Unitary Coupled-Channel Model for Heavy Meson Decays into Three Mesons

Satoshi Nakamura

Excited Baryon Analysis Center (EBAC), JLab

Collaborators

Hiroyuki Kamano	(RCNP, Osaka U.)
Harry Lee	(Argonne National Lab)
Toru Sato	(Osaka Univ.)

Hadron Spectroscopy

$\Rightarrow$ Key information for confinement physics

Hadron Spectroscopy

data analysis $\Rightarrow$ hadron properties

(J^{PC}, mass, width, branching ratios, ..)

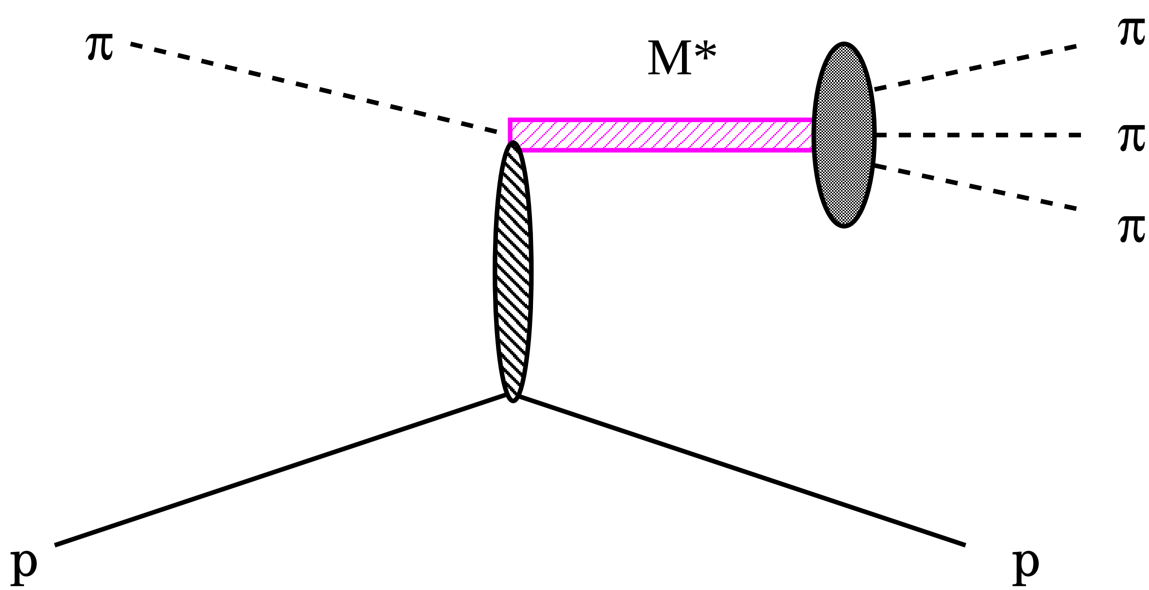
Reliable analysis tool is essential !

Light Flavor Meson Spectroscopy (relevant to JLab 12 GeV programs)

- data
- analysis tool

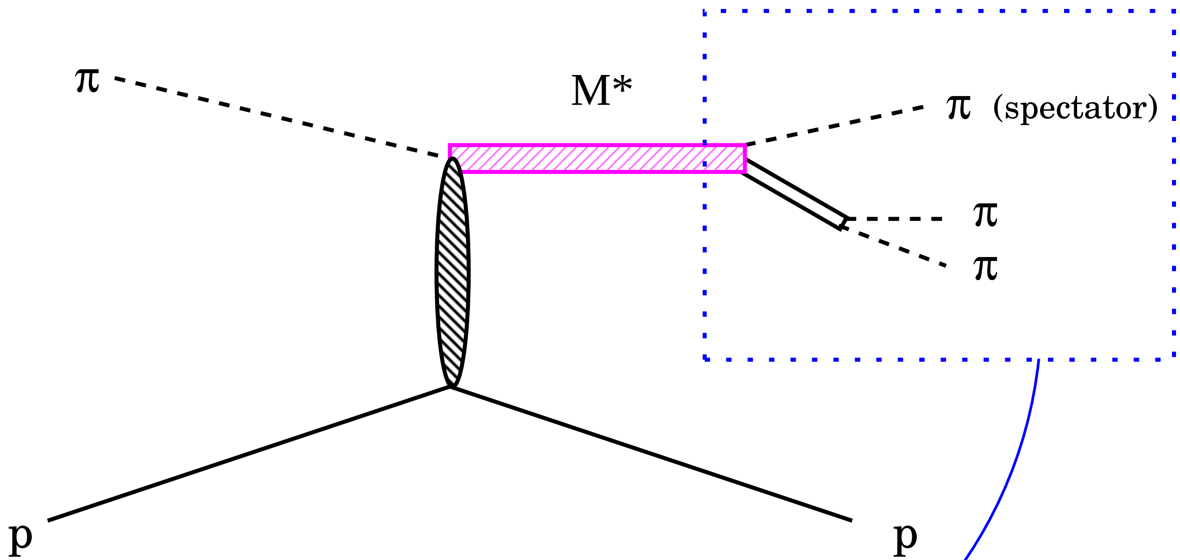
Light Flavor Meson Spectroscopy (relevant to JLab 12 GeV programs)

e.g., E852 (BNL) $\pi^- p \to \pi^+\pi^-\pi^- p$ Chung et al., PRD **65**, 072001 (2001)



Light Flavor Meson Spectroscopy (relevant to JLab 12 GeV programs)

e.g., E852 (BNL) $\pi^- p \to \pi^+ \pi^- \pi^- p$ Chung et al., PRD **65**, 072001 (2001)



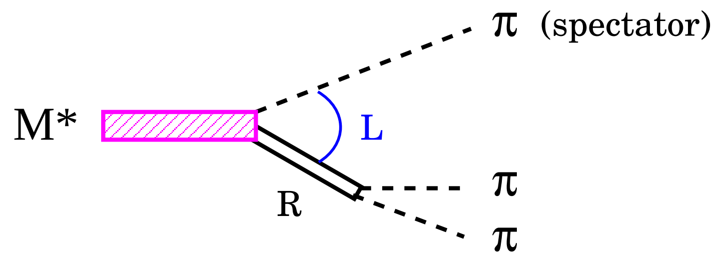
isobar model

$*$ $\pi\pi$ subsystem forms a resonance

$*$ 3rd π is a spectator

Isobar model

E852 (BNL) , Chung et al., PRD **65**, 072001 (2001)



$* \; L = 0, \; 1, \; 2$

$*$ For $R = f_0(980), \; \rho(770), \; f_2(1270), \; \rho_3(1690)$

$\Longrightarrow$ Breit-Wigner form $\quad A_R = \dfrac{F_{R\to\pi\pi}}{m_R^2 - m_{\pi\pi}^2 - i\, m_R\, \Gamma_R(m_{\pi\pi})}$

$*$ For $R = \sigma$

$\Longrightarrow$ K-matrix model [e.g., Au, Morgan, Pennington, PRD **35**, 1633 (1987)]

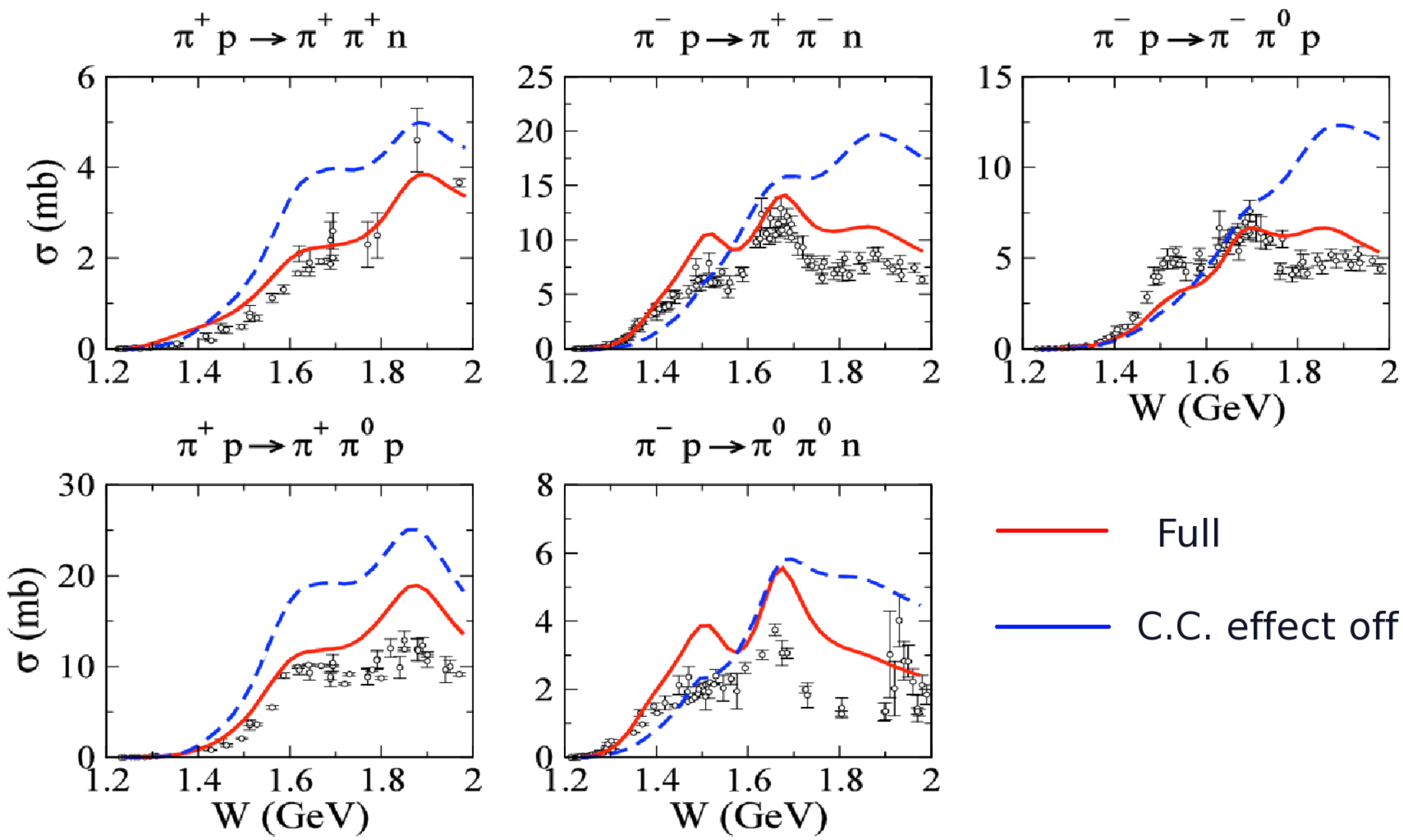
$* \; A_{M^*\to\pi\pi\pi} \; = \; \sum_R \, a_R \, e^{i\phi_R} \, A_R +$ (background)

