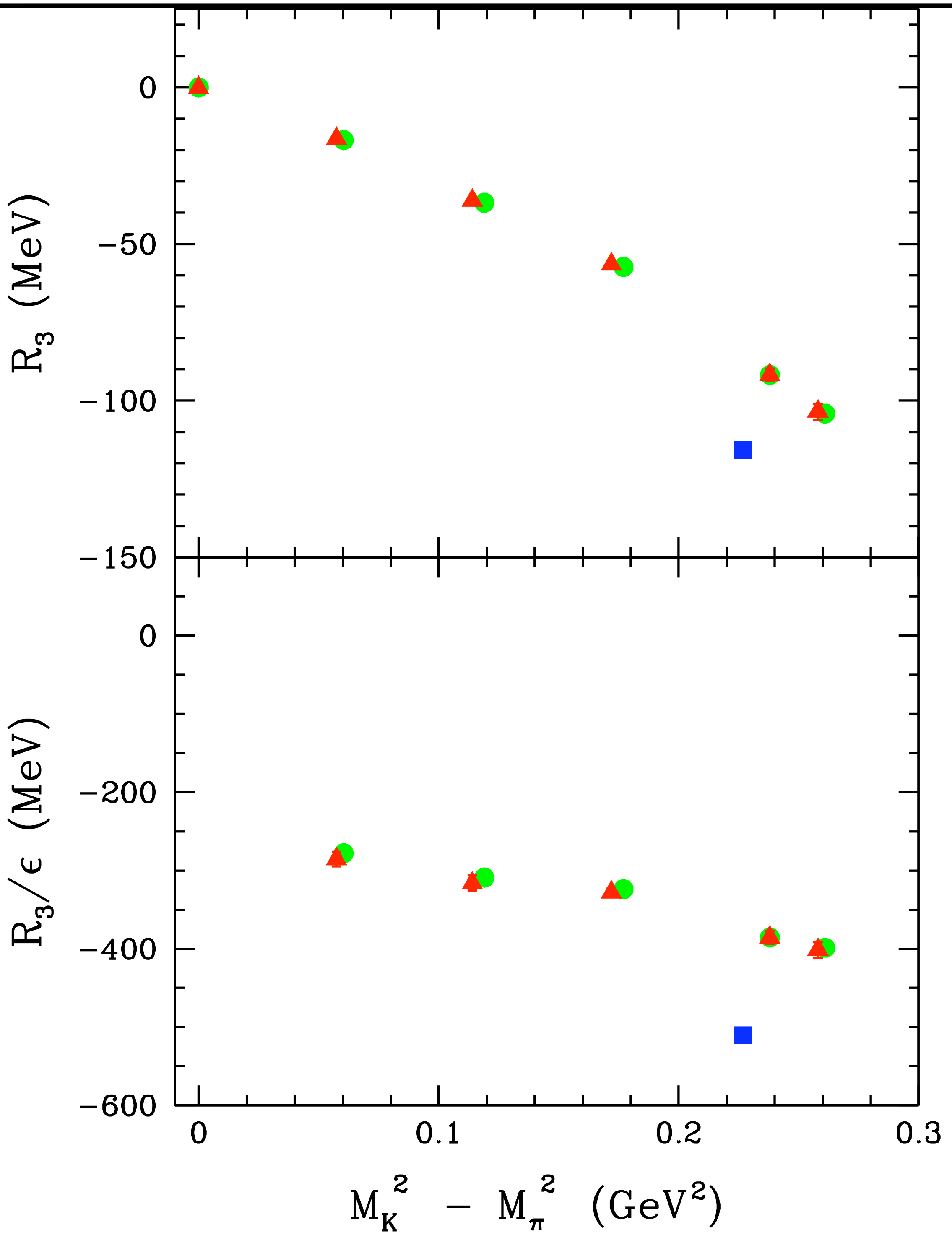


$$R_1 \sim \mathcal{O}(N_c) \times \mathcal{O}(\epsilon^0)$$

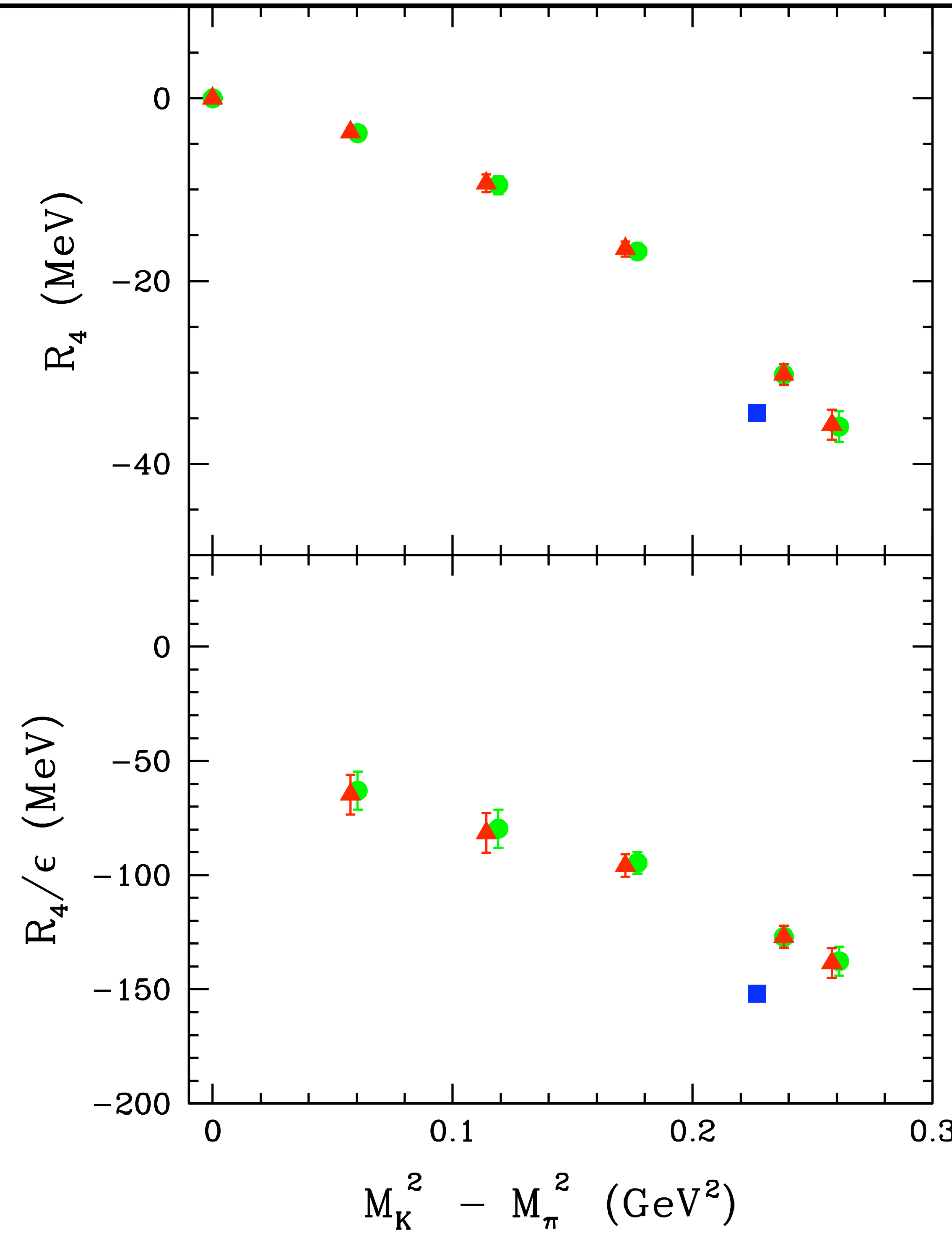
$$R_2 \sim \mathcal{O}(1/N_c) \times \mathcal{O}(\epsilon^0)$$

Jenkins, Manohar, Negele + AWL arXiv:0907.0529

Large N_c and SU(3) Chiral Perturbation Theory



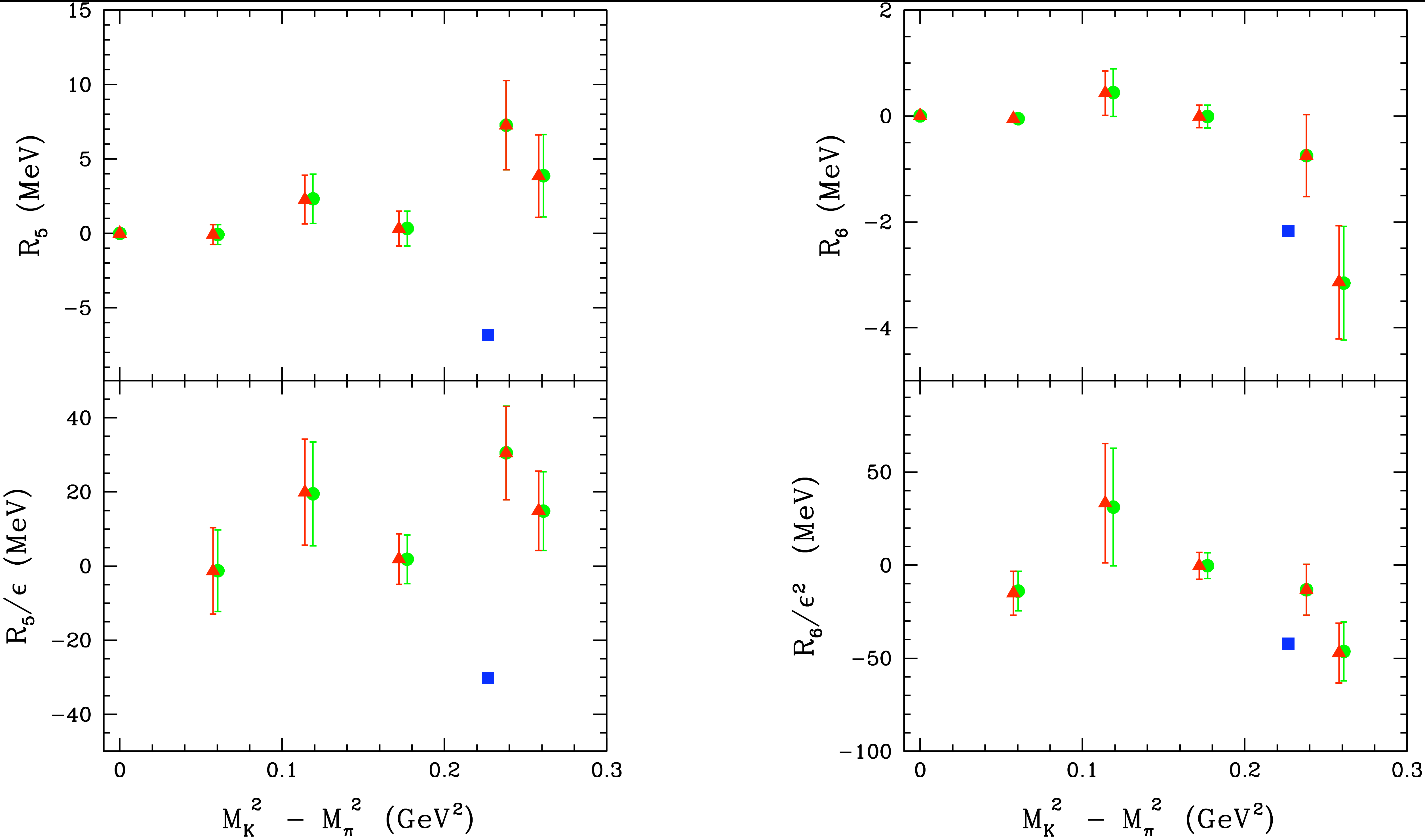
$$R_3 \sim \mathcal{O}(N_c^0) \times \mathcal{O}(\epsilon)$$



$$R_4 \sim \mathcal{O}(1/N_c) \times \mathcal{O}(\epsilon)$$

Jenkins, Manohar, Negele + AWL arXiv:0907.0529

Large N_c and SU(3) Chiral Perturbation Theory



$$R_5 \sim \mathcal{O}(1/N_c^2) \times \mathcal{O}(\epsilon) \qquad R_6 \sim \mathcal{O}(1/N_c) \times \mathcal{O}(\epsilon^2)$$

