Determining the Hadron Spectrum Using Lattice QCD

Robert Edwards
Jefferson Lab

Twin Approaches to Confinement Physics 2012

Collaborators (Hadron Spectrum Collaboration):
J. Dudek, P. Guo, B. Joo, D. Richards (JLab), S. Wallace (Maryland)
N. Mathur (Tata), L. Liu, M. Peardon, S. Ryan, C. Thomas (Trinity College)

Determining the Hadron Spectrum Using Lattice QCD

Robert Edwards
Jefferson Lab

Twin Approaches to Confinement Physics
2012

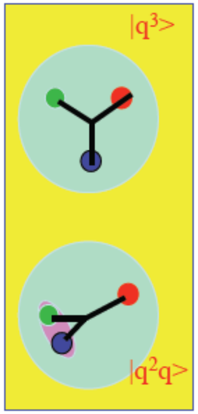
Collaborators (Hadron Spectrum Collaboration):
J. Dudek, P. Guo, B. Joo, D. Richards (JLab), S. Wallace (Maryland)
N. Mathur (Tata), L. Liu, M. Peardon, S. Ryan, C. Thomas (Trinity College)

Recent publications:

"Hybrid baryons", in press PRD, 1201.2349
"Helicity operators for mesons in flight", PRD85, 1107.1930
"Lightest hybrid meson supermultiplet", PRD84, 1106.5515
"Excited state baryon spectroscopy", PRD84, 1104.5152
"Isoscalar meson spectroscopy", PRD83, 1102.4299
"Phase shift of isospin-2 scattering", PRD83, 1011.6352
"Toward the excited meson spectrum", PRD82, 1004.4930
"Highly excited and exotic meson spectrum", PRL103, 0909.0200

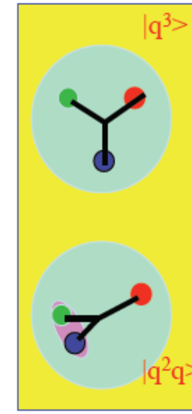
Where are the "Missing" Baryon Resonances?

- What are collective modes?
- Is there "freezing" of degrees of freedom?
- What is the structure of the states?
- Where is the glue?



Where are the "Missing" Baryon Resonances?

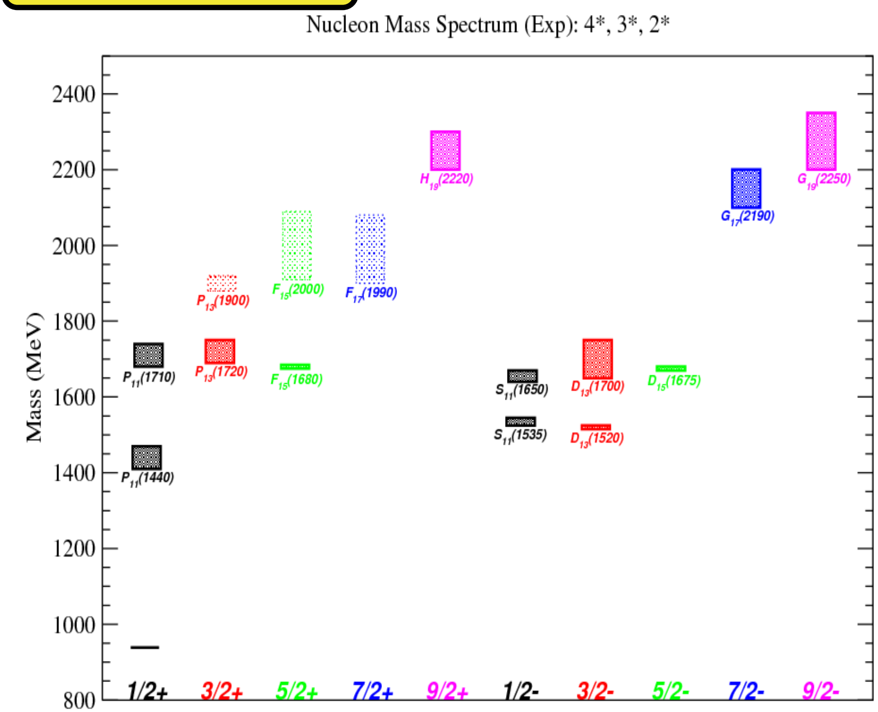
- What are collective modes?
- Is there "freezing" of degrees of freedom?
- What is the structure of the states?
- Where is the glue?

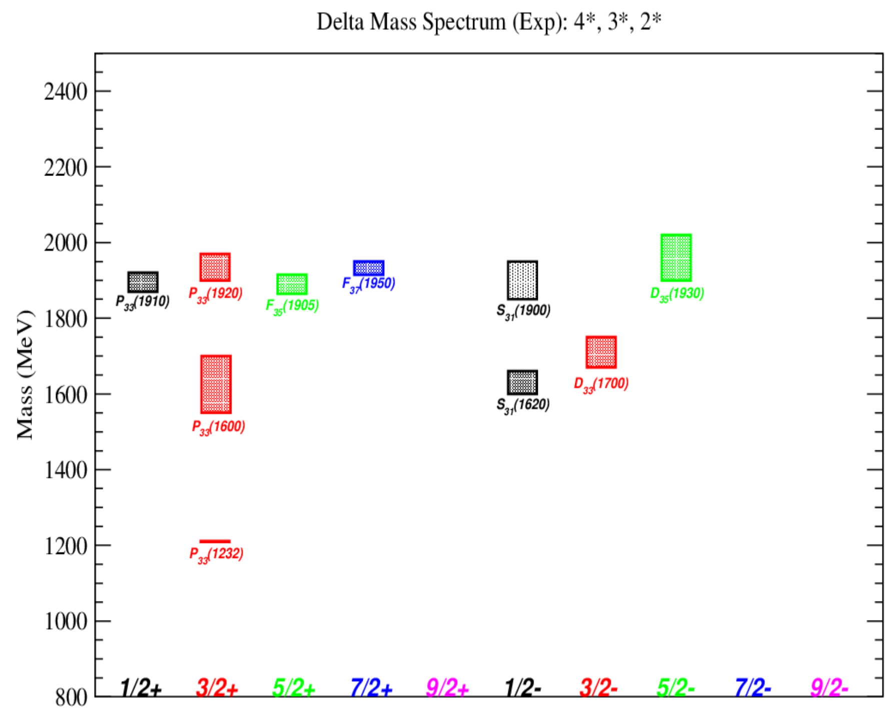


QM predictions

Nucleon & Delta spectrum

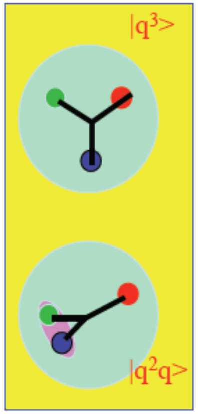
PDG uncertainty on B-W mass





Where are the "Missing" Baryon Resonances?

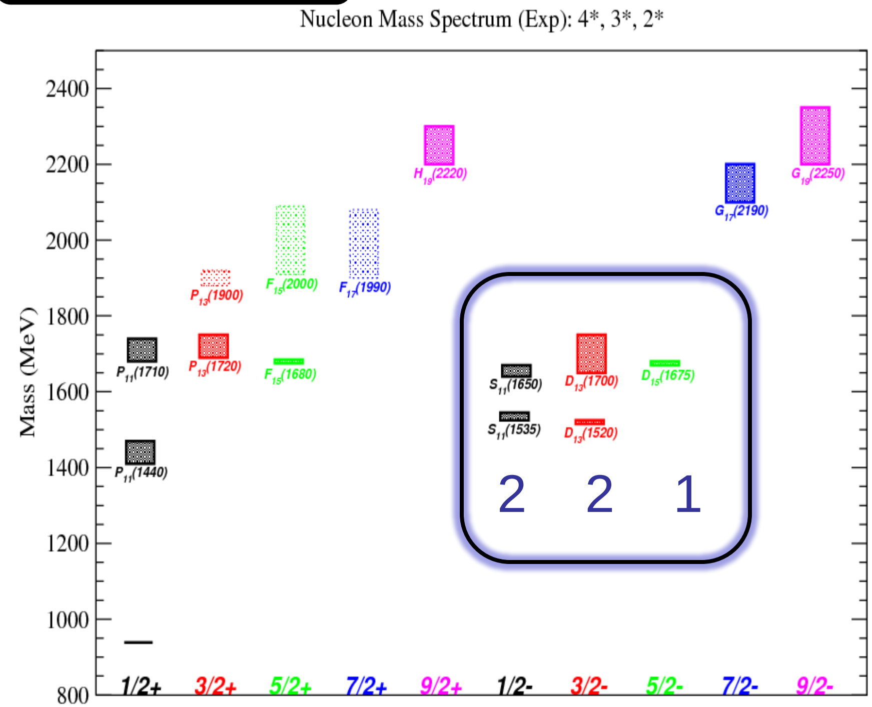
- What are collective modes?
- Is there "freezing" of degrees of freedom?
- What is the structure of the states?
- Where is the glue?

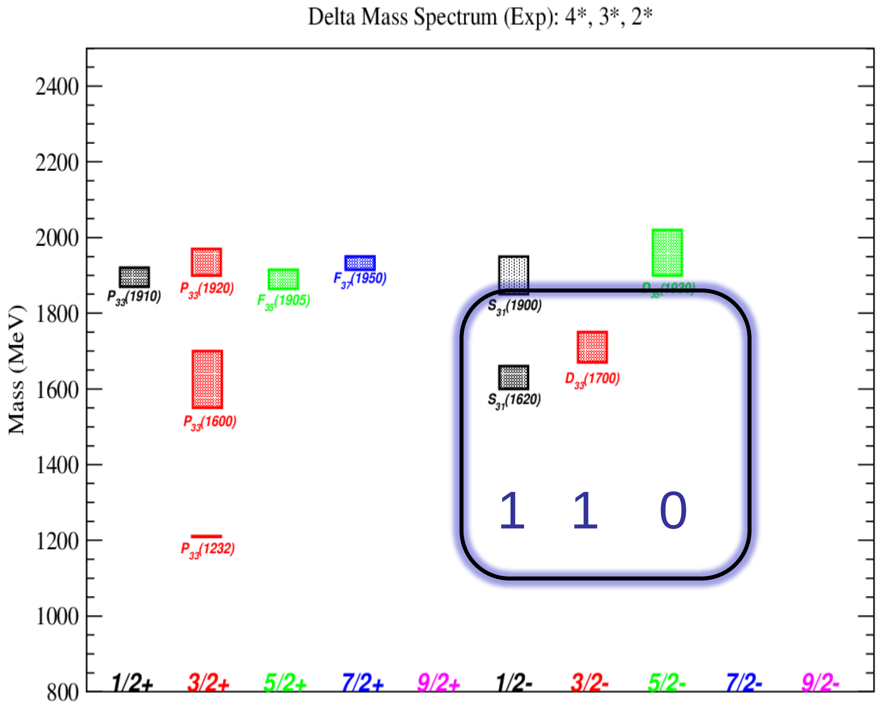


QM predictions

Nucleon & Delta spectrum

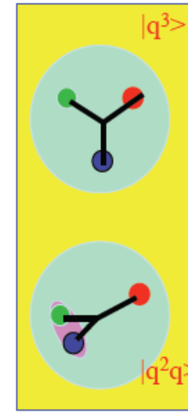
PDG uncertainty on B-W mass





Where are the "Missing" Baryon Resonances?

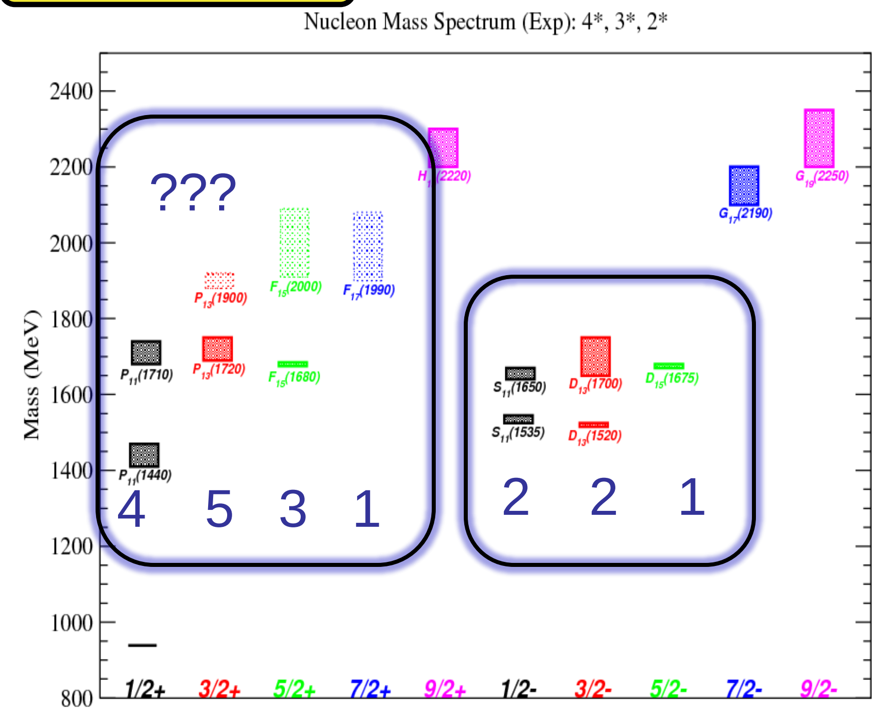
- What are collective modes?
- Is there "freezing" of degrees of freedom?
- What is the structure of the states?
- Where is the glue?

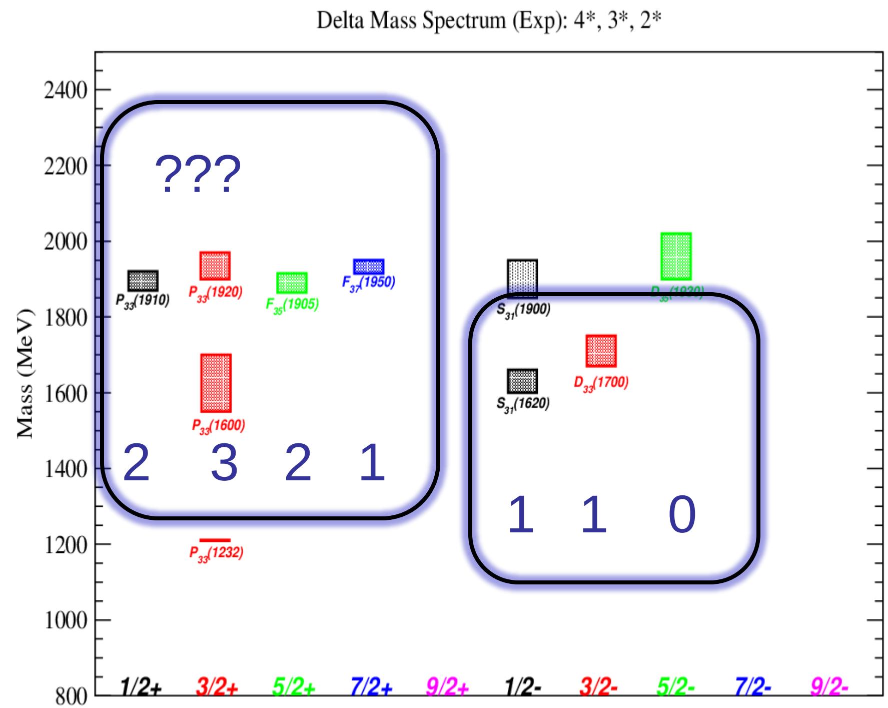


QM predictions

Nucleon & Delta spectrum

PDG uncertainty on B-W mass





Spectrum from variational method

Two-point correlator

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t}\langle 0|\Phi_i(0)|\mathfrak{n}\rangle\langle \mathfrak{n}|\Phi_j^\dagger(0)|0\rangle$$

$$Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n}|\ \Phi_i^\dagger\ |0\rangle$$

Spectrum from variational method

Two-point correlator

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t}\langle 0|\Phi_i(0)|\mathfrak{n}\rangle\langle \mathfrak{n}|\Phi_j^\dagger(0)|0\rangle$$

$$Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n}|\ \Phi_i^\dagger\ |0\rangle$$

