(De-)Confinement from QCD Green's functions

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JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HIC for FAIR
Helmholtz International Center

Content

1. Introduction
2. Gluon screening masses and analytic structure
   - $T = 0$
   - $T \neq 0$
3. Chiral and deconfinement transitions in QCD

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1 Introduction

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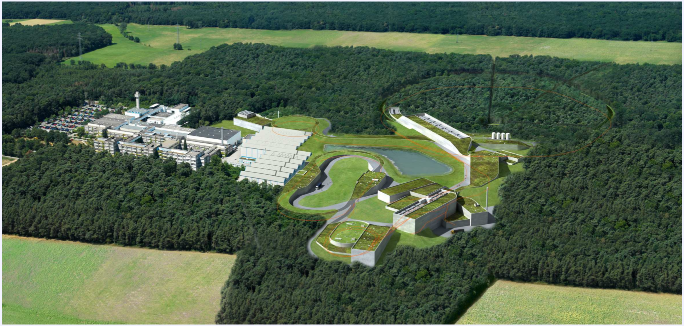
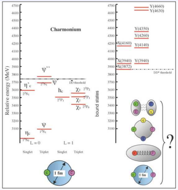
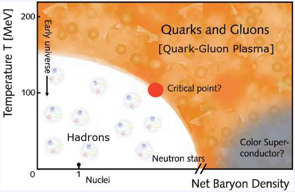
3 Chiral and deconfinement transitions in QCD

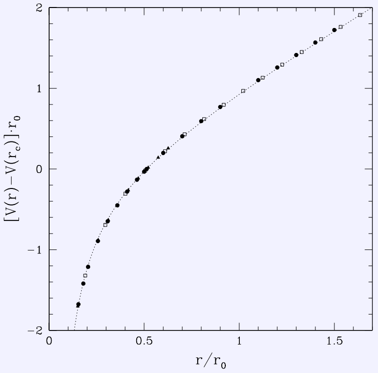
FAIR: PANDA and CBM







Confinement - string (breaking)



S. Necco and R. Sommer, Nucl. Phys. B **622** (2002) 328

$r_0 \approx 0.5fm$

Quarks:

- YM-theory: linear rising potential $V(r) \sim r$
- QCD: dynamical quarks in fundamental representation $\rightarrow$ string breaking $\rightarrow$ mesons

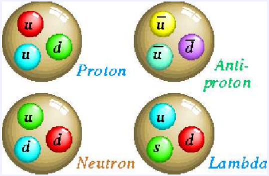
Gluons:

- Adjoint representation $\rightarrow$ string breaking $\rightarrow$ glueballs

Driving mechanism for linear potential ?

Greensite, Lect. Notes Phys. **821** (2011) 1.

State space of QCD:



- contains colorless physical states with positive norm (Hilbert space!)

- but also 'unphysical' states with indefinite norm

$$\Delta(t) := \int \frac{dp_4}{2\pi} e^{-itp_4}\, D(p^2)_{|\vec{p}^2=0}$$

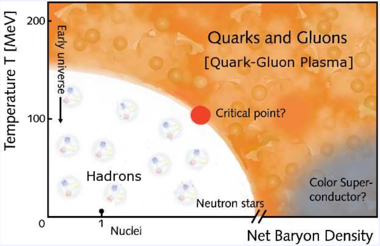
$\Delta(t) < 0 \longrightarrow$ unphysical state $\longrightarrow$ Color Confinement

Open problem: how to construct physical state space wo BRST...

QCD phase transitions I

Open questions:

- Existence and location of CEP
- Gluons and quarks in QGP



Phase transitions:

- Chiral limit ($M_{weak} \to 0$): order parameter chiral condensate

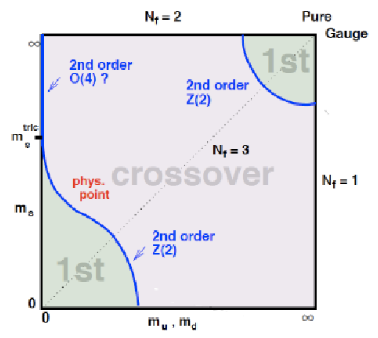
$$\langle \bar{\psi}\psi \rangle = Z_2 N_c Tr_D \int \frac{d^4p}{(2\pi)^4} S(p)$$

