

Partons to Reality: A Matter of Substance

Craig Roberts



Physics Division



Published collaborations: 2010-present

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Students

*Early-career
scientists*

QCD's Challenges

Understand emergent phenomena

➤ Quark and Gluon Confinement

No matter how hard one strikes the proton, one cannot liberate an individual gluon or quark

➤ Dynamical Chiral Symmetry Breaking

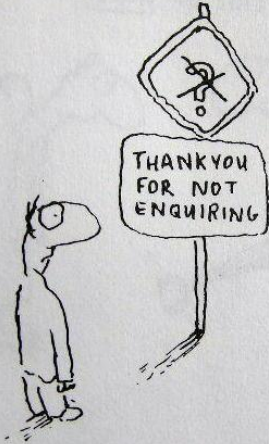
Very unnatural pattern of bound state masses; e.g., Lagrangian (pQCD) quark mass is small but ... no degeneracy between $J^P=+$ and $J^P=-$ (parity partners)

J^P	$\frac{1}{2}^+$ (p)	$\frac{1}{2}^-$
Mass	940 MeV	1535 MeV

➤ Neither of these phenomena is apparent in QCD's Lagrangian **Yet** they are the dominant determining characteristics of real-world QCD.

➤ QCD

– Complex behaviour from apparently simple rules.



Dyson-Schwinger Equations

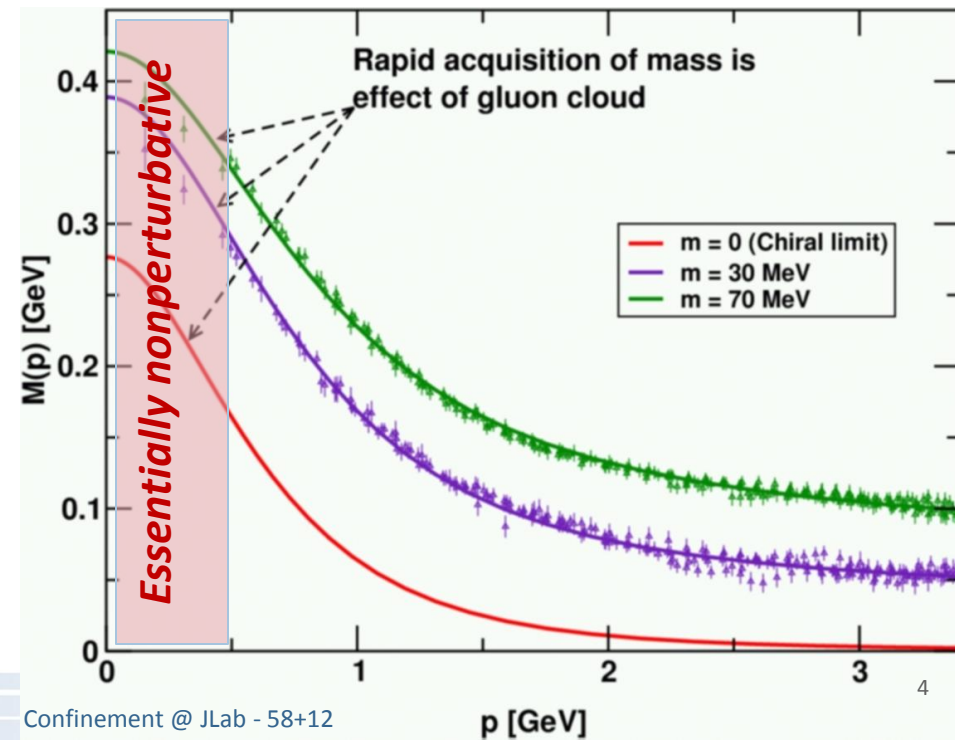
- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory . . . Materially Reduces Model-Dependence ... Statement about long-range behaviour of quark-quark interaction
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
 - ❖ Dynamical Chiral Symmetry Breaking
 - Generation of fermion mass from *nothing*
 - ❖ Quark & Gluon Confinement
 - Coloured objects not detected, *Not detectable?*
- Approach yields Schwinger functions; i.e., propagators and vertices
- Cross-Sections built from Schwinger Functions
- Hence, method connects observables with long-range behaviour of the running coupling
- Experiment \leftrightarrow Theory comparison leads to an understanding of long-range behaviour of strong running-coupling

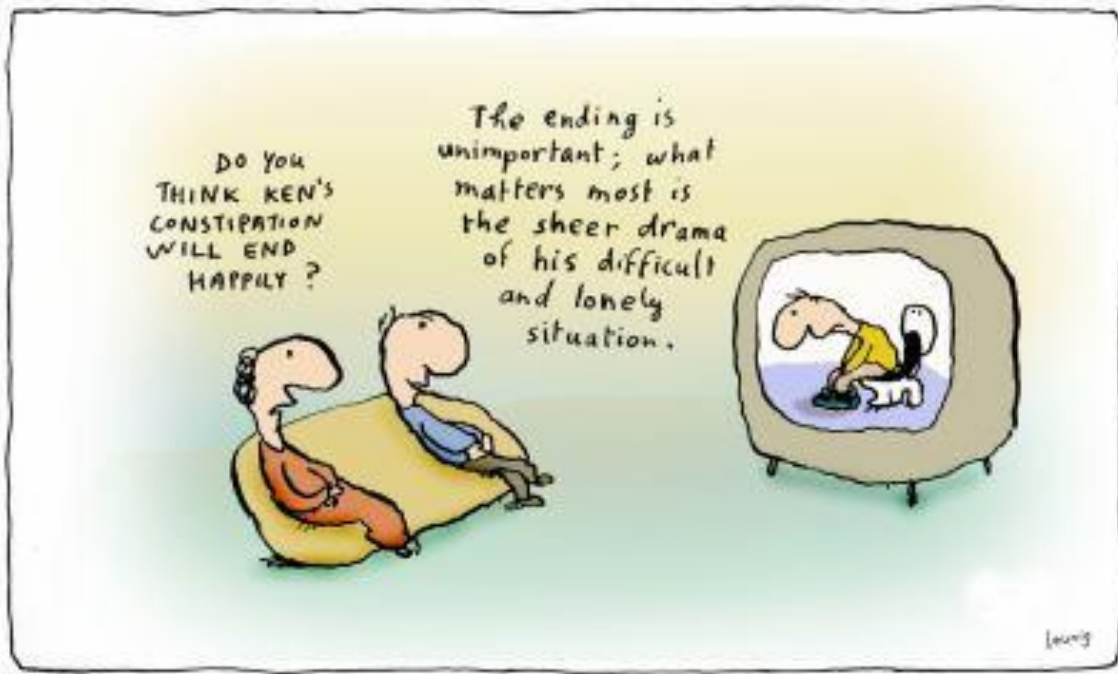
Necessary Precondition

- Experiment \leftrightarrow Theory comparison leads to an understanding of long-range behaviour of strong running-coupling
- However, if one wants to draw reliable conclusions about Q^2 -dependence of QCD's running coupling,
- Then, approach must veraciously express Q^2 -dependence of QCD's running masses
- True for ALL observables
 - ✓ From spectrum ...
 - ✓ through elastic & transition form factors ...
 - ✓ to PDFs and GPDs ... etc.

- Mass function exhibits inflexion point at $Q_{IR} \approx 0.5 \text{ GeV}$
- So ... **pQCD is definitely invalid for momenta $Q < Q_{IR}$**
- E.g., use of **DGLAP** equations cannot be justified in QCD at $Q < Q_{IR} = 0.5 \text{ GeV}$, irrespective of order.

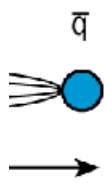
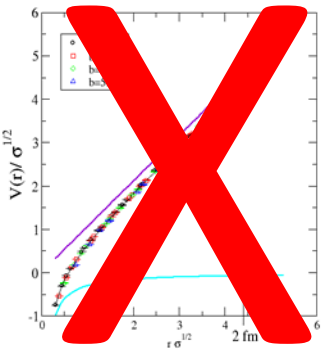
Distribution Functions of the Nucleon and Pion in the Valence Region, Roy J. Holt and Craig D. Roberts
[arXiv:1002.4666 \[nucl-th\]](https://arxiv.org/abs/1002.4666),
[Rev. Mod. Phys. 82 \(2010\) pp. 2991-3044](https://doi.org/10.1093/rmp/82/3/2991)
& **presentations by Adnan Bashir and Peter Tandy on Thursday**





Confinement

Confinement



$$\mathcal{H}_{\text{QCD}} = \sum_n E_n |H_n^{1c}\rangle \langle H_n^{1c}|$$

Colour singlets

➤ Gluon and Quark Confinement

- No coloured states have yet been observed to reach a detector

➤ Empirical fact. However

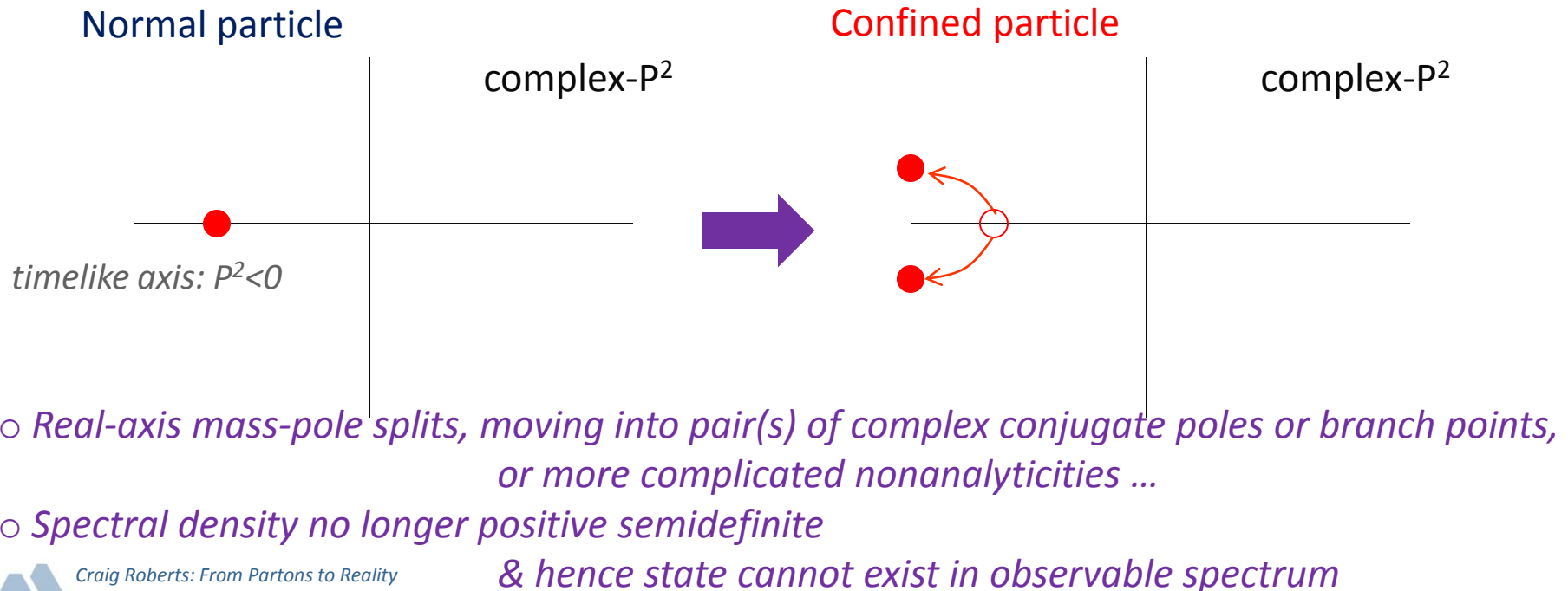
- There is no agreed, theoretical definition of light-quark confinement
- Static-quark confinement is irrelevant to real-world QCD
 - *There are no long-lived, very-massive quarks*
 - *But light-quarks are ubiquitous*

➤ Confinement entails *quark-hadron duality*; i.e., that *all observable consequences of QCD can, in principle, be computed using an hadronic basis.*



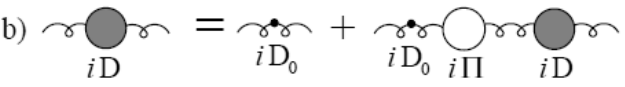
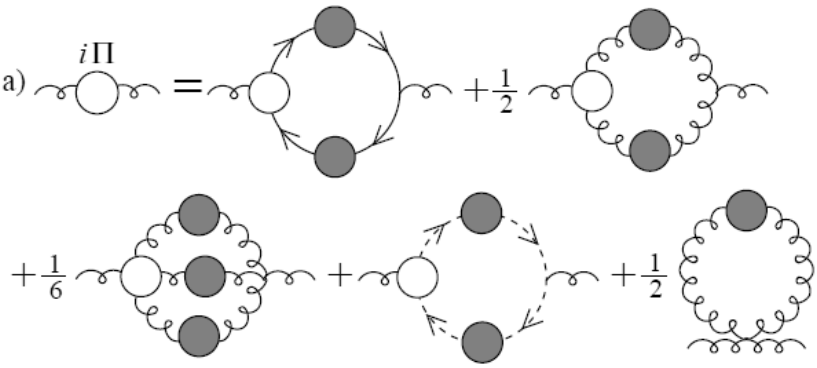
Confinement

- Confinement is expressed through a *dramatic* change in the analytic structure of propagators for coloured particles & can almost be read from a plot of a states' dressed-propagator
 - Gribov (1978); Munczek (1983); Stingl (1984); Cahill (1989); Roberts, Williams & Krein (1992); Tandy (1994); ...