Questions

- $\ast$ coupled-channels ?
- $\ast$ 3-body unitarity ?

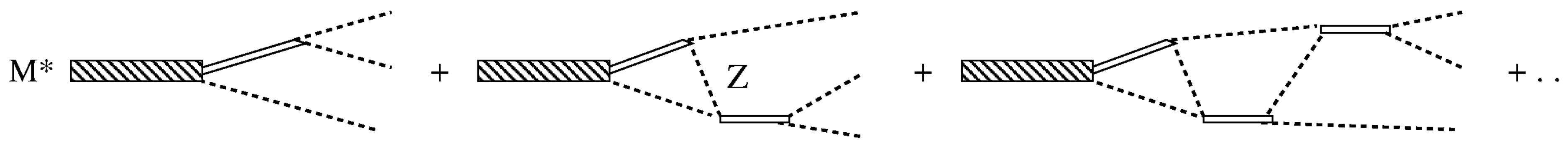
Coupled-channel effect

e.g., $\pi N \to \pi\pi N$ $[\pi N, \eta N, \pi\Delta, \rho N, \sigma N$ coupled-channels]



Kamano et al., PRC79 025206 (2009)

3-body unitarity requires ... Z-diagrams



Question to be addressed

How 3-body unitarity makes a difference in extracting hadron properties from data ?

Method

1. Construct a unitary and an isobar models

2. Fit them to the same Dalitz plot

3. Extract and compare M^* properties from them

   (pole position, coupling strength to decay channels)

Coupled-Channels Model

Matsuyama, Sato, Lee, Phys. Rept. **439**, 193 (2007)

Kamano, Nakamura, Sato, Lee, PRD **84** 114019 (2011)

$\underline{M^* \to \pi R \to \pi\pi\pi}$

Channels R : $f_0(600)$, $f_0(980)$, $\rho(760)$, $f_2(1270)$, ..

R : resonance in $\pi\pi$ scattering amplitude (not Breit-Wigner form)

(I) Develop $\pi\pi$ model

(II) Develop πR interaction

(III) Solve πR scattering equation

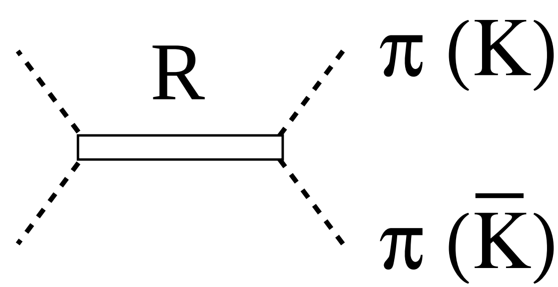
Simple $\pi\pi$ model

Coupled-channel scattering equation for $\pi\pi$ partial wave (L, I)

$$t^{LI}_{i,j}(p', p; W) = V^{LI}_{i,j} + \sum_k \int_0^\infty q^2 dq\, V^{LI}_{i,k}(p', q; W) \frac{1}{W - E_k(q) + i\epsilon} t^{LI}_{k,j}(q, p; W)$$

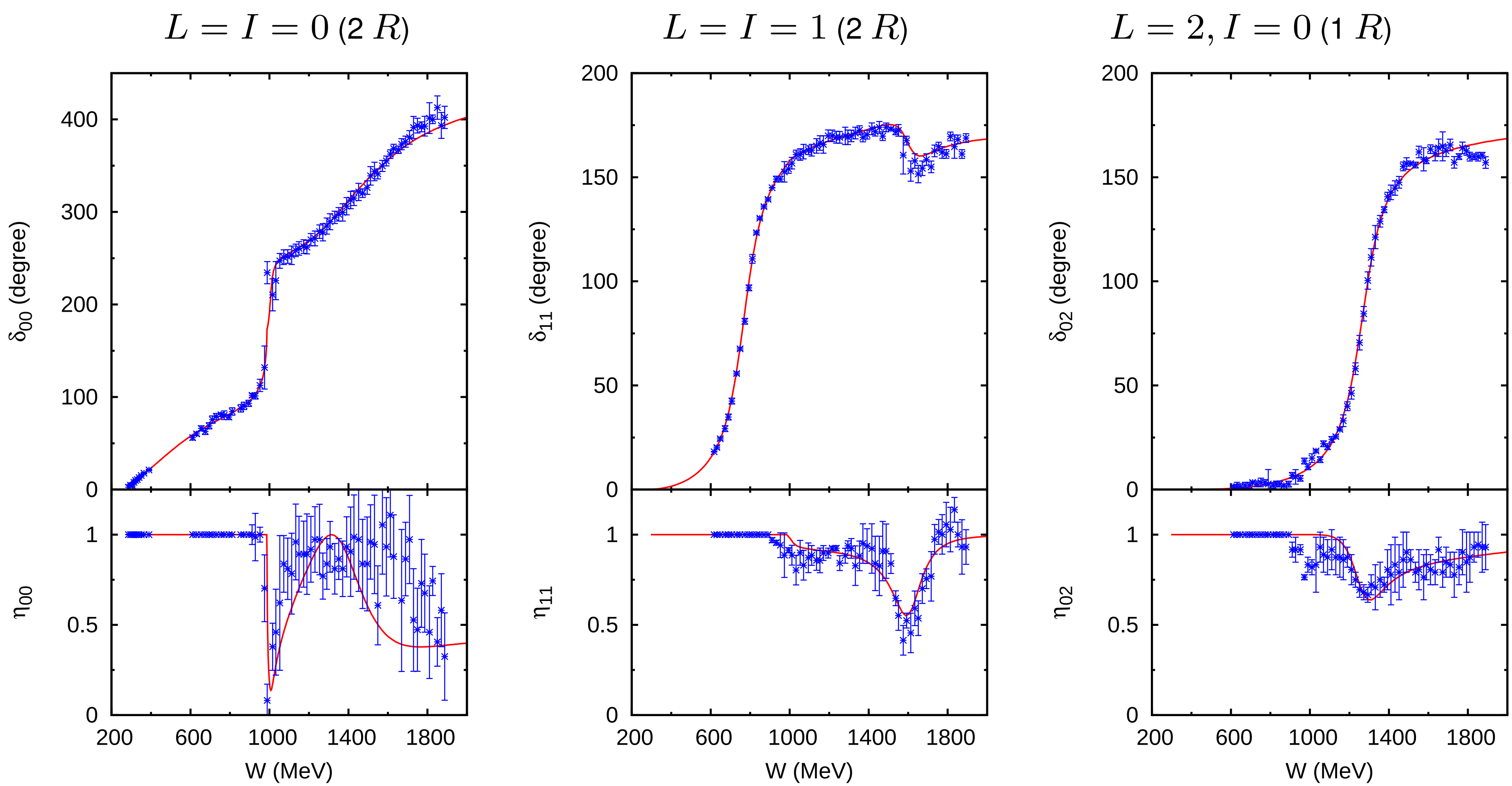
$$E_{\pi\pi}(q) = 2\sqrt{m_\pi^2 + q^2} \qquad\qquad (i, j, k = \pi\pi, K\bar{K})$$

$$V^{LI}_{i,j}(p', p; W) = \sum_R f^{LI}_{R,i}(p') \frac{1}{W - m_R} f^{LI}_{R,j}(p)$$



$$f^{LI}_{R,i}(p) = \frac{g_{R,i}}{\sqrt{m_\pi}} \frac{1}{(1 + (c_{R,i}p)^2)} \left(\frac{p}{m_\pi}\right)^L$$