- Static quarks ($M_{weak} \to \infty$): order parameter Polyakov-loop

$$\Phi \sim e^{-F_q/T}$$

QCD phase transitions II

Quark mass dependence:



- Deconfinement transition at large masses
- Chiral transition at small masses

in this talk: pure gauge, $N_f = 2$ and $N_f = 2 + 1$

QCD in covariant gauge

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \exp\left\{ -\int_0^{1/T} dt \int d^3x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \text{gauge fixing} \right) \right\}$$

Landau gauge propagators in momentum space, $p = (\vec{p}, \omega_p)$:

$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{Z_T(p)}{p^2} P^T_{\mu\nu}(p) + \frac{Z_L(p)}{p^2} P^L_{\mu\nu}(p)$$

$$S^{\text{Quark}}(p) = \left[-i\,\vec{\gamma}\vec{p}\,A(p) - i\,\gamma_4\omega_n\,C(p) + B(p) \right]^{-1}$$

The Goal:

Gauge invariant information from gauge fixed functional approach

Lattice QCD vs. DSE/FRG: Complementary!

- Lattice simulations
  - Ab initio
  - Gauge invariant
- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  - Analytic solutions at small momenta
    CF, J. Pawlowski, PRD 80 (2009) 025023
  - Space-Time-Continuum
  - Chiral symmetry: light quarks and mesons
  - Multi-scale problems feasible: e.g. $(g\text{-}2)_\mu$
    T. Goecke, C.F., R. Williams, PLB 704 (2011); PRD 83 (2011)
  - Chemical potential: no sign problem

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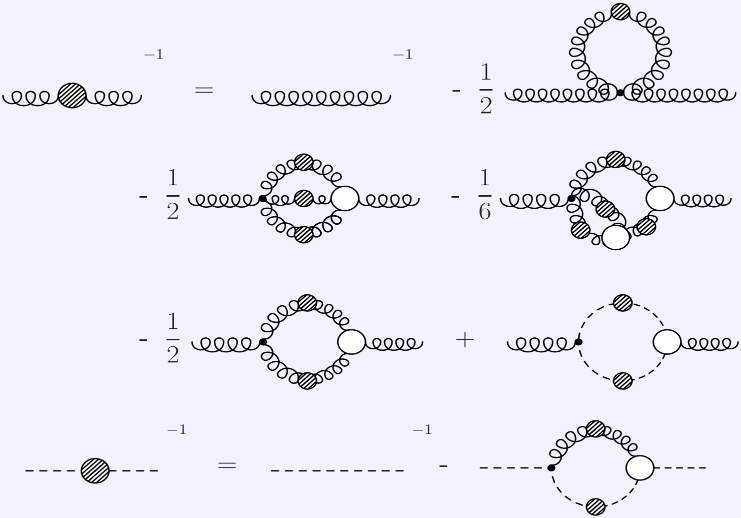
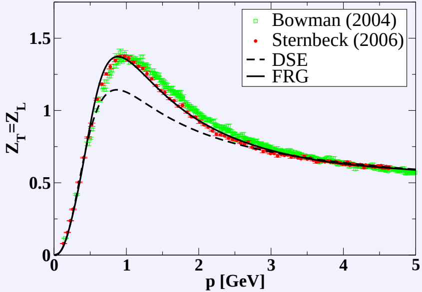
Dyson-Schwinger equations (DSEs)



DSEs vs Lattice ($T = 0$)



C.F., A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408-2437.

- Small momenta: $Z(p^2) \sim p^2$, i.e. gluon mass generation

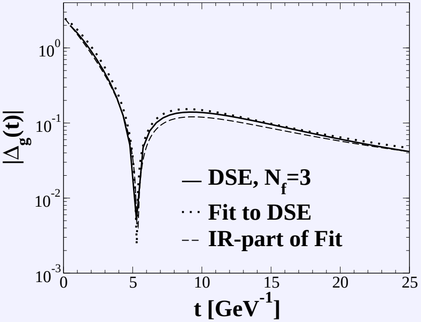
  Cornwall PRD **26** (1982) 1453; Cucchieri, Mendes, PoS **LAT2007** (2007) 297.
  Aguilar, Binosi, Papavassiliou, PRD **78**, 025010 (2008); Boucaud, et al. JHEP **0806** (2008) 099

- Deep infrared: subtle questions related to gauge fixing...