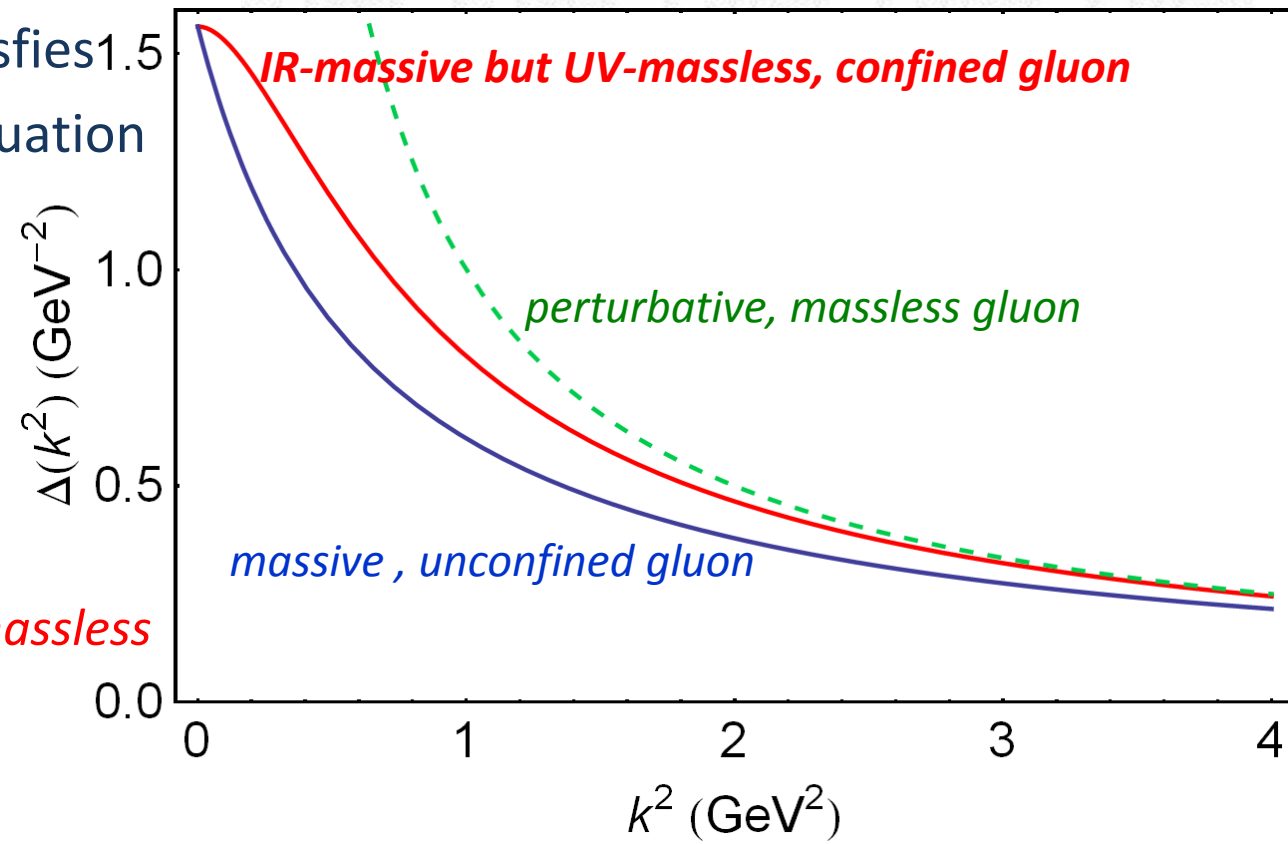


Dressed-gluon propagator

A.C. Aguilar et al., [Phys.Rev. D80 \(2009\) 085018](#)



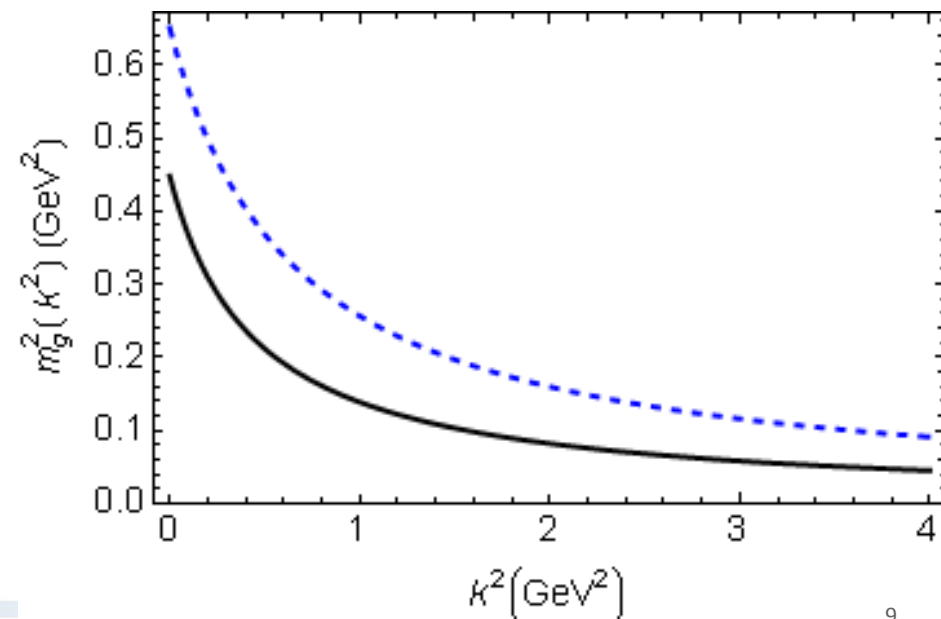
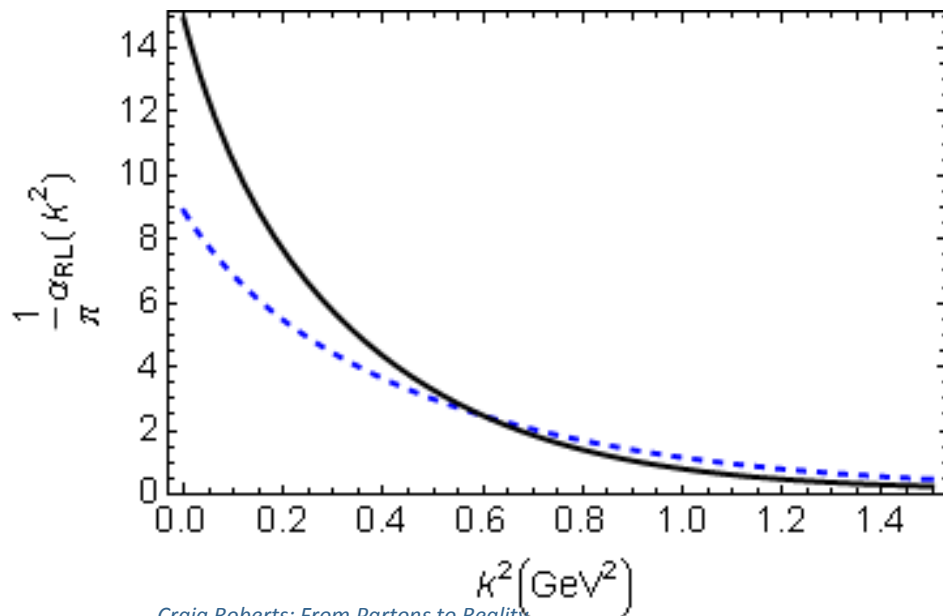
- Gluon propagator satisfies a Dyson-Schwinger Equation
- Plausible possibilities for the solution
- DSE and lattice-QCD agree on the result
 - *Confined gluon*
 - *IR-massive but UV-massless*
 - $m_G \approx 2-4 \Lambda_{\text{QCD}}$



DSE Studies

- Phenomenology of gluon

- Wide-ranging study of π & ρ properties
- Effective coupling
 - Agrees with pQCD in ultraviolet
 - Saturates in infrared
 - $\alpha(0)/\pi = 8-15$
 - $\alpha(m_G^2)/\pi = 2-4$
- Running gluon mass
 - Gluon is massless in ultraviolet in agreement with pQCD
 - Massive in infrared
 - $m_G(0) = 0.67-0.81$ GeV
 - $m_G(m_G^2) = 0.53-0.64$ GeV





Dynamical Chiral Symmetry Breaking

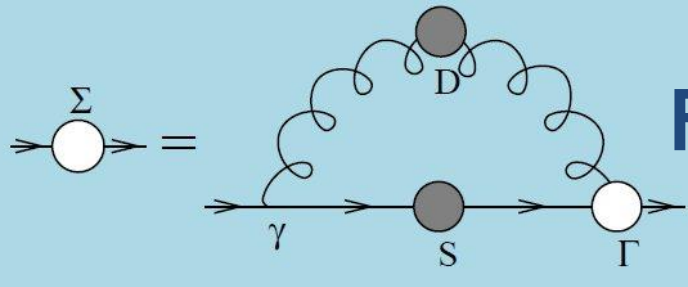
Craig Roberts: From Partons to Reality



Dynamical Chiral Symmetry Breaking

- Strong-interaction: **QCD**
- Confinement
 - Empirical feature
 - Modern theory and lattice-QCD support conjecture
 - that light-quark confinement is a fact
 - associated with violation of reflection positivity;
i.e., novel analytic structure for propagators and vertices
 - Still circumstantial, no proof yet of confinement
- On the other hand, *DCSB is a fact in QCD*
 - It is the most important mass generating mechanism for visible matter in the Universe.

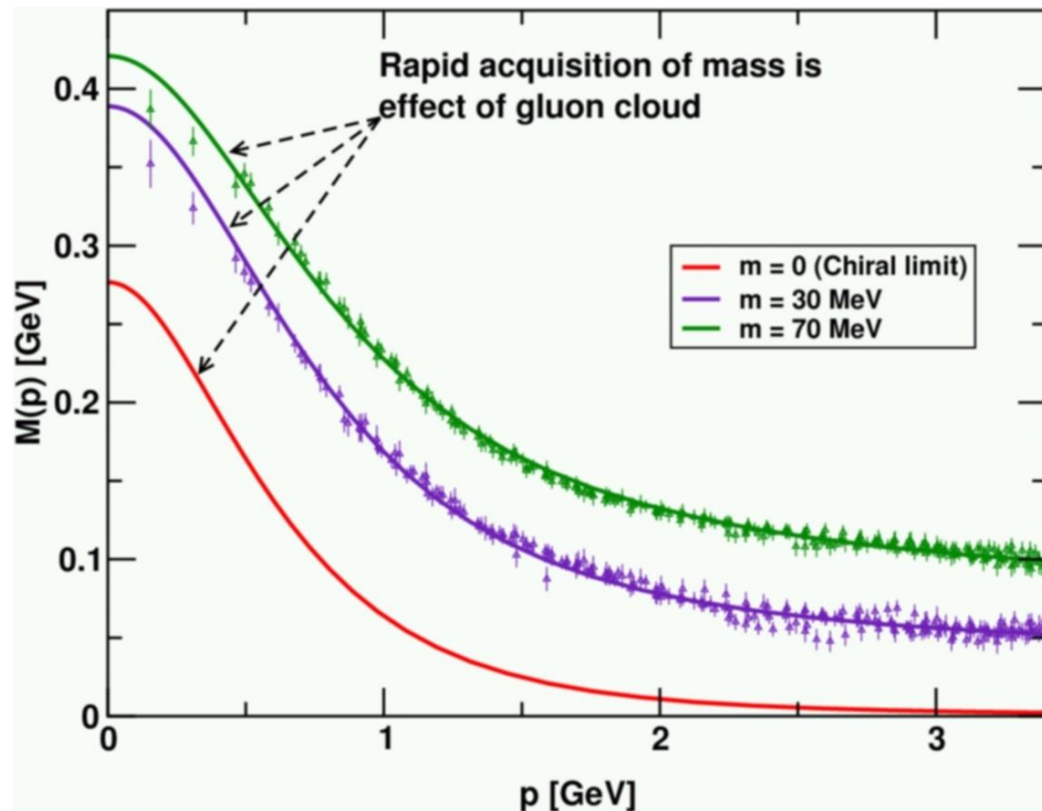
Responsible for approximately 98% of the proton's mass.
Higgs mechanism is (*almost*) irrelevant to light-quarks.



