UNIVERSITAT DE BARCELONA
B·KC Barcelona Knowledge Campus
Campus d'Excel·lència Internacional
ICCUB
Institut de Ciències del Cosmos

Exotic Nuclear Systems from Lattice QCD

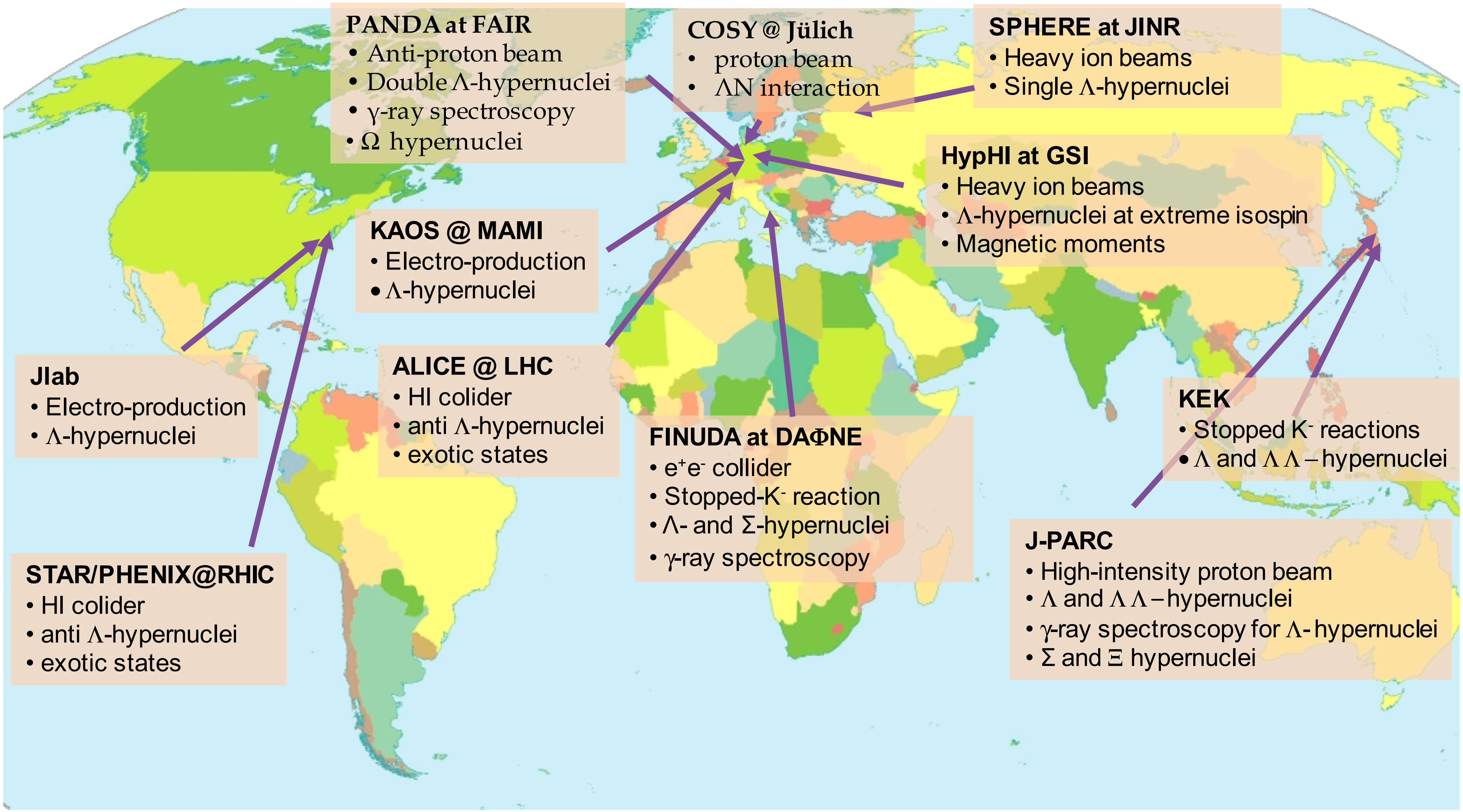
Assumpta Parreño (U Barcelona)