Jenkins, Manohar, Negele + AWL arXiv:0907.0529

Evidence for non-analytic light quark mass dependence arXiv:1112.2658

$$\mathcal{L} = i\ \mathrm{Tr}\ \bar{B}_v\left(v\cdot\mathcal{D}\right)B_v - i\ \bar{T}^\mu_v\left(v\cdot\mathcal{D}\right)T_{v\,\mu} - \frac{1}{4}\Delta_0\ Tr\ \bar{B}_v B_v + \frac{5}{4}\Delta_0\ \bar{T}^\mu_v T_{v\,\mu}$$
$$+ 2D\,\mathrm{Tr}\left(\bar{B}_v S^\mu_v\left\{\mathcal{A}_\mu, B_v\right\}\right) + 2F\,\mathrm{Tr}\left(\bar{B}_v S^\mu_v\left[\mathcal{A}_\mu, B_v\right]\right)$$
$$+ \mathcal{C}\ \left(\bar{T}^\mu_v\mathcal{A}_\mu B_v + \bar{B}_v\mathcal{A}_\mu T^\mu_v\right) + 2\mathcal{H}\,\bar{T}^\mu_v S^\nu_v\mathcal{A}_\nu T_{v\,\mu}$$
$$+ 2\sigma_B\ \mathrm{Tr}\left(\bar{B}_v B_v\right)\mathrm{Tr}\mathcal{M}_+ - 2\sigma_T\ \bar{T}^\mu_v T_{v\,\mu}\mathrm{Tr}\mathcal{M}_+$$
$$+ 2b_D\mathrm{Tr}\left(\bar{B}_v\left\{\mathcal{M}_+, B_v\right\}\right) + 2b_F\mathrm{Tr}\left(\bar{B}_v\left[\mathcal{M}_+, B_v\right]\right) + 2b_T\,\bar{T}^\mu_v\mathcal{M}_+ T_{v\,\mu}$$

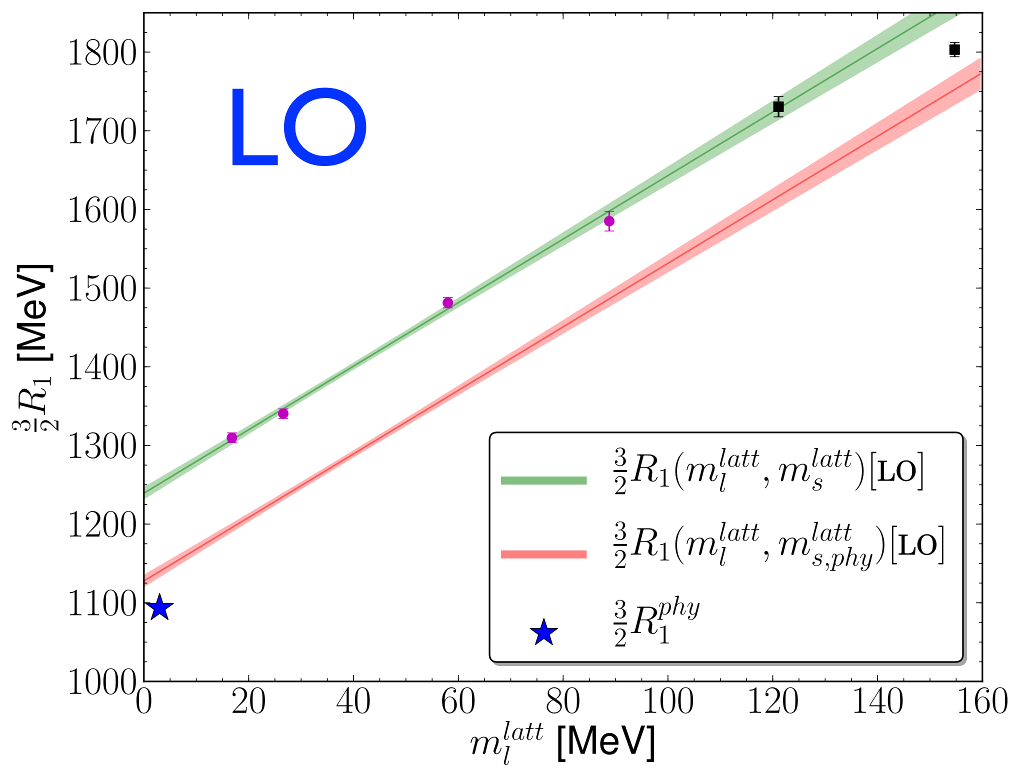
Large N_c expansion simplifies operators: Jenkins hep-ph/9509433

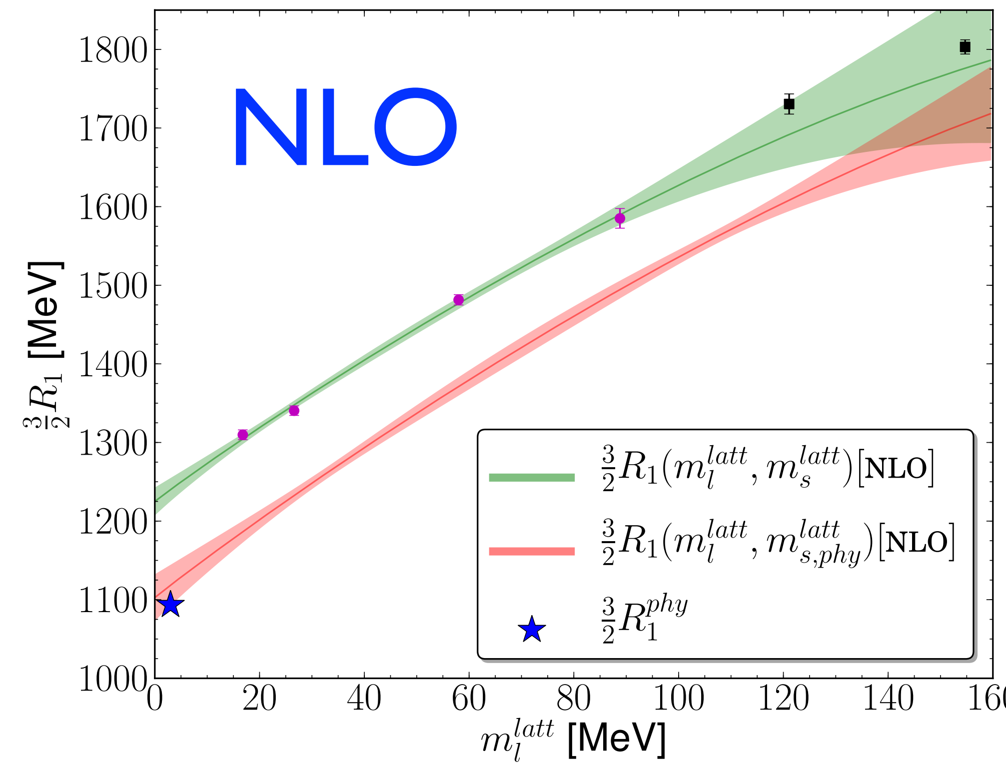
$$b_D = \frac{1}{4}b_{(2)}\,, \qquad b_F = \frac{1}{2}b_{(1)} + \frac{1}{6}b_{(2)}\,, \qquad b_T = -\frac{3}{2}b_{(1)} - \frac{5}{4}b_{(2)}$$
$$\sigma_B = \frac{1}{2}b_{(1)} + \frac{1}{12}b_{(2)}\,, \qquad \sigma_T = \frac{1}{2}b_{(1)} + \frac{5}{12}b_{(2)}\,.$$

$$D = \frac{1}{2}a_{(1)}\,, \qquad F = \frac{1}{3}a_{(1)} + \frac{1}{6}a_{(2)}\,,$$
$$\mathcal{C} = -a_{(1)}\,, \qquad \mathcal{H} = -\frac{3}{2}a_{(1)} - \frac{3}{2}a_{(2)}$$

$$\mathcal{C} = -2D,$$
$$\mathcal{H} = 3D - F.$$

Evidence for non-analytic light quark mass dependence arXiv:1112.2658



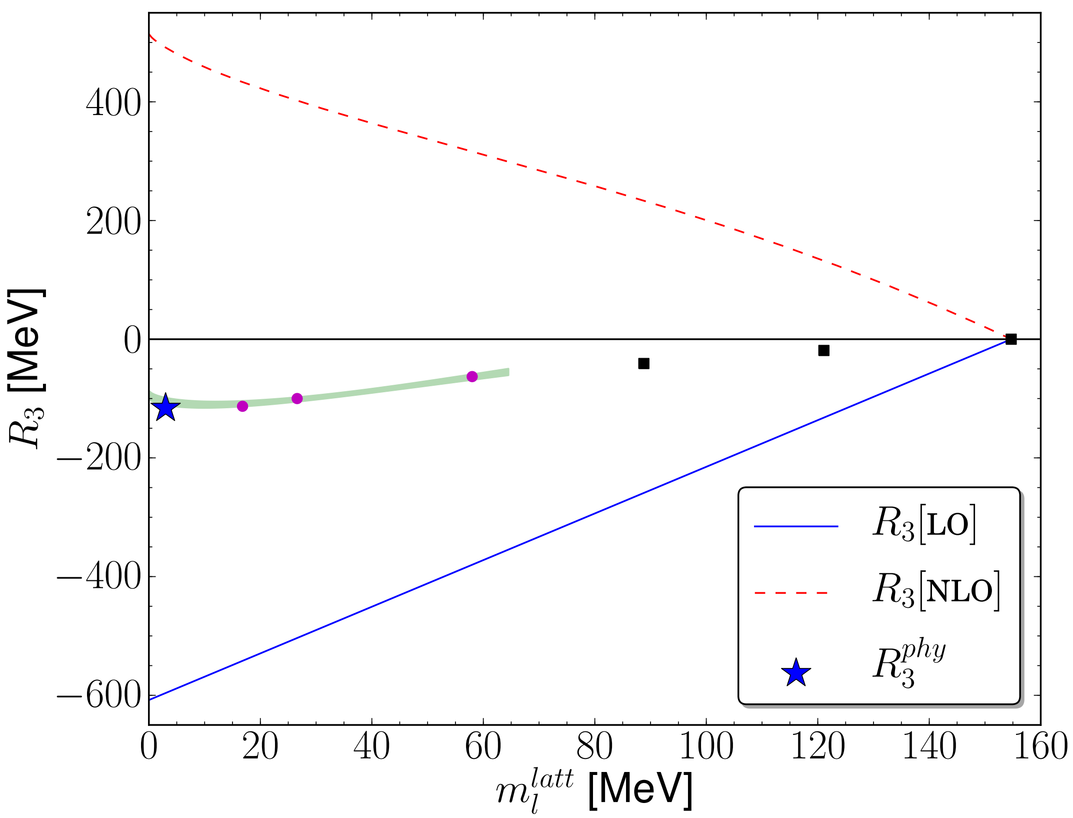


$$
\begin{aligned}
\frac{3}{2}\, R_1(m_l, m_s) = M_0 &- \left(\frac{3}{4}b_{(1)} + \frac{5}{24}b_{(2)}\right)(2m_l + m_s) \\
&- \frac{1}{12}\left(35a_{(1)}^2 - 5a_{(2)}^2\right)\left(\frac{3\mathcal{F}(m_\pi,0,\mu) + 4\mathcal{F}(m_K,0,\mu) + \mathcal{F}(m_\eta,0,\mu)}{8(4\pi f)^2}\right) \\
&- \frac{1}{12}a_{(1)}^2\left[50\left(\frac{3\mathcal{F}(m_\pi,\Delta,\mu) + 4\mathcal{F}(m_K,\Delta,\mu) + \mathcal{F}(m_\eta,\Delta,\mu)}{8(4\pi f)^2}\right)\right. \\
&\left. - 4\left(\frac{3\mathcal{F}(m_\pi,-\Delta,\mu) + 4\mathcal{F}(m_K,-\Delta,\mu) + \mathcal{F}(m_\eta,-\Delta,\mu)}{8(4\pi f)^2}\right)\right]
\end{aligned}
$$