Frontiers of Nuclear Science: Theoretical Advances

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. **Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates.** In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, **red curve**) acquires a large constituent mass at low energies.

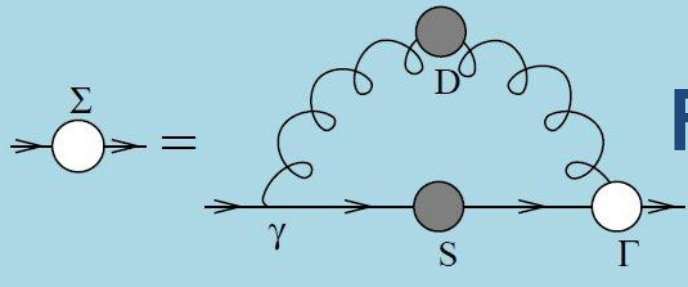
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



C.D. Roberts, [Prog. Part. Nucl. Phys. 61 \(2008\) 50](#)

M. Bhagwat & P.C. Tandy, [AIP Conf.Proc. 842 \(2006\) 225-227](#)

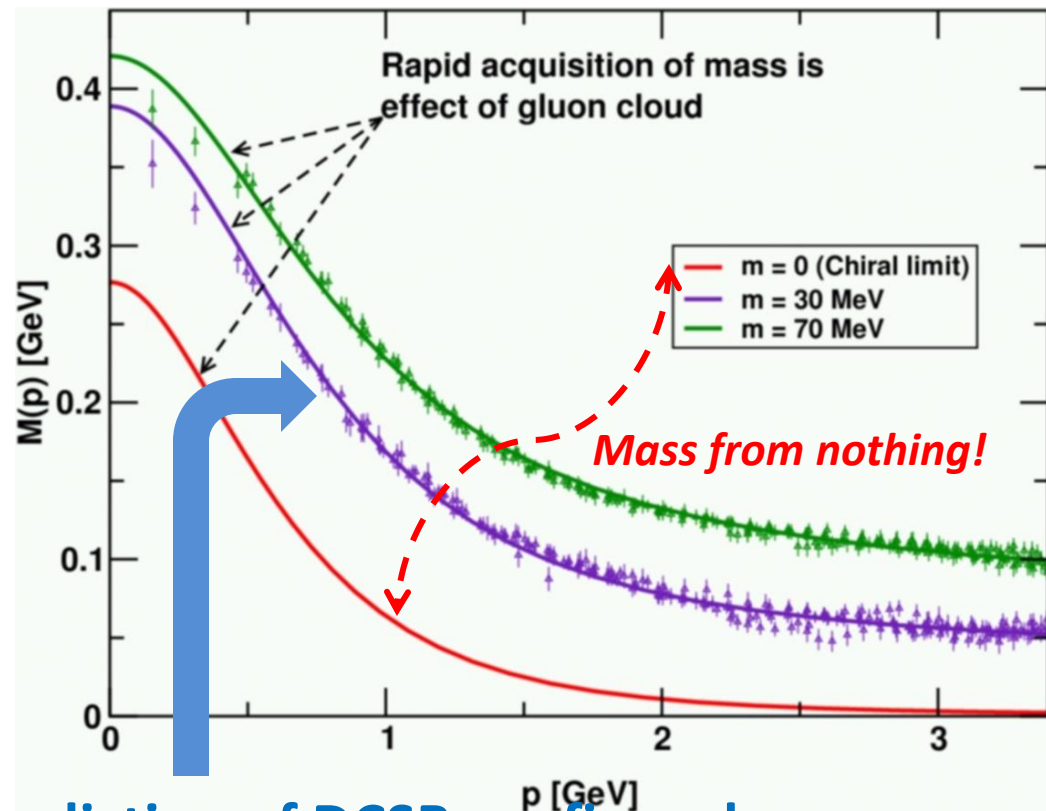




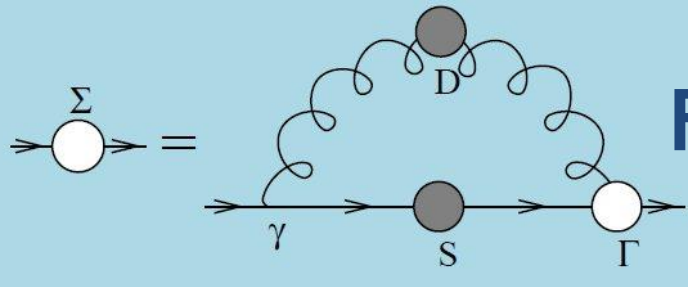
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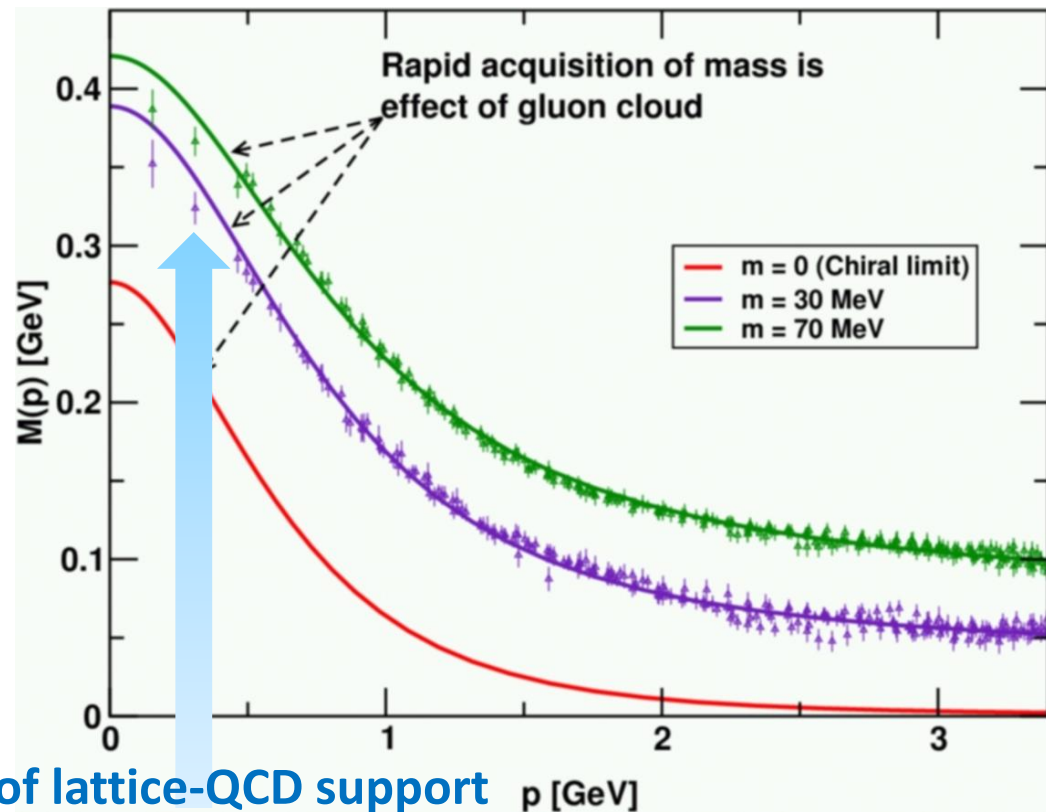
DSE prediction of DCSB confirmed



Frontiers of Nuclear Science: Theoretical Advances

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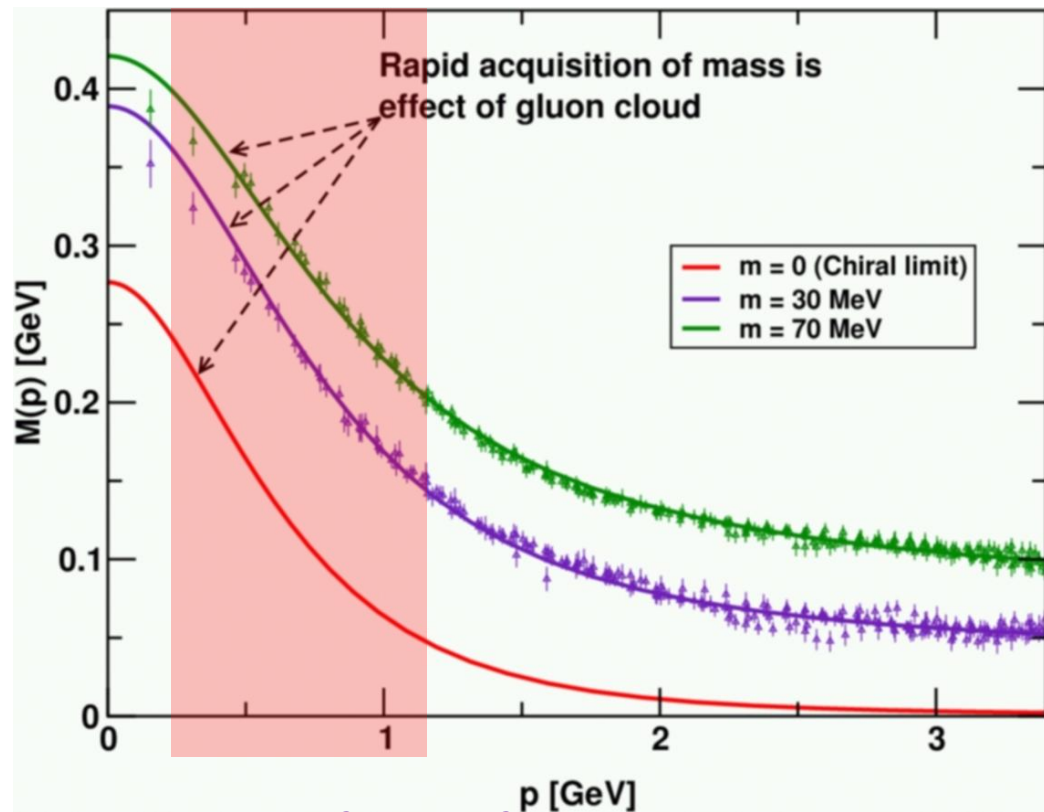