  Maas, PLB 689 (2010) 107; Sternbeck, Smekal, EPJC 68 (2010) 487

Gluon: positivity violation

$$\Delta_g(t) := \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_4+\vec{p}\vec{x})} \frac{Z(p^2)}{p^2}$$



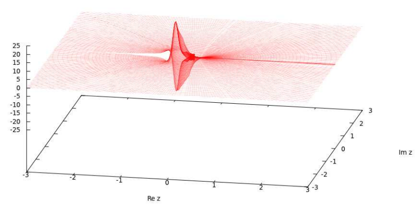
- Violation of positivity $\Rightarrow$ **no physical asymptotic gluons**
- Cut on the timelike momentum axis ?

R. Alkofer, W. Detmold, C. F., P. Maris, Phys. Rev. D **70** (2004) 014014

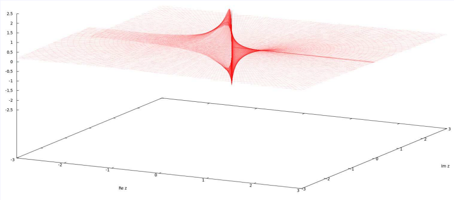
C.F., A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408-2437.

Gluon: analytic structure

$\Im[D(p^2)]$



$\Im[G(p^2)]$

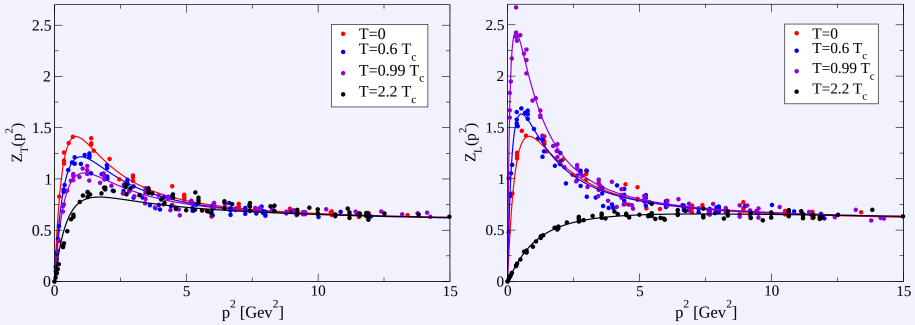


- Ghost and gluon DSEs solved in the complex p^2-plane
- No non-analytic structure outside real axis
- Cut/singularity for timelike real momenta $p^2 < 0$

CF, Kellermann, Strauss, in preparation

Glue at finite temperature $T \neq 0$

T-dependent gluon propagator from lattice simulations:



- Difference between electric and magnetic gluon
- Maximum of electric gluon around T_c

Cucchieri, Maas, Mendes, PRD 75 (2007)

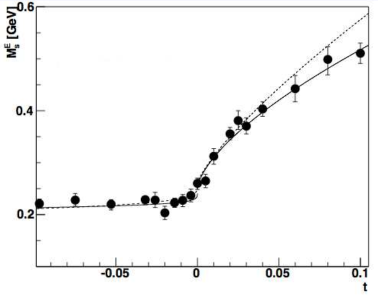
C.F., Maas and Mueller, EPJC 68 (2010)

Cucchieri, Mendes, PoS FACESQCD (2010) 007.

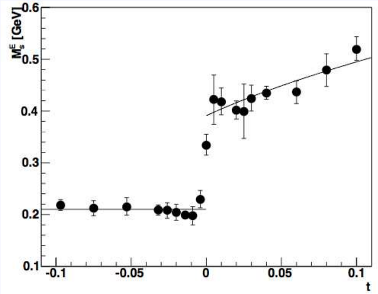
Aouane, Bornyakov, Ilgenfritz, Mitrjushkin, Muller-Preussker, Sternbeck, [arXiv:1108.1735 [hep-lat]].