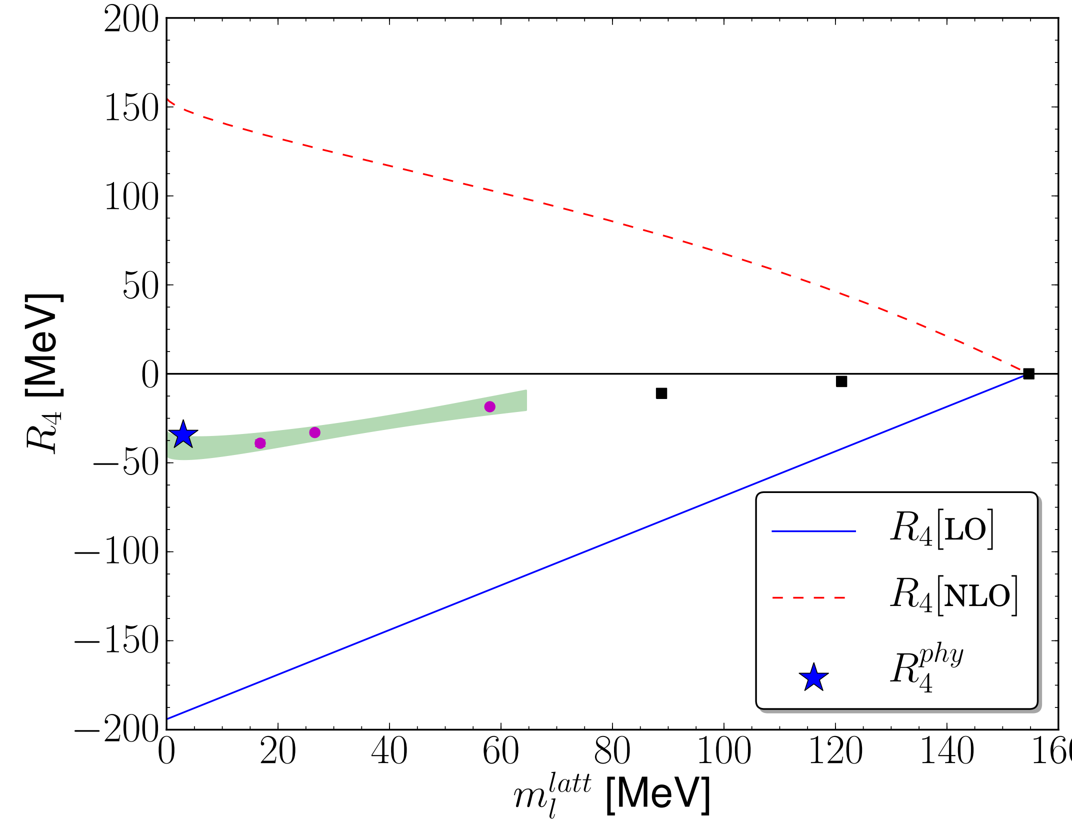
$a_{(1)} = 0.2(5)$ → $D = 0.10(25)$

Evidence for non-analytic light quark mass dependence arXiv:1112.2658

$R_3 \propto m_s - m_l$



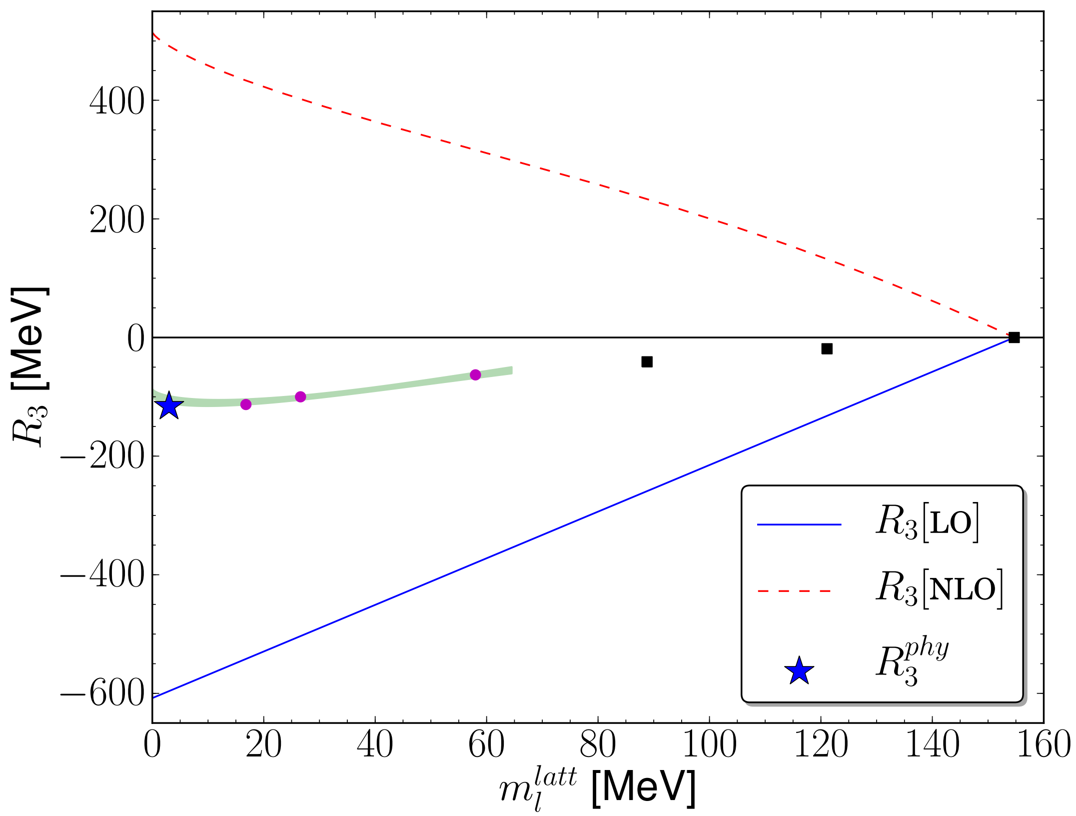
$R_4 \propto (m_s - m_l)/N_c$

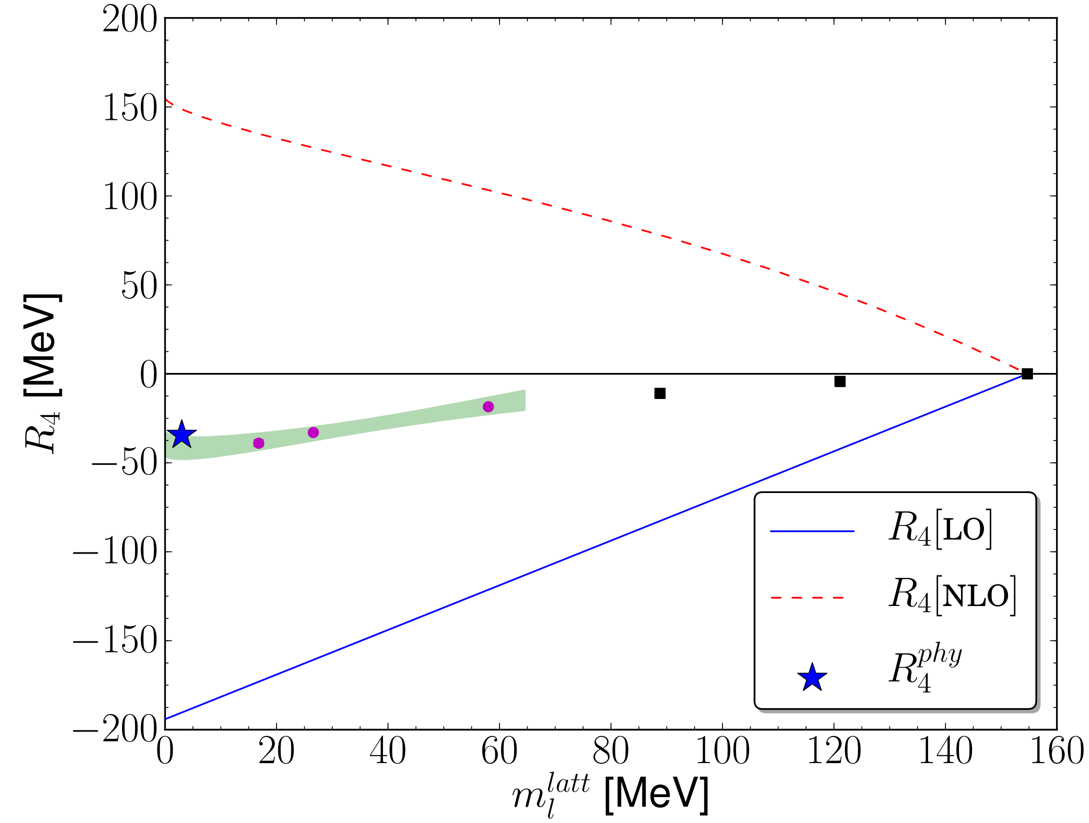


$$R_3(m_l, m_s) = \frac{20}{39}\, b_1\,(m_s - m_l) - \frac{20a_1^2 - 5a_2^2}{117}\,\frac{3\mathcal{F}_\pi^0 - 2\mathcal{F}_K^0 - \mathcal{F}_\eta^0}{(4\pi f)^2} - \frac{a_1^2}{117}\left[35\frac{3\mathcal{F}_\pi^\Delta - 2\mathcal{F}_K^\Delta - \mathcal{F}_\eta^\Delta}{(4\pi f)^2} - \frac{3\mathcal{F}_\pi^{-\Delta} - 2\mathcal{F}_K^{-\Delta} - \mathcal{F}_\eta^{-\Delta}}{(4\pi f)^2}\right],$$

$$R_4(m_l, m_s) = -\frac{5}{18}\, b_2\,(m_s - m_l) + \frac{a_1^2 + 4a_1a_2 + a_2^2}{36}\,\frac{3\mathcal{F}_\pi^0 - 2\mathcal{F}_K^0 - \mathcal{F}_\eta^0}{(4\pi f)^2} - \frac{2a_1^2}{9}\,\frac{3\mathcal{F}_\pi^\Delta - 2\mathcal{F}_K^\Delta - \mathcal{F}_\eta^\Delta}{(4\pi f)^2}$$