NPLQCD Collaboration

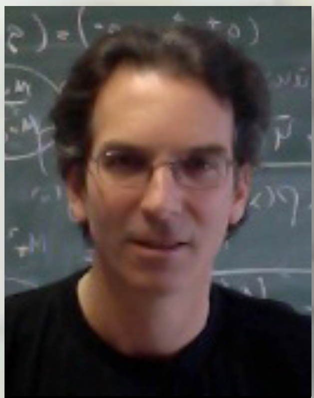
ELBA XIV workshop on Lepton-nucleus scattering
June 27 - July 1, 2016 @ Marciana Marina, Isola d'Elba

GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN
Generalitat de Catalunya
Departament d'Economia i Coneixement
NPLQCD

“strange” experimental program



updated from J. Pochodzalla, Int. Journal Modern Physics E, Vol 16, no. 3 (2007) 925-936



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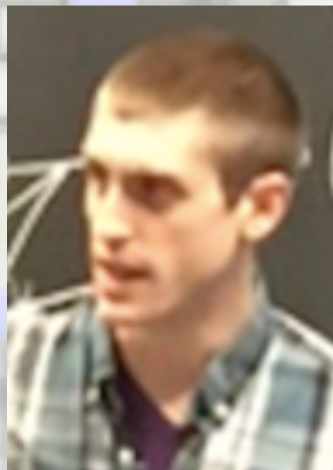

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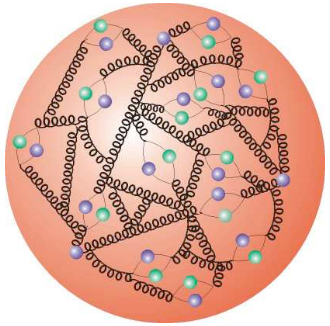
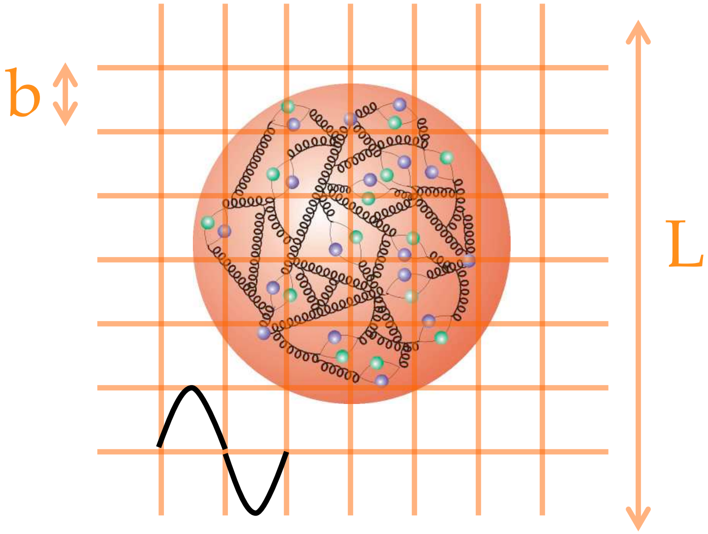
Calculation of the properties and interactions of nuclear systems ($A = 1, 2, 3, \ldots$) including exotics (strange and hidden-charm)





$L \gg$ relevant scales $\gg b$

finite number of d.o.f
(finite volume)

$L^3 \times T$

$\lambda_{\min} = 2b$ shortest wave length

$x = b\,(n_1, n_2, n_3, n_4) \qquad n_j \in \mathbb{Z}$

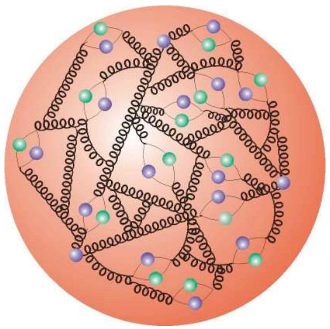
$$\left.\begin{array}{r} \vec{p} = \dfrac{2\pi}{L}\vec{n}, \quad n_\mu \in \mathbb{Z} \\ x_\mu = m_\mu b, \quad \text{with } m_\mu = 0,1,2,\ldots N-1, \quad \text{and } L = Nb \end{array}\right\} \Rightarrow p_{\max} = n_{\max}\frac{2\pi}{L} = \frac{N}{2}\frac{2\pi}{Nb} = \frac{\pi}{b}$$