**Hint of lattice-QCD support
for DSE prediction of violation of reflection positivity**



12GeV The Future of JLab

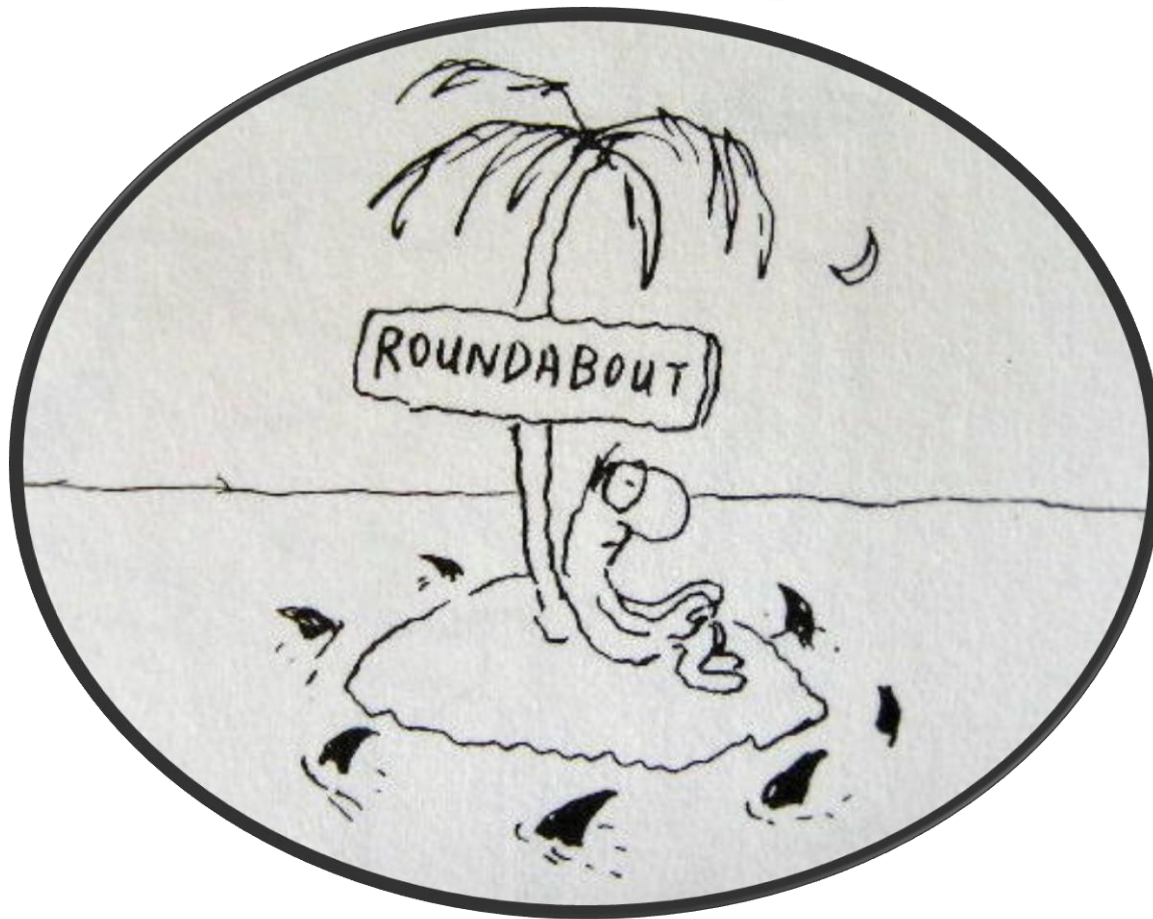
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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Jlab 12GeV: Scanned by $2 < Q^2 < 9 \text{ GeV}^2$

elastic & transition form factors.



Persistent Challenge Truncation



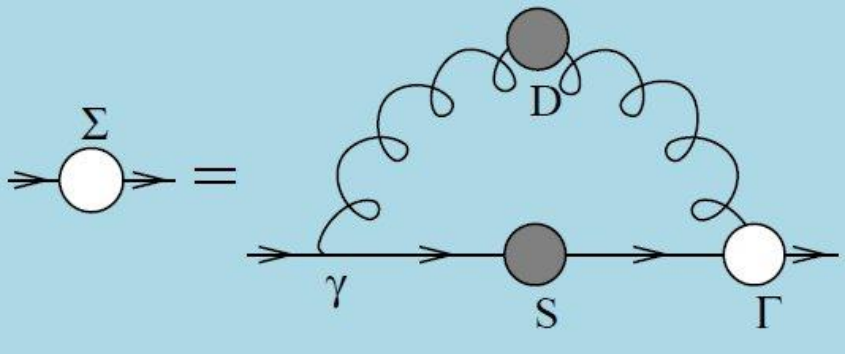
Dichotomy of the pion Goldstone mode and bound-state

- The *correct understanding* of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a
- **well-defined** and **valid** chiral limit;
 - and an **accurate realisation** of dynamical chiral symmetry breaking.

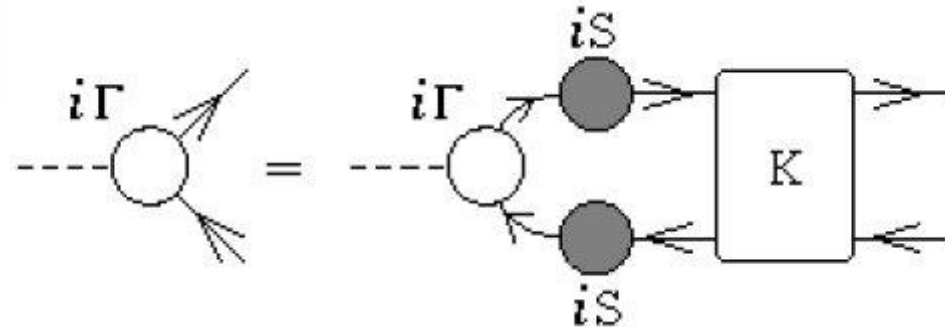
HIGHLY NONTRIVIAL

Impossible in quantum mechanics

Only possible in asymptotically-free gauge theories



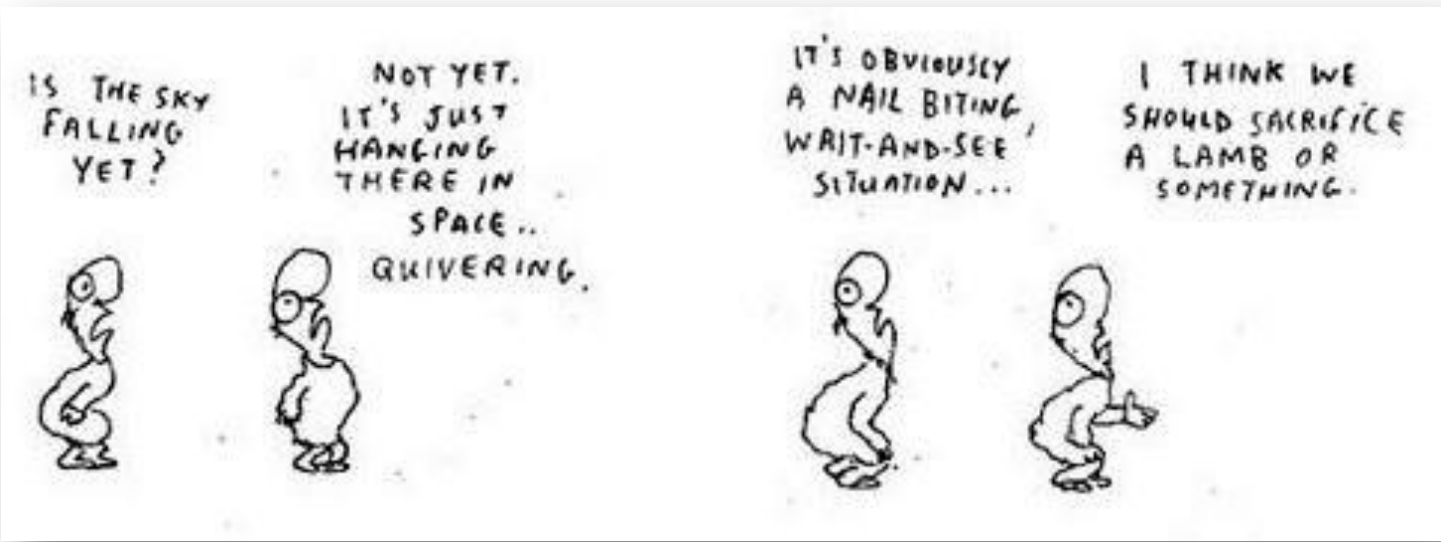
Persistent challenge - truncation scheme



➤ There are now two **nonperturbative & symmetry preserving** truncation schemes

1. **1995** – H.J. Munczek, [Phys. Rev. D 52 \(1995\) 4736](#), *Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*
- 1996 – A. Bender, C.D. Roberts and L. von Smekal, [Phys.Lett. B 380 \(1996\) 7](#), *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*
2. **2009** – Lei Chang and C.D. Roberts, [Phys. Rev. Lett. 103 \(2009\) 081601, 0903.5461 \[nucl-th\]](#), *Sketching the Bethe-Salpeter kernel*

➤ Enables proof of numerous exact results



Some of many Exact Results

Pion's Goldberger-Treiman relation

- Pion's Bethe-Salpeter amplitude

Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k; P) = \tau^{\pi^j} \gamma_5 \left[iE_{\pi}(k; P) + \gamma \cdot P F_{\pi}(k; P) + \gamma \cdot k k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} H_{\pi}(k; P) \right]$$

Pseudovector components necessarily nonzero. Cannot be ignored!

- Dressed-quark propagator $S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_{\pi} E_{\pi}(k; P = 0) = B(p^2)$$

Exact in Chiral QCD

Miracle: two body problem solved, almost completely, once solution of one body problem is known



Dichotomy of the pion Goldstone mode and bound-state

$$f_{\pi} E_{\pi}(p^2) = B(p^2)$$

- Goldstone's theorem
has a pointwise expression in QCD;

Namely, in the chiral limit the wave-function for the two-body bound-state Goldstone mode is intimately connected with, and almost completely specified by, the fully-dressed one-body propagator of its characteristic constituent

- The one-body momentum is equated with the relative momentum of the two-body system

Dichotomy of the pion Mass Formula for 0^- Mesons

$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^\zeta \mathcal{M}_{H_5}^\zeta$$

- Mass-squared of the pseudoscalar hadron
- Sum of the current-quark masses of the constituents;
e.g., pion = $m_u^\zeta + m_d^\zeta$, where “ ζ ” is the renormalisation point

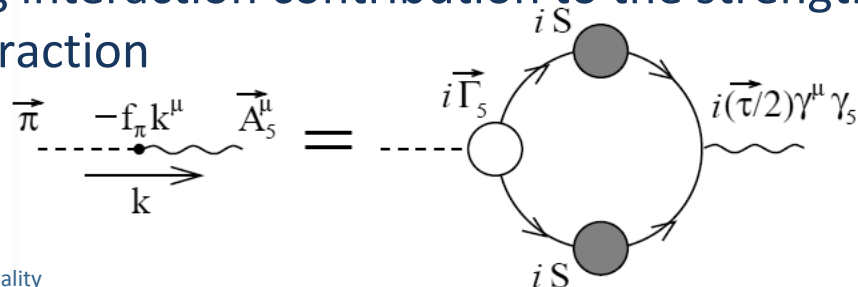
Dichotomy of the pion Mass Formula for 0^- Mesons

$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^\zeta \mathcal{M}_{H_5}^\zeta$$

$$f_{H_5} P_\mu = Z_2 \text{tr} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{2} (T^{H_5})^t \gamma_5 \gamma_\mu S(q + \frac{1}{2}P) \Gamma_{H_5}(q; P) S(q - \frac{1}{2}P)$$

➤ **Pseudovector** projection of the **Bethe-Salpeter** wave function onto the origin in configuration space

- Namely, the pseudoscalar meson's leptonic decay constant, which is the strong interaction contribution to the strength of the meson's weak interaction

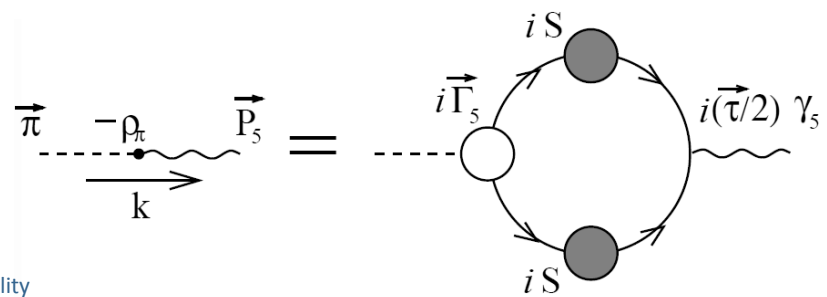


Dichotomy of the pion Mass Formula for 0^- Mesons

$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^\zeta \mathcal{M}_{H_5}^\zeta$$

$$i\rho_{H_5} = Z_4 \text{tr} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{2} (T^{H_5})^t \gamma_5 S(q + \frac{1}{2}P) \Gamma_{H_5}(q; P) S(q - \frac{1}{2}P)$$

- **Pseudoscalar** projection of the **Bethe-Salpeter wave function** onto the origin in configuration space
 - Namely, a pseudoscalar analogue of the meson's leptonic decay constant



Dichotomy of the pion Mass Formula for 0^- Mesons

$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^\zeta \mathcal{M}_{H_5}^\zeta$$

➤ Consider the case of light quarks; namely, $m_q \approx 0$

– If chiral symmetry is dynamically broken, then

- $f_{H_5} \rightarrow f_{H_5}^0 \neq 0$
- $\rho_{H_5} \rightarrow -\langle \bar{q}q \rangle / f_{H_5}^0 \neq 0$

both of which are independent of m_q

➤ Hence, one arrives at the corollary *Gell-Mann, Oakes, Renner relation*

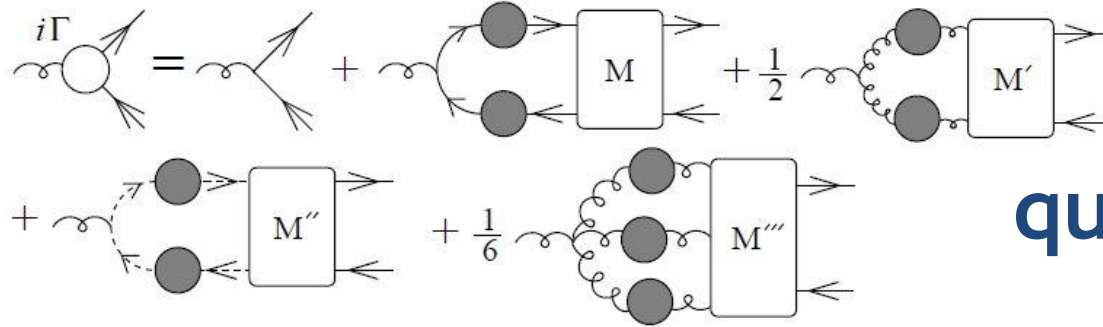
$$m_{H_5}^2 = 2m_q \frac{-\langle \bar{q}q \rangle}{f_{H_5}^0}$$

$$m_\pi^2 \propto m \quad 1968$$

The so-called “vacuum quark condensate.” More later about this.