Evidence for non-analytic light quark mass dependence arXiv:1112.2658





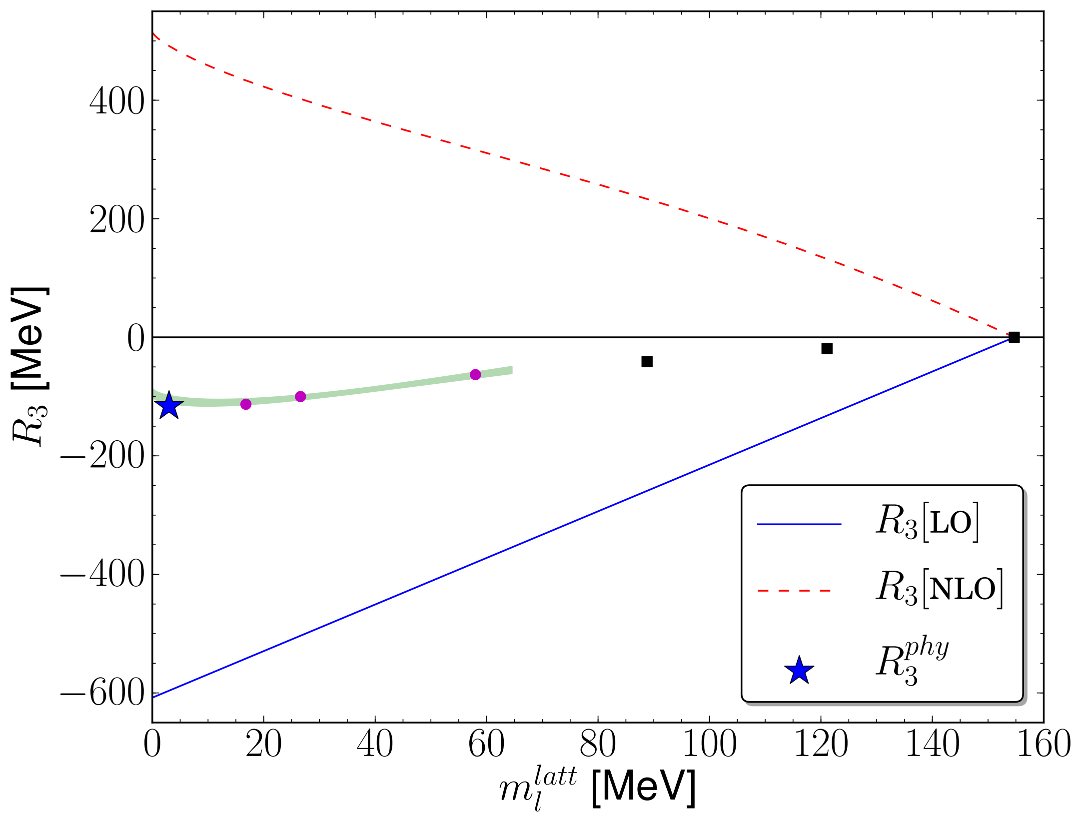
Fit yields

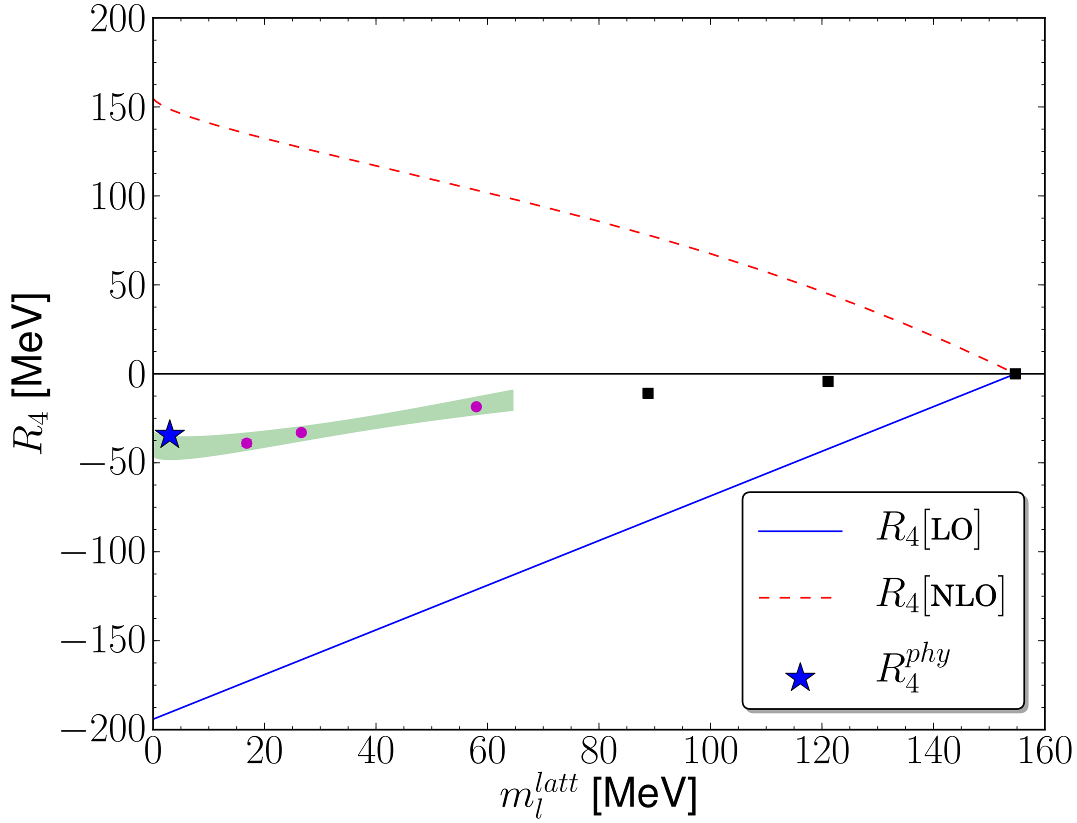
$$b_1[\mathrm{NLO}] = -6.6(5)\,, \qquad b_2[\mathrm{NLO}] = 4.3(4)\,, \qquad a_1[\mathrm{NLO}] = 1.4(1)\,.$$

$$D = 0.70(5)\,, \qquad F = 0.47(3)\,, \qquad C = -1.4(1)\,, \qquad H = -2.1(2)$$

First time axial couplings left as free parameters and: values consistent with phenomenological determinations

Evidence for non-analytic light quark mass dependence arXiv:1112.2658





Fit yields

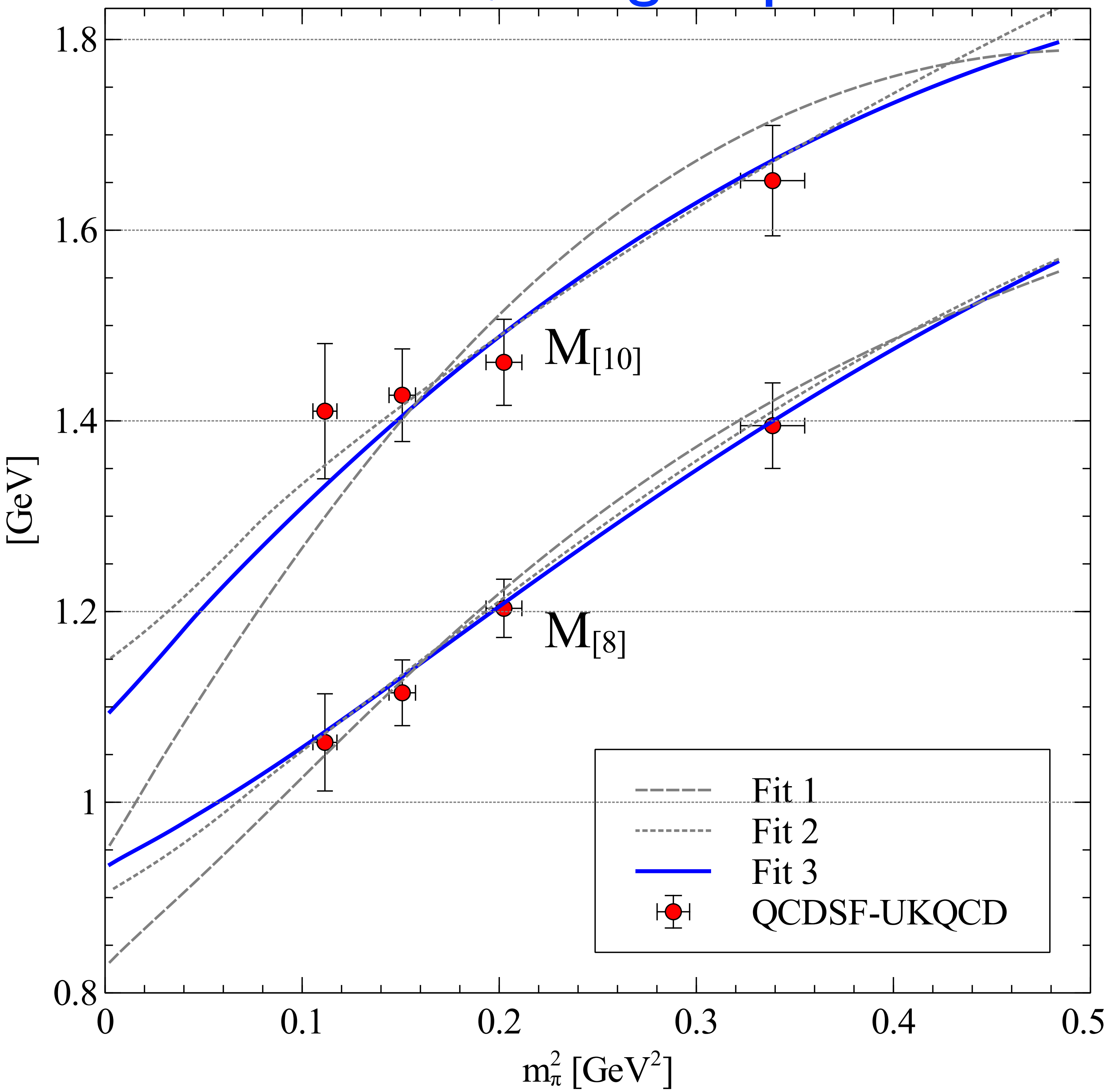
$$b_1[\text{NLO}] = -6.6(5)\,, \qquad b_2[\text{NLO}] = 4.3(4)\,, \qquad a_1[\text{NLO}] = 1.4(1)\,.$$

$$D = 0.70(5)\,, \qquad F = 0.47(3)\,, \qquad C = -1.4(1)\,, \qquad H = -2.1(2)$$

but still observe large cancellations between LO and NLO

Evidence for non-analytic light quark mass dependence arXiv:1112.2658

Also - I would like to draw attention to the work of Mathias Lutz and Alexandre Semke who fit the masses (not mass splittings) of 4 different lattice QCD groups, and obtained similar axial couplings



Fit i:

