Energy per particle of a N-boson system using soft potentials

A. Kievsky

INFN, Sezione di Pisa (Italy)

Lepton-Nucleus Scattering XIV
Marciana Marina, 27 June - 1 July 2016
The Legacy of Adelchi Fabrocini

Collaborators

- N. Timofeyuk - *University of Surrey, Surrey (UK)*
- A. Polls - *University of Barcelona, Barcelona (Spain)*
- B. Juliá Díaz *University of Barcelona, Barcelona (Spain)*

Energy per particle of a N-boson system using soft potentials

A. Kievsky

INFN, Sezione di Pisa (Italy)

Lepton-Nucleus Scattering XIV
Marciana Marina, 27 June - 1 July 2016
The Legacy of Adelchi Fabrocini

Collaborators

- N. Timofeyuk - *University of Surrey, Surrey (UK)*
- A. Polls - *University of Barcelona, Barcelona (Spain)*
- B. Juliá Díaz *University of Barcelona, Barcelona (Spain)*

Historical context

Studies on liquid ^{4}He at zero temperature in the 80's

- High quality potential curves for the Helium-Helium interaction: For example the Aziz and Nain potential: HFDHE2
- High quality methods to solve the many-body problem: Variational methods, GFMC or DMC
- main interest: to compare as best as possible theory (methods and potentials) and experimental results
- Exp.:
  E/N= -7.14 K at $\rho_0 = 0.022\ \text{Å}^{-3}$
- theory (GFMC using the HFDHE2 potential):
  E/N=-7.11 K at $\rho_0 = 0.022\ \text{Å}^{-3}$

A great success of the many body theories

Historical context

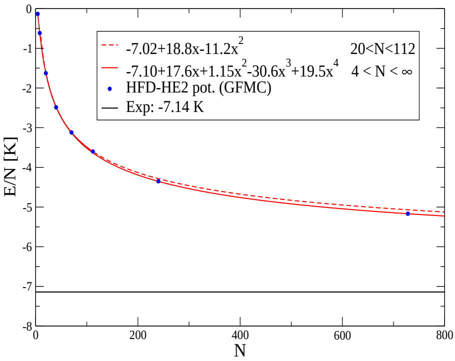
Studies on liquid ^{4}He at zero temperature in the 80's

- High quality potential curves for the Helium-Helium interaction: For example the Aziz and Nain potential: HFDHE2
- High quality methods to solve the many-body problem: Variational methods, GFMC or DMC
- main interest: to compare as best as possible theory (methods and potentials) and experimental results
- Exp.:
  E/N= -7.14 K at $\rho_0 = 0.022$ Å^{-3}
- theory (GFMC using the HFDHE2 potential):
  E/N=-7.11 K at $\rho_0 = 0.022$ Å^{-3}

A great success of the many body theories

Studies on liquid ^{4}He droplets in the 80's

- main interest: to obtain the density and energy of liquid ^{4}He from energies and radii of droplets having a few hundred atoms
- A liquid-drop model for the energy:
  $E(N)/N = E_v + E_s x + E_c x^2$ with $x = N^{-1/3}$
- To support liquid-drop formulas to obtain nuclear matter properties



Changing perspective: The Helium dimer

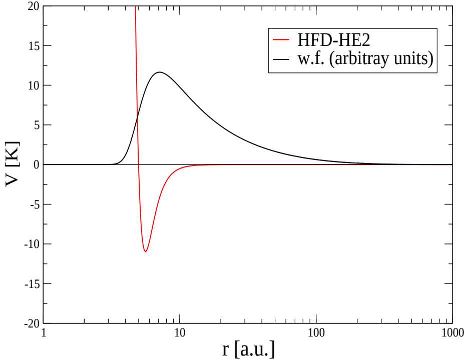
The energy: $E_d \approx -1$ mK

The two-body scattering length $a \approx 220$ a.u.

