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The Legacy of Adelchi Fabrocini

**Short-range correlations and
Double-Beta Decay**

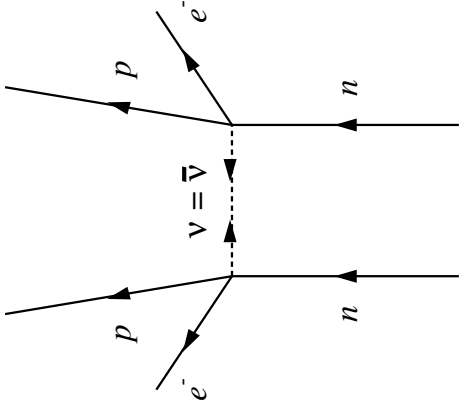
Short-range correlation effects on the nuclear matrix element of neutrinoless double- β decayOmar Benhar,^{1,*} Riccardo Biondi,^{2,3,†} and Enrico Speranza^{2,3,‡}¹*Center for Neutrino Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061, USA*²*Dipartimento di Fisica, "Sapienza" Università di Roma, I-00185 Roma, Italy*³*INFN, Sezione di Roma, I-00185 Roma, Italy*

(Received 26 May 2014; published 29 December 2014)

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TABLE I. Ratio between the $0\nu\beta\beta$ NME of Eq. (4), computed including central and central plus spin-dependent correlations and the corresponding quantity obtained setting $f(r_{12}) = 1$ and $g(r_{12}) = 0$.

	$f(r_{12})$	$f(r_{12}) + g(r_{12})(\sigma_1 \cdot \sigma_2)$
M/M_{SM}	0.77	0.79

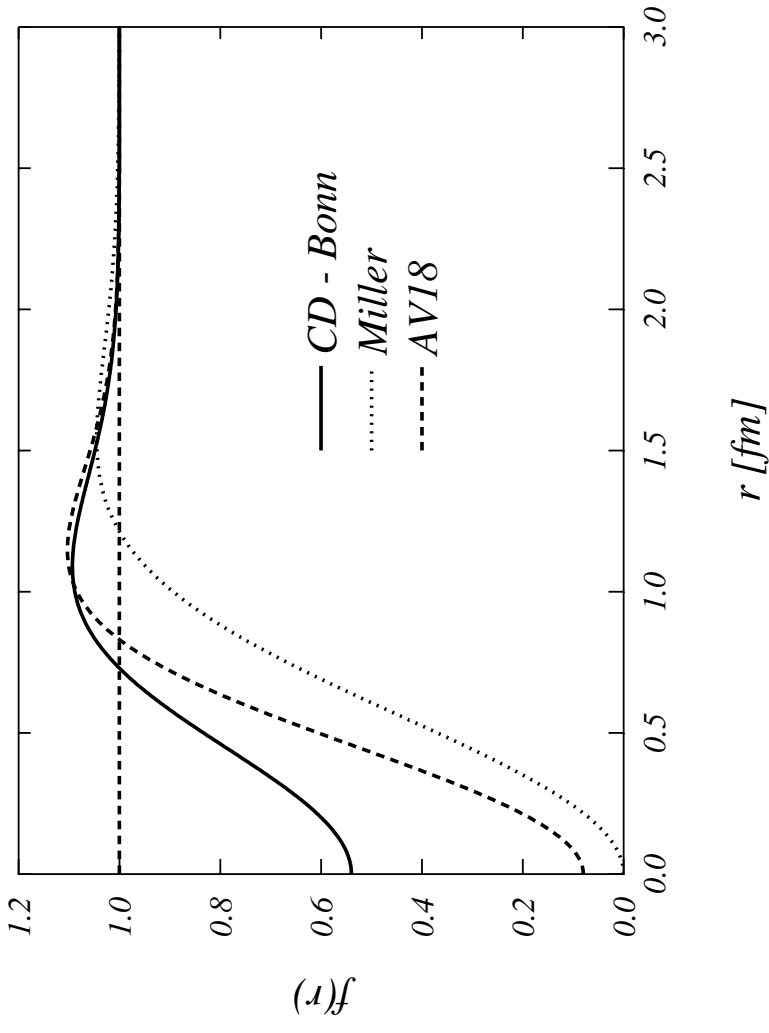


$$\frac{1}{\tau} = G |M|^2$$

$$\begin{aligned}
 M &= \langle A, Z + 2 | \int d^3 q e^{i\mathbf{q} \cdot (\mathbf{r}_1 - \mathbf{r}_2)} \mathcal{O}(1) \sum_i |i\rangle \frac{1}{|\mathbf{q}|(|\mathbf{q}| + \omega_i/\hbar)} \langle i | \mathcal{O}(2) | A, Z \rangle \\
 &\simeq \langle A, Z + 2 | \int d^3 q \frac{e^{i\mathbf{q} \cdot (\mathbf{r}_1 - \mathbf{r}_2)}}{|\mathbf{q}|(|\mathbf{q}| + \omega_i/\hbar)} \mathcal{O}(1) \mathcal{O}(2) | A, Z \rangle \\
 &= \langle A, Z + 2 | H(\mathbf{r}_{12}) \mathcal{O}(1) \mathcal{O}(2) | A, Z \rangle = \langle A, Z + 2 | \mathcal{T}(1, 2) | A, Z \rangle
 \end{aligned}$$

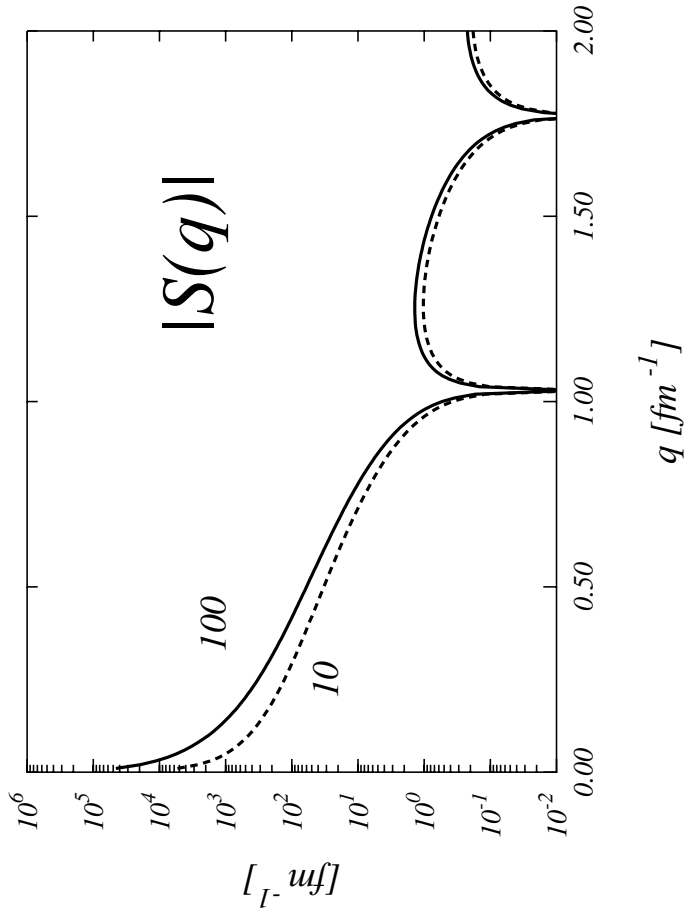
$$\langle A, Z + 2 | \mathcal{T}(1, 2) | A, Z \rangle \longrightarrow \langle A, Z + 2 | f(r_{12}) \mathcal{T}(1, 2) f(r_{12}) | A, Z \rangle$$