Gluon screening mass at T_c: SU(2) vs. SU(3)

SU(2)



SU(3)



$t = (T - T_c)/T_c$

Maas, Pawlowski, Smekal, Spielmann, arXiv:1110.6340.

C.F., Maas and Mueller, EPJC 68 (2010)

- phase transition of second and first order clearly visible in electric screening mass

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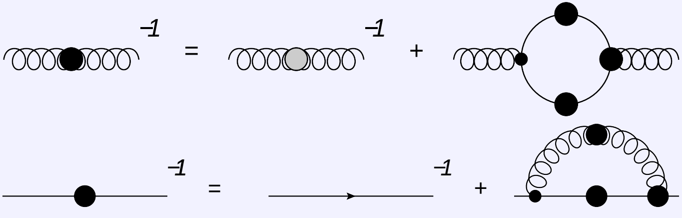
The ordinary chiral condensate



- quenched lattice gluon propagator $+$ DSE-quark-loop
- $T = 0$: quark-gluon vertex studied via DSEs

  Alkofer, C.F., Llanes-Estrada, Schwenzer, Annals Phys.324:106-172,2009.

  C.F, R. Williams, PRL **103** (2009) 122001

  $T \neq 0$: ansatz, T, μ and mass dependent (STI)
- Order parameter for chiral symmetry breaking:

$$\langle\bar{\psi}\psi\rangle = Z_2 N_c T \sum_{n_p} \int \frac{d^3p}{(2\pi)^3} \, Tr_D \, S(\vec{p}, \omega_p)$$

The dual condensate/dressed Polykaov loop

Then define dual condensate Σ_n:

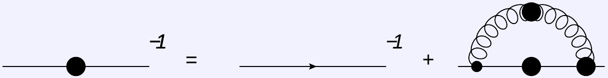
$$\Sigma_n = -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi n}\, \langle\overline{\psi}\psi\rangle_\varphi$$

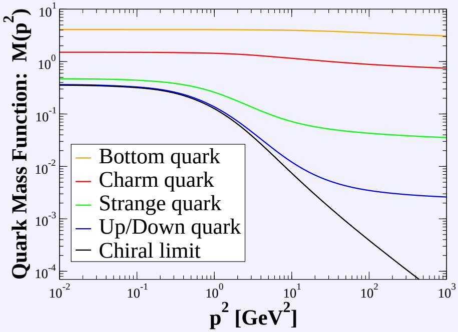
- $n = 1$ projects out loops with $n(l) = 1$: dressed Polyakov loop
- transforms under center transformation exactly like ordinary Polyakov loop: order parameter for center symmetry breaking
- Σ_1 is accessible with functional methods

  C.F., PRL **103** (2009) 052003

C. Gattringer, PRL **97**, 032003 (2006)
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** 094007 (2008).
F. Synatschke, A. Wipf and K. Langfeld, PRD **77**, 114018 (2008).
J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, PRL 106 (2011)

$T = 0$: Explicit vs. dynamical chiral symmetry breaking

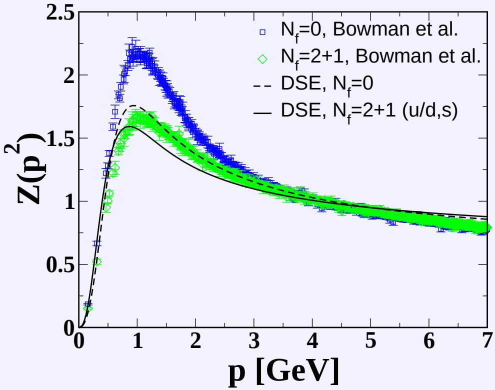




C.F. J.Phys.G G32 (2006) R253-R291

- $M(p^2) = B(p^2)/A(p^2)$: momentum dependent!
- Dynamical masses $M_{strong}(0) \approx 350$ MeV
- Flavour dependence because of M_{weak}
- $\langle\bar{\psi}\psi\rangle \approx (250\text{MeV})^3$