$a >> r_0 \longrightarrow a$ emerges as a control parameter

$E_d \approx -\hbar^2/ma^2 \approx -1$ mK $\longrightarrow$ unnatural state $\longrightarrow$ universal behavior

$\longrightarrow$ zero-range theory $\longrightarrow$ Efimov Physics



Universality in systems having large scattering length

Shallow states ⟶ zero-range theory ⟶ Efimov Physics

low energy physics:

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_{eff} k^2 + \ldots$$

The binding wave number k_d ($\hbar^2 k_d^2/m = E_d$) is the solution of the pole equation $ik_d \cot \delta(ik_d) + k_d = 0$ and therefore

$$k_d = \frac{1}{a} + \frac{1}{2} r_{eff} k_d^2 + \ldots$$

for shallow states ($a >> r_{eff}$)

$$k_d = \frac{1}{a} + \frac{1}{2} r_{eff} k_d^2$$

or

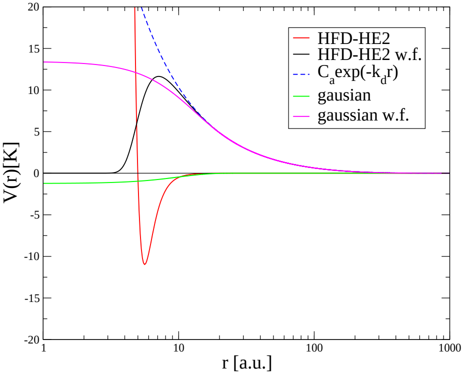
$$E_d = \frac{\hbar^2}{ma^2} \left(1 + \frac{r_{eff}}{a} + \frac{5 r_{eff}^2}{4a^2} + \ldots \right) \longrightarrow \lim_{r_{eff} \to 0} E_d = \frac{\hbar^2}{ma^2}$$

Effective description of the Helium dimer

At LO the dimer is described by a (regularized) contact interaction

Gaussian form: $V(1,2) \to V_0 e^{-r^2/r_0^2}$

	a(a.u.)	r_{eff}(a.u.)	E_d(mK)	C_a(a.u.$^{-1/2}$)
HFD-HE2	235.5	13.978	-0.83012	0.096
gaussian	235.5	13.978	-0.83012	0.096



Effective description of the Helium trimer: Efimov physics

At LO the trimer is described by a (regularized) two-body plus a three-body contact interaction

Gaussian form: $V(1,2) + V(1,2,3) \rightarrow V_0 e^{-r^2/r_0^2} + W_0 e^{-\rho^2/\rho_0^2}$
with the hyperadius $\rho^2 = 2/3(r_{12}^2 + r_{23}^2 + r_{31}^2)$

	a(a.u.)	r_{eff}(a.u.)	E_d(mK)	E_3^0(mK)	E_3^1(mK)
HFD-HE2	235.5	13.978	-0.83012	-117.3	-1.67
TBG	235.5	13.978	-0.83012	-139.8	-1.77
TBG+ 3BH				-117.3	-1.67(1)

- V_0 and r_0 has been fixed to describe a, r_{eff} and E_d
- W_0 and ρ_0 has been fixed to describe E_3^0, E_3^1
- we have explored 5 a.u. < ρ_0 < 14 a.u.
- correspondingly 60 K < W_0 < 0.4 K
- there is a low sensibility to the range ρ_0

Effective description of the Helium trimer: Efimov physics

At LO the trimer is described by a (regularized) two-body plus a three-body contact interaction

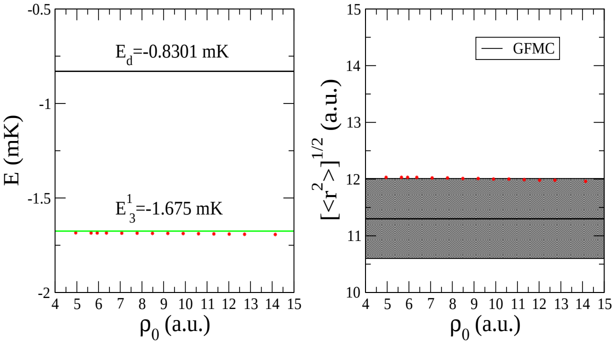
Gaussian form: $V(1,2) + V(1,2,3) \rightarrow V_0 e^{-r^2/r_0^2} + W_0 e^{-\rho^2/\rho_0^2}$
with the hyperadius $\rho^2 = 2/3(r_{12}^2 + r_{23}^2 + r_{31}^2)$

	a(a.u.)	r_{eff}(a.u.)	E_d(mK)	E_3^0(mK)	E_3^1(mK)
HFD-HE2	235.5	13.978	-0.83012	-117.3	-1.67
TBG	235.5	13.978	-0.83012	-139.8	-1.77
TBG+ 3BH				-117.3	-1.67(1)