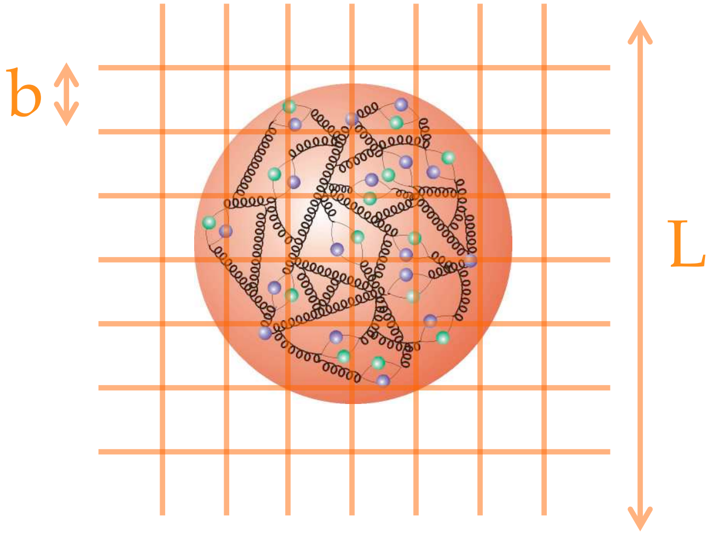
cut-off

(larger wave vector)

For numerical calculations in QCD, the theory is formulated on a (Euclidean) space-time lattice

((anti) periodic (time) spatial boundary conditions) $N_s \times N_s \times N_s \times N_t$





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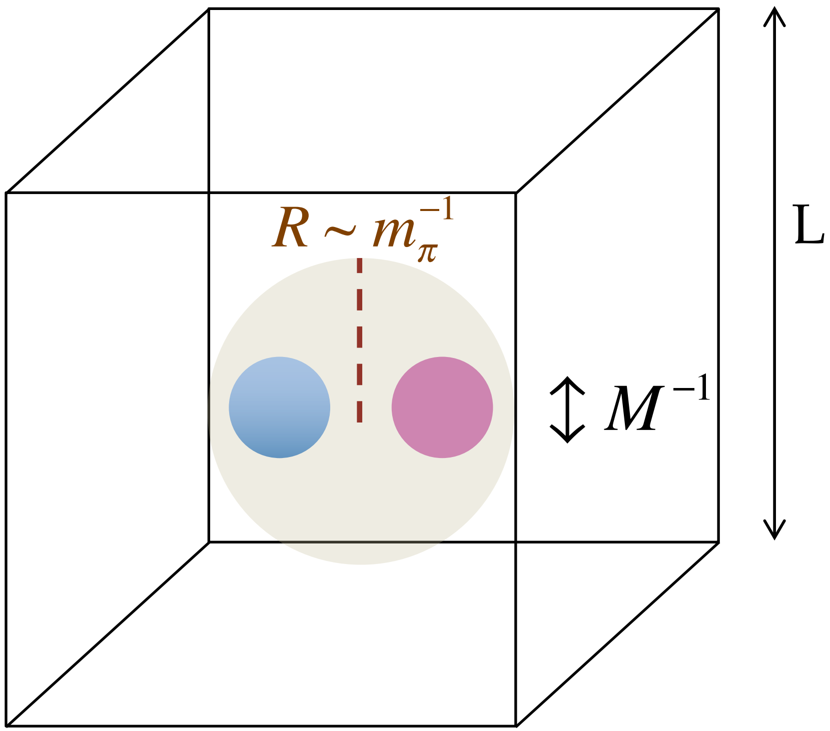


