$\eta \to 3\pi$ and quark masses

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International Workshop on Chiral Dynamics,

August 6 – 10, 2012, Jefferson Lab

Outline

1. Introduction
2. Dalitz plot measurements
3. Theoretical work
4. Our dispersive analysis
5. Comparison of results

Outline

Light quark masses

- not directly accessible to experiment due to confinement
- $m_{u,d,s} \ll$ scale of QCD $\Rightarrow$ small contribution to hadronic quantities

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quark condensate $\langle \bar{q}q \rangle$ → spontaneous χSB

quark masses m_q → explicit χSB

Gell-Mann–Oakes–Renner relations

- $m_{\pi^+}^2 = B_0(m_u + m_d)$
- $m_{\pi^0}^2 = B_0(m_u + m_d)$
- $m_{K^+}^2 = B_0(m_u + m_s)$
- $m_{K^0}^2 = B_0(m_d + m_s)$
- $m_\eta^2 = B_0 \frac{m_u + m_d + 4m_s}{3}$

Gell-Mann–Oakes–Renner relations

- $m^2_{\pi^+} = B_0(m_u + m_d)$
- $m^2_{\pi^0} = B_0(m_u + m_d) + \frac{2\epsilon}{\sqrt{3}} B_0(m_u - m_d) + \ldots$
- $m^2_{K^+} = B_0(m_u + m_s)$ $\qquad \epsilon \sim 0.015$
- $m^2_{K^0} = B_0(m_d + m_s)$
- $m^2_{\eta} = B_0 \frac{m_u + m_d + 4m_s}{3} - \frac{2\epsilon}{\sqrt{3}} B_0(m_u - m_d) + \ldots$

Gell-Mann–Oakes–Renner relations

- $m^2_{\pi^+} = B_0(m_u + m_d) + \Delta^{\pi}_{em} + \ldots$
- $m^2_{\pi^0} = B_0(m_u + m_d) + \frac{2\epsilon}{\sqrt{3}} B_0(m_u - m_d) + \ldots$
- $m^2_{K^+} = B_0(m_u + m_s) + \Delta^{K}_{em} + \ldots$ $\qquad \Delta^{\pi/K}_{em} \sim (35\text{ MeV})^2$
- $m^2_{K^0} = B_0(m_d + m_s)$ $\qquad \Delta^{\pi}_{em} = \Delta^{K}_{em}$ [Dashen '69]
- $m^2_{\eta} = B_0 \frac{m_u + m_d + 4m_s}{3} - \frac{2\epsilon}{\sqrt{3}} B_0(m_u - m_d) + \ldots$

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- $m^2_{\eta} = B_0 \frac{m_u + m_d + 4m_s}{3} - \frac{2\epsilon}{\sqrt{3}} B_0(m_u - m_d) + \dots$
- $\Rightarrow (m_u - m_d)$ well hidden

Quark masses from the lattice

- more on this from others [talks by Bernard, Lellouch, Sachrajda, Izubuchi, . . .]
- relations between meson masses and quark masses from QCD
- $m_u - m_d$ needs handle on e.m. effects
  - input from phenomenology (e.g., Kaon mass difference)
  - put photons on the lattice
- recent review from FLAG [Colangelo et al. '11]

What has $\eta \to 3\pi$ to do with quark masses?

- $\eta \to 3\pi$ depends on m_q in special way:
  - violates isospin
  - generated by $\mathscr{L}_{IB} = -\dfrac{m_u - m_d}{2}(\bar{u}u - \bar{d}d)$
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  - $\Delta I = 1$ operator
- $\Rightarrow$ decay amplitude proportional to $(m_u - m_d)$
- $\Rightarrow$ measure for strength of isospin breaking in QCD

Electromagnetic corrections

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- one-loop contributions known and small [Baur, Kambor, Wyler '96, Ditsche, Kubis, Meißner '09]
- recent claim that $\eta \to 3\pi^0$ is mainly e.m. based on incomplete 2-loop calculation [Nehme, Zein '11]

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- recent claim that $\eta \to 3\pi^0$ is mainly e.m. based on incomplete 2-loop calculation [Nehme, Zein '11]
- $\Rightarrow$ clean access to $(m_u - m_d)$

The quark mass ratio Q

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- $$\mathcal{A}_{\eta\to 3\pi} \propto B_0(m_u - m_d) = \begin{cases} \frac{1}{Q^2}\frac{m_K^2(m_K^2 - m_\pi^2)}{m_\pi^2} + \mathcal{O}(\mathcal{M}^3) \\ \\ \end{cases}$$

  - $$Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

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    - $$Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$
    - $$R = \frac{m_s - \hat{m}}{m_d - m_u}$$

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  - $Q^2 = \dfrac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$
  - $R = \dfrac{m_s - \hat{m}}{m_d - m_u}$
  - define normalised amplitude: $\mathcal{A}(s,t,u) = -\dfrac{1}{Q^2}\,\dfrac{m_K^2(m_K^2 - m_\pi^2)}{2\sqrt{3}m_\pi^2 F_\pi^2}\,\mathcal{M}(s,t,u)$
  - $\Gamma_{exp} \propto \int |\mathcal{A}(s,t,u)| \propto 1/Q^4$

What else is interesting?