Matrix of correlators

$$C(t) = \begin{pmatrix} \langle 0|\Phi_1(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \langle 0|\Phi_2(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \vdots & & \ddots \end{pmatrix}$$

Spectrum from variational method

Two-point correlator

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}}t}\langle 0|\Phi_i(0)|\mathfrak{n}\rangle\langle \mathfrak{n}|\Phi_j^\dagger(0)|0\rangle$$

$$Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n}|\,\Phi_i^\dagger\,|0\rangle$$

Matrix of correlators

$$C(t) = \begin{pmatrix} \langle 0|\Phi_1(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \langle 0|\Phi_2(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \vdots & & \ddots \end{pmatrix}$$

"Rayleigh-Ritz method"
Diagonalize:
- eigenvalues $\rightarrow$ spectrum
- eigenvectors $\rightarrow$ spectral "overlaps" Z_i^n

Spectrum from variational method

Two-point correlator

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}}t}\langle 0|\Phi_i(0)|\mathfrak{n}\rangle\langle\mathfrak{n}|\Phi_j^\dagger(0)|0\rangle$$

$$Z_i^{\mathfrak{n}} \equiv \langle\mathfrak{n}|\,\Phi_i^\dagger\,|0\rangle$$

Matrix of correlators

$$C(t) = \begin{pmatrix} \langle 0|\Phi_1(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \langle 0|\Phi_2(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \vdots & & \ddots \end{pmatrix}$$

"Rayleigh-Ritz method"
Diagonalize:
- eigenvalues → spectrum
- eigenvectors → spectral "overlaps" Z_i^n

Each state optimal combination of Φ_i

$$\Omega^{(\mathfrak{n})} = \sum_i v_i^{(\mathfrak{n})}\Phi_i$$

Spectrum from variational method

Two-point correlator

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t}\langle 0|\Phi_i(0)|\mathfrak{n}\rangle\langle\mathfrak{n}|\Phi_j^\dagger(0)|0\rangle \qquad Z_i^{\mathfrak{n}} \equiv \langle\mathfrak{n}|\,\Phi_i^\dagger\,|0\rangle$$

Matrix of correlators

$$C(t) = \begin{pmatrix} \langle 0|\Phi_1(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \langle 0|\Phi_2(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \vdots & & \ddots \end{pmatrix}$$

"Rayleigh-Ritz method"
Diagonalize:
- eigenvalues $\rightarrow$ spectrum
- eigenvectors $\rightarrow$ spectral "overlaps" Z_i^n

Each state optimal combination of Φ_i

$$\Omega^{(\mathfrak{n})} = \sum_i v_i^{(\mathfrak{n})}\Phi_i$$

Benefit: orthogonality for near degenerate states

Baryon operators

Construction :   permutations of 3 objects

1104.5152

Baryon operators

Construction : permutations of 3 objects

- Symmetric:
  - e.g., uud+udu+duu
- Antisymmetric:
  - e.g., uud-udu+duu-…
- Mixed: (antisymmetric & symmetric)
  - e.g., udu - duu & 2duu - udu - uud

1104.5152

Baryon operators

Construction : permutations of 3 objects

- Symmetric:
  - e.g., uud+udu+duu
- Antisymmetric:
  - e.g., uud-udu+duu-…
- Mixed: (antisymmetric & symmetric)
  - e.g., udu - duu & 2duu - udu - uud

Multiplication rules:

- Symmetric Antisymmetric $\rightarrow$ Antisymmetric
- MixedxMixed $\rightarrow$ Symmetric$\oplus$Antisymmetric$\oplus$Mixed
- ….

1104.5152

Baryon operators

Construction : permutations of 3 objects

- Symmetric:
  - e.g., uud+udu+duu
- Antisymmetric:
  - e.g., uud-udu+duu-…
- Mixed: (antisymmetric & symmetric)
  - e.g., udu - duu & 2duu - udu - uud

Multiplication rules:

- Symmetric Antisymmetric $\rightarrow$ Antisymmetric
- MixedxMixed $\rightarrow$ Symmetric$\oplus$Antisymmetric$\oplus$Mixed
- ….

Color antisymmetric $\rightarrow$ Require **Space x [Flavor x Spin]** symmetric

1104.5152

Baryon operators

Construction : permutations of 3 objects

- Symmetric:
  - e.g., uud+udu+duu
- Antisymmetric:
  - e.g., uud-udu+duu-…
- Mixed: (antisymmetric & symmetric)
  - e.g., udu - duu & 2duu - udu - uud

Multiplication rules:
- Symmetric Antisymmetric $\rightarrow$ Antisymmetric
- MixedxMixed $\rightarrow$ Symmetric$\oplus$Antisymmetric$\oplus$Mixed
- ….

Color antisymmetric $\rightarrow$ Require **Space x [Flavor x Spin]** symmetric

Space: couple covariant derivatives onto single-site spinors - build any J,M

$$\Phi^{JM} \leftarrow (CGC's)_{i,j,k} \left[\vec{D}\right]_i \left[\vec{D}\right]_j [\Psi]_k$$

$$J \leftarrow \mathbf{1} \otimes \mathbf{1} \otimes S$$

1104.5152

Baryon operators

Construction : permutations of 3 objects

- Symmetric:
  - e.g., uud+udu+duu
- Antisymmetric:
  - e.g., uud-udu+duu-…
- Mixed: (antisymmetric & symmetric)
  - e.g., udu - duu & 2duu - udu - uud

Multiplication rules:

- Symmetric Antisymmetric $\rightarrow$ Antisymmetric
- MixedxMixed $\rightarrow$ Symmetric$\oplus$Antisymmetric$\oplus$Mixed
- ….

Color antisymmetric $\rightarrow$ Require **Space x [Flavor x Spin]** symmetric

Space: couple covariant derivatives onto single-site spinors - build any J,M

$$\Phi^{JM} \leftarrow (CGC's)_{i,j,k} \left[\vec{D}\right]_i \left[\vec{D}\right]_j [\Psi]_k$$

$$J \leftarrow \mathbf{1} \otimes \mathbf{1} \otimes S$$

Classify operators by permutation symmetries:

- Leads to rich structure

1104.5152

Baryon operator basis

All possible 3-quark operators up to two covariant derivatives: some J^P

$$\left(\left[\mathrm{Flavor} \otimes \mathrm{Dirac} \right] \otimes \mathrm{Space}_{\mathrm{symmetry}} \right)^{J^P}$$

Baryon operator basis

All possible 3-quark operators up to two covariant derivatives: some J^P

$$\left(\left[\text{Flavor} \otimes \text{Dirac}\right] \otimes \text{Space}_{\text{symmetry}}\right)^{J^P}$$

Spatial symmetry classification:

e.g., Nucleons: $N\ {}^{2S+1}L_{\pi}\ J^P$

J^P	#ops	E.g., spatial symmetries	
J=1/2⁻	32	$N\ {}^2P_M\ \frac{1}{2}^-$	$N\ {}^4P_M\ \frac{1}{2}^-$
J=3/2⁻	36	$N\ {}^2P_M\ 3/2^-$	$N\ {}^4P_M\ 3/2^-$
J=5/2⁻	19	$N\ {}^4P_M\ 5/2^-$	
J=1/2⁺	32	$N\ {}^2S_S\ \frac{1}{2}^+$ $N\ {}^2S_M\ \frac{1}{2}^+$ $N\ {}^2P_M\ \frac{1}{2}^+$	$N\ {}^4D_M\ \frac{1}{2}^+$ $N\ {}^2P_A\ \frac{1}{2}^+$ $N\ {}^4P_M\ \frac{1}{2}^+$
J=3/2⁺	36	$N\ {}^2D_S\ 3/2^+$ $N\ {}^2D_M\ 3/2^+$ $N\ {}^2P_A\ 3/2^+$ $N\ {}^2P_M\ 3/2^+$	$N\ {}^4S_M\ 3/2^+$ $N\ {}^4D_M\ 3/2^+$ $N\ {}^4P_M\ 3/2^+$
J=5/2⁺	19	$N\ {}^2D_S\ 5/2^+$ $N\ {}^2D_M\ 5/2^+$	$N\ {}^4D_M\ 5/2^+$ $N\ {}^4P_M\ 5/2^+$
J=7/2⁺	4	$N\ {}^4D_M\ 7/2^+$	

Baryon operator basis

All possible 3-quark operators up to two covariant derivatives: some J^P

$$\left(\left[\mathrm{Flavor} \otimes \mathrm{Dirac}\right] \otimes \mathrm{Space}_{\mathrm{symmetry}}\right)^{J^P}$$

Spatial symmetry classification:

e.g., Nucleons: $N\ {}^{2S+1}L_{\pi}\ J^P$

By far the largest operator basis ever used for such calculations

Hold on, what are those P_M ??

J^P	#ops	E.g., spatial symmetries	
$J=1/2^-$	32	$N\ {}^2P_M\ \frac{1}{2}^-$	$N\ {}^4P_M\ \frac{1}{2}^-$
$J=3/2^-$	36	$N\ {}^2P_M\ 3/2^-$	$N\ {}^4P_M\ 3/2^-$
$J=5/2^-$	19	$N\ {}^4P_M\ 5/2^-$	
$J=1/2^+$	32	$N\ {}^2S_S\ \frac{1}{2}^+$ $N\ {}^2S_M\ \frac{1}{2}^+$ $N\ {}^2P_M\ \frac{1}{2}^+$	$N\ {}^4D_M\ \frac{1}{2}^+$ $N\ {}^2P_A\ \frac{1}{2}^+$ $N\ {}^4P_M\ \frac{1}{2}^+$
$J=3/2^+$	36	$N\ {}^2D_S 3/2^+$ $N\ {}^2D_M 3/2^+$ $N\ {}^2P_A\ 3/2^+$ $N\ {}^2P_M\ 3/2^+$	$N\ {}^4S_M 3/2^+$ $N\ {}^4D_M 3/2^+$ $N\ {}^4P_M\ 3/2^+$
$J=5/2^+$	19	$N\ {}^2D_S 5/2^+$ $N\ {}^2D_M 5/2^+$	$N\ {}^4D_M 5/2^+$ $N\ {}^4P_M\ 5/2^+$
$J=7/2^+$	4	$N\ {}^4D_M 7/2^+$	

Operators featuring explicit glue

At two derivatives & L=1, have operators featuring a chromomagnetic B field

$$\Phi \sim \varepsilon_{abc}(B_k\psi)_a\psi_b\psi_c \; + \; \ldots$$

$$B_k = \frac{1}{2}\varepsilon_{ijk}[D_i, D_j] \quad \text{where } D_i = \partial_i - A_i$$

Operators featuring explicit glue

At two derivatives & L=1, have operators featuring a chromomagnetic B field

$$\Phi \sim \varepsilon_{abc} (B_k \psi)_a \psi_b \psi_c \; + \; \ldots$$

$$B_k = \frac{1}{2} \varepsilon_{ijk} [D_i, D_j] \quad \text{where } D_i = \partial_i - A_i$$

Occurs only in *Mixed* spatial symmetry with L=1 → P_M with *positive* parity

Hybrid operator: is 0 unless in a nontrivial background gauge field $A \neq 0$

Operators featuring explicit glue

At two derivatives & L=1, have operators featuring a chromomagnetic B field

$$\Phi \sim \varepsilon_{abc}(B_k\psi)_a\psi_b\psi_c \ + \ \ldots$$

$$B_k = \tfrac{1}{2}\varepsilon_{ijk}[D_i, D_j] \quad \text{where } D_i = \partial_i - A_i$$

Occurs only in *Mixed* spatial symmetry with L=1 → P_M with *positive* parity

Hybrid operator: is 0 unless in a nontrivial background gauge field $A \neq 0$

Note: Antisymmetric P_A operators not of this form. Is not 0 if $A = 0$

Just a basis

Recall: two-point correlator matrix

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

Just a basis

Recall: two-point correlator matrix

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

Spectral decomposition

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0|\Phi_i(0)|\mathfrak{n}\rangle \langle \mathfrak{n}|\Phi_j^\dagger(0)|0\rangle$$

$$Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n}| \, \Phi_i^\dagger \, |0\rangle$$

Just a basis

Recall: two-point correlator matrix

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

Spectral decomposition

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t}\langle 0|\Phi_i(0)|\mathfrak{n}\rangle\langle\mathfrak{n}|\Phi_j^\dagger(0)|0\rangle$$

$$Z_i^{\mathfrak{n}} \equiv \langle\mathfrak{n}|\ \Phi_i^\dagger\ |0\rangle$$

Choose a random linear combination of operators

$$\Phi_i' = \sum_j \eta_{ij}\Phi_j$$

Just a basis

Recall: two-point correlator matrix

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

Spectral decomposition

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t}\langle 0|\Phi_i(0)|\mathfrak{n}\rangle\langle \mathfrak{n}|\Phi_j^\dagger(0)|0\rangle$$

$$Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n}|\,\Phi_i^\dagger\,|0\rangle$$