$$\mathcal{T}(1, 2) \longrightarrow \mathcal{T}^{\text{eff}}(1, 2) = f(r_{12}) \mathcal{T}(1, 2) f(r_{12})$$



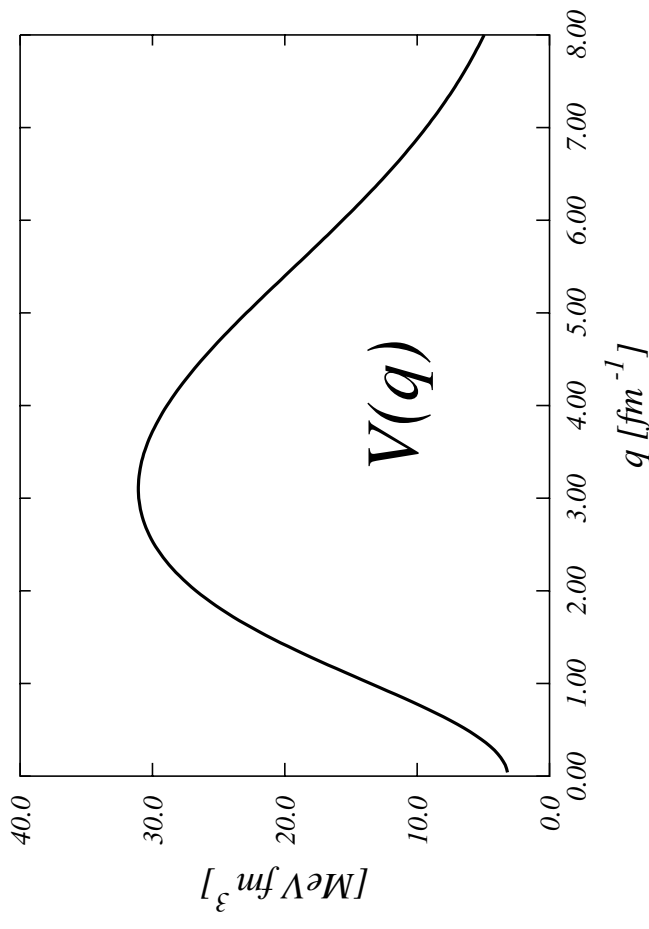
$f(r) = 1 - e^{-ar} (1 - br^2)$ Miller and Spencer, Ann.Phys. 100(1976)562.

Authors	Method	Ref.	M/M_{MF}
M. Kortelainen <i>et al.</i>	SM	PLB 647 (2007) 128	0.7
E. Caurier <i>et al.</i>	SM	EPJA 36 (2008) 195	0.59
F. Šimkovic <i>et al.</i>	QRPA	PRC 77(2008) 045503	~ 0.8
F. Šimkovic <i>et al.</i>	QRPA	PRC 79(2009) 055501	0.7, 0.5
M. Horoi S. Stoica	QRPA	PRC 81 (2010) 024321	0.8, 1.2, 1.1



Neutrino potential

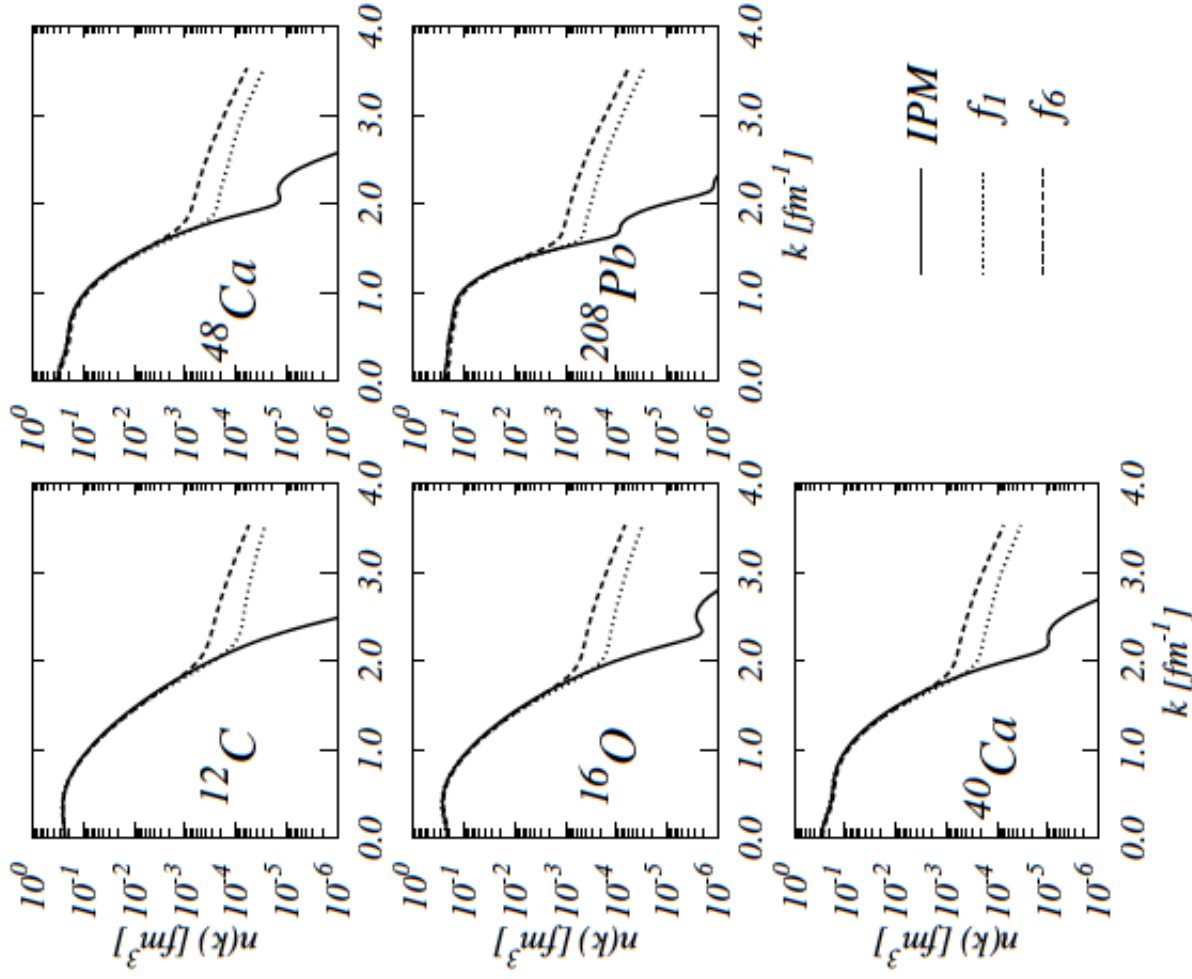
$$S(q) = \int dr r^2 \frac{j_0(qr)}{q(q + \omega)}$$



