Gluonic Excitations - a view from lattice QCD

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for the **Hadron Spectrum Collaboration**

"Hybrid baryons from QCD" - arXiv:1201.2349 (PRD in press)
"Helicity operators for mesons in flight on the lattice" - PRD.85.014507 (2012)
"The lightest hybrid meson supermultiplet in QCD" - PRD.84.074023 (2011)
"Excited state baryon spectroscopy from lattice QCD" - PRD.84.074508 (2011)
"Isoscalar meson spectroscopy from lattice QCD" - PRD.83.071504 (2011)
"The phase-shift of isospin-2 ππ scattering from lattice QCD" - PRD.83.071504 (2011)
"Toward the excited meson spectrum of dynamical QCD" - PRD.82.034508 (2010)
"Highly excited and exotic meson spectrum from dynamical lattice QCD" - PRL.103.262001 (2009)
"A novel quark-field creation operator construction for hadronic physics in lattice QCD" - PRD.80.054506 (2009)

hybrid mesons

observed meson state flavor & J^{PC} systematics suggest $q\bar{q}$

"constituent quarks"

$$q\bar{q}[S,L] \to (J = L \otimes S)^{P=(-1)^{L+1}, C=(-1)^{L+S}}$$

$$\begin{array}{cccc} \cdot & 0^{-+} & 0^{++} & \cdot \\ 1^{--} & \cdot & 1^{++} & 1^{+-} \\ 2^{--} & 2^{-+} & 2^{++} & \cdot \\ & & \vdots & \end{array}$$

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exotic quantum numbers

$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$$

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.	0^{-+}	0^{++}	.
1^{--}	.	1^{++}	1^{+-}
2^{--}	2^{-+}	2^{++}	.
	$\vdots$		

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but what if excited gluonic fields play a role - a *hybrid* meson, $q\bar{q}G$?

possibly exotic J^{PC} & extra 'non-exotic' states

must be 'heavier' or 'harder to produce' ?

hybrid mesons - models

with minimal quark content, $q\bar{q}G$, gluonic field could be color singlet or octet

- 'constituent' gluon $G \sim 1^{--}_{\mathbf{8}}$

$q\bar{q}_{L=0}$ $(0,1,2)^{++},\ 1^{+-}$

$q\bar{q}_{L=1}$ $0^{--},(1^{-+})^3,3^{-+}\ldots$

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- bag model / 'constituent' gluon in *P*-wave

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- flux-tube model

  $(0,1,2)^{-+},\ 1^{--},\ (0,1,2)^{+-},\ 1^{++}$

hadron spectrum

spectrum extraction in lattice QCD

write down 'any old' set of interpolating fields with the right quantum numbers

$\mathcal{O}_j$

hadron spectrum

spectrum extraction in lattice QCD

write down 'any old' set of interpolating fields with the right quantum numbers

$$\mathcal{O}_j$$

form the matrix of correlation functions

$$C_{ij}(t) = \langle 0 \big| \mathcal{O}_i(t) \mathcal{O}_j(0) \big| 0 \rangle$$

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'diagonalise' this - find linear combinations of operators optimal for creation of each state

$$C(t) v^{(\mathfrak{n})} = \lambda_\mathfrak{n}(t, t_0) C(t_0) v^{(\mathfrak{n})}$$

$$\Omega_\mathfrak{n} = \sum_j v_j^{(\mathfrak{n})} \mathcal{O}_j$$

$$\Omega_\mathfrak{n} \big| 0 \big\rangle \approx \big| \mathfrak{n} \big\rangle$$

hadron spectrum

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form the matrix of correlation functions $C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle$

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any random 'rotation' of the operators won't affect the spectrum

don't need to use a basis which is close to diagonal

hadron spectrum

what did we actually use ...

large basis of operators of fermion bilinear type

$\bar{\psi}\Gamma\psi$ $\rightsquigarrow 0,1$

$\bar{\psi}\Gamma\overleftrightarrow{D}\psi$ $\rightsquigarrow 0,1,2$

$\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\psi$ $\rightsquigarrow 0,1,2,3$

$\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\overleftrightarrow{D}\psi$ $\rightsquigarrow 0,1,2,3,4$

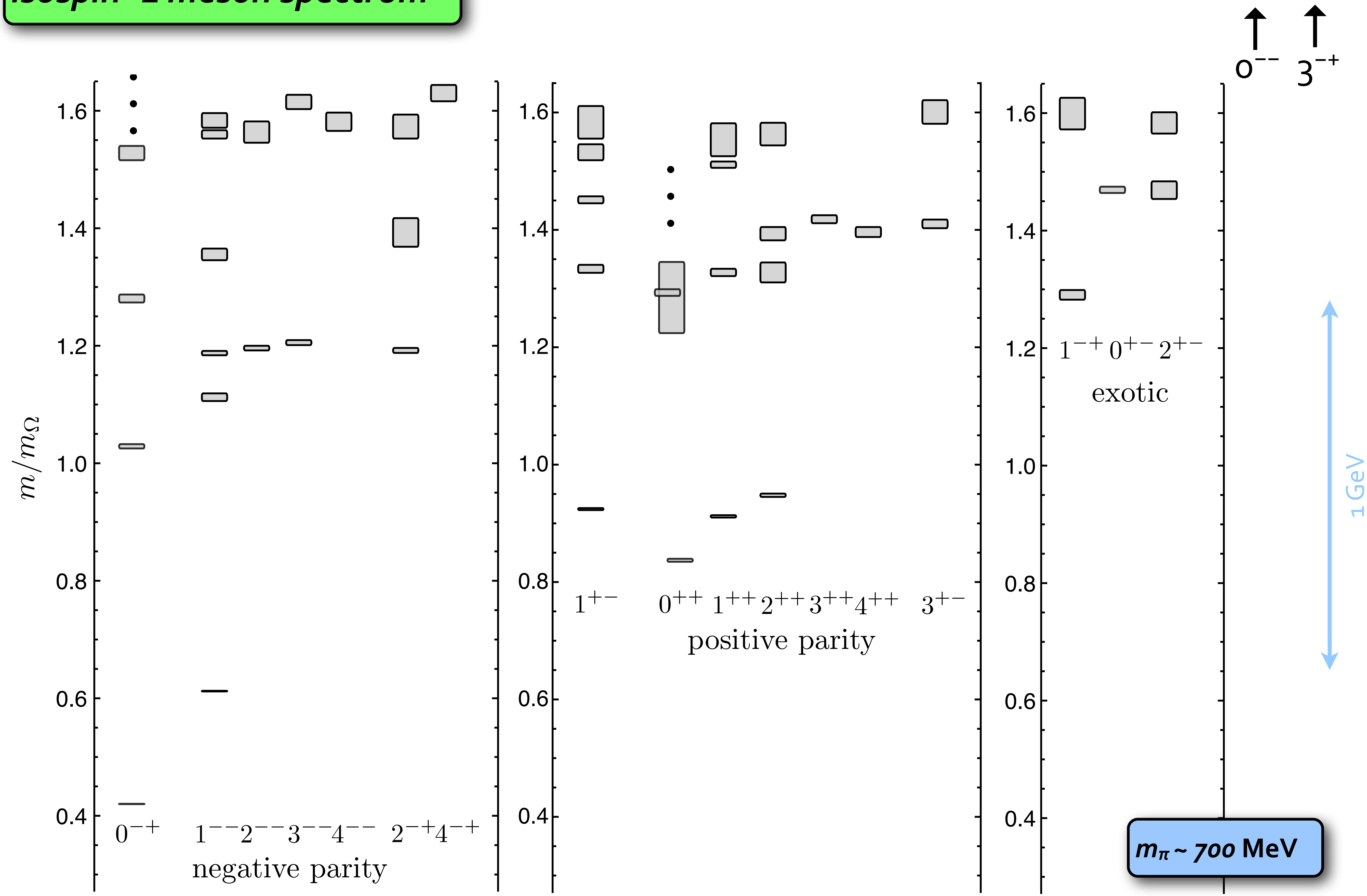
smeared quark fields

gauge-covariant derivatives

coupling $\langle 1m_1;1m_2|j_{12}m_{12}\rangle\overleftrightarrow{D}_{m_1}\overleftrightarrow{D}_{m_2}$

e.g. ~20 operators with $J^{PC}=1^{--}$

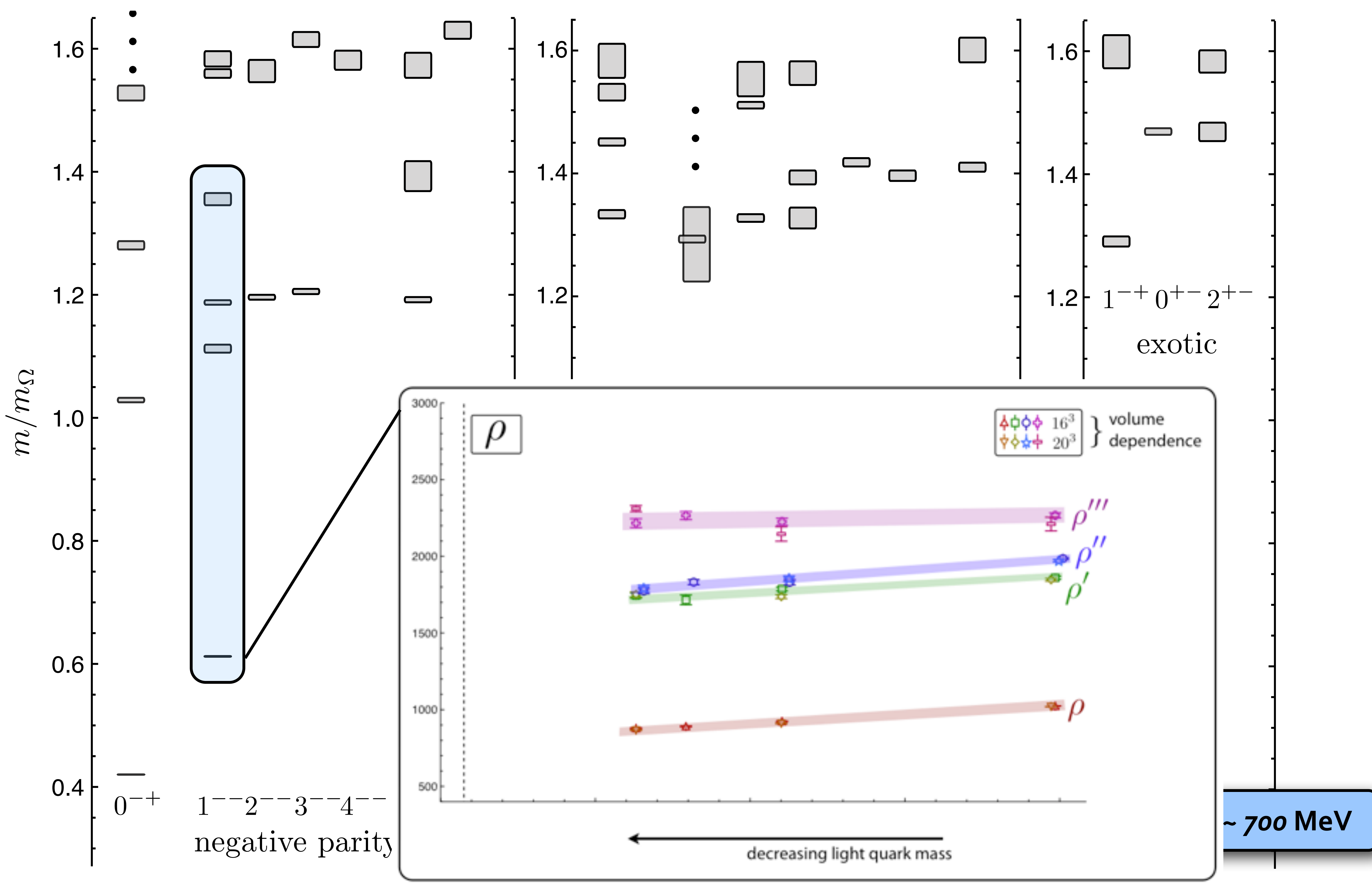
isospin=1 meson spectrum



$m_\pi \sim 700$ MeV

isospin=1 meson spectrum

decreasing the (u,d) quark mass



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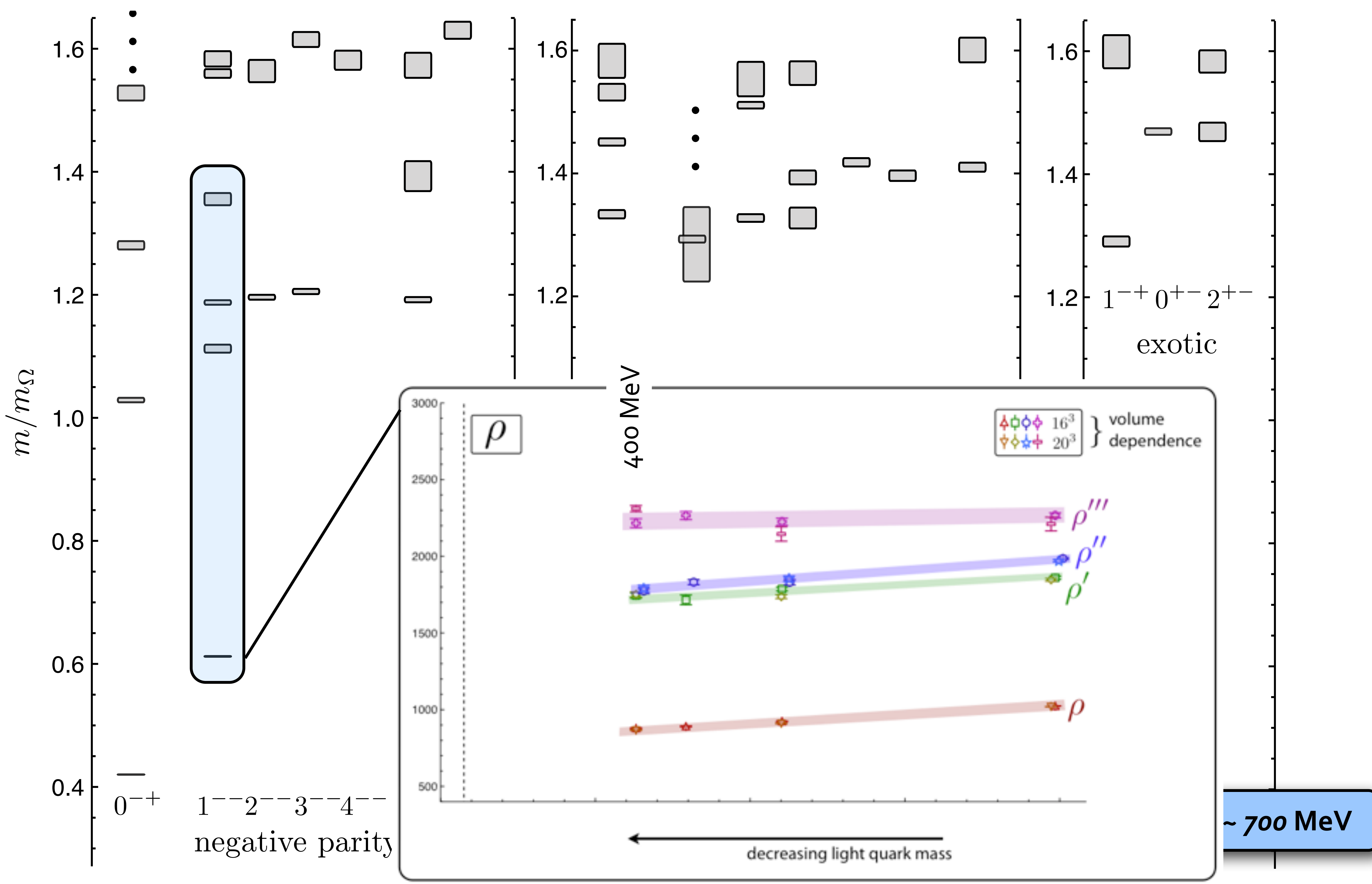
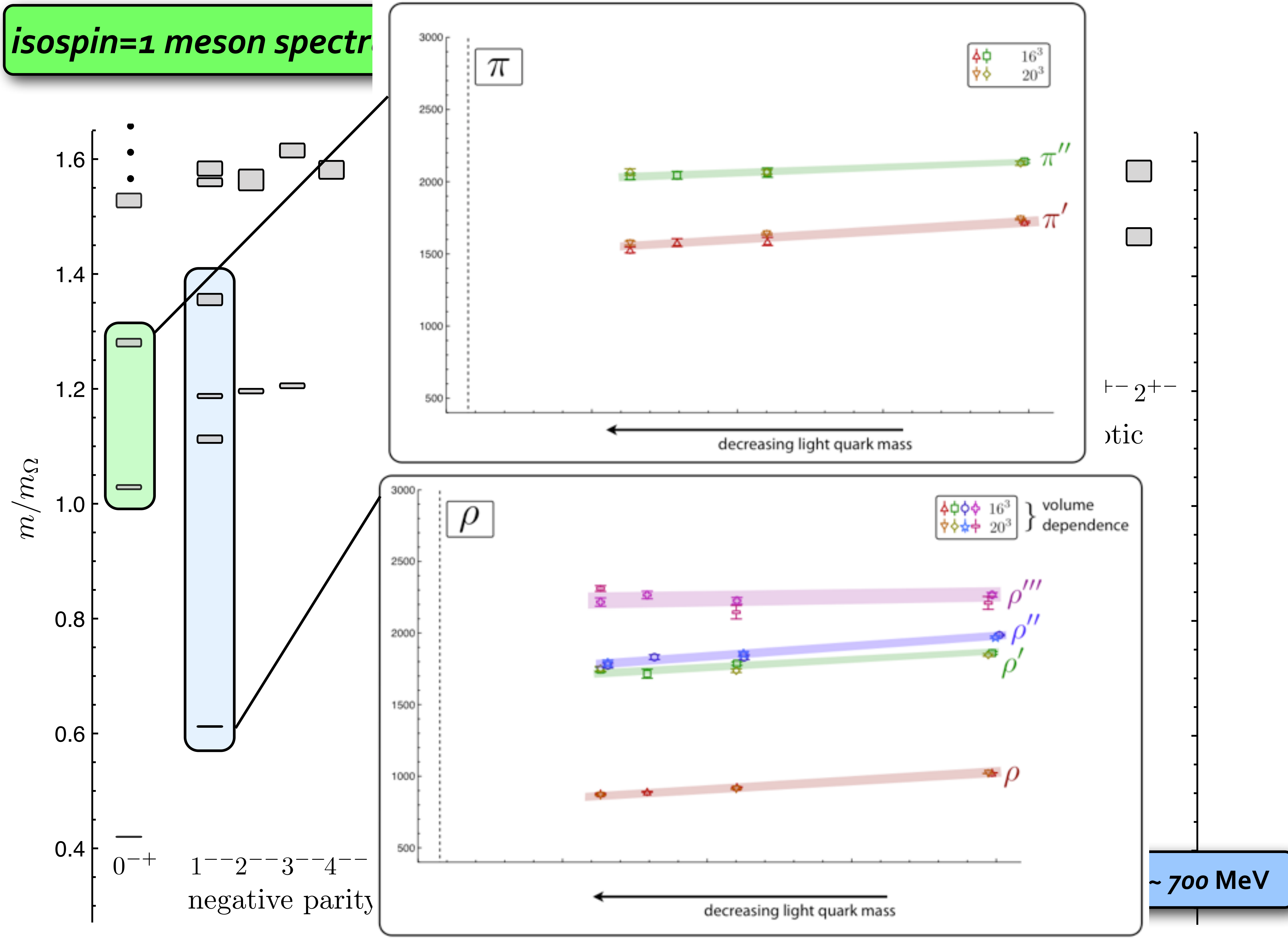
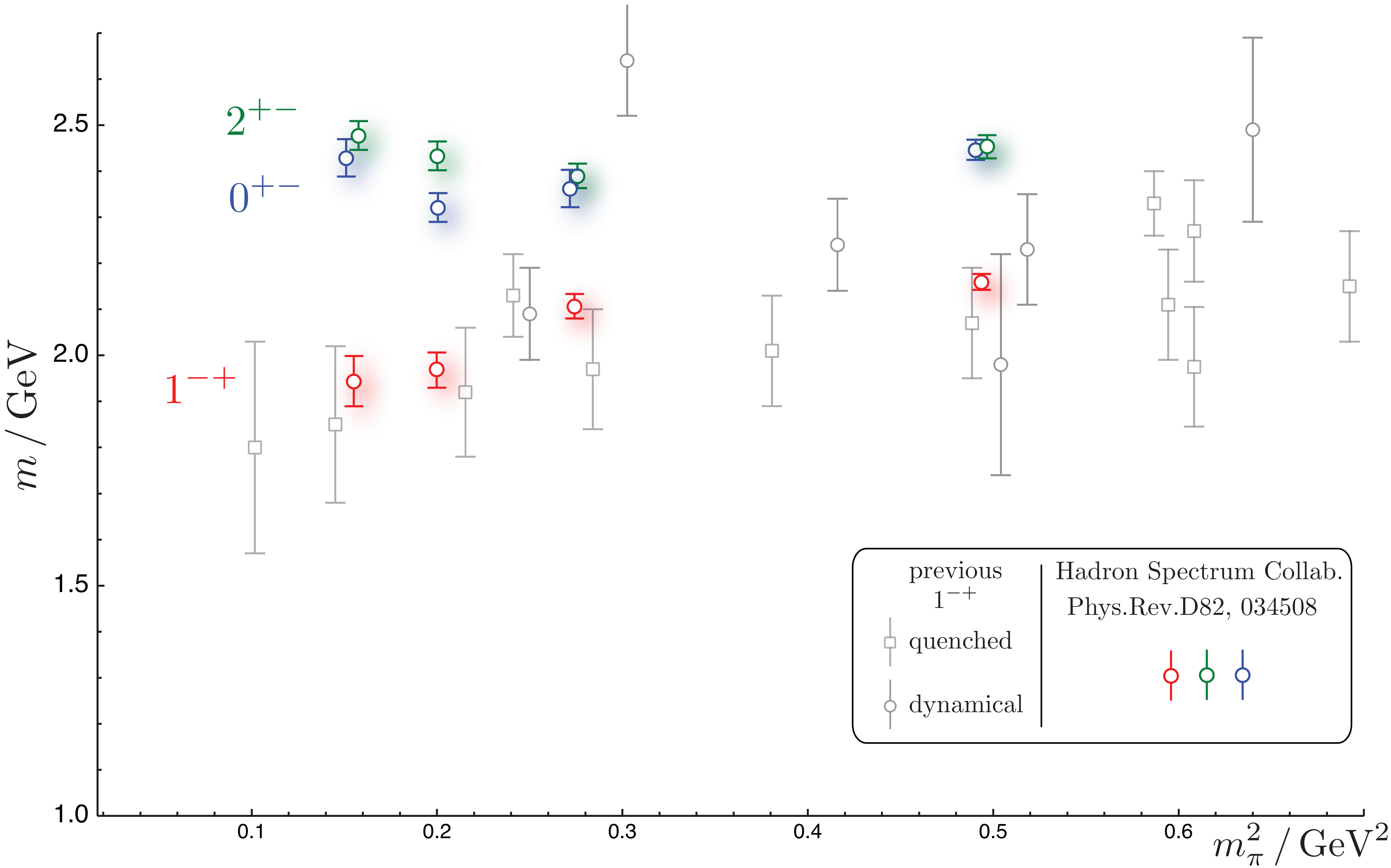


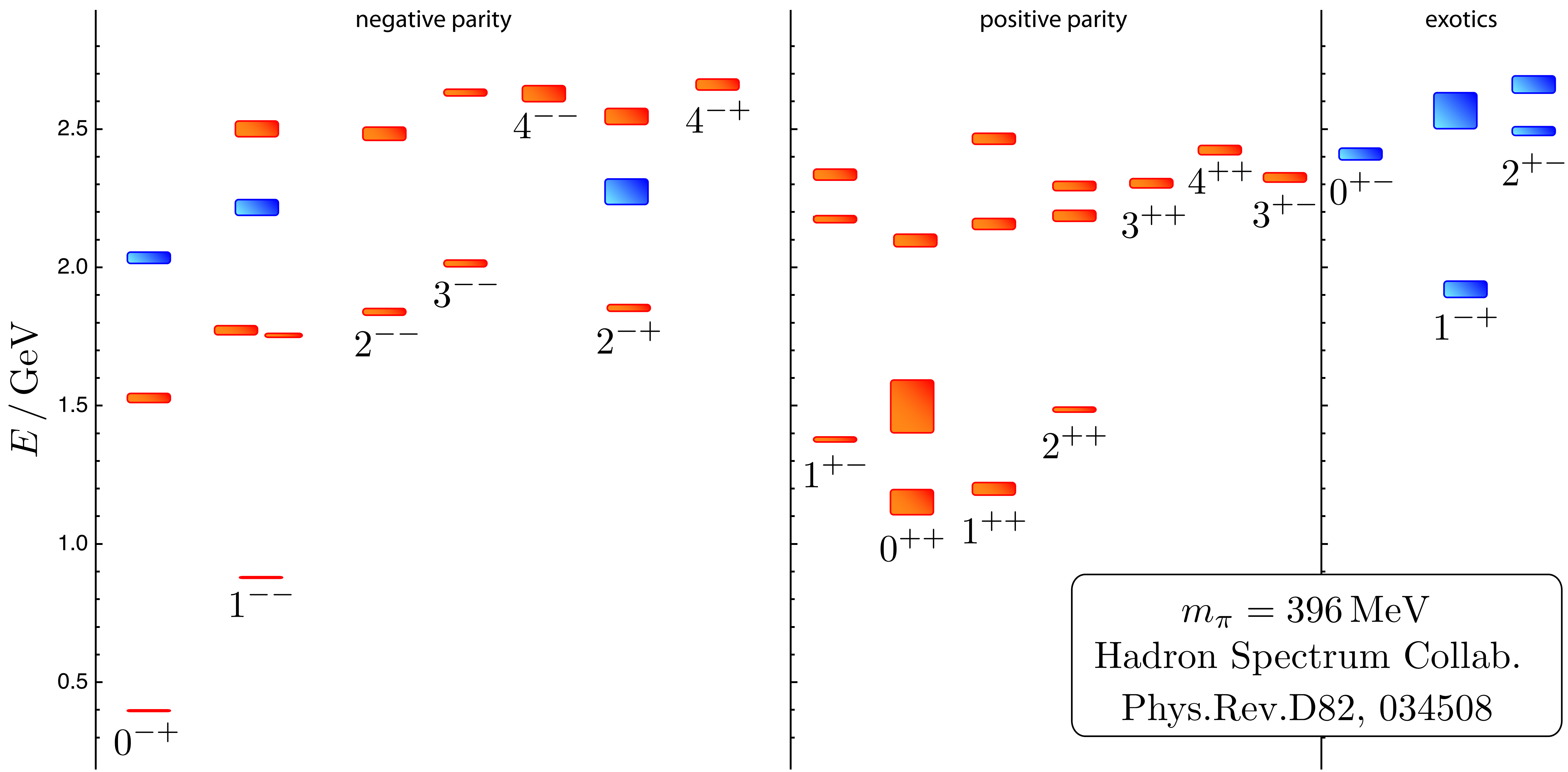


isospin=1 meson spectr
m/mΩ
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0−+
1−−2−−3−−4−−
negative parity
π
16³
20³
π″
π′
decreasing light quark mass
ρ
16³
20³
volume dependence
ρ‴
ρ″
ρ′
ρ
decreasing light quark mass
~ 700 MeV