Choose a random linear combination of operators

$$\Phi_i' = \sum_j \eta_{ij}\Phi_j$$

Spectral "overlaps" Z_i^n change

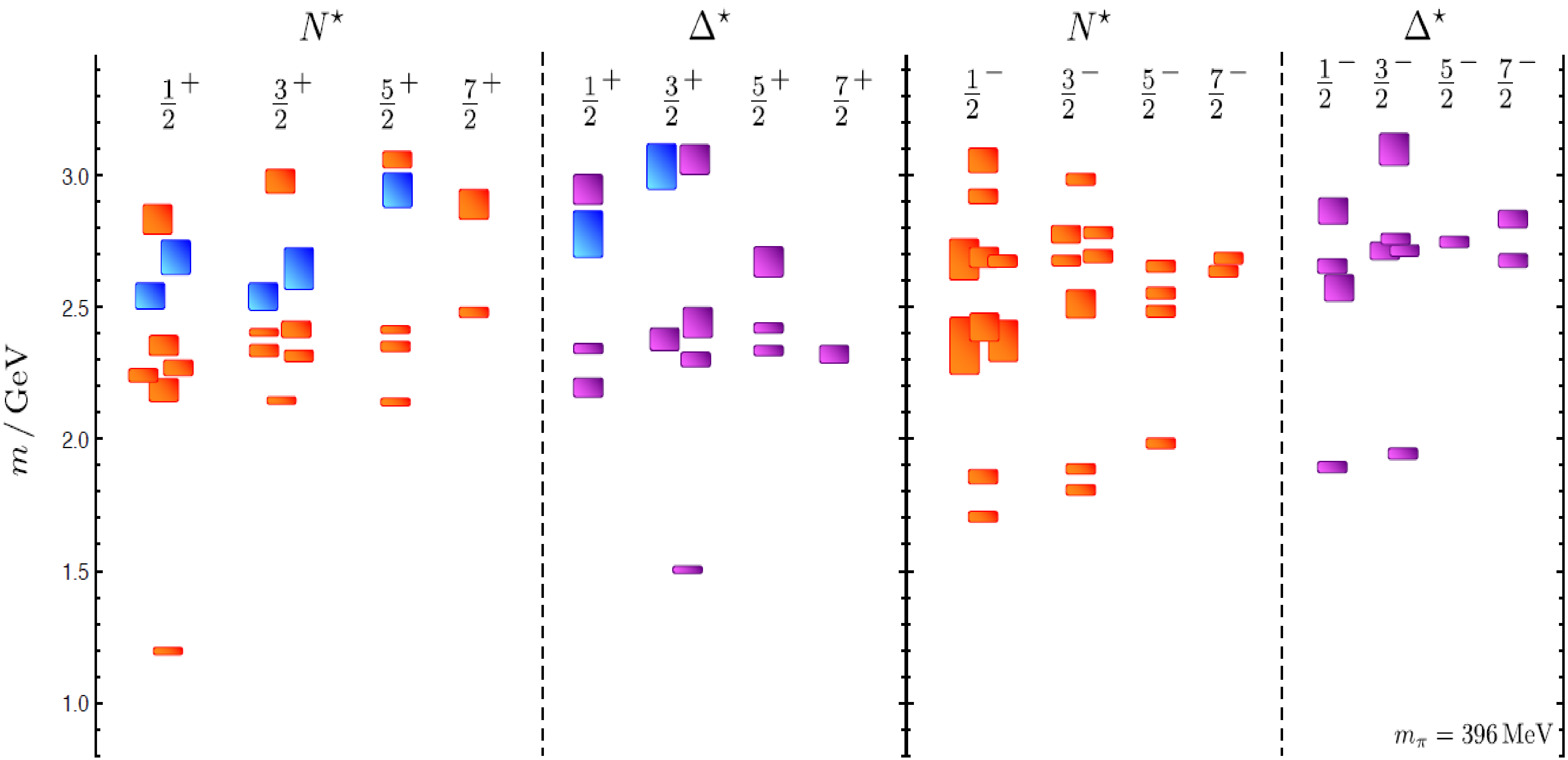
Energies E_n unchanged

Spin identified Nucleon & Delta spectrum

Statistical errors < 2%

arXiv:1104.5152, 1201.2349

m_π ~ 396MeV

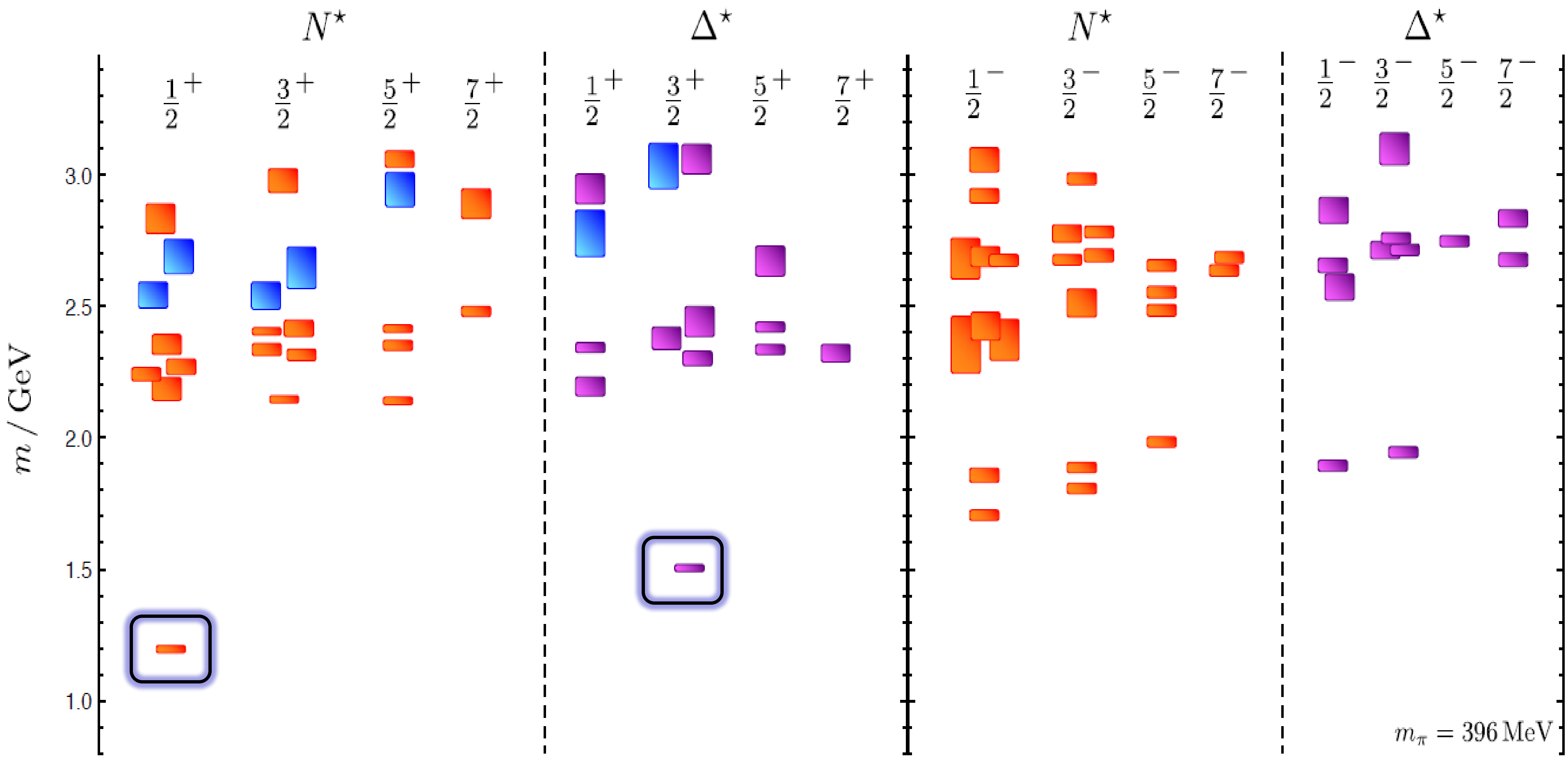


Spin identified Nucleon & Delta spectrum

Statistical errors < 2%

arXiv:1104.5152, 1201.2349

m_π ~ 396MeV

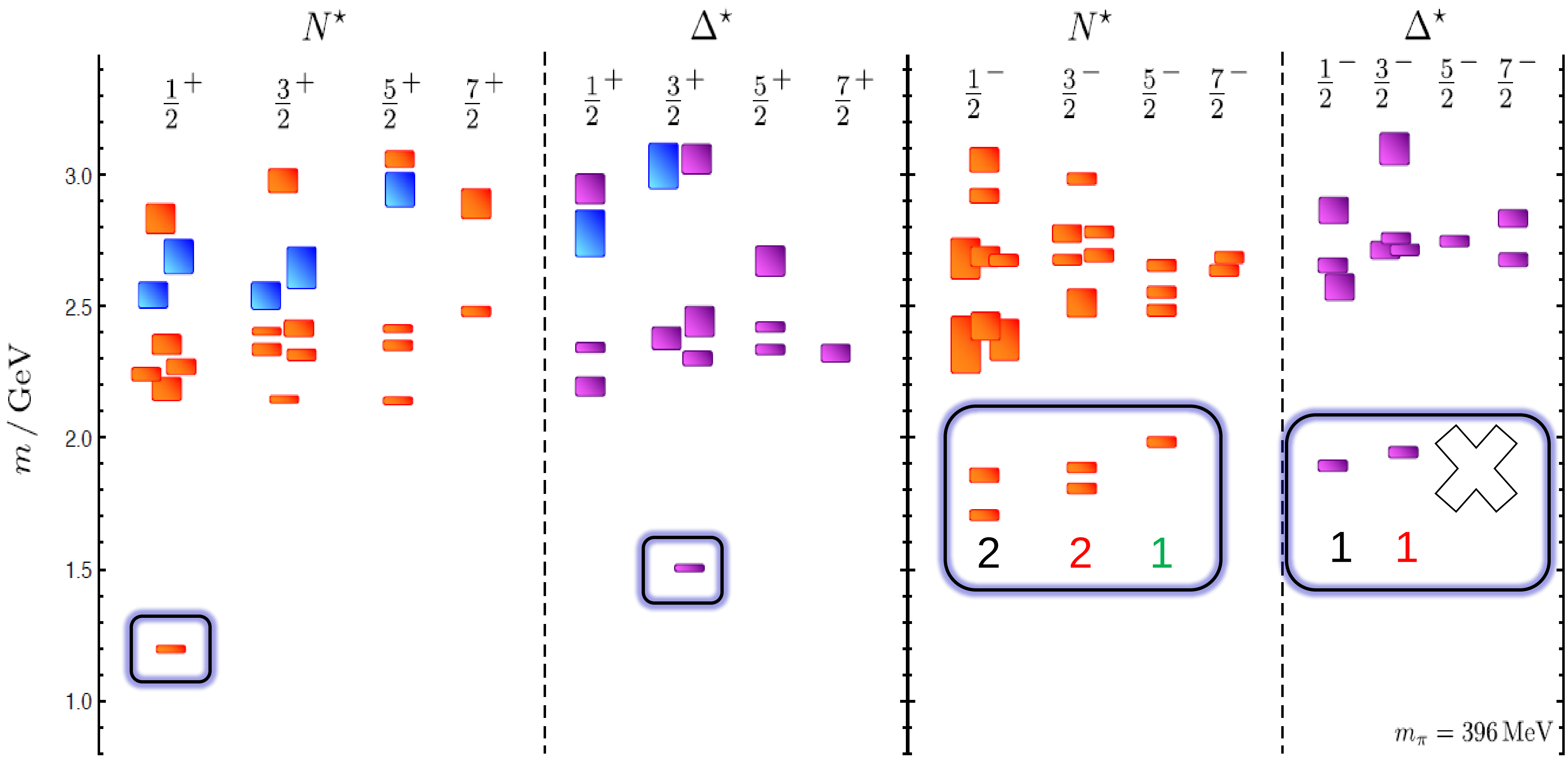


Spin identified Nucleon & Delta spectrum

Statistical errors < 2%

arXiv:1104.5152, 1201.2349

m_π ~ 396MeV

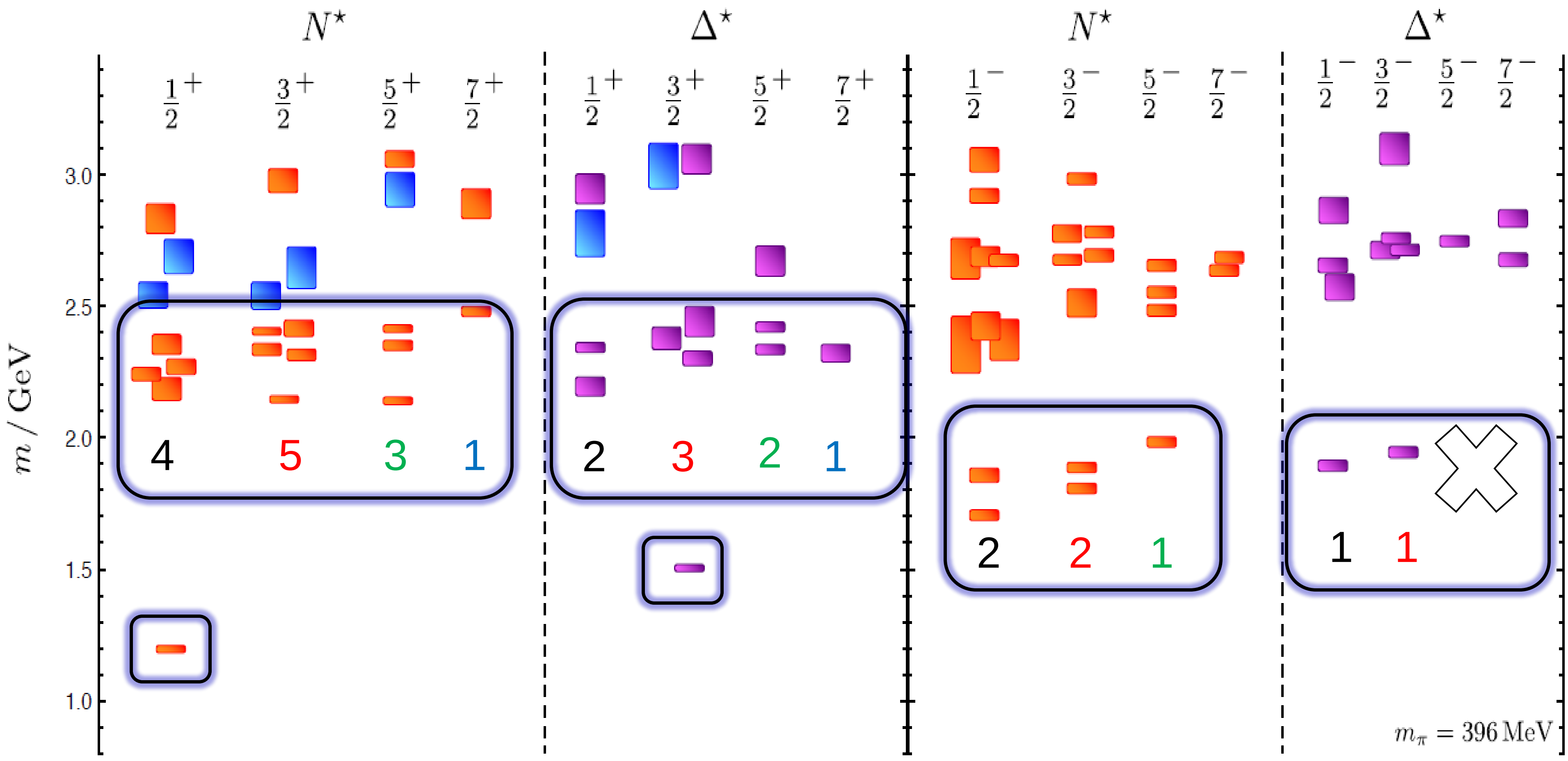


Spin identified Nucleon & Delta spectrum

Statistical errors < 2%

arXiv:1104.5152, 1201.2349

m_π ~ 396MeV

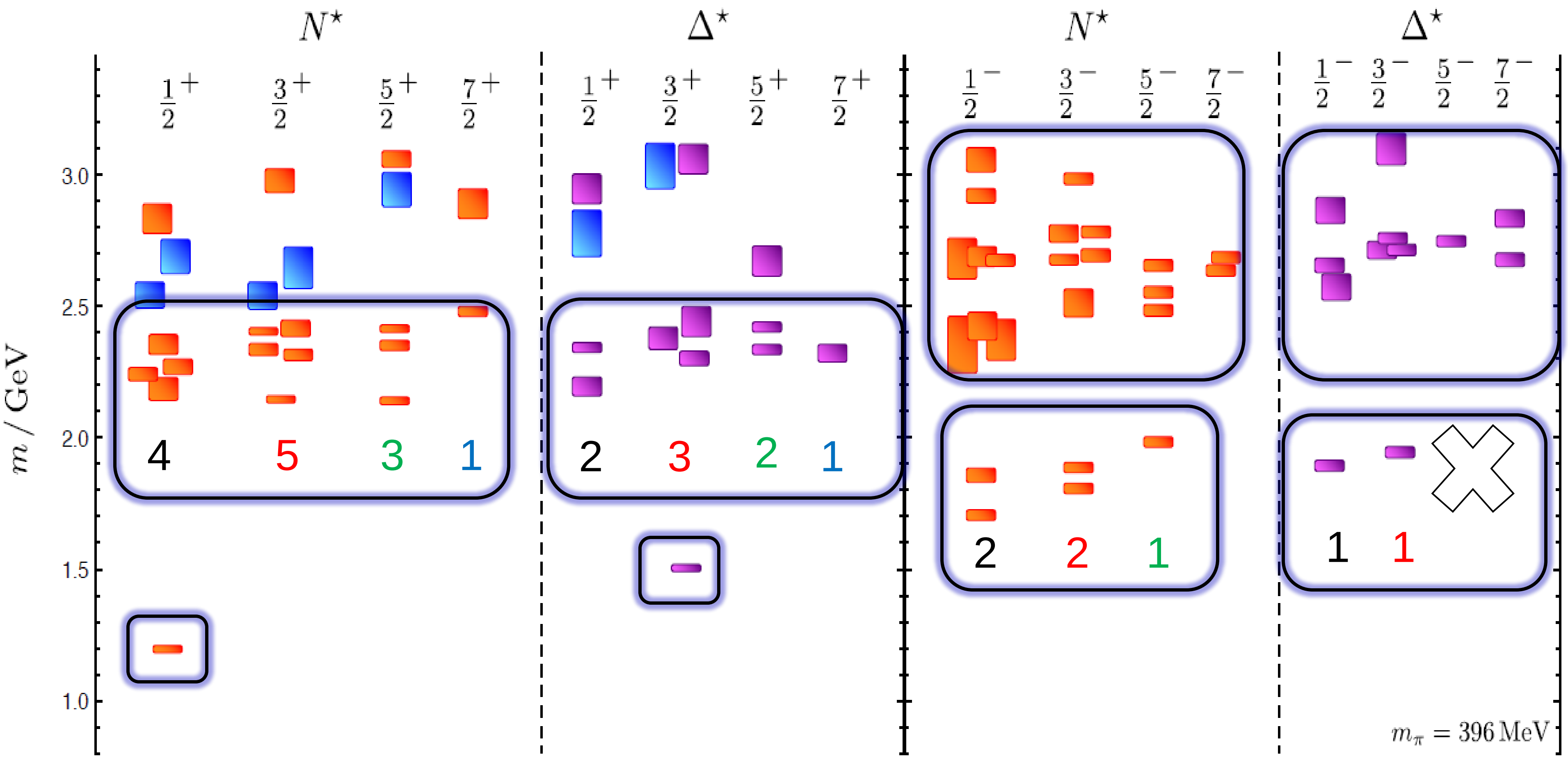


Spin identified Nucleon & Delta spectrum