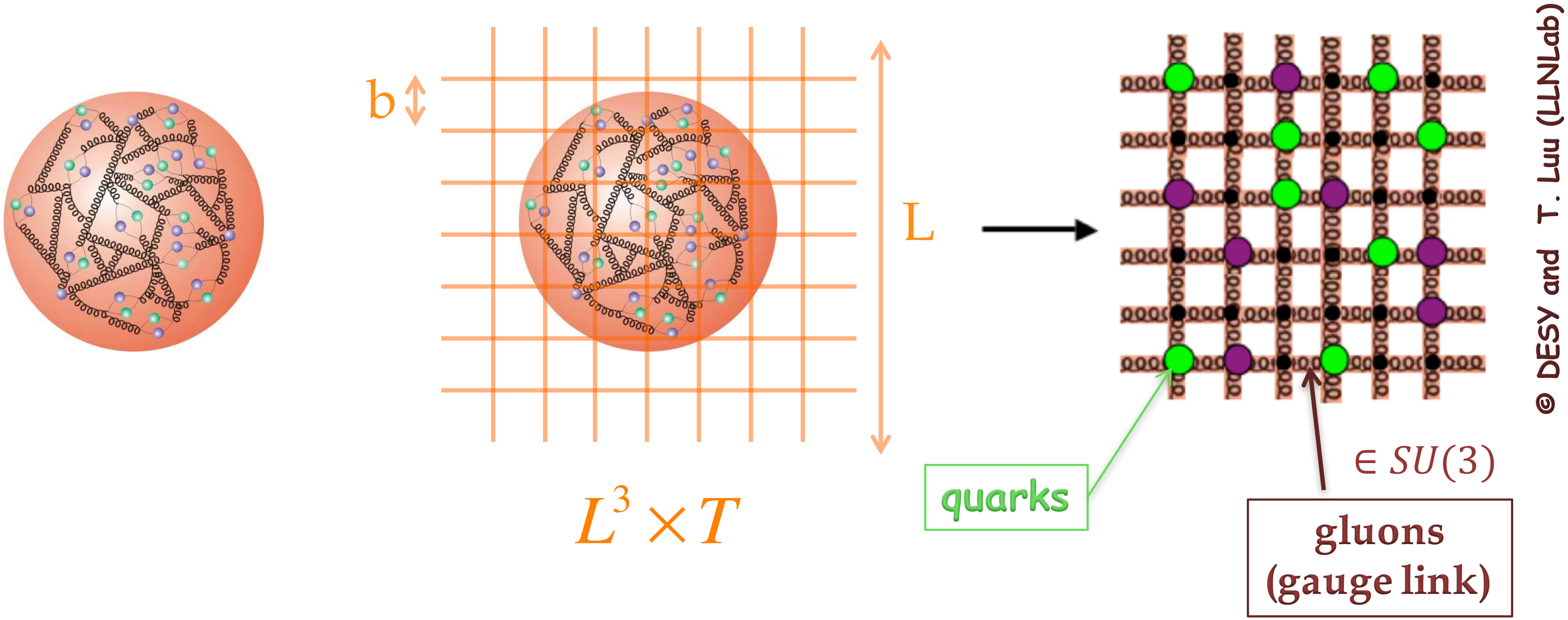
Lüscher, 1990

$R < \frac{L}{2}$

nucleon-nucleon scattering

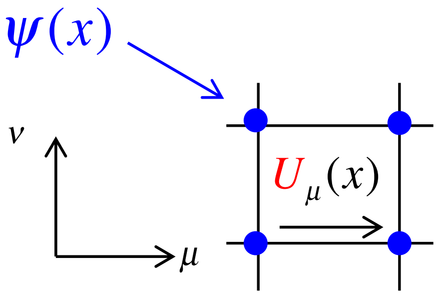
$\Rightarrow m_\pi L >> 1$ (infrared cutoff)

$b << \frac{1}{M_N}$ (ultraviolet cutoff)