Phase and inelasticity of $\pi\pi$ amplitude



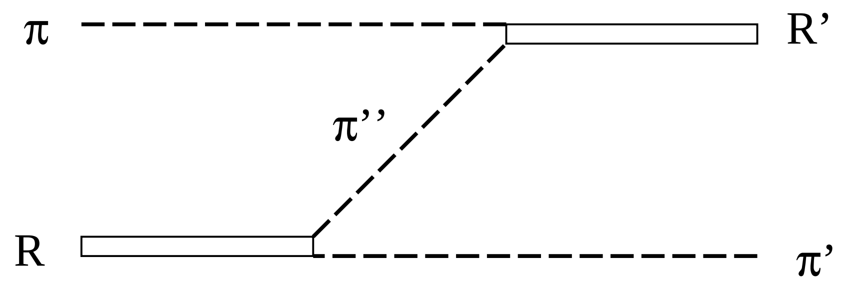
[Data: Gayer et al. (1974); Hyams et al. (1973); Batley et al. (2008)]

Pole positions in $\pi\pi$ amplitude

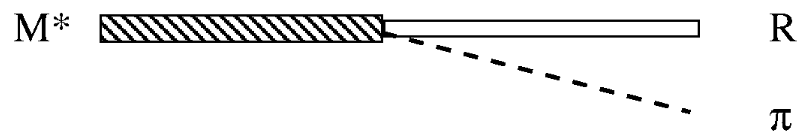
	Re[M_R] (MeV)		$-$Im[M_R] (MeV)	
	Ours	PDG	Ours	PDG
f_0 (600)	430	$400 - 1200$	270	$250 - 500$
f_0 (980)	1000	980 ± 10	9	$20 - 50$
f_0 (1370)	1350	$1200 - 1500$	170	$150 - 250$
ρ (760)	770	775.5 ± 0.3	81	74.5 ± 0.4
ρ (1700)	1610	$1550 - 1780$	120	$80 - 300$
f_2 (1270)	1250	1275 ± 1.2	100	92.5 ± 1.3

Quasi two-particle (πR) interaction

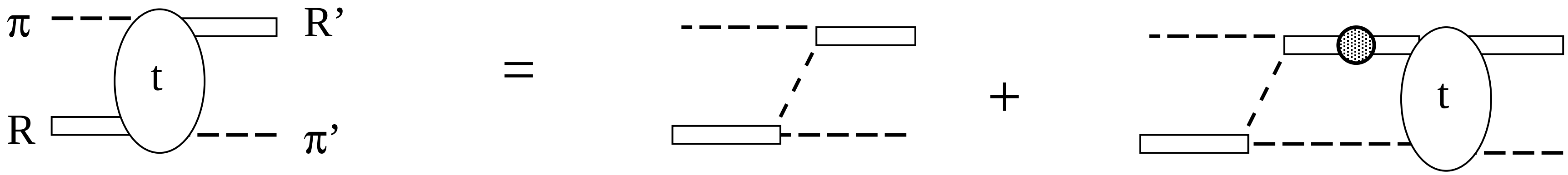
3 π Z-graph



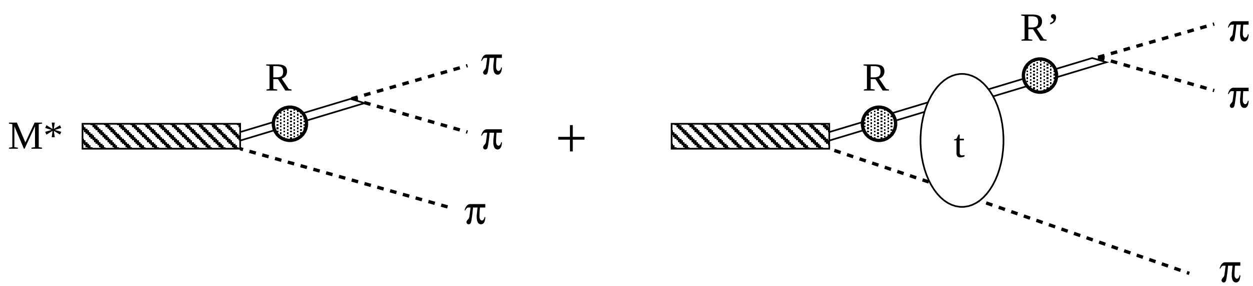
M^* graph



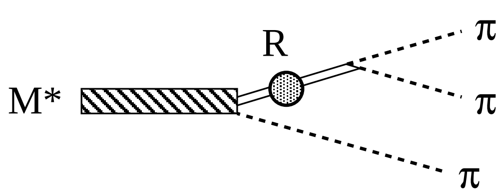
Quasi two-particle (πR) scattering equation



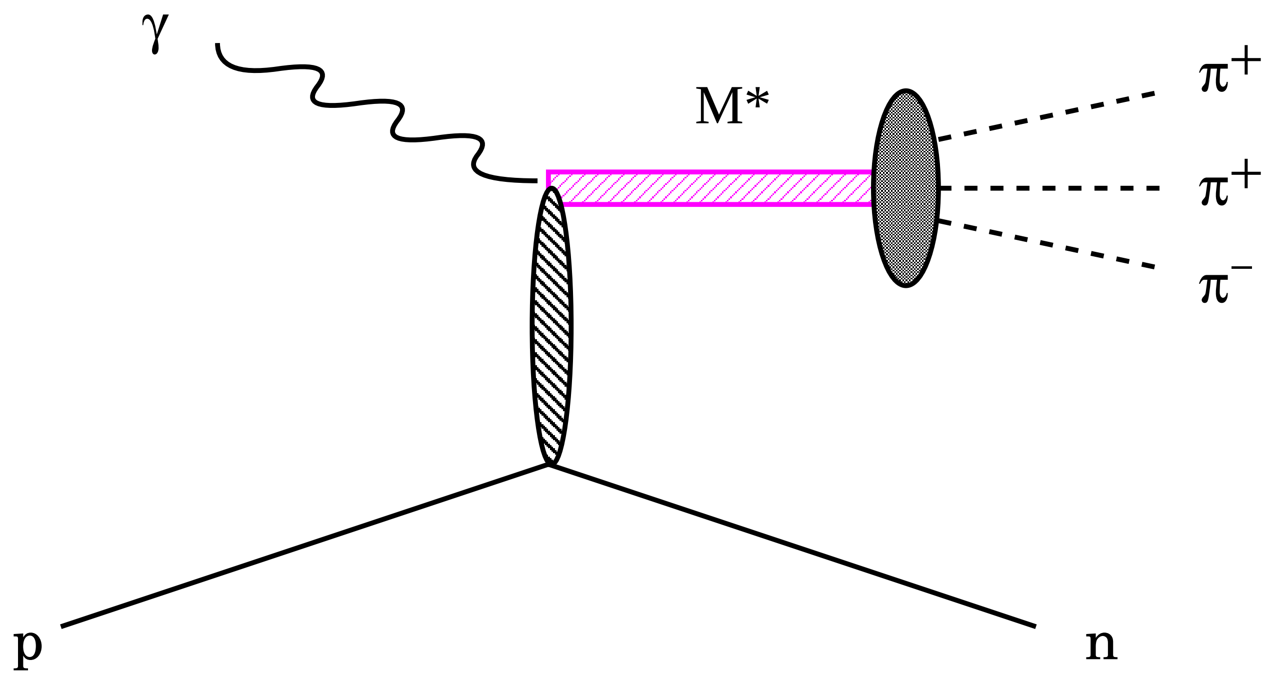
M^* decay amplitude (unitary model)



M^* decay amplitude (isobar model)



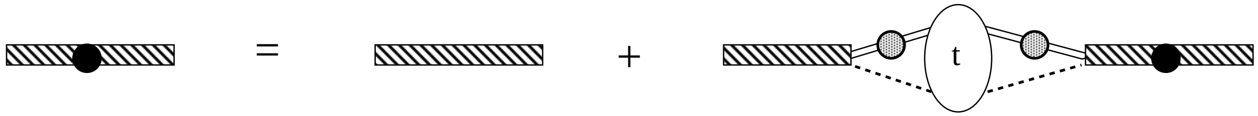
Case Study : $\gamma p \to M^* n \to \pi^+\pi^+\pi^- n$ (CLAS 6, GlueX)

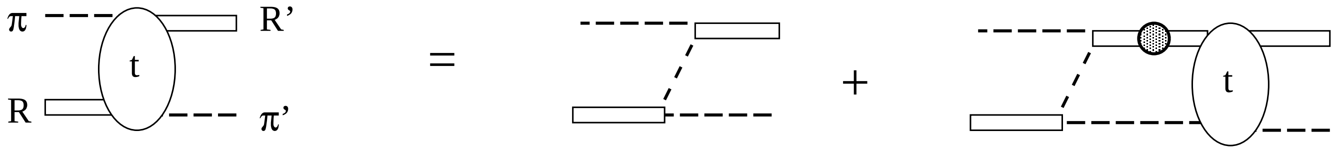


Nakamura, Kamano, Sato, Lee, in preparation

How 3-body unitary makes a difference in extracting M^* properties from data ?