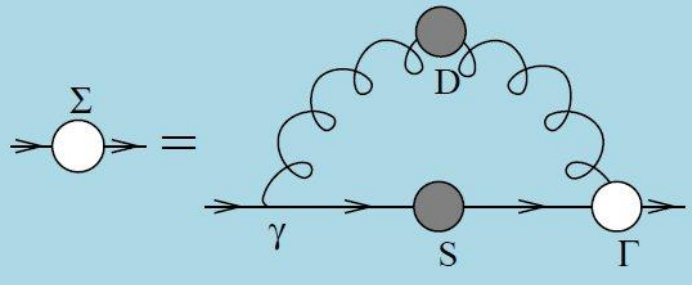
Dynamical Chiral Symmetry Breaking Importance of being well- dressed for quarks & mesons



Dressed- quark-gluon vertex

➤ Single most important feature

- Perturbative vertex is helicity-conserving:
 - Cannot cause spin-flip transitions
- *However, DCSB introduces nonperturbatively generated structures that very strongly break helicity conservation*
- These contributions
 - Are large when the dressed-quark mass-function is large
 - Therefore vanish in the ultraviolet; i.e., on the perturbative domain
- Exact form of the contributions is still the subject of debate *but* their *existence* is model-independent - *a fact*.



Gap Equation General Form

$$S_f(p)^{-1} = Z_2 (i\gamma \cdot p + m_f^{\text{bm}}) + \Sigma_f(p),$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma_\nu^f(q,p)$$

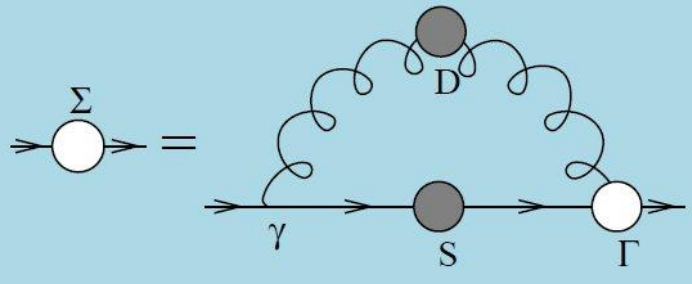
- $D_{\mu\nu}(k)$ – dressed-gluon propagator
- $\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex
- Until 2009, all studies of other hadron phenomena used the leading-order term (or LO+NLO) in a symmetry-preserving truncation scheme; viz.,

*Bender, Roberts & von Smekal
[Phys.Lett. B380 \(1996\) 7-12](#)*

- $D_{\mu\nu}(k)$ = fully-dressed

- $\Gamma_\nu(q,p) = \gamma_\mu$

- ... plainly, key nonperturbative effects are missed and cannot be recovered through any step-by-step improvement procedure



Gap Equation General Form

$$S_f(p)^{-1} = Z_2 (i\gamma \cdot p + m_f^{\text{bm}}) + \Sigma_f(p),$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma_\nu^f(q,p)$$

➤ $D_{\mu\nu}(k)$ – dressed-gluon propagator

➤ good deal of information available

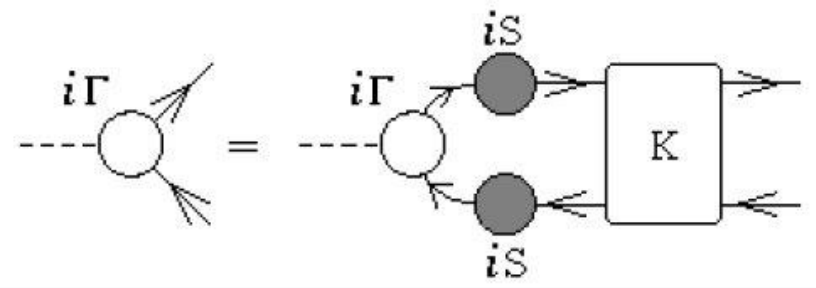
➤ $\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex

➤ Information accumulating

➤ Suppose one has in hand – from anywhere – the exact form of the dressed-quark-gluon vertex

If kernels of Bethe-Salpeter and gap equations don't match, one won't even get right charge for the pion.

➔ What is the associated symmetry-preserving Bethe-Salpeter kernel?!



Bethe-Salpeter Equation Bound-State DSE

$$[\Gamma_{\pi}^j(k; P)]_{tu} = \int_q^{\Lambda} [S(q + P/2)\Gamma_{\pi}^j(q; P)S(q - P/2)]_{sr} K_{tu}^{rs}(q, k; P)$$

- *$K(q, k; P)$ – fully amputated, two-particle irreducible, quark-antiquark scattering kernel*
- Textbook material.
- Compact. Visually appealing. Correct

Blocked progress for more than 60 years.



Bethe-Salpeter Equation

General Form

Lei Chang and C.D. Roberts

0903.5461 [nucl-th]

Phys. Rev. Lett. 103 (2009) 081601

$$\Gamma_{5\mu}^{fg}(k; P) = Z_2 \gamma_5 \gamma_\mu$$

$$- \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \Gamma_{5\mu}^{fg}(q; P) S_g(q_-) \frac{\lambda^a}{2} \Gamma_\beta^g(q_-, k_-)$$

$$+ \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P),$$

- Equivalent exact bound-state equation **but** in this form

$$K(q, k; P) \rightarrow \Lambda(q, k; P)$$

which is **completely determined by dressed-quark self-energy**

- Enables derivation of a Ward-Takahashi identity for $\Lambda(q, k; P)$

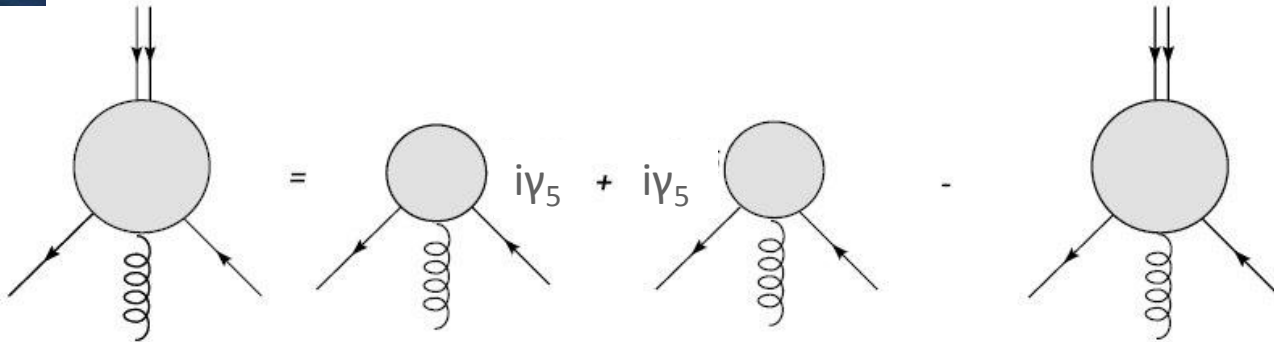


Ward-Takahashi Identity Bethe-Salpeter Kernel

Lei Chang and C.D. Roberts

0903.5461 [nucl-th]

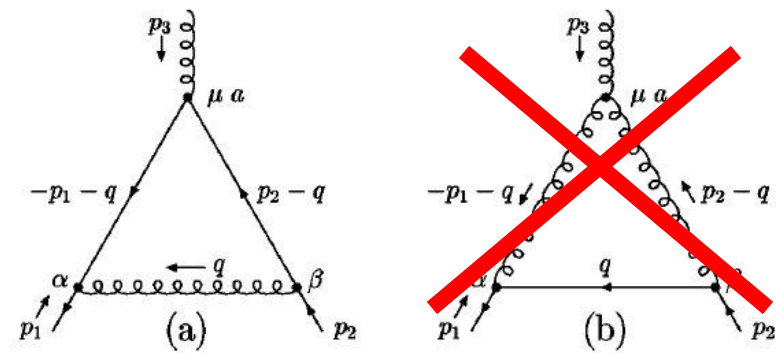
Phys. Rev. Lett. 103 (2009) 081601



$$P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_\beta^f(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma_\beta^g(q_-, k_-) - i[m_f(\zeta) + m_g(\zeta)] \Lambda_{5\beta}^{fg}(k, q; P),$$

- Now, for first time, it's possible to formulate an *Ansatz* for Bethe-Salpeter kernel given *any form* for the dressed-quark-gluon vertex by using this identity
- This enables the identification and elucidation of a wide range of novel consequences of DCSB

QCD and dressed-quark anomalous magnetic moments



➤ Schwinger's result for QED:

$$\frac{q}{2m} \rightarrow \left(1 + \frac{\alpha}{2\pi}\right) \frac{q}{2m}$$

➤ pQCD: two diagrams

- (a) is QED-like
- (b) is only possible in QCD – involves 3-gluon vertex

➤ Analyse (a) and (b)

- (b) vanishes identically: the 3-gluon vertex does *not* contribute to a quark's anomalous chromomag. moment at leading-order
- (a) Produces a finite result: “ $-\frac{1}{6} \alpha_s/2\pi$ ”
 $\sim (-\frac{1}{6})$ QED-result

➤ But, in QED and QCD, the *anomalous chromo- and electro-magnetic moments vanish identically in the chiral limit!*

➤ In QCD, chiral symmetry is dynamically broken, strongly
 – What then?

Dressed-quark anomalous magnetic moments

➤ **DCSB** → Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:

Ball-Chiu term →

- Vanishes if no DCSB
- Appearance driven by STI

$$\lambda_\mu^3(p, q) = 2(p + q)_\mu \Delta_B(p, q)$$

$$\Delta_F(p, q) = \frac{F(p^2) - F(q^2)}{p^2 - q^2}$$

Anom. chrom. mag. mom. → contribution to vertex

- Similar properties to BC term
- Strength commensurate with lattice-QCD

$$\Gamma_\mu^{\text{acm}_5} = \sigma_{\mu\nu} k_\nu \tau_5(p_1, p_2),$$

$$\Gamma_\mu^{\text{acm}_4} = [\ell_\mu^\text{T} \gamma \cdot k + i \gamma_\mu^\text{T} \sigma_{\nu\rho} \ell_\nu k_\rho] \tau_4(p_1, p_2),$$

$$\tau_4 = \frac{2 \tau_5(p_1, p_2)}{\mathcal{M}(p_1^2, p_2^2)},$$

Skullerud, Bowman, Kizilersu *et al.*
hep-ph/0303176

$$\tau_5 = \eta \Delta_B(p_1^2, p_2^2) \text{ and } \mathcal{M}(x, y) = [x + M(x)^2 + y + M(y)^2] / (2[M(x) + M(y)]).$$