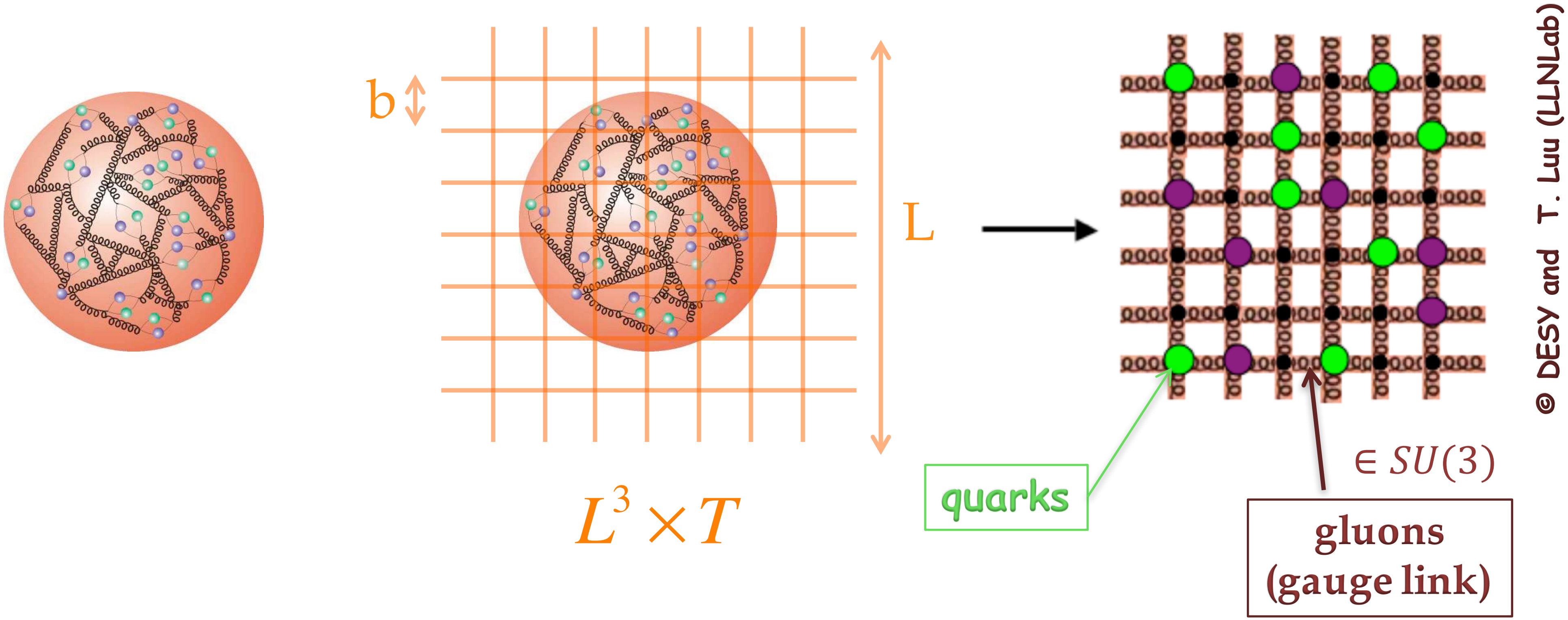


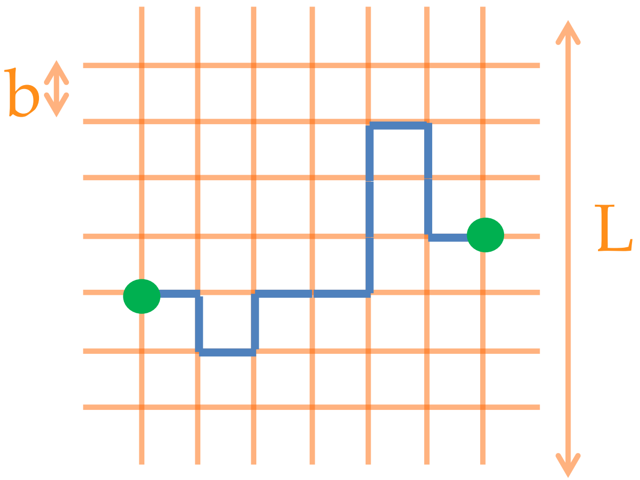
$$P \exp\left(i \int_x^{x+b\hat{\mu}} dx_\mu \; A_\mu^{cont}(x) \right)$$

(in the continuum)



$U_\mu(x) \in SU(3)$





LQCD is a non-perturbative implementation of Field Theory, which uses the Feynman path-integral approach to evaluate transition matrix elements