Argonne V18 potential

$$\tilde{V}(q) = \int dr r^2 j_0(qr) V(r)$$

Momentum distribution



F. Arias de Saavedra, C. Biscconti,
 G. Co', **A. Fabrocini**,
 Phys. Rep. 450 (2007) 1-95.

$$\Delta x \Delta p c \simeq \hbar c \simeq 2.0 \text{fm}^{-1} \cdot 0.5 \text{fm}$$

$$\rho^{\text{II}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\langle \Psi | \Psi \rangle} \int d^3 r_3 \cdots d^3 r_A \Psi^*(\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_A) \Psi(\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_A)$$

$$\rho^{\text{II,MF}}(\mathbf{r}_1, \mathbf{r}_2) = \rho^{\text{I,MF}}(\mathbf{r}_1, \mathbf{r}_1) \rho^{\text{I,MF}}(\mathbf{r}_2, \mathbf{r}_2) - \rho^{\text{I,MF}}(\mathbf{r}_1, \mathbf{r}_2) \rho^{\text{I,MF}}(\mathbf{r}_2, \mathbf{r}_1)$$

$$\rho^{\text{I,MF}}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{i < \epsilon_F} \phi_i^*(\mathbf{r}_1) \phi_i(\mathbf{r}_2)$$

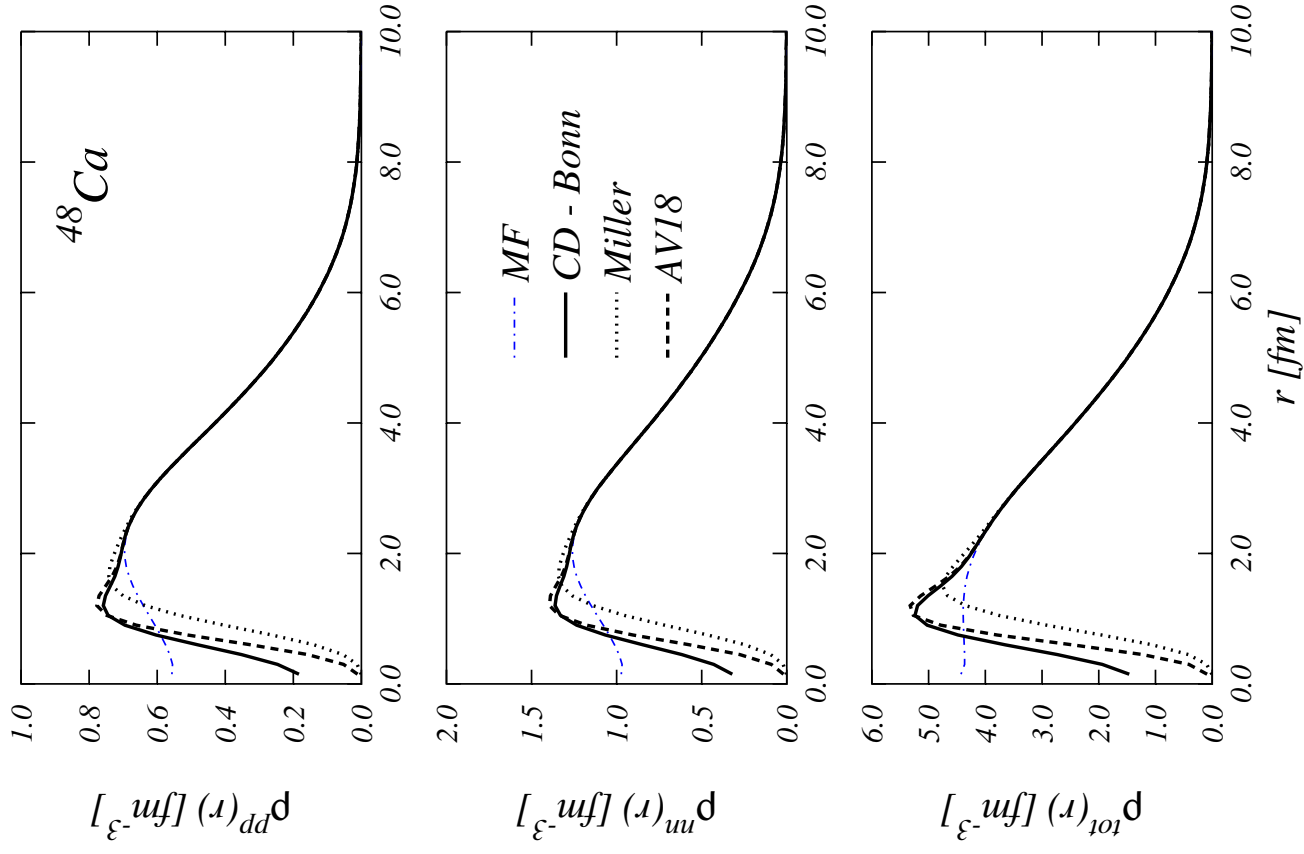
$$\int d^3 r_1 \rho^{\text{I,MF}}(\mathbf{r}_1, \mathbf{r}_1) = A ; \quad \int d^3 r_3 \rho^{\text{I,MF}}(\mathbf{r}_1, \mathbf{r}_3) \rho^{\text{I,MF}}(\mathbf{r}_3, \mathbf{r}_2) = \rho^{\text{I,MF}}(\mathbf{r}_1, \mathbf{r}_2)$$

$$\int d^3 r_1 \int d^3 r_2 \rho^{\text{II,MF}}(\mathbf{r}_1, \mathbf{r}_2) = A(A - 1)$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 ; \quad \mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2) ; \quad \rho^{\text{II,MF}}(\mathbf{r}_1, \mathbf{r}_2) \rightarrow \rho^{\text{II,MF}}(\mathbf{r}, \mathbf{R})$$