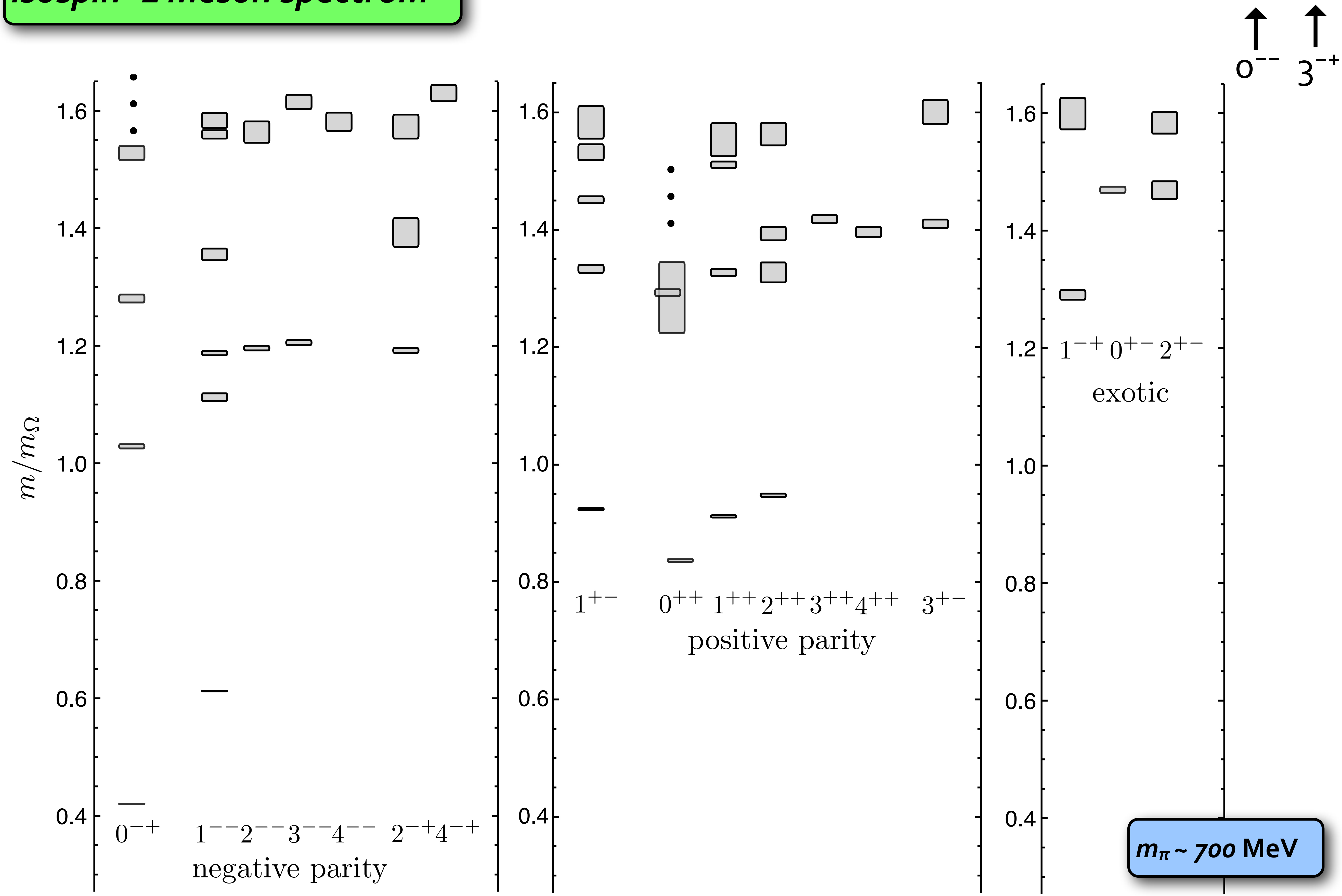
exotic J^{PC} mesons



isovector spectrum at m_π = 396 MeV



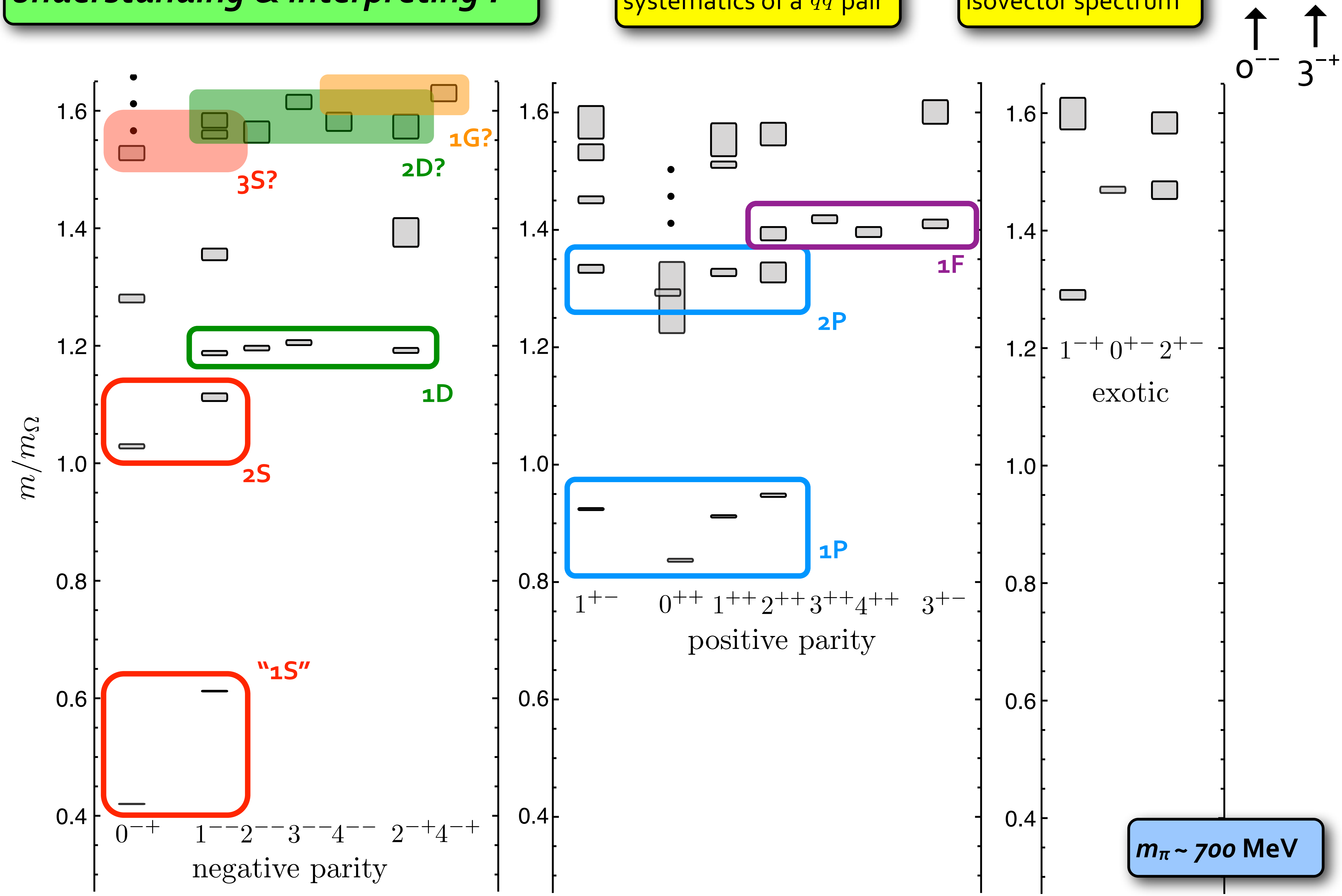
isospin=1 meson spectrum

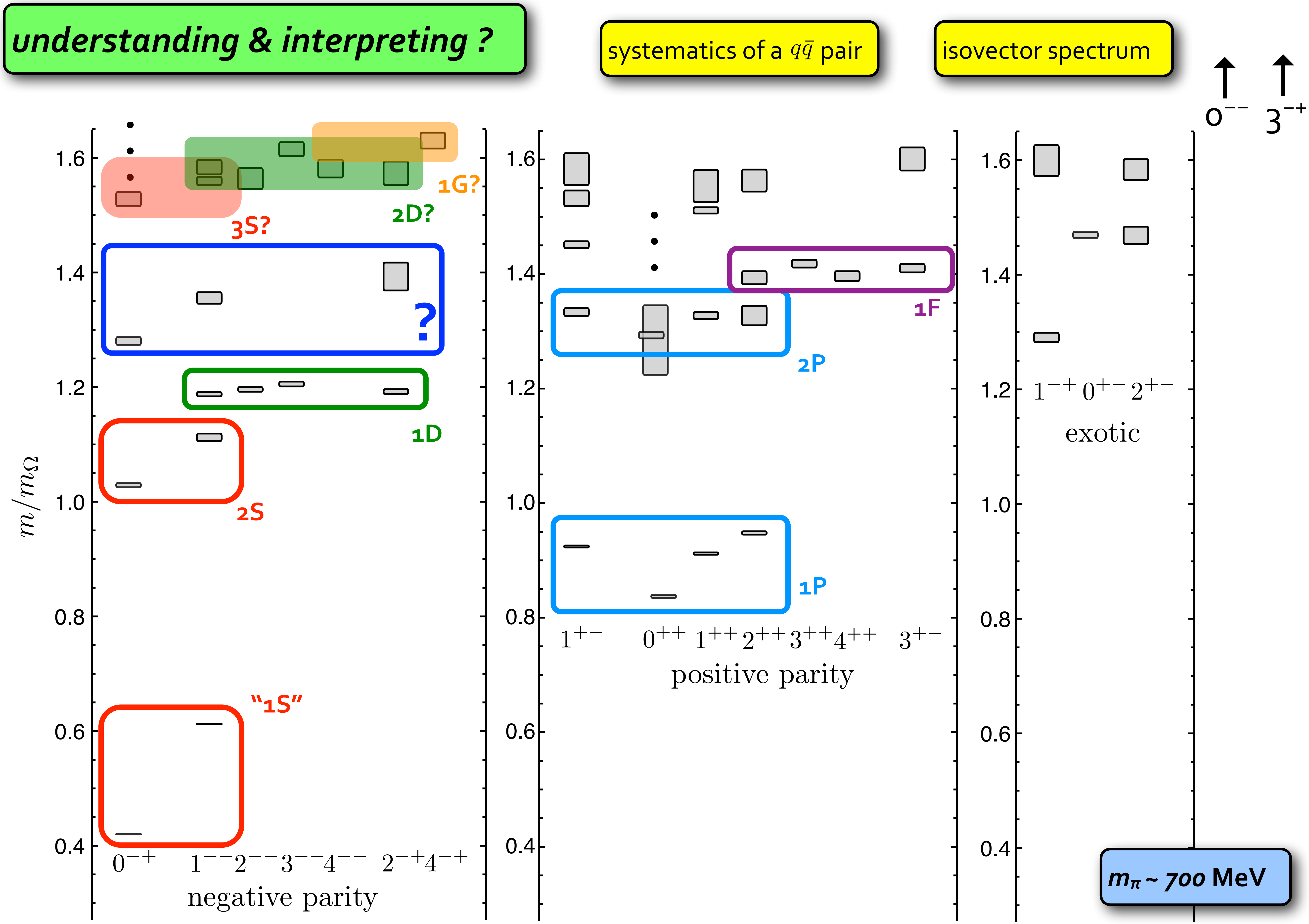


understanding & interpreting ?

systematics of a $q\bar{q}$ pair

isovector spectrum





understanding & interpreting ?
systematics of a $q\bar{q}$ pair
isovector spectrum
0^{--}
3^{-+}
m/m_Ω
1.6
1.4
1.2
1.0
0.8
0.6
0.4
3S?
2D?
1G?
?
1D
2S
"1S"
0^{-+} 1^{--} 2^{--} 3^{--} 4^{--} 2^{-+} 4^{-+}
negative parity
1F
2P
1P
1^{+-} 0^{++} 1^{++} 2^{++} 3^{++} 4^{++} 3^{+-}
positive parity
1^{-+} 0^{+-} 2^{+-}
exotic
$m_\pi \sim 700$ MeV

understanding & interpreting ?

remarkable considering the caveats that Robert raised about resonances in finite volume ...

... let's roll with it and see if we can learn anything qualitative ?

understanding & interpreting ?

try (model-dependent) analysis of matrix elements

1^{--}

a (tiny) subset of operators used :

$$Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n} | \mathcal{O}_i | 0 \rangle$$

$\rho_{\rm NR}$ $\quad$ $\bar{\psi}\gamma_i \frac{1}{2}(1-\gamma_0)\psi$

$\left(\rho_{\rm NR} \times D^{[2]}_{J=2}\right)^{J=1}$ $\quad$ $D^{[2]}_{J=2,M} = \langle 1 m_1; 1 m_2 | 2M \rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}$

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$$\sim \vec{\epsilon}_M \cdot \vec{B}^a$$

chromomagnetic field strength

understanding & interpreting ?

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$$\sim\ {}^1\mathrm{hyb}_1\ ?$$

understanding & interpreting ?

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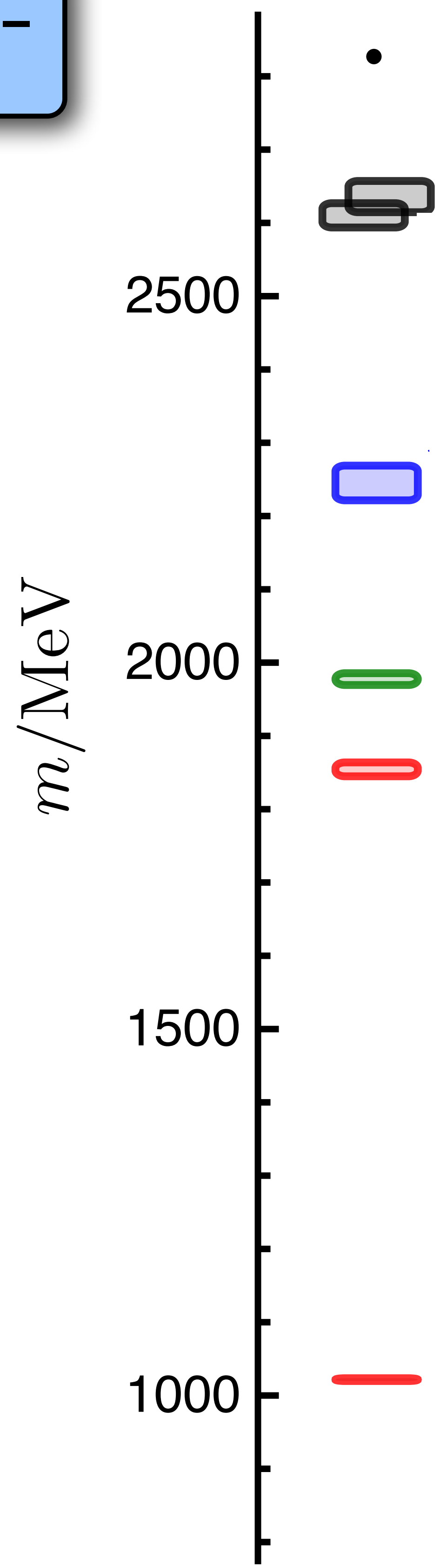

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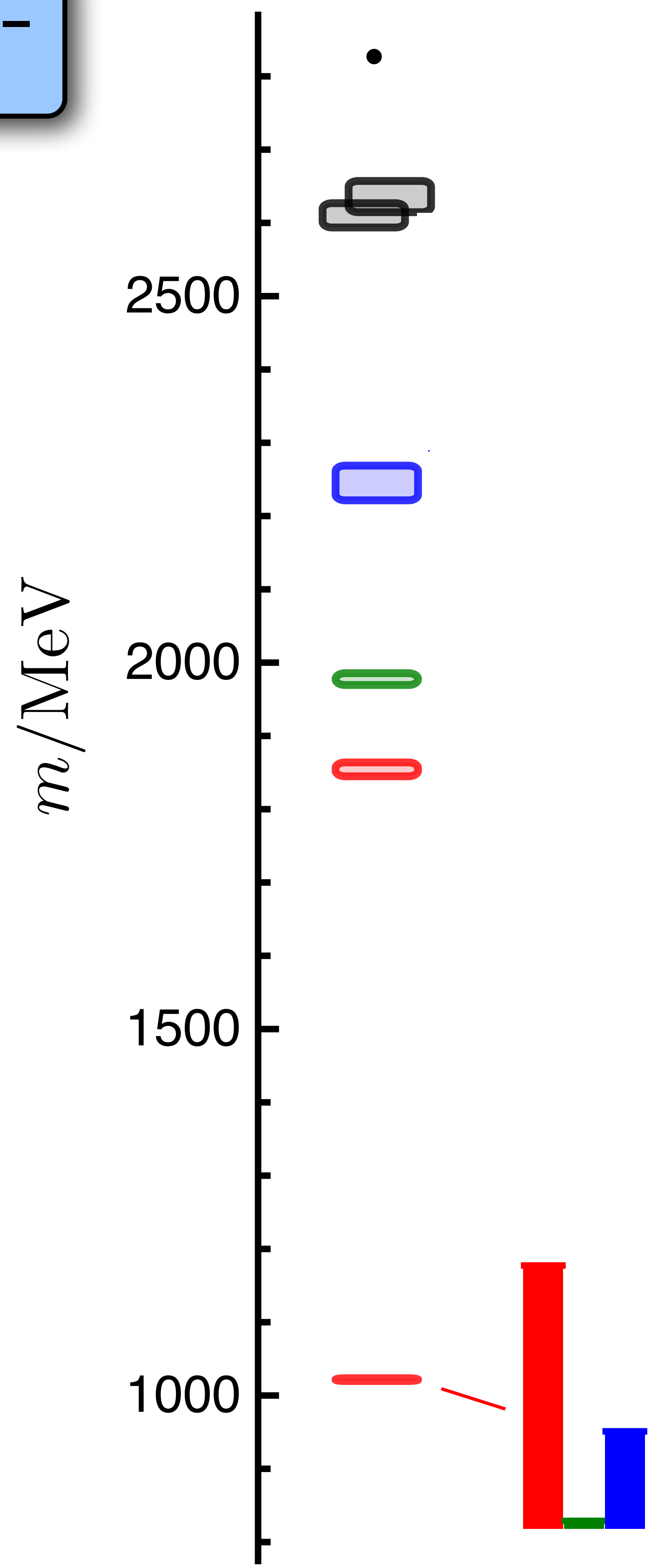

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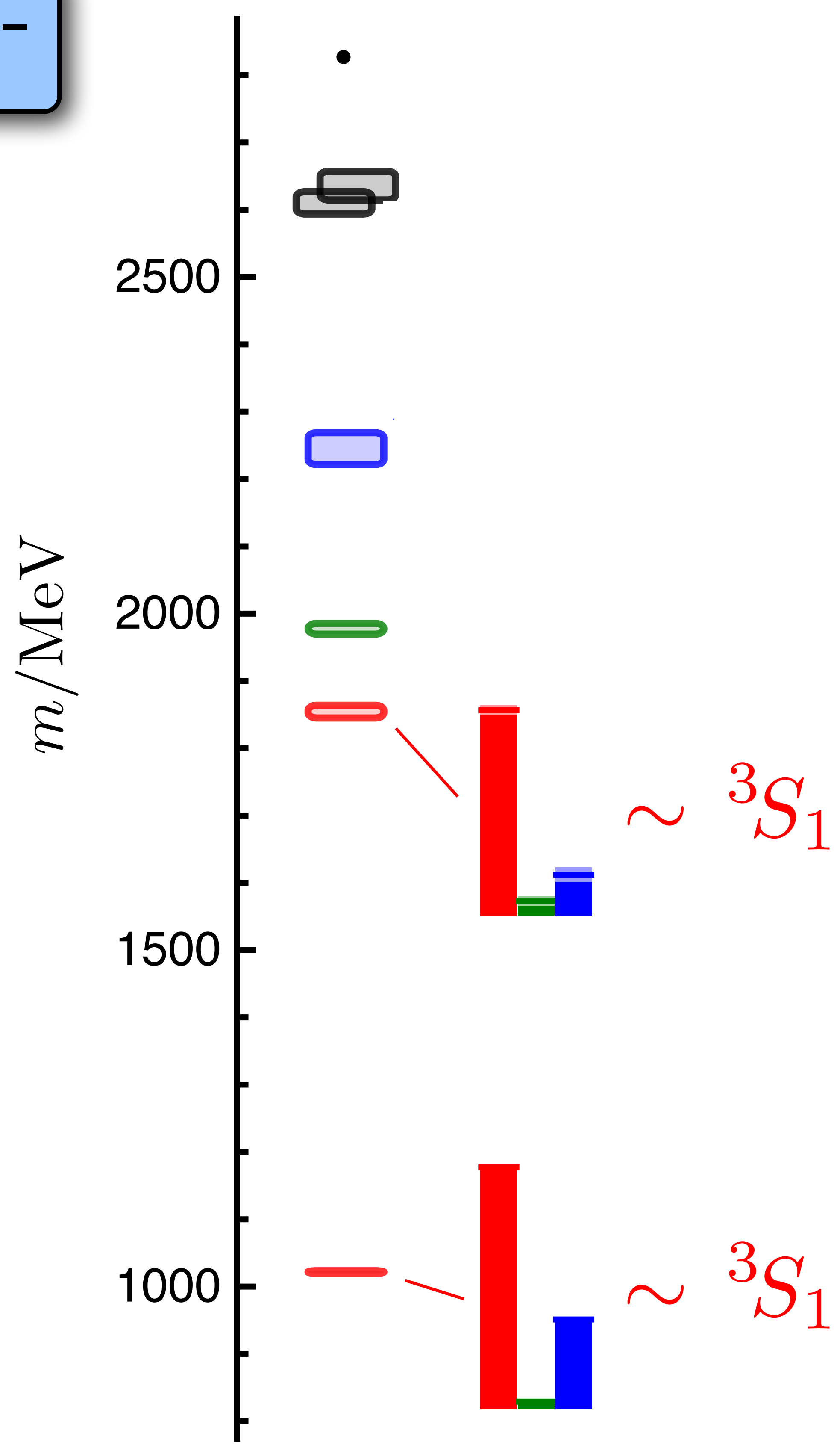

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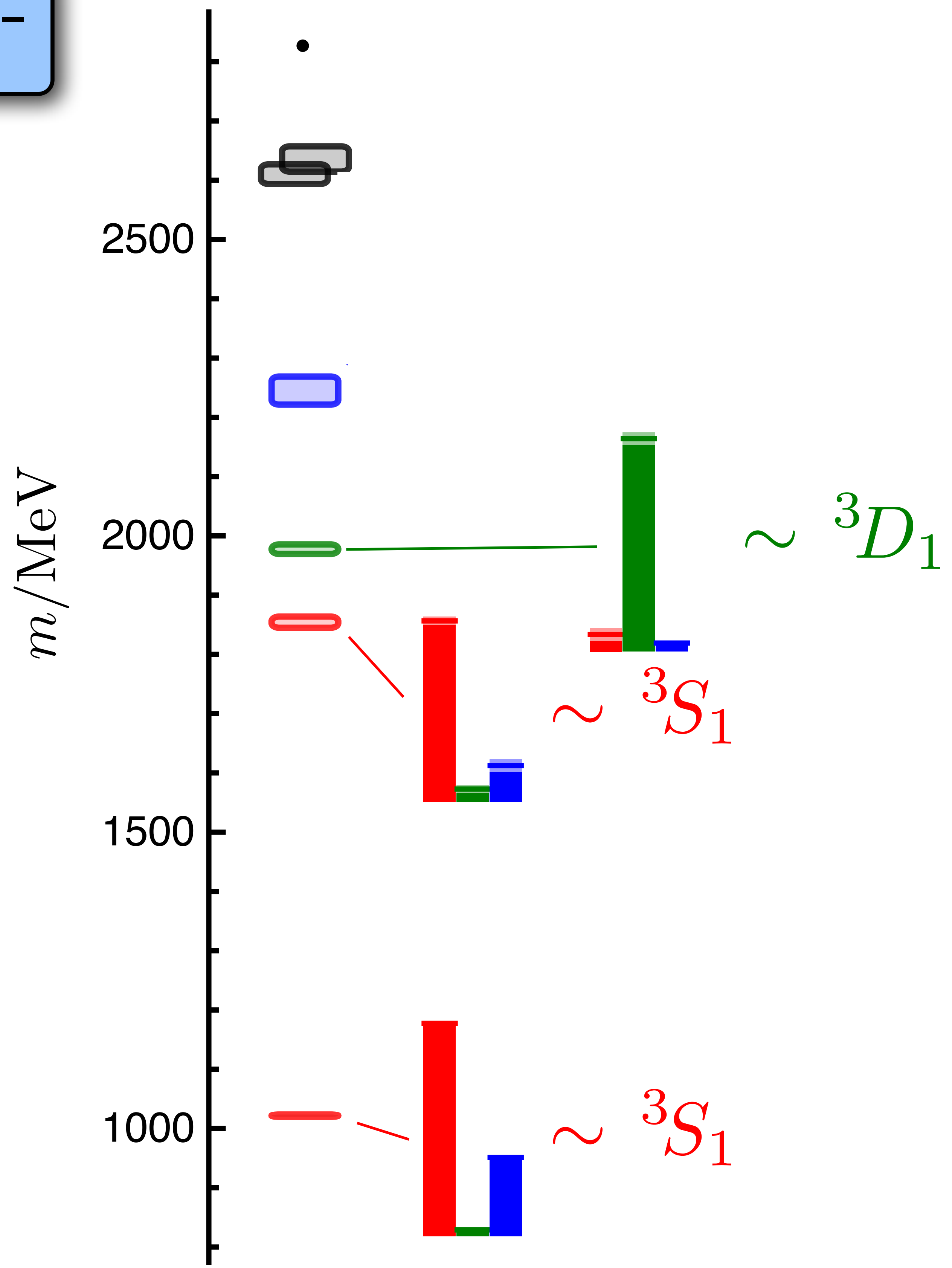

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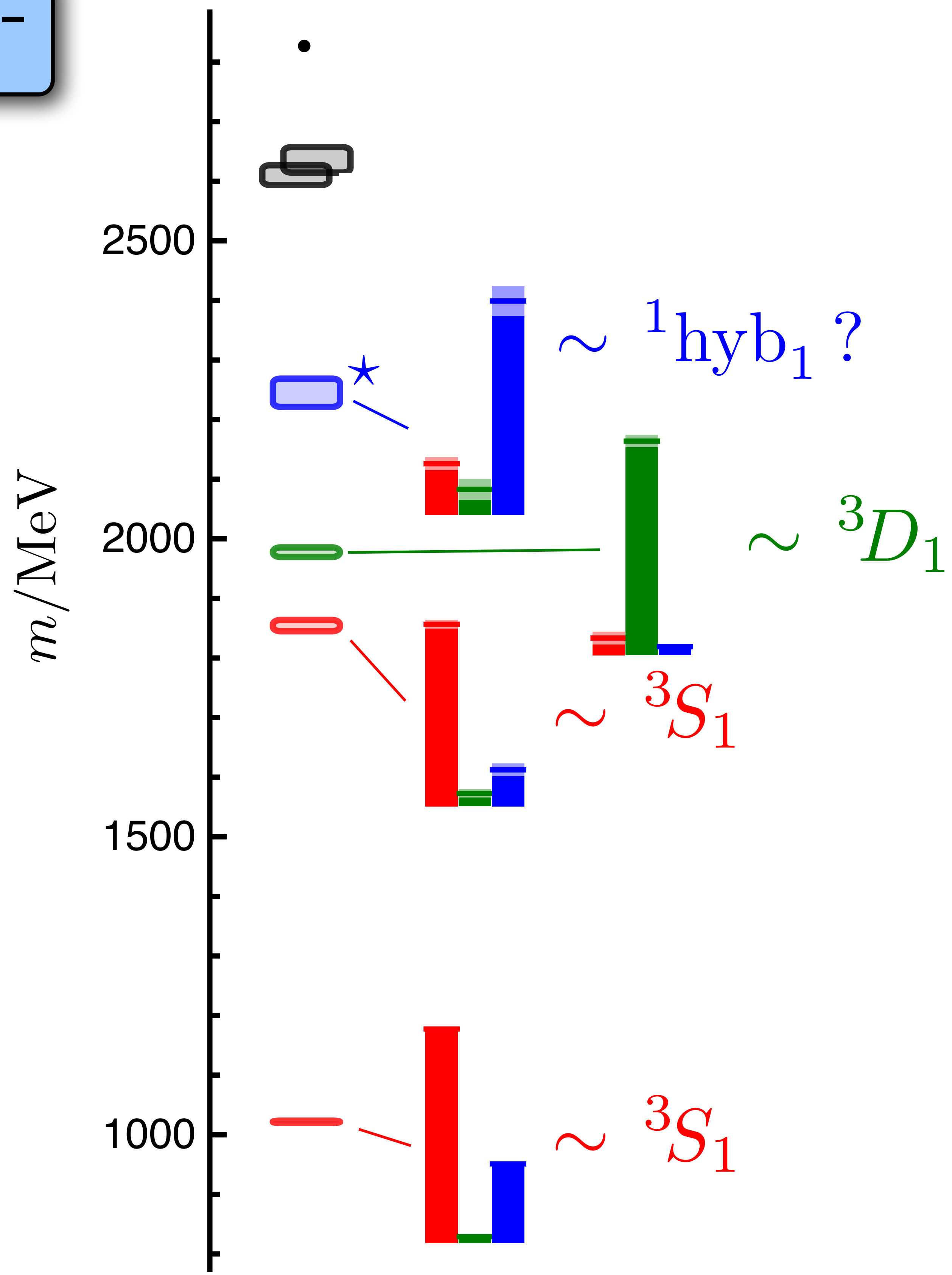

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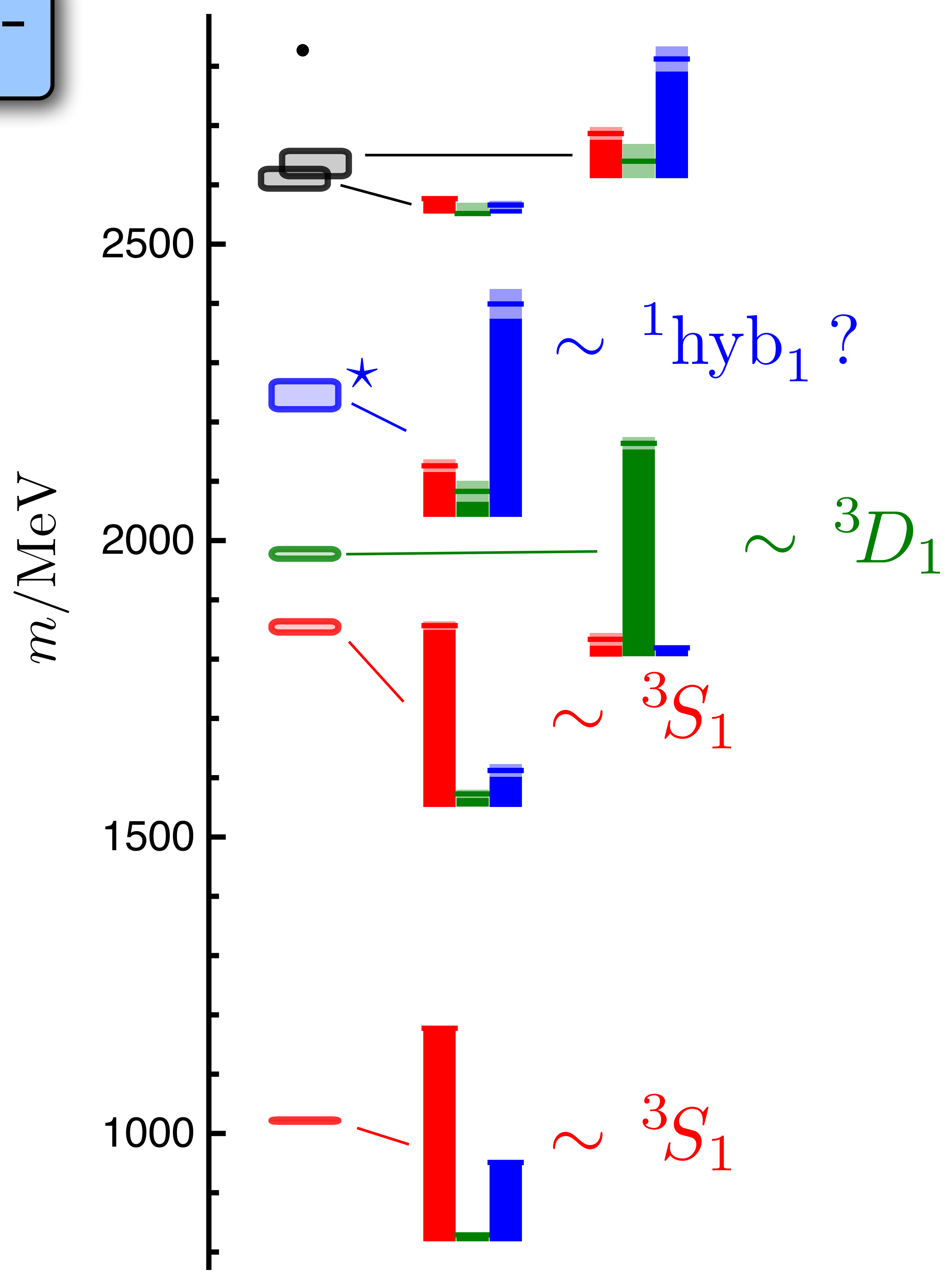