Statistical errors < 2%

arXiv:1104.5152, 1201.2349

m_π ~ 396MeV

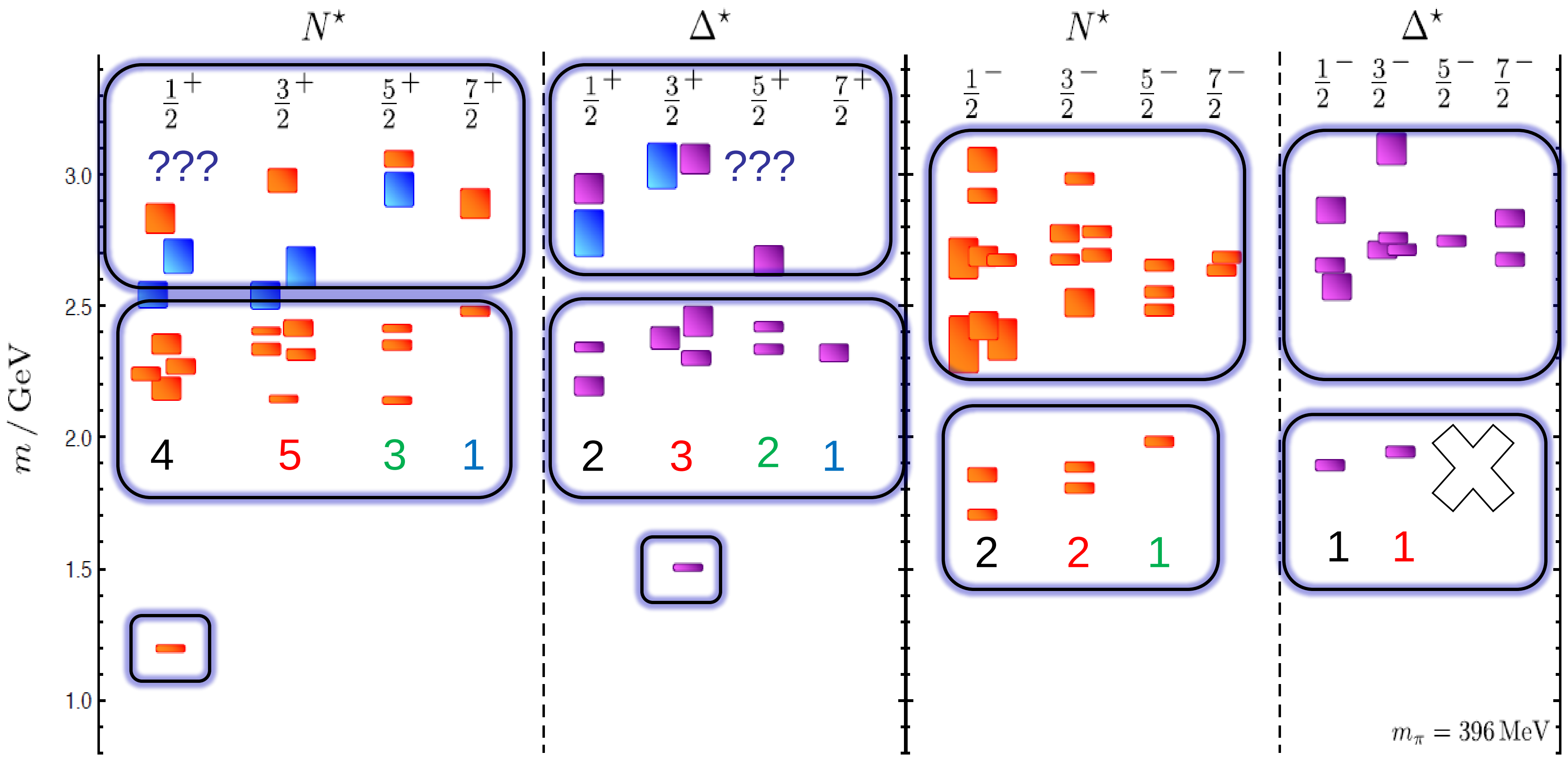


Spin identified Nucleon & Delta spectrum

Statistical errors < 2%

arXiv:1104.5152, 1201.2349

m_π ~ 396MeV

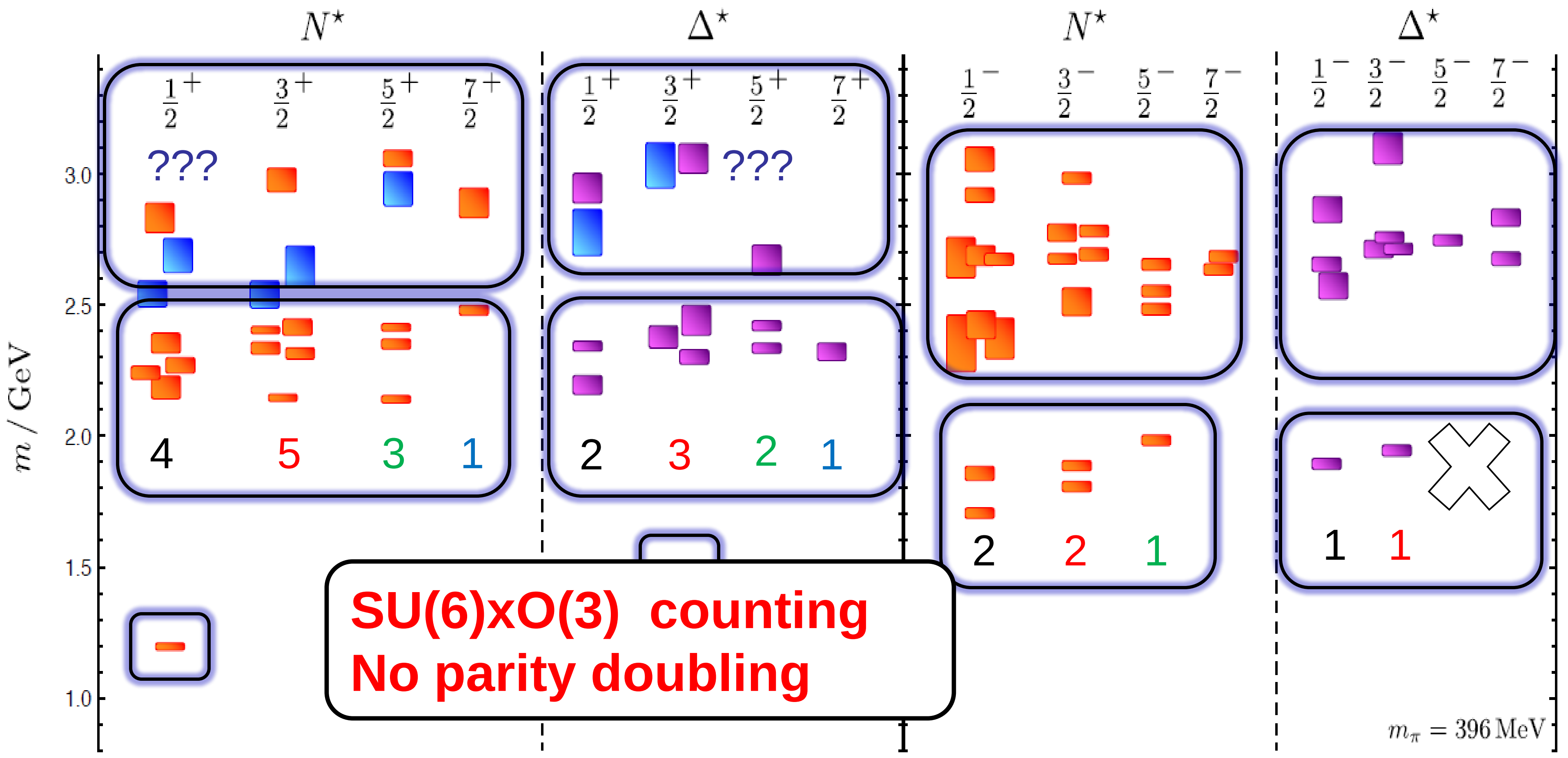


Spin identified Nucleon & Delta spectrum

Statistical errors < 2%

arXiv:1104.5152, 1201.2349

m_π ~ 396MeV



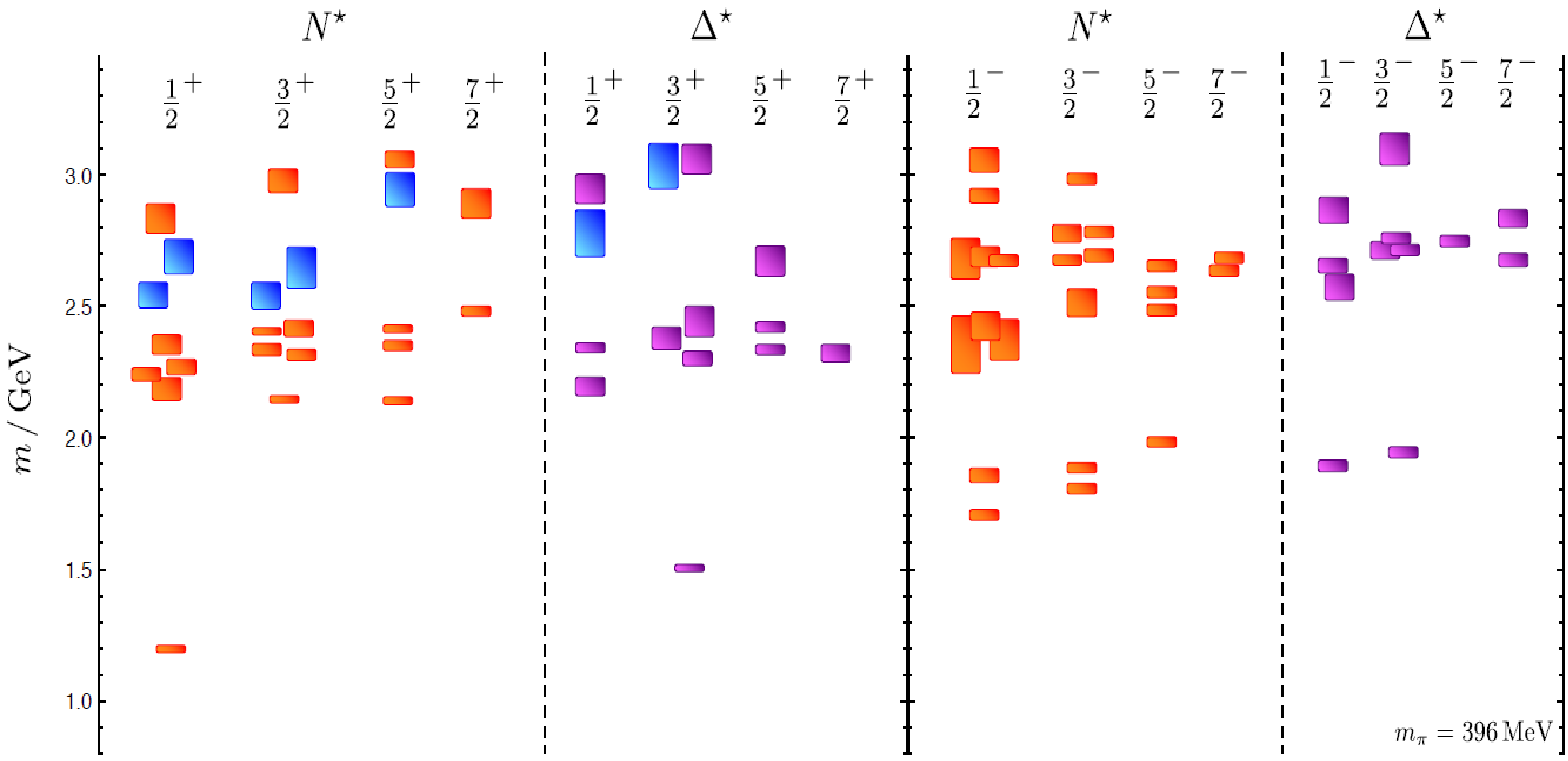
SU(6)xO(3) counting
No parity doubling

Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

arXiv:1104.5152, 1201.2349

$m_\pi \sim 396$MeV

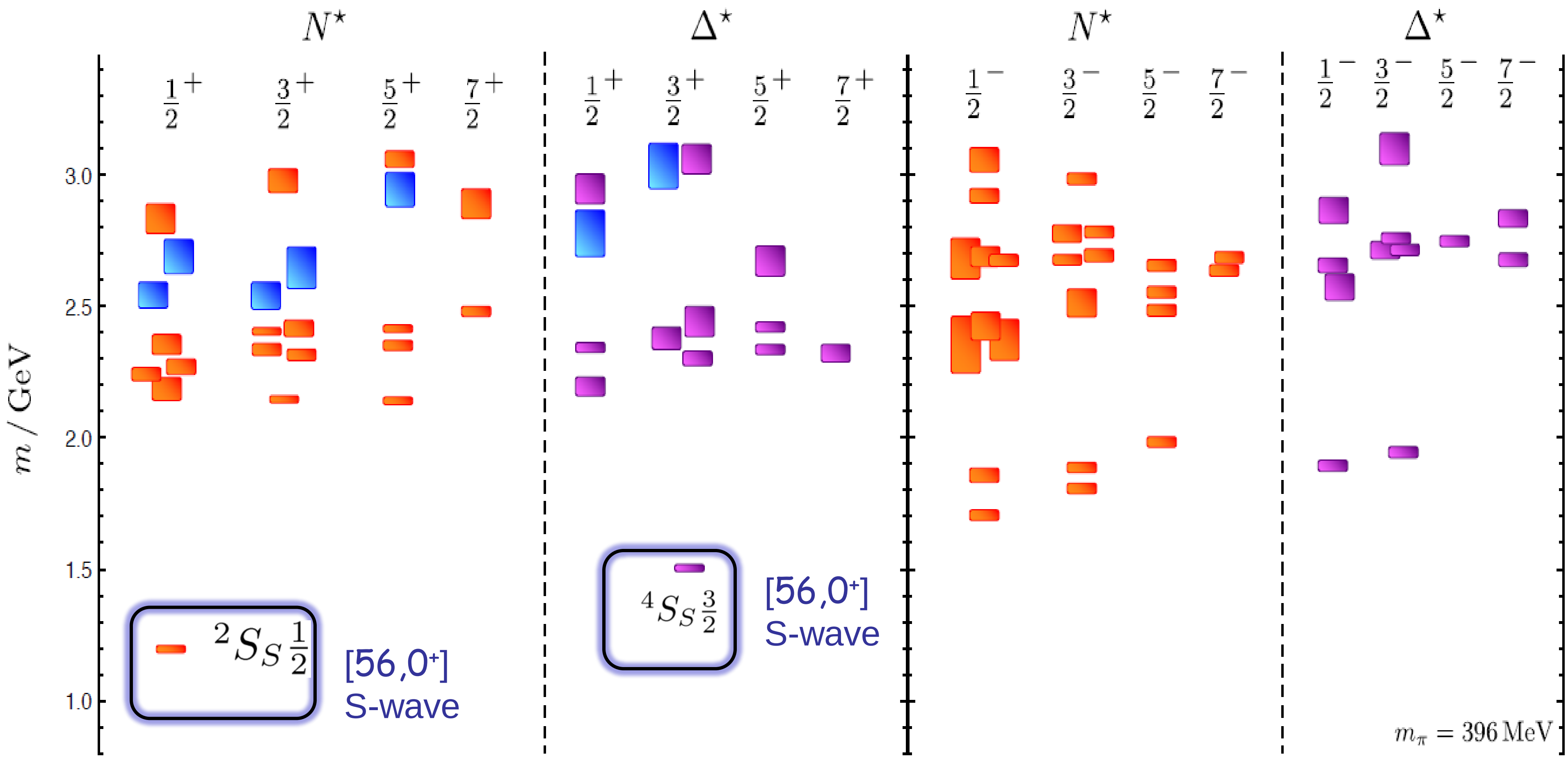


Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

arXiv:1104.5152, 1201.2349

$m_\pi \sim 396$MeV

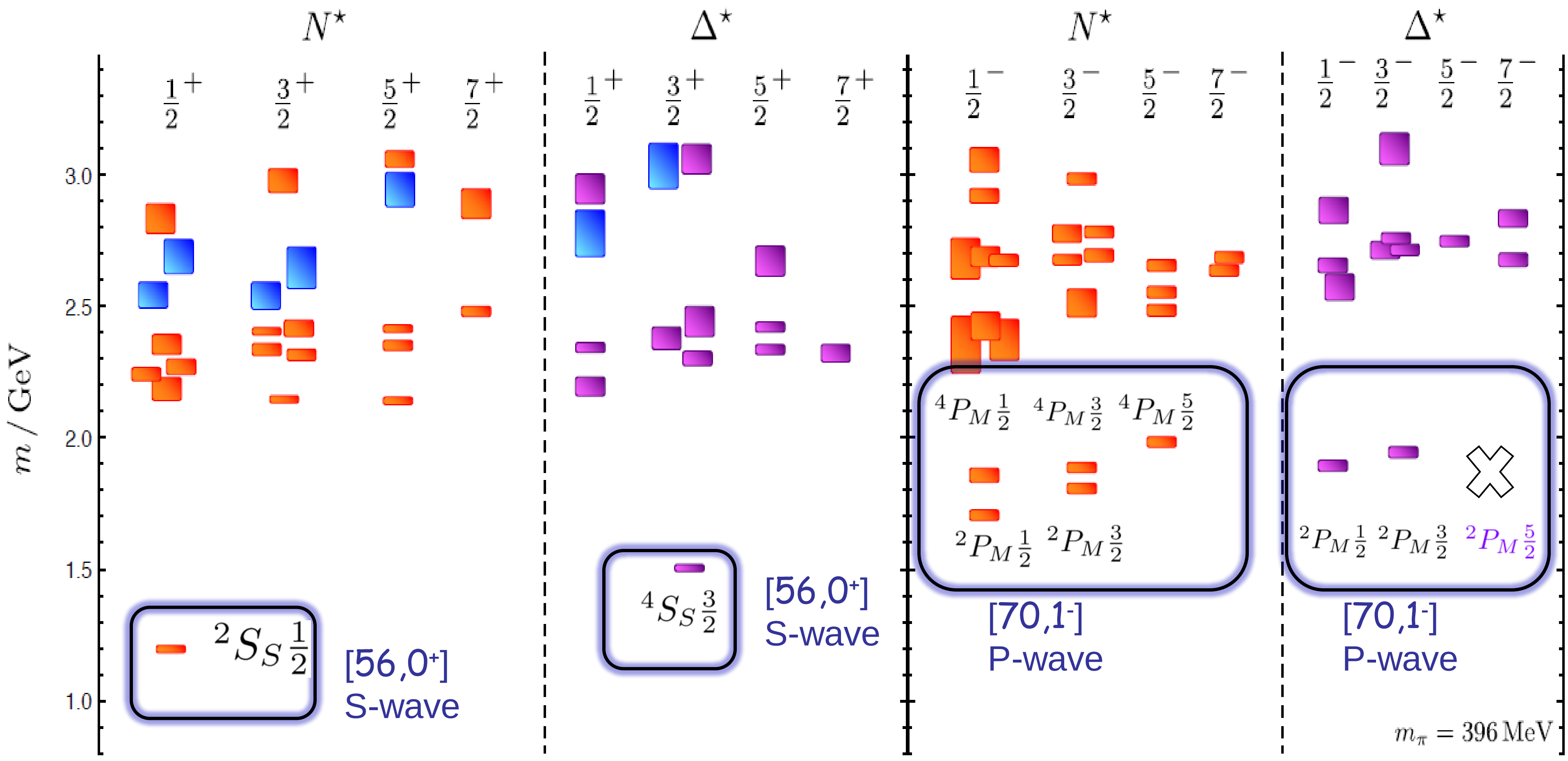


Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