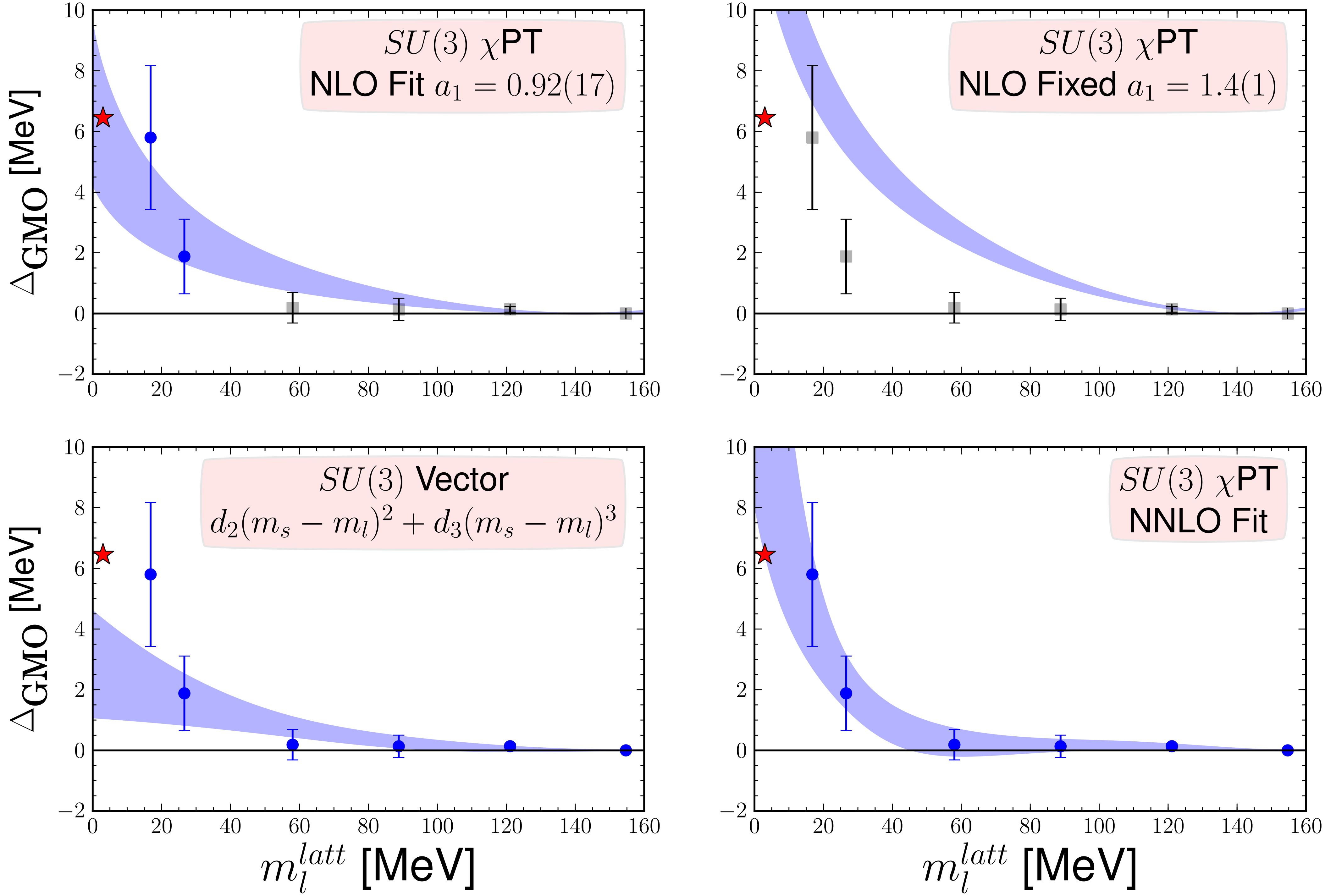
each fit is to set of BMW, LHPC, PACS-CS

none of the fits include QCDSF-UKQCD, who computed masses in SU(3) limit as well as SU(3)-broken (with similar agreement)

I do not understand - but this agreement is remarkable

Evidence for non-analytic light quark mass dependence arXiv:1112.2658

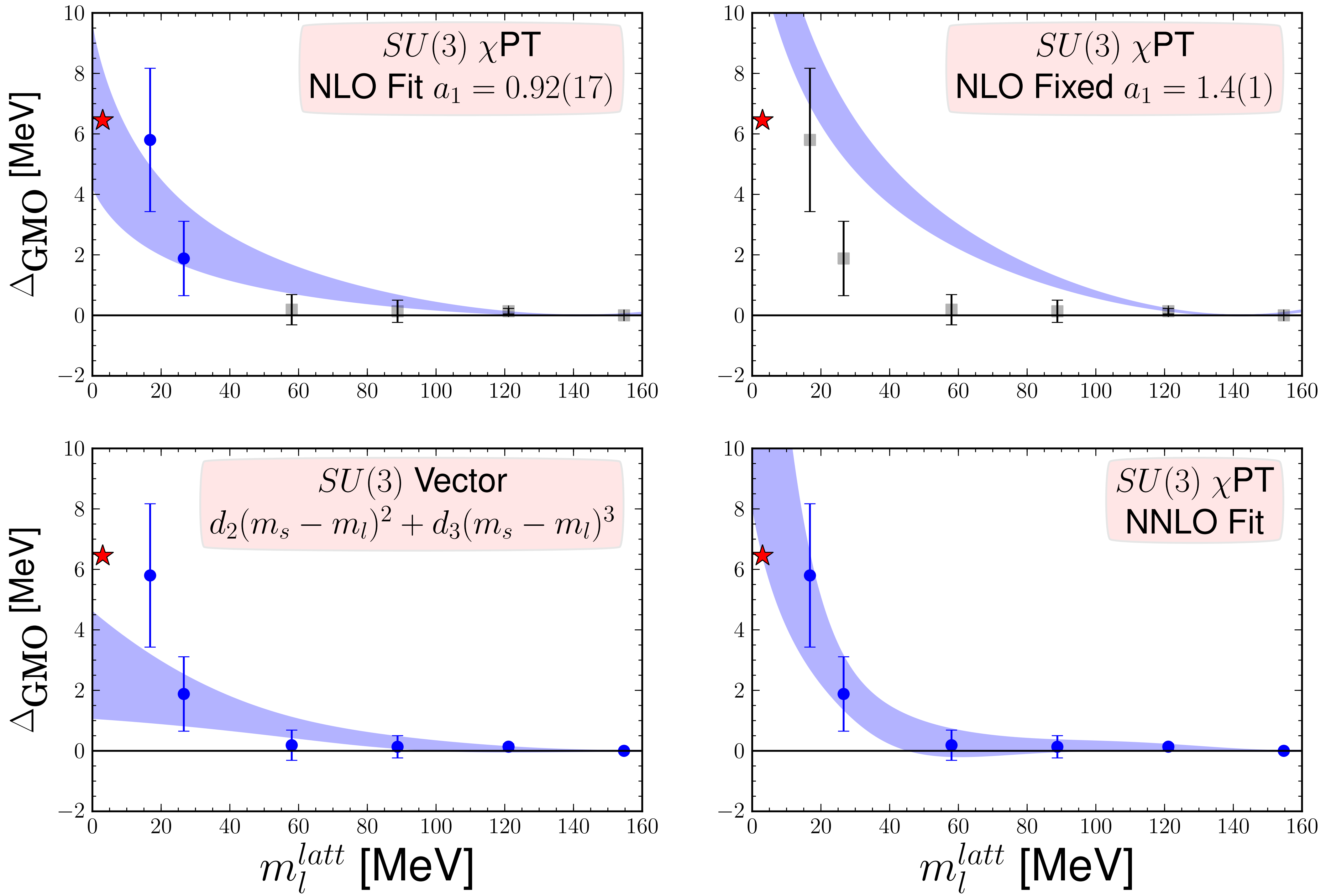
Gell-Mann--Okubo Relation



Only NNLO SU(3) naturally supports strong light quark mass dependence

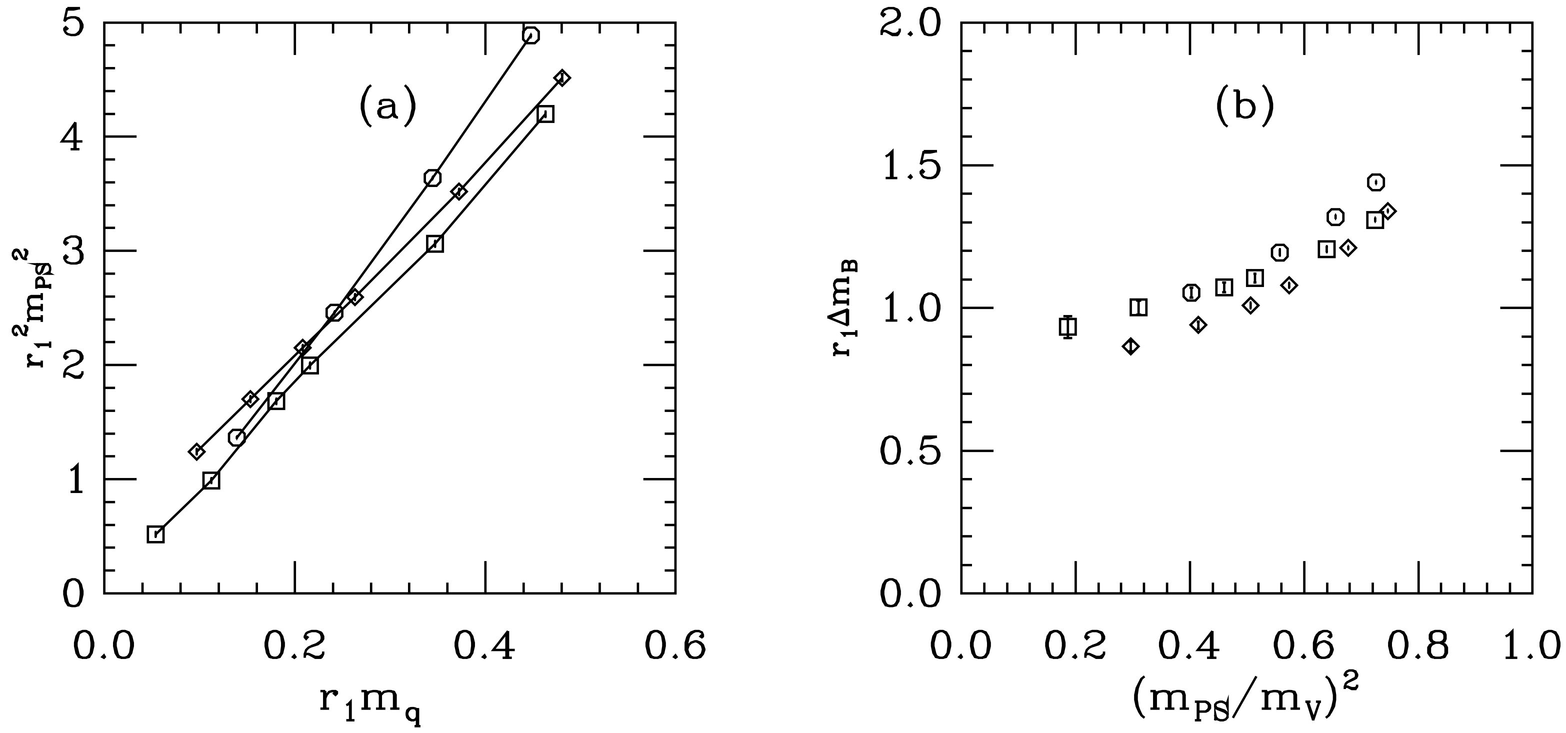
Evidence for non-analytic light quark mass dependence arXiv:1112.2658

Gell-Mann--Okubo Relation



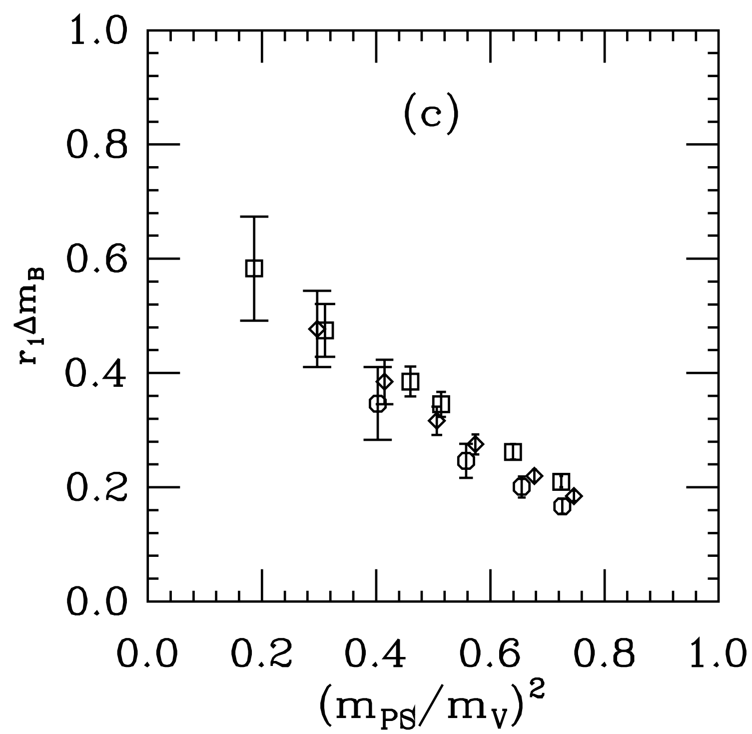
Combined with R3 and R4 - provides first compelling evidence of non-analytic light quark mass dependence in the baryon spectrum