M^* propagator (unitary model)

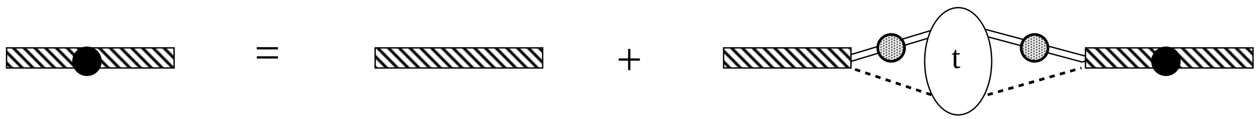


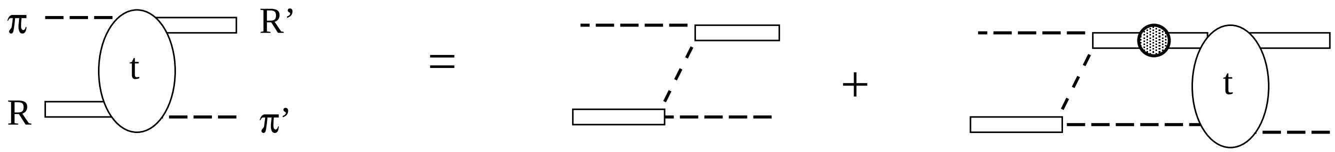


$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W)$$

3-body unitarity requires consistency with M^* decay amplitude

M^* propagator (unitary model)





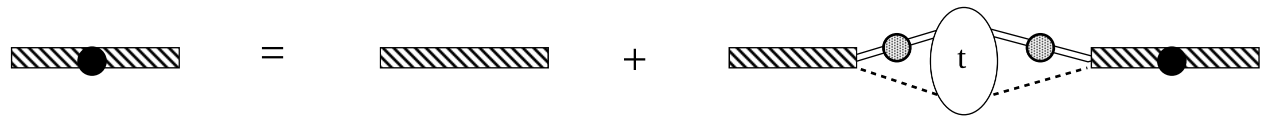
$$G^{-1}(W) \;=\; W \;-\; M^0_{M^*} \;-\; \Sigma_{M^*}(W)$$

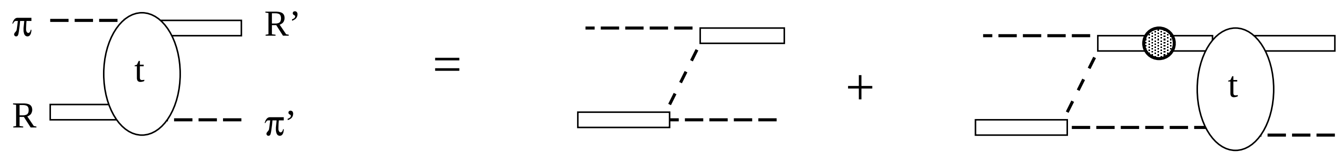
Pole position : $M_R \;\Rightarrow\; G^{-1}(M_R) \;=\; 0$

Solved with analytic continuation to the unphysical Riemann sheet

Suzuki, Sato, Lee, PRC **79**, 025205 (2009); ibid, **82**, 045206 (2010)

M^* propagator (unitary model)





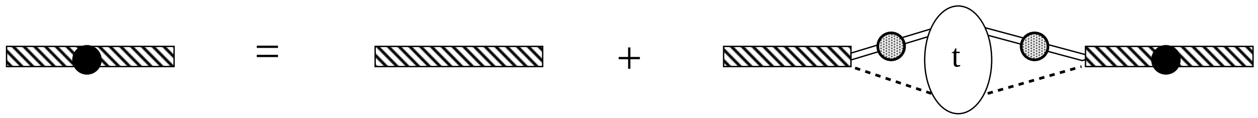
$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W)$$

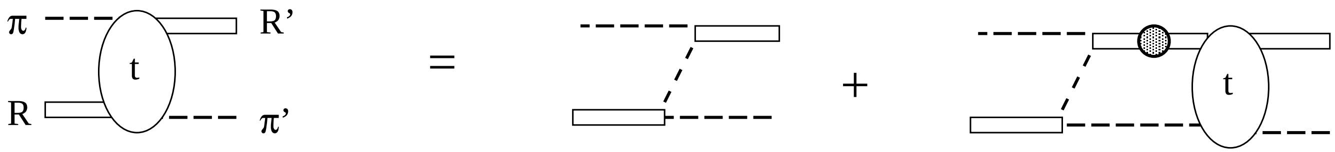
M^* propagator (isobar model)

Breit-Wigner : $$G^{-1}(W) = W - M_{M^*} - i\,\frac{\Gamma(W)}{2}$$

but any phenomenological parametrization should be fine ...

M^* propagator (unitary model)





$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W)$$

M^* propagator (isobar model)

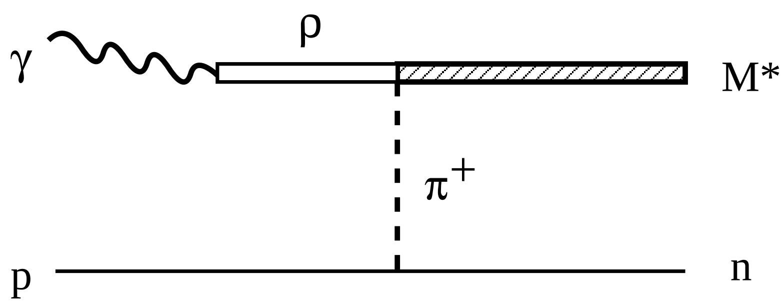
This work : $$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W) - \Delta M^0_{M^*}$$

$\uparrow$

complex constant

Pole position : $M_R(\text{isobar}) \sim M_R(\text{unitary}) + \Delta M^0_{M^*}$

Production amplitude



Simple assumptions :

- $*$ t-channel π-exchange
- $*$ vector-dominance of $\gamma\pi M^*$ coupling

Not realistic but good enough

interested in effect of 3-body unitarity implemented in M^* propagation and decay

Kinematics

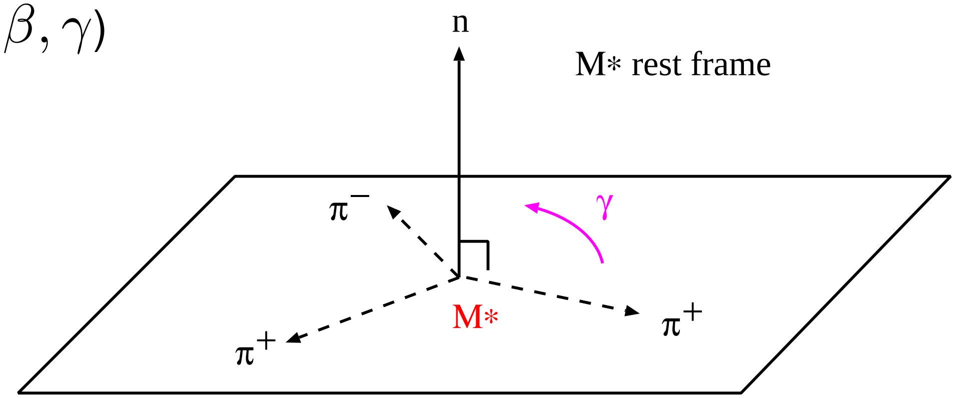
[cf. CLAS 6, PRL **102**, 102002 (2009)]

$*$ E_γ = 5 GeV

$*$ t = $-$ 0.4 GeV^2

$*$ 0.8 GeV $\leq W \leq$ 2 GeV ; W : 3 π invariant mass

$*$ 3 π orientation
(Euler angles : α, β, γ)



$\alpha,\ \beta$ fixed ; $0 \leq \gamma \leq 2\pi$ Dalitz plot depends on γ

$\Leftarrow$ Photon excites a polarized M^* that decays to a certain γ more often

Procedure

1. Determine parameters of unitary model with reasonable input
2. Generate *mock data* with the unitary model
3. Fit the data with isobar model
4. Compare M^* properties from the two models

Partial wave, M^*'s in unitary model

[cf. CLAS 6, PRL **102**, 102002 (2009)]

J^{PC}	M^*
1^{++}	$a_1(1230)$, $a_1(1700)$
2^{++}	$a_2(1320)$, $a_2(1700)$
2^{-+}	$\pi_2(1670)$, $\pi_2(1800)$
1^{-+}	$\pi_1(1600)$

Determination of M^* parameters for unitary model

- $*$ M^* bare mass
- $*$ $M^* \to \pi R$ bare coupling and cutoff

M^* propagator

$$G^{-1}(W) \;=\; W \;-\; M^0_{M^*} \;-\; \Sigma_{M^*}(W)$$

$M^* \to \pi R$ vertex function

$$F(p) \;\propto\; \frac{C}{\sqrt{E_R E_\pi}} \left(\frac{\Lambda^2}{p^2+\Lambda^2}\right)^{2+(L/2)} p^L$$

Determination of M^* parameters for unitary model

- $*$ M^* bare mass
- $*$ $M^* \to \pi R$ bare coupling and cutoff

$^3P_0$ model — Barnes et al., PRD **55**, 4157 (1997)

Flux-tube model for $\pi_1(1600)$ — Isgur et al., PRL **54**, 869 (1985)

- Partial width $\Rightarrow$ coupling
- Cutoff is set to 1 GeV

J^{PC}	decay modes		$\Gamma_{q\bar{q}}$	Γ_{hybrid}
1^{++}	$a_1(1230) \rightarrow$	$\pi\rho(770)$	540.	-
	$a_1(1700) \rightarrow$	$\pi f_0(1300)$	2.	6.
		$\pi\rho(770)$	57.	30.
		$\pi\rho(1465)$	41.	0.
		$\pi f_2(1275)$	39.	70.
2^{++}	$a_2(1318) \rightarrow$	$\pi\rho(770)$	55 .	-
	$a_2(1700) \rightarrow$	$\pi\rho(770)$	104.	-
		$\pi f_2(1275)$	20.	-
2^{-+}	$\pi_2(1670) \rightarrow$	$\pi\rho(770)$	118.	-
		$\pi f_2(1275)$	75.	-
	$\pi_2(1800) \rightarrow$	$\pi f_0(1300)$	1.	1.
		$\pi\rho(770)$	162.	8.
		$\pi f_2(1275)$	86.	50.
1^{-+}	$\pi_1(1600) \rightarrow$	$\pi\rho(770)$	-	8.