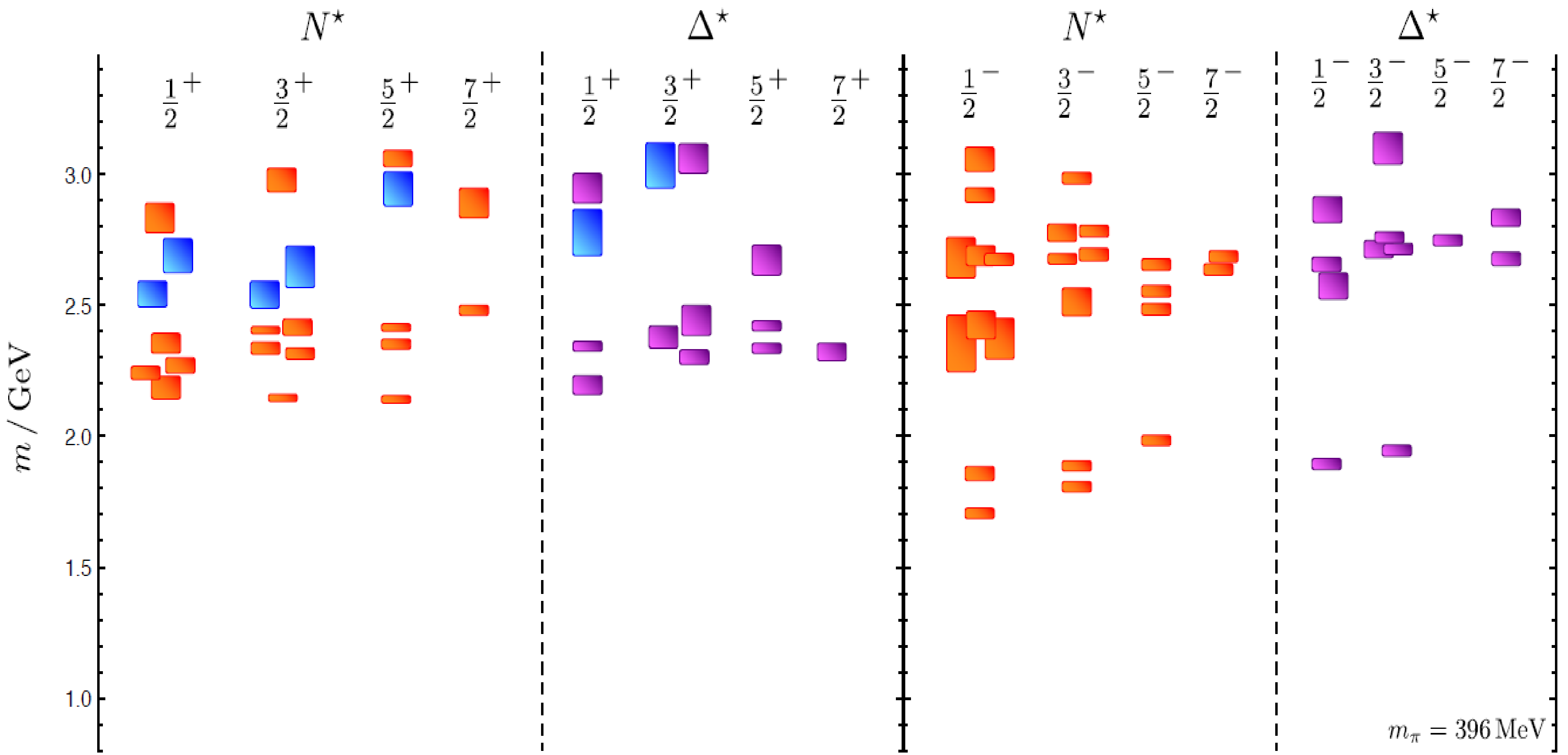
arXiv:1104.5152, 1201.2349

$m_\pi \sim 396$MeV



Spin identified Nucleon & Delta spectrum

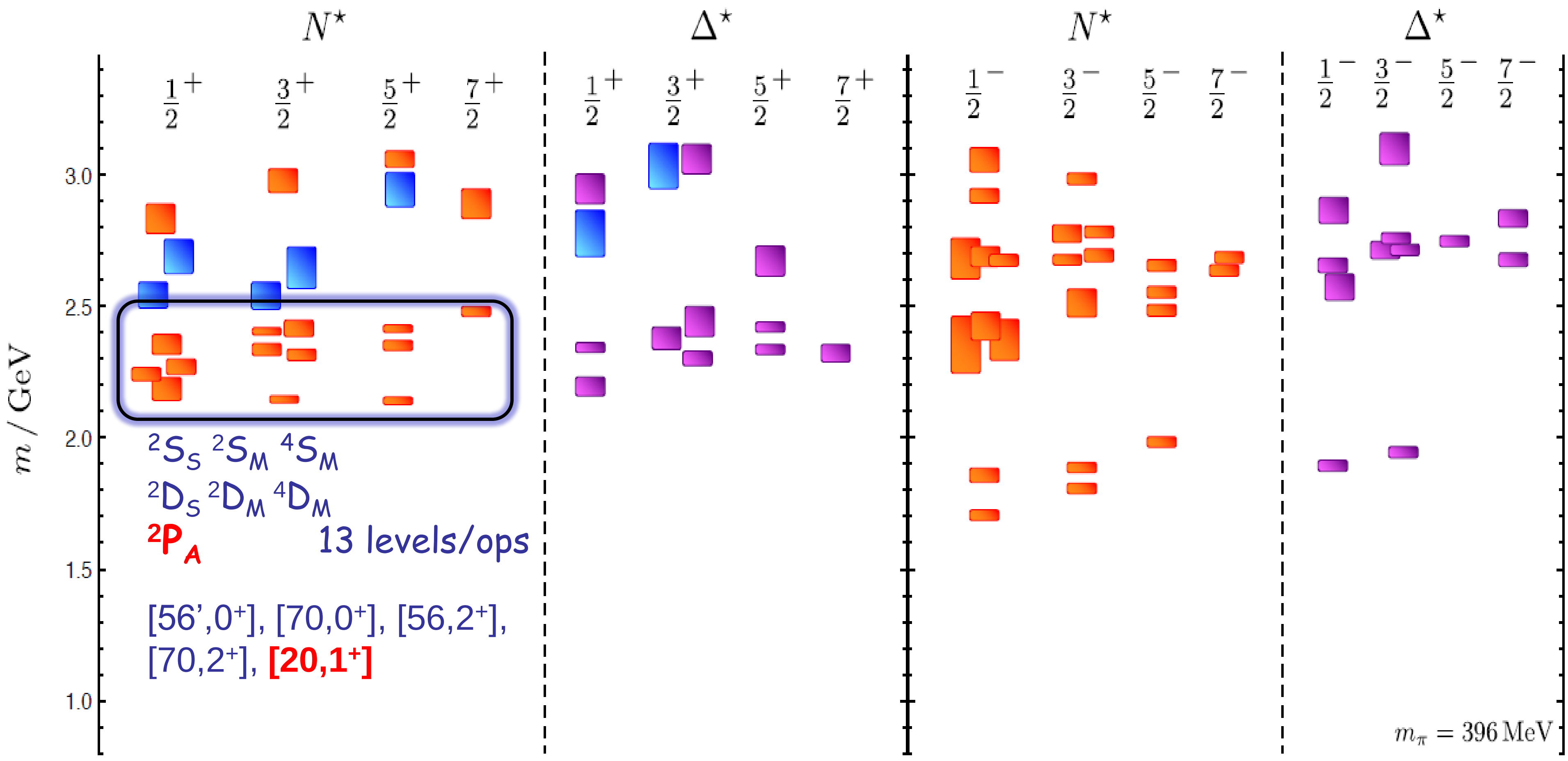
Discern structure: spectral overlaps



Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

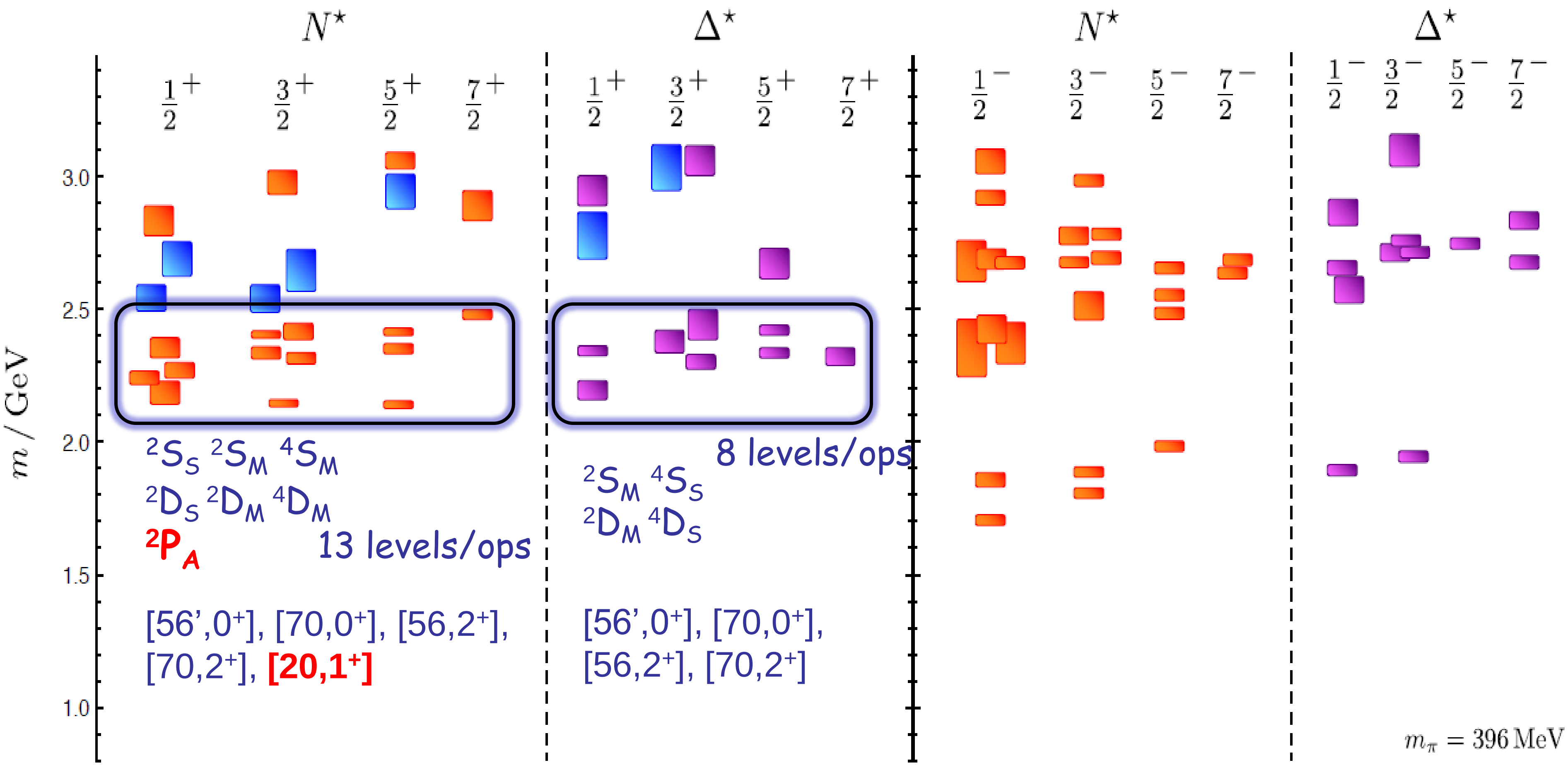
Significant mixing in J^+



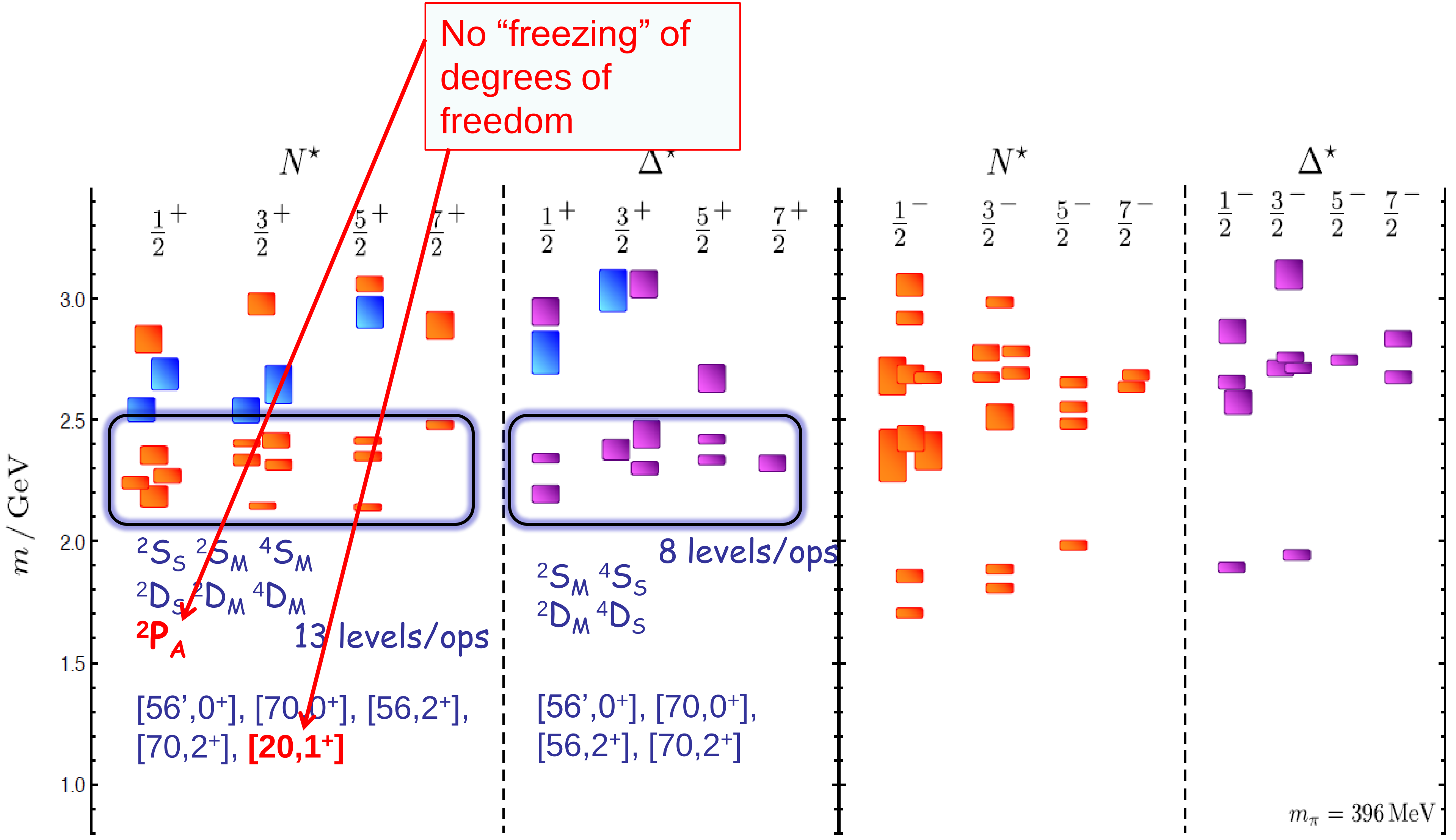
Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

Significant mixing in J^+

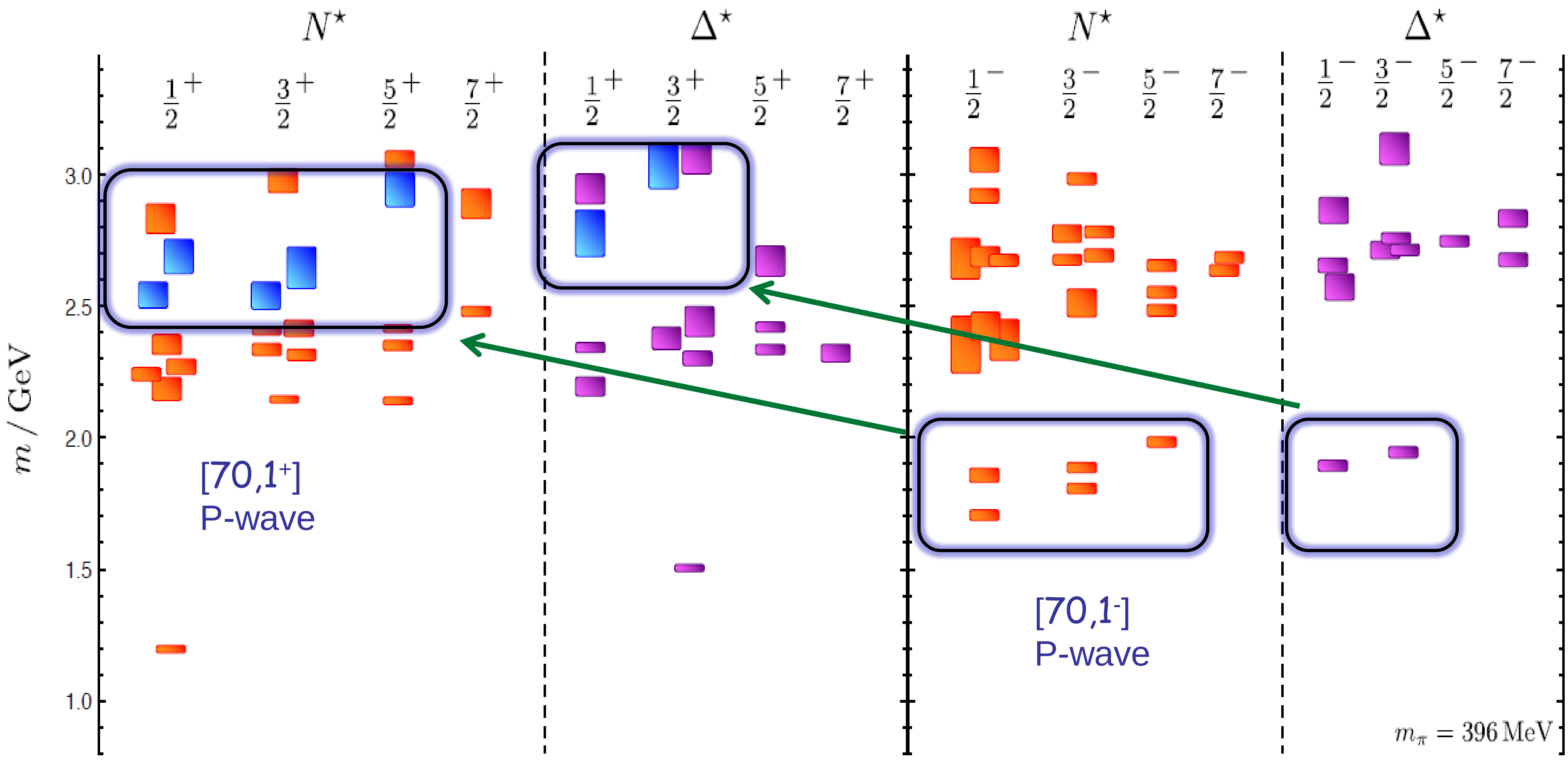


Spin identified Nucleon & Delta spectrum



Hybrid baryons

Negative parity structure replicated: gluonic components (*hybrid* baryons)

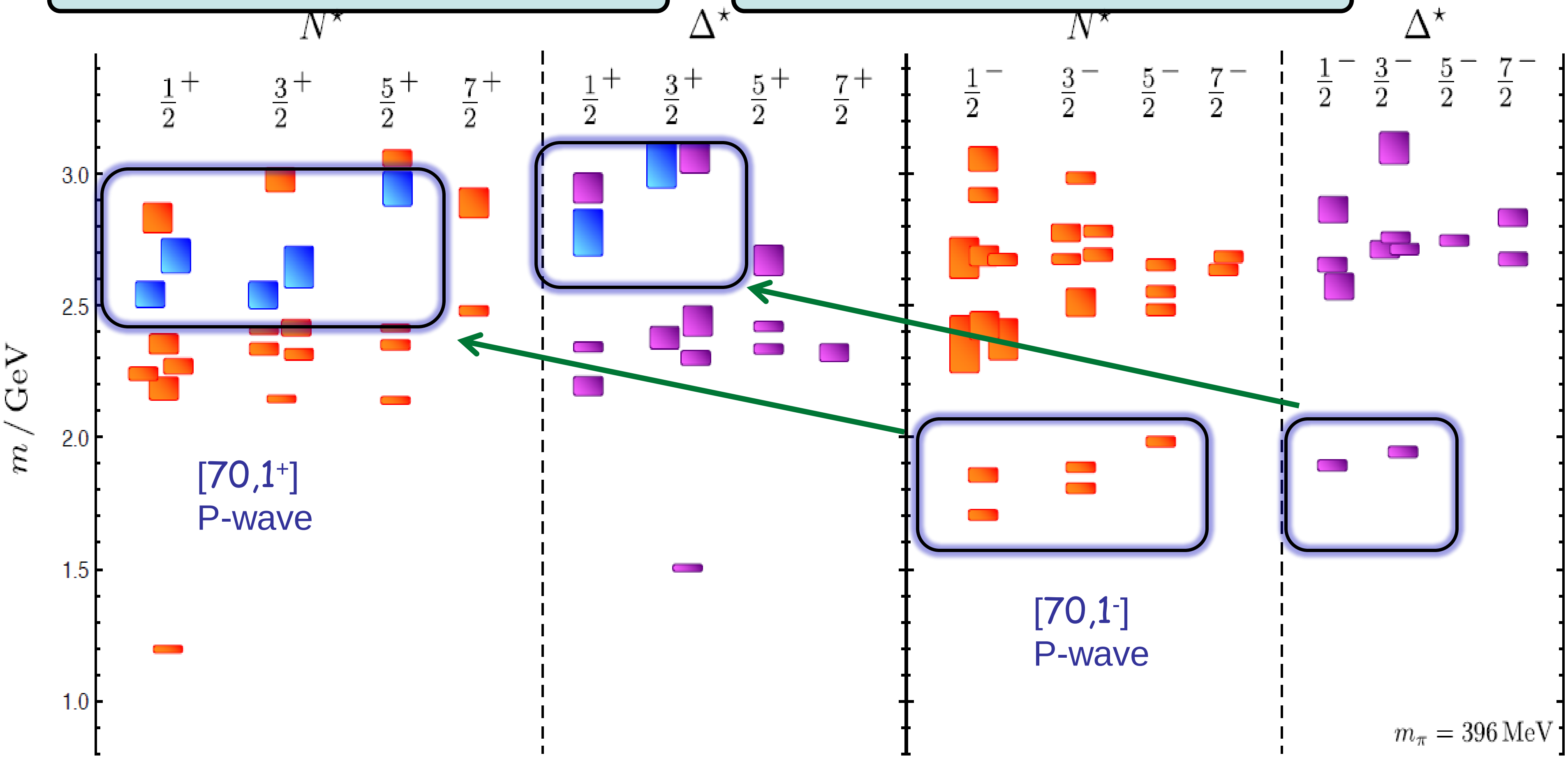


Hybrid baryons

Negative parity structure replicated: gluonic components (*hybrid* baryons)

Predicted by Barnes & Close, 1983