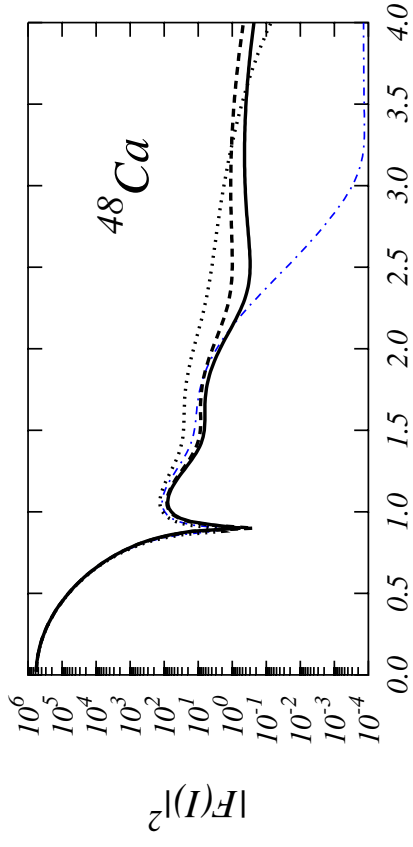
$$\tilde{\rho}^{\text{II,MF}}(r) = \int d(\cos(\theta_{rR})) \int dR R^2 \rho^{\text{II,MF}}(r, R, \cos(\theta_{rR}))$$

$$\tilde{\rho}^{\text{II,c1}}(r) = f(r) \tilde{\rho}^{\text{II,MF}}(r) f(r)$$



Normalisations

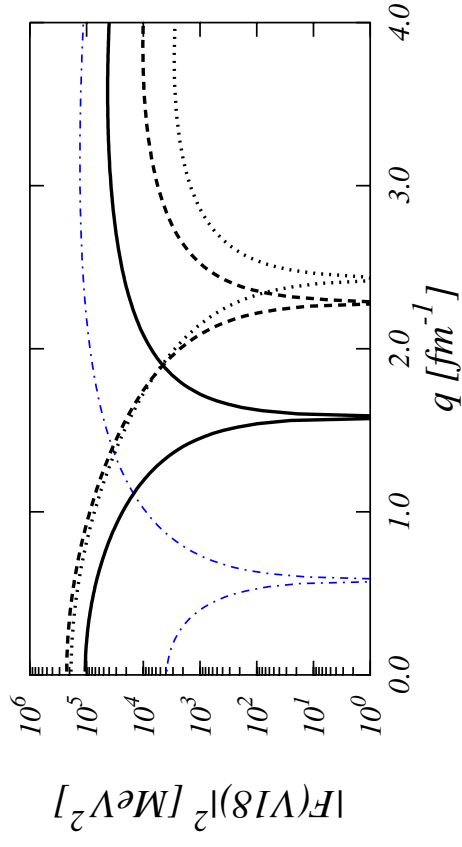
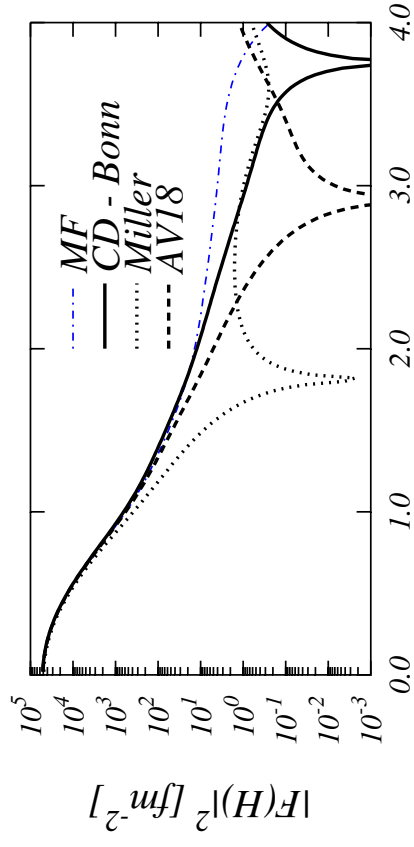
	pp	nn	tot
expct.	380.00	756.00	2256.00
MF	380.44	756.74	2258.40
Bonn	382.10	759.73	2269.14
Miller	380.70	757.27	2258.19
AV18	382.00	760.00	2268.08



$$F(\mathbf{I}) = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \tilde{\rho}^{\text{nn}}(r)$$

$$F(\mathbf{H}) = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} H(r) \tilde{\rho}^{\text{nn}}(r)$$

$$F(\mathbf{V18}) = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} V_{18}(r) \tilde{\rho}^{\text{nn}}(r)$$



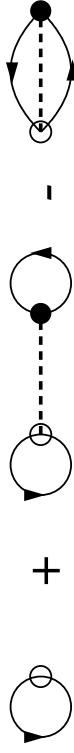
$$\begin{aligned}
\rho^I(\mathbf{r}_1, \mathbf{r}_1) &= \frac{1}{\langle \Psi | \Psi \rangle} \int d^3 r_2 d^3 r_3 \cdots d^3 r_A \Psi^*(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) \Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) \\
&= \frac{1}{\langle \Psi | \Psi \rangle} \int d^3 r_2 d^3 r_3 \cdots d^3 r_A \Phi^*(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) F^\dagger F \Phi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)
\end{aligned}$$

$$F^\dagger F = \prod_{ij} f^2(r_{ij}) = \prod_{ij} [1 + h(r_{ij})]$$



$h(r_{12}) \rightarrow$

$$\rho^{\text{I,MF}}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{\alpha < \epsilon_F} \phi_{\alpha}^*(\mathbf{r}_1) \phi_{\alpha}(\mathbf{r}_2) \rightarrow$$



$$+ \left[\begin{array}{c} \text{Diagram 1} \\ + \\ \text{Diagram 2} \\ - \\ \text{Diagram 3} \\ - \\ \text{Diagram 4} \end{array} \right]$$