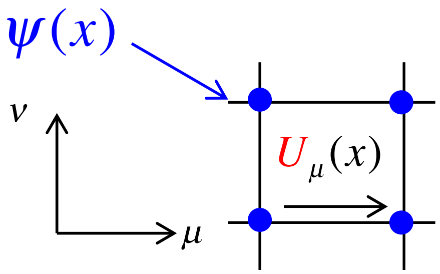
Our continuous Path-Integral (QCD partition function):

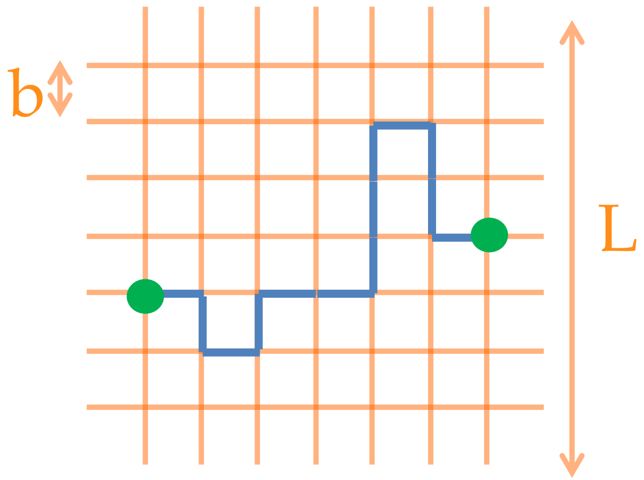
$$Z = \int D\varphi(x)\, e^{-iS_{QCD}[\varphi(x)]} = \int DU\, D\bar{\psi}\, D\psi\, e^{-iS_{QCD}[U,\bar{\psi},\psi]} \xrightarrow[t\to -i\tilde{t}]{} \int DU\, D\bar{\psi}\, D\psi\, e^{-\tilde{S}_{QCD}[U,\bar{\psi},\psi]}$$

$(U_{x\mu} \sim e^{igbA_{x\mu}})$ ↵

oscillating phase — decaying exponential

The weight of each path is a real positive quantity





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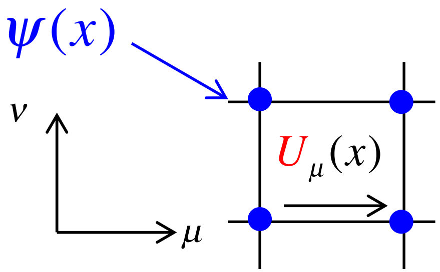


$$Z = \int [dU] \prod_f [dq_f][d\bar{q}_f]\; e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f)\, q_f}$$

$$= \int [dU]\; e^{-S_g[U]} \prod_f [dq_f][d\bar{q}_f]\; e^{-\sum_f \bar{q}_f (D[U]+m_f)\, q_f}$$

$$= \int [dU] \boxed{e^{-S_g[U]} \prod_f \det\left(D[U] + m_f\right)}$$

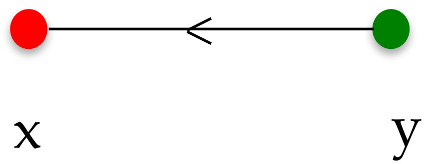
~ P(U)
Boltzmann weight

BASIS OF NUMERICAL SIMULATIONS

expectation values

When computing expectation values of any given operator O, the quark fields in O are re-expressed in terms of quark propagators using Wick's Theorem: **write all possible contractions for the fields** (removing the dependence of quarks as dynamical fields)

$$Q_u^{-1}(x,y) = \overbrace{u(y)\bar{u}(x)}$$



$$\langle O(U,q,\bar{q})\rangle = \frac{1}{Z}\int [dU] \prod_f [dq_f][d\bar{q}_f]\, O(U,q,\bar{q})\, e^{-S_g[U]-\sum_f \bar{q}_f (D[U]+m_f)\, q_f}$$

$$= \frac{1}{Z}\int [dU] \prod_f \left(D[U]+m_f\right)^{-1} \det\left(D[U]+m_f\right)\, e^{-S_g[U]}$$