See talk by J. Dudek



Hadronic decays

Current spectrum calculations:
no evidence of multi-particle levels

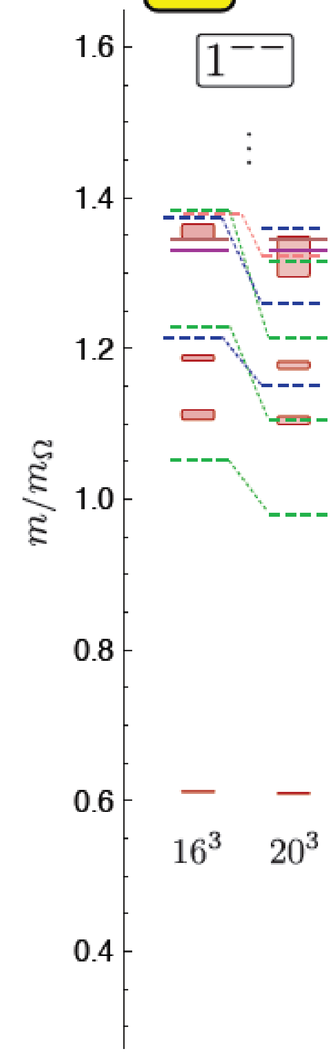
Hadronic decays

Current spectrum calculations:
no evidence of multi-particle levels

Plot the non-interacting meson levels as a guide

$$|A(\vec{p})B(-\vec{p})\rangle \qquad m_{AB} = \sqrt{m_A^2 + \vec{p}^2} + \sqrt{m_B^2 + \vec{p}^2}$$

e.g.



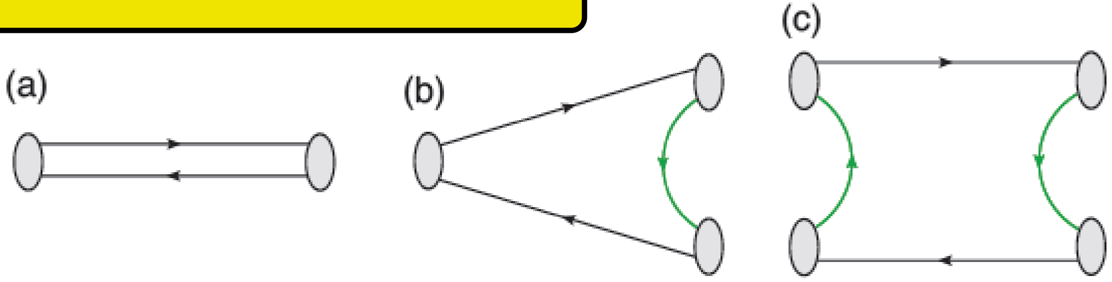
Hadronic decays

Current spectrum calculations:
no evidence of multi-particle levels

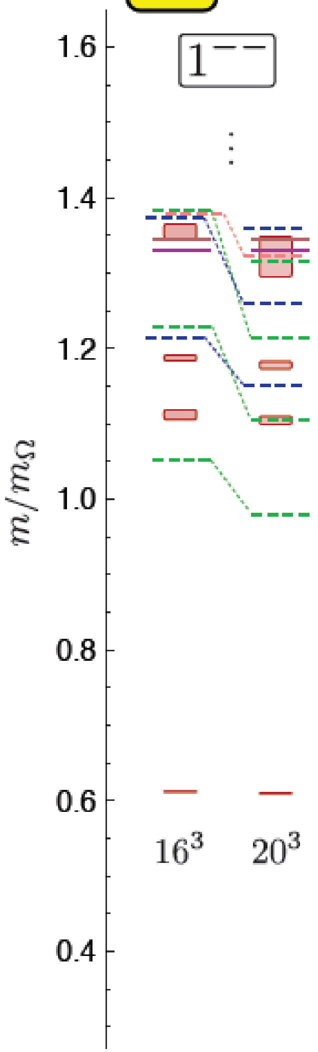
Plot the non-interacting meson levels as a guide

$$\left|A(\vec{p})B(-\vec{p})\right\rangle \qquad m_{AB} = \sqrt{m_A^2 + \vec{p}^2} + \sqrt{m_B^2 + \vec{p}^2}$$

Require multi-particle operators
- (lattice) helicity construction
- annihilation diagrams



e.g.