- slow convergence of chiral series:

experiment [PDG '12]

$$\Gamma_c = 66\ \text{eV} + 94\ \text{eV} + \ldots = 296\ \text{eV}$$

current algebra (66 eV) [Cronin '67, Osborn & Wallace '70]

one-loop χPT (94 eV) [Gasser & Leutwyler '84]

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296 eV: experiment [PDG '12]

- $\Rightarrow$ enhanced by large final state rescattering effects [Roiesnel & Truong '81]

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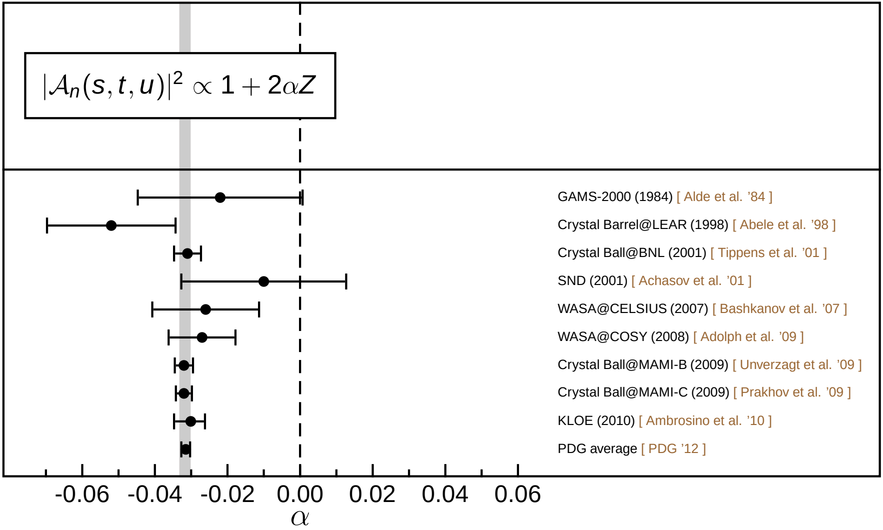
- possible tension among charged and neutral channel experiments
- charged and neutral channel amplitudes are related:

$$\mathcal{A}_n(s,t,u) = \mathcal{A}_c(s,t,u) + \mathcal{A}_c(t,u,s) + \mathcal{A}_c(u,s,t)$$

- $\Rightarrow$ allows for consistency check among measurements
- more on this later...

What else is interesting?



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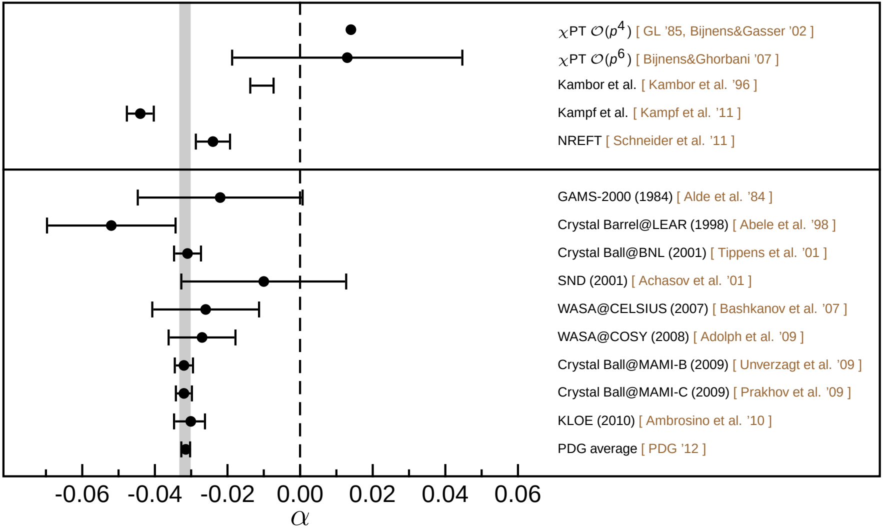
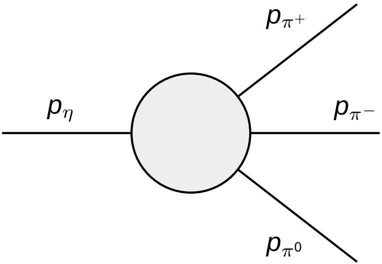