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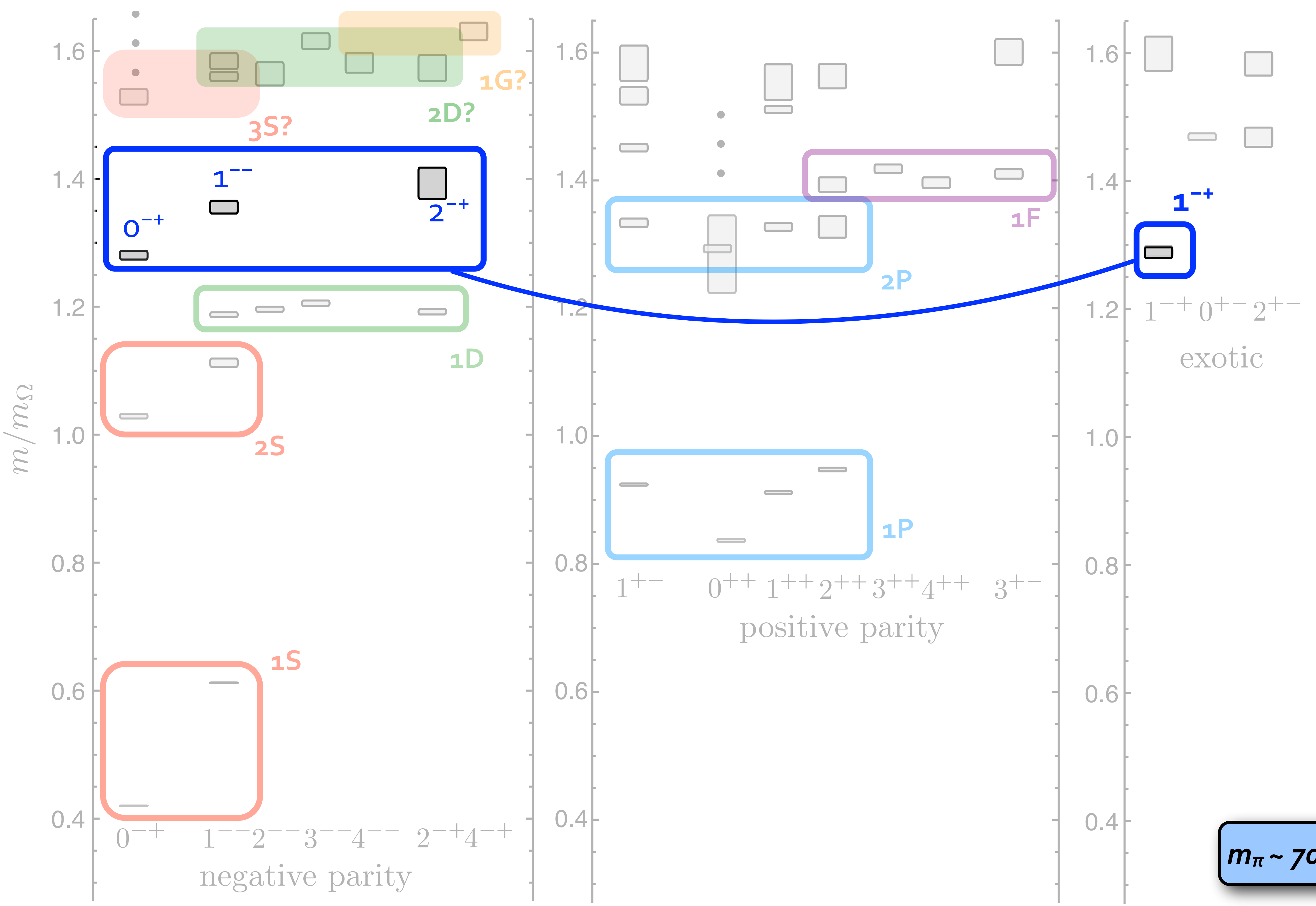
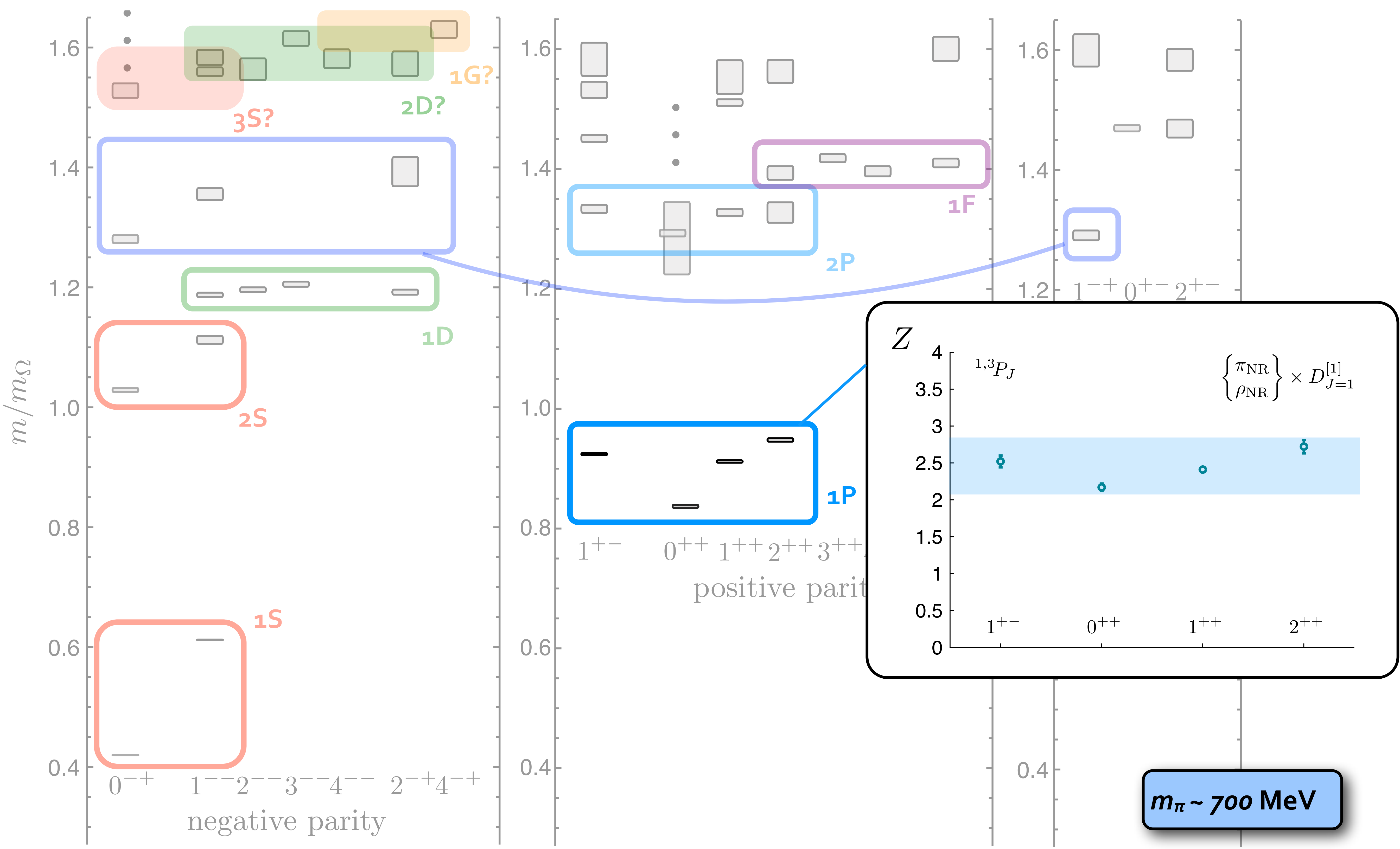
the lightest hybrid supermultiplet ?



$m_\pi \sim 700$ MeV

supermultiplets ?



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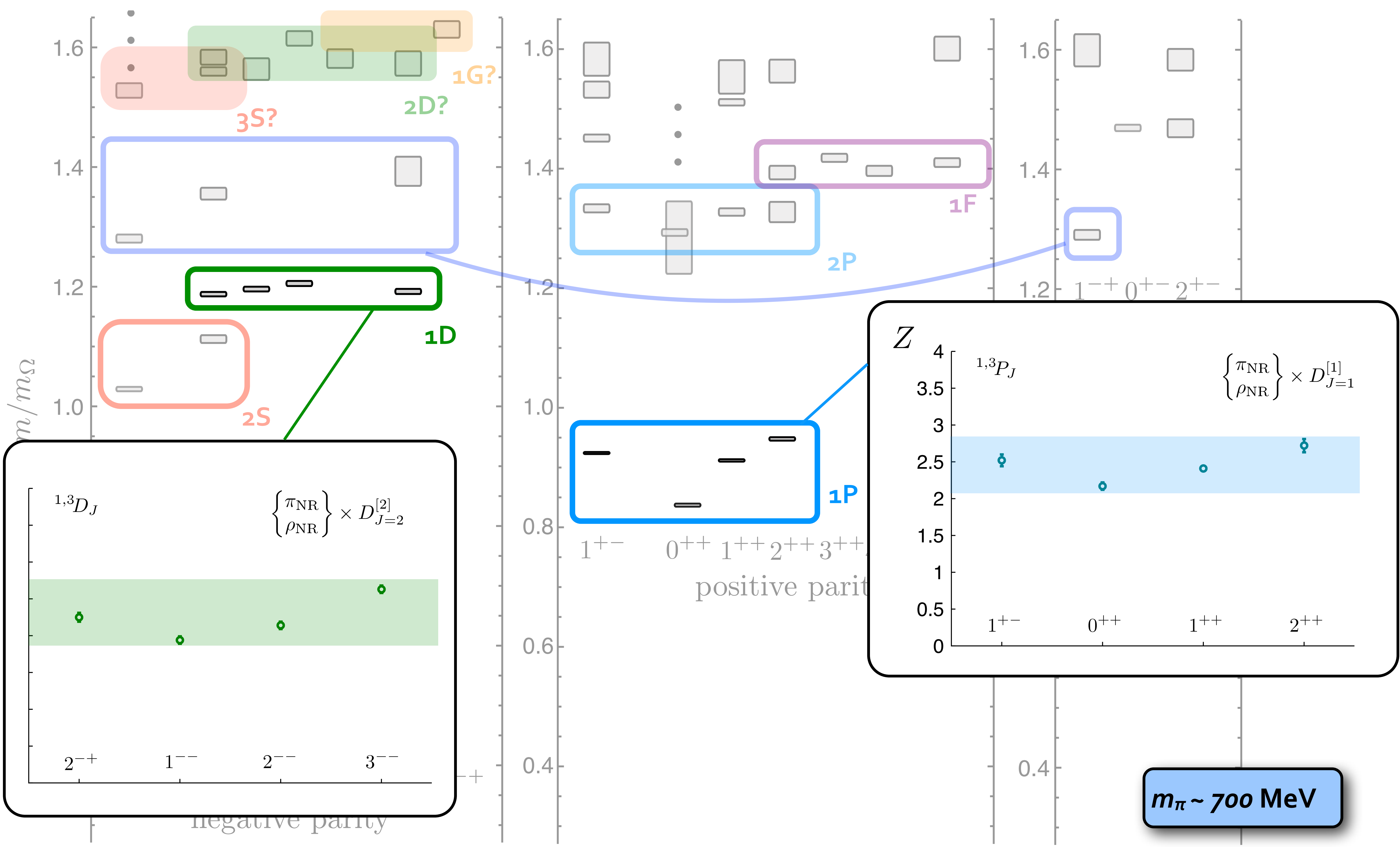

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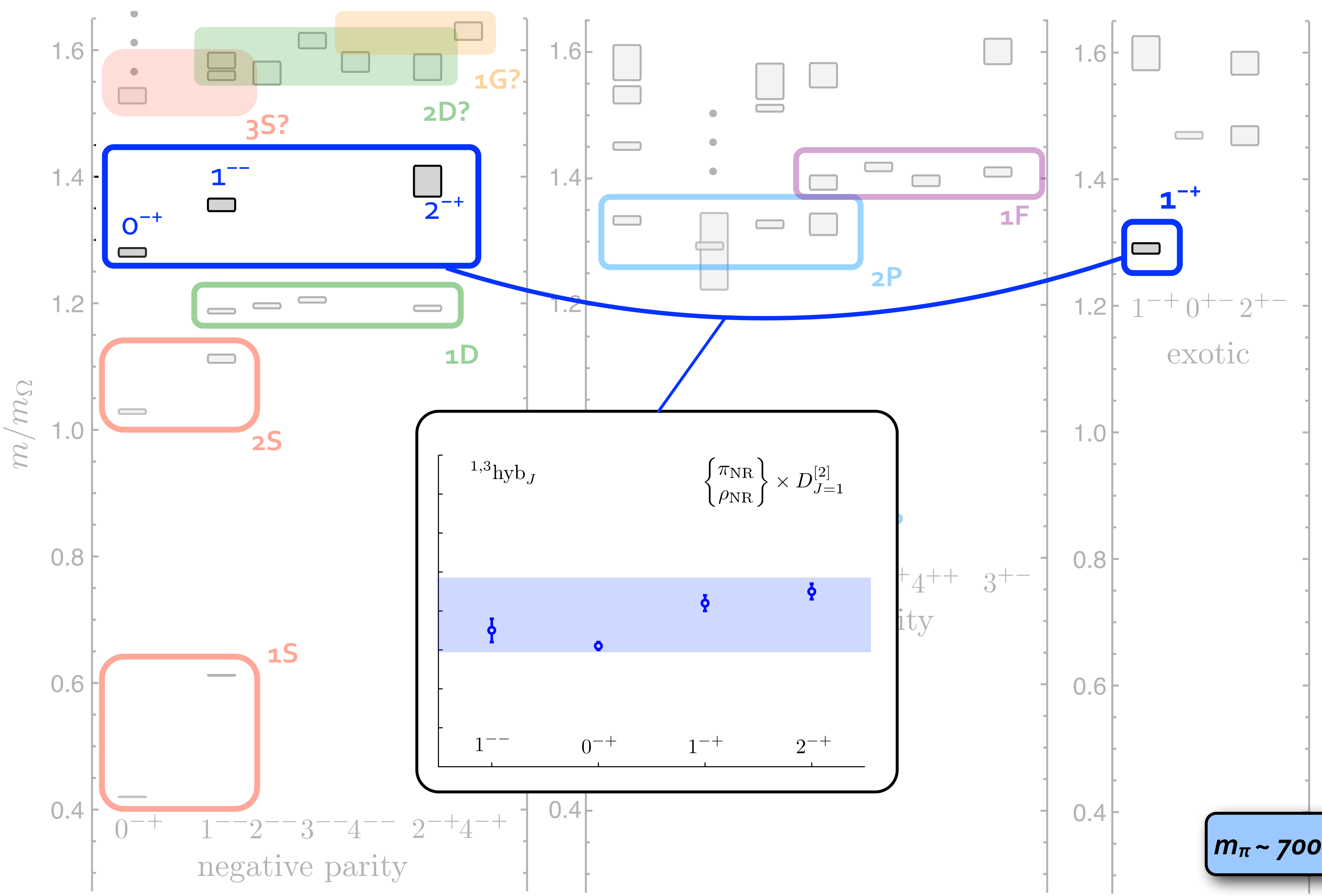
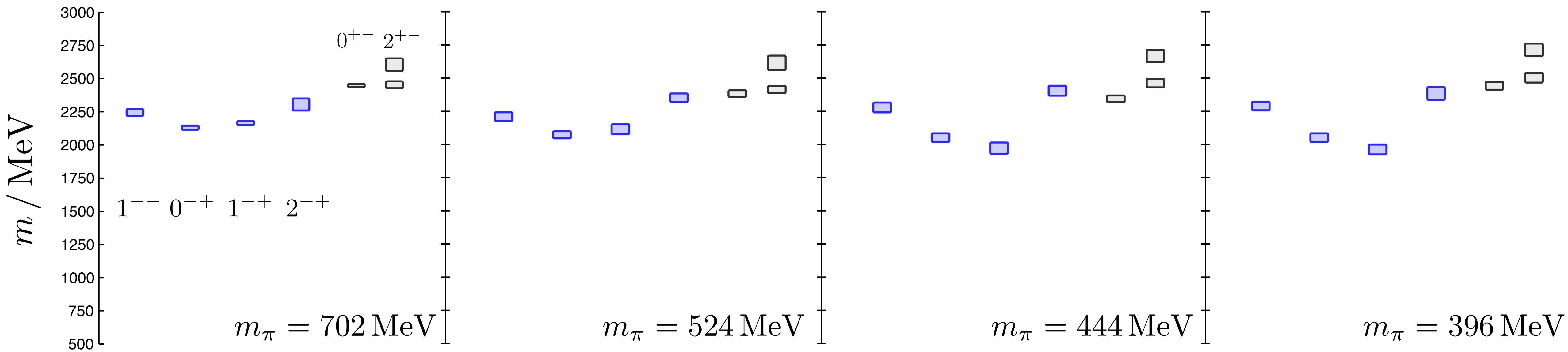


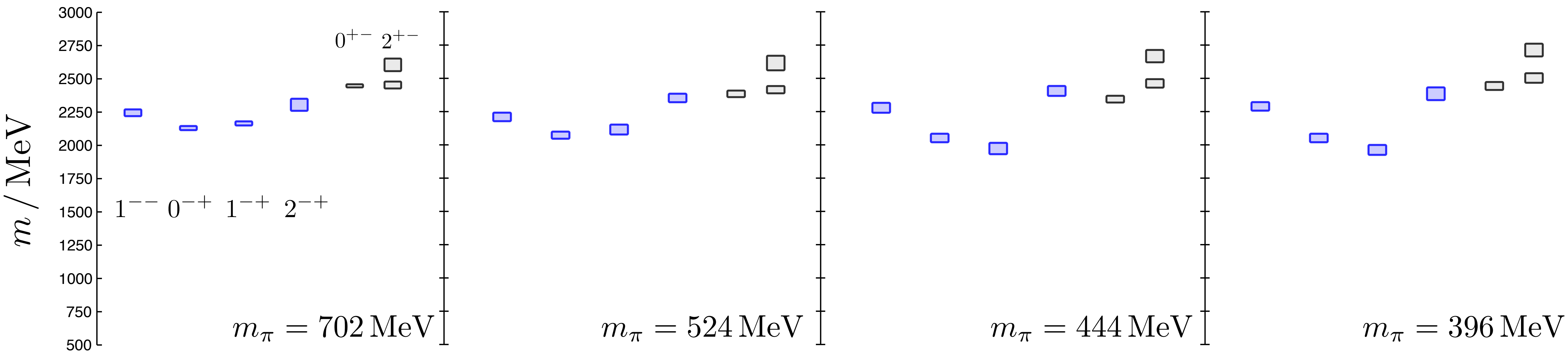
mπ ~ 700 MeV

hybrid mesons



can we build a plausible picture that describes this particular pattern of states & overlap preferences ?

hybrid mesons



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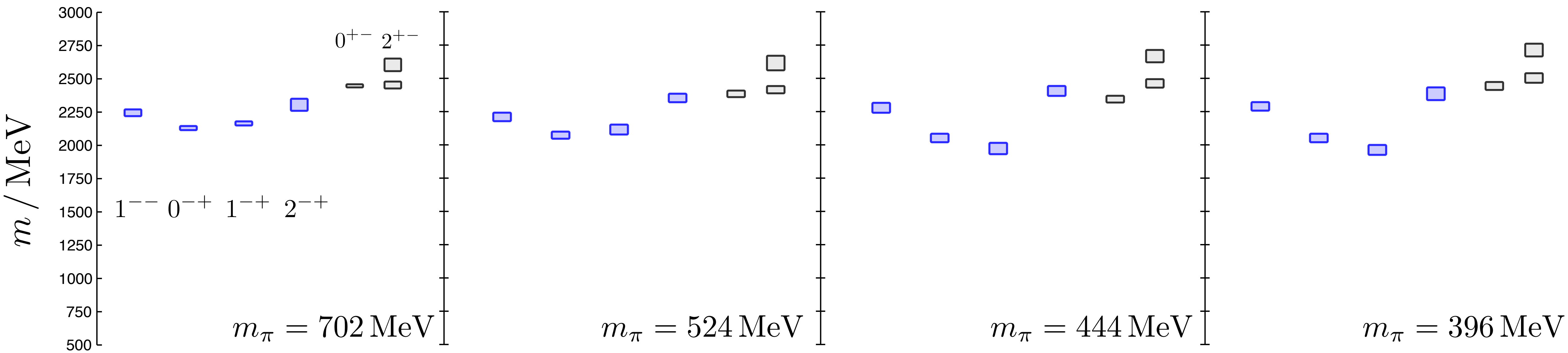

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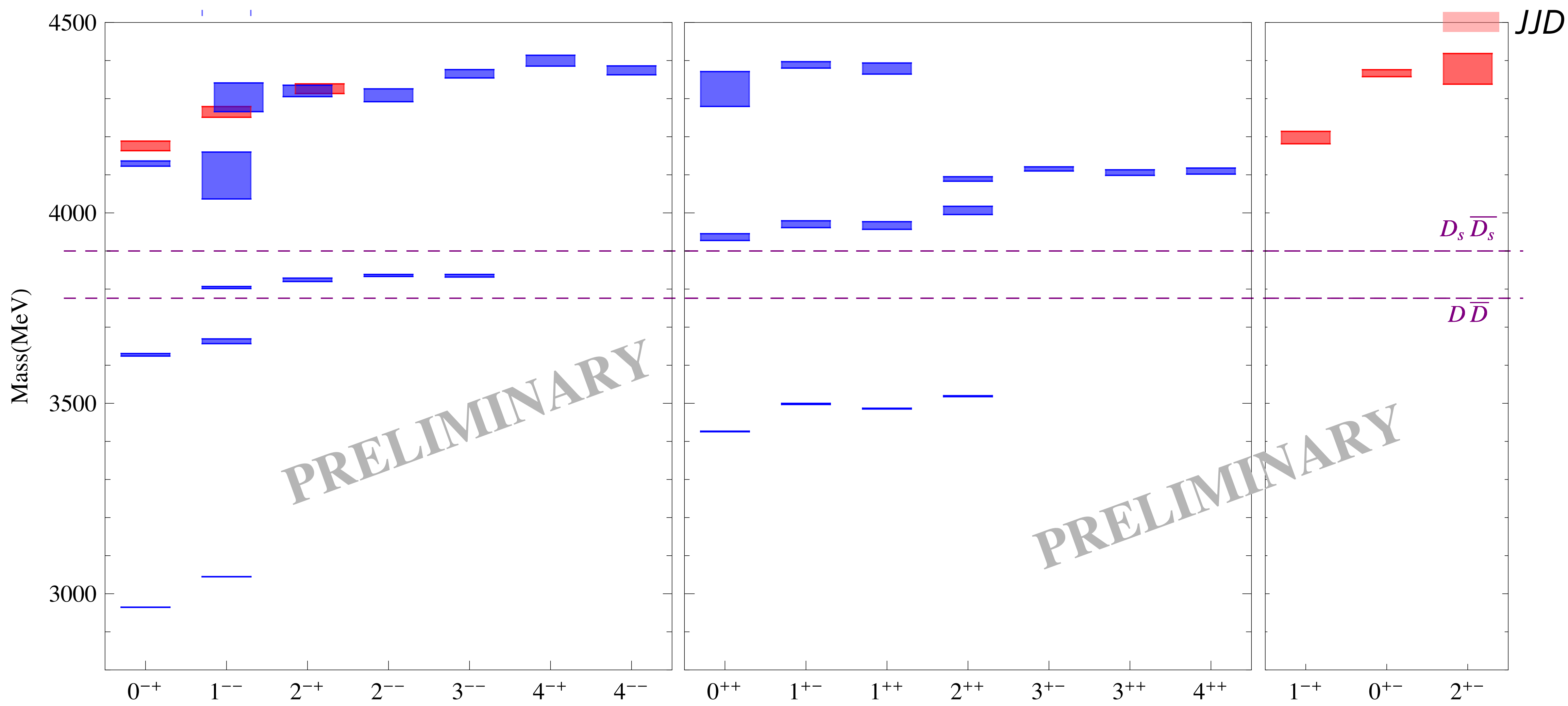
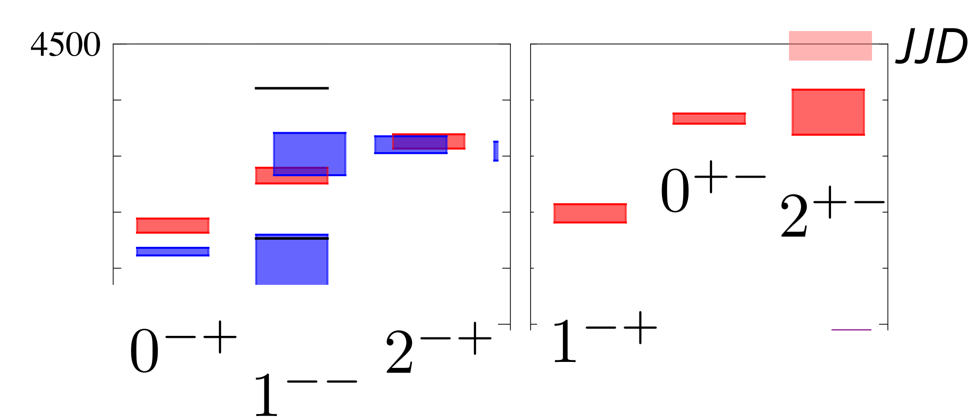
chromomagnetic excitation ?

charmonium

thanks to Dublin group (Liu, Peardon, Ryan, Thomas ...)





baryon operators

three-quark field constructions, obeying permutation (anti-)symmetry

$$\epsilon_{abc}\left(D^{n_1}\tfrac{1}{2}(1\pm\gamma^0)\psi\right)^a\left(D^{n_2}\tfrac{1}{2}(1\pm\gamma^0)\psi\right)^b\left(D^{n_3}\tfrac{1}{2}(1\pm\gamma^0)\psi\right)^c$$

derivative constructions

$$D^{[1]}_{\mathsf{MS},m}=\frac{1}{\sqrt{6}}\big(2D^{(3)}_m-D^{(1)}_m-D^{(2)}_m\big)\qquad\sim\vec{\epsilon}_m\cdot\vec{\lambda}$$

$$D^{[1]}_{\mathsf{MA},m}=\frac{1}{\sqrt{2}}\big(D^{(1)}_m-D^{(2)}_m\big)\qquad\sim\vec{\epsilon}_m\cdot\vec{\rho}$$

$$D^{[2]}_{\mathsf{S};L,M}=\langle 1m;1m'|LM\rangle\frac{1}{\sqrt{2}}\big(D^{[1]}_{\mathsf{MS},m}D^{[1]}_{\mathsf{MS},m'}+D^{[1]}_{\mathsf{MA},m}D^{[1]}_{\mathsf{MA},m'}\big)\qquad L=0,2$$