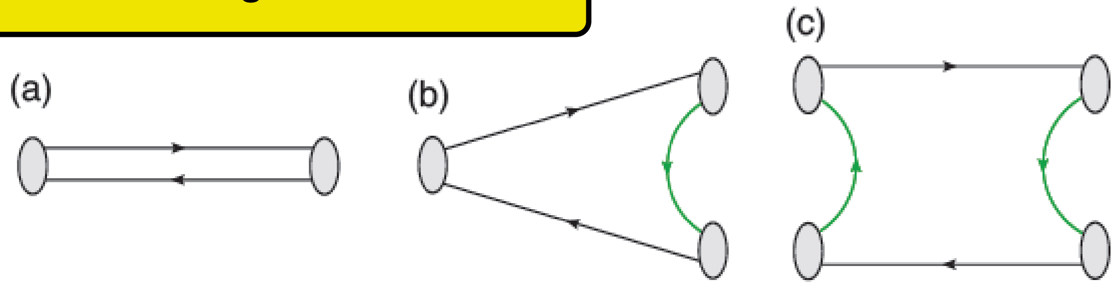
Hadronic decays

Current spectrum calculations:
no evidence of multi-particle levels

Plot the non-interacting meson levels as a guide

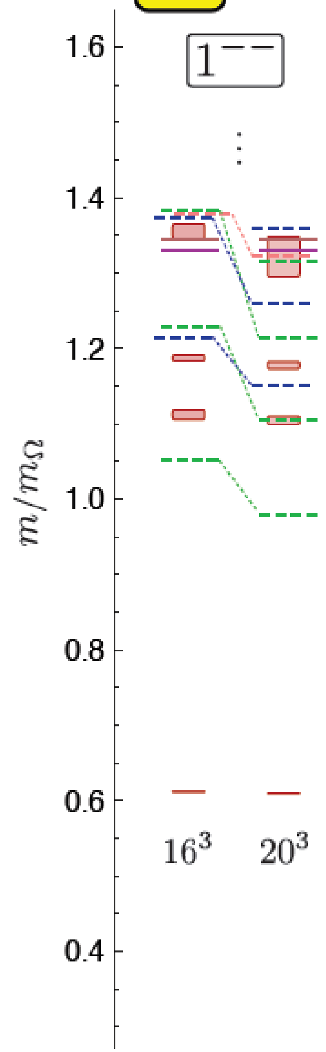
$$|A(\vec{p})B(-\vec{p})\rangle \qquad m_{AB} = \sqrt{m_A^2 + \vec{p}^2} + \sqrt{m_B^2 + \vec{p}^2}$$

Require multi-particle operators
- (lattice) helicity construction
- annihilation diagrams



Extract δ(E) at discrete E

e.g.



Spectrum of finite volume field

The idea: 1 dim quantum mechanics

Two spin-less bosons: ψ(x,y) = f(x-y) $\rightarrow$ f(z)

$$\left[-\frac{1}{m}\frac{d^2}{dz^2} + V(z)\right] f(z) = E\, f(z)$$

Solutions

$$f(z) \rightarrow \cos\left[k|z| + \delta(k)\right], \qquad E = k^2/m$$

Quantization condition when -L/2 < z < L/2

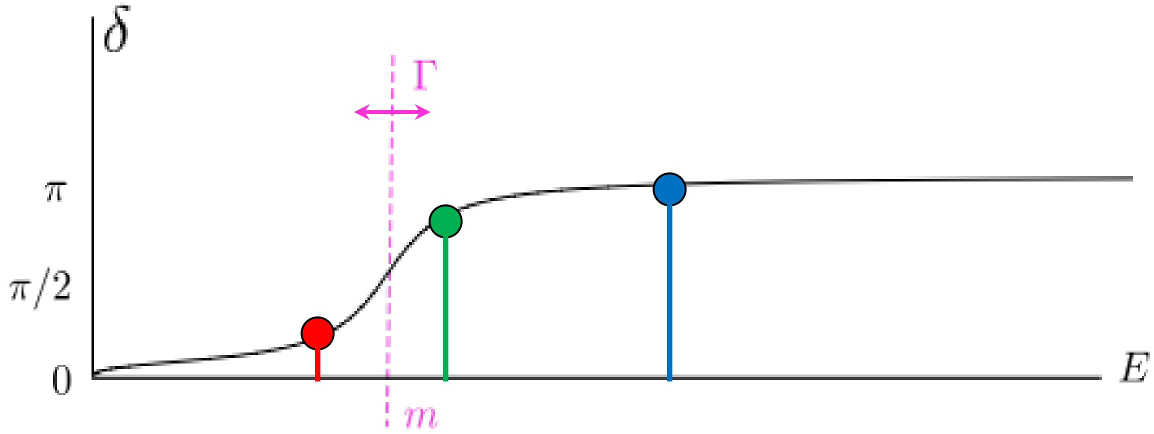
$$kL + 2\delta(k) = 0 \quad \text{mod } 2\pi$$

Same physics in 4 dim version (but messier)
Provable in a QFT (and relativistic)

Finite volume scattering

Scattering in a periodic cubic box (length **L**)
- Discrete energy levels in finite volume

E.g. just a single elastic resonance



e.g.

$$\pi\pi \to \rho \to \pi\pi$$

$$\pi N \to \Delta \to \pi N$$

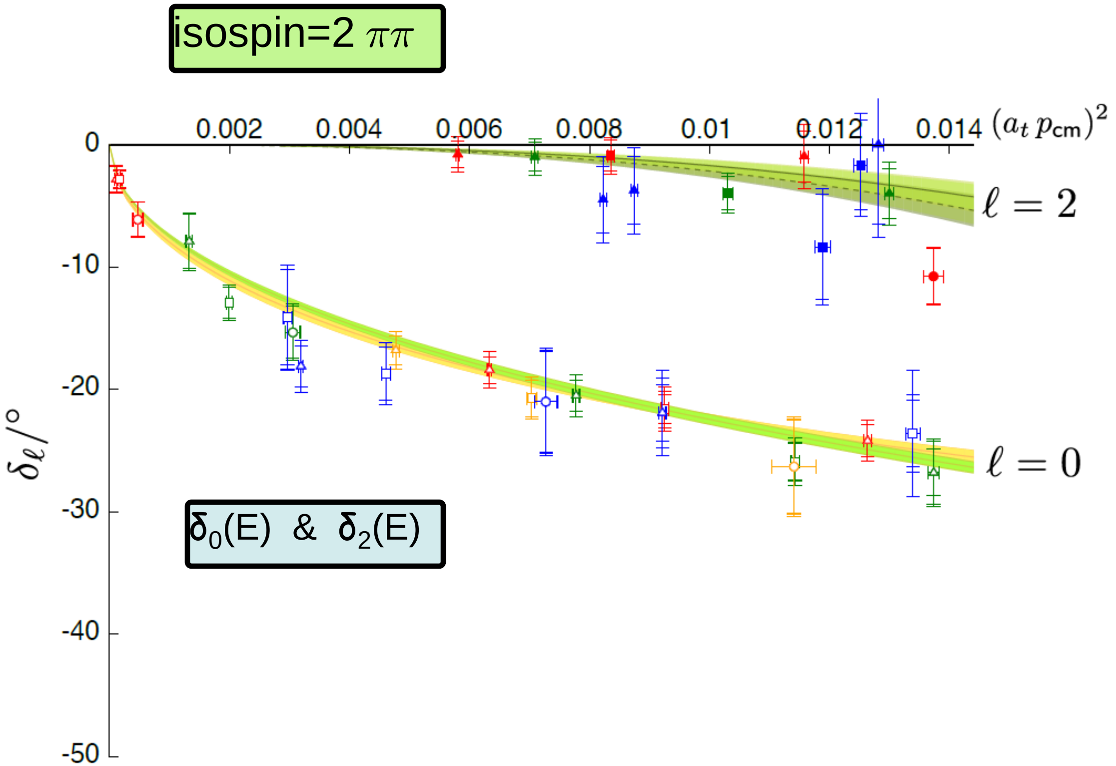
At some **L** **,** have discrete excited energies

$$E \to k; \quad kL + 2\delta(k) = 0 \mod 2\pi$$

- T-matrix amplitudes $\to$ partial waves
- Finite volume energy levels **E(L)** $\leftrightarrow$ **δ(E)**

Resonances

Scattering of composite objects in non-perturbative field theory

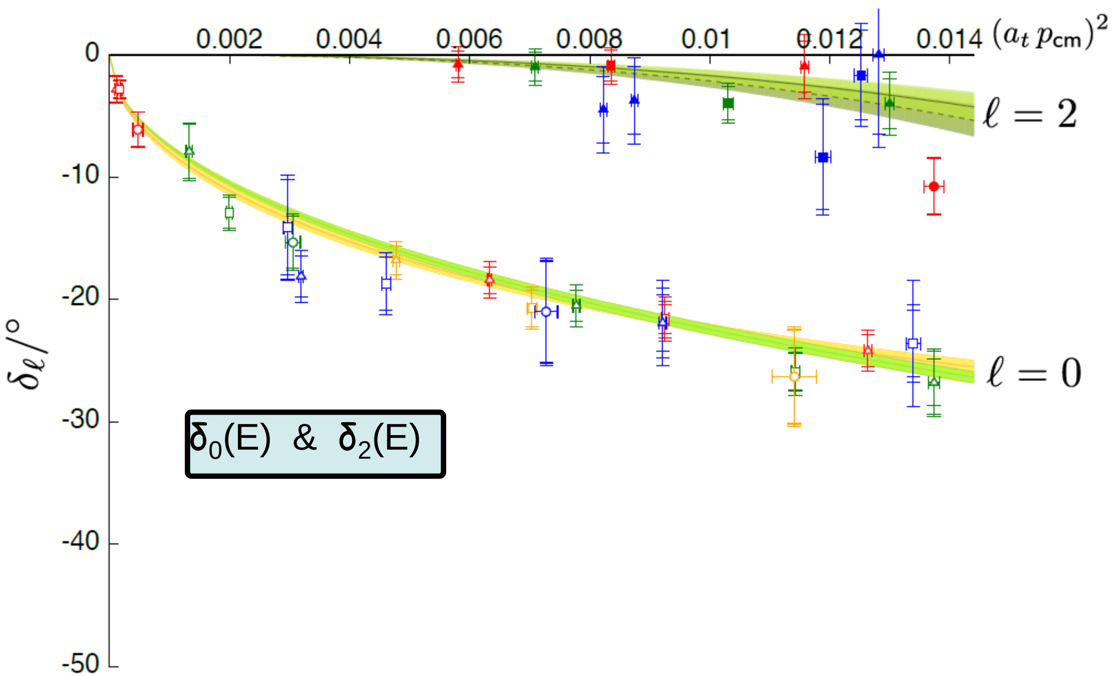


1011.6352, 1203.????

Resonances

Scattering of composite objects in non-perturbative field theory

isospin=2 $\pi\pi$

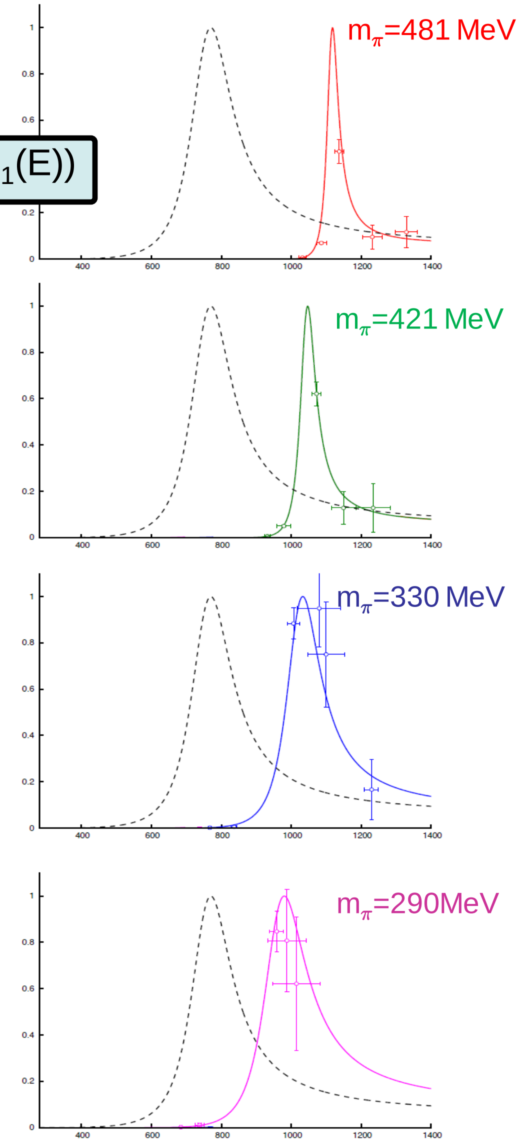


$\delta_0(E)$ & $\delta_2(E)$

1011.6352, 1203.????