Large N_c and SU(3) Chiral Perturbation Theory



Tom Degrand: arXiv:1205.0235
heroically performing quenched QCD calculations for
N_c = 3, 5, 7

$$(b): \quad A \quad \text{(constituent quark mass)}$$



$$M(N_c, J) = N_c A + \frac{J(J+1)}{N_c} B$$

$$(c): \; M(N_c, J = N_c/2) - M(N_c, J = N_c/2 - 1)$$

Baryons in lattice QCD: Conclusions II

- the more I study baryons, the more confused I get
  - there now seems to be un-ignorable evidence for entirely unexpected light quark mass dependence in the nucleon (baryon) spectrum, basically down to the physical pion mass

$$M_N = \alpha_0 + \alpha_1 m_\pi$$

  - combining large N_c with SU(2) and SU(3) flavor symmetry is showing promise - at least qualitatively
  - what is clearly (still) needed is high statistics study of baryons with (with the aim of understanding chiral perturbation theory)

$$120 \leq m_\pi \leq 400 \text{ MeV}$$

Baryons and lattice QCD

electromagnetic self-energy of M_p - M_n: Cottingham Formula

- self-energy related to forward Compton scattering
- in principle, allows for robust, model independent determination of self-energy through dispersion theory
- two challenges in realizing this method

  requires renormalization — Collins Nucl.Phys. B149 (1979)

  requires subtracted dispersion integral

  Harari PRL 17(1966)
  Abarbanel and Nussinov Phys.Rev. 158 (1967)

→ unknown subtraction function

AWL, Carl Carlson, Jerry Miller: PRL 108 (2012)

Baryons and lattice QCD

composition of early universe, exponentially sensitive to isovector nucleon mass:

primordial ratio $$\frac{X_n}{X_p} = e^{-(m_n - m_p)/kT}$$

$$m_n - m_p = 1.29333217(42)\ \mathrm{MeV}$$

$$m_n - m_p = \underbrace{\delta M^{\gamma}_{n-p} + \delta M^{m_d - m_u}_{n-p}}$$

this separation only
at LO in isospin breaking

$\langle N|(m_d - m_u)\bar{q}q|N\rangle$ needed to renormalize EM self-energy

Baryons and lattice QCD

my original interest was to use lattice QCD calculations of

$$m_n - m_p = \alpha(m_d - m_u) + \dots$$

as an independent method to determine $m_d - m_u$

However, this requires subtracting from experimental value the electromagnetic self-energy contribution

$$\delta M^{\gamma}_{p-n} = 0.76(30)\ \mathrm{MeV},$$

Gasser and Leutwyler
Nucl. Phys. B94 (1975)
Phys. Rept. 87 (1982)

the uncertainty in this determination of the electromagnetic self-energy dominates the determination of $m_d - m_u$

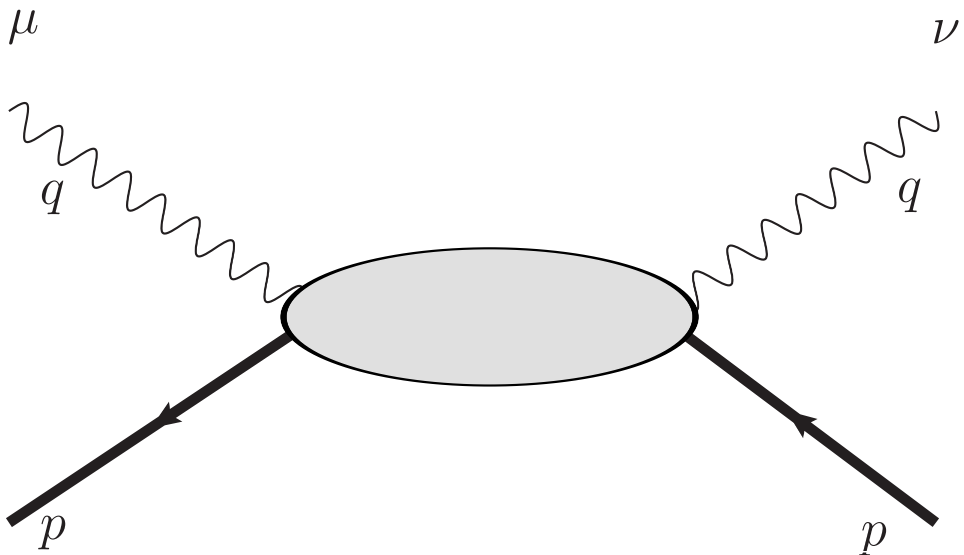
so lets try and improve this with modern knowledge of nucleon Compton scattering

Cottingham's Formula

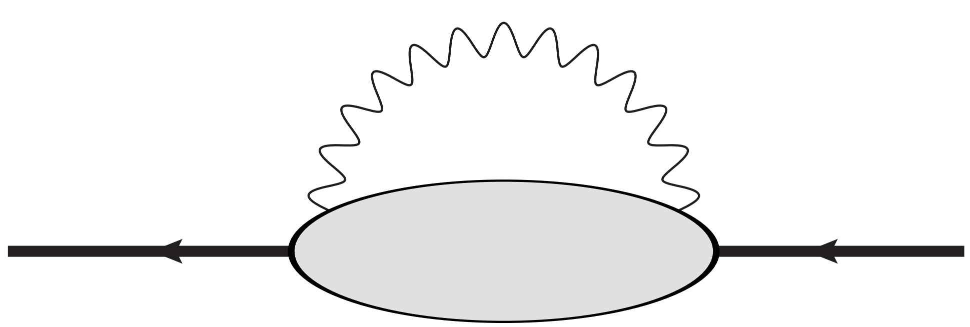
AWL, C.Carlson, G.Miller: PRL 108 (2012)

Cini, Ferrari, Gato: PRL 2 (1959)

Cottingham: Annals Phys 25 (1963)



$$T_{\mu\nu} = \frac{i}{2}\sum_{\sigma}\int d^4\xi \; e^{iq\cdot\xi}\langle p\sigma|T\left\{J_\mu(\xi)J_\nu(0)\right\}|p\sigma\rangle$$



$$\delta M^{\gamma} = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int_R d^4q\frac{T^{\mu}_{\mu}(p,q)}{q^2+i\epsilon}$$

$$\alpha = \frac{e^2}{4\pi}$$

Integral diverges and must be renormalized

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$\delta M^\gamma = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int_R d^4q \frac{T^\mu_\mu(p,q)}{q^2+i\epsilon}$$

- Wick rotate $\quad q^0 \to i\nu$
- variable transform $\quad Q^2 = \mathbf{q}^2 + \nu^2$

$$\delta M^\gamma = \frac{\alpha}{8\pi^2}\int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\frac{T^\mu_\mu}{M} + \delta M^{ct}(\Lambda)$$

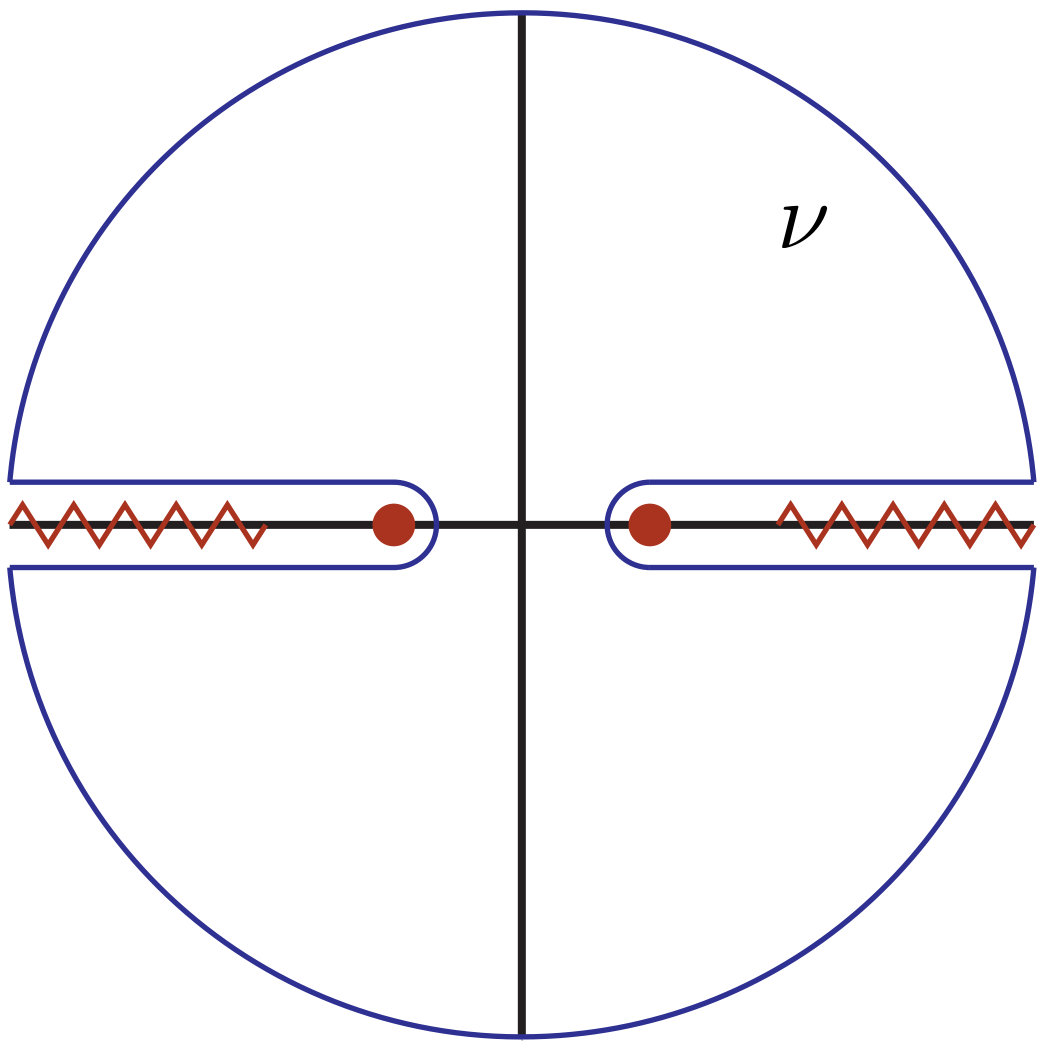
$$T^\mu_\mu = -3\,T_1(i\nu,Q^2) + \left(1-\frac{\nu^2}{Q^2}\right)T_2(i\nu,Q^2)\,, \qquad (7a)$$

$$= -3Q^2\,t_1(i\nu,Q^2) + \left(1+2\frac{\nu^2}{Q^2}\right)Q^2 t_2(i\nu,Q^2)\,. \qquad (7b)$$

use dispersion integrals to evaluate scalar functions

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)



$$T_i(\nu, Q^2) = \frac{1}{2\pi} \oint d\nu' \, \frac{T_i(\nu', Q^2)}{\nu' - \nu}$$