Role and importance is novel discovery

$M_E = \{s | s > 0, s = M^2(s)\}$ is the Euclidean constituent-quark mass

- Essential to recover pQCD
- Constructive interference with Γ^5

Dressed-quark anomalous magnetic moments

➤ Formulated and solved general Bethe-Salpeter equation

➤ Obtained dressed electromagnetic vertex

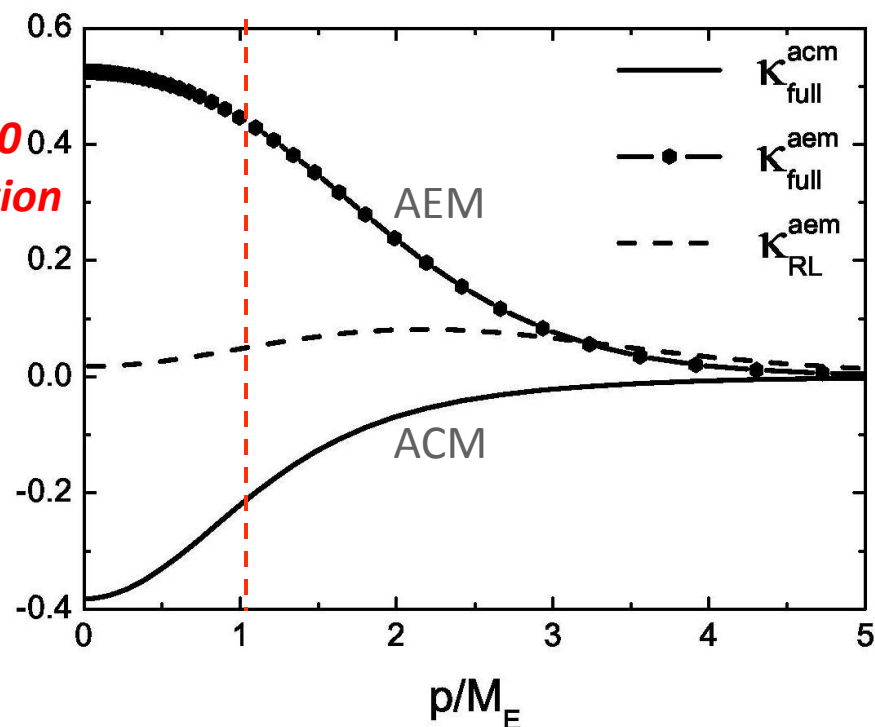
➤ Confined quarks don't have a mass-shell

- Can't unambiguously define magnetic moments

- But can define

magnetic moment distribution

Factor of 10 magnification



➤ AEM is opposite in sign but of roughly equal magnitude as ACM

	M^E	K^{ACM}	K^{AEM}
Full vertex	0.44	-0.22	0.45
Rainbow-ladder	0.35	0	0.048

Dressed-quark anomalous magnetic moments

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➤ Obtained dressed electromagnetic vertex

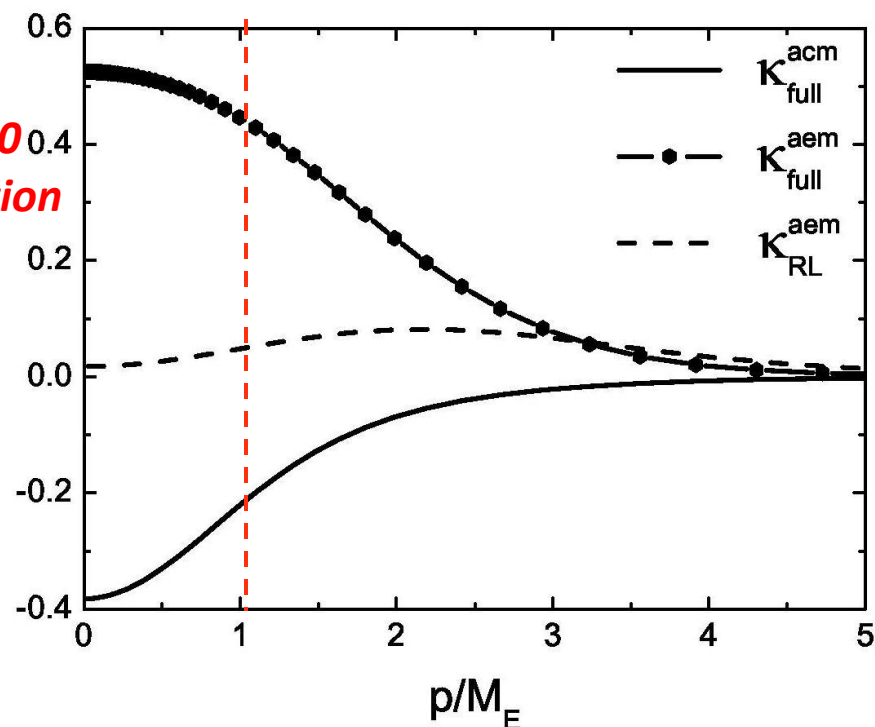
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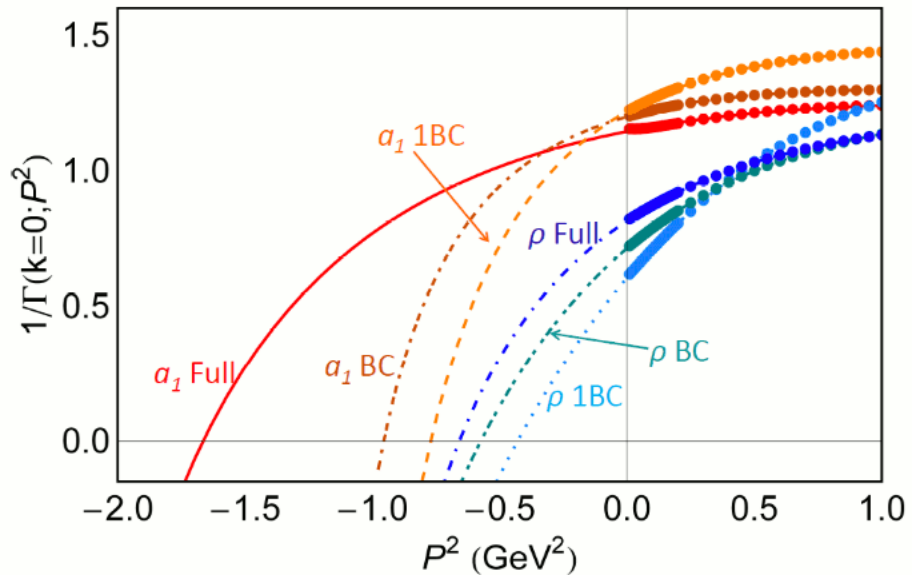


➤ Potentially important for elastic and transition form factors, etc.

➤ **Indeed, for any process involving photons coupling to a dressed-quark**

Solves problem of $a_1 - \rho$ mass splitting

Lei Chang & C.D. Roberts,
[arXiv:1104.4821 \[nucl-th\]](https://arxiv.org/abs/1104.4821)
 Tracing mass of ground-state
 light-quark mesons



$M(p^2)$ magnifies spin orbit splitting here,
 precisely as in $\sigma - \pi$ comparison

- Fully nonperturbative BSE kernel that incorporates and expresses DCSB: establishes unambiguously that a_1 & ρ are parity-partner bound-states of dressed light valence-quarks.

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a_1	1230	759	885	1020	1280
ρ	770	644	764	800	840
Mass splitting	455	115	121	220	440



Dynamical Chiral Symmetry Breaking Vacuum Condensates?

Craig Roberts: From Partons to Reality



Dichotomy of the pion Mass Formula for 0^- Mesons

$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^\zeta \mathcal{M}_{H_5}^\zeta$$

- C *We now have sufficient information to address the question of just what is this so-called “vacuum quark condensate.”*
- H

cum
e.” More

elation

1968

$$m_{H_5}^2 = 2m_q \frac{-\langle \bar{q}q \rangle}{f_{H_5}^0}$$

$$m_\pi^2 \propto m$$



Spontaneous(Dynamical) Chiral Symmetry Breaking

The **2008 Nobel Prize in Physics**
was divided, one half awarded to
Yoichiro Nambu

*"for the discovery of the mechanism
of spontaneous broken symmetry in
subatomic physics"*





Nambu - Jona-Lasinio Model

Dynamical Model of Elementary Particles

Based on an Analogy with Superconductivity. I

Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345–358

Dynamical Model Of Elementary Particles

Based On An Analogy With Superconductivity. II

Y. Nambu, G. Jona-Lasinio, Phys.Rev. 124 (1961) 246-254

➤ Treats a chirally-invariant four-fermion Lagrangian & solves the gap equation in Hartree-Fock approximation (analogous to rainbow truncation)

➤ Poss The following Lagrangian density will be assumed:

➤ *Esse* ($\hbar = c = 1$):

$$L = -\bar{\psi}\gamma_{\mu}\partial_{\mu}\psi + g_0[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]. \quad (2.6)$$

The coupling parameter g_0 is positive, and has dimensions $[\text{mass}]^{-2}$. The γ_5 invariance property of the interaction is evident from Eq. (2.5). According to the Nambu vacuum, related by a chiral rotation

which form another orthogonal set. Since the original total H commutes with X , it will have no matrix elements connecting different “worlds.” Moreover, as

➤ **Nontrivial Vacuum is “Born”**

Craig Roberts: From Partons to Reality



Gell-Mann - Oakes - Renner Relation

Behavior of current divergences under $SU(3) \times SU(3)$.

Murray Gell-Mann, R.J. Oakes, B. Renner

Phys.Rev. 175 (1968) 2195-2199

- This paper derives a relation between m_π^2 and the expectation-value $\langle \pi | u_0 | \pi \rangle$, where u_0 is an operator that is linear in the putative Hamiltonian's explicit chiral-symmetry breaking term
 - NB. QCD's current-quarks were not yet invented, so u_0 was not expressed in terms of current-quark fields
- PCAC-hypothesis (partial conservation of axial current) is used in the derivation
- Subsequently, the concepts of soft-pion theory
 - Operator expectation values do not change as $t=m_\pi^2 \rightarrow t=0$ to take $\langle \pi | u_0 | \pi \rangle \rightarrow \langle 0 | u_0 | 0 \rangle \dots$ *in-pion* \rightarrow *in-vacuum*



Gell-Mann - Oakes - Renner Relation

$$m_{\pi}^2 \propto m \langle 0 | \bar{q}q | 0 \rangle - (0.25\text{GeV})^3$$

- Theoretical physics at its best.
- But no one is thinking about how properly to consider or define what will come to be called the *vacuum quark condensate*

- So long as the condensate is just a mass-dimensional constant, which approximates value of another well-defined matrix element, there is no problem.
- Problem arises if one over-interprets this number, which textbooks have been doing for a **VERY LONG TIME.**





Note of Warning

Chiral Magnetism (or Magnetohydrochironics)
A. Casher and L. Susskind, Phys. Rev. D9 (1974) 436

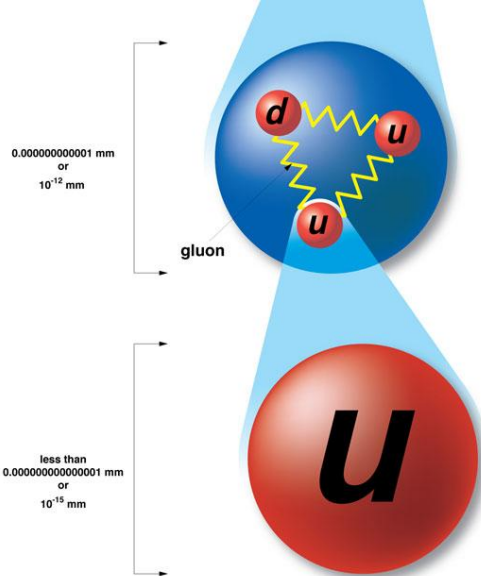
The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.¹ Because of an instability of the chirally invariant vacuum, the real vacuum is “aligned” into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.² A number of investigations

➤ *These authors argue that dynamical chiral-symmetry breaking can be realised as a property of hadrons, instead of via a nontrivial vacuum exterior to the measurable degrees of freedom*

The essential ingredient required for a spontaneous symmetry breakdown in a composite system is the existence of a divergent number of constituents

– DIS provided evidence for divergent sea of low-momentum partons – parton model.



Quarks and
Gluons in
Proton

QCD

$$\langle 0 | \bar{q}q | 0 \rangle$$

1973-1974

Quark

➤ How should one approach this problem to understand it, within Quantum Chromodynamics?

- 1) Are the quark and gluon "asymptotic states" theoretically well-defined?
- 2) Is there a physical meaning to this quantity or is it merely just a dimensioned parameter in a theoretical procedure?

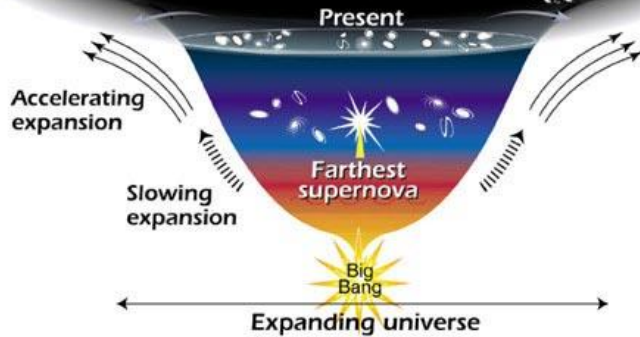
Why does it matter?