$T = 0$: Unquenched gluon propagator



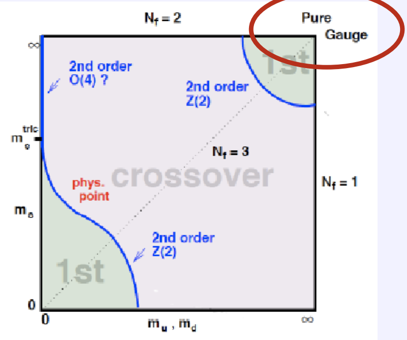
- Quark effects are screening

C.F. and Alkofer, Phys. Rev. D **67** (2003) 094020

P. O. Bowman *et al.* (CSSM), Phys. Rev. D **70** (2004) 034509.

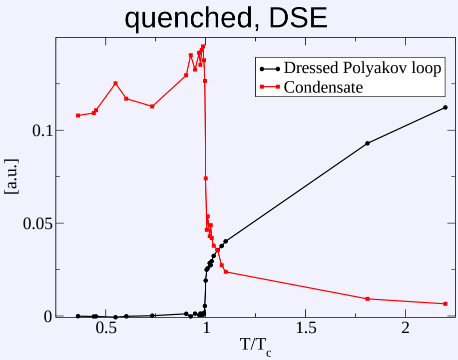
QCD phase transitions: quenched

Quark mass dependence:



- Expect: Transitions controlled by deconfinement
- SU(2) second order, SU(3) first order

Transition temperatures, quenched



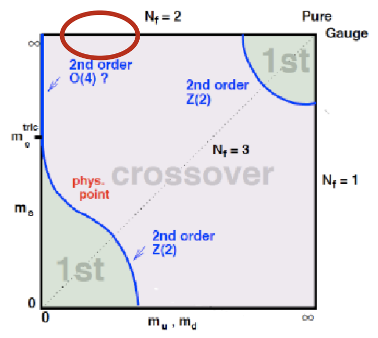
Luecker, C.F., arXiv:1111.0180; C.F., Maas, Mueller, EPJC 68 (2010).

- SU(2): $T_c \approx 305$ MeV
  SU(3): $T_c \approx 270$ MeV
- $T \leq T_c$: increasing condensate due to electric part of gluon

cf. Buividovich, Luschevskaya, Polikarpov, PRD 78 (2008) 074505.
cf. Braun, Gies, Pawlowski, PLB 684 (2010) 262-267.

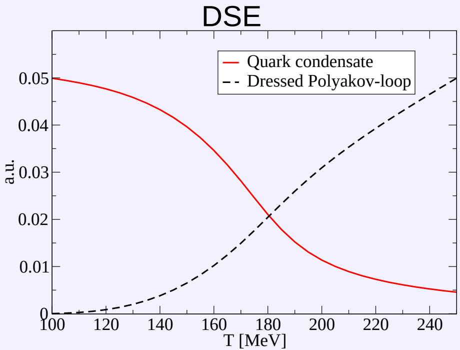
QCD phase transitions: Nf=2

Quark mass dependence:



- $N_f = 2$, physical up/down quark masses
- Transition controlled by chiral dynamics

$N_f = 2$: Transition temperatures at $\mu = 0$

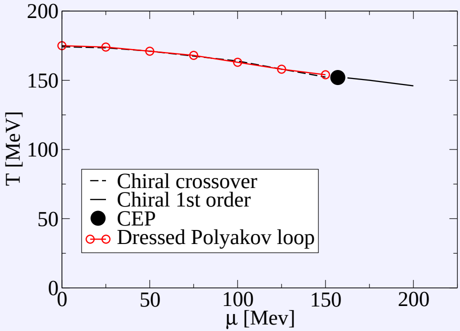


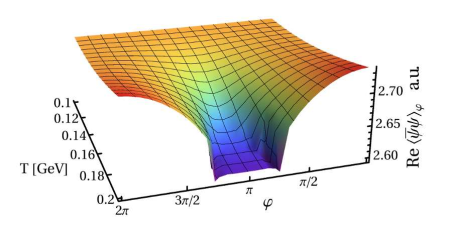
C.F., J. Luecker, J. A. Mueller, PLB 702 (2011) 438-441.