J^{PC}	decay modes		$\Gamma_{q\bar{q}}$	$\Gamma_{\rm hybrid}$
1^{++}	$a_1(1700) \to$	$\pi f_0(1300)$	2.	6.
		$\pi\rho(770)$	57.	30.
		$\pi\rho(1465)$	41.	0.
		$\pi f_2(1275)$	39.	70.
2^{-+}	$\pi_2(1800) \to$	$\pi f_0(1300)$	1.	1.
		$\pi\rho(770)$	162.	8.
		$\pi f_2(1275)$	86.	50.

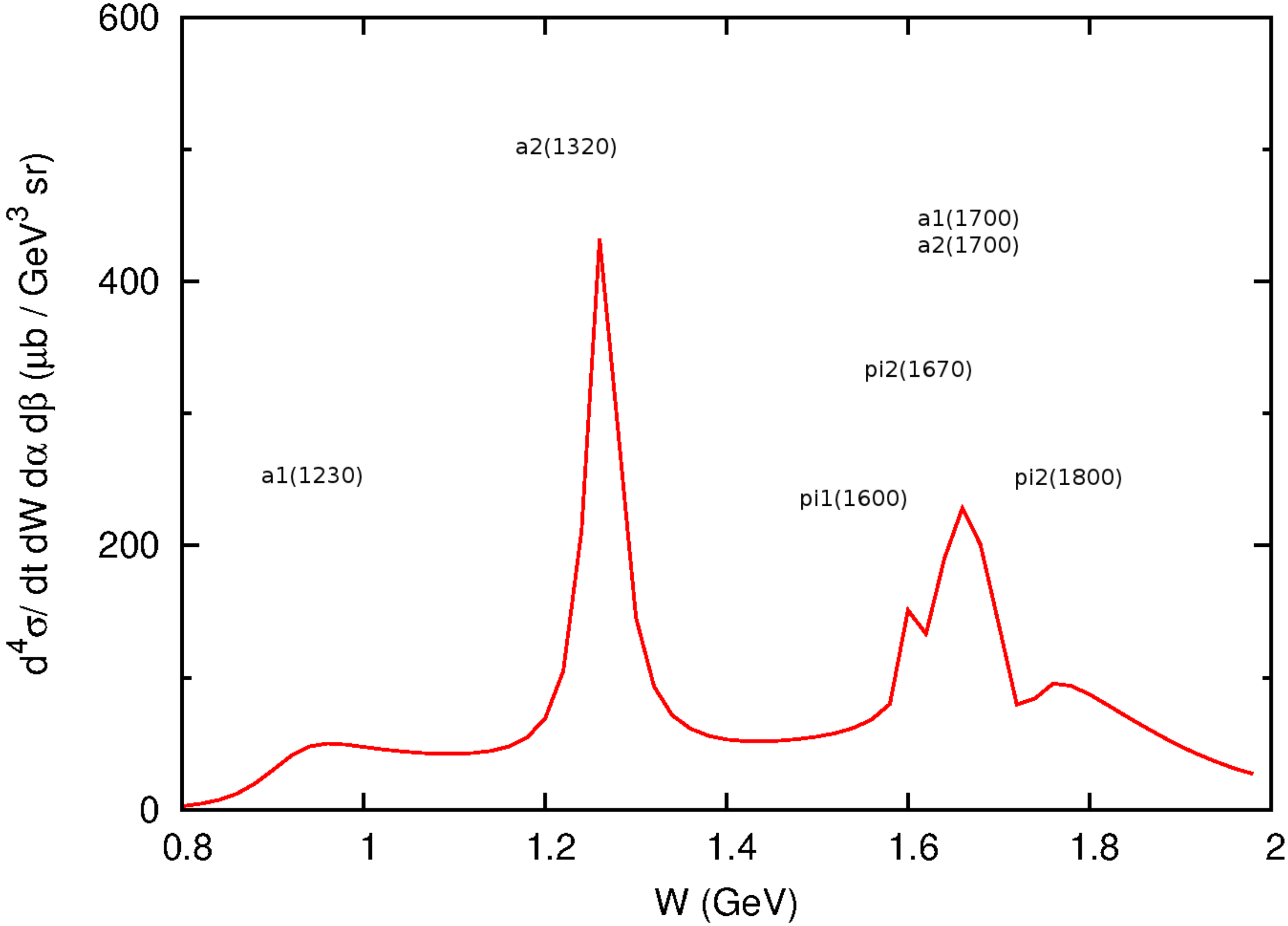
Difference between $\Gamma_{q\bar{q}}$ and $\Gamma_{\rm hybrid}$

$\Rightarrow$ Coupling strength to decay channel is a key information

to understand the nature of hadron structure

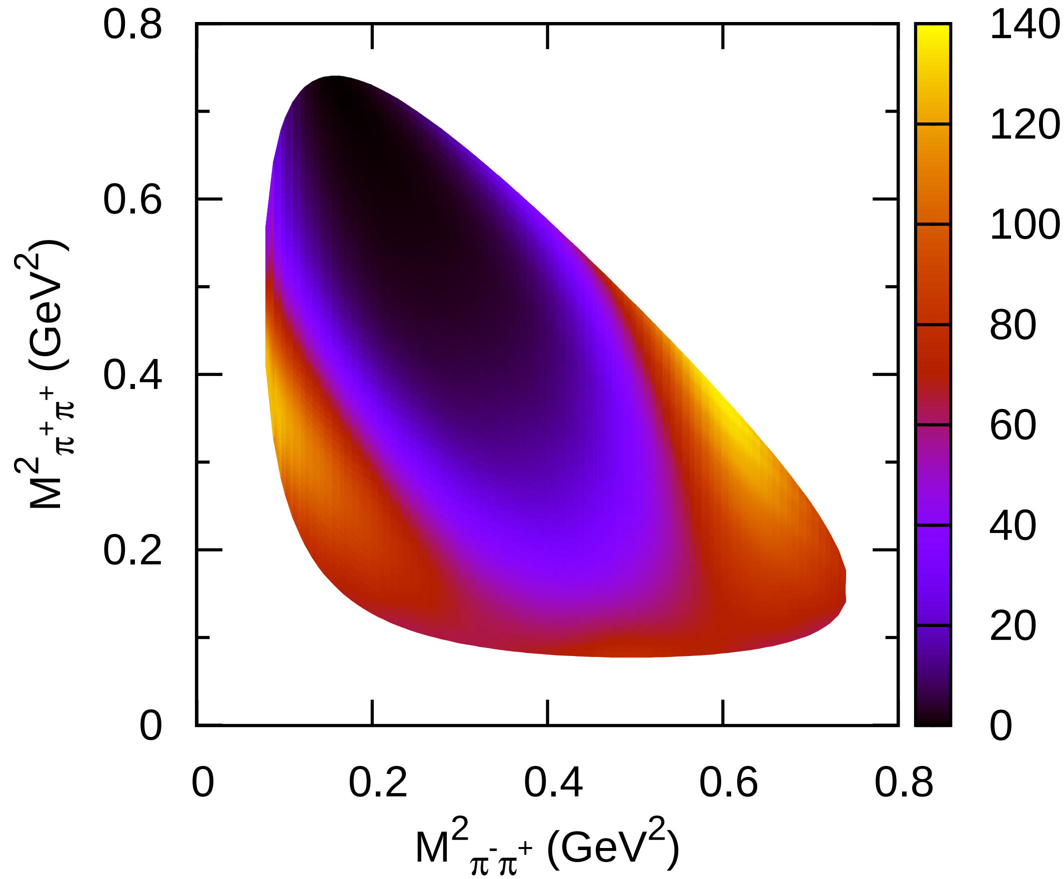
We use $\Gamma_{q\bar{q}}$ except for $\pi_1(1600)$