- V_0 and r_0 has been fixed to describe a, r_{eff} and E_d
- W_0 and ρ_0 has been fixed to describe E_3^0, E_3^1
- we have explored 5 a.u. < ρ_0 < 14 a.u.
- correspondingly 60 K < W_0 < 0.4 K
- there is a low sensibility to the range ρ_0

Analysis of the three-body force parameters W_0 and ρ_0

(W_0, ρ_0) fixed to describe $E_3^0 = -117.3$ mK



Two different points of view of Helium systems

- In the 80's strong efforts have been done to construct high quality potentials
- A fit to low and high energy data has been used
- These potentials describe very well properties of the infinite liquid

- The extremely large value of a suggests universal behavior
- The Helium dimer can be described using the value of a plus first order corrections in r_{eff}/a
- The Helium trimer can be described by an effective hamiltonian introducing and independent constant: W_0.

Could this description based on low energy data be extended to systems with $N > 3$?

To which extend the saturation properties are determined by the values of a, E_d and E_3^0?

Two different points of view of Helium systems

- In the 80's strong efforts have been done to construct high quality potentials
- A fit to low and high energy data has been used
- These potentials describe very well properties of the infinite liquid

- The extremely large value of a suggests universal behavior
- The Helium dimer can be described using the value of a plus first order corrections in r_{eff}/a
- The Helium trimer can be described by an effective hamiltonian introducing and independent constant: W_0.

Could this description based on low energy data be extended to systems with $N > 3$?

To which extend the saturation properties are determined by the values of a, E_d and E_3^0?

Two different points of view of Helium systems

- In the 80's strong efforts have been done to construct high quality potentials
- A fit to low and high energy data has been used
- These potentials describe very well properties of the infinite liquid

- The extremely large value of a suggests universal behavior
- The Helium dimer can be described using the value of a plus first order corrections in r_{eff}/a
- The Helium trimer can be described by an effective hamiltonian introducing and independent constant: W_0.

Could this description based on low energy data be extended to systems with $N > 3$?

To which extend the saturation properties are determined by the values of a, E_d and E_3^0?

Saturation properties using soft potentials

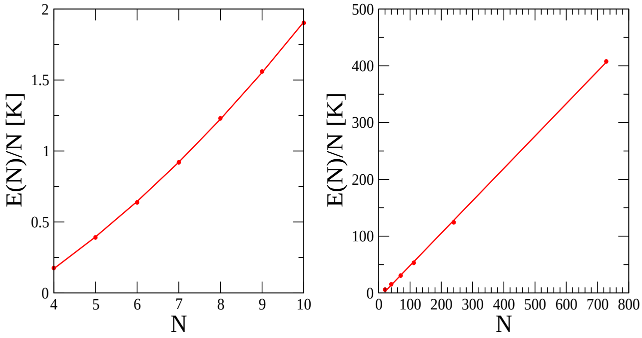
- The Hamiltonian of the system is:

$$\sum_i -\frac{\hbar^2\nabla_i^2}{m} + \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k)$$

- $V(i,j) = V_0 e^{-r_{ij}^2/r_0^2}$ and $W(i,j,k) = W_0 e^{-\rho_{ijk}^2/\rho_0^2}$
- V_0 and r_0 determined from the two-body data
- W_0 and ρ_0 determined from a fit to the trimer energy
- To solve the N-body problem we use the Hyperspherical Harmonic basis
- For $N \leq 10$ the convergence is fine
- For $N \geq 10$ a good convergence depends on ρ_0
- For $N \geq 10$ we present preliminary results using a DMC computation

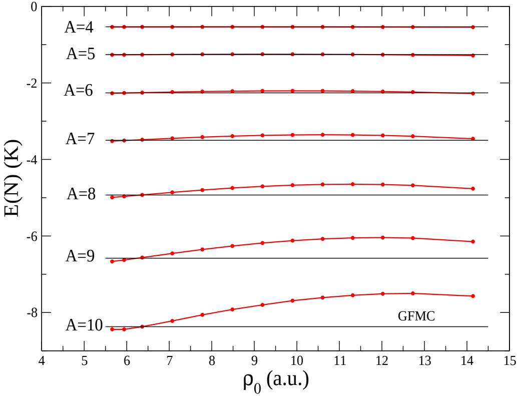
Energy per particle with a soft two-body force