Crossing Symmetric

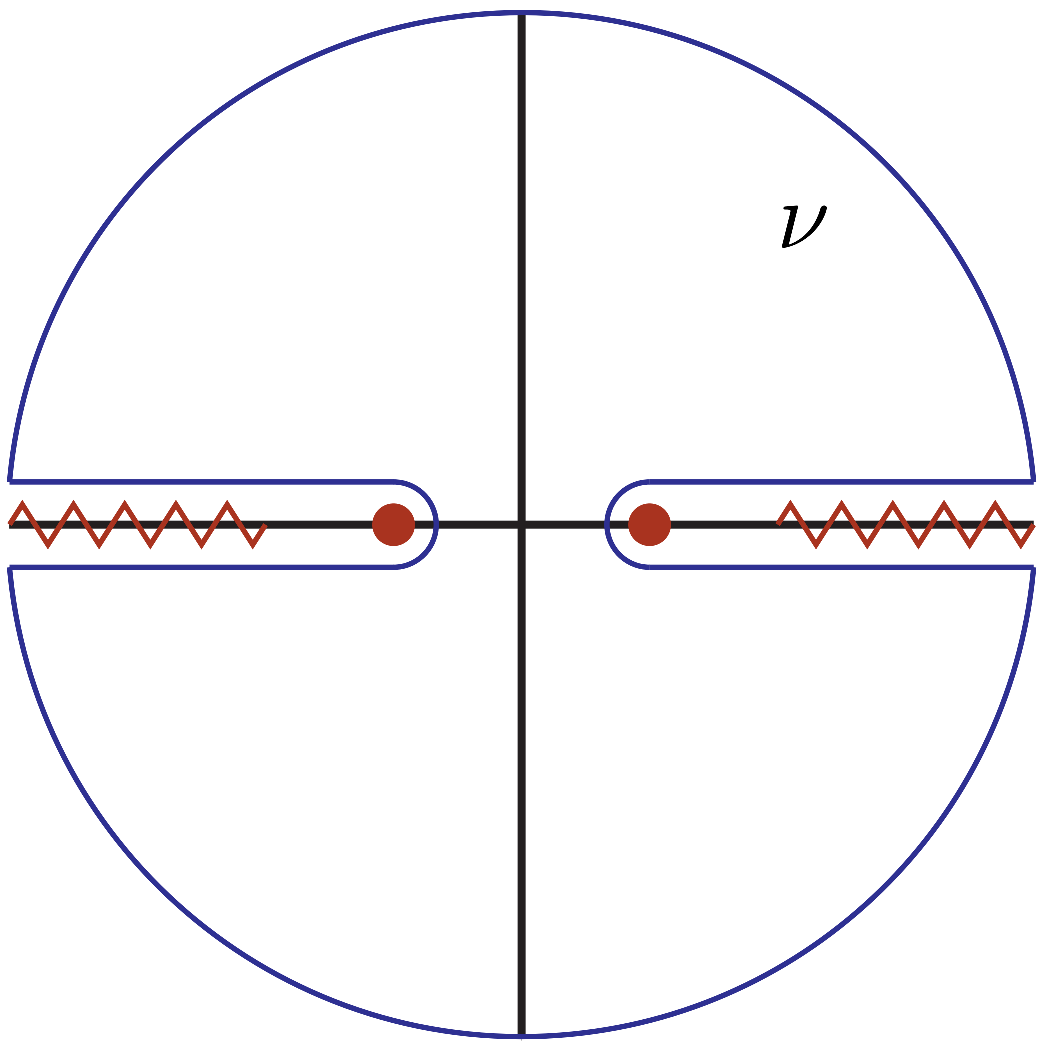
$$T_i(\nu, Q^2) = T_i(-\nu, Q^2)$$

$$T_i(\nu, Q^2) = \frac{1}{2\pi} \int_{\nu_t}^{\infty} d\nu' \, \frac{2\nu'}{(\nu')^2 - \nu^2} 2\mathrm{Im} T_i(\nu' + i\epsilon, Q^2)$$

(provided contour and infinity vanishes)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)



if contour at infinity does not vanish

subtracted dispersion integral

$$g(\nu) = \frac{T_i(\nu, Q^2)}{\nu^2}$$

introduces new pole at $\nu = 0$
which you need to subtract

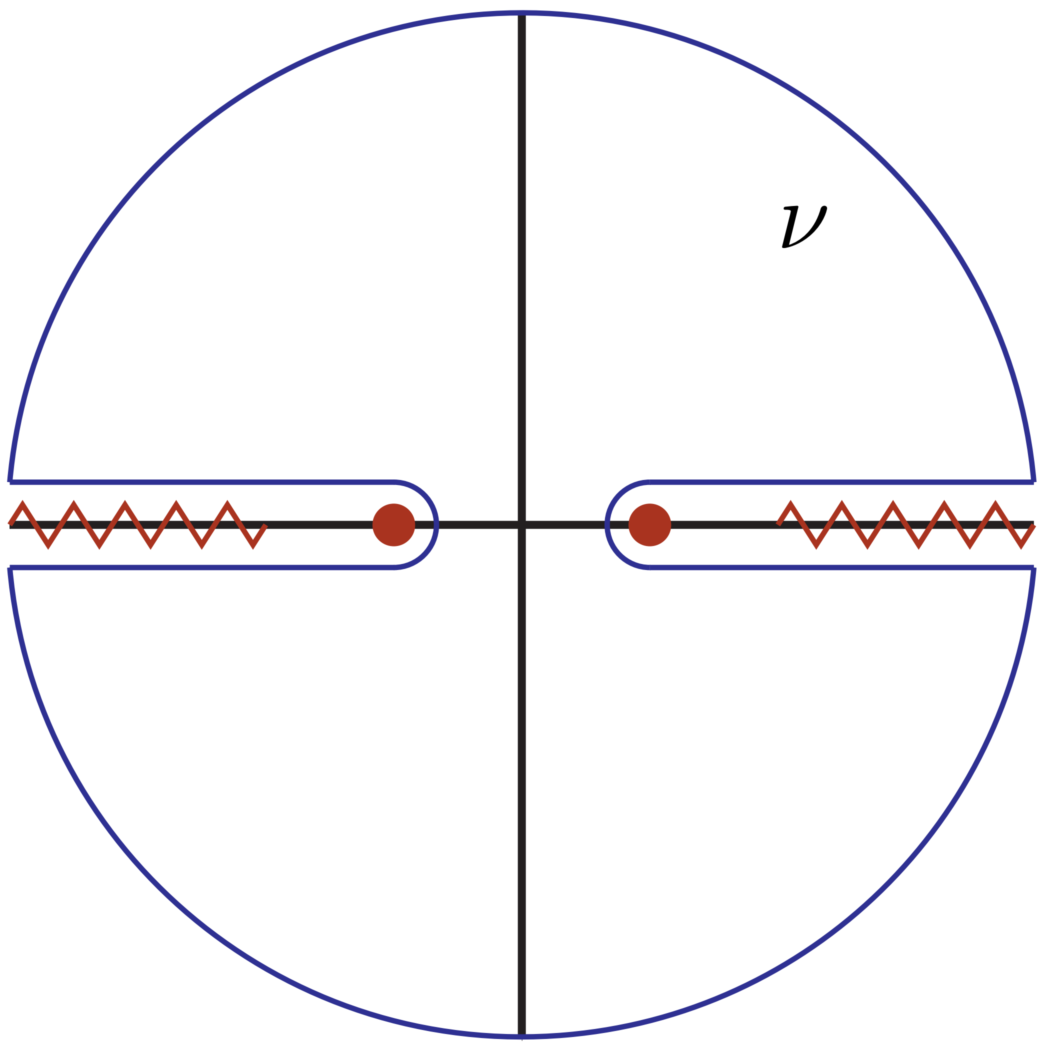
$$T_i(\nu, Q^2) = \frac{\nu^2}{2\pi} \int_{\nu_t}^{\infty} d\nu' \frac{2\nu'}{\nu'^2(\nu'^2 - \nu^2)} 2\mathrm{Im}T_i(\nu' + i\epsilon, Q^2) + T_i(0, Q^2)$$

measured experimentally

unknown function

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)



It is known that

$$T_2(\nu, Q^2) \quad [t_2(\nu, Q^2)]$$

satisfies unsubtracted dispersion integral while

$$T_1(\nu, Q^2) \quad [t_1(\nu, Q^2)]$$

requires a subtraction

Regge behavior

$$\mathrm{Im} t_1 [T_1] \Big|_{p-n} \propto \nu^{1/2}$$

H. Harari: PRL 17 (1966)

H.D. Abarbanel S. Nussinov: Phys.Rev. 158 (1967)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

at the time, introducing an unknown subtraction function would be disastrous for getting a precise value:

they provided an argument based upon various assumptions to avoid the subtracted dispersive integral

$$\delta M^{\gamma}_{p-n} = 0.76(30)\ \mathrm{MeV}$$

central value: from elastic contribution

uncertainty: estimates of inelastic structure contributions

however, one can show their arguments are incorrect:

one must face the subtraction function

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

$$\delta M^{\gamma} = \frac{\alpha}{8\pi^2} \int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2} \frac{T_\mu^\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T_\mu^\mu = -3\, T_1(i\nu, Q^2) + \left(1 - \frac{\nu^2}{Q^2}\right) T_2(i\nu, Q^2)\,, \qquad (7a)$$

$$= -3Q^2\, t_1(i\nu, Q^2) + \left(1 + 2\frac{\nu^2}{Q^2}\right) Q^2 t_2(i\nu, Q^2)\,. \qquad (7b)$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$T_{\mu\nu} = \frac{i}{2}\sum_\sigma \int d^4\xi\ e^{iq\cdot\xi}\langle p\sigma|T\left\{J_\mu(\xi)J_\nu(0)\right\}|p\sigma\rangle$$

Insert complete set of states:

isolate elastic contributions

$$1 = \sum_\Gamma |\Gamma\rangle\langle\Gamma|$$

$$\delta M^{el}_{unsub,a} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ \Big\{ \left[G_E^2(Q^2) - 2\tau_{el}G_M^2(Q^2)\right] \frac{(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}}}{1+\tau_{el}} - \frac{3}{2}G_M^2(Q^2)\frac{{\tau_{el}}^{3/2}}{1+\tau_{el}}\Big\}, \quad (8a)$$

$$\delta M^{el}_{unsub,b} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ \Big\{ \left[G_E^2(Q^2) - 2\tau_{el}\, G_M^2(Q^2)\right] \frac{(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2}}{1+\tau_{el}} + 3G_M^2(Q^2)\frac{{\tau_{el}}^{3/2}}{1+\tau_{el}}\Big\}, \quad (8b)$$

typically quoted as elastic Cottingham

$$\delta M^\gamma = \frac{\alpha}{8\pi^2}\int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\frac{T^\mu_\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T^\mu_\mu = -3\,T_1(i\nu,Q^2) + \left(1 - \frac{\nu^2}{Q^2}\right)T_2(i\nu,Q^2)\,, \quad (7a)$$

$$= -3Q^2\,t_1(i\nu,Q^2) + \left(1 + 2\frac{\nu^2}{Q^2}\right)Q^2 t_2(i\nu,Q^2)\,. \quad (7b)$$