W-dependence of integrated Dalitz plots from unitary model



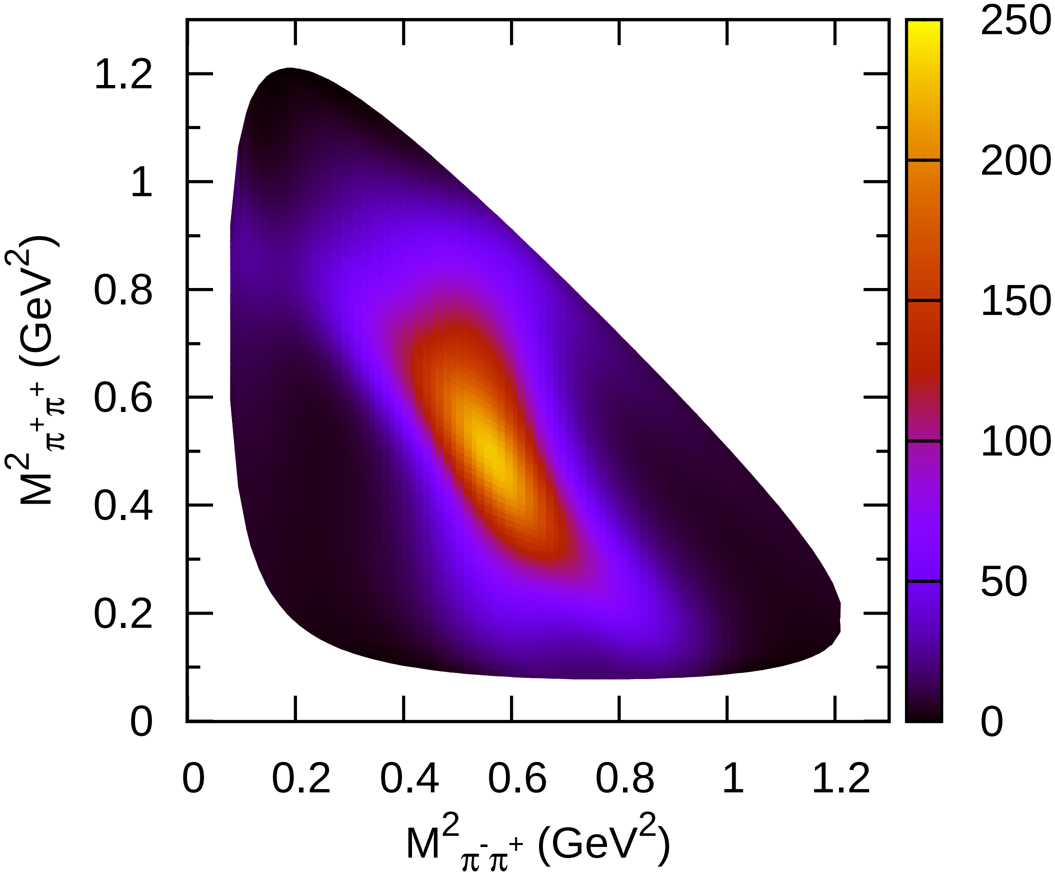
Dalitz plot from unitary model

$W = 1$ GeV near $a_1(1230)$ peak



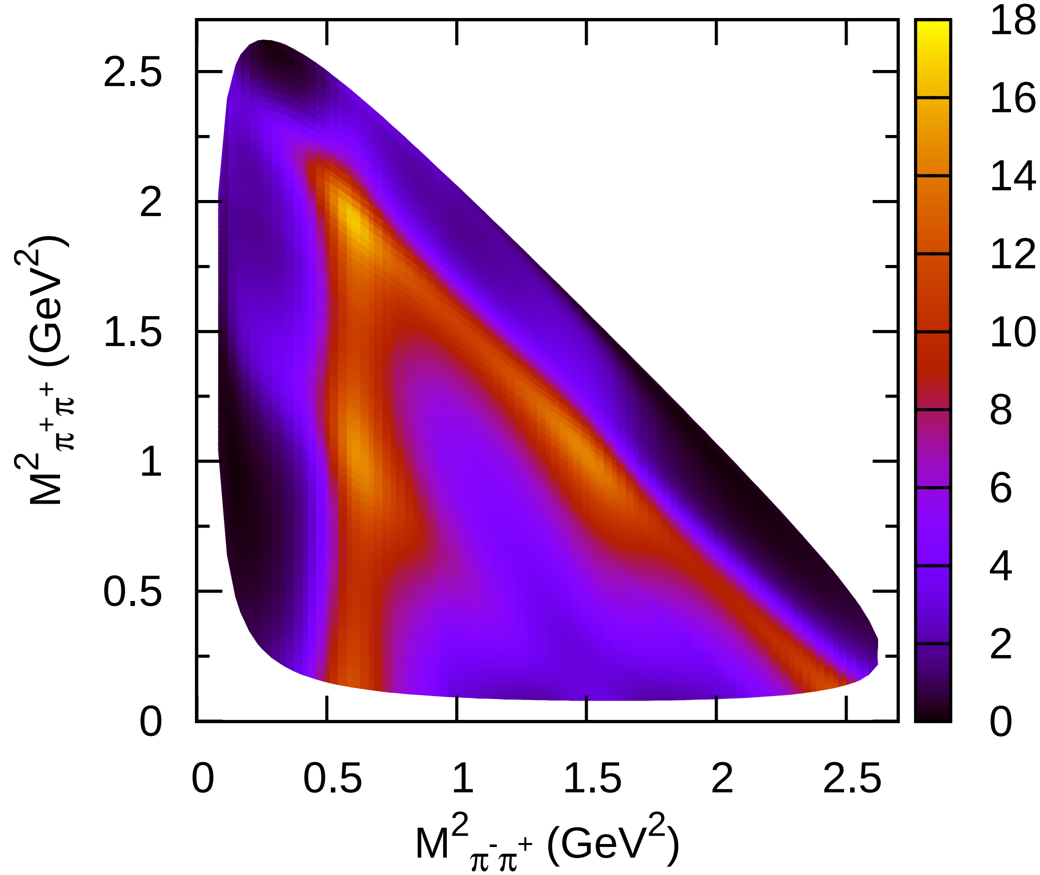
Dalitz plot from unitary model

$W = 1.24$ GeV near $a_2(1320)$ peak



Dalitz plot from unitary model

$W = 1.76$ GeV near $\pi_2(1800)$ peak



Fit with isobar model

Error :

- Data for the same W have the same error
- At each W, the error is assigned by 5% of the highest peak

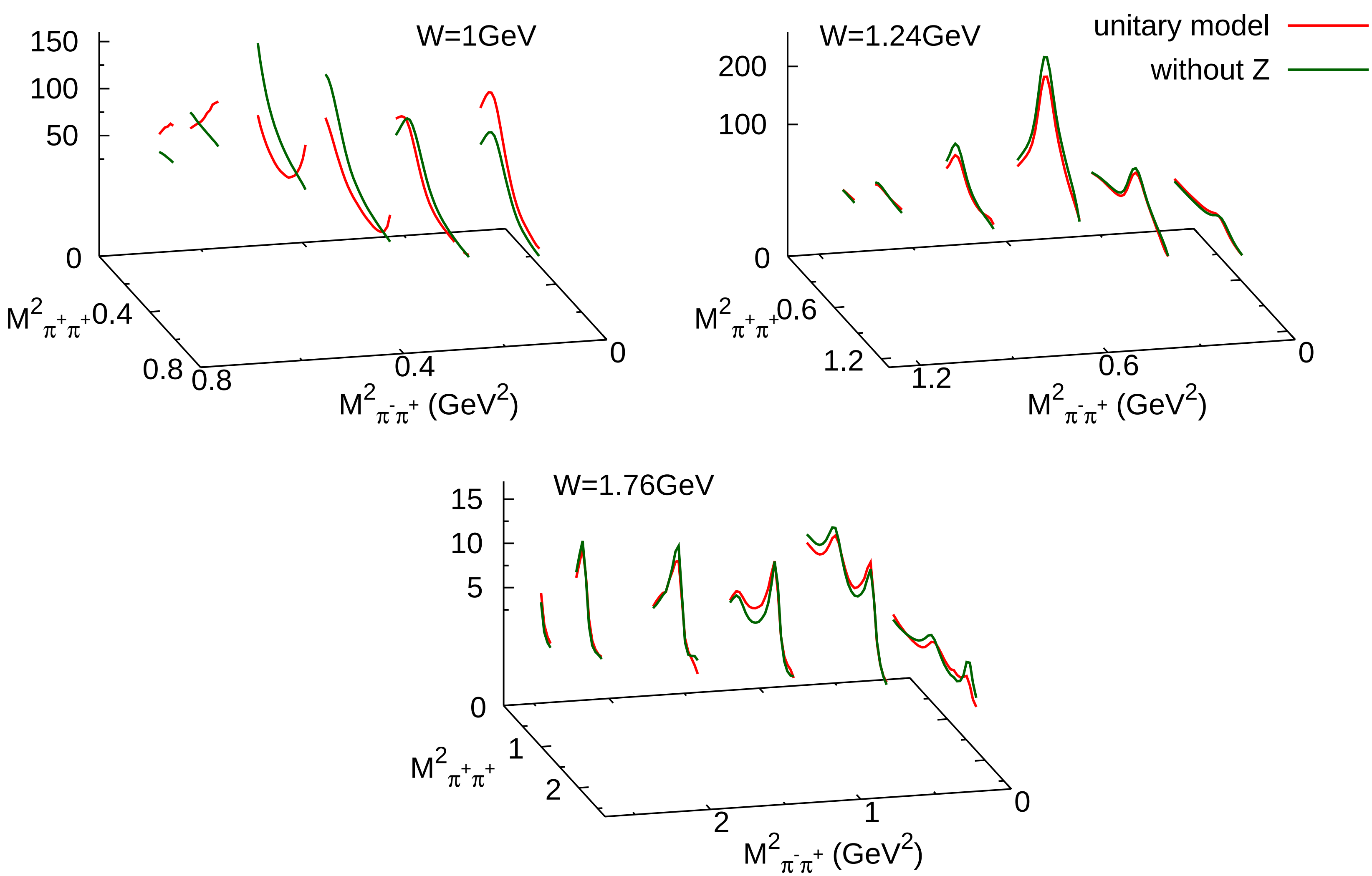
Fit :