- $E(N)/N \to$ increases linearly with $N \to$ system collapse

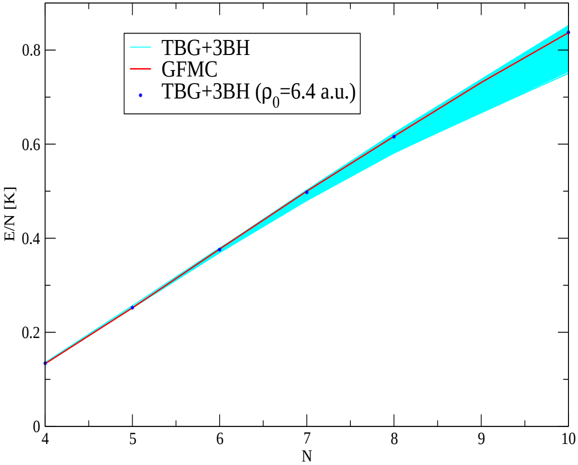


Studies in the region $4 \leq N \leq 10$

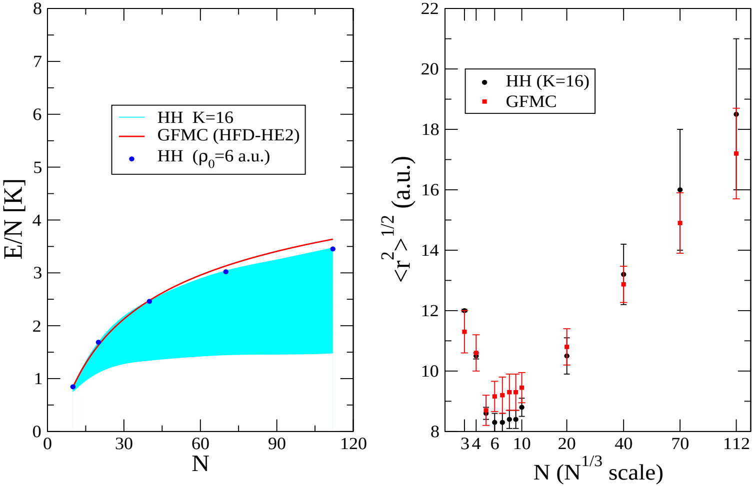
- We compute the energies using the Hyperspherical Harmonic basis
- We study the energy as a function of the 3BH range



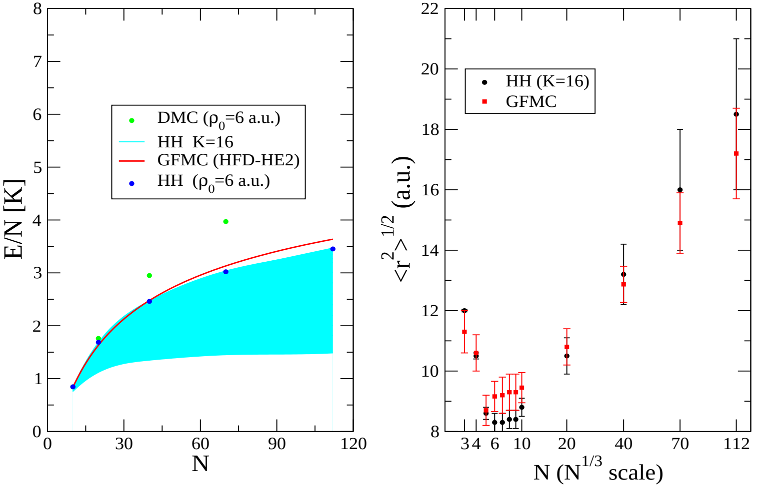
Studies in the region $4 \leq N \leq 10$



Studies in the region $10 \leq N \leq 112$



Studies in the region $10 \leq N \leq 112$



Studies in the region $10 \leq N \leq 112$

Preliminary conclusions

- Calculations in the sector $N \leq 10$ tend to prefer small values of ρ_0
- Calculations in the sector $10 \leq N \leq 112$ tend to prefer higher values of ρ_0
- This seems to be a manifestation of two different scales: a repulsive three-body force with longer range, even with a smaller strength, tends to maintain the particles well apart.
- This behavior mimic better the short range repulsion of the original two-body force
- The universal behavior observed in small clusters makes a transition to a natural state in which the bosons are close to each other and feel the internal part of the interaction
- Perhaps the inclusion of a four-body force could help
- Different scales in the original interaction could be represented by many-body force