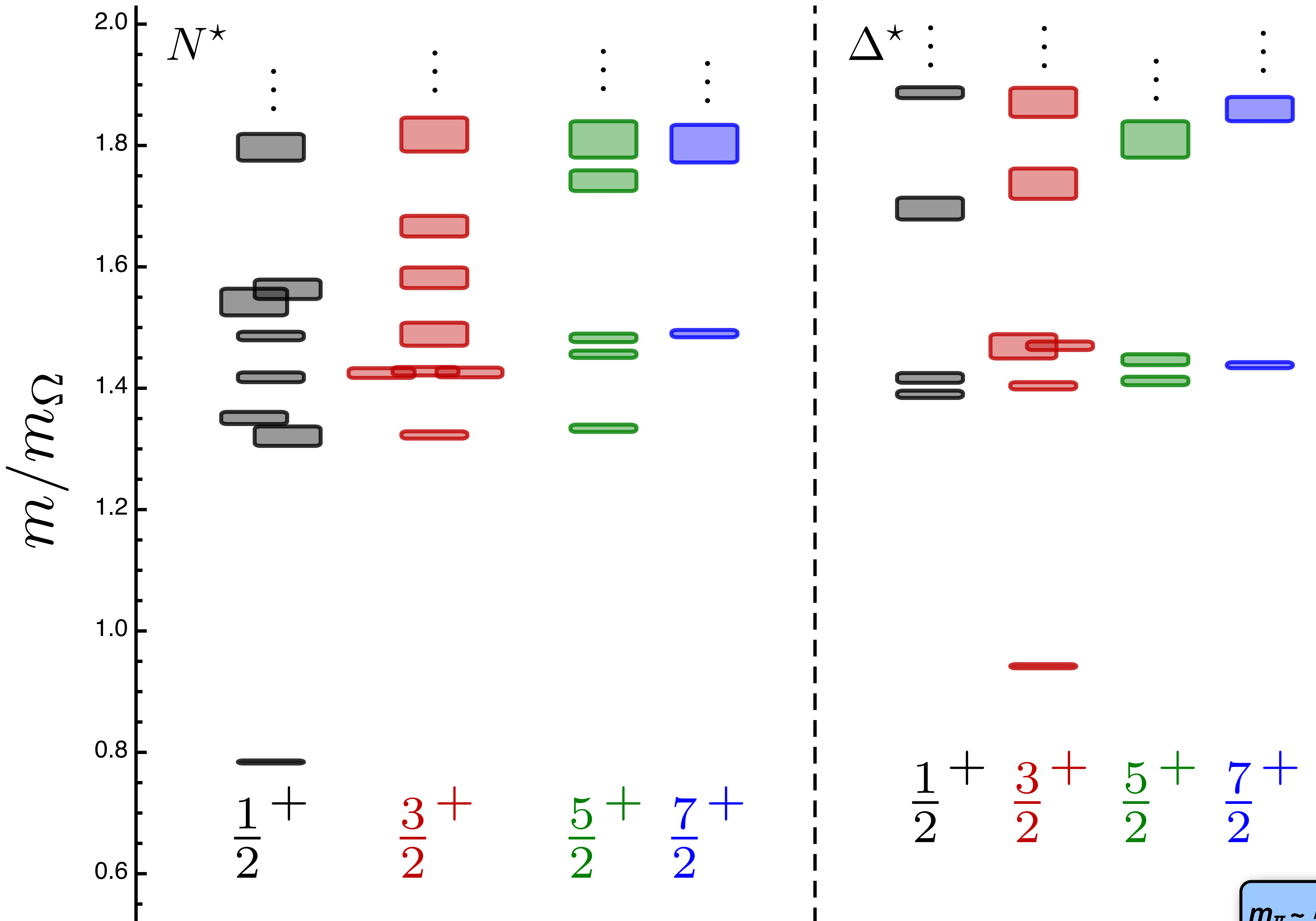
$$D^{[2]}_{\mathsf{A};L,M}=\langle 1m;1m'|LM\rangle\frac{1}{\sqrt{2}}\big(D^{[1]}_{\mathsf{MS},m}D^{[1]}_{\mathsf{MA},m'}-D^{[1]}_{\mathsf{MA},m}D^{[1]}_{\mathsf{MS},m'}\big)\qquad L=1$$

$$D^{[2]}_{\mathsf{MS};L,M}=\langle 1m;1m'|LM\rangle\frac{1}{\sqrt{2}}\big(-D^{[1]}_{\mathsf{MS},m}D^{[1]}_{\mathsf{MS},m'}+D^{[1]}_{\mathsf{MA},m}D^{[1]}_{\mathsf{MA},m'}\big)$$

$$D^{[2]}_{\mathsf{MA};L,M}=\langle 1m;1m'|LM\rangle\frac{1}{\sqrt{2}}\big(D^{[1]}_{\mathsf{MS},m}D^{[1]}_{\mathsf{MA},m'}+D^{[1]}_{\mathsf{MA},m}D^{[1]}_{\mathsf{MS},m'}\big)$$

L=0,1,2

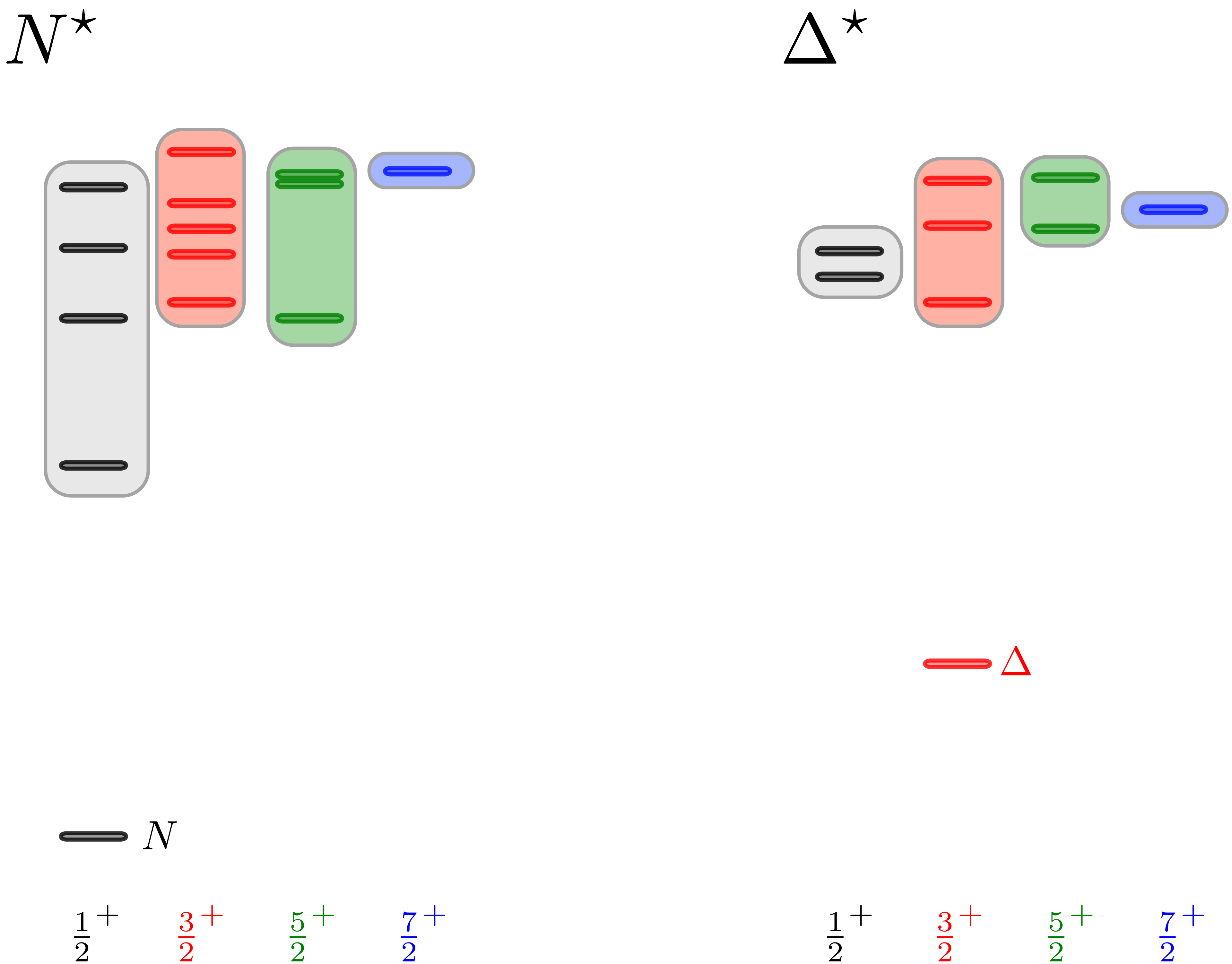
lattice QCD baryon spectrum

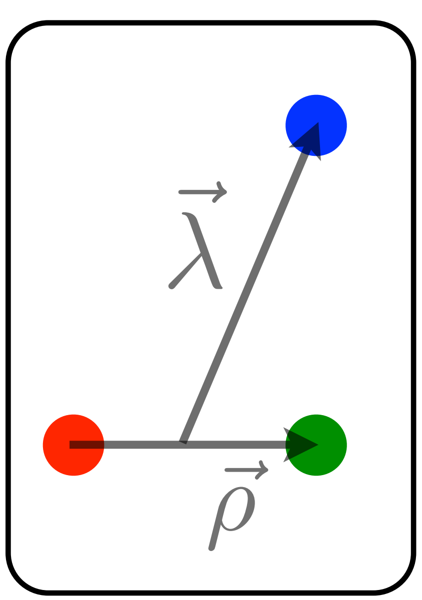


$m_\pi \sim 524$ MeV

three 'quark' baryons

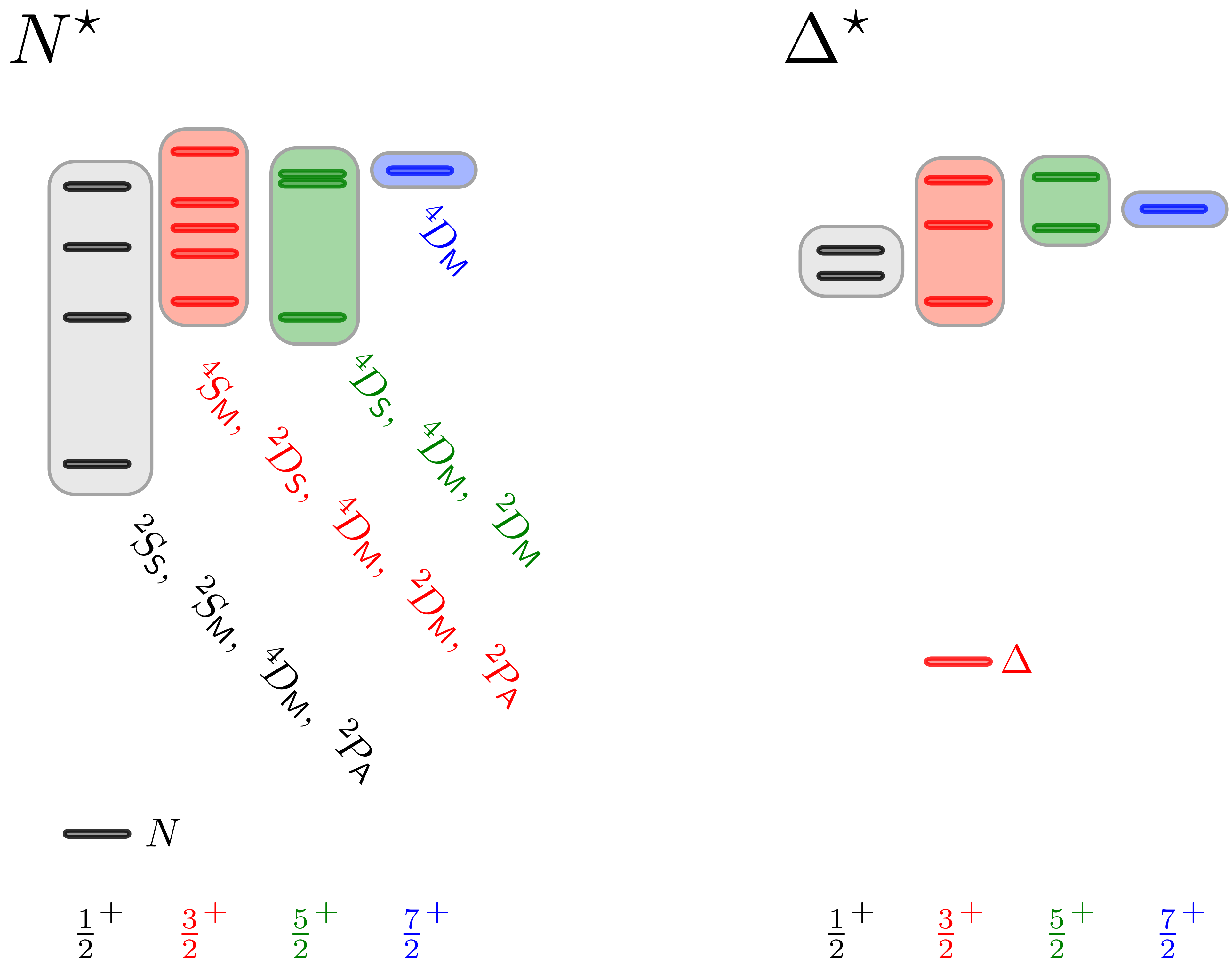
Capstick-Isgur as an example

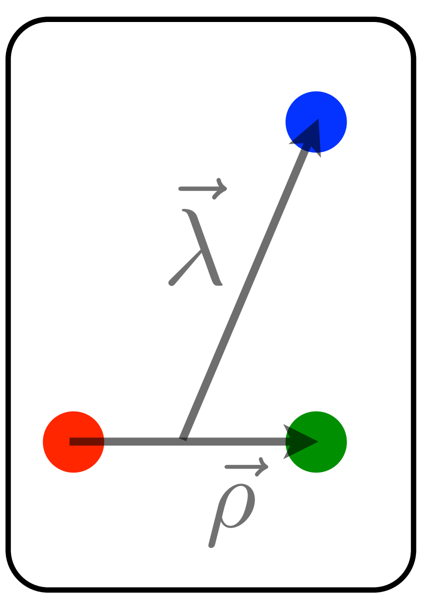




three 'quark' baryons

Capstick-Isgur as an example





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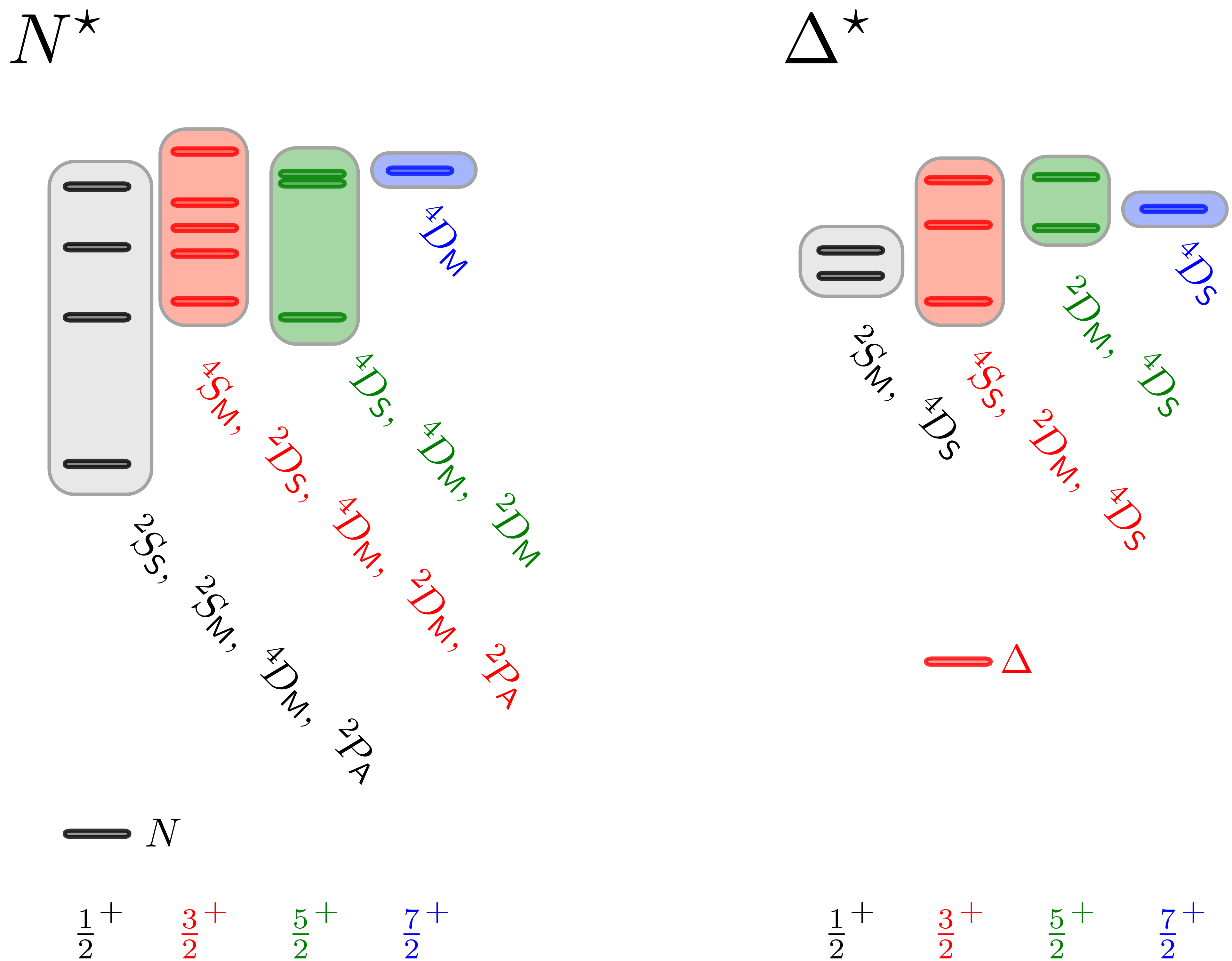
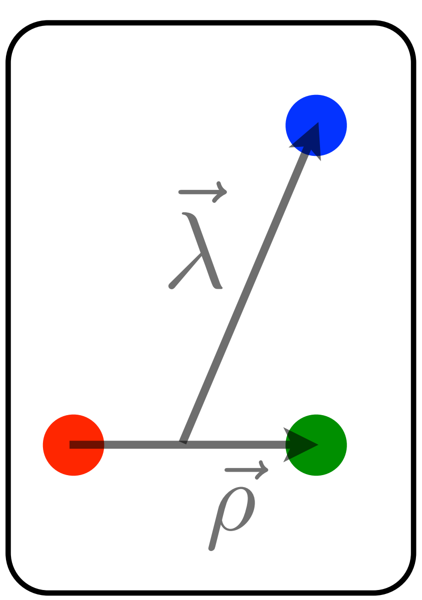
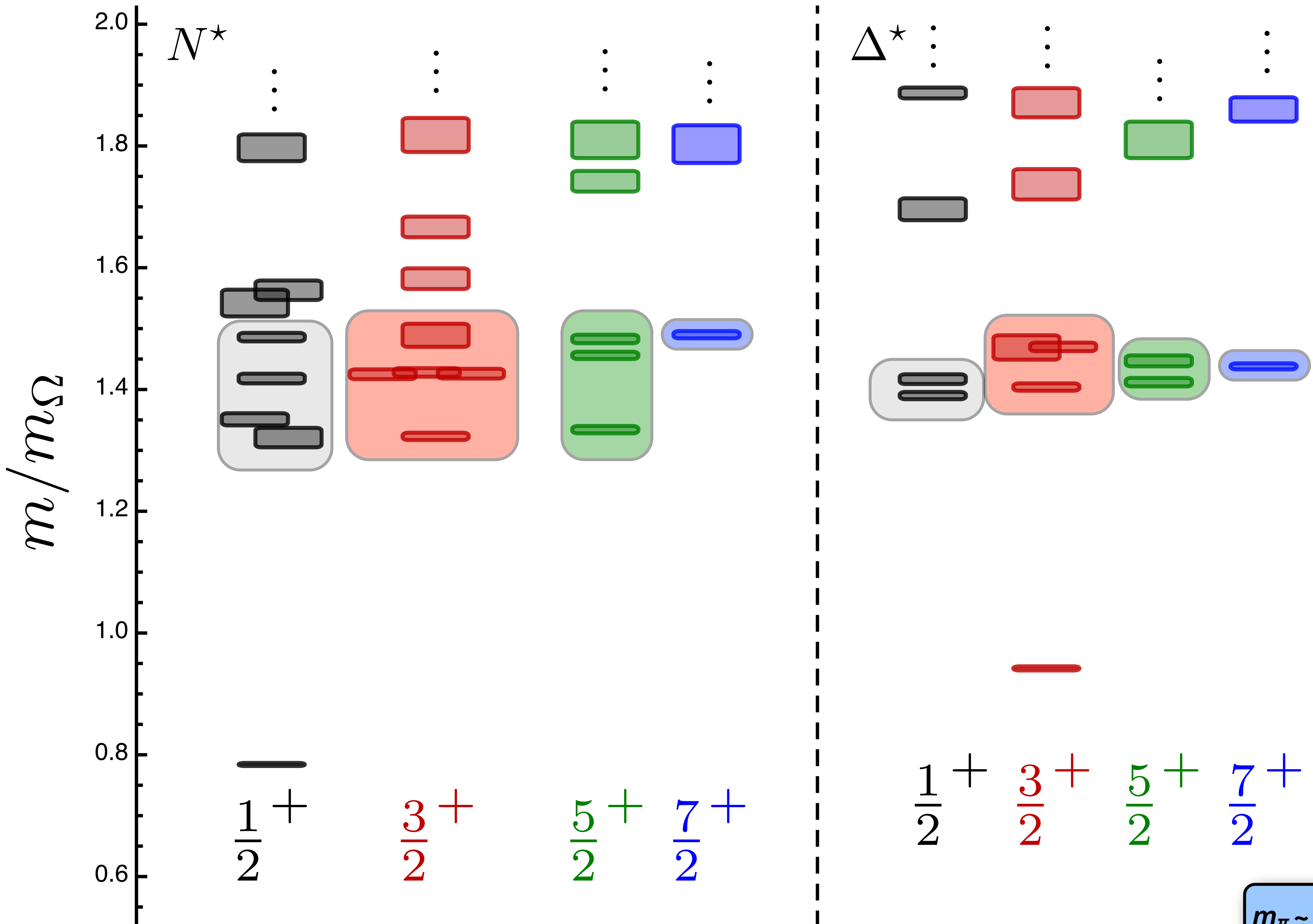




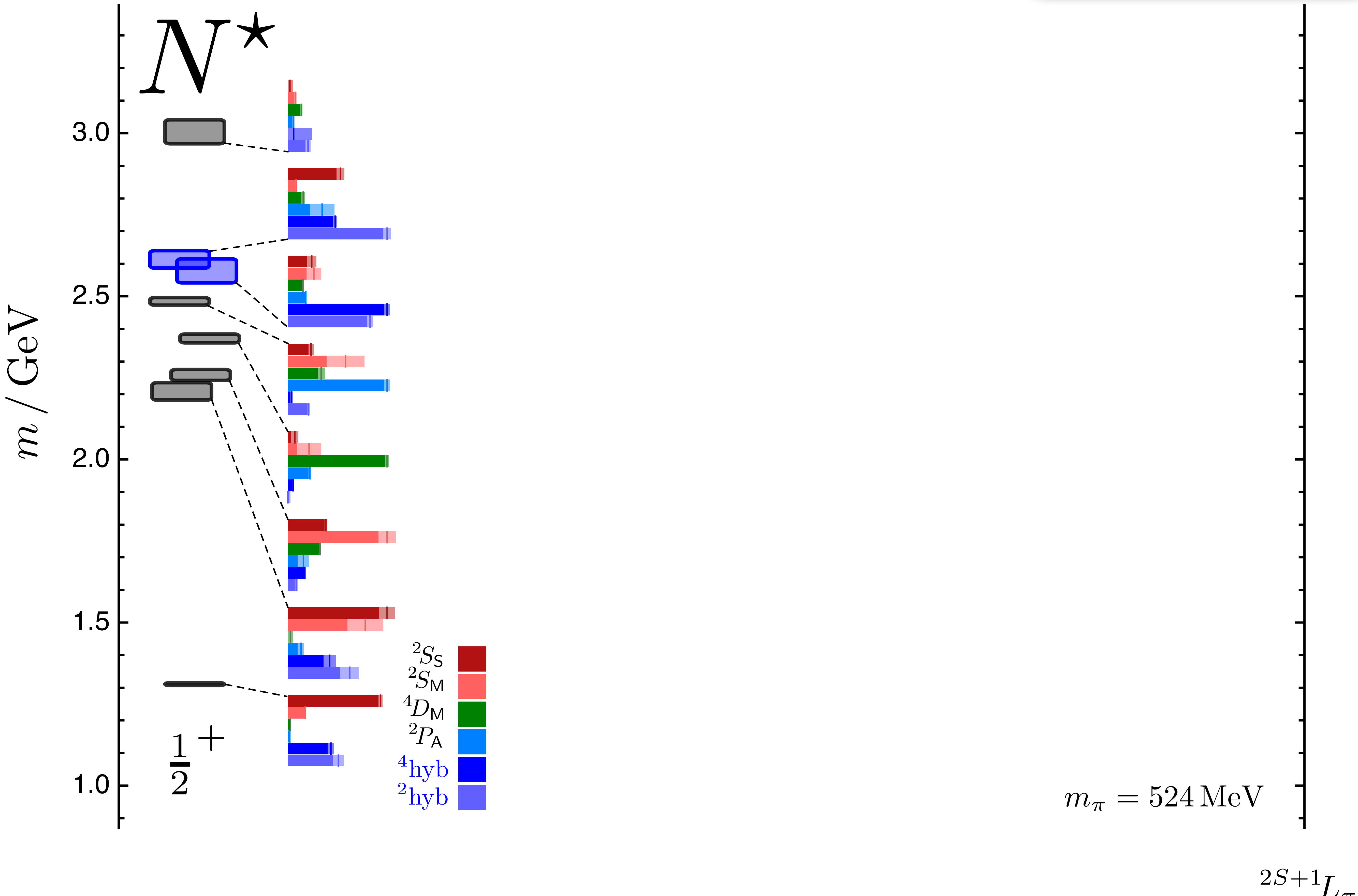
lattice QCD spectrum



m_π ~ 524 MeV

baryon matrix elements

$Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n} | \mathcal{O}_i | 0 \rangle$



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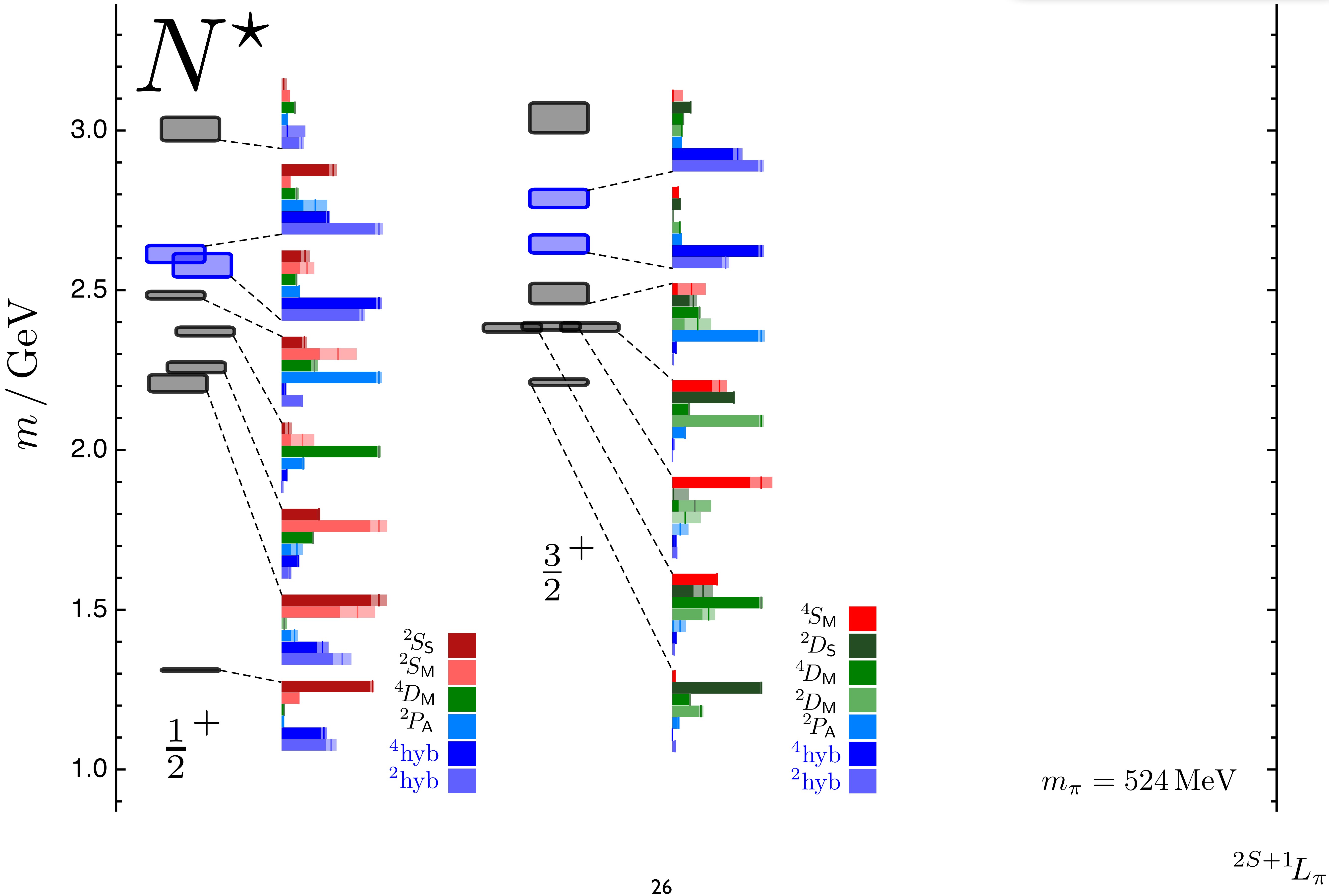