1. Fit with real couplings of $M^* \to \pi R$
2. Allow couplings complex
3. Include W-dependent flat non-interfering background

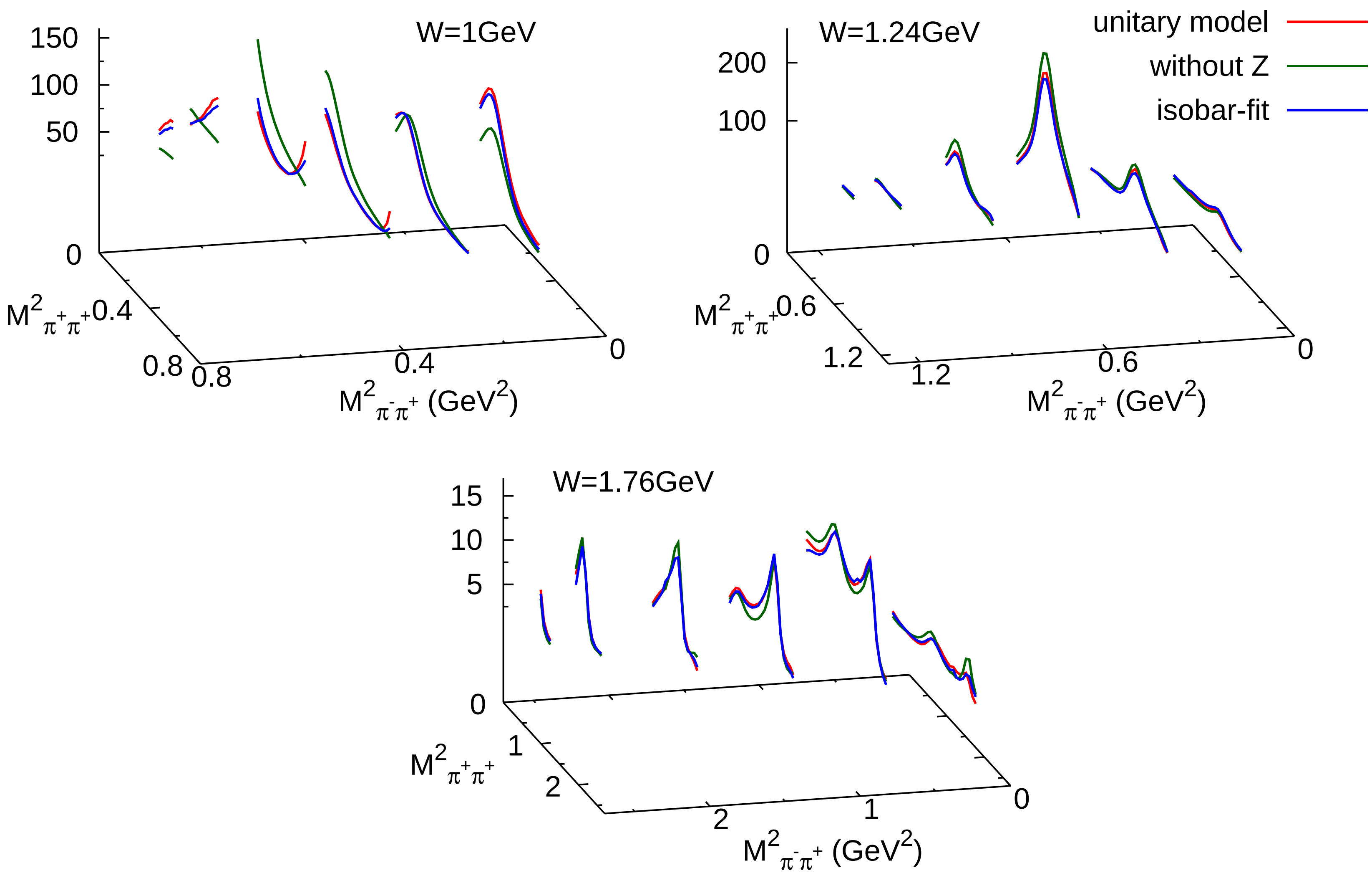
   (2, 3 are common in isobar-model analysis)