Kinematics



- $s = (p_{\pi^+} + p_{\pi^-})^2$
- $t = (p_{\pi^0} + p_{\pi^-})^2$
- $u = (p_{\pi^0} + p_{\pi^+})^2$
- $s + t + u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0$

  $\Rightarrow$ only two independent variables

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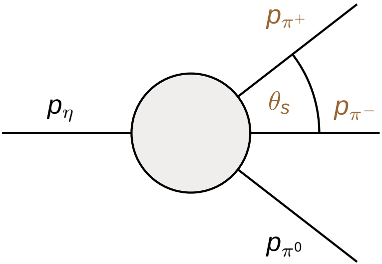

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  $\Rightarrow$ only two independent variables ,

  e.g., s & $t - u \propto \cos\theta_S$

Adler Zero

- soft pion theorem, i.e., valid in $SU(2)$ chiral limit [Adler '65]
- decay amplitude has a zero if
  - $p_{\pi^+} \to 0 \quad \Leftrightarrow \quad s = u = 0,\ t = m_\eta^2$
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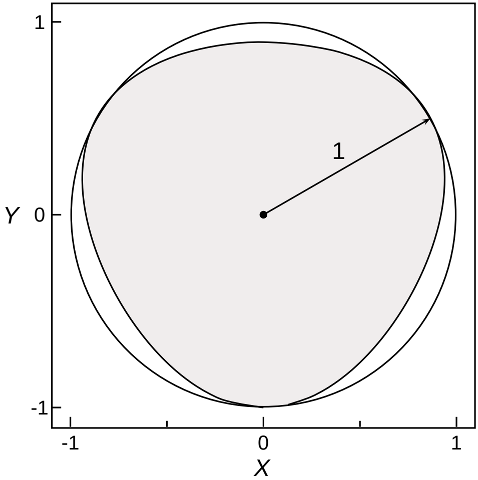
  - $p_{\pi^-} \to 0 \quad \Leftrightarrow \quad s = t = 0,\ u = m_\eta^2$
- for $m_\pi \neq 0$ Adler zeros at
  - $s = u = \frac{4}{3} m_\pi^2,\ t = m_\eta^2 + m_\pi^2/3$
  - $s = t = \frac{4}{3} m_\pi^2,\ u = m_\eta^2 + m_\pi^2/3$
- protected by $SU(2)$ chiral symmetry $\Rightarrow$ no $\mathcal{O}(m_s)$ corrections

Outline

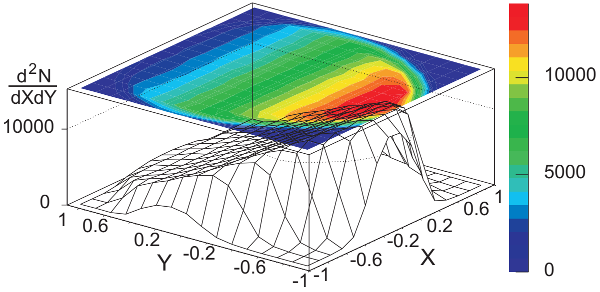
Dalitz plot variables



- $X = \frac{\sqrt{3}}{2m_\eta Q_c}(u - t)$
- $Y = \frac{3}{2m_\eta Q_c}\left((m_\eta - m_{\pi^0})^2 - s\right) - 1$
- $Q_c = m_\eta - 2m_{\pi^+} - m_{\pi^0}$
- $Z = X^2 + Y^2$

KLOE measurement of the charged channel

- only modern high-statistics Dalitz plot measurement [KLOE '08]
- $\sim 1.3 \times 10^6$ $\eta \to \pi^+\pi^-\pi^0$ events from $e^+e^- \to \phi \to \eta\gamma$



[Figure from Ambrosino et al. '08]

KLOE result for Dalitz plot parameters

- Dalitz plot parametrisation:

$$|\mathcal{A}_c(s,t,u)|^2 \propto 1 + aY + bY^2 + cX + dX^2 + eXY + fY^3 + gX^3 + hX^2Y + lXY^2$$

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- result: [KLOE '08]

$$a = -1.090^{+0.009}_{-0.020} \qquad b = 0.124 \pm 0.012$$

$$d = 0.057^{+0.009}_{-0.017} \qquad f = 0.14 \pm 0.02$$

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- Da

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- ch
- h
- re … [KLOE '08]

- older experiments:
  - AGS@BNL [Gormley et al. '70]
  - Princeton-Pennsylvania Accelerator [Layter et al. '73]
  - Crystal Barrel@LEAR [Abele et al. '98]
- upcoming analyses:
  - WASA@COSY [Adlarson, tbp]
  - KLOE [Caldeira Balkeståhl, tbp]

MAMI-C measurement of the neutral channel

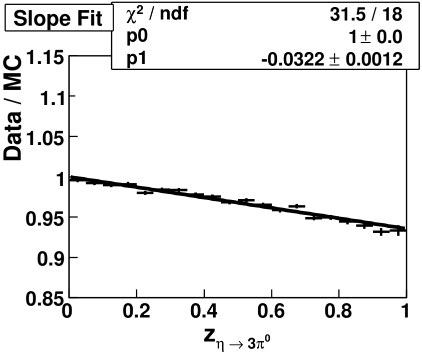
- $\sim 3 \times 10^6$ $\eta \to 3\pi^0$ events

  from $\gamma p \to \eta p$

- smallest uncertainties on α

- similar but independent

  measurement from MAMI-B



[figure from Prakhov et al. '09]

- $|\mathcal{A}_n(s,t,u)|^2 \propto 1 + 2\alpha Z + 6\beta Y \left(X^2 - \frac{Y^2}{3}\right) + 2\gamma Z^2$

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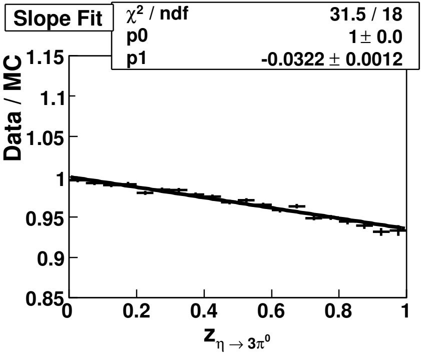

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Outline

What has been done?

	Q	α	tension	
e.m. contributions in χPT	✓	✓	✗	[Baur et al., '96, Ditsche et al. '09]
two-loop χPT	✓	✓	✗	[Bijnens & Ghorbani '07]
non-relativistic EFT	✗	✓	✓	[Schneider, Kubis & Ditsche '11]
analytical dispersive	✓	✓	(✓)	[Kampf, Knecht, Novotný & Zdráhal '11]
resummed χPT	✗	(✓)	(✓)	[Kolesar et al. '11]
numerical dispersive	✓	✓	✓	[Colangelo, SL, Leutwyler, Passemar tbp]

NREFT analysis

	Q	α	tension	
non-relativistic EFT	×	✓	✓	[Schneider, Kubis & Ditsche '11]

- expansion in small π three momenta in η rest frame
- explicitly includes two pion rescattering processes
- inputs:
  - $\mathcal{O}(p^4)$ $\eta \to 3\pi$ amplitude from χPT
  - empirical $\pi\pi$ scattering phases
- results only for shape, but not normalisation

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- $\alpha = -0.025 \pm 0.005 \Rightarrow$ correct sign, marginal agreement with experiment
- tension between charged and neutral channel experiments:

$$\alpha \leq \frac{1}{4}(b + d - \frac{1}{4}a^2)$$

[Bijnens & Ghorbani '07]

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- Δ can be calculated in NREFT (no $\eta \to 3\pi$ input from χPT needed!)

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- Δ can be calculated in NREFT (no $\eta \to 3\pi$ input from χPT needed!)
- from KLOE Dalitz plot parameters: $\alpha = -0.059 \pm 0.007$
- main reason for disagreement: $b_{NREFT} = 0.308 > b_{KLOE} = 0.124$

Dispersive analysis by Kampf et al.

	Q	α	tension	
analytical dispersive	✓	✓	(✓)	[Kampf, Knecht, Novotný & Zdráhal '11]

- analytical dispersive analysis relying on two-loop χPT and KLOE data
- 6 subtraction constants
- two rescattering processes $\Rightarrow$ reproduces two-loop result

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- 6 subtraction constants
- two rescattering processes $\Rightarrow$ reproduces two-loop result
- main result: subtraction constants from fit to KLOE data

  (normalisation fixed by imaginary part of two-loop result along $t = u$)

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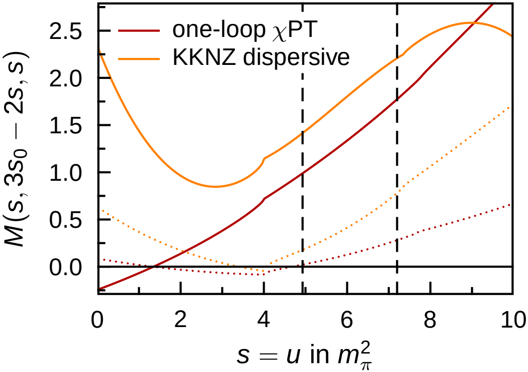
- Adler zero strongly violated ⇒ incompatible with $SU(2)$ chiral symmetry



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- includes arbitrary number of rescattering processes
- two main steps:
  - derive & solve dispersion relations [Anisovich & Leutwyler '96]
  - fix subtraction constants

Dispersion relations

- relies on decomposition [Fuchs, Sazdijan & Stern '93, Anisovich & Leutwyler '96]

$$\mathcal{M}(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \tfrac{2}{3}M_2(s)$$

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- dispersion relation for each $M_I(s)$: [Anisovich & Leutwyler '96]

$$M_I(s) = \Omega_I(s)\left\{P_I(s) + \frac{s^{n_I}}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^{n_I}}\frac{\sin\delta_I(s')\hat{M}_I(s')}{|\Omega_I(s')|(s'-s-i\epsilon)}\right\}$$

- Omnès function: $\Omega_I(s) = \exp\left\{\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta_I(s')}{s'(s'-s-i\epsilon)}\right\}$ [Omnès '58]

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- input needed for
  - $\pi\pi$ phase shifts [Ananthanarayan, Colangelo, Gasser & Leutwyler '01]
  - subtraction constants

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- dispersion relation for each $M_I(s)$: [Anisovich & Leutwyler '96]

$$M_I(s) = \Omega_I(s)\left\{P_I(s) + \frac{s^{n_I}}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^{n_I}}\frac{\sin\delta_I(s')\hat{M}_I(s')}{|\Omega_I(s')|(s'-s-i\epsilon)}\right\}$$

- Omnès function: $\Omega_I(s) = \exp\left\{\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta_I(s')}{s'(s'-s-i\epsilon)}\right\}$ [Omnès '58]

- input needed for
  - $\pi\pi$ phase shifts [Ananthanarayan, Colangelo, Gasser & Leutwyler '01]
  - subtraction constants

Taylor coefficients

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- $a_I, b_I, \dots \in \mathbb{R}$, but $\alpha_I, \beta_I, \dots \in \mathbb{C}$
- imaginary parts of subtraction constants suppressed
- splitting into $M_I(s)$ not unique because of $s + t + u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2$

  $\Rightarrow$ gauge freedom to fix some Taylor coefficients arbitrarily

Matching to one-loop χPT

- Subtraction constants from theory alone:
  - $M_0(s) = a_0 + b_0 s + c_0 s^2 + d_0 s^3 + \ldots$
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use χPT at low energy

extrapolate to physical region using dispersion relations

Fit to data

- use data to further constrain subtraction constants:
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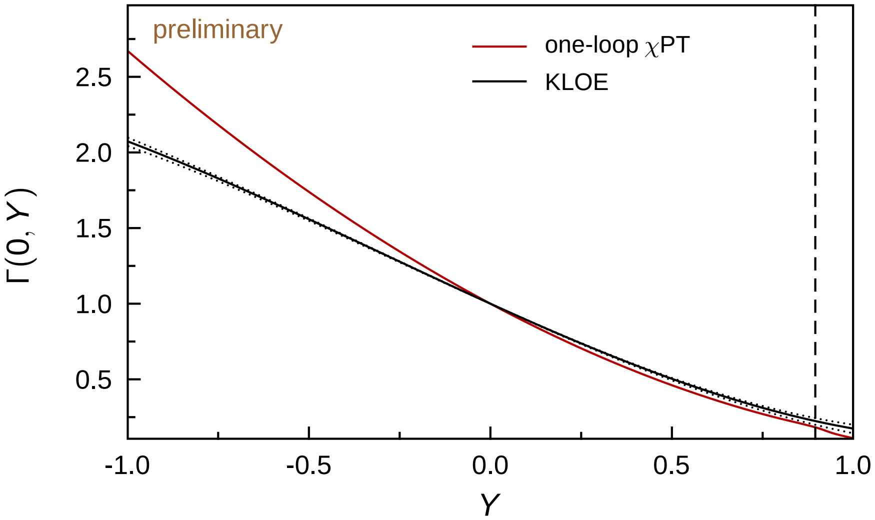
Dalitz distribution for $\eta \to \pi^+\pi^-\pi^0$



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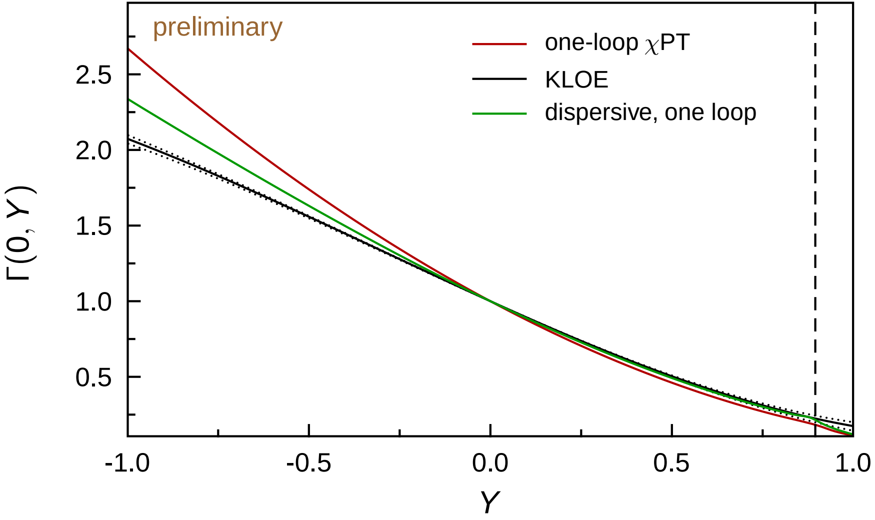

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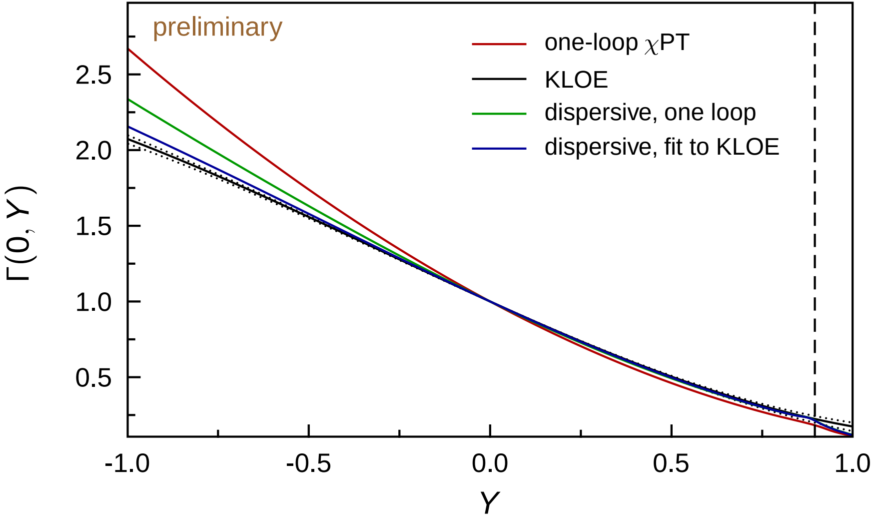
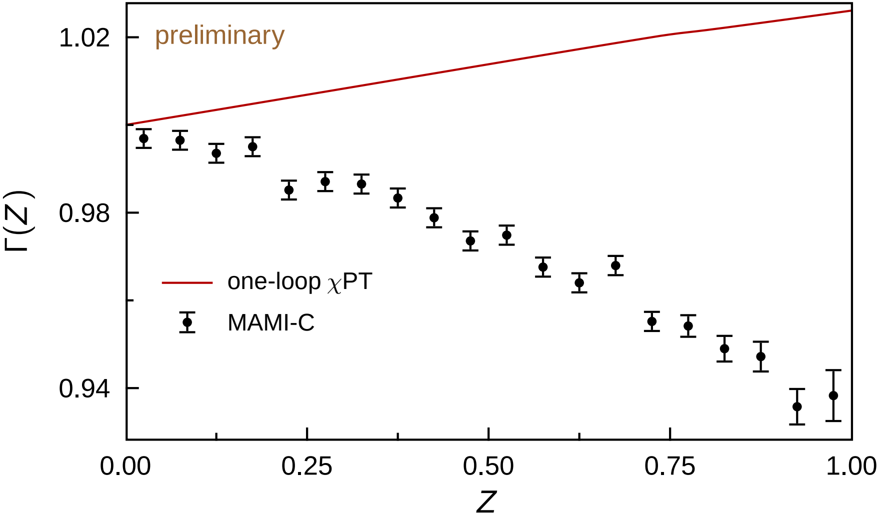


Dalitz distribution for $\eta \to 3\pi^0$



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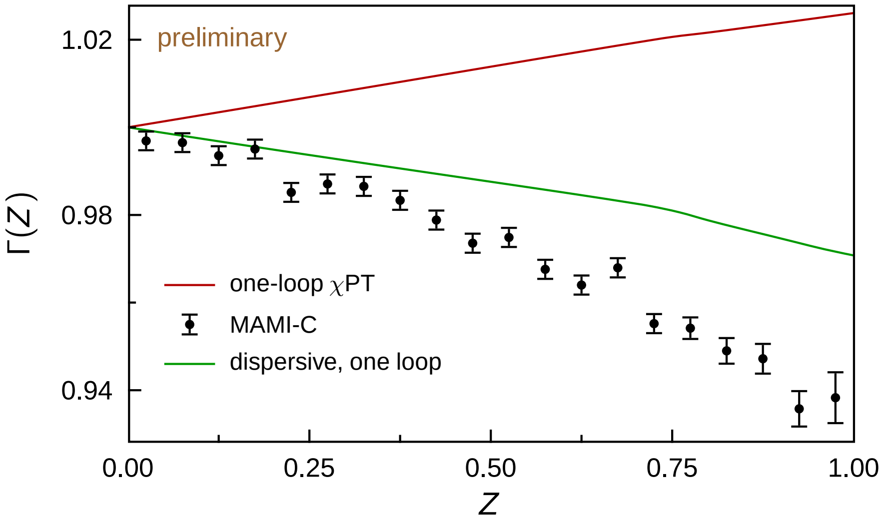

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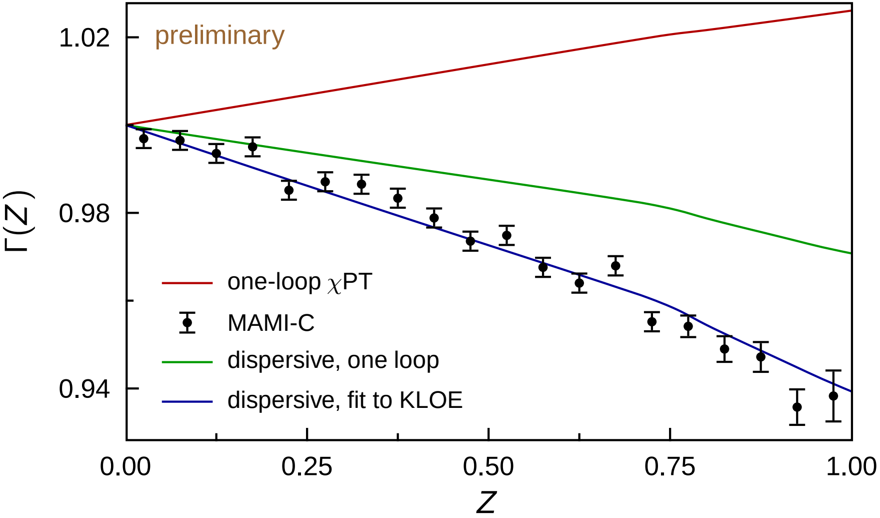

But: electromagnetic corrections

- dispersive amplitude in isospin limit $\Rightarrow$ needs to be accounted for in fits:
  - Dalitz plot distribution: kinematic effects most important (position of cusps)
  - decay rate: kinematic effects not enough (size of phase space)

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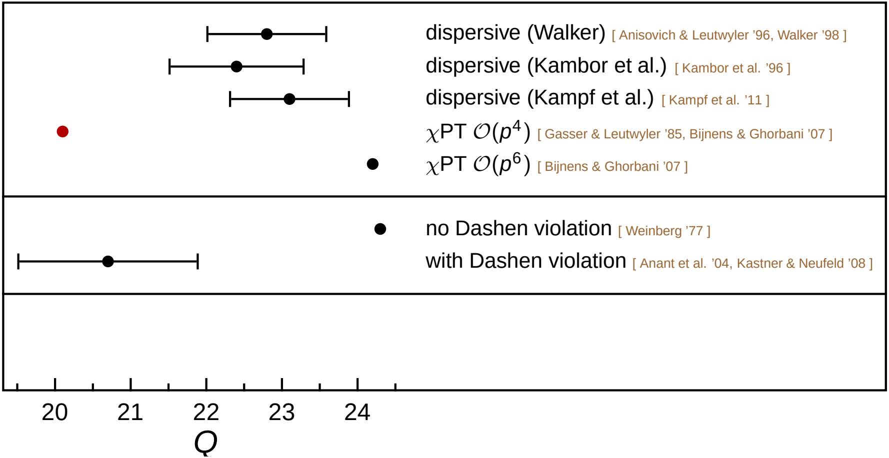
  $\Rightarrow$ e.m. corrections can amend BR
- needs to be improved

Outline

Comparison of Q



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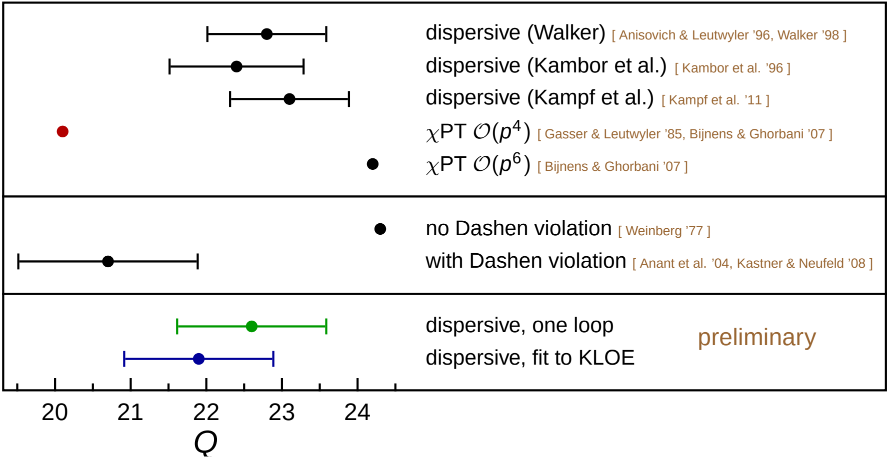
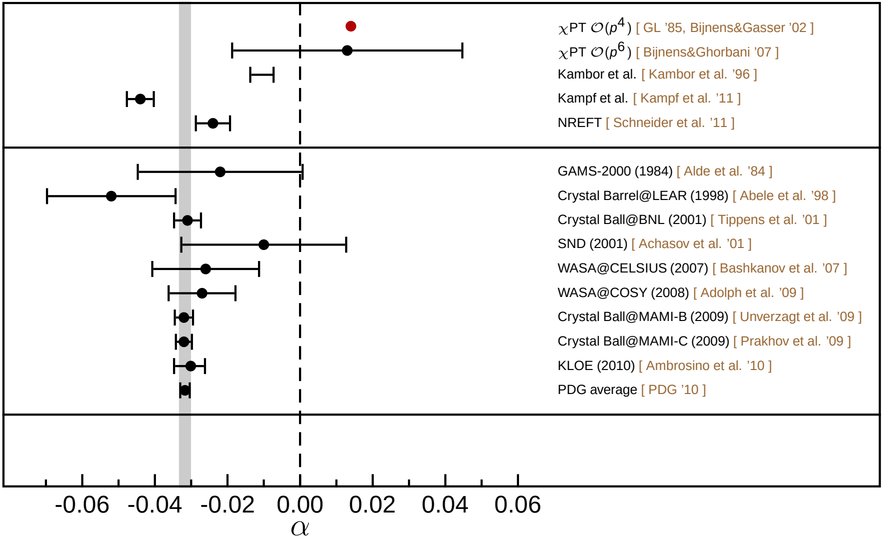


Comparison of α



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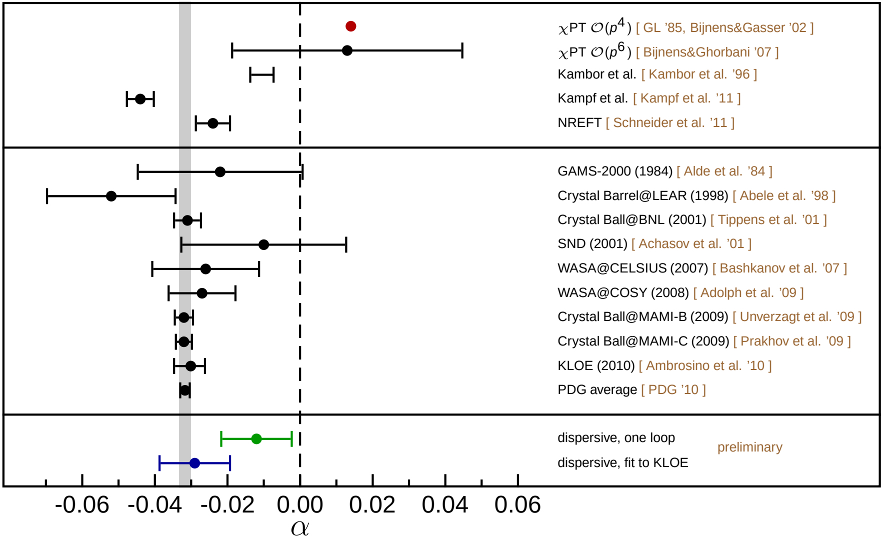

Conclusion & Outlook

- $\eta \to 3\pi$ very well suited to gain information on isospin breaking in QCD
- dispersion relations allow to treat rescattering effects properly
- dispersive treatment significantly improves one-loop result
- neutral channel slope parameter can be understood based on charged channel data
- no clear sign of a tension among experiments
- more careful treatment of electromagnetic effects needed