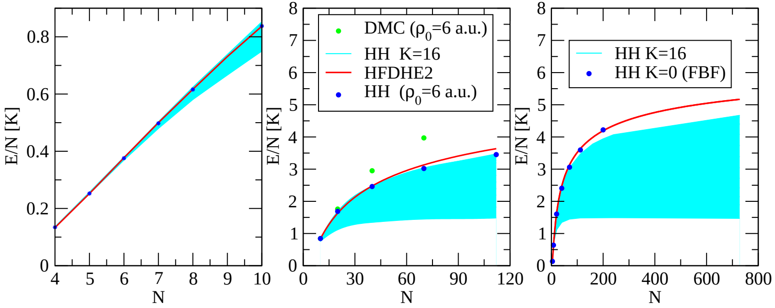
Studies in the region $10 \leq N \leq 112$

Preliminary conclusions

- Calculations in the sector $N \leq 10$ tend to prefer small values of ρ_0
- Calculations in the sector $10 \leq N \leq 112$ tend to prefer higher values of ρ_0
- This seems to be a manifestation of two different scales:
  a repulsive three-body force with longer range, even with a smaller strength, tends to maintain the particles well apart.
- This behavior mimic better the short range repulsion of the original two-body force
- The universal behavior observed in small clusters makes a transition to a natural state in which the bosons are close to each other and feel the internal part of the interaction
- Perhaps the inclusion of a four-body force could help
- Different scales in the original interaction could be represented by many-body force

Studies in the region $10 \le N \le 728$

- In this region the saturation properties can be predicted
- Convergence properties of the HH by using DMC results
- Possibility of including a four-body force



Conclusions

- Liquid ^{4}He has been studied using a soft potential model
- The shallow characteristic of the ^{4}He-^{4}He system has been considered a key element to construct an effective soft interaction
- the soft potential has been fixed from the low energy 2N data and the trimer energy
- The intention is to see if the effective description can predict the saturation properties
- Using the HH expansion convergence problems appeared as N increases
- Preliminary results using a DMC technique have been shown
- It seems possible to reproduce the HFD-HE2 saturation curve
- The possibility of including a four-body force will be explored
- Could this means that the saturation properties are determined by the low energy data and not necessarily from the high energy data?

Conclusions

- Liquid ^{4}He has been studied using a soft potential model
- The shallow characteristic of the ^{4}He-^{4}He system has been considered a key element to construct an effective soft interaction
- the soft potential has been fixed from the low energy 2N data and the trimer energy
- The intention is to see if the effective description can predict the saturation properties
- Using the HH expansion convergence problems appeared as N increases
- Preliminary results using a DMC technique have been shown
- It seems possible to reproduce the HFD-HE2 saturation curve
- The possibility of including a four-body force will be explored
- Could this means that the saturation properties are determined by the low energy data and not necessarily from the high energy data?

Final remark

- Question:

Why we spend time and efforts to construct a soft potential model that imitates the original potential?

- Answer:
  - There is an intense experimental activity to understand the behavior of a boson system close to the unitary limit $a \longrightarrow \infty$
  - The interaction between atoms is modified using Feshbach resonances
  - Moving the system from its original position (value of a), the complete validity of the original interaction is not guarantied
  - The present analysis will help to understand the behavior of boson gas (or liquid) close to the unitary limit
  - In particular if some universal aspects observed in small clusters are relevant as the number of particles increases

Final remark

- Question:

Why we spend time and efforts to construct a soft potential model that imitates the original potential?

- Answer:

  - There is an intense experimental activity to understand the behavior of a boson system close to the unitary limit $a \to \infty$
  - The interaction between atoms is modified using Feshbach resonances
  - Moving the system from its original position (value of a), the complete validity of the original interaction is not guarantied
  - The present analysis will help to understand the behavior of boson gas (or liquid) close to the unitary limit
  - In particular if some universal aspects observed in small clusters are relevant as the number of particles increases