“Dark Energy”

“The advent of quantum field theory made consideration of the cosmological constant obligatory not optional.”

Michael Turner, “Dark Energy and the New Cosmology”

Time
(~15 billion years)



- The only possible covariant form for the energy of the (quantum) vacuum; viz.,

$$T_{\text{VAC}}^{\mu\nu} = \rho_{\text{VAC}} g^{\mu\nu}$$

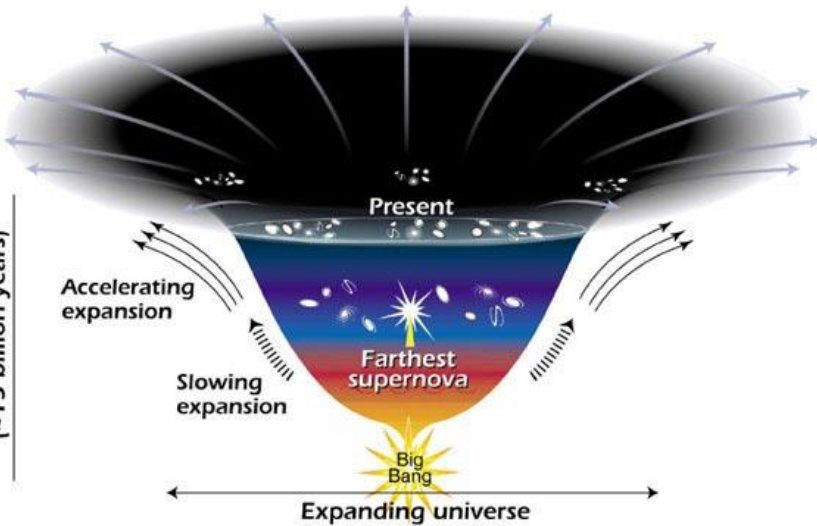
is mathematically equivalent to the cosmological constant.

“It is a perfect fluid and precisely spatially uniform”

“Vacuum energy is almost the perfect candidate for dark energy.”

“Dark Energy”

Time
(~15 billion years)



$$T_{\text{VAC}}^{\mu\nu} = \rho_{\text{VAC}} g^{\mu\nu}$$

Enormous and even greater contribution from Higgs VEV!

➤ QCD vacuum contribution

- If chiral symmetry breaking is expressed in a nonzero expectation value of the quark bilinear, then the energy difference between the symmetric and broken phases is of order

$$M_{\text{QCD}} \approx 0.3 \text{ GeV}$$

Mass-scale generated by spacetime-independent condensate

- One obtains therefrom:

$$\rho_{\Lambda}^{\text{QCD}} = 10^{46} \rho_{\Lambda}^{\text{obs}}$$

“The biggest embarrassment in theoretical physics.”



Resolution?

Relevant References

- [arXiv:1202.2376](#)
Confinement contains condensates
Stanley J. Brodsky, Craig D. Roberts, Robert Shrock, Peter C. Tandy
- [arXiv:1109.2903 \[nucl-th\]](#), [Phys. Rev. C85 \(2012\) 012201\(RapCom\)](#),
Expanding the concept of in-hadron condensates
Lei Chang, Craig D. Roberts and Peter C. Tandy
- [arXiv:1005.4610 \[nucl-th\]](#), [Phys. Rev. C82 \(2010\) 022201\(RapCom.\)](#)
New perspectives on the quark condensate,
Brodsky, Roberts, Shrock, Tandy
- [arXiv:0905.1151 \[hep-th\]](#), [PNAS 108, 45 \(2011\)](#)
Condensates in Quantum Chromodynamics and the Cosmological Constant , Brodsky and Shrock,
- [hep-th/0012253](#)
The Quantum vacuum and the cosmological constant problem,
Svend Erik Rugh and Henrik Zinkernagel.



QCD

1973-1974

$$\langle 0 | \bar{q}q | 0 \rangle$$

Are the condensates real?

*Or ... just mass-dimensioned parameters in
a theoretical computation procedure?*

Dichotomy of the pion Mass Formula for 0^- Mesons

$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^\zeta \mathcal{M}_{H_5}^\zeta$$

- C *We now have sufficient information to address the question of just what is this so-called “vacuum quark condensate.”*
- H

cum
e.” More

relation

1968

$$m_{H_5}^2 = 2m_q \frac{-\langle \bar{q}q \rangle}{f_{H_5}^0}$$

$$m_\pi^2 \propto m$$

$$f_\pi m_\pi^2 = 2 m(\zeta) \rho_\pi^\zeta$$

In-meson condensate

Maris & Roberts

[nucl-th/9708029](https://arxiv.org/abs/nuc-th/9708029)

- Pseudoscalar projection of pion's Bethe-Salpeter wavefunction onto the origin in configuration space: $|\Psi_\pi^{PS}(0)|$
– or the pseudoscalar pion-to-vacuum matrix element

$$\begin{aligned} i\rho_\pi &= -\langle 0 | \bar{q} i \gamma_5 q | \pi \rangle \\ &= Z_4(\zeta, \Lambda) \text{tr}_{\text{CD}} \int \frac{d^4 q}{(2\pi)^4} \gamma_5 S(q_+) \Gamma_\pi(q; P) S(q_-) \end{aligned}$$

- Rigorously defined in QCD – gauge-independent, cutoff-independent, etc.
 - For arbitrary current-quark masses
 - For any pseudoscalar meson

$$f_\pi m_\pi^2 = 2 m(\zeta) \rho_\pi^\zeta$$

In-meson condensate

Maris & Roberts

[nucl-th/9708029](https://arxiv.org/abs/nucl-th/9708029)

- Pseudovector projection of pion's Bethe-Salpeter wave-function onto the origin in configuration space: $|\Psi_\pi^{AV}(0)|$
 - or the pseudoscalar pion-to-vacuum matrix element
 - or the pion's leptonic decay constant

$$\begin{aligned} i f_\pi P_\mu &= \langle 0 | \bar{q} \gamma_5 \gamma_\mu q | \pi \rangle \\ &= Z_2(\zeta, \Lambda) \text{tr}_{\text{CD}} \int \frac{d^4 q}{(2\pi)^4} i \gamma_5 \gamma_\mu S(q_+) \Gamma_\pi(q; P) S(q_-) \end{aligned}$$

- Rigorously defined in QCD – gauge-independent, cutoff-independent, etc.
 - For arbitrary current-quark masses
 - For any pseudoscalar meson

$$f_\pi m_\pi^2 = 2 m(\zeta) \rho_\pi^\zeta$$

In-meson condensate

Maris & Roberts

[nucl-th/9708029](https://arxiv.org/abs/nuc-th/9708029)

➤ Define $-\langle \bar{q}q \rangle_\zeta^\pi \equiv -f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m}; \zeta)$.

➤ Then, using the pion Goldberger-Treiman relations (equivalence of 1- and 2-body problems), one derives, in the chiral limit

Chiral limit

$$\kappa_\pi(0; \zeta) = - \langle \bar{q}q \rangle_\zeta^0 \quad | \Psi_\pi^{PS}(0) | * | \Psi_\pi^{AV}(0) |$$

➤ Namely, the so-called *vacuum quark condensate* is the *chiral-limit value* of the *in-pion condensate*

➤ The *in-pion condensate* is the *only well-defined function of current-quark mass* in QCD that is *smoothly connected to the vacuum quark condensate*.

$$f_\pi m_\pi^2 = 2 m(\zeta) \rho_\pi^\zeta$$

There is *only one* condensate

Langeld, Roberts *et al.*
[nucl-th/0301024](https://arxiv.org/abs/nucl-th/0301024),
 Phys.Rev. C67 (2003) 065206

I. Casher Banks formula:

Density of eigenvalues
of Dirac operator

$$-\langle 0 | \bar{q}q | 0 \rangle = 2m \int_0^\infty d\lambda \frac{\rho(\lambda)}{\lambda^2 + m^2}$$

II. Constant in the Operator Product Expansion:

$$M(p^2) \stackrel{\text{large } -p^2}{=} \frac{2\pi^2 \gamma_m}{3} \frac{(-\langle \bar{q}q \rangle^0)}{p^2 \left(\frac{1}{2} \ln \left[\frac{p^2}{\Lambda_{\text{QCD}}^2} \right] \right)^{1-\gamma_m}}$$

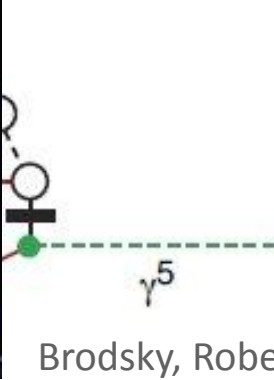
III. Trace of the dressed-quark propagator:

$$\tilde{\sigma}(m) := N_c \text{tr}_D \int_p^\Lambda \tilde{S}_m(p)$$

$m \rightarrow 0$

*Algebraic proof
that these are
all the same.
So, no matter
how one
chooses to
calculate it,
one is always
calculating the
same thing; viz.,*

$$|\Psi_\pi^{PS}(0)|^* |\Psi_\pi^{AV}(0)|$$



Paradigm shift: In-Hadron Condensates

Brodsky, Roberts, Shrock, Tandy, [Phys. Rev. C82 \(Rapid Comm.\) \(2010\) 022201](#)

Brodsky and Shrock, [PNAS 108, 45 \(2011\)](#)

➤ Resolution

- Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, “condensates” do not exist as spacetime-independent mass-scales that fill all spacetime.
- *So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

- **GMOR**
cf.

QCD

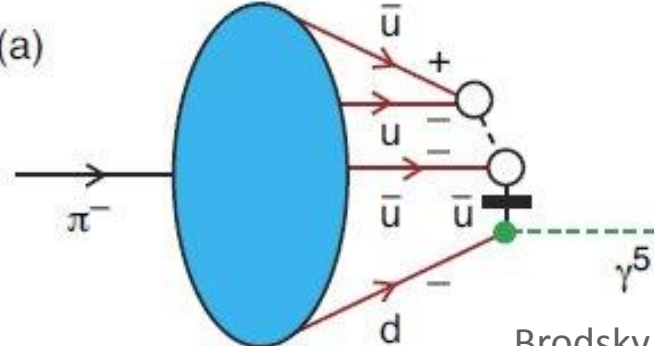
$$f_{\pi}^2 m_{\pi}^2 = -2 m(\zeta) \langle \bar{q}q \rangle_0^{\zeta}$$

$$f_{\pi} m_{\pi}^2 = 2 m(\zeta) \rho_{\pi}^{\zeta}$$

The diagram shows two equations. In the first, the '2' in f_{π}^2 and the $\langle \bar{q}q \rangle_0^{\zeta}$ term are circled in red. A red dashed arrow points from the circled '2' to the circled $\langle \bar{q}q \rangle_0^{\zeta}$ term. In the second equation, the ρ_{π}^{ζ} term is circled in red. A red dashed arrow points from the circled ρ_{π}^{ζ} term back to the circled $\langle \bar{q}q \rangle_0^{\zeta}$ term in the first equation.