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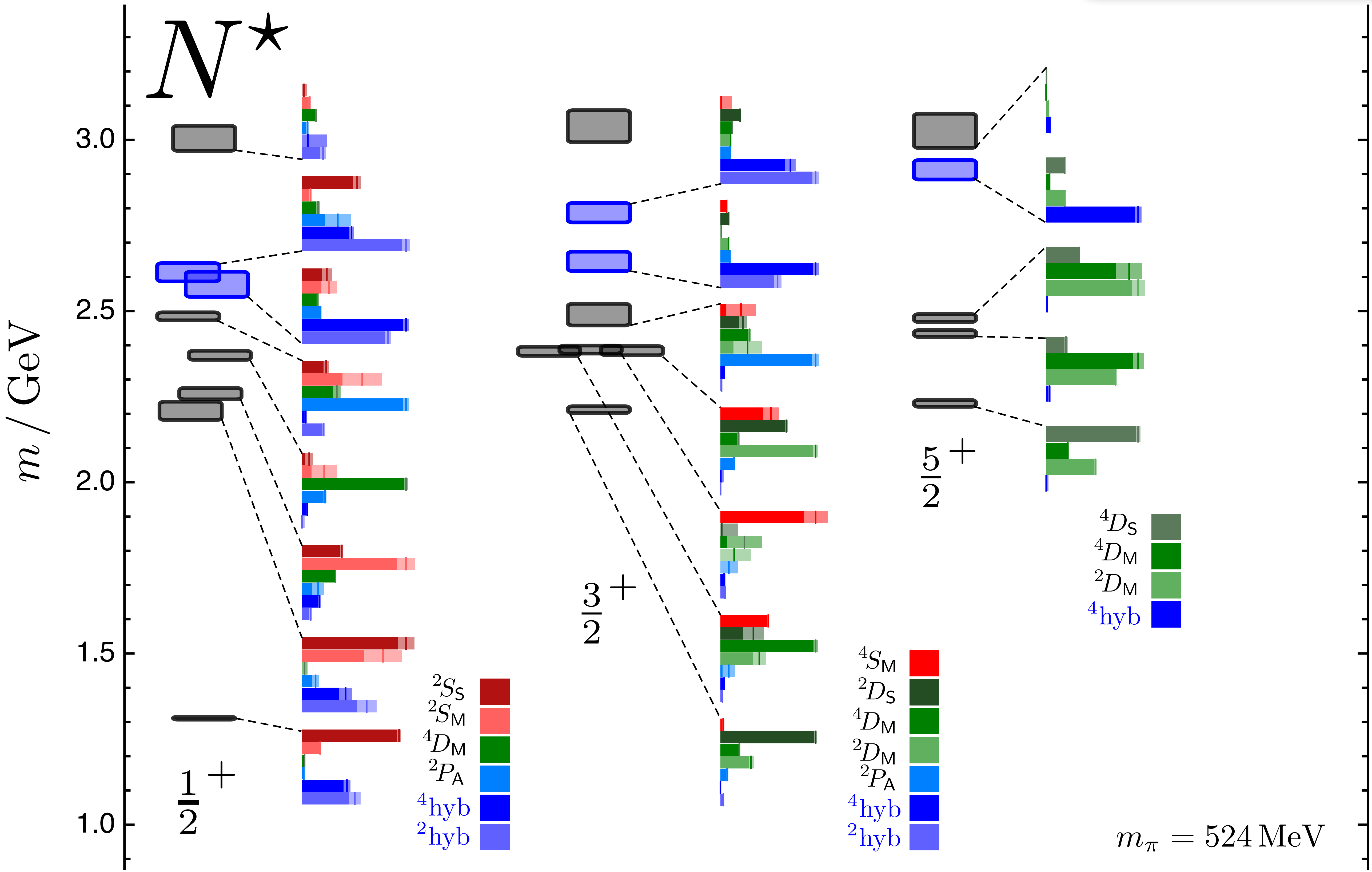
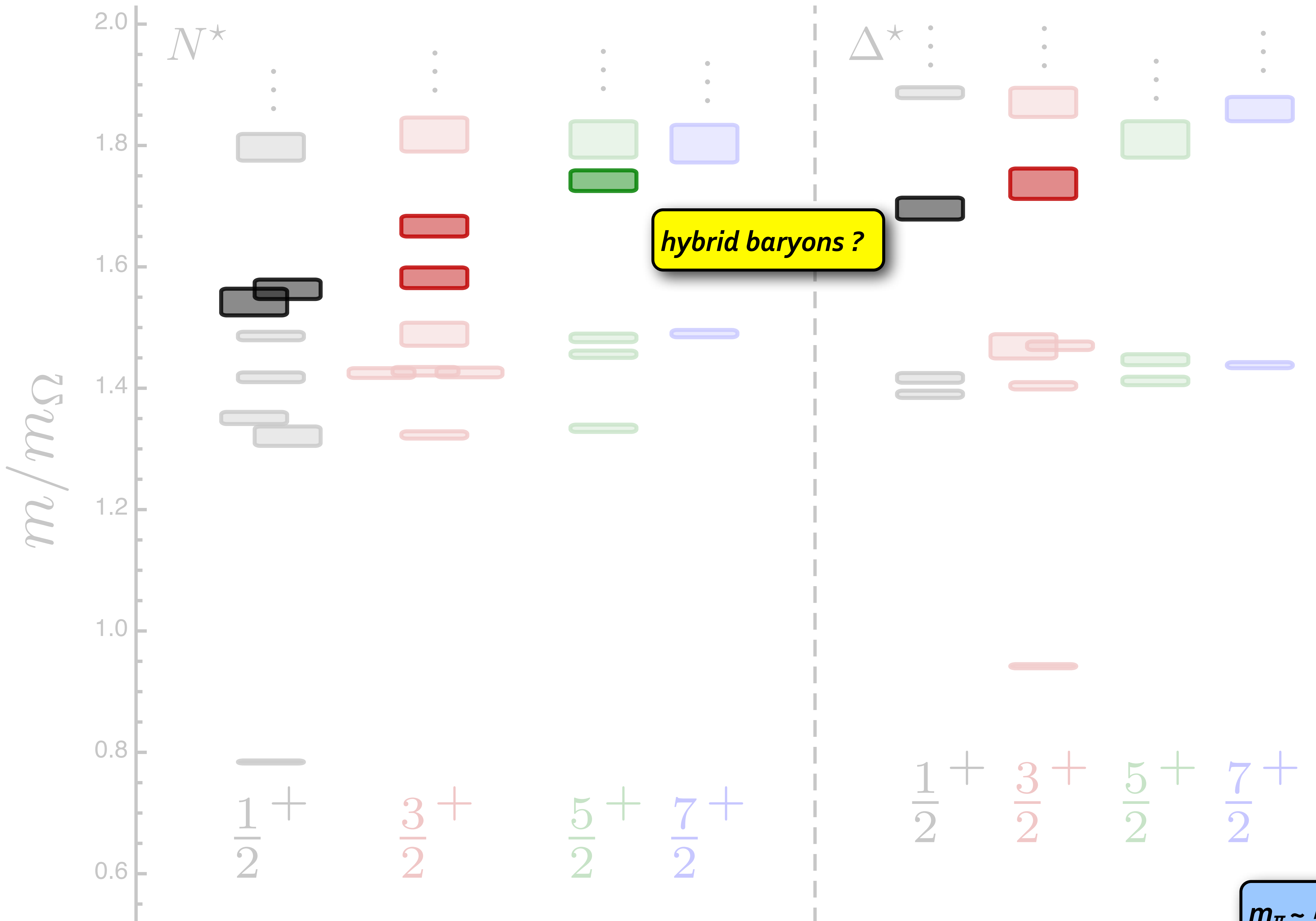


$^{2S+1}L_{\pi}$

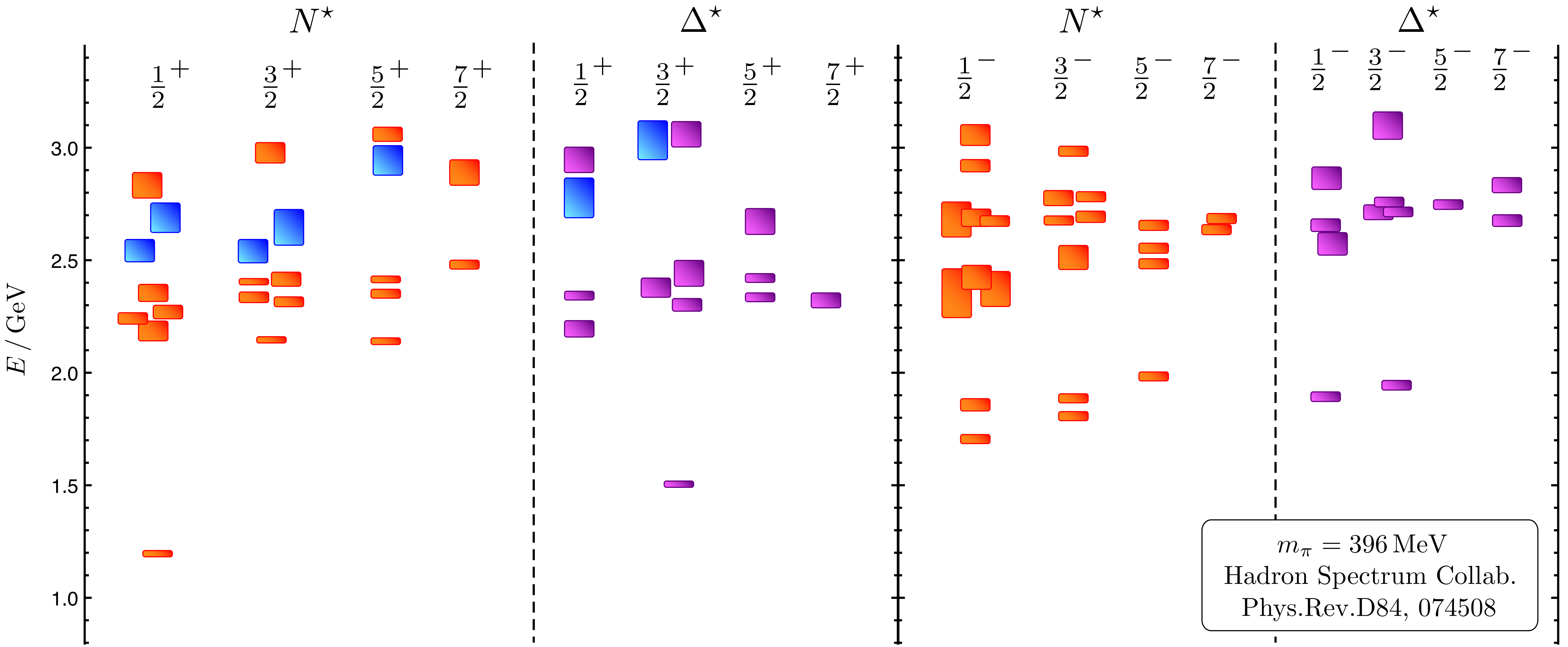
lattice QCD spectrum



hybrid baryons ?

m_π ~ 524 MeV

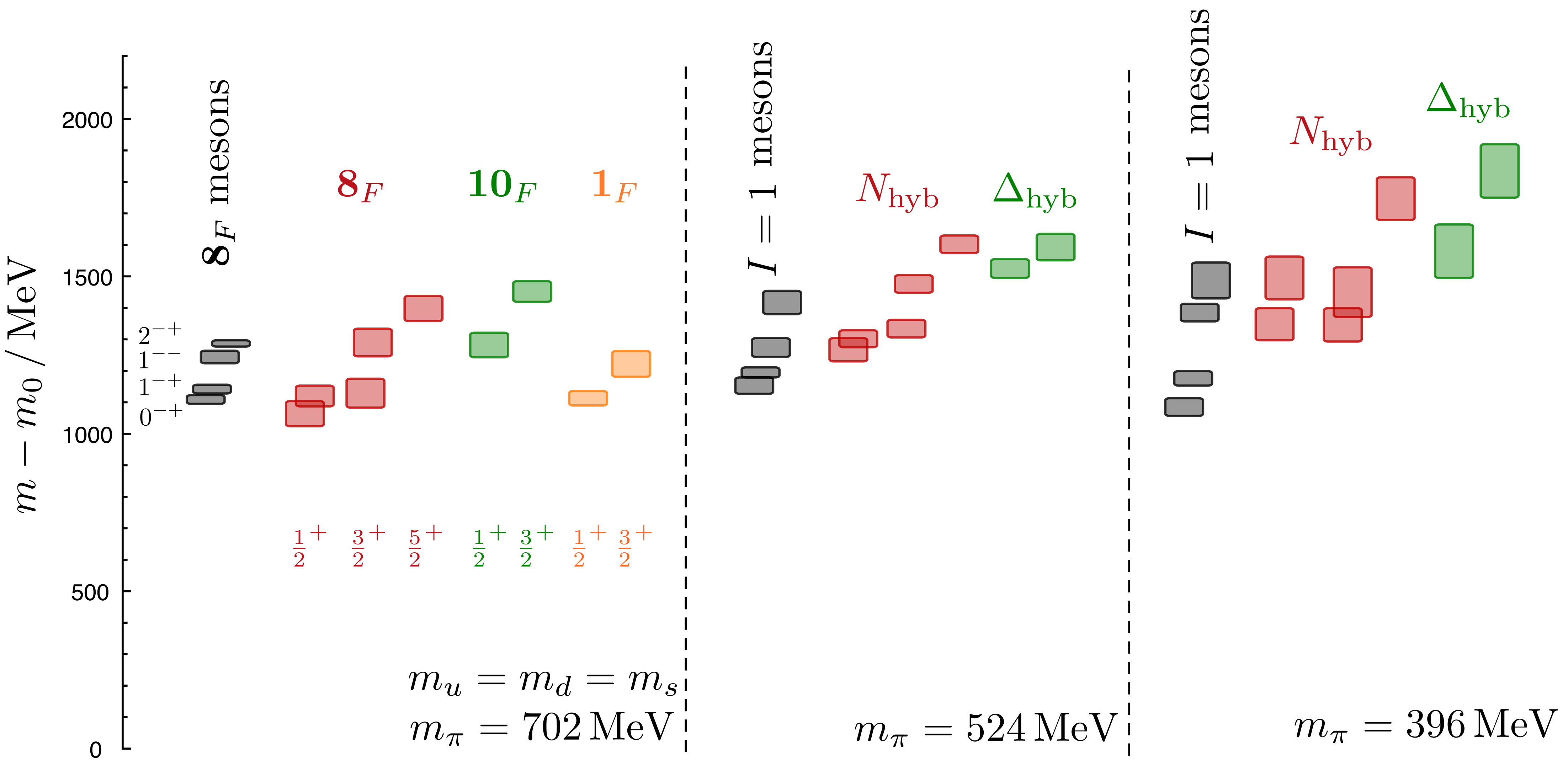
lattice QCD spectrum



$m_\pi \sim 396$ MeV

hybrid hadrons

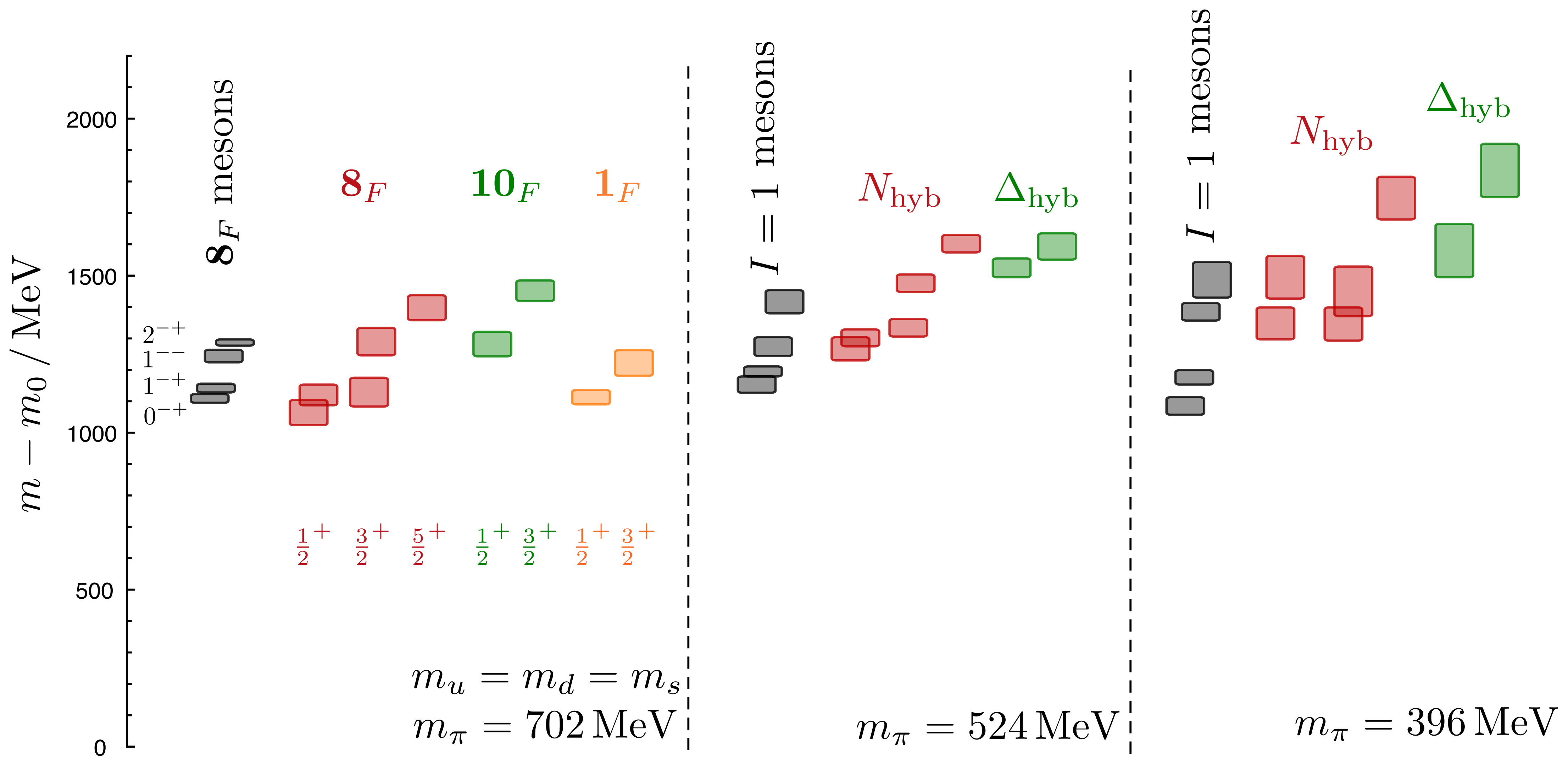
'subtract off' the quark masses



$$m_0 = \begin{cases} m_\rho & \text{mesons} \\ m_N & \text{baryons} \end{cases}$$

hybrid hadrons

'subtract off' the quark masses



$$m_0 = \begin{cases} m_\rho & \text{mesons} \\ m_N & \text{baryons} \end{cases}$$

appears to be a single scale for gluonic excitations in mesons or baryons
~ 1.3 GeV

gluonic excitation transforming like a color ***octet*** with ***J^{PC}=1^{+-}***

summary

excited state spectra of mesons and baryons extracted from lattice QCD

clear signals for hybrid hadrons in the excited spectra

gluonic excitation form appears to be common to mesons and baryons

might the gluonic excitation sector turn out to be relatively simple ?

this is definitely not the end of the story ...

... just the beginning of exploring gluonic excitations starting from QCD

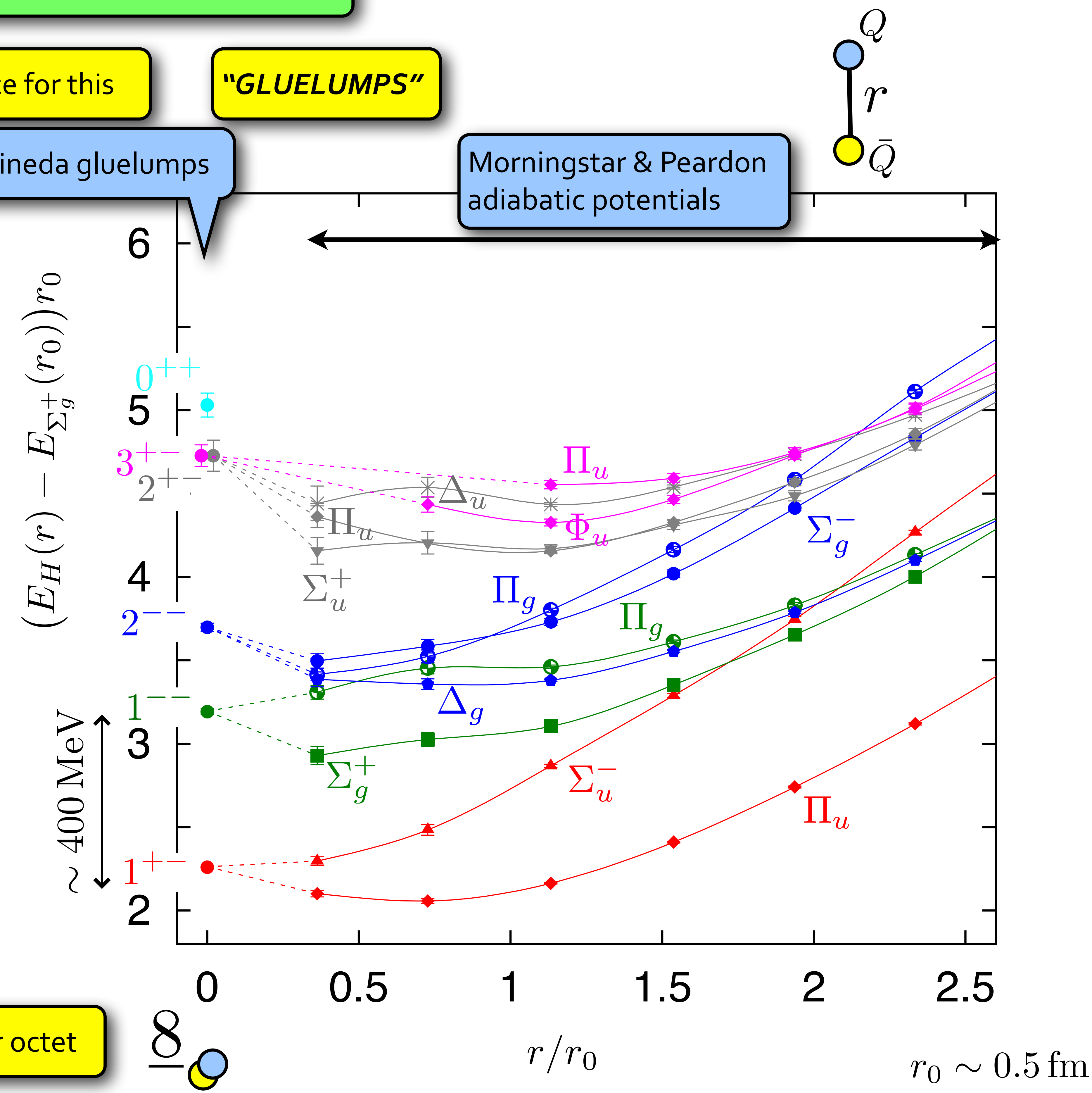
$J_g^{P_g} = 1^+$ **lighter than** $J_g^{P_g} = 1^-$ **?**

this is not the first evidence for this

"GLUELUMPS"

Bali & Pineda gluelumps

Morningstar & Peardon adiabatic potentials



energy of a static color octet

lattice QCD

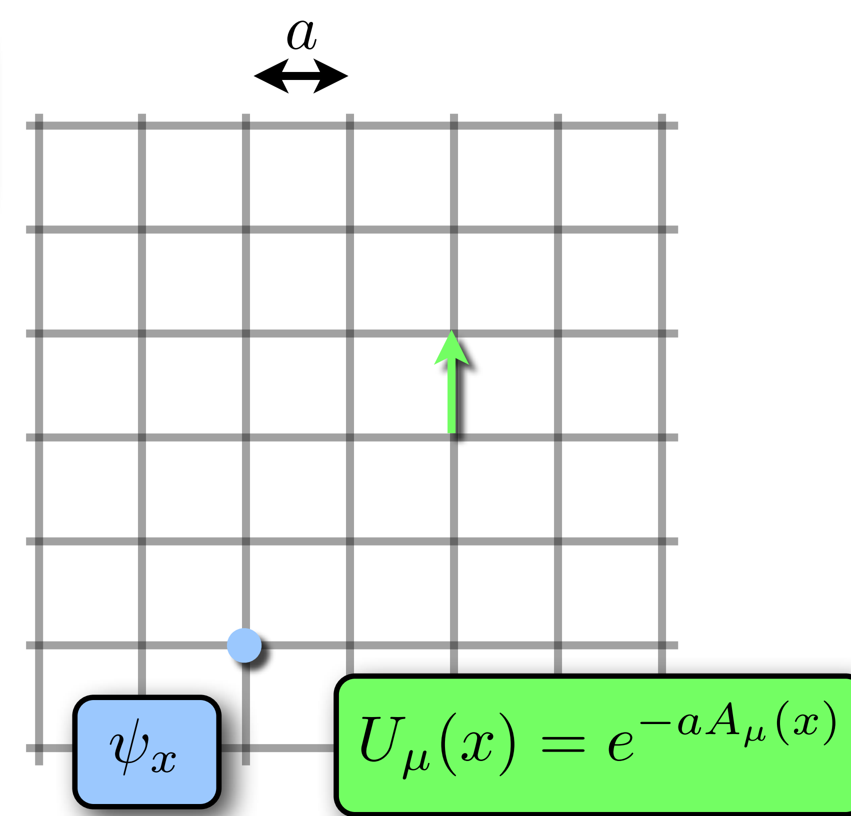
QCD action : $S_{\text{QCD}} = \int d^4x \left[\bar{\psi}(x) \left(i\not{D} - m \right) \psi(x) + \frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} \right]$

lattice QCD

QCD action : $S_{\mathrm{QCD}} = \int d^4x \left[\bar{\psi}(x) \left(i\not{D} - m \right) \psi(x) + \frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} \right]$

discretise on a finite grid

$$S^{[a]}_{\mathrm{QCD}} = \sum_{x,y} \bar{\psi}_x \mathsf{Q}[U]_{xy} \psi_y + S_g[U]$$



lattice QCD

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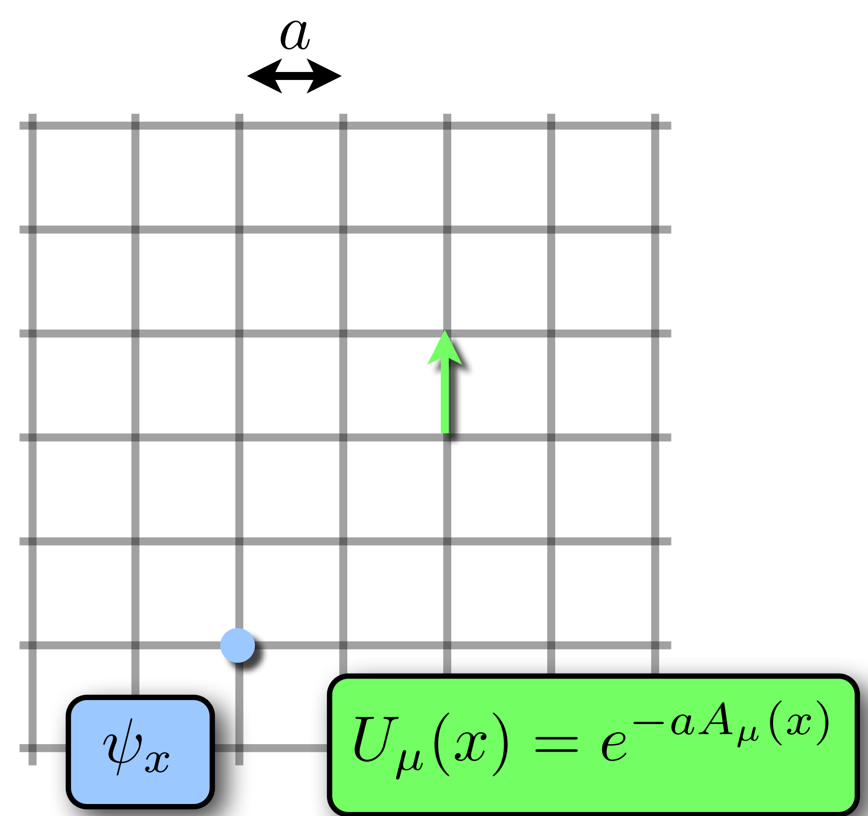
$$S^{[a]}_{\mathrm{QCD}} = \sum_{x,y} \bar{\psi}_x \mathsf{Q}[U]_{xy}\psi_y + S_g[U]$$

QCD path integral : $\mathcal{Z} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \; e^{iS_{\mathrm{QCD}}[\bar{\psi},\psi,A_\mu]}$

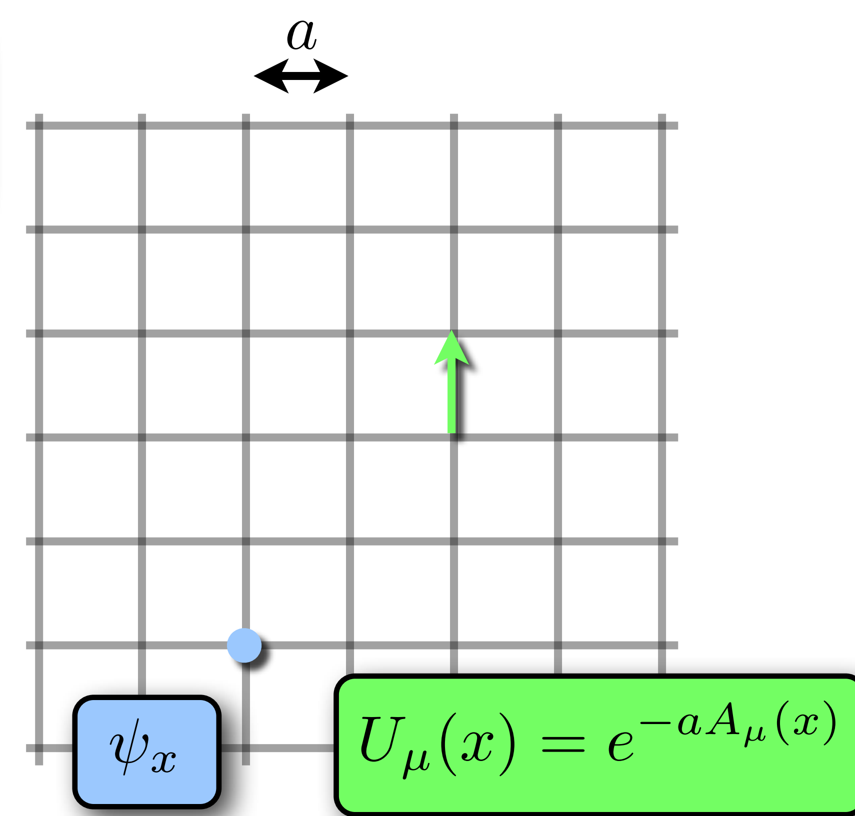


lattice QCD

QCD action : $S_{\mathrm{QCD}} = \int d^4x \left[\bar{\psi}(x) \left(i\not{D} - m \right) \psi(x) + \frac{1}{2g^2} F_{\mu\nu}F^{\mu\nu} \right]$

discretise on a finite grid

$$S_{\mathrm{QCD}}^{[a]} = \sum_{x,y} \bar{\psi}_x \mathsf{Q}[U]_{xy} \psi_y + S_g[U]$$