One must use a subtracted dispersive integral even for elastic terms

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

perform once subtracted dispersion integral for $T_1(t_1)$
and unsubtracted dispersion integral for $T_2(t_2)$

$$\delta M^{\gamma} = \delta M^{el} + \delta M^{inel} + \delta M^{sub} + \delta \tilde{M}^{ct}$$

$$\delta M^{el} = \frac{\alpha}{\pi} \int_0^{\Lambda_0^2} dQ \left\{ \frac{3\sqrt{\tau_{el}} G_M^2}{2(1+\tau_{el})} + \frac{\left[G_E^2 - 2\tau_{el}\, G_M^2\right]}{1+\tau_{el}} \left[(1+\tau_{el})^{3/2} - \tau_{el}{}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}} \right] \right\}$$

$$\delta M^{inel} = \frac{\alpha}{\pi} \int_0^{\Lambda_0^2} \frac{dQ^2}{2Q} \int_{\nu_{th}}^{\infty} d\nu \left\{ \frac{3F_1(\nu, Q^2)}{M} \left[\frac{\tau^{3/2} - \tau\sqrt{1+\tau} + \sqrt{\tau}/2}{\tau} \right] + \frac{F_2(\nu, Q^2)}{\nu} \left[(1+\tau)^{3/2} - \tau^{3/2} - \frac{3}{2}\sqrt{\tau} \right] \right\},$$

$$\tau_{el} = \frac{Q^2}{4M^2} \qquad \tau = \frac{\nu^2}{Q^2}$$

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M} \int_0^{\Lambda_0^2} dQ^2 \; T_1(0, Q^2)\,,$$

$$\delta \tilde{M}^{ct} = -\frac{3\alpha}{16\pi M} \int_{\Lambda_0^2}^{\Lambda_1^2} dQ^2 \sum_i C_{1,i} \langle \mathcal{O}^{i,0} \rangle\,,$$

OPE: operators and Wilson coeffic.
J.C. Collins: Nucl. Phys. B149 (1979)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

what is the flaw in the argument?

$$\delta M^{\gamma} = \frac{\alpha}{8\pi^2} \int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2 - \nu^2}}{Q^2} \frac{T_\mu^\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T_\mu^\mu = -3\, T_1(i\nu, Q^2) + \left(1 - \frac{\nu^2}{Q^2}\right) T_2(i\nu, Q^2)\,, \qquad (7a)$$

$$= -3 Q^2\, t_1(i\nu, Q^2) + \left(1 + 2\frac{\nu^2}{Q^2}\right) Q^2 t_2(i\nu, Q^2)\,. \qquad (7b)$$

is there some motivation to pick t_i vs T_i ?

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

what is the flaw in the argument?

in the point limit (electron) $t_1(\nu, Q^2) = 0!$

for the nucleon (with motivated resummations) the elastic contribution is

$$t_1(\nu, Q^2) = \frac{2}{Q^2}\left[\frac{Q^4\frac{G_M^2 - G_E^2}{1+\tau}}{(Q^2 - i\epsilon)^2 - 4M^2\nu^2} - \underbrace{\left(F_1^2 - \frac{G_E^2 + \tau G_M^2}{1+\tau}\right)}\right]$$

"Fixed-Pole" missed by unsubtracted dispersion relation

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

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numerically, this term is negligible

Cottingham's Formula

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

what is the flaw in the argument?

in the point limit (electron) $t_1(\nu, Q^2) = 0!$

real problem comes in the Regge limit: Q^2 fixed, $\nu \to \infty$

$$\mathrm{Im}t_1(\nu, Q^2) = \frac{\pi M \nu}{Q^4}\left[2xF_1(x, Q^2) - F_2(x, Q^2)\right] \qquad x = \frac{Q^2}{2M\nu}$$

in the strict DIS limit: Callan-Gross relation

$$2xF_1(x) - F_2(x) = 0$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

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Gasser and Leutwyler assumed

$$2xF_1(x, Q^2) - F_2(x, Q^2) = \frac{H_1(x)}{\nu}$$

if this were true, their argument would go through, however...

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

what is the flaw in the argument?

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Zee, Wilczek and Treiman Phys.Rev. D10 (1974)

$$2xF_1(x) - F_2(x) = \frac{-32}{9}\frac{\alpha_s(Q^2)}{4\pi}F_2(x)$$

Both IR and UV safe

This criticism first given by J.C. Collins: Nucl. Phys. B149 (1979)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

what is the flaw in the argument?

in the point limit (electron) $t_1(\nu, Q^2) = 0!$

real problem comes in the Regge limit: Q^2 fixed, $\nu \to \infty$

$$\lim_{x \to 0} F_2^{p-n}(x) \propto x^{1/2} \qquad x = \frac{Q^2}{2M\nu}$$

$$\mathrm{Im}\, t_1^{p-n}(\nu, Q^2) \propto \alpha_s(Q^2) \frac{\sqrt{M\nu}}{Q^3}$$

→ subtracted dispersion integral is unavoidable evaluation of various contributions

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

elastic contribution: use well measured form factors

$$\delta M^{el} = \frac{\alpha}{\pi}\int_0^{\Lambda_0^2} dQ \left\{ \frac{3\sqrt{\tau_{el}} G_M^2}{2(1+\tau_{el})} + \frac{\left[G_E^2 - 2\tau_{el}\, G_M^2\right]}{1+\tau_{el}} \left[(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}}\right] \right\}$$

$$\delta M^{el}\Big|_{p-n} = 1.39(02)\ \text{MeV}$$

- insensitive to value of Λ_0 since form factors fall as $1/Q^4$
- uncertainty from Monte Carlo evaluation of parameters describing form factors

central values: $\Lambda_0^2 = 2\ \text{GeV}^2$

uncertainties: $1.5\ \text{GeV}^2 \leq \Lambda_0^2 \leq 2.5\ \text{GeV}^2$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

inelastic terms: use modern knowledge of structure functions to improve determination of inelastic contributions

$$\delta M^{inel} = \frac{\alpha}{\pi} \int_0^{\Lambda_0^2} \frac{dQ^2}{2Q} \int_{\nu_{th}}^{\infty} d\nu \Bigg\{ \frac{3F_1(\nu,Q^2)}{M} \left[\frac{\tau^{3/2} - \tau\sqrt{1+\tau} + \sqrt{\tau}/2}{\tau} \right] + \frac{F_2(\nu,Q^2)}{\nu} \left[(1+\tau)^{3/2} - \tau^{3/2} - \frac{3}{2}\sqrt{\tau} \right] \Bigg\},$$

$$\delta M^{inel}\Big|_{p-n} = 0.057(16)\ \mathrm{MeV}$$

- contributions from two regions:
  - resonance region — Bosted and Christy: Phys.Rev. C77, C81
  - scaling region — Capella et al: PLB 337; Sibirtsev et al: Phys. Rev. D82
- uncertainty dominated by choice of transition between two regions

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

renormalization: no time to discuss properly

quark mass operator renormalizes EM self-energy: can not cleanly separate these two contributions (but mixing is higher order in isospin breaking)

summary: (J.C. Collins) with Naive Dimensional Analysis and suitable renormalization (dim. reg.) one can show the contribution from the operator is numerically second order in isospin breaking

$$\delta \tilde{M}^{ct}_{p-n} = 3\alpha \ln\left(\frac{\Lambda_0^2}{\Lambda_1^2}\right) \frac{e_u^2 m_u - e_d^2 m_d}{8\pi M \delta} \langle p|\delta(\bar{u}u - \bar{d}d)|p\rangle$$

$$\left|\delta \tilde{M}^{ct}_{p-n}\right| < 0.02 \text{ MeV}$$

$$2\delta = m_d - m_u$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\ T_1(0,Q^2)\,,$$

- low energy: constrained by effective field theory

$$T_1(0,Q^2) = 2\kappa(2+\kappa) - Q^2\left\{\frac{2}{3}\left[(1+\kappa)^2 r_M^2 - r_E^2\right] + \frac{\kappa}{M^2} - 2M\frac{\beta_M}{\alpha}\right\} + \mathcal{O}(Q^4)\,,$$

most of these contributions come from Low Energy Theorems and are "elastic" (arising from a photon striking an on-shell nucleon)

intimately related to the proton size puzzle which suffers from the same subtracted dispersive problem

K. Pachucki: Phys. Rev. A53 (1996); A. Pineda: Phys. Rev. C67 (2003); Phys. Rev. C71 (2005); R.J. Hill, G. Paz: PRL 107 (2011); C. Carlson, M. Vanderhaeghen: Phys.Rev.A84 (2011); arXiv1109.3779; M.. Birse, J. McGovern: arXiv:1206.3030

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\; T_1(0,Q^2)\,,$$

- high energy: OPE (perturbative QCD) constrains

$$\lim_{Q^2\to\infty} T_1(0,Q^2) \propto \frac{1}{Q^2}$$

$$T_1(0,Q^2) \simeq 2G_M^2(Q^2) - 2F_1^2(Q^2) + Q^2 2M\frac{\beta_M}{\alpha}\left(\frac{m_0^2}{m_0^2+Q^2}\right)^2$$

$\mathcal{O}(Q^4)$ inelastic terms known

Birse and McGovern arXiv:1206.3030

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub}_{el} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2 \left[2G_M^2 - 2F_1^2\right], \qquad \delta M^{sub}_{el}\Big|_{p-n} = -0.62 \text{ MeV}$$

$$\delta M^{sub}_{inel} = -\frac{3\beta_M}{8\pi}\int_0^{\Lambda_0^2} dQ^2 Q^2 \left(\frac{m_0^2}{m_0^2+Q^2}\right)^2$$

$$\beta_M^{p-n} = -1.0 \pm 1.0 \times 10^{-4} \text{ fm}^3$$

H.W. Griesshammer, J.A. McGovern, D.R. Phillips, G. Feldman: Prog.Nucl.Part.Phys. (2012)

taking $m_0^2 = 0.71 \text{ GeV}^2$

$$\delta M^{sub}_{inel}\Big|_{p-n} = 0.47 \pm 0.47 \text{ MeV}$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

adding it all up:

$$\delta M^{\gamma}|_{p-n} = +1.39(02)$$

$$-0.62(02)$$

(the first two terms together) $= 0.77(03)$ MeV — elastic terms

$$+0.057(16)$$

inelastic terms

$$+0.47(47) \text{ MeV}$$

unknown subtraction term

$$= 1.30(03)(47) \text{ MeV}$$

recall the fixed pole in the elastic contribution makes a negligible contribtion adding it all up:

$$\delta M^{\gamma}\Big|_{p-n} = 1.30(03)(47)\ \mathrm{MeV}$$

AWL, C.Carlson, G.Miller:
PRL 108 (2012)

$$= 0.76(30)\ \mathrm{MeV}$$

J. Gasser and H. Leutwyler:
Nucl Phys B94 (1975)

We reduced the uncertainty from structure by an order of magnitude! But we uncovered an oversight that dominates the uncertainty :(

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

adding it all up:

$$\delta M^{\gamma}\Big|_{p-n} = 1.30(03)(47)\ \text{MeV}$$

AWL, C.Carlson, G.Miller:
PRL 108 (2012)

$$= 0.76(30)\ \text{MeV}$$

J. Gasser and H. Leutwyler:
Nucl Phys B94 (1975)

expectation from experiment + lattice QCD

$$\delta M^{\gamma}\Big|_{p-n} = -1.29333217(42) + 2.53(40)\ \text{MeV}$$

$$= 1.24(40)\ \text{MeV}$$

average of 3 independent lattice results

Baryons and lattice QCD: Conclusions

- attempt to improve the old determination of nucleon iso-vector EM self-energy uncovered an oversight
  - no avoiding the subtraction (dispersion integral)
  - modeling was necessary to control uncertainty subtraction function
  - a central value was found in much better agreement with expectations from lattice QCD + experiment
  - comparison with independent determinations of iso-vector nucleon magnetic polarizability show the modeling is not crazy
- improvements will come from three areas
  - improved measurement of β_M^{p-n}
  - lattice QCD calculation of β_M^{p-n} — Brian Tiburzi's Talk
  - including EM effects with lattice QCD: — Taku Izubuchi's Talk

Fin