↑ quark propagator

$$\xleftrightarrow{\lim N\to\infty} \langle \hat{O}\rangle = \frac{1}{N}\sum_{i=1}^{N} \hat{O}\,[Q(U_i)^{-1}]$$

gluon cfgs

$$\text{error} \sim \frac{1}{\sqrt{N}}$$

LQCD algorithm

1. Generate an ensemble of N gauge-field configurations $\{U_i\}$ according to the probability distribution P(U)

$$Z = \int [dU] \; e^{-S_g[U]} \prod_f \det\left(D[U] + m_f\right)$$

~ P(U)
Boltzmann weight

2. Use the N gauge-field configurations previously generated to calculate the quark propagators on each configuration $Q^{-1}[U_i] \sim \left(D[U] + m_f\right)^{-1}$

3. Compute correlation functions (expectation values of local gauge-invariant operators):

$$\langle O(U, q, \bar{q}) \rangle = \frac{1}{Z} \int [dU] \prod_f \underbrace{\left(D[U] + m_f\right)^{-1}}_{\text{propagators}} \underbrace{\det\left(D[U] + m_f\right) \; e^{-S_g[U]}}_{\text{configurations } (\sim P(U))}$$

valence quarks → $\left(D[U] + m_f\right)^{-1}$; sea quarks → $\det\left(D[U] + m_f\right)$

$(D[U] + m_f)^{-1}$

Repeated inversions are done using iterative solvers (CG)
Computational cost ~ condition number ~ $1/m_f$
For light quark masses (u,d) this factor the cost is very large

→ Use large values of the light quark masses

$$\text{Cost} \approx \left[\frac{1}{m_q}\right] [L]^a \left[\frac{1}{b}\right]^\gamma$$

USE UNPHYSICAL VALUES OF THESE PARAMETERS
LATTICE ARTIFACTS

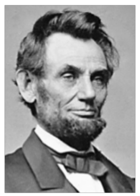
sources of systematic errors in the numerical calculation

finite volume L, discretization (finite spacing) b, value of the light quark masses

EFT Extrapolations

$L \to \infty$

$b \to 0$



$m_q \to m_q^{physical}$

Ground-state hadronic energies

Our method consists in a direct extraction of the energy levels from LQCD calculations of correlation functions for the one, two-, three- and 4 baryon systems in the non-strange and strange sectors.

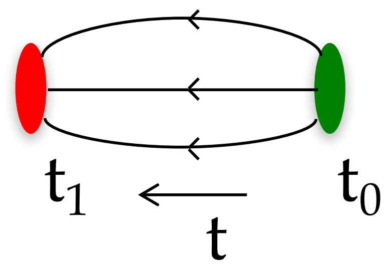
One can use these energy extractions to obtain information about the binding energy of the system, on the scattering parameters, magnetic moments, polarizabilitites, etc.

We use different method analysis to ensure a robust extraction of the g.s.:

- ✓ Multiple exponential fits
- ✓ Matrix-Prony method
- ✓ Generalized-pencil-of-function method

Formalism: Direct Lattice QCD extraction ⟷ Compute correlation functions

$$C(\Gamma^{\nu},\vec{p},t)=\sum_{\vec{x}_1} e^{-i\vec{p}\vec{x}_1}\,\Gamma^{\nu}\left\langle J(\vec{x}_1,t)\bar{J}(\vec{x}_0,0)\right\rangle$$



$$C(t)=\langle 0|\phi(t)\phi^{\dagger}(0)|0\rangle \longrightarrow \langle\phi|e^{-Ht}|\phi\rangle=\sum_n\langle\phi|e^{-Ht}|n\rangle\langle n|\phi\rangle=\sum|\langle\phi|n\rangle|^2 e^{-E_n t}$$