QCD path integral : $\mathcal{Z} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \; e^{iS_{\mathrm{QCD}}[\bar{\psi},\psi,A_\mu]}$

to Euclidean time

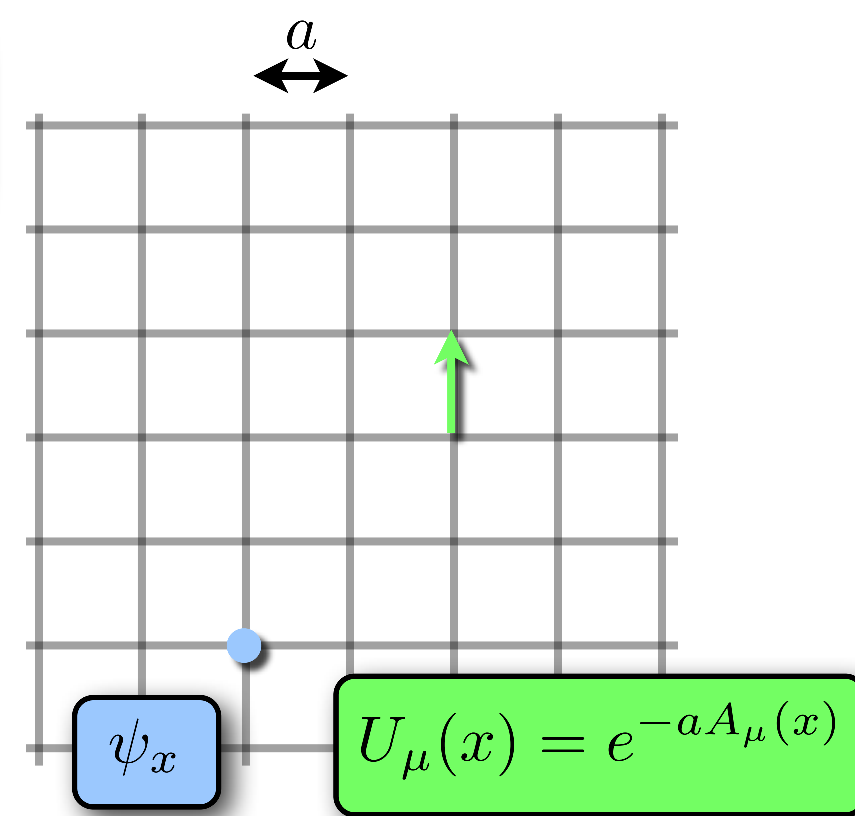
$$\mathcal{Z}_{[\mathrm{E}]} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \; e^{-S_{\mathrm{QCD}}^{[\mathrm{E}]}[\bar{\psi},\psi,A_\mu]}$$

lattice QCD

QCD action : $S_{\text{QCD}} = \int d^4x \left[\bar{\psi}(x) \left(i\not{D} - m \right) \psi(x) + \frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} \right]$

discretise on a finite grid

$$S^{[a]}_{\text{QCD}} = \sum_{x,y} \bar{\psi}_x \mathsf{Q}[U]_{xy} \psi_y + S_g[U]$$



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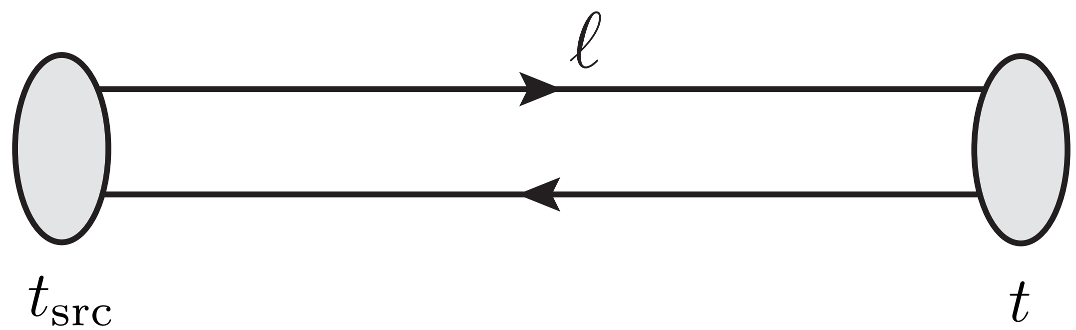
to Euclidean time

$$\mathcal{Z}_{[\text{E}]} = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_\mu \; e^{-S^{[\text{E}]}_{\text{QCD}}[\bar{\psi}, \psi, A_\mu]}$$

perform quark integration

$$\mathcal{Z}_{[\text{E}]} = \sum_{\{U\}} \det \mathsf{Q}[U] \, e^{-S_g[U]}$$

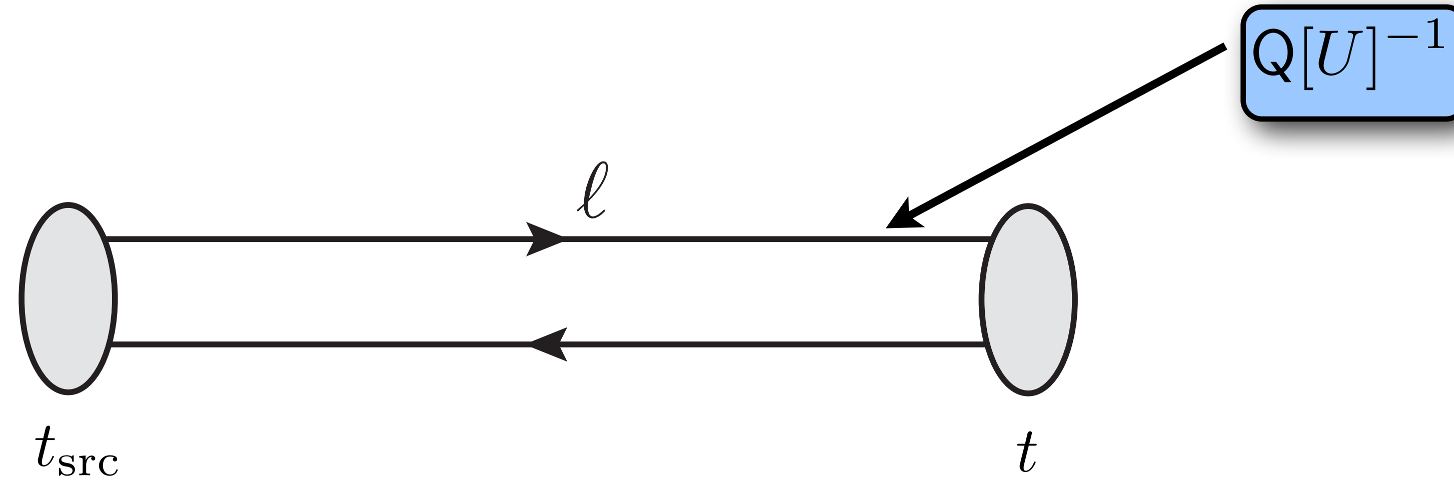
isoscalar mesons

isovector correlator : $C_{ij}^{[I=1]}(t, t_{\rm src}) =$ 

isoscalar mesons

isovector correlator :

$$C_{ij}^{[I=1]}(t, t_{\rm src}) =$$



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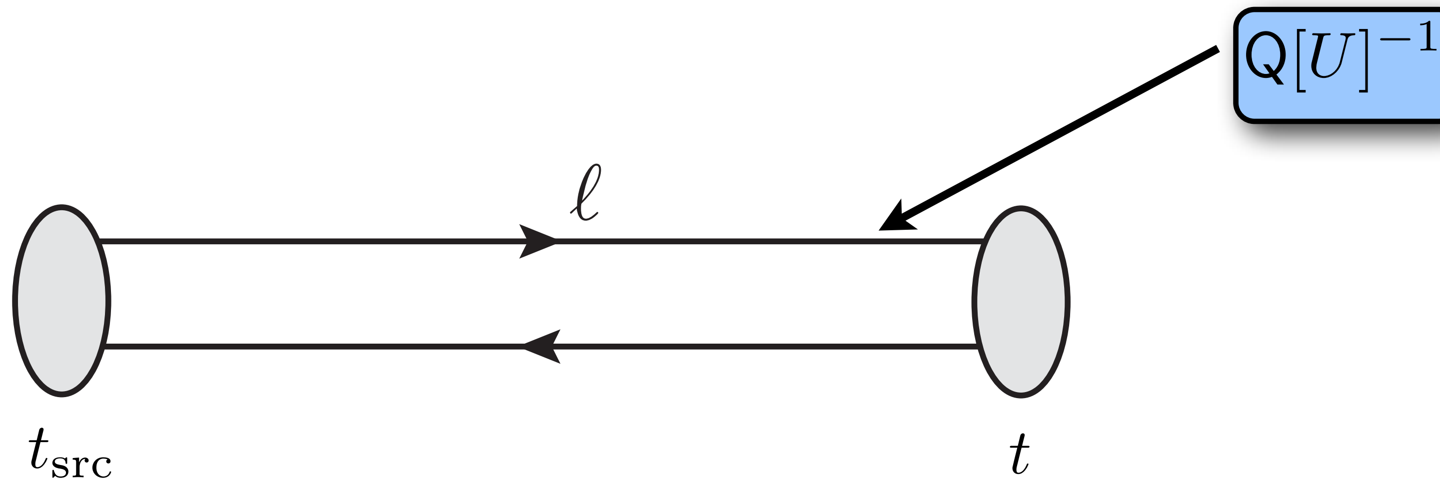
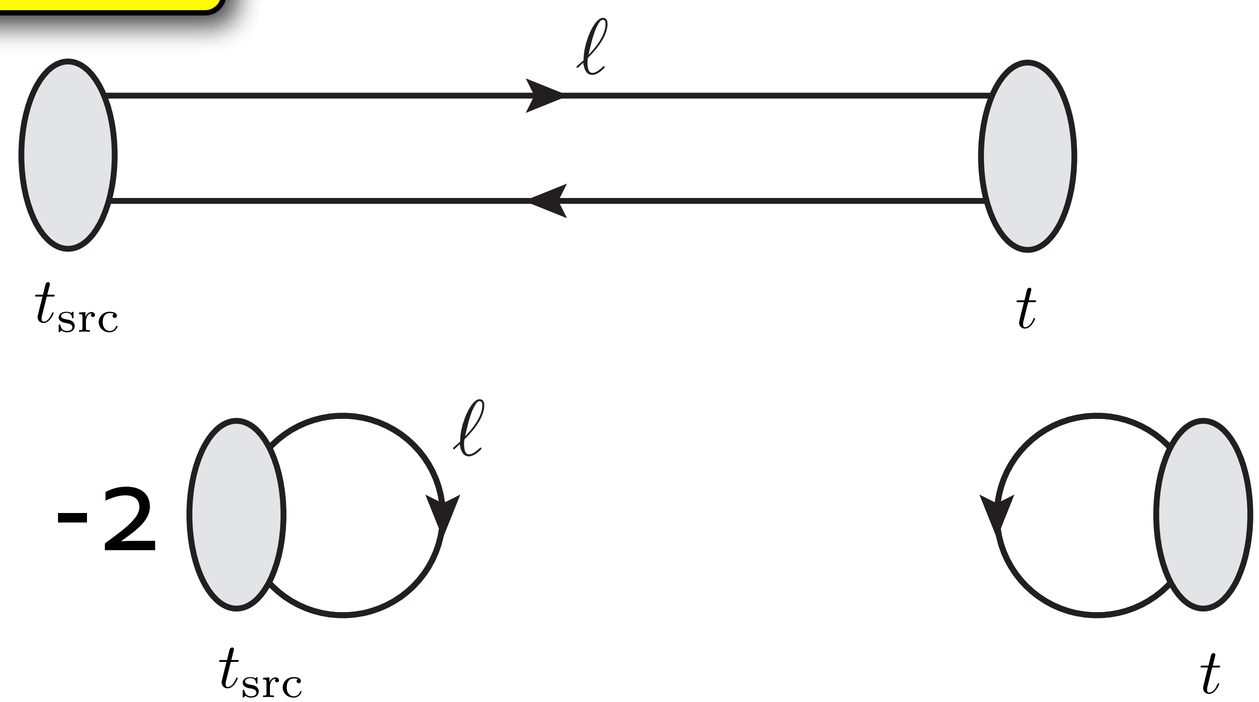


isoscalar (with just light quarks) correlator :

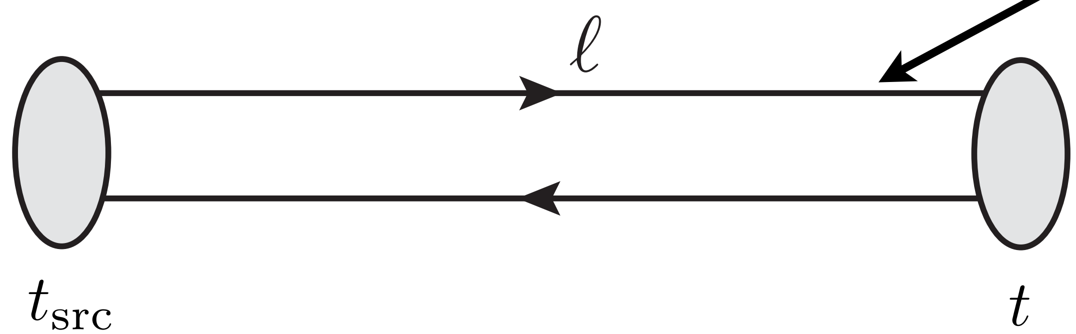
$$C_{ij}^{[I=0]}(t, t_{\rm src}) =$$



isoscalar mesons

isovector correlator :

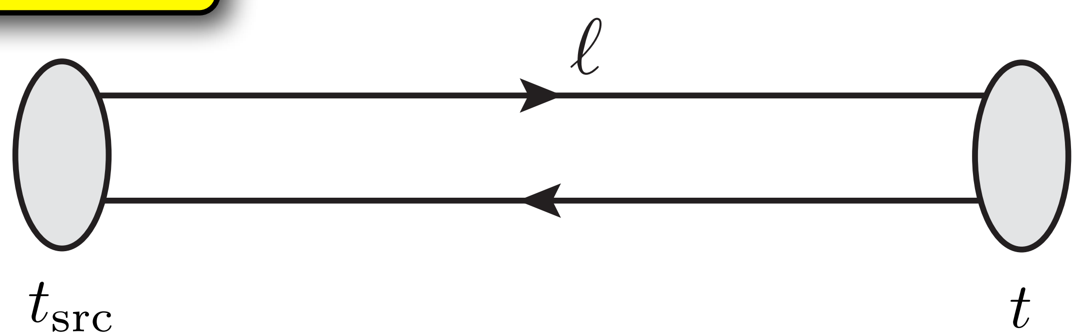
$$C_{ij}^{[I=1]}(t, t_{\rm src}) =$$



$\mathrm{Q}[U]^{-1}$

isoscalar (with just light quarks) correlator :

$$C_{ij}^{[I=0]}(t, t_{\rm src}) =$$



-2

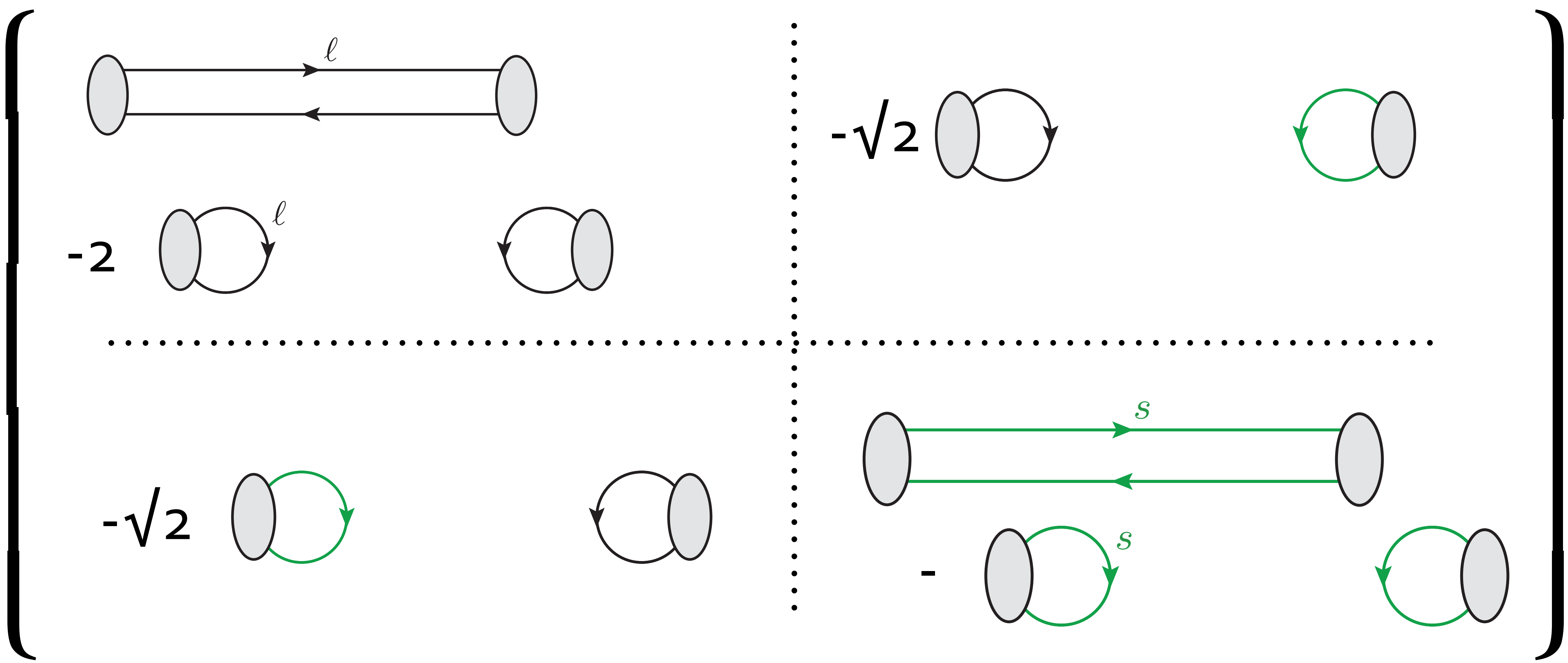
noisy

need inversion from all ***t***

GPUs ideal for the ‘grunt’ work

isoscalar mesons

isoscalar (with light & strange) :



diagonalising gives the $\ell\bar{\ell}, s\bar{s}$ mixing

isoscalar spectrum

very few results due to the difficulty of calculation

C. Michael et al (UKQCD, 2001) [heavy quarks, 2-flavour theory] found f_1/a_1 , b_1/h_1 , ρ/ω splittings consistent with zero

ETMC (2009) [2-flavour, extrap. to phys. quark mass] ρ/ω splitting of 27(10) MeV

isoscalar spectrum

RBC/UKQCD (2010)

very few results due to the difficulty of calculation

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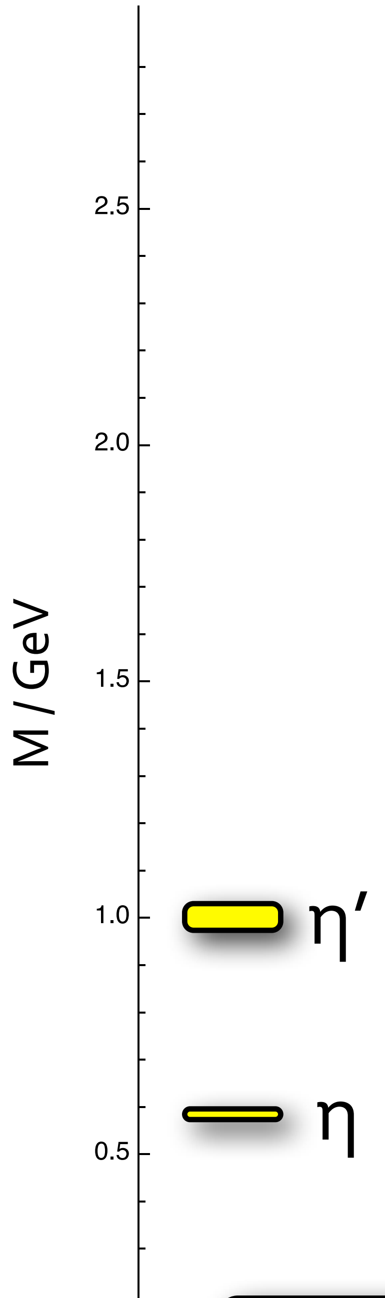
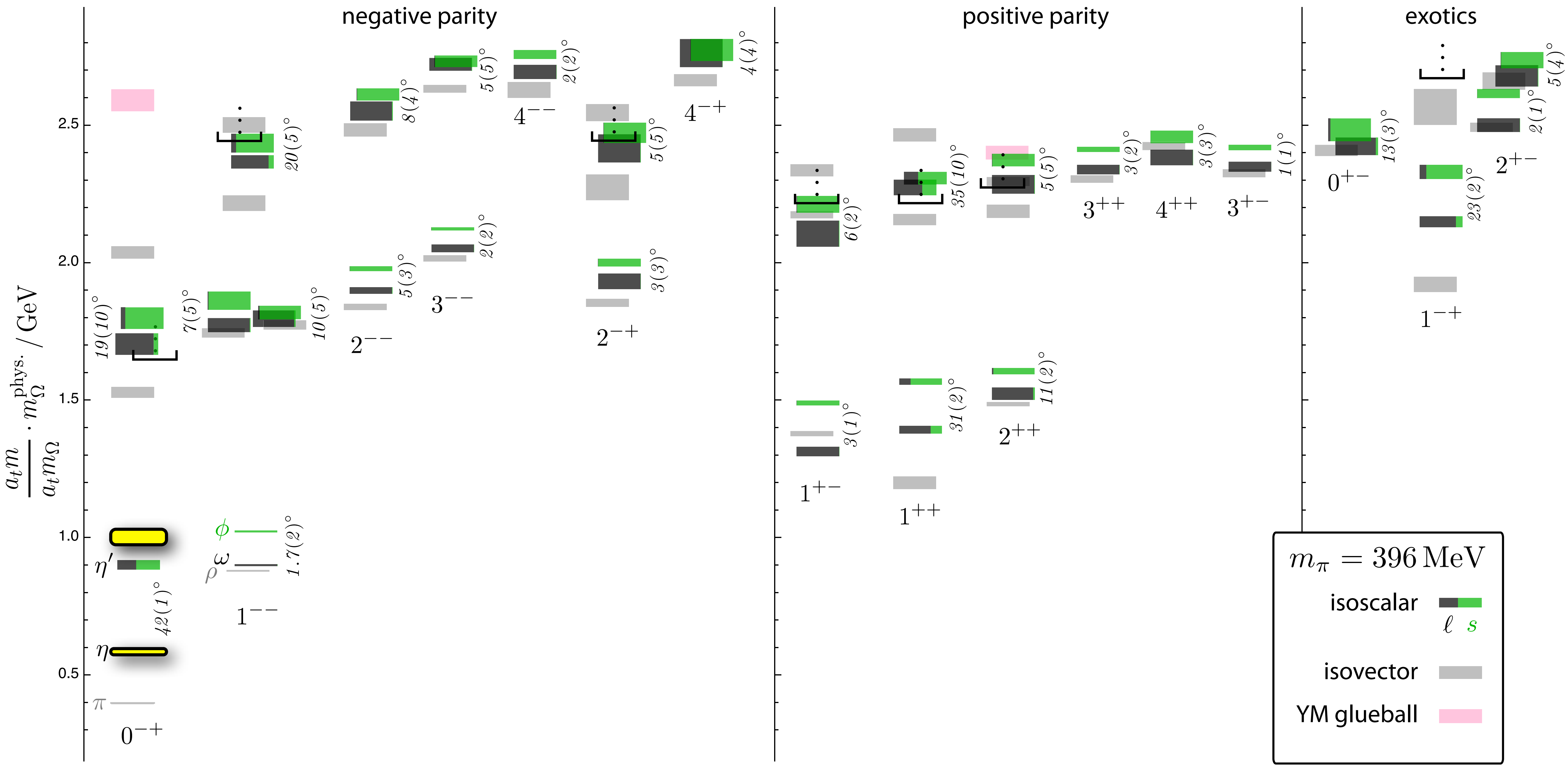


***m_π ~ 420* MeV**

isoscalar spectrum



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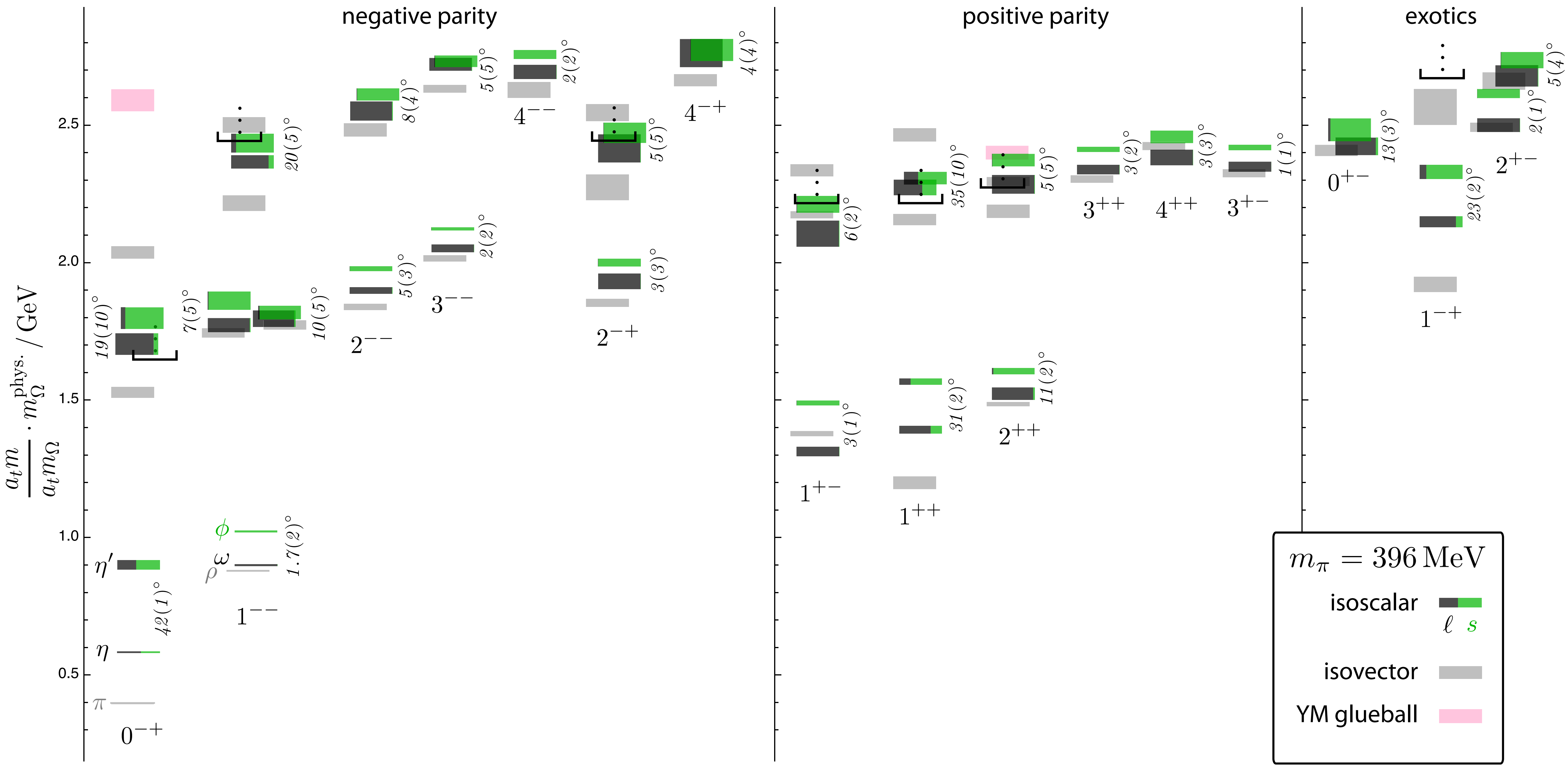