The diagrams in the brackets are:

- Diagram 1: Two particles (one positive, one negative) connected by a dashed line. A circular arrow loops around the positive charge, and another circular arrow loops around the negative charge.
- Diagram 2: Two particles (one positive, one negative) connected by a dashed line. A circular arrow loops around the positive charge, and another circular arrow loops around the negative charge.
- Diagram 3: Two particles (one positive, one negative) connected by a dashed line. A circular arrow loops around the positive charge, and another circular arrow loops around the negative charge.
- Diagram 4: Two particles (one positive, one negative) connected by a dashed line. A circular arrow loops around the positive charge, and another circular arrow loops around the negative charge.

$$\int d^3 r_3 \rho^I(\mathbf{r}_1, \mathbf{r}_3) \rho^I(\mathbf{r}_3, \mathbf{r}_2) = \rho^I(\mathbf{r}_1, \mathbf{r}_2)$$

$$\int d^3 r_1 \rho^I(\mathbf{r}_1, \mathbf{r}_1) = \int d^3 r_1 \rho(\mathbf{r}_1) =$$

$$\begin{aligned}
 & \text{[Diagram: a circle with a dot on the left and a dot on the right, connected by a dashed line, with a self-loop on the left dot]} + \text{[Diagram: a circle with a dot on the left and a dot on the right, connected by a dashed line, with a self-loop on the right dot]} - \text{[Diagram: a circle with a dot on the left and a dot on the right, connected by a dashed line, with self-loops on both dots]} \\
 & + \frac{1}{2} \left[\text{[Diagram: a circle with a dot on the left and a dot on the right, connected by a dashed line, with self-loops on both dots]} + \text{[Diagram: a circle with a dot on the left and a dot on the right, connected by a dashed line, with self-loops on both dots]} - \text{[Diagram: a circle with a dot on the left and a dot on the right, connected by a dashed line, with self-loops on both dots]} - \text{[Diagram: a circle with a dot on the left and a dot on the right, connected by a dashed line, with self-loops on both dots]} \right] \\
 & = A
 \end{aligned}$$

$$\int d^3 r_1 \rho^{\text{MF}}(\mathbf{r}_1) = \int d^3 r_1 \sum_{\alpha} |\phi_{\alpha}(\mathbf{r}_1)|^2 = A$$

S. Fantoni and V. R. Pandharipande
 Nucl. Phys. A 473 (1987), 234.

$$D(\mathbf{q}, \omega) = \sum_n \frac{|\langle \tilde{\Psi}_n | O(\mathbf{q}) | \tilde{\Psi}_0 \rangle|^2}{E_n - E_0 - \omega + i\eta} = \sum_n \xi_n^+(\mathbf{q}) (E_n - E_0 - \omega + i\eta)^{-1} \xi_n(\mathbf{q}),$$

$$\xi_n(\mathbf{q}) = \frac{\langle \Psi_n | O(\mathbf{q}) | \Psi_0 \rangle}{\langle \Psi_n | \Psi_n \rangle^{\frac{1}{2}} \langle \Psi_0 | \Psi_0 \rangle^{\frac{1}{2}}}.$$

$$\begin{aligned} \xi_n(\mathbf{q}) &= \frac{\langle \Phi_n | \prod_{i<j} f_{ij}^+ O(\mathbf{q}) \prod_{i<j} f_{ij} | \Phi_0 \rangle}{\langle \Phi_0 | \prod_{i<j} f_{ij}^+ \prod_{i<j} f_{ij} | \Phi_0 \rangle} \left[\frac{\langle \Phi_0 | \prod_{i<j} f_{ij}^+ \prod_{i<j} f_{ij} | \Phi_0 \rangle}{\langle \Phi_n | \prod_{i<j} f_{ij}^+ \prod_{i<j} f_{ij} | \Phi_n \rangle} \right]^{\frac{1}{2}} \\ &= \frac{\langle \Phi_n | O(\mathbf{q}) \prod_{i<j} (1 + h_{ij}) | \Phi_0 \rangle}{\langle \Phi_0 | \prod_{i<j} (1 + h_{ij}) | \Phi_0 \rangle} \left[\frac{\langle \Phi_0 | \prod_{i<j} (1 + h_{ij}) | \Phi_0 \rangle}{\langle \Phi_n | \prod_{i<j} (1 + h_{ij}) | \Phi_n \rangle} \right]^{\frac{1}{2}}, \end{aligned}$$

our ansatz

$$\xi_n(\mathbf{q}) \rightarrow \xi_n^1(\mathbf{q}) = \langle \Phi_n | O(\mathbf{q}) \sum_{i<j} (1 + h_{ij}) | \Phi_0 \rangle_L,$$

$$+ \frac{1}{2} \left[\begin{array}{c} \text{Diagram (1.1)} \\ \text{(1.1)} \end{array} \right]$$

$$+ \frac{1}{2} \left[\begin{array}{c} \text{Diagram (2.1)} \\ \text{(2.1)} \end{array} \right]$$

$$- \left[\begin{array}{c} \text{Diagram (2.2)} \\ \text{(2.2)} \end{array} \right]$$

$$+ \left[\begin{array}{c} \text{Diagram (2.3)} \\ \text{(2.3)} \end{array} \right]$$

$$- \left[\begin{array}{c} \text{Diagram (2.4)} \\ \text{(2.4)} \end{array} \right]$$

$$+ \frac{1}{3} \left[\begin{array}{c} \text{Diagram (3.1)} \\ \text{(3.1)} \end{array} \right]$$