$$\phi(t)=e^{Ht}\phi\, e^{-Ht}$$

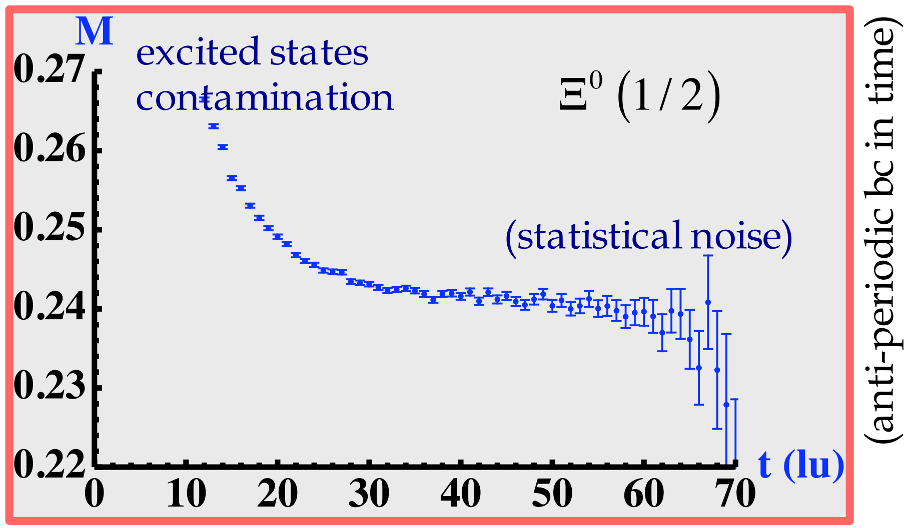
large t ↓

$$Z_0\, e^{-E_0 t}$$

Effective mass plot → extract the g.s. energy (mass) from plateau

$$\frac{1}{t_J}\log\frac{C_A(t)}{C_A(t+t_J)}=E_{0A}\quad (m_A)$$

$$\Xi^0_{\alpha}(\vec{x},t)=\varepsilon^{ijk}s^i_{\alpha}(\vec{x},t)\left(u^{j^T}_{\alpha}(\vec{x},t)C\gamma_5\, s^k(\vec{x},t)\right)$$



noise-to-signal

Lepage, 1989

pions:

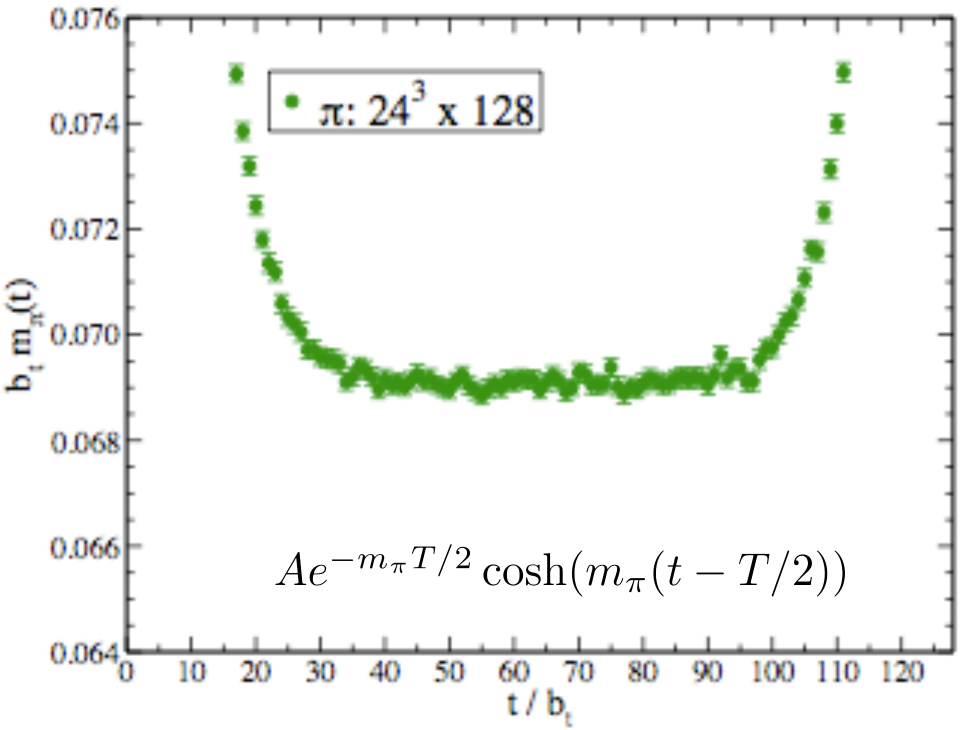
$$\frac{\sigma}{\langle C \rangle} \to \frac{1}{\sqrt{N}}$$