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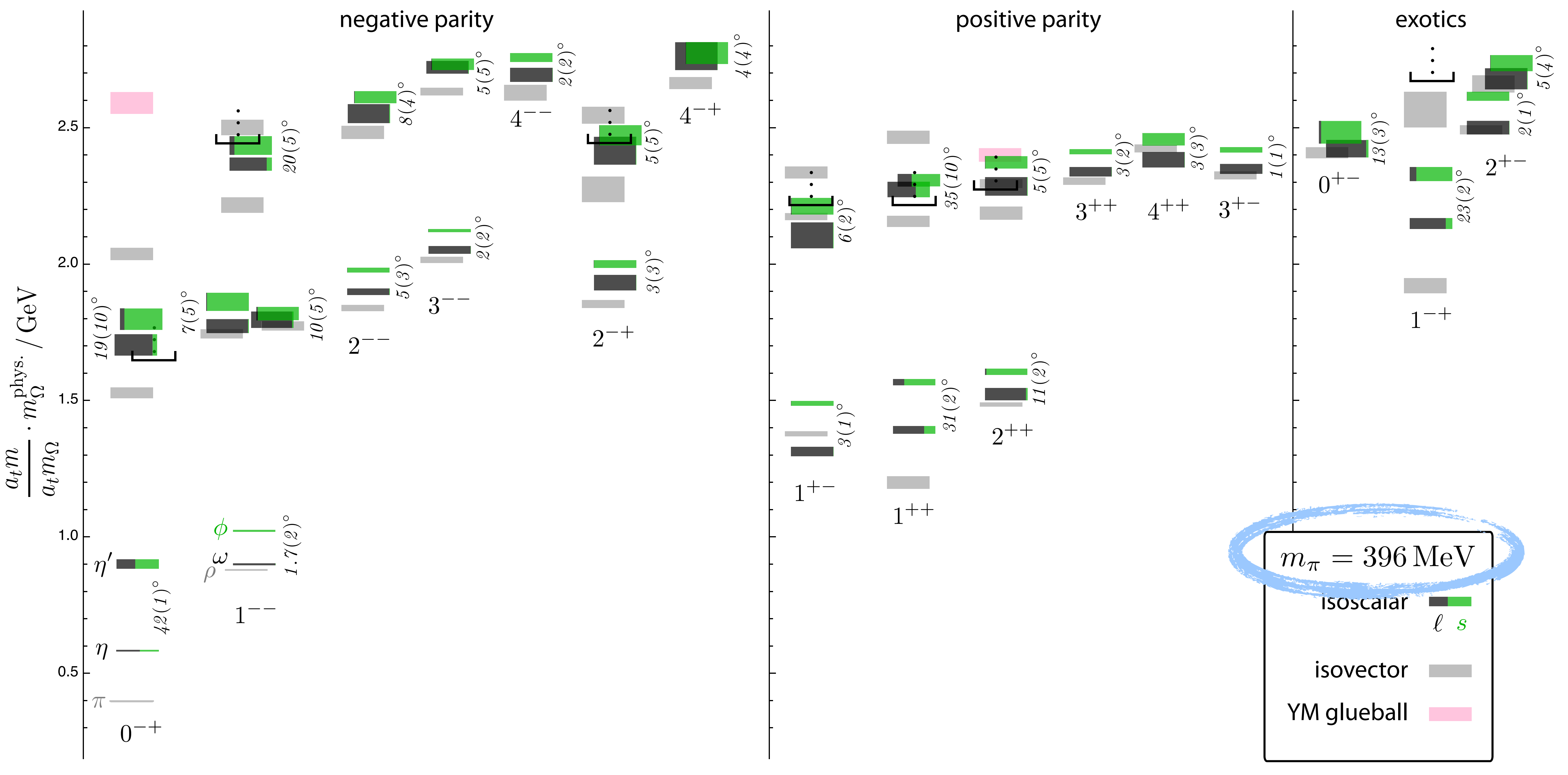
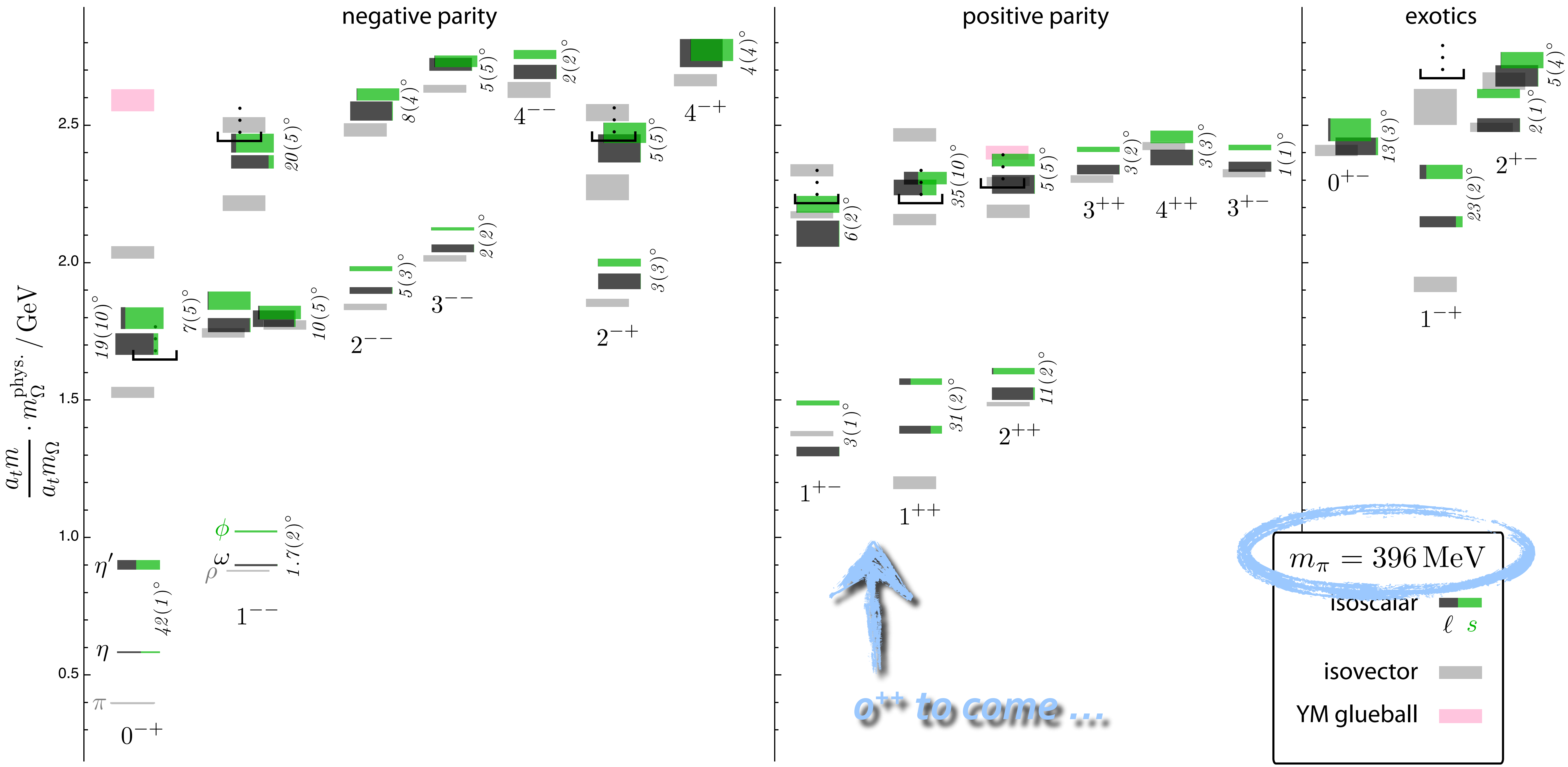
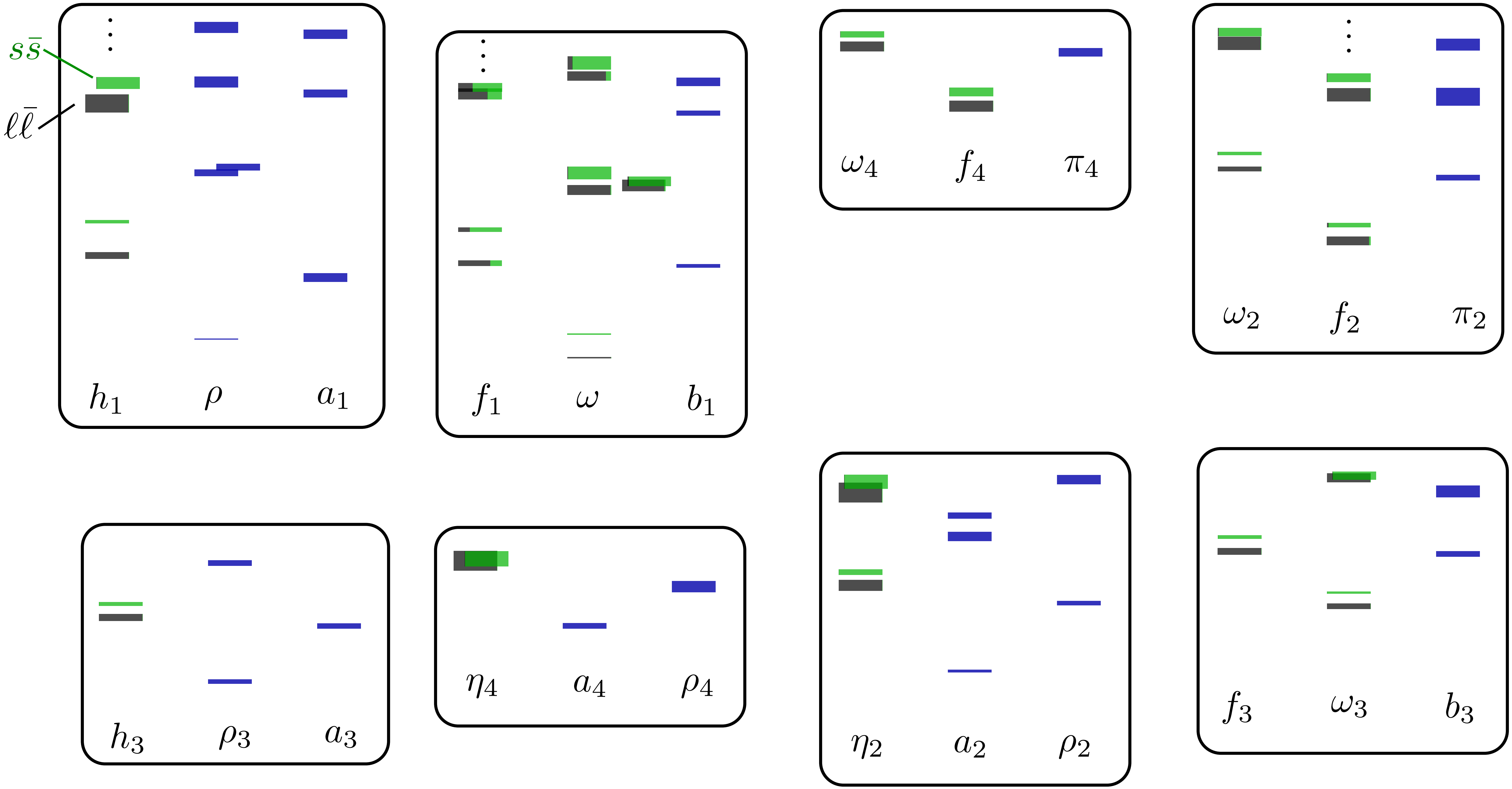


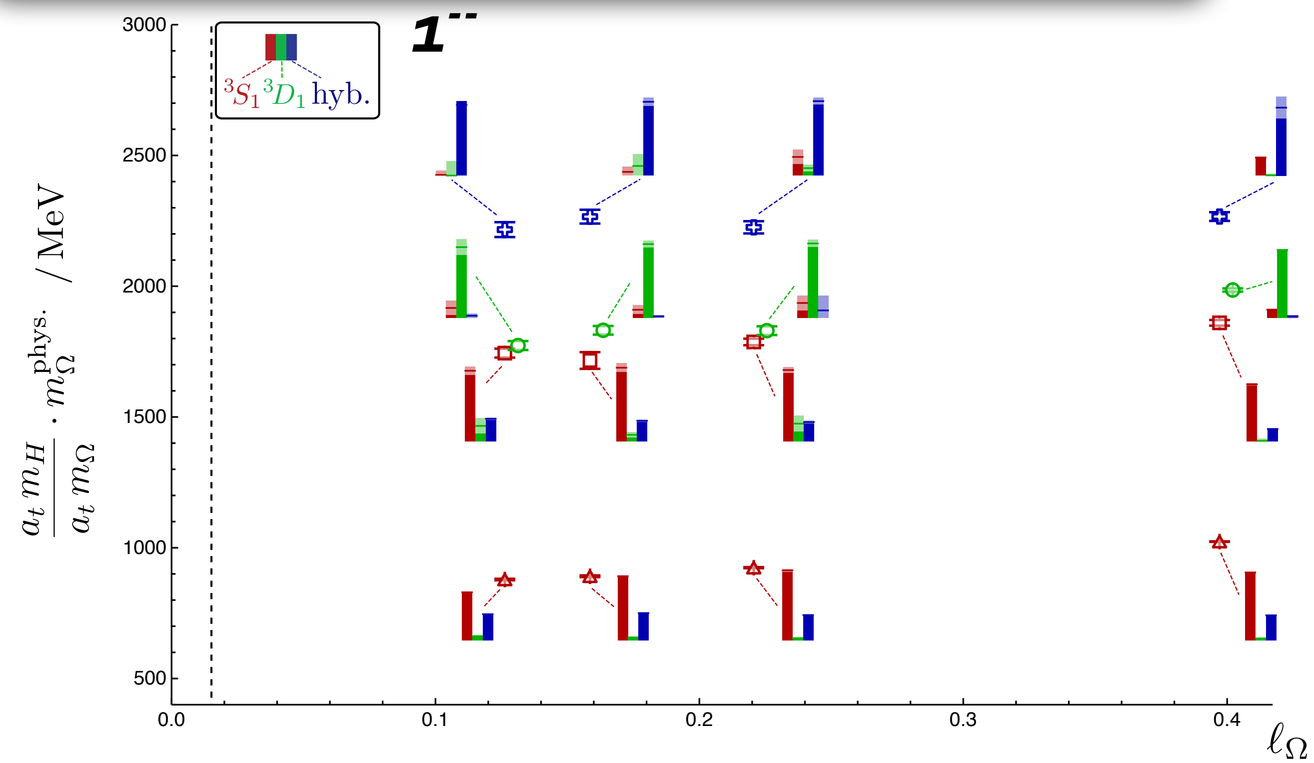
isoscalar spectrum

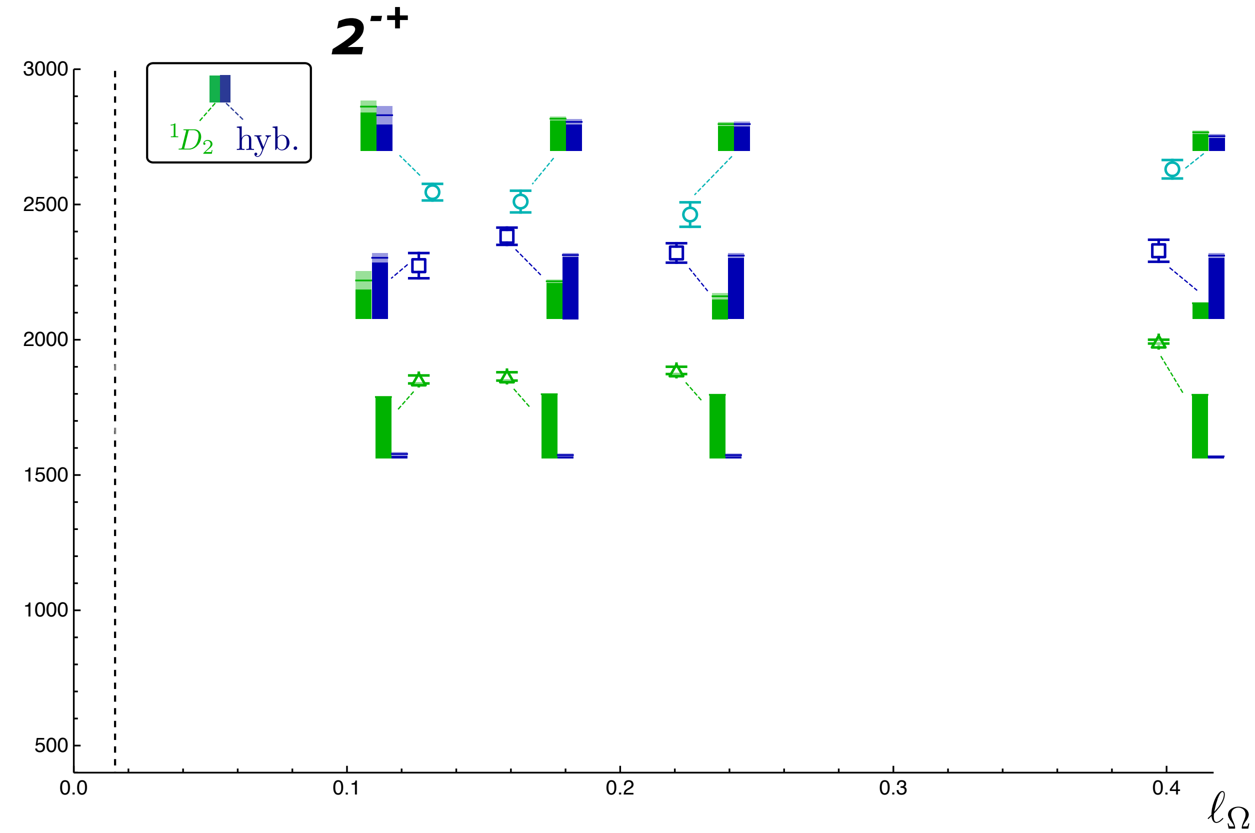


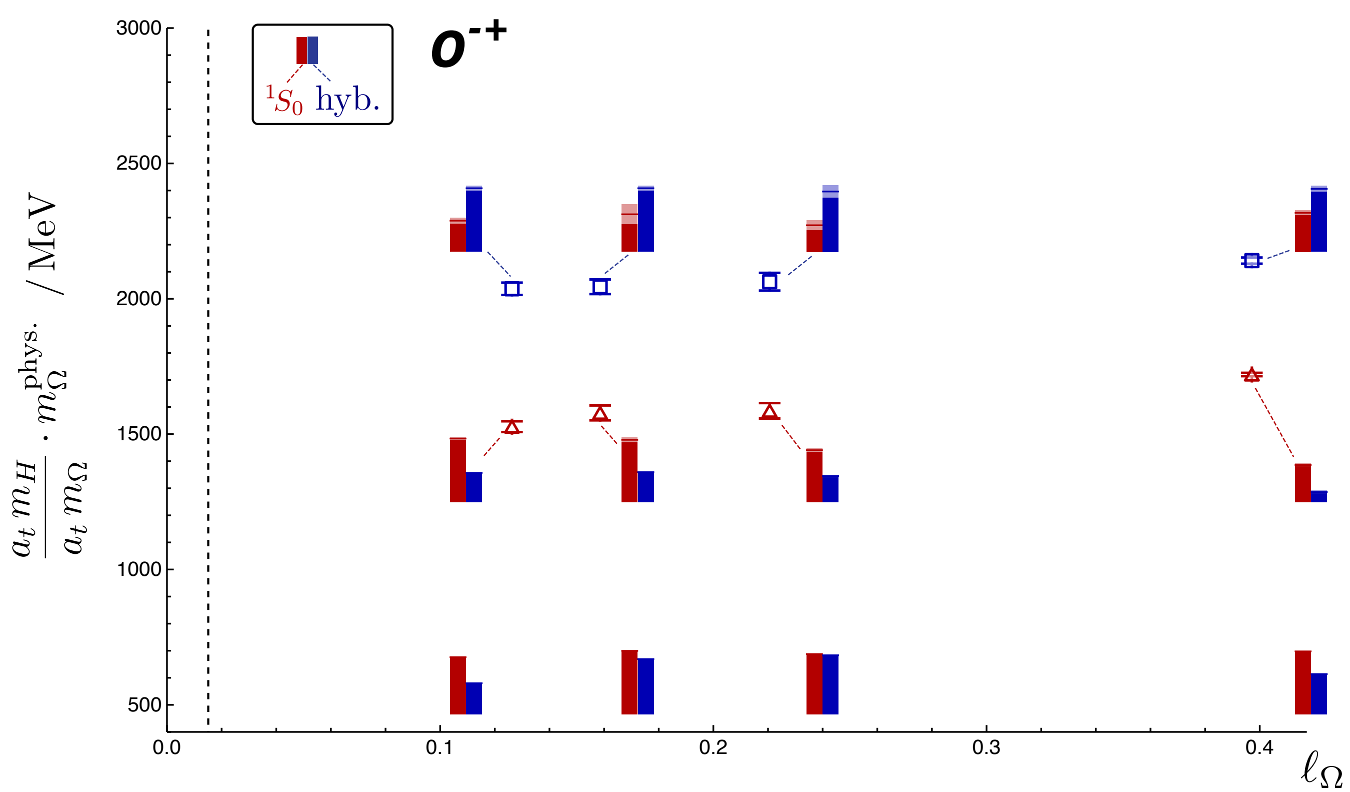
parity doubling & chiral symmetry restoration ?



overlaps with decreasing quark mass







cubic complications ...

integer spin not a good quantum number

restricted rotational symmetry of a cube

A_1	$0, 4 \ldots$
T_1	$1, 3, 4 \ldots$
T_2	$2, 3, 4 \ldots$
E	$2, 4 \ldots$
A_2	$3 \ldots$

cubic complications ...

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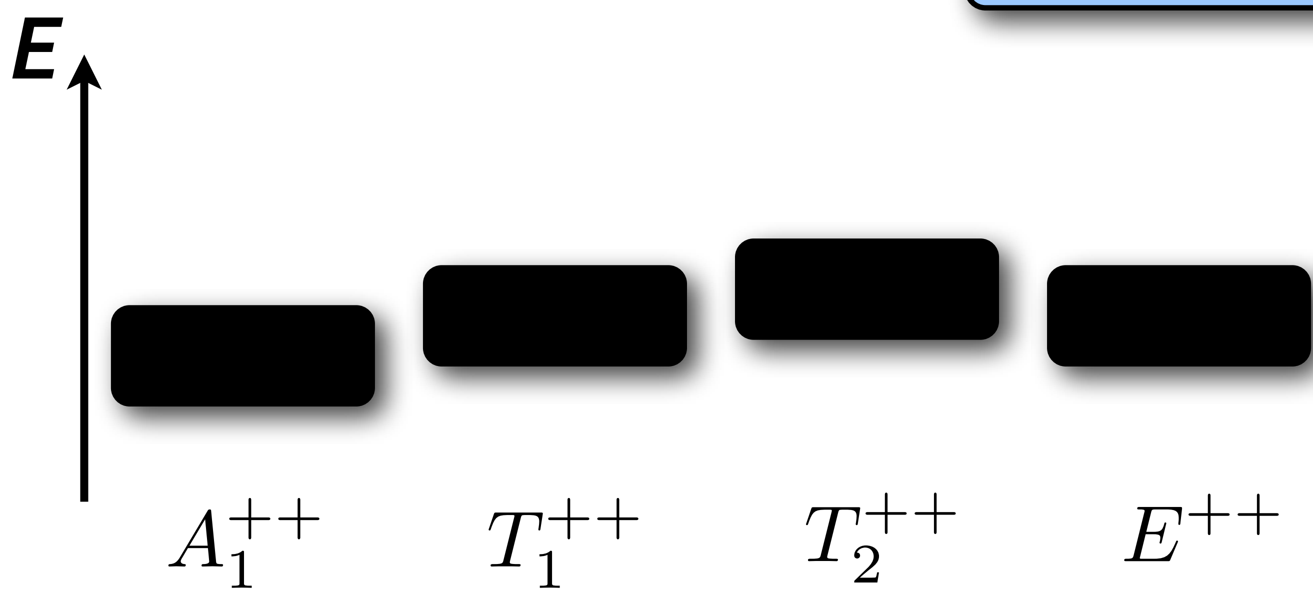

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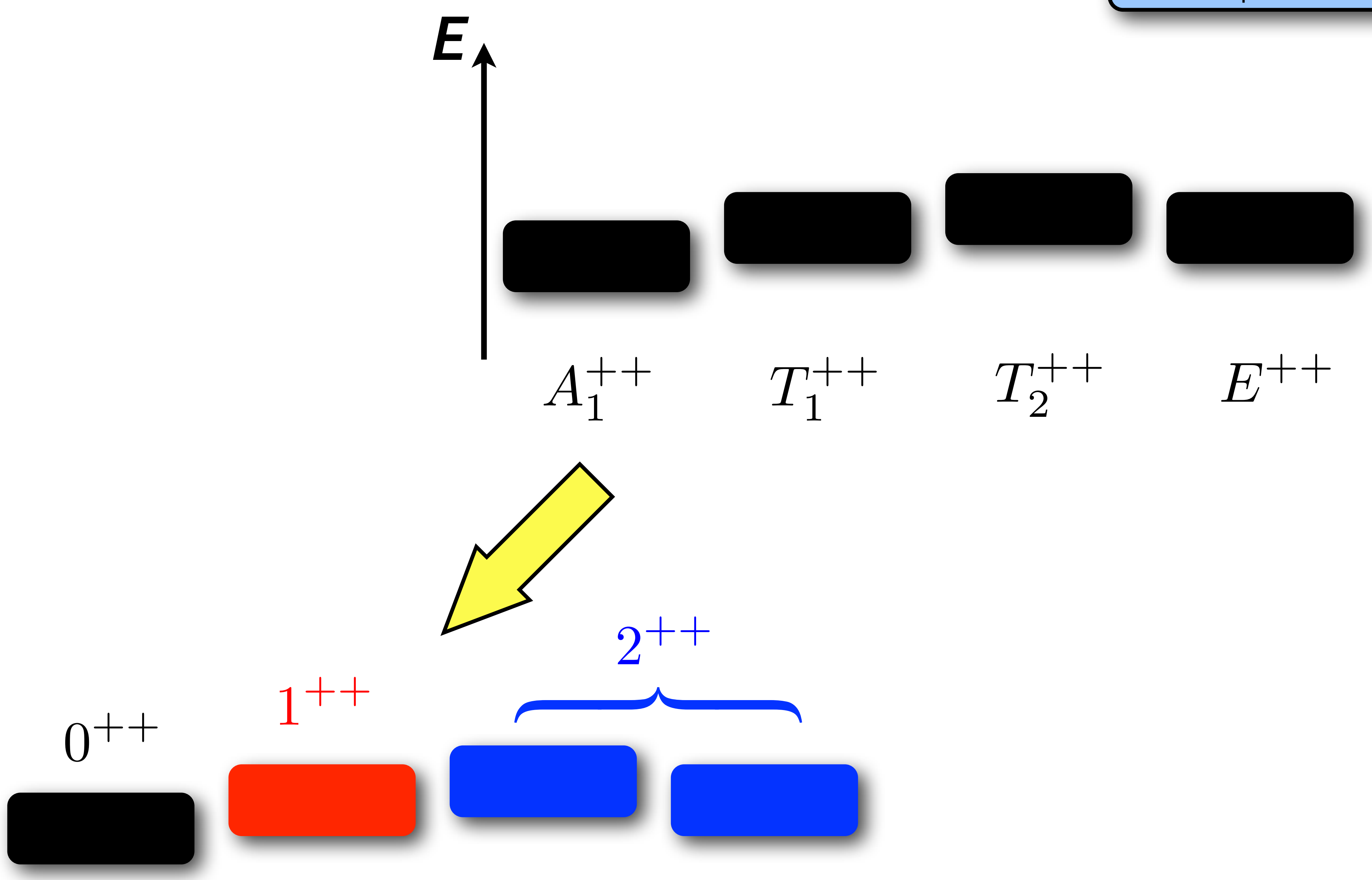

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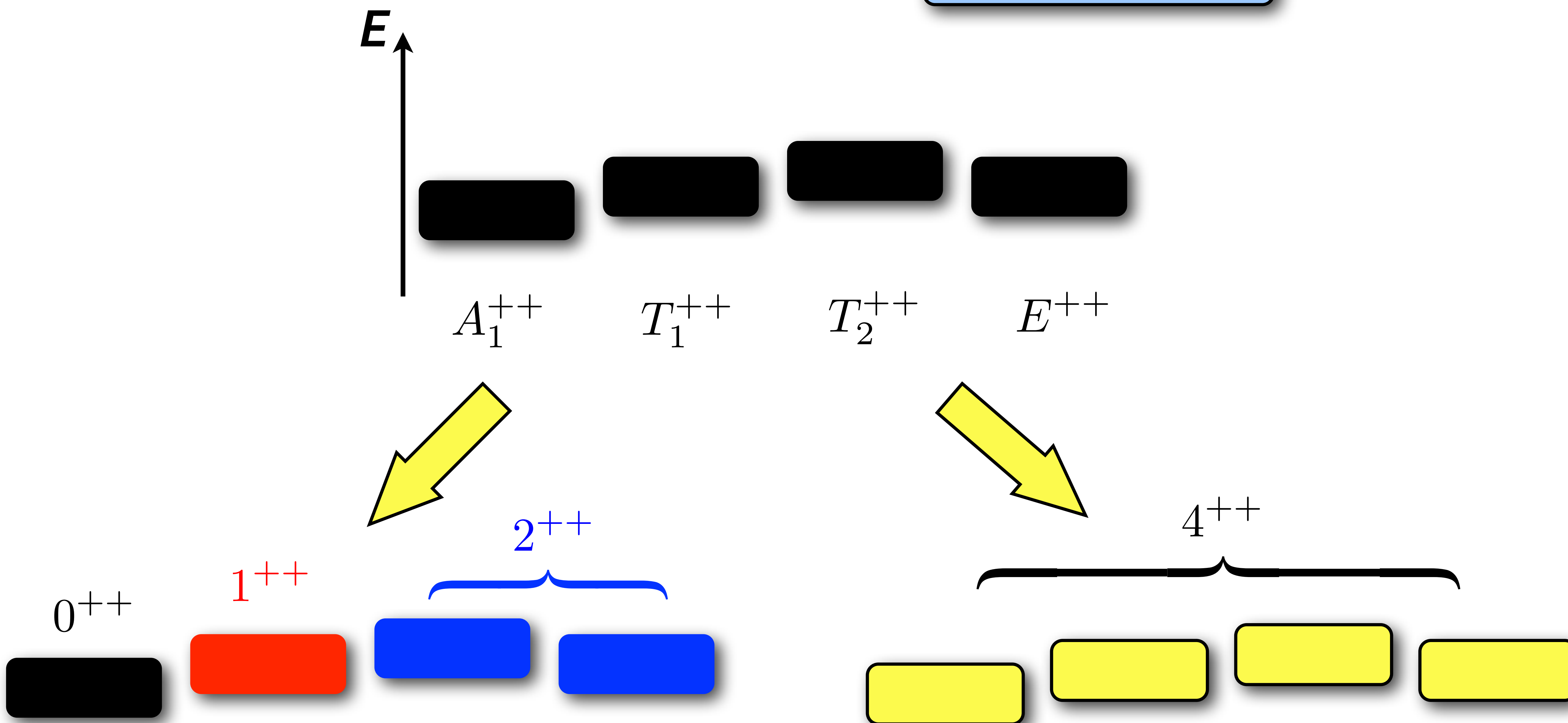


cubic complications ...