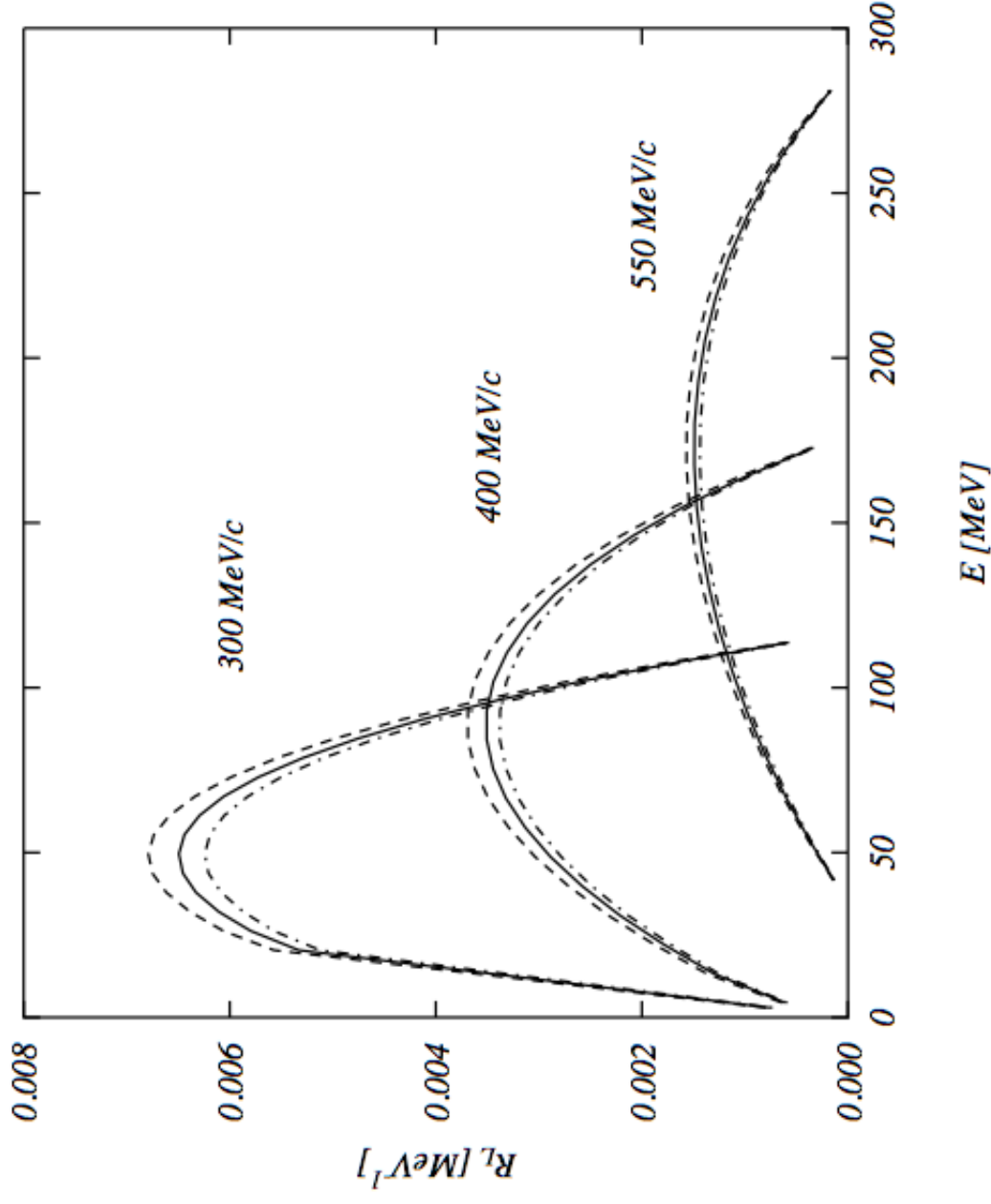
$$+ \left[\begin{array}{c} \text{Diagram (3.2)} \\ \text{(3.2)} \end{array} \right]$$

$$- \left[\begin{array}{c} \text{Diagram (3.3)} \\ \text{(3.3)} \end{array} \right]$$

$$+ \left[\begin{array}{c} \text{Diagram (3.4)} \\ \text{(3.4)} \end{array} \right]$$

$$+ \left[\begin{array}{c} \text{Diagram (3.5)} \\ \text{(3.5)} \end{array} \right]$$

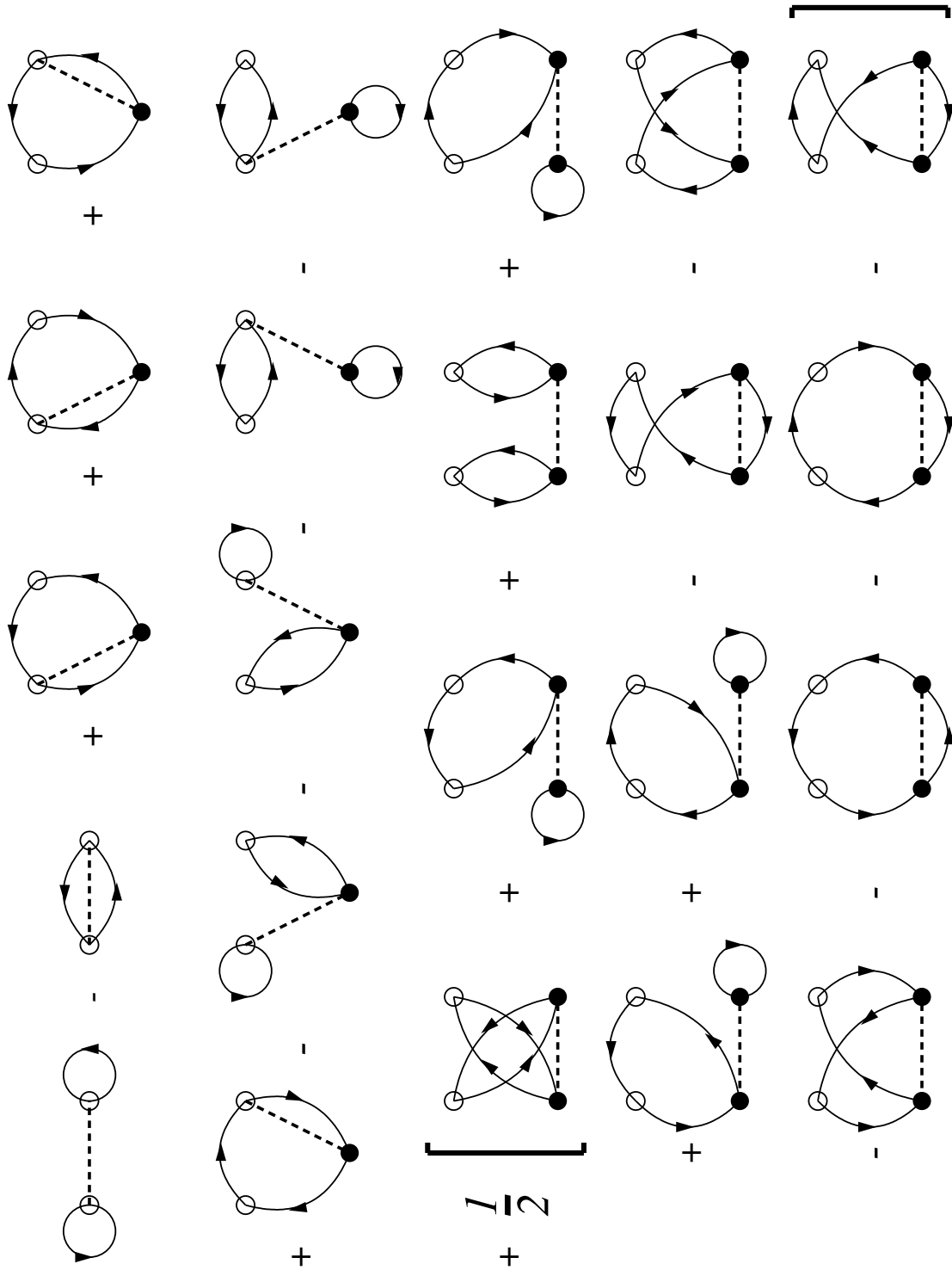
$$- \left[\begin{array}{c} \text{Diagram (3.6)} \\ \text{(3.6)} \end{array} \right]$$