- $T_\chi \approx 185$ MeV
- $T_{conf} \approx 195$ MeV
- condensate in qualitative agreement with χPT

cf. J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, PRL 106 (2011) 022002

$N_f = 2$: QCD phase diagram





C.F., J. Luecker, J. A. Mueller, PLB 702 (2011) 438-441.

C.F., J. Luecker, in preparation

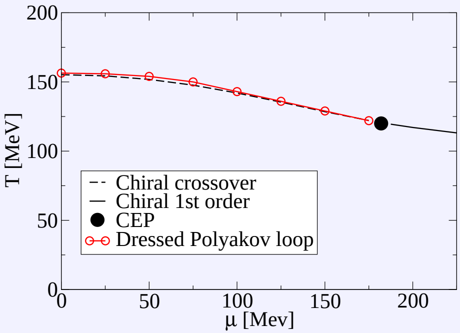
- chiral CEP
- crucial: backreaction of quark onto gluon
- qualitative agreement with RG-improved PQM model

  Herbst, Pawlowski, Schaefer, PLB **696** (2011)
- no CEP at $\mu_c/T_c < 1$ in agreement with lattice

  de Forcrand, Philipsen, JHEP **0811** (2008) 012; Nucl. Phys. **B642** (2002) 290-306.

  Endrodi, Fodor, Katz, Szabo, JHEP **1104** (2011) 001.

$N_f = 2 + 1$: QCD phase diagram (preliminary)



CF., J. Luecker, in preparation

- CEP: $\mu_c/T_c \simeq 1.1 \;\; (N_f = 2) \quad \longrightarrow \quad \mu_c/T_c \simeq 1.3 \;\; (N_f = 2+1)$
- no quarkyonic region
- But: need to include

Conclusions

Summary:

- Gluon propagator:
  - color confinement via positivity violation
  - non-analytic structures for real time-like momenta only
  - characteristic behavior of electric screening mass at T_c
- QCD phase diagram: chiral and deconfinement transitions
  - backreaction of quarks onto gluon crucial
  - $N_f = 2 + 1$: CEP at $\mu_c^q / T_c > 1$

Other topics:

- Meson structure (pion cloud, form factors etc)
  CF and R. Williams, PRL **103** 122001 (2009).
- Baryon structure (3-body problem, form factors etc.)
  G. Eichmann and CF, PRD in press, [arXiv:1111.0197 [hep-ph]].
- Hadronic contributions to $(g - 2)_\mu$
  T. Goecke, CF, R. Williams. PLB 704 (2011); PRD 83 (2011)

Thank you for your attention!

Helmholtz Young Investigator Group ”Nonperturbative Phenomena in QCD”

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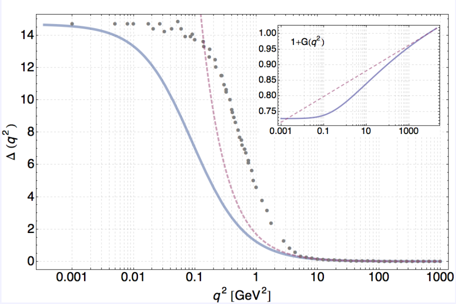
Gluon and Quark-Gluon-Vertex

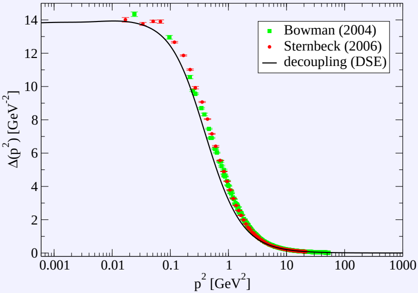
Ansatz for Quark-Gluon-Vertex:

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3 \left(\delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times$$
$$\times \left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right) .$$

Gluon propagator ($T = 0$)



Aguilar, Binosi, Papavassiliou, PRD **78**, 025010 (2008).



C.F., Maas and Pawlowski, Annals Phys. **324** (2009) 2408.