'solved' by careful operator construction

construct operators of definite ***J*** in the continuum $\mathcal{O}^{JM}$

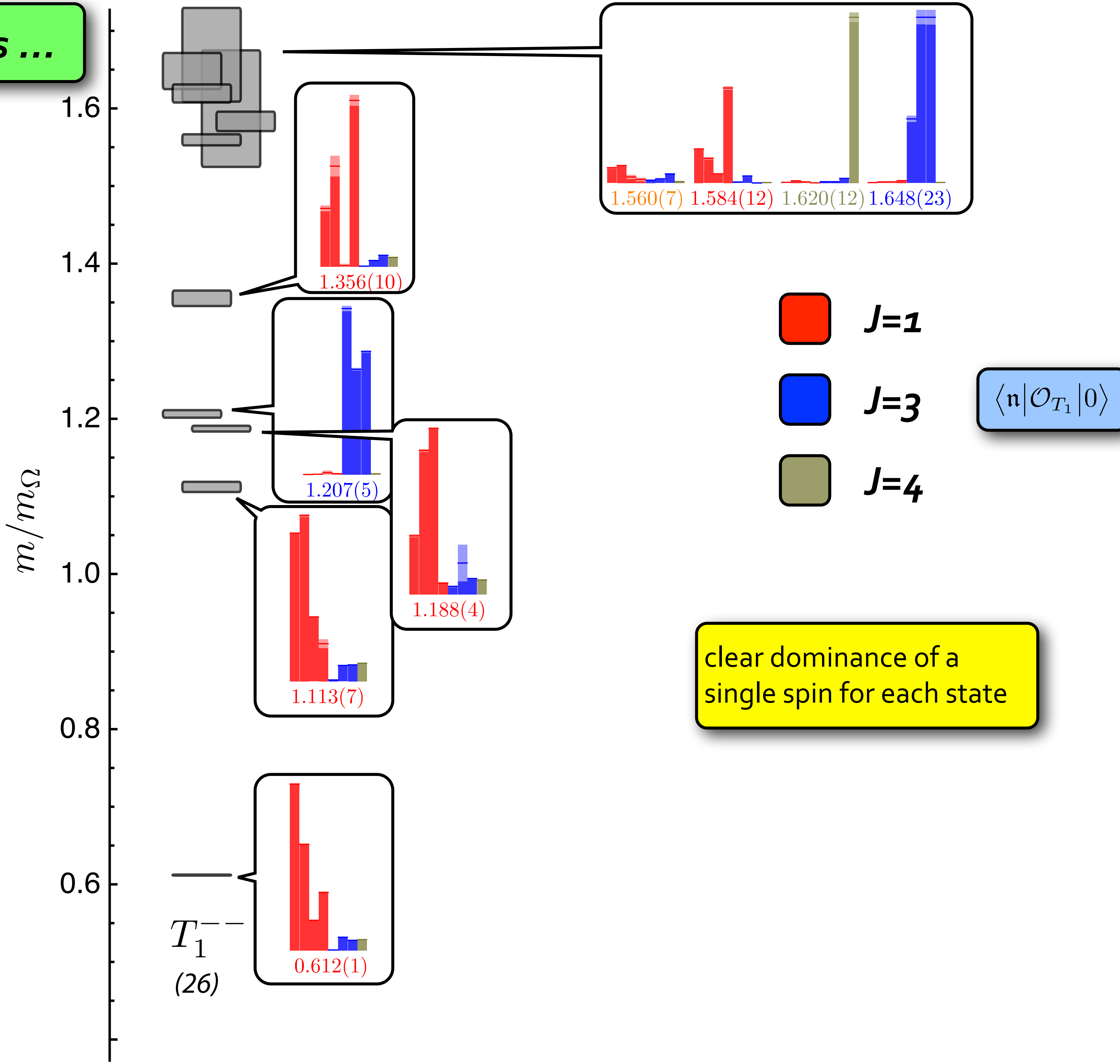
"subduce" into the cubic group irreps $\mathcal{O}^{[J]}_{\Lambda,\lambda} \equiv \sum_M \mathcal{S}^{JM}_{\Lambda,\lambda} \cdot \mathcal{O}^{JM}$

and then $\langle \mathfrak{n}(J) | \mathcal{O}^{[J']}_{\Lambda} | 0 \rangle \approx Z^{[J]}_{\mathfrak{n}} \cdot \delta_{J' J}$ if the rotational symmetry is "restored"

operators respect cubic symmetry, but are 'preconditioned' to be ***J***-diagonal

... but does it work in practice ?

cubic complications ...



clear dominance of a single spin for each state

baryon operators

three-quark field constructions, obeying permutation (anti-)symmetry

$$\epsilon_{abc}\left(D^{n_1}\tfrac{1}{2}(1\pm\gamma^0)\psi\right)^a\left(D^{n_2}\tfrac{1}{2}(1\pm\gamma^0)\psi\right)^b\left(D^{n_3}\tfrac{1}{2}(1\pm\gamma^0)\psi\right)^c$$

derivative constructions

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$$D^{[1]}_{\mathsf{MA},m}=\frac{1}{\sqrt{2}}\left(D^{(1)}_m-D^{(2)}_m\right)\qquad\sim\vec{\epsilon}_m\cdot\vec{\rho}$$

$$D^{[2]}_{\mathsf{S};L,M}=\langle 1m;1m'|LM\rangle\frac{1}{\sqrt{2}}\left(D^{[1]}_{\mathsf{MS},m}D^{[1]}_{\mathsf{MS},m'}+D^{[1]}_{\mathsf{MA},m}D^{[1]}_{\mathsf{MA},m'}\right)\qquad L=0,2$$

$$D^{[2]}_{\mathsf{A};L,M}=\langle 1m;1m'|LM\rangle\frac{1}{\sqrt{2}}\left(D^{[1]}_{\mathsf{MS},m}D^{[1]}_{\mathsf{MA},m'}-D^{[1]}_{\mathsf{MA},m}D^{[1]}_{\mathsf{MS},m'}\right)\qquad L=1$$

$$D^{[2]}_{\mathsf{MS};L,M}=\langle 1m;1m'|LM\rangle\frac{1}{\sqrt{2}}\left(-D^{[1]}_{\mathsf{MS},m}D^{[1]}_{\mathsf{MS},m'}+D^{[1]}_{\mathsf{MA},m}D^{[1]}_{\mathsf{MA},m'}\right)$$

$$D^{[2]}_{\mathsf{MA};L,M}=\langle 1m;1m'|LM\rangle\frac{1}{\sqrt{2}}\left(D^{[1]}_{\mathsf{MS},m}D^{[1]}_{\mathsf{MA},m'}+D^{[1]}_{\mathsf{MA},m}D^{[1]}_{\mathsf{MS},m'}\right)$$

L=0,1,2

real* resonances

**complex*

but we can’t be satisfied with this …

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the spectrum should not be this simple

excited states should be **resonances**

enhancements in the meson-meson scattering continuum

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in finite volume only **discrete meson-meson** states

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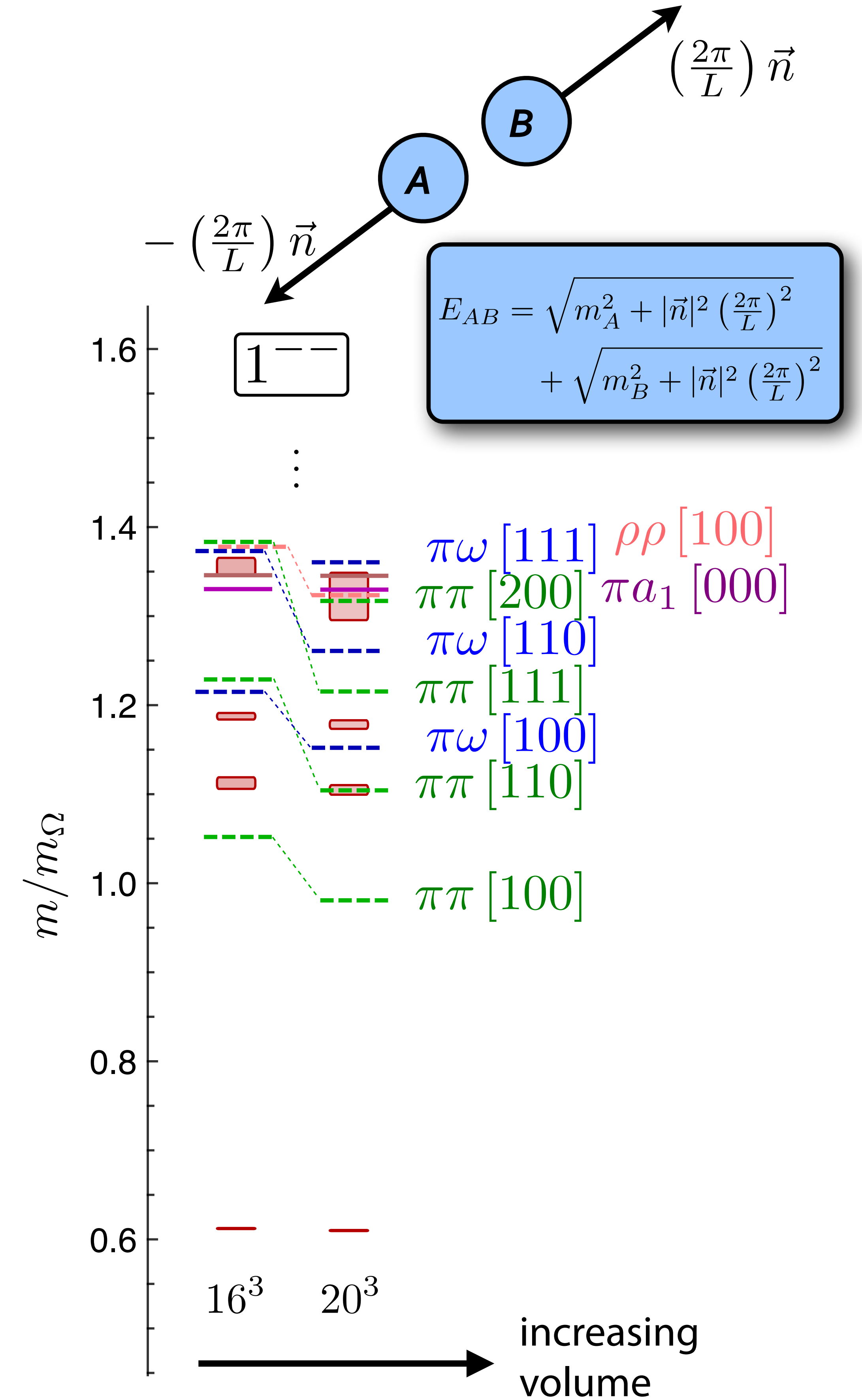

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enhancements in the meson-meson scattering continuum

in finite volume only **discrete meson-meson** states

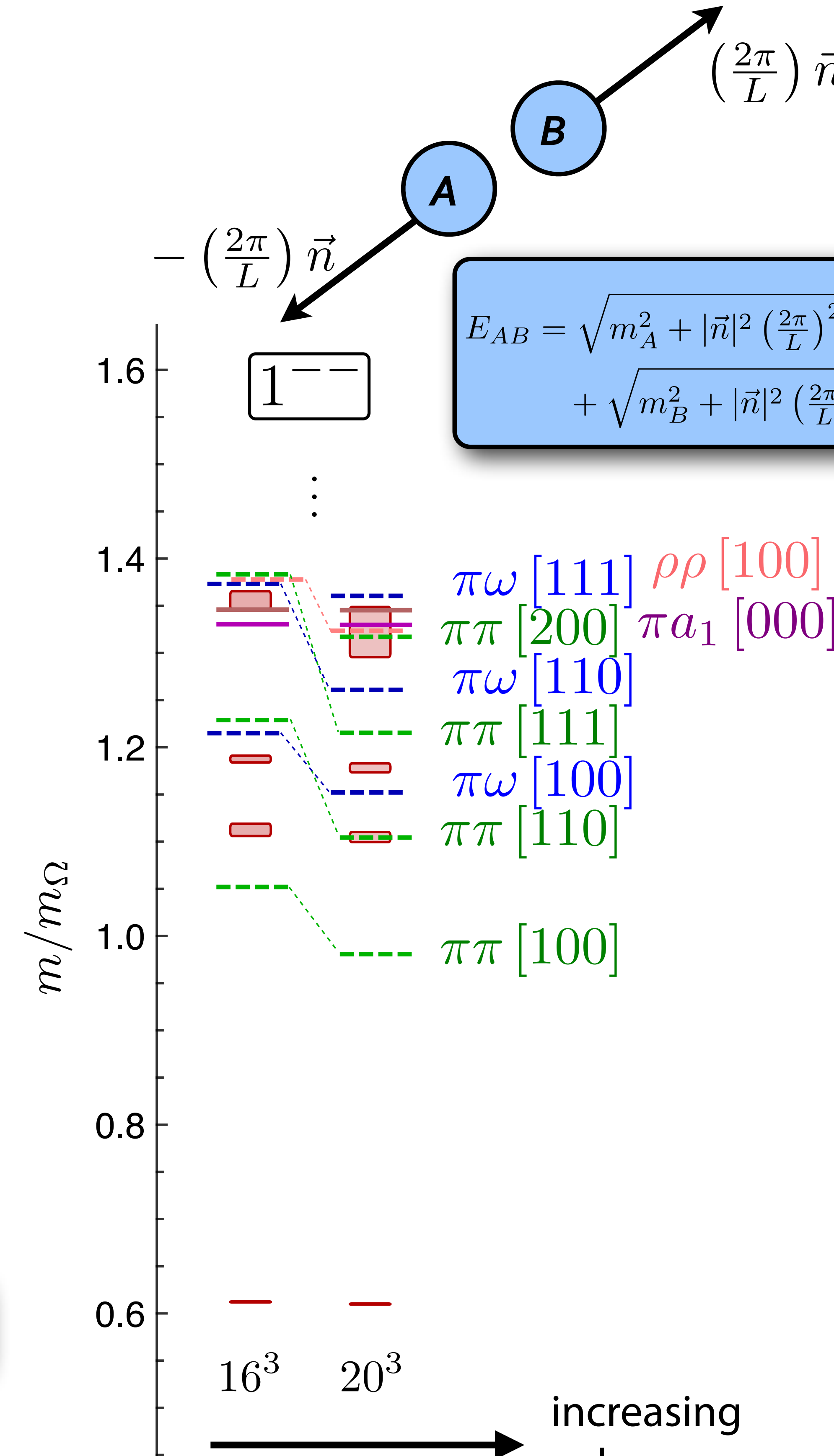
but we aren't seeing them !

our operators closely resemble single hadrons ...

$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \psi$

and not meson-meson pairs ~

$\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \left(\bar{\psi}\Gamma\psi\right)_{\vec{x}} \cdot \sum_{\vec{y}} e^{i(-\vec{p})\cdot\vec{y}} \left(\bar{\psi}\Gamma\psi\right)_{\vec{y}}$

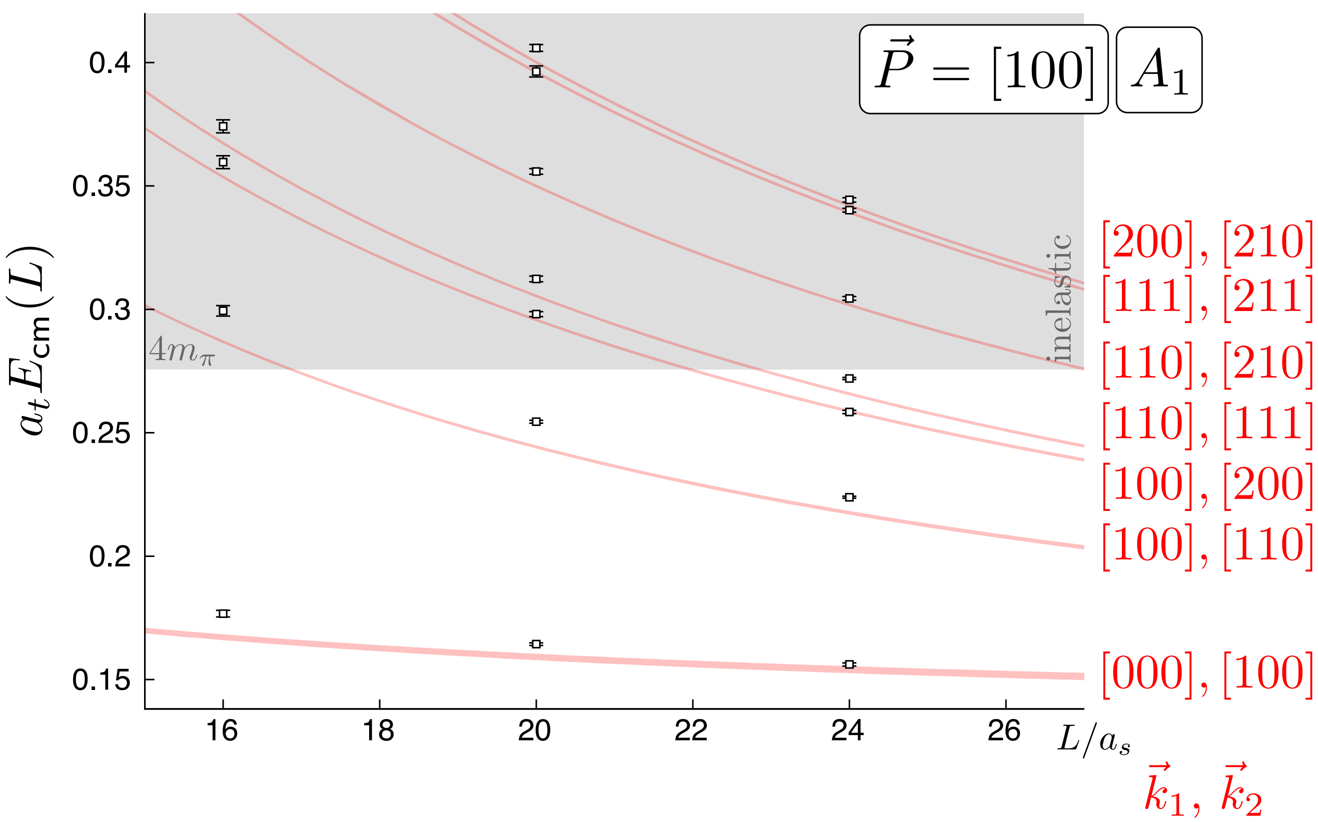


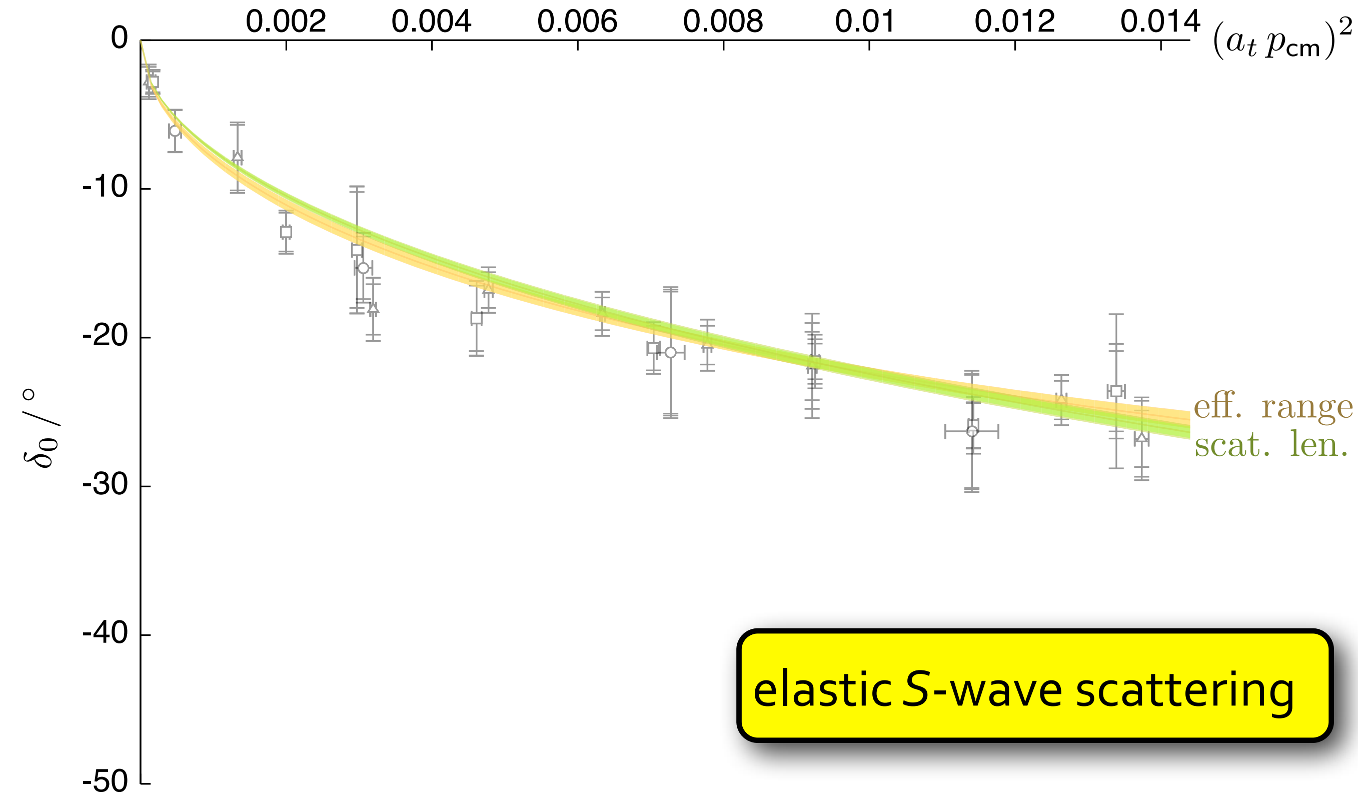
$$E_{AB} = \sqrt{m_A^2 + |\vec{n}|^2 \left(\tfrac{2\pi}{L}\right)^2} + \sqrt{m_B^2 + |\vec{n}|^2 \left(\tfrac{2\pi}{L}\right)^2}$$

ππ I=2 scattering

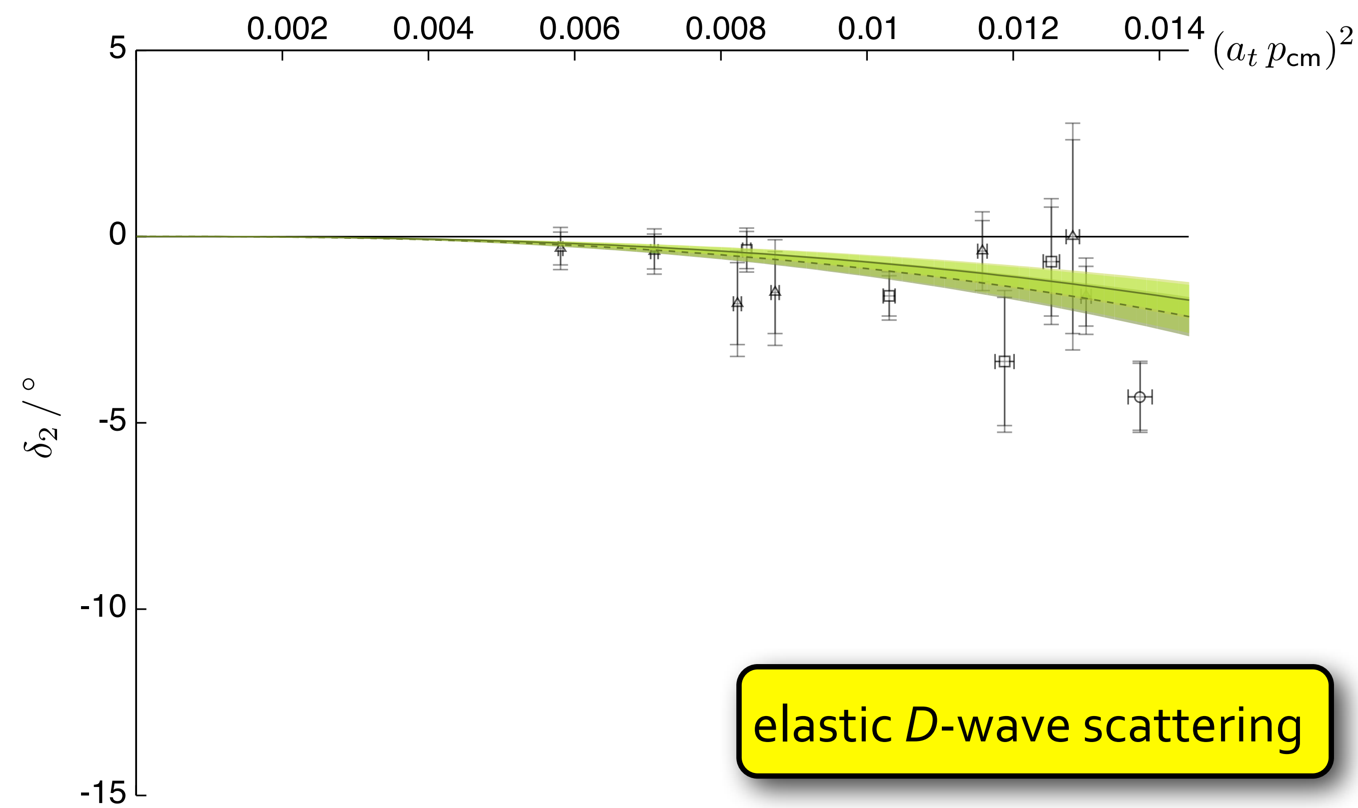
finite-volume analysis - “Lüscher”

$$(\pi\pi)_{\vec{P}\Lambda} = \sum_{\vec{k}_1,\vec{k}_2} \mathcal{C}(\vec{P}\Lambda;\vec{k}_1,\vec{k}_2)\,\pi(\vec{k}_1)\pi(\vec{k}_2)$$





elastic *S*-wave scattering



elastic *D*-wave scattering

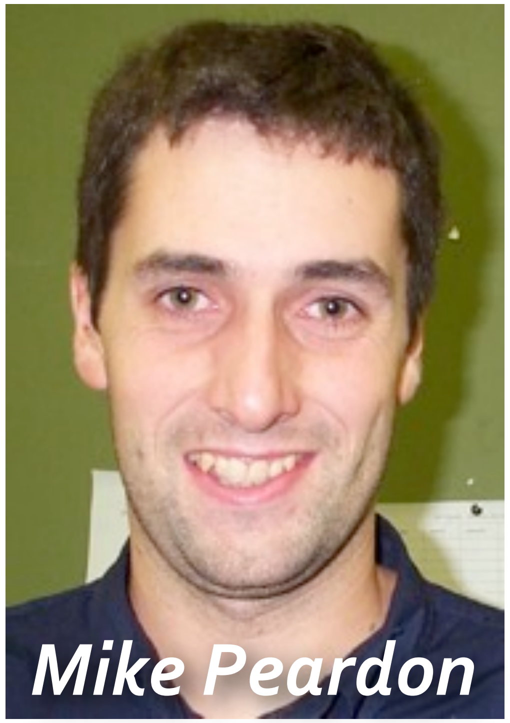
$m_\pi \sim 396$ MeV

distillation

novel method for correlator construction

essentially a (very) smart choice of quark field smearing

smeared quark field : $\Box\psi$



meson correlation function

$$\Big\langle \bar{\psi}_t \Box_t \mathbf{\Gamma}_t \Box_t \psi_t \;\cdot\; \bar{\psi}_0 \Box_0 \mathbf{\Gamma}_0 \Box_0 \psi_0 \Big\rangle$$

"distillation" : $\Box = \sum_n^N \xi_n \xi_n^\dagger$

a simple choice : $-\nabla^2 \xi_n = \lambda_n \xi_n$

meson correlation function

$$-\,\xi_q^\dagger \psi_0 \bar{\psi}_t \xi_n \;\cdot\; \xi_n^\dagger \mathbf{\Gamma}_t \xi_m \;\cdot\; \xi_m^\dagger \psi_t \bar{\psi}_0 \xi_p \;\cdot\; \xi_p^\dagger \mathbf{\Gamma}_0 \xi_q$$

$$\tau_{qn}(0,t) \qquad \Phi_{nm}(t) \qquad \tau_{mp}(t,0) \qquad \Phi_{pq}(0)$$

"perambulator"

factorises the problem !