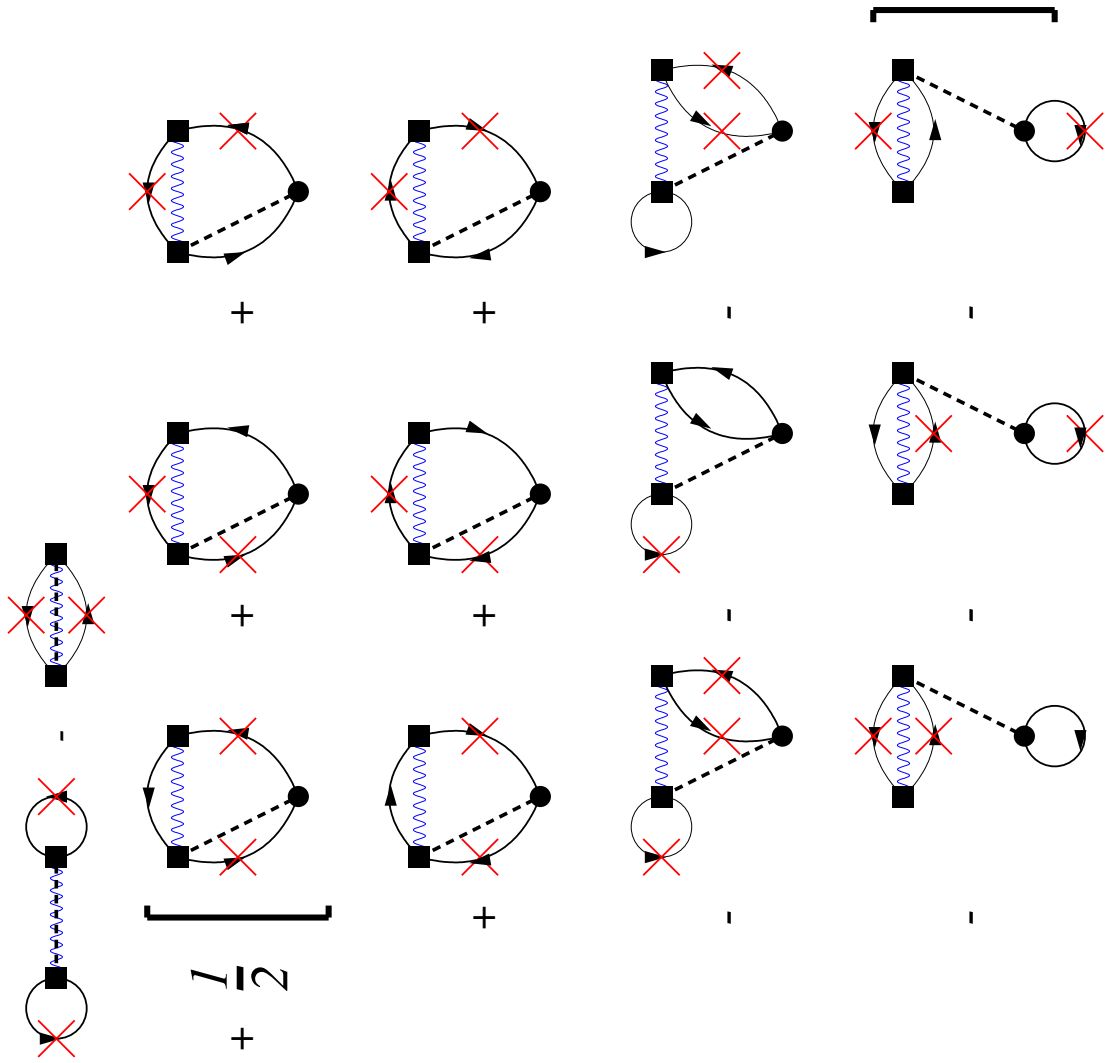


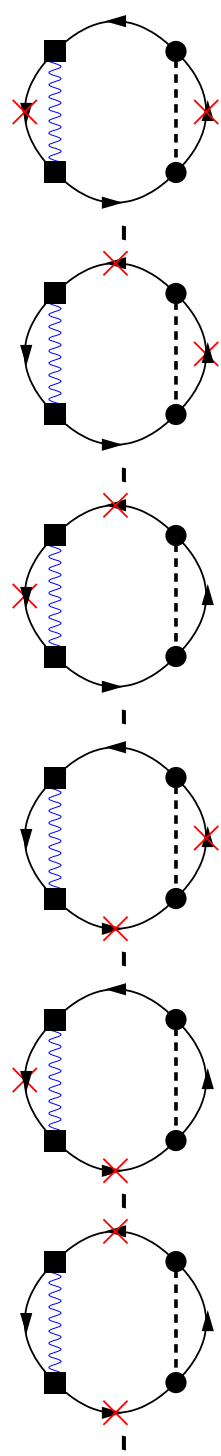
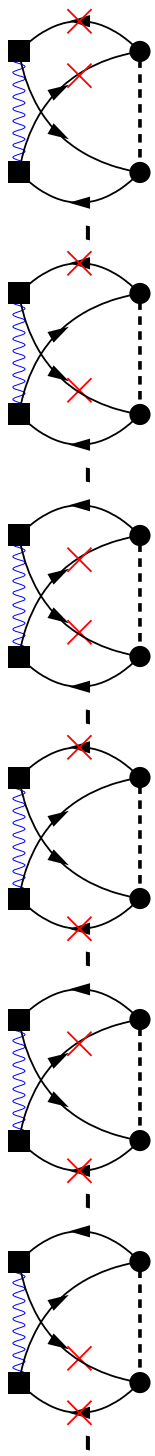
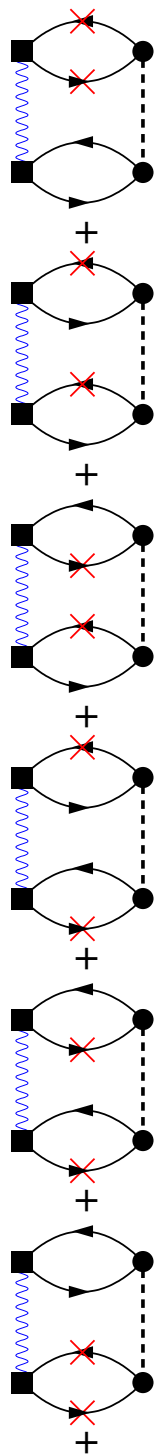
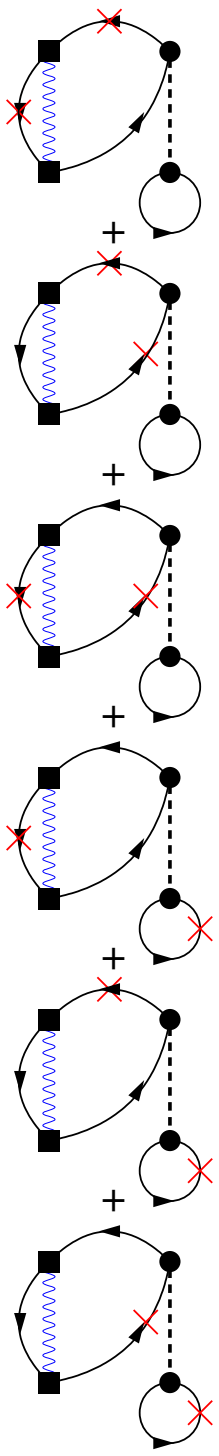
Nuclear matter
charge response
Dashed lines - MF
D-d lines - 2 points