Cornwall, PRD **26** (1982) 1453.

$$\Delta(p^2) = \frac{Z(p^2)}{p^2}$$

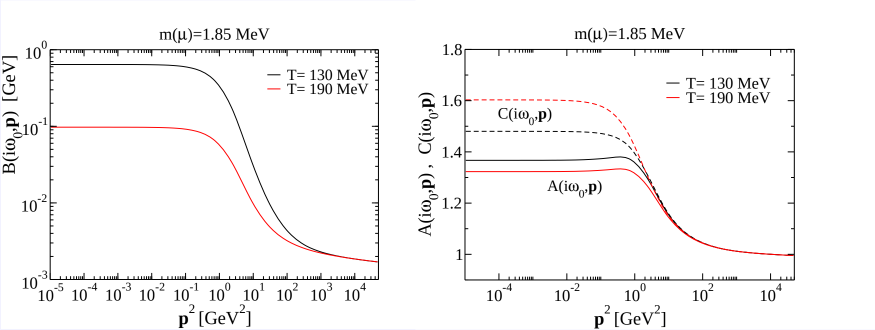
- Gluon mass generation in agreement with lattice results

  Cucchieri, Mendes, PoS **LAT2007** (2007) 297.

- see however
  Sternbeck, L. von Smekal, Eur. Phys. J. C68 (2010) 487;
  Cucchieri, Mendes, Phys. Rev. **D81** (2010) 016005.
  Maas, Phys. Lett. **B689** (2010) 107-111.

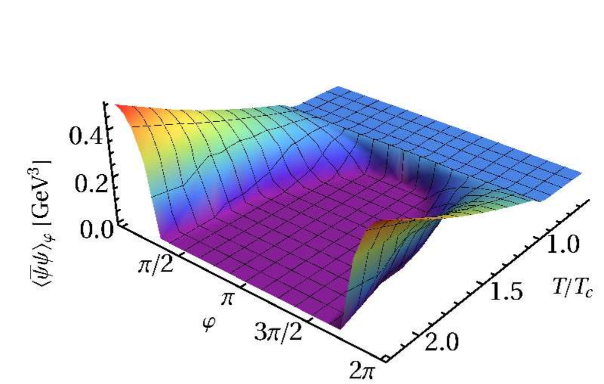
$T \neq 0$: Chiral symmetry restauration

$$S^{\text{Quark}}(q) = \frac{1}{-i\,\vec{\gamma}\vec{q}\,A(q) - i\,\gamma_4\omega_n\,C(q) + B(q)}$$



- dynamical effects below T_c $\leftrightarrow$ 'HTL-ish' above T_c

Condensate: angular dependence in chiral limit



$$\Sigma_1 \quad = \quad -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \langle\overline{\psi}\psi\rangle_\varphi$$

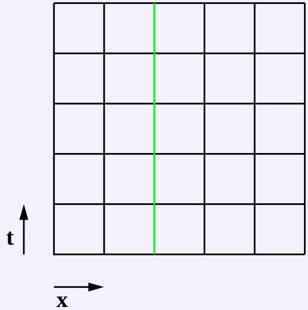
- Width of plateau is T-dependent, $\langle\overline{\psi}\psi\rangle_\varphi(\varphi = 0) \sim T^2$

C.F. and Jens Mueller, PRD **80** (2009) 074029.

The Polyakov Loop

$$\Phi = \left\langle \frac{1}{N_c} Tr_D \mathcal{P} \exp \left\{ i \int_0^{1/T} A_4 \, dt \right\} \right\rangle \sim e^{-F_q/T}$$

Lattice:



Order parameter for center symmetry breaking:

$\Phi = 0$: confined

$\Phi \neq 0$: deconfined

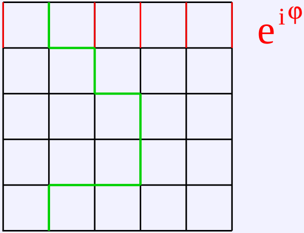
The dual condensate I

Consider general $U(1)$-valued boundary conditions in temporal direction for quark fields ψ:

$$\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0)$$

Matsubara frequencies: $\omega_p(n_t) = (2\pi T)(n_t + \varphi/2\pi)$

Lattice:



$$\langle\overline{\psi}\psi\rangle_\varphi \sim \sum \frac{exp[i\,\varphi\, n]}{(am)^l} \text{ Closed Loops}$$

E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).