Paradigm shift:

In-Hadron Condensates



Brodsky, Roberts, Shrock, Tandy, [Phys. Rev. C82 \(Rapid Comm.\) \(2010\) 022201](#)

Brodsky and Shrock, [PNAS 108, 45 \(2011\)](#)

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– *So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

– No qualitative difference between f_π and ρ_π

$$if_\pi P_\mu = \langle 0 | \bar{q} \gamma_5 \gamma_\mu q | \pi \rangle$$

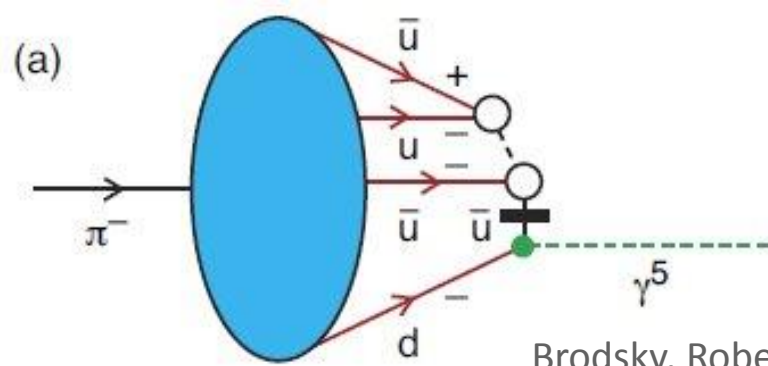
$$= Z_2(\zeta, \Lambda) \text{tr}_{\text{CD}} \int \frac{d^4 q}{(2\pi)^4} i \gamma_5 \gamma_\mu S(q_+) \Gamma_\pi(q; P) S(q_-), \quad (5)$$

$$i\rho_\pi = -\langle 0 | \bar{q} i \gamma_5 q | \pi \rangle$$

$$= Z_4(\zeta, \Lambda) \text{tr}_{\text{CD}} \int \frac{d^4 q}{(2\pi)^4} \gamma_5 S(q_+) \Gamma_\pi(q; P) S(q_-). \quad (6)$$

Paradigm shift:

In-Hadron Condensates



Brodsky, Roberts, Shrock, Tandy, [Phys. Rev. C82 \(Rapid Comm.\) \(2010\) 022201](#)

Brodsky and Shrock, [PNAS 108, 45 \(2011\)](#)

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- *So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.
- No qualitative difference between f_π and ρ_π
- And

Chiral limit

$$\kappa_\pi(0; \zeta) = - \langle \bar{q} q \rangle_\zeta^0$$

$$- \langle \bar{q} q \rangle_\zeta^\pi \equiv - f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m}; \zeta).$$

Behavior of Current Divergences under $SU_3 \times SU_3^{*†}$

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GMOR Relation

GMOR Relation

- Valuable to highlight the precise form of the Gell-Mann–Oakes–Renner (GMOR) relation: Eq. (3.4) in [Phys.Rev. 175 \(1968\) 2195](https://doi.org/10.1103/PhysRev.175.2195)

$$m_{\pi}^2 = \lim_{P' \rightarrow P \rightarrow 0} \langle \pi(P') | \mathcal{H}_{\chi sb} | \pi(P) \rangle$$

- m_{π} is the pion's mass
- $H_{\chi sb}$ is that part of the hadronic Hamiltonian density which explicitly breaks chiral symmetry.
- Crucial to observe that the operator expectation value in this equation is evaluated between pion states.
- Moreover, the virtual low-energy limit expressed in the equation is purely formal. It does not describe an achievable empirical situation.

GMOR Relation

- In terms of QCD quantities, GMOR relation entails

$$\forall m_{ud} \sim 0, m_{\pi^\pm}^2 = m_{ud}^\zeta \mathcal{S}_\pi^\zeta(0),$$

$$\mathcal{S}_\pi^\zeta(0) = -\langle \pi(P) | \frac{1}{2}(\bar{u}u + \bar{d}d) | \pi(P) \rangle$$

- $m_{ud}^\zeta = m_u^\zeta + m_d^\zeta \dots$ the current-quark masses
- $\mathcal{S}_\pi^\zeta(0)$ is the pion's scalar form factor at zero momentum transfer, $Q^2=0$
- RHS is proportional to the pion σ -term
- Consequently, using the connection between the σ -term and the Feynman-Hellmann theorem, GMOR relation is actually the statement

$$\forall m_{ud} \sim 0, m_\pi^2 = m_{ud}^\zeta \frac{\partial}{\partial m_{ud}^\zeta} m_\pi^2$$

GMOR Relation

➤ Using
$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^\zeta \mathcal{M}_{H_5}^\zeta$$

Maris, Roberts and Tandy
[nucl-th/9707003](https://arxiv.org/abs/nucl-th/9707003),
 Phys.Lett. B420 (1998) 267-273

it follows that

$$\mathcal{S}_\pi^\zeta(0) = \frac{\partial}{\partial m_{ud}^\zeta} m_\pi^2 = \frac{\partial}{\partial m_{ud}^\zeta} \left[m_{ud}^\zeta \frac{\rho_\pi^\zeta}{f_\pi} \right]$$

➤ This equation is valid for any values of $m_{u,d}$ including the neighbourhood of the chiral limit, wherein

$$\frac{\partial}{\partial m_{ud}^\zeta} \left[m_{ud}^\zeta \frac{\rho_\pi^\zeta}{f_\pi} \right]_{m_{ud}=0} = \frac{\rho_\pi^{\zeta 0}}{f_\pi^0}$$

GMOR Relation

- Consequently, in the neighbourhood of the chiral limit

$$m_{\pi^{\pm}}^2 = -m_{ud}^{\zeta} \frac{\langle \bar{q}q \rangle^{\zeta_0}}{(f_{\pi}^0)^2} + \mathcal{O}(m_{ud}^2)$$

- This is a QCD derivation of the commonly recognised form of the GMOR relation.
- Neither PCAC nor soft-pion theorems were employed in the analysis.
- Nature of each factor in the expression is abundantly clear; viz., **chiral limit values of matrix elements that explicitly involve the hadron.**

PHYSICAL REVIEW C 85, 012201(R) (2012)

Expanding the concept of in-hadron condensates

Lei Chang,¹ Craig D. Roberts,^{1,2,3} and Peter C. Tandy⁴¹*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*²*Department of Physics, Center for High Energy Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*³*Department of Physics, Illinois Institute of Technology, Chicago, Illinois 60616-3793, USA*⁴*Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

(Received 13 September 2011; published 23 January 2012)

The in-pseudoscalar-meson condensate can be represented through the pseudoscalar meson's scalar form factor at zero-momentum transfer. With the aid of a mass formula for scalar mesons, revealed herein, the analog is shown to be true for in-scalar-meson condensates. The concept is readily extended to all hadrons so that, via the zero-momentum-transfer value of any hadron's scalar form factor, one can readily extract the value for a quark condensate in that hadron which is a measure of dynamical chiral symmetry breaking.

DOI: [10.1103/PhysRevC.85.012201](https://doi.org/10.1103/PhysRevC.85.012201)

PACS number(s): 12.38.Aw, 11.30.Rd, 11.15.Tk, 24.85.+p

Expanding the Concept ...

In-Hadron Condensates

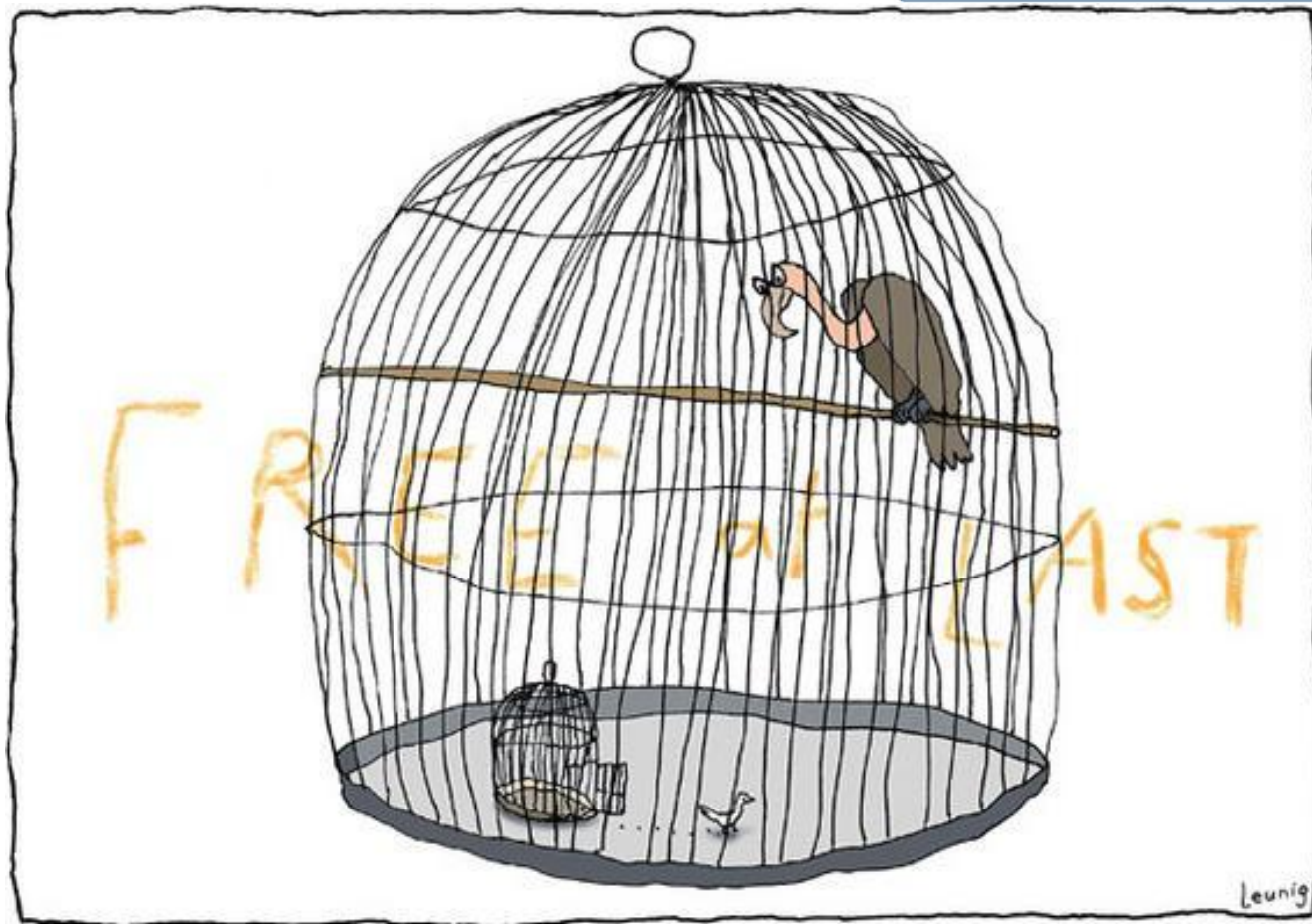
- Plainly, the in-pseudoscalar-meson condensate can be represented through the pseudoscalar meson's scalar form factor at zero momentum transfer $Q^2 = 0$.
- Using an *exact mass formula for scalar mesons*, one proves the in-scalar-meson condensate can be represented in precisely the same way.

$$f_{S_{qQ}} m_{S_{qQ}}^2 = -\check{m}_q Q \rho_{S_{qQ}}^\zeta$$
- By analogy, and with appeal to demonstrable results of heavy-quark symmetry, the $Q^2 = 0$ values of vector- and pseudovector-meson scalar form factors also determine the in-hadron condensates in these cases.
- This expression for the concept of in-hadron quark condensates is readily extended to the case of baryons.
- Via the $Q^2 = 0$ value of any hadron's scalar form factor, one can extract the value for a quark condensate in that hadron which is a reasonable and realistic measure of dynamical chiral symmetry breaking.

$$\langle H(p') | \bar{q} \mathcal{O} q | H(p) \rangle$$

Hadron Charges

- Hadron Form factor matrix elements
- *Scalar charge of a hadron is an intrinsic property of that hadron ... no more a property of the vacuum than the hadron's electric charge, axial charge, tensor charge, etc. ...*



Confinement

Confinement

- Confinement is essential to the validity of the notion of in-hadron condensates.
- Confinement makes it impossible to construct gluon or quark quasiparticle operators that are nonperturbatively valid.
- So, although one can define a perturbative (bare) vacuum for QCD, it is impossible to rigorously define a ground state for QCD upon a foundation of gluon and quark quasiparticle operators.
- Likewise, it is impossible to construct an interacting vacuum – a BCS-like trial state – and hence DCSB in QCD cannot rigorously be expressed via a spacetime-independent coherent state built upon the ground state of perturbative QCD.
- Whilst this does not prevent one from following this path to build practical models for use in hadron physics phenomenology, it does invalidate any claim that theoretical artifices in such models are empirical.



Paradigm shift: In-Hadron Condensates

“Void that is truly empty
solves dark energy puzzle”

Rachel Courtland, New Scientist 4th Sept. 2010

~~“EMPTY space may really be empty. Though quantum theory suggests that a vacuum should be fizzing with particle activity, it turns out that this paradoxical picture of nothingness may not be so bad. A calmer view of the vacuum would also help resolve a nagging inconsistency with dark energy, the elusive force thought to be speeding up the expansion of the universe.”~~

$\Omega_{QCD-condensates} = 8\omega \frac{\Lambda_{QCD}^4}{\Lambda_N^4} \approx 10^{46}$

$\frac{\Lambda_{QCD}^4}{3H_0^2}$

Cosmological Constant:

- ✓ **Putting QCD condensates back into hadrons reduces the mismatch between experiment and theory by a factor of 10^{46}**
- ✓ **Possibly by far more, if technicolour-like theories are the correct paradigm for extending the Standard Model**

Craig Roberts: From Partons to Reality



This is not the end