Full lines 2+3 point.

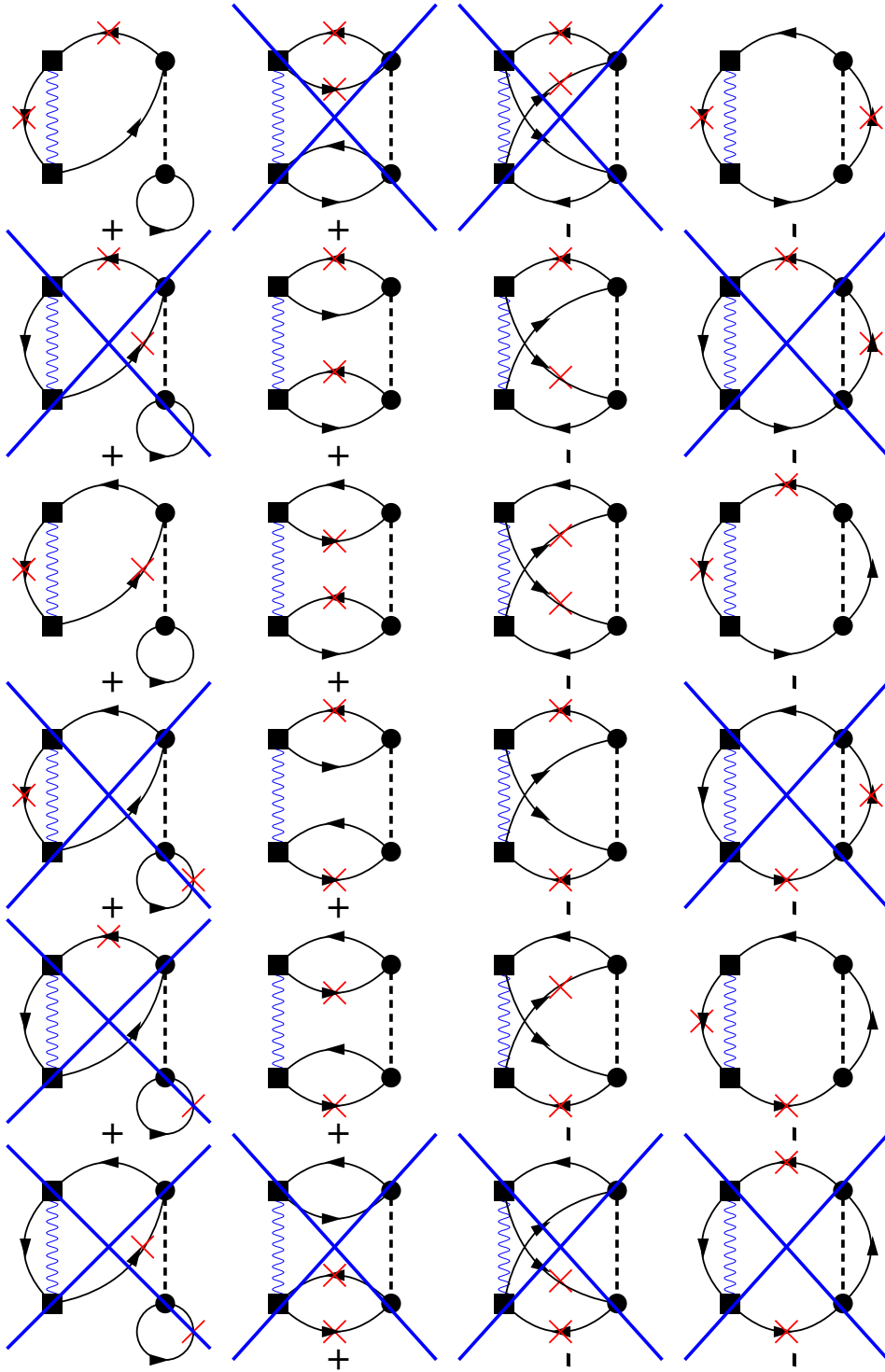
J.E.Amaro,
A.M.Lallena,
G. Co' and **A.Fabrocini**
Phys. Rev. C 57
(1998) 3473;

$$\rho^{\text{II}}(\mathbf{r}_1, \mathbf{r}_2) = \rho^{\text{II, MF}}(\mathbf{r}_1, \mathbf{r}_2) +$$











Adelchi Fabrocini 1951 - 2006