isospin=1 $\pi\pi$

$\sin^2(\delta_1(E))$



Feng, et.al, 1011.5288

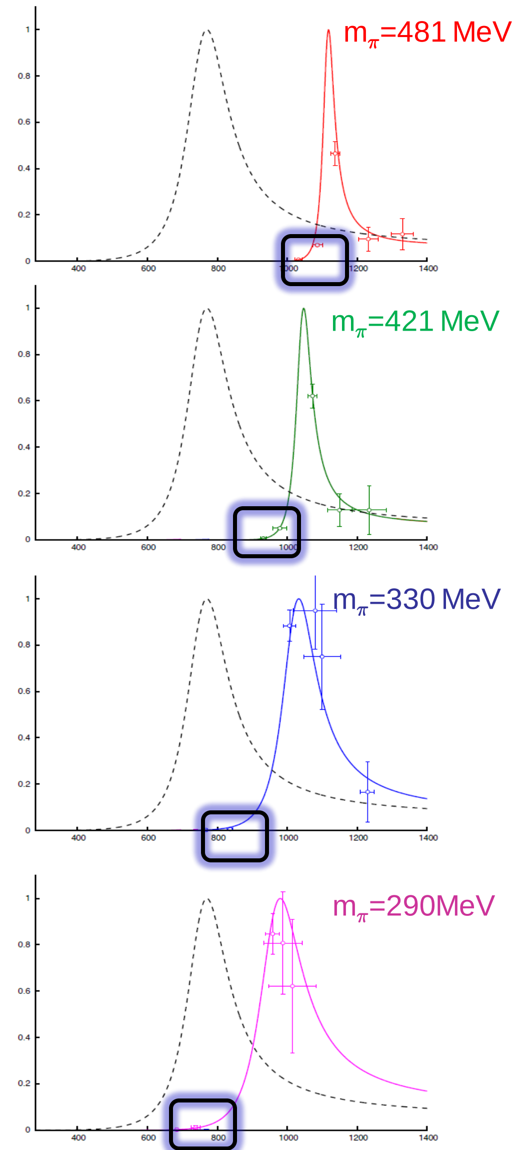
Resonances

isospin=1 $\pi\pi$

Scattering of composite objects in non-perturbative field theory

Manifestation of "decay" in Euclidean space

Can extract pole position



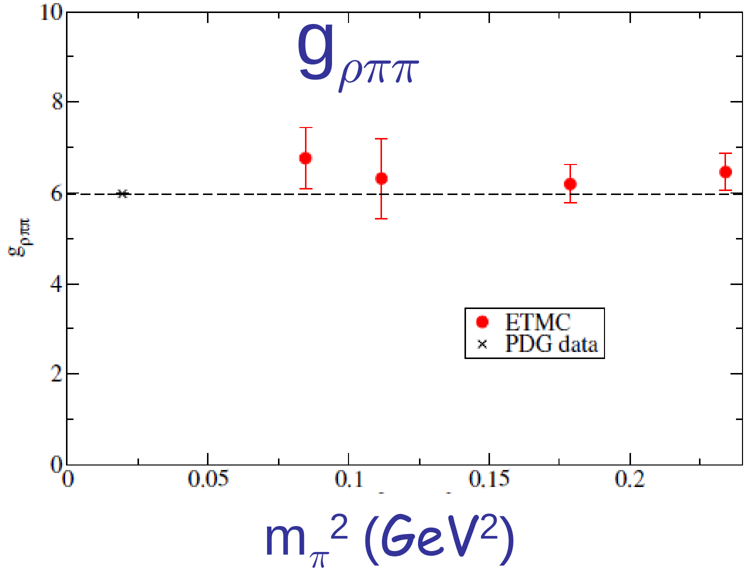
Feng, et.al, 1011.5288

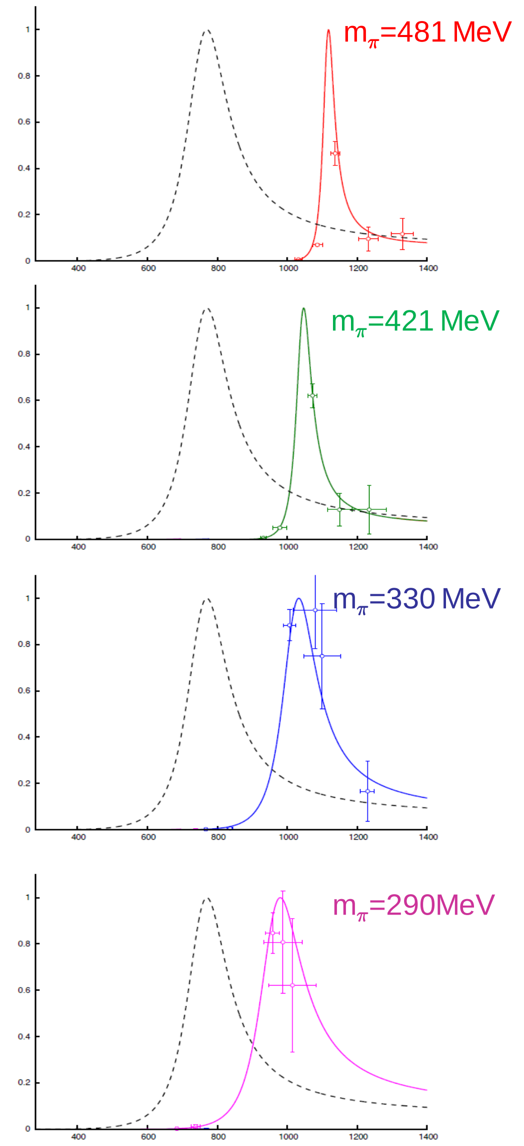
Resonances

isospin=1 $\pi\pi$

Scattering of composite objects in non-perturbative field theory

Extracted coupling: stable in pion mass



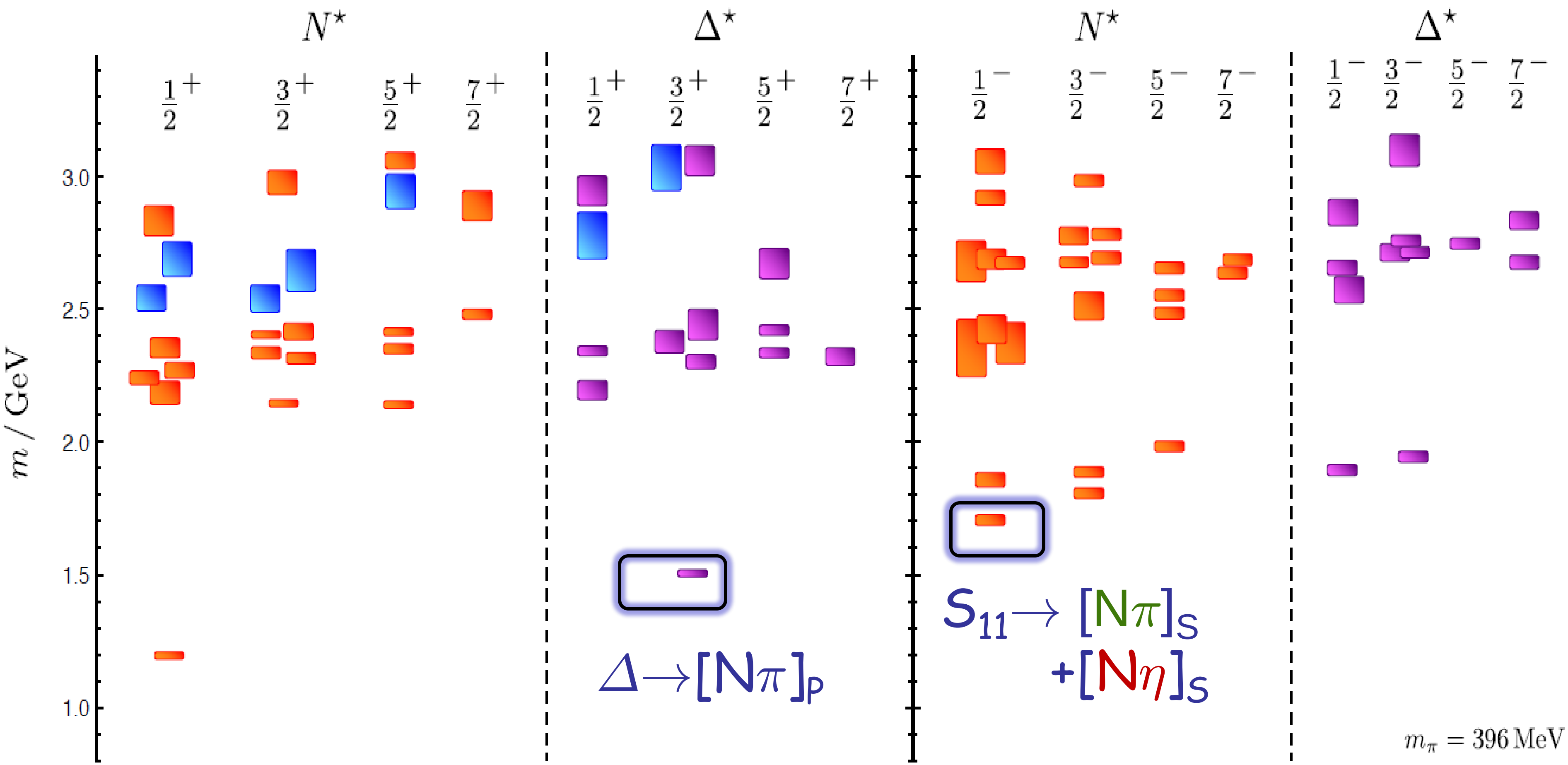


Stability a generic feature of couplings??

Feng, et.al, 1011.5288

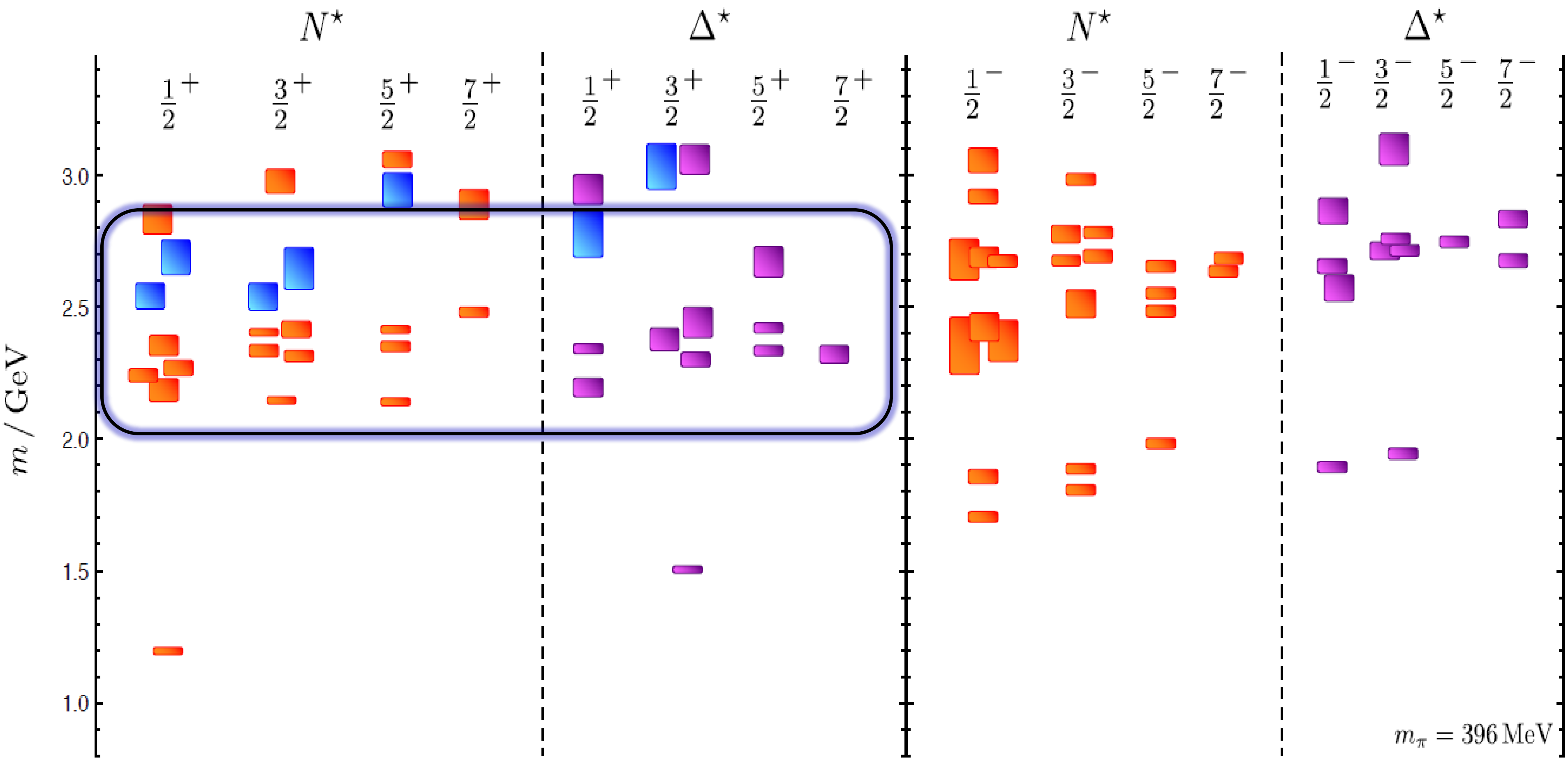
Hadronic decays

Some candidates: determine phase shift
Somewhat elastic



Hadronic decays – the full job

Need a lattice program of "amplitude analysis" - lots of room for help!



Summary & prospects

Summary & prospects

Results for baryon excited state spectrum:

- No “freezing” of degrees of freedom nor parity doubling
- Broadly consistent with non-relativistic quark model
- Extra bits interpreted as hybrid baryons
- Add multi-particle ops $\rightarrow$ baryon spectrum becomes denser

Summary & prospects

Results for baryon excited state spectrum:

- No “freezing” of degrees of freedom nor parity doubling
- Broadly consistent with non-relativistic quark model
- Extra bits interpreted as hybrid baryons
- Add multi-particle ops $\rightarrow$ baryon spectrum becomes denser

Short-term plans: resonance determination!

- Lighter pion masses (230MeV available)
- Extract couplings in multi-channel systems (with π, η, K…)

Summary & prospects

Results for baryon excited state spectrum:

- No "freezing" of degrees of freedom nor parity doubling
- Broadly consistent with non-relativistic quark model
- Extra bits interpreted as hybrid baryons
- Add multi-particle ops $\rightarrow$ baryon spectrum becomes denser

Short-term plans: resonance determination!

- Lighter pion masses (230MeV available)
- Extract couplings in multi-channel systems (with π, η, K…)

T-matrix "poles" from Euclidean space?

Summary & prospects

Results for baryon excited state spectrum:

- No "freezing" of degrees of freedom nor parity doubling
- Broadly consistent with non-relativistic quark model
- Extra bits interpreted as hybrid baryons
- Add multi-particle ops $\rightarrow$ baryon spectrum becomes denser

Short-term plans: resonance determination!

- Lighter pion masses (230MeV available)
- Extract couplings in multi-channel systems (with π, η, K…)

T-matrix "poles" from Euclidean space?

Yes! [with caveats]

- Also complicated
- But all Minkowski information is there

Summary & prospects

Results for baryon excited state spectrum:

- No "freezing" of degrees of freedom nor parity doubling
- Broadly consistent with non-relativistic quark model
- Extra bits interpreted as hybrid baryons
- Add multi-particle ops $\rightarrow$ baryon spectrum becomes denser

Short-term plans: resonance determination!

- Lighter pion masses (230MeV available)
- Extract couplings in multi-channel systems (with π, η, K…)

T-matrix "poles" from Euclidean space?

Yes! [with caveats]

- Also complicated
- But all Minkowski information is there

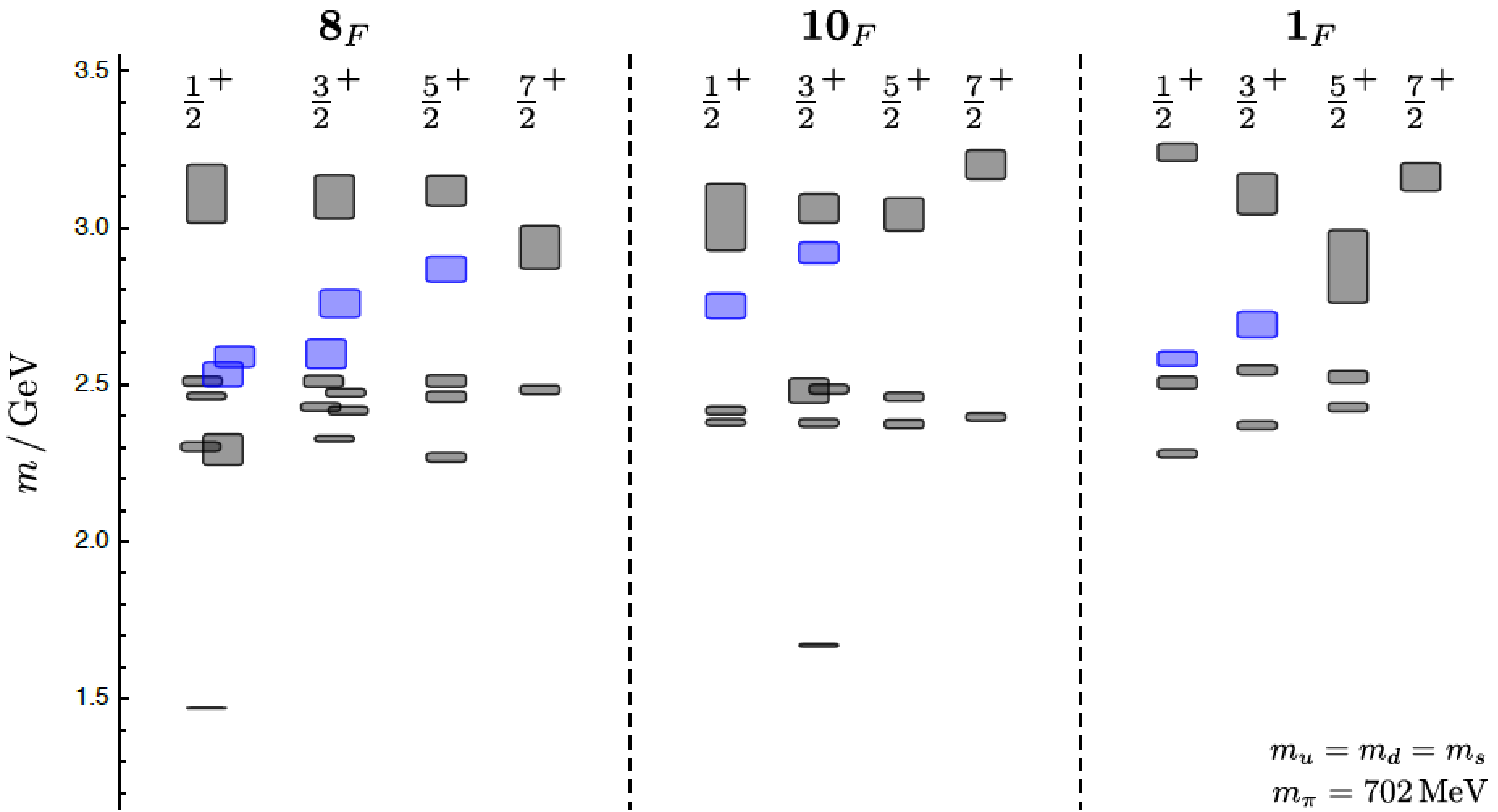
Optimistic: see confluence of methods (an "amplitude analysis")

- Develop techniques concurrently with decreasing pion mass

Backup slides

- The end

SU(3) limit


$\mathbf{8}_F$
$\mathbf{10}_F$
$\mathbf{1}_F$
$\frac{1}{2}^+$ $\frac{3}{2}^+$ $\frac{5}{2}^+$ $\frac{7}{2}^+$
3.5
3.0
2.5
2.0
1.5
m / GeV
$m_u = m_d = m_s$
$m_\pi = 702\,\mathrm{MeV}$