$\chi^2/$(# of data) < 0.5 is achieved

Fit with isobar model



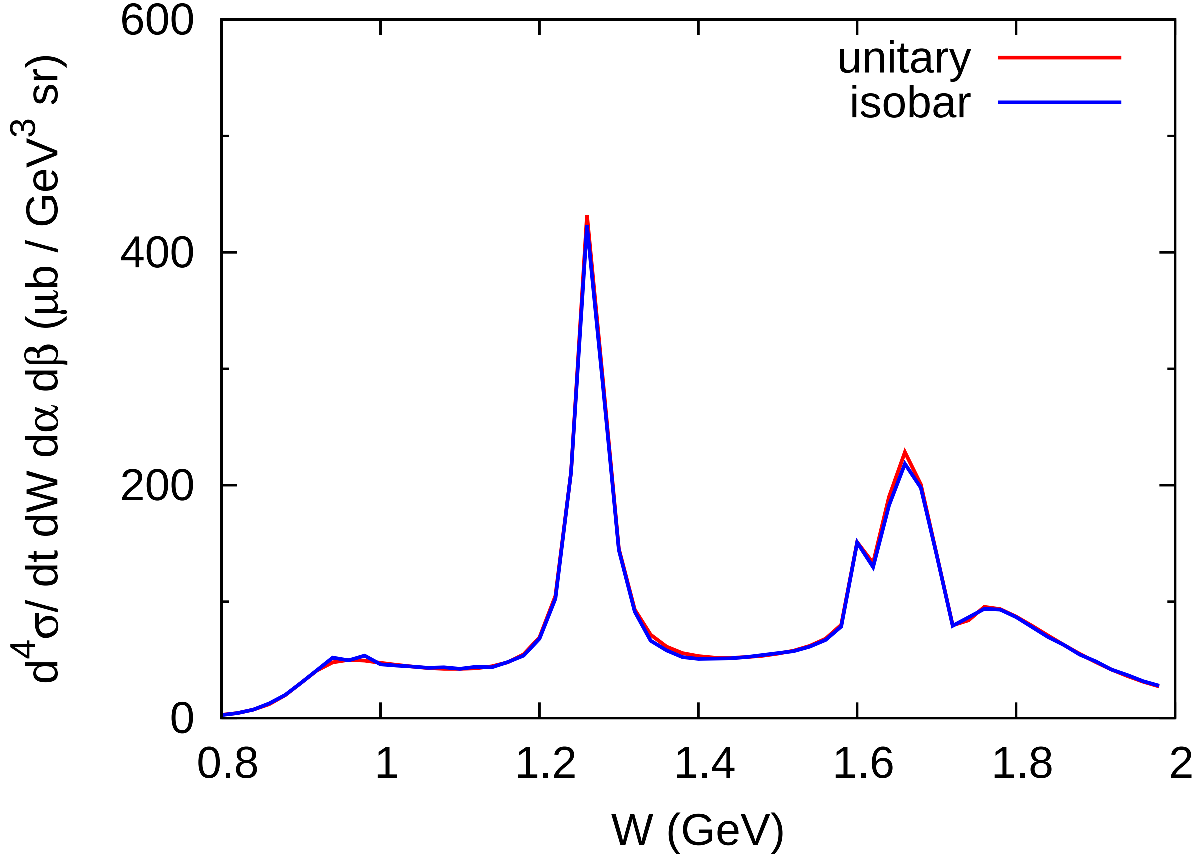
Fit with isobar model

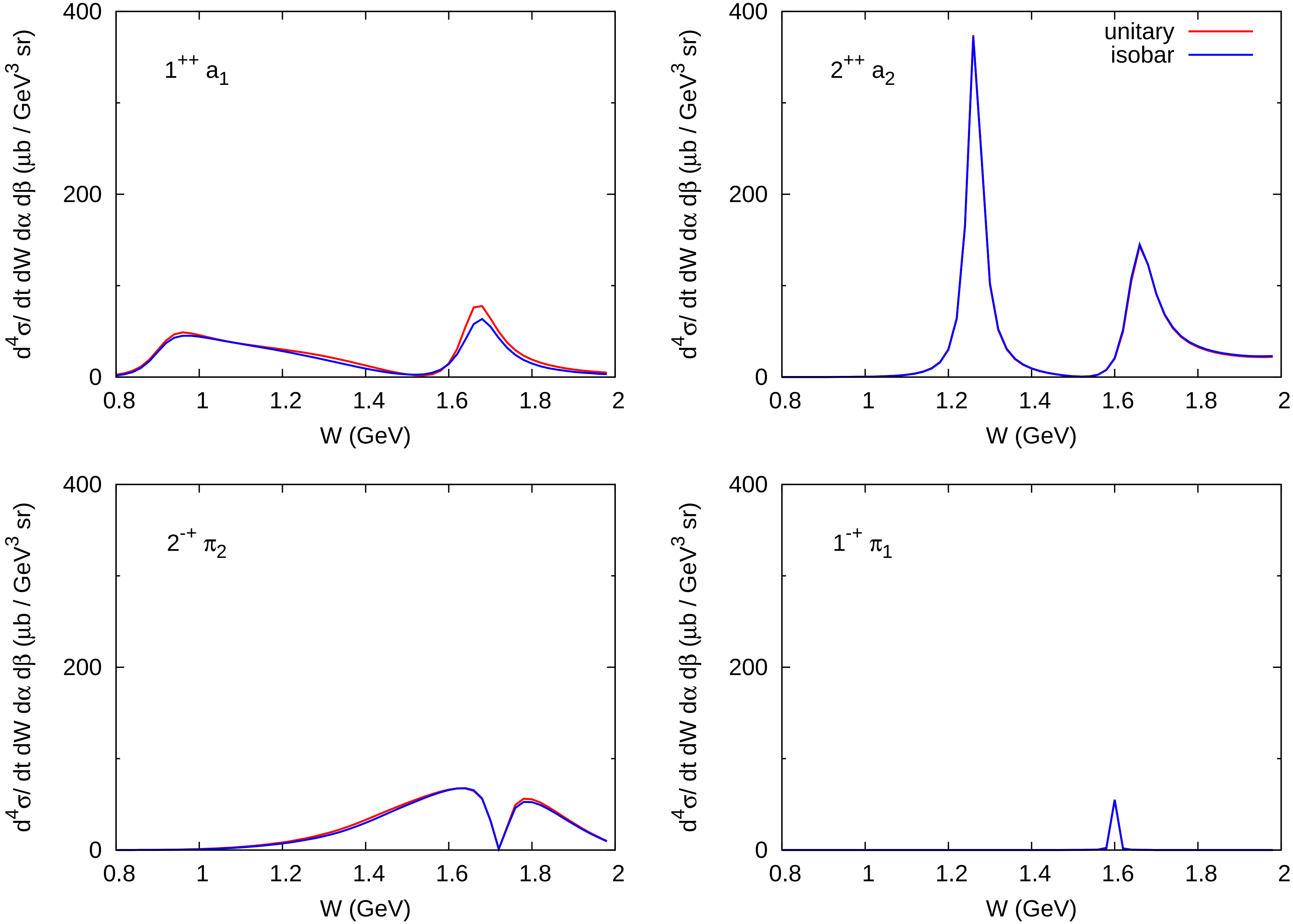


Question I didn't explicitly ask

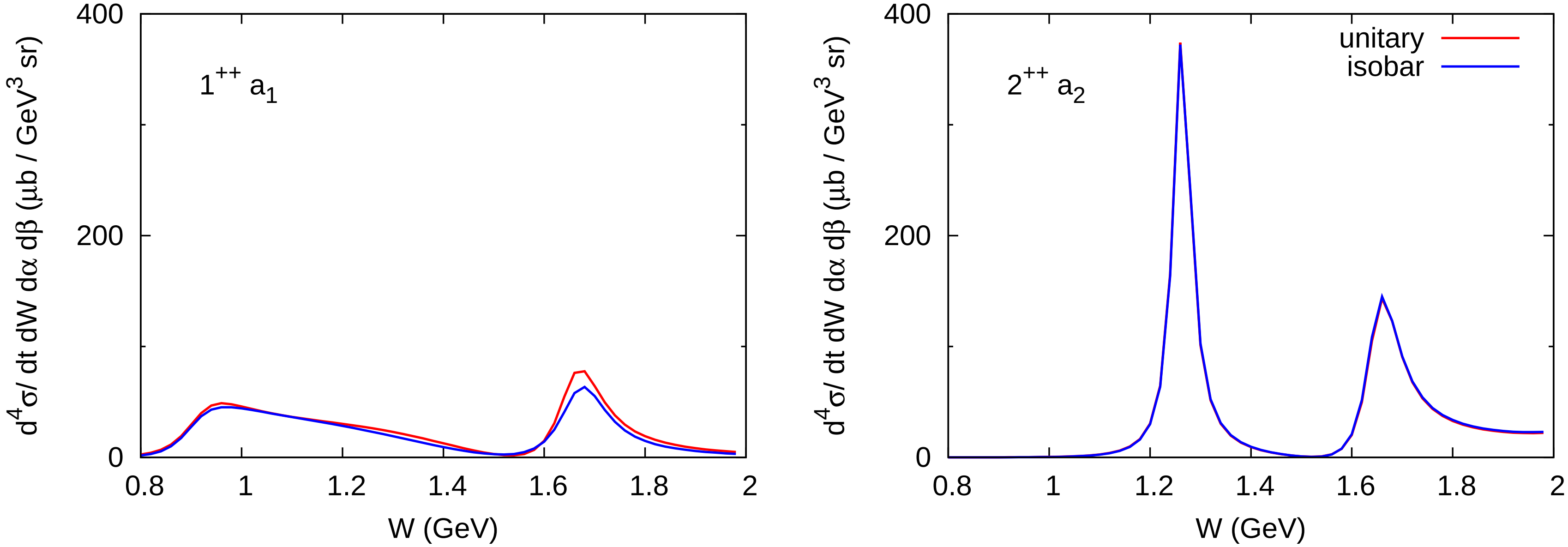
Can isobar model extract partial wave amplitudes of the unitary model ?

W-dependence of integrated Dalitz plot





1^{++} a_1
2^{++} a_2
2^{-+} π_2
1^{-+} π_1
unitary
isobar
$d^4\sigma/ dt\, dW\, d\alpha\, d\beta$ (μb / GeV^3 sr)
W (GeV)
0
200
400
0.8
1
1.2
1.4
1.6
1.8
2



Q : *Can isobar model extract partial wave amplitudes of the unitary model ?*

A : To a good extent, yes.

Comments

- Not so in kinematics where a partial wave amplitude plays a minor role
- Analyzing polarized observables may have helped

M^* pole positions

$\Delta M^0_{M^*}$ (MeV) : pole position shift in the isobar model

$$G^{-1}(W) = W - M^0_{M^*} - \Sigma_{M^*}(W) - \Delta M^0_{M^*}$$

$a_1(1260)$	$a_1(1700)$	$a_2(1320)$	$a_2(1700)$
$-22.37 - 23.21i$	$8.48 - 4.46i$	$0.15 - 0.04i$	$-1.03 - 0.30i$

$\pi_2(1670)$	$\pi_2(1800)$	$\pi_1(1600)$
$-0.51 + 0.38i$	$-3.07 - 3.42i$	$0.57 + 0.33i$

Here, 3-body unitarity effect is moderate

Couplings of $a_1(1260)$ to decay channels

		Unitary	Isobar
$a_1(1260)$	$\to \pi f_0(1300)$	-	$-$ 2.0 + 8.2 i
	$\to \pi f_0(2400)$	-	7.5 $-$ 2.3 i
	$\to \pi\rho(770)$	24.6	31.9 $-$ 3.7 i
	$\to \pi\rho(1700)$	-	11.0 + 5.2 i
	$\to \pi f_2(1270)$	-	$-$ 2.7 + 4.4 i

Rather large change in M^* couplings to decay channels

$\Leftarrow$ Large Z-graph effect in $a_1(1260)$ region

Couplings of $a_2(1320)$ to decay channels

		Unitary	Isobar
$a_2(1320)$	$\to \pi\rho(770)$	1.0	0.9 $-$ 0.1 i
	$\to \pi f_2(1270)$	-	1.4 $-$ 0.1 i

Still rather large change in M^* couplings to decay channels

even though Z-graph effect on Dalitz plot in $a_2(1320)$ region seems moderate

Conclusion

Q : How 3-body unitary makes a difference in extracting M^* properties from data ?

Method

1. Construct a unitary and an isobar models

2. Fit them to the same Dalitz plot

3. Extract and compare M^* properties from them

Conclusion

Q : How 3-body unitary makes a difference in extracting M^* properties from data ?

A : It (and thus Z-diagrams) makes a significant difference in extracting

dynamical aspect of M^* properties , i.e., coupling strength to decay channel

Key information to understand the hadron structure

Conclusion

Q : How 3-body unitary makes a difference in extracting M^* properties from data ?

A : It (and thus Z-diagrams) makes a significant difference in extracting

dynamical aspect of M^* properties , i.e., coupling strength to decay channel

Q : What about pole position ?

A : Moderate. Sometimes non-negligible.