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Introduction

WHY HADRON-HADRON SCATTERING?

- Weinberg's 1966 paper "Pion scattering lengths"

Weinberg, Phys. Rev. Lett. **17** (1966) 616

- pion scattering on a target with mass m_t and isospin T_t :

$$a_T = -\frac{L}{1 + M_\pi/m_t} [T(T + 1) - T_t(T_t + 1) - 2]$$

- pion scattering on a pion ["the more complicated case"]:

$$a_0 = \frac{7}{4}L, \quad a_2 = -\frac{1}{2}L$$

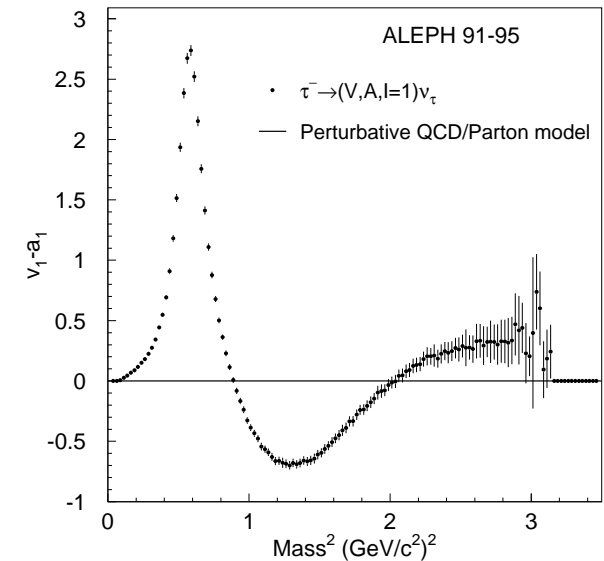
$$L = \frac{g_V^2 M_\pi}{2\pi F_\pi^2} \simeq 0.1 M_\pi^{-1}$$

- amazing predictions - witness to the power of chiral symmetry
- what have we learned since then?

THE FATE of QCD's CHIRAL SYMMETRY

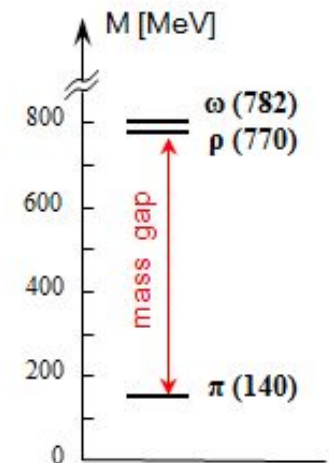
- the chiral symmetry is not “visible” (spontaneously broken)

- no parity doublets
- $\langle 0|AA|0\rangle \neq \langle 0|VV|0\rangle$
- scalar condensate $\bar{q}q$ acquires v.e.v.
- Vafa-Witten theorem
- (almost) massless pseudoscalar bosons



- the chiral symmetry is realized in the Nambu-Goldstone mode

- weakly interacting massless pseudoscalar excitations
- approximate symmetry (small quark masses)
 - π, K, η as Pseudo-Goldstone Bosons
- calls for an effective field theory
 - ⇒ Chiral Perturbation Theory (CHPT)



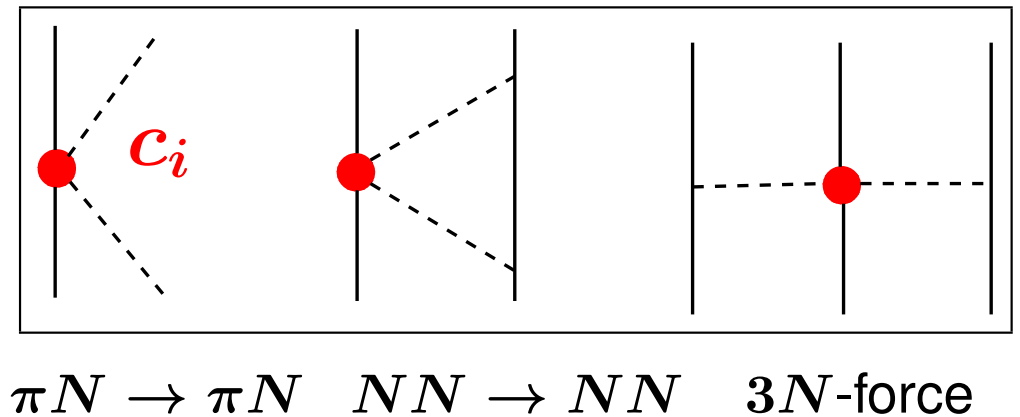
THE ESSENCE of CHIRAL PERTURBATION THEORY + ⁷

- explores the consequences of chiral symmetry breaking in QCD
- has an underlying power counting, based on the scale separation
- relates *many* processes

- increasing number of LECs w/ higher orders, but: NOT prolific for basic processes

e.g. $\pi\pi \rightarrow \pi\pi$ at one loop:
has 4 LECs

$\pi\pi \rightarrow \pi\pi$ at two loops:
has only 2 new LECs



- and the + : predictions can be sharpened by combining with dispersion relations, coupled-channels, lattice, . . . , but: there is no free lunch!
- and now lets see how this works and what we can learn...

Lesson 1

ELASTIC PION-PION SCATTERING

- Purest process in two-flavor chiral dynamics (really light quarks)
- scattering amplitude at threshold: two numbers (a_0, a_2)
- History of the prediction for a_0 :

LO (tree): $a_0 = 0.16$ Weinberg 1966

NLO (1-loop): $a_0 = 0.20 \pm 0.01$ Gasser, Leutwyler 1983

NNLO (2-loop): $a_0 = 0.217 \pm 0.009$ Bijmens et al. 1996

- even better: match 2-loop representation to Roy equation solution

Roy + 2-loop: $a_0 = 0.220 \pm 0.005$ Colangelo et al. 2000

⇒ this is an *amazing* prediction!

- same precision for a_2 , but corrections very small . . .

HOW ABOUT EXPERIMENT?

- Kaon decays (K_{e4} and $K^0 \rightarrow 3\pi^0$): most precise
- Lifetime of pionium: experimentally more difficult

Kaon decays:

$$a_0^0 = 0.2210 \pm 0.0047_{\text{stat}} \pm 0.0040_{\text{sys}}$$

$$a_0^2 = -0.0429 \pm 0.0044_{\text{stat}} \pm 0.0028_{\text{sys}}$$

J. R. Batley et al. [NA48/2 Coll.] EPJ C 79 (2010) 635

Pionium lifetime:

$$|a_0^0 - a_0^2| = 0.264^{+0.033}_{-0.020}$$

B. Adeva et al. [DIRAC Coll.] PL B 619 (2005) 50

- and how about the lattice?

⇒ direct and indirect determinations of the scattering lengths

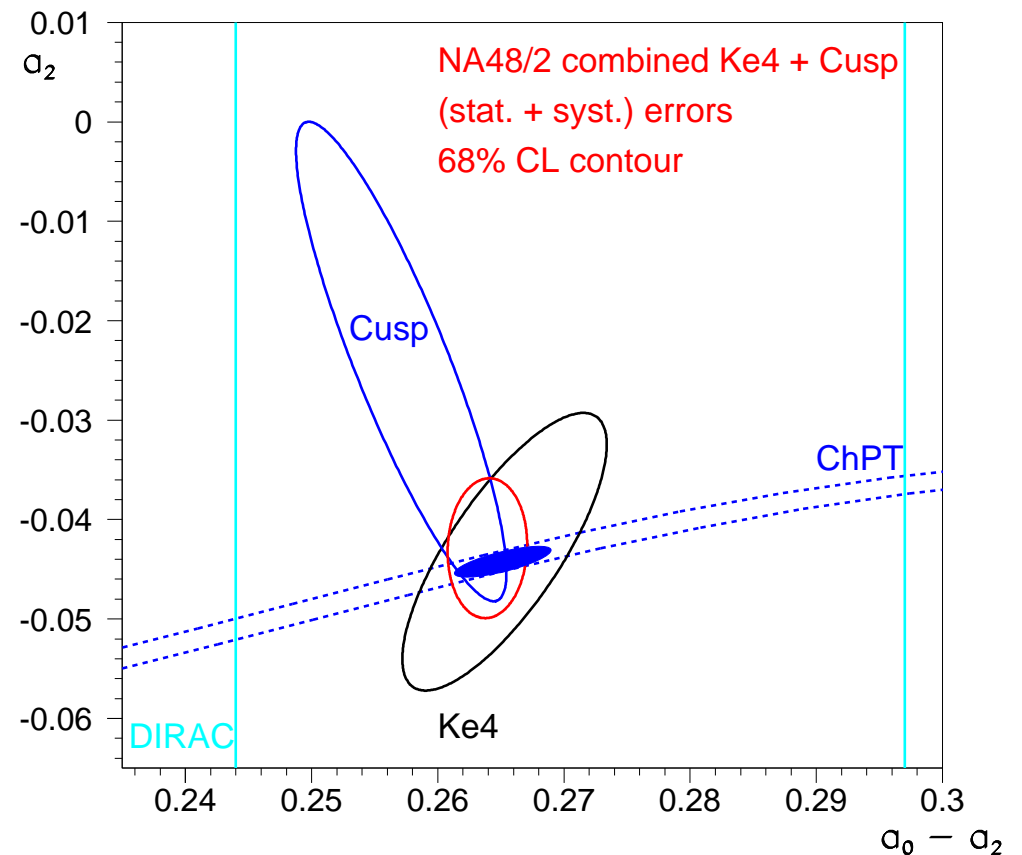
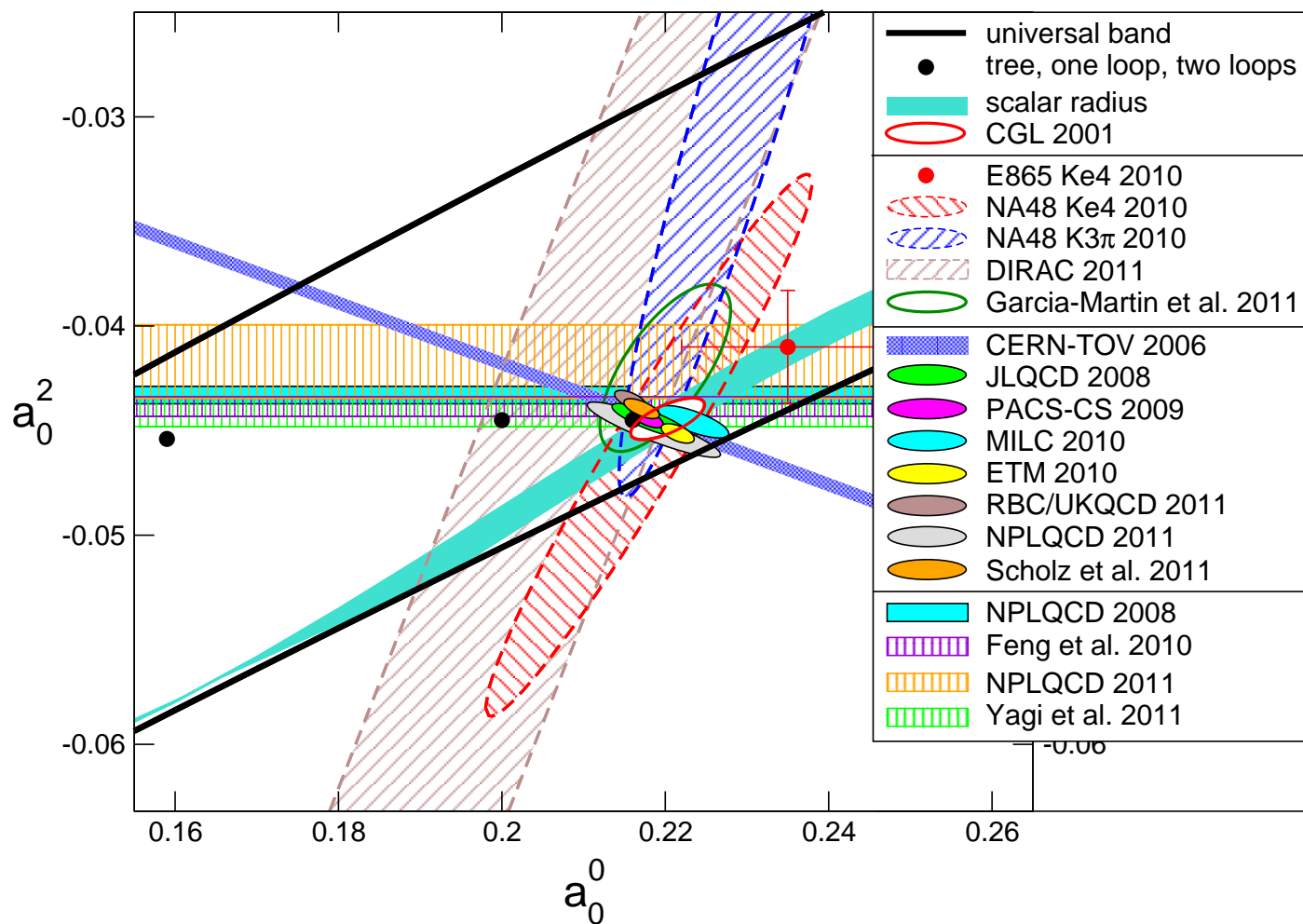


Fig. courtesy Heiri Leutwyler 2012



• one of the finest tests of the Standard Model (but: direct lattice a_0 missing)

Lesson 2

- Is the strange quark really light?

$$m_s \sim \Lambda_{\text{QCD}}$$

→ expansion parameter: $\xi_s = \frac{M_K^2}{(4\pi F_\pi)^2} \simeq 0.18$ [SU(2): $\xi = \frac{M_\pi^2}{(4\pi F_\pi)^2} \simeq 0.014$]

- many predictions of SU(3) CHPT work quite well, but:

↪ indications of bad convergence in some recent lattice calculations:

★ masses and decay constants, $K_{\ell 3}$ -decays Allton et al. 2008, Boyle et al. 2008

↪ suppression of the three-flavor condensate?

★ sum rule: $\Sigma(3) = \Sigma(2)[1 - 0.54 \pm 0.27]$ Moussallam 2000

★ lattice: $\Sigma(3) = \Sigma(2)[1 - 0.23 \pm 0.39]$ Fukuya et al. 2011

- possible solution: *resummed* CHPT

Bernard, Descotes-Genon, ... 2011

ELASTIC PION-KAON SCATTERING

- Purest process in three-flavor chiral dynamics
- scattering amplitude at threshold: two numbers ($a_0^{1/2}$, $a_0^{3/2}$)
- History of the chiral predictions:

	CA [1]	1-loop [2]	2-loop [3]
$a_0^{1/2}$	0.14	0.18 ± 0.03	0.220 [0.17 ... 0.225]
$a_0^{3/2}$	-0.07	-0.05 ± 0.02	-0.047[-0.075 ... -0.04]

[1] Weinberg 1966, Griffith 1969

[2] Bernard, Kaiser, UGM 1990

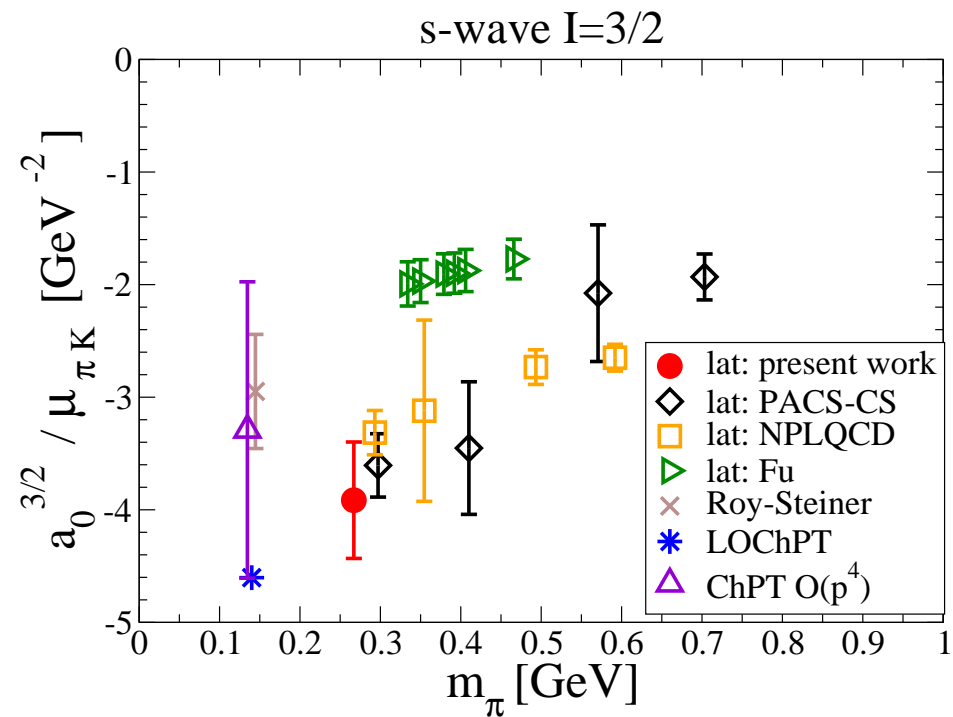
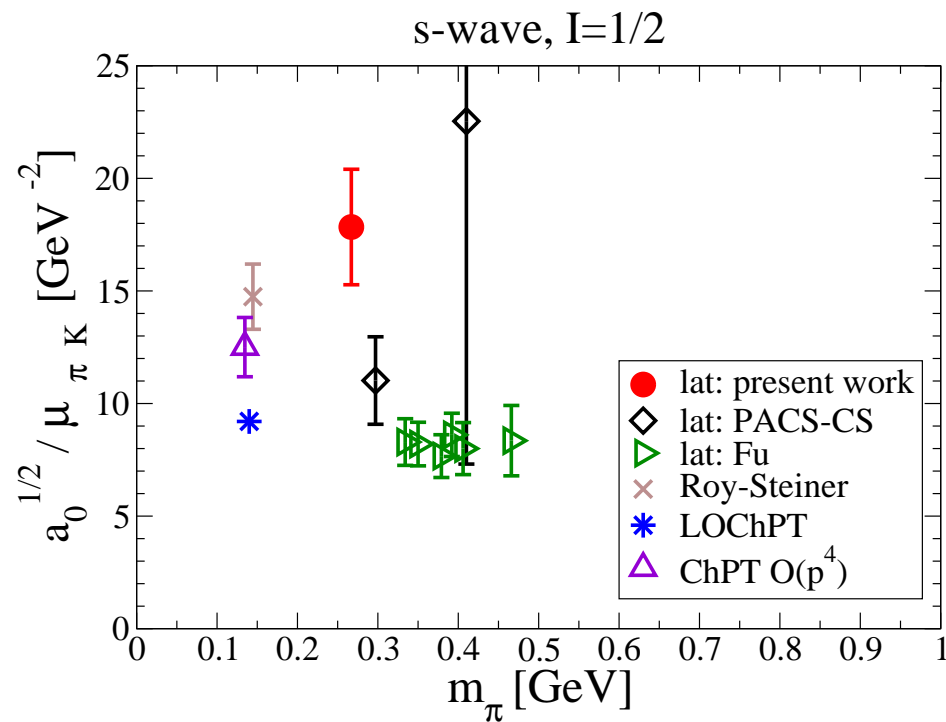
[3] Bijnens, Dhonte, Talavera 2004

- match 1-loop representation to Roy-Steiner equation solution

$$a_0^{1/2} = 0.224 \pm 0.022, \quad a_0^{3/2} = -0.0448 \pm 0.0077$$

Büttiker et al. 2003

Fig. from Lang et al. [1207.3204]



- tension between lattice results and/or Roy-Steiner
- need improved lattice results (more direct calculations)

⇒ work required

Lesson 3

PION-NUCLEON SCATTERING

- simplest scattering process involving nucleons
- intriguing LO prediction for isoscalar/isovector scattering length:

$$a_{CA}^+ = 0, \quad a_{CA}^- = \frac{1}{1 + M_\pi/m_p} \frac{M_\pi^2}{8\pi F_\pi^2} = 79.5 \cdot 10^{-3} / M_\pi,$$

- chiral corrections:

- chiral expansion for a^- converges fast Bernard, Kaiser, UGM 1995
- large cancellations in a^+ , even sign not known from scattering data

	$\mathcal{O}(q)$	$\mathcal{O}(q^2)$	$\mathcal{O}(q^3)$	$\mathcal{O}(q^4)$
fit to KA85	0.0	0.46	-1.00	-0.96
fit to EM98	0.0	0.24	0.49	0.45
fit to SP98	0.0	1.01	0.14	0.27

Fettes, UGM 2000

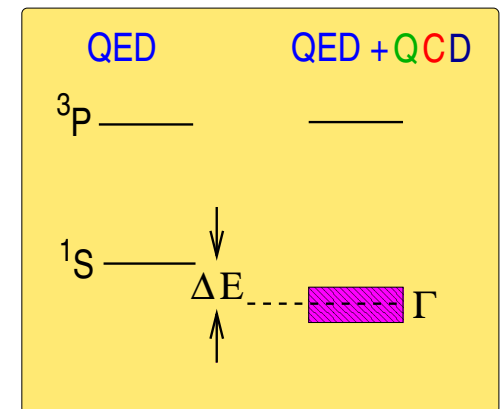
A WONDERFUL ALTERNATIVE: HADRONIC ATOMS

- Hadronic atoms: bound by the static Coulomb force (QED)
- Many species: $\pi^+\pi^-$, $\pi^\pm K^\mp$, π^-p , π^-d , K^-p , K^-d , ...
- Observable effects of QCD: strong interactions as **small** perturbations

★ energy shift ΔE

★ decay width Γ

⇒ access to scattering at zero energy!
= S-wave scattering lengths



- can be analyzed in suitable NREFTs

Pionic hydrogen

Pionic deuterium

Gasser, Rusetsky, ... 2002

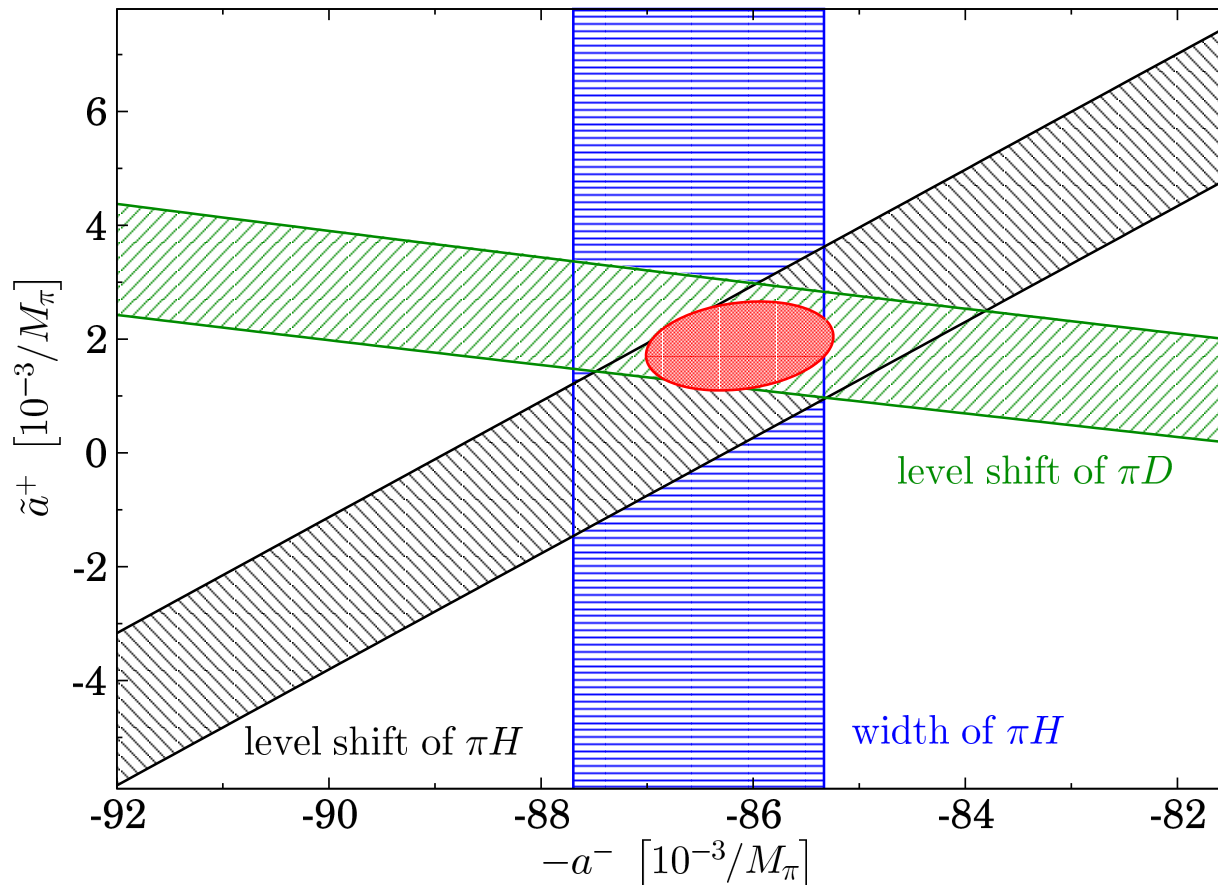
Baru, Hoferichter, Kubis ... 2011

→ talk by Martin Hoferichter

PION-NUCLEON SCATTERING LENGTHS

- superbe experiments performed at PSI

Gotta et al.



$$a^+ = (7.6 \pm 3.1) \cdot 10^{-3} / M_\pi$$

$$a^- = (86.1 \pm 0.9) \cdot 10^{-3} / M_\pi$$

GMO sum rule:

$$\frac{g_{\pi N}^2}{4\pi} = 13.69(12)(15)$$

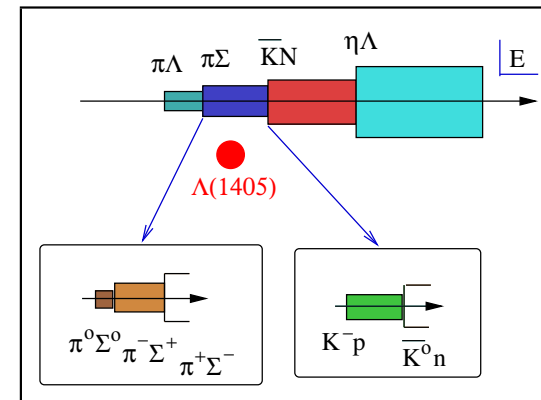
⇒ very precise value for a^- & first time definite sign for a^+

for details, see Hoferichter's talk

Lesson 4

ANTIKAON-NUCLEON SCATTERING

- $K^- p \rightarrow K^- p$: fundamental scattering process with strange quarks
- coupled channel dynamics
- dynamic generation of the $\Lambda(1405)$
Dalitz, Tuan 1960
- major playground of **unitarized CHPT**
- chiral Lagrangian + unitarization leads to generation of certain resonances
like e.g. the $\Lambda(1405)$, $S_{11}(1535)$, $S_{11}(1650)$, ...
Kaiser, Siegel, Weise, Oset, Ramos, Oller, UGM, Lutz, ...
- loopholes: convergence a posteriori, crossing symmetry, on-shell approximation, unphysical poles, ...



A PUZZLE RESOLVED

- DEAR data inconsistent with scattering data

UGM, Raha, Rusetsky 2004

⇒ vaste number of papers ...

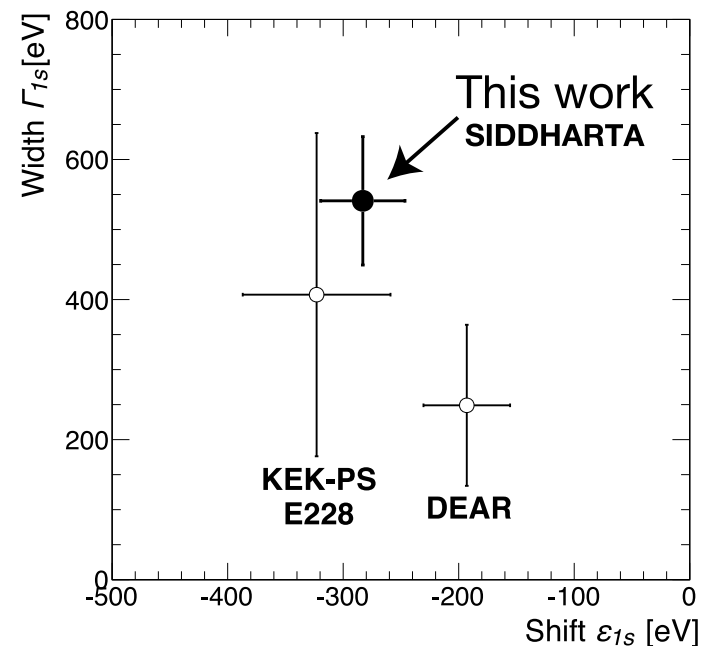
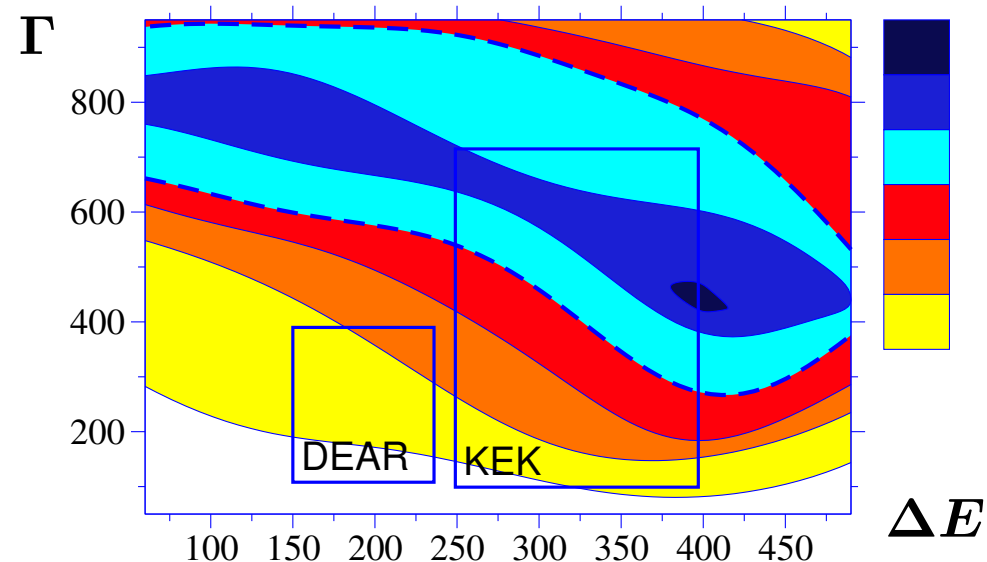
- SIDDHARTA to the rescue

Bazzi et al. 2011

⇒ more precise, consistent with KpX

$$\epsilon_{1s} = -283 \pm 36(\text{stat}) \pm 6(\text{syst}) \text{ eV}$$

$$\Gamma_{1s} = 541 \pm 89(\text{stat}) \pm 22(\text{syst}) \text{ eV}$$



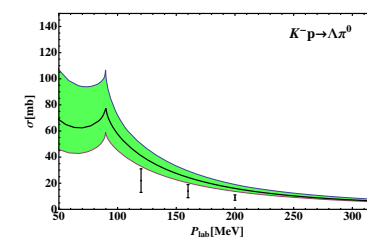
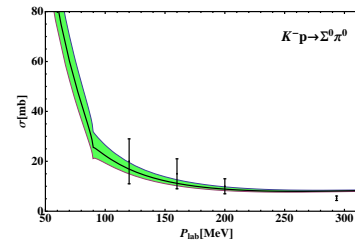
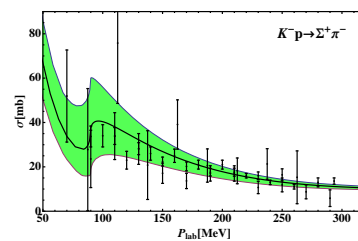
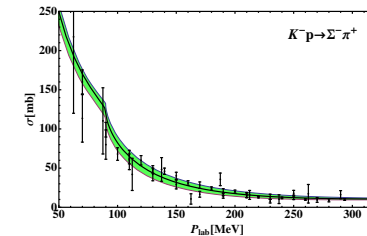
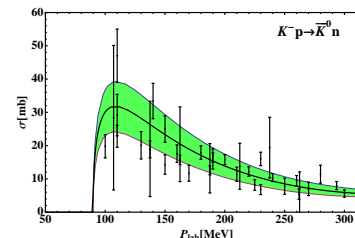
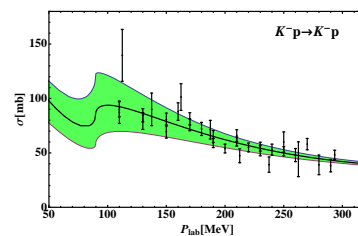
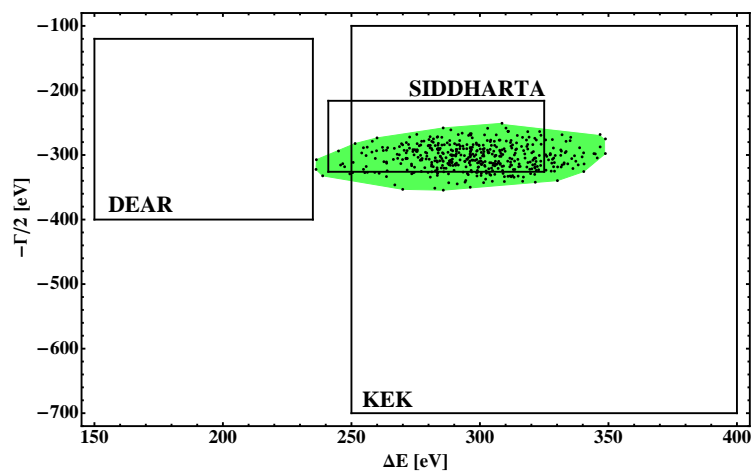
CONSISTENT ANALYSIS

- kaonic hydrogen + scattering data can now be analyzed consistently
- use the chiral Lagrangian at NLO, two groups (different schemes)

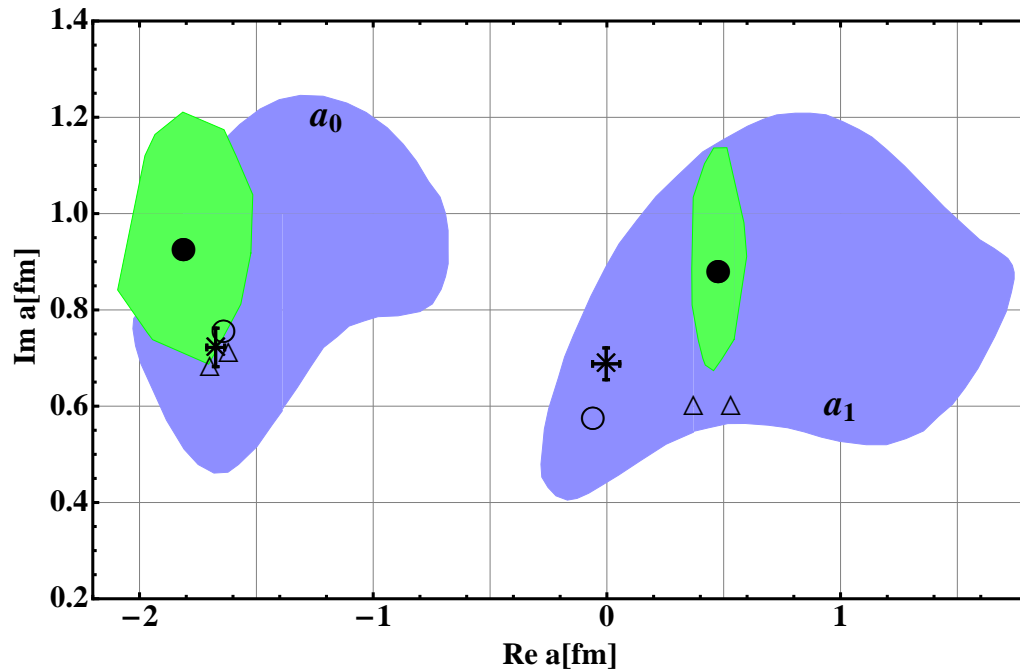
Ikeda, Hyodo, Weise 2011; UGM, Mai 2012

- 14 LECs and 3 subtraction constants to fit

⇒ simultaneous description of the SIDDHARTA and the scattering data



KAON-NUCLEON SCATTERING LENGTHS



$$a_0 = -1.81_{-0.28}^{+0.30} + i 0.92_{-0.23}^{+0.29} \text{ fm}$$

$$a_1 = +0.48_{-0.11}^{+0.12} + i 0.87_{-0.20}^{+0.26} \text{ fm}$$

$$a_{K-p} = -0.68_{-0.17}^{+0.18} + i 0.90_{-0.13}^{+0.13} \text{ fm}$$

SIDDHARTA only:

$$a_{K-p} = -0.65_{-0.15}^{+0.15} + i 0.81_{-0.18}^{+0.18} \text{ fm}$$

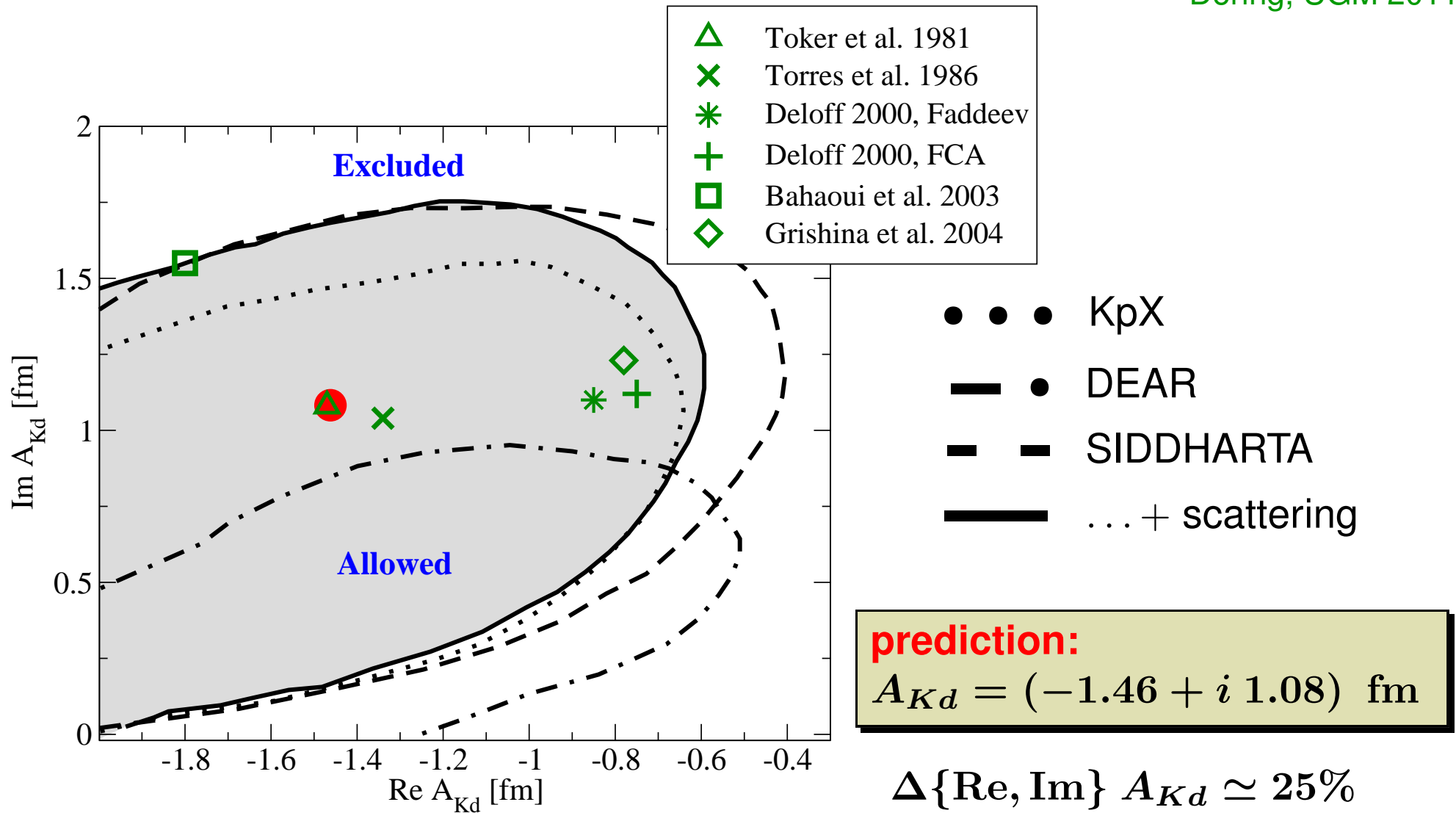
- clear improvement compared to scattering data only

⇒ fundamental parameters to within about 15% accuracy

KAON-DEUTERON SCATTERING LENGTH

- analyze K^-d , imposing consistency with the $\bar{K}N$ scattering lengths

Döring, UGM 2011



Lesson 5

EFFECTIVE LAGRANGIAN for $\phi D \rightarrow \phi D$

- Goldstone boson octet (π, K, η) scatters off D -meson triplet (D^0, D^+, D_s^+)
- multi-scale/multi-faceted problem:
 - light particles, chiral symmetry \rightarrow chiral expansion in (p, m_q)
 - heavy particles, heavy quark symmetry \rightarrow expansion in $1/m_c$
 - isospin-violation \rightarrow strong = quark mass difference $m_d \neq m_u$
 \rightarrow electromagnetic = quark charge difference $q_u \neq q_d$
- 16 channels with different total strangeness and isospin
 - some are perturbative
 - some are non-perturbative, require resummation \rightarrow possible molecules

- $T(\phi D \rightarrow \phi D)$ depends on two LECs at NLO h_3, h_5 using large- N_c :

h_3 the mass of the $D_{s0}^*(2317)$ as a DK molecule

h_5 from naturalness, $h_5/M_D^2 \in [-1, +1]$

$$\Rightarrow \Gamma(D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0) = (180 \pm 110) \text{ keV} \quad \text{testable prediction}$$

note: much smaller in quark models (a few keV)

see also Gutsche et al., Lutz, Soyeur, ...

- expectation for the scattering length for $DK(I=0)$ in the molecular picture:

$$a_{DK}^{I=0} = -g_{\text{eff}}^2 \Delta_{DK} = -\frac{1}{2\sqrt{\mu_{DK}\epsilon}} \simeq -1 \text{ fm}$$

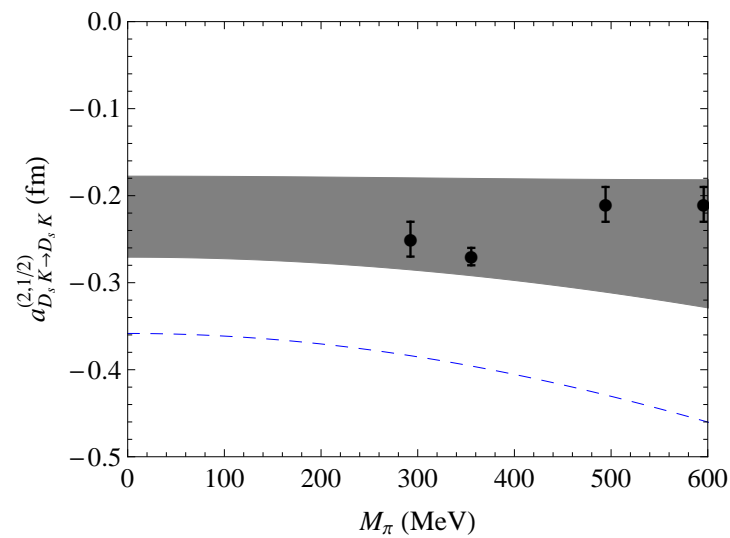
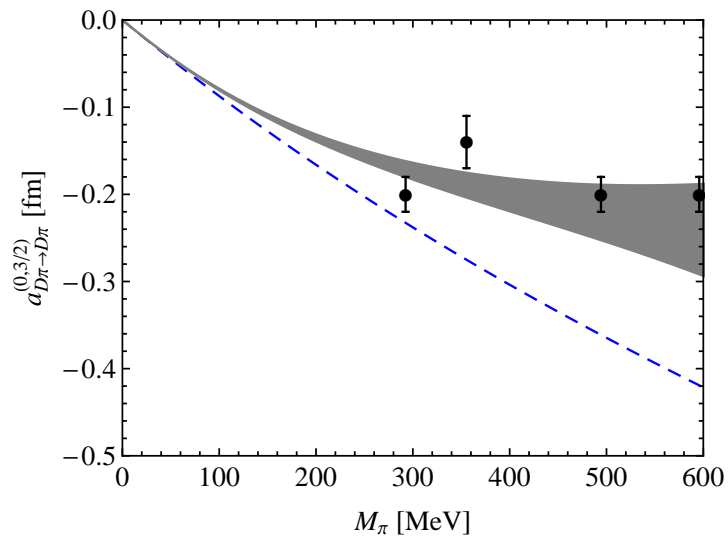
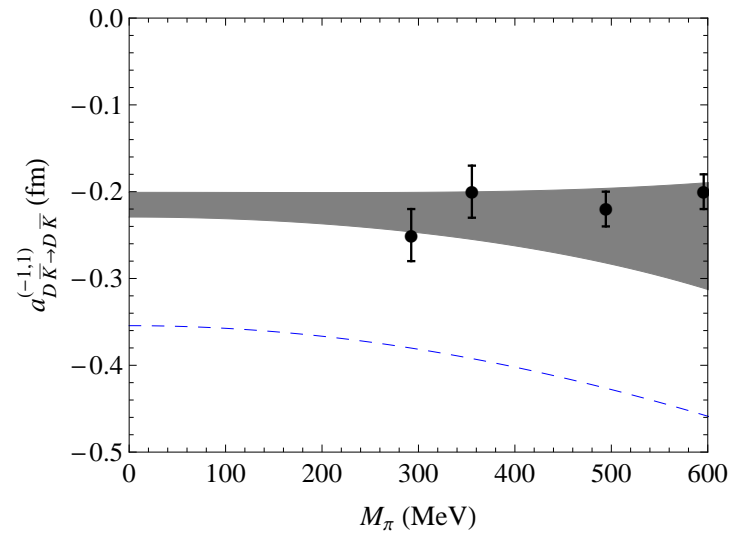
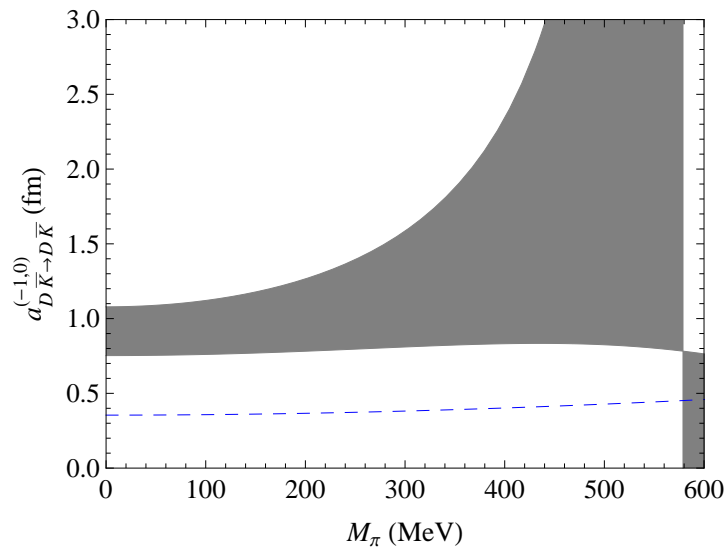
- no data, but first lattice investigations at varying quark masses

Liu, Lin, Orginos, PoS LATTICE 2008, 112 and more to come!

QUARK MASS DEPENDENCE

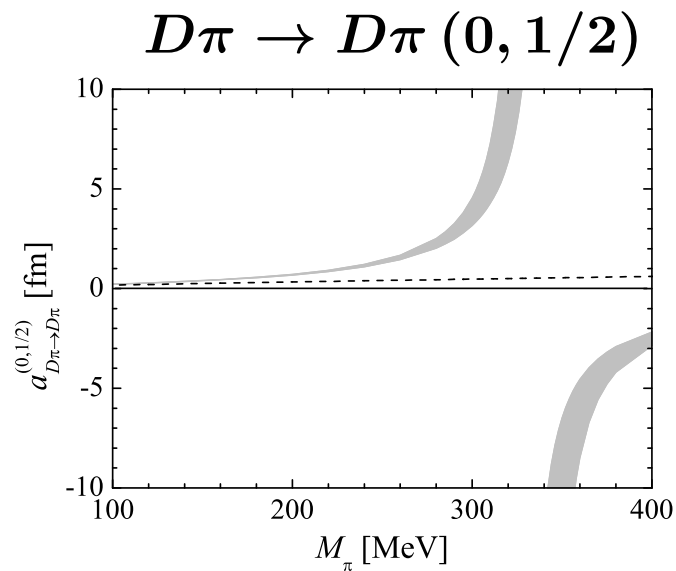
- *predictions: channels with no poles*

Guo, Hanhart, UGM 2009; Wang, Wang 2012



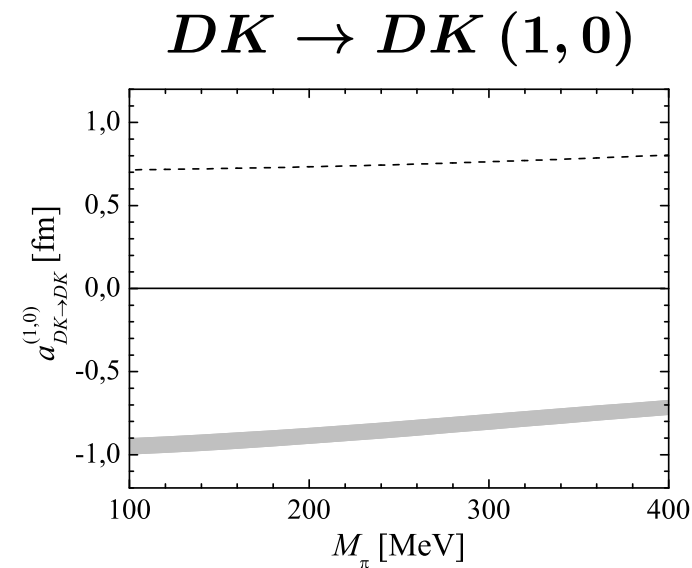
QUARK MASS DEPENDENCE cont'd

- *predictions:* channels with poles \rightarrow resonances or molecular states



a pair of poles above thr.

$$a_{D\pi}^{(0,1/2)} = 0.35(1) \text{ fm}$$



a bound states below thr. $D_{s0}^* (2317)$

$$a_{DK}^{(1,0)} = -0.93(5) \text{ fm}$$

\Rightarrow lattice test of the molecular nature

NEW RESULTS for $\phi D \rightarrow \phi D$

- new lattice data using MILC plus Fermilab actions L. Liu, K. Orginos 2012
- more data \rightarrow allow for inclusion of N_c -suppressed operators
- fix again the pole of the $D_{s0}^*(2317) \rightarrow 5$ fit parameters
- Results: Liu, Orginos, Guo, Hanhart, UGM 2012
 - all parameters of natural size, large- N_c hierarchy obeyed!
 - $a(DK(I = 0)) = -0.85_{-0.05}^{+0.07}$ fm
 - improved prediction for the IV width

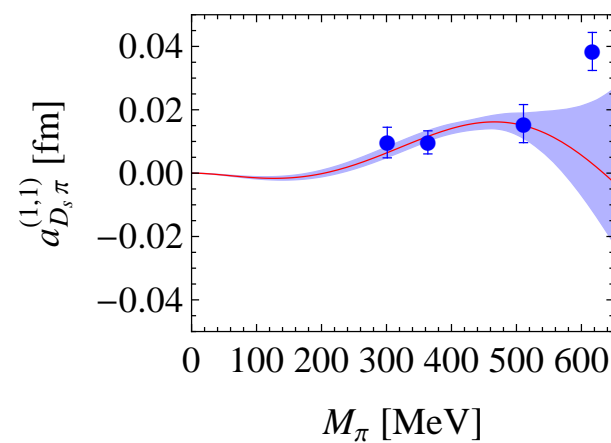
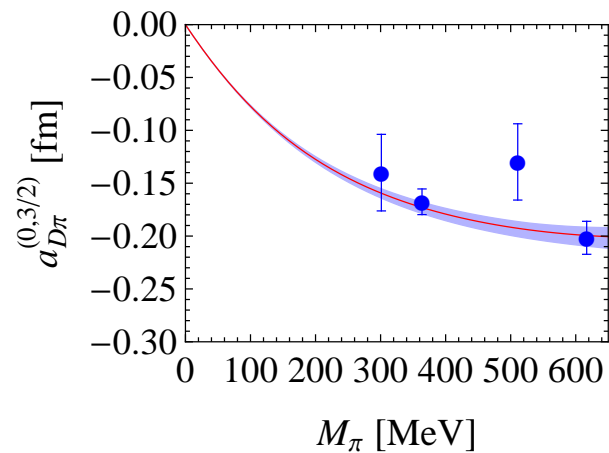
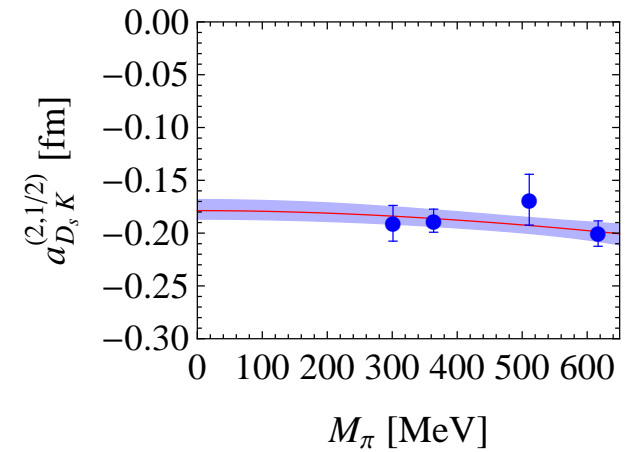
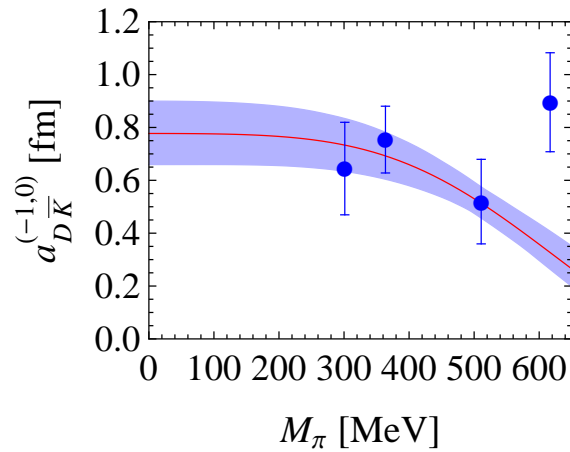
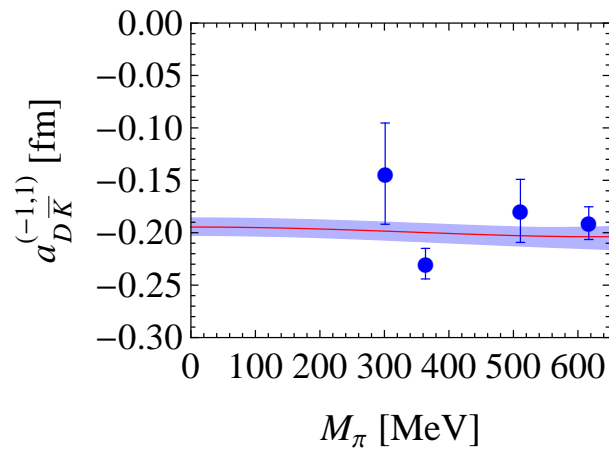
$$\Gamma(D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0) = (89 \pm 27) \text{ keV}$$

 - also unconstrained fits (6 param.), larger uncertainties

NEW RESULTS for ϕD -SCATTERING

- 5 parameter fit (last points not included)

Liu, Orginos, Guo, Hanhart, UGM 2012

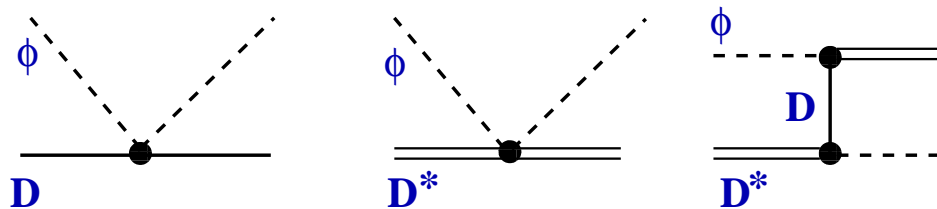


NATURE of the $D_{s1}(2460)$

- Nature of the $D_{s1}(2460)$: $M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \simeq M_{D^*} - M_D$
 - ⇒ most likely a D^*K molecule (if the $D_{s0}^*(2317)$ is DK)
 - ⇒ study Goldstone boson scattering off D - and D^* -mesons
- Use heavy meson chiral perturbation theory Wise, Donoghue et al., Casalbuoni et al., . . .

$$H_v = \frac{1 + \not{v}}{2} [V_v^* + iP_v \gamma_5]$$
$$P = (D^0, D^+, D_s^+), \quad V_\mu^* = (D_{\mu}^{*0}, D_{\mu}^{*+}, D_{s,\mu}^{*+})$$

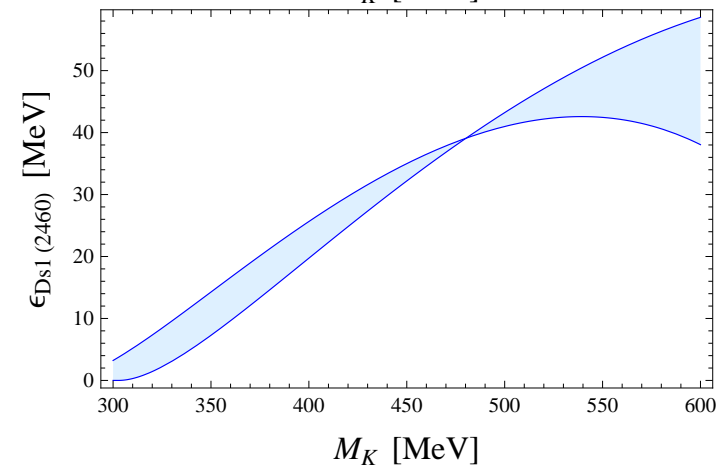
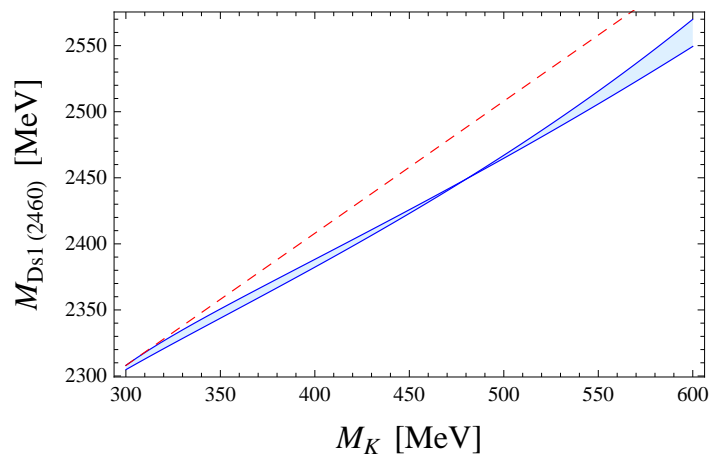
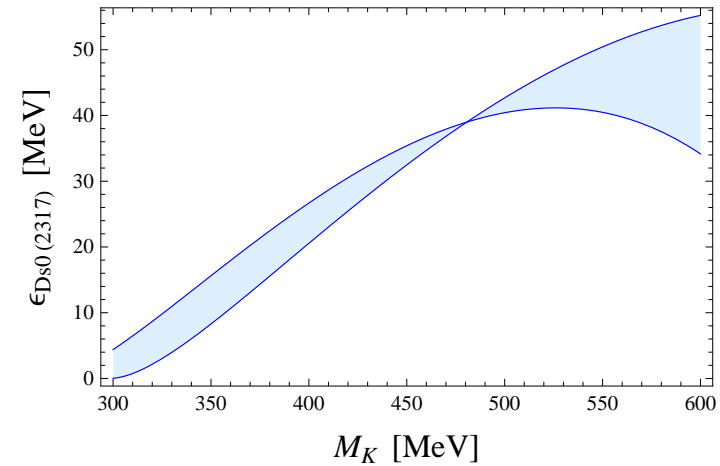
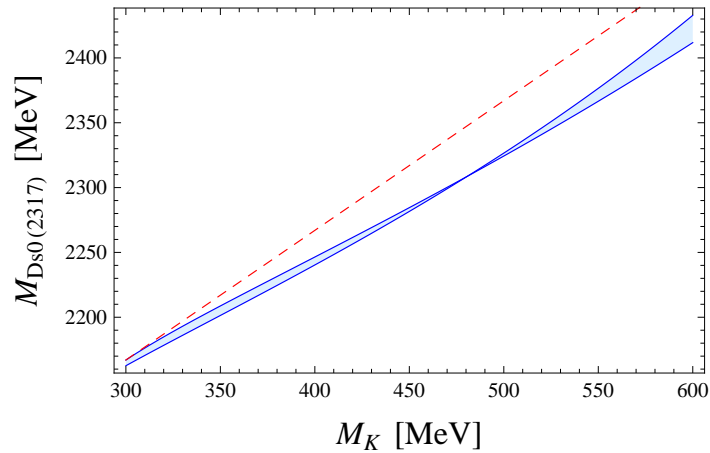
- T-matrix:



- Unitarization (as before) → find poles in the complex plane

KAON MASS DEPENDENCE

- Mass and binding energy: $M_{\text{mol}} = M_K + M_H - \epsilon$ Cleven, Guo, Hanhart, UGM 2009



⇒ typical for a molecule → test in LQCD

- Hadron-hadron scattering: important role of chiral symmetry (CHPT)
→ combine with dispersion relations, unitarization, lattice
- Pion-pion scattering
→ a fine test of the Standard Model
- Pion-kaon scattering
→ tension between lattice results and/or Roy-Steiner solution
- Pion-nucleon scattering
→ superbe accuracy from EFTs for pionic hydrogen/deuterium
- Antikaon-nucleon scattering
→ consistent determination of the scattering lengths possible
- Goldstone-boson scattering off D, D^* -mesons
→ lattice test of molecular states possible ⇒ exciting times ahead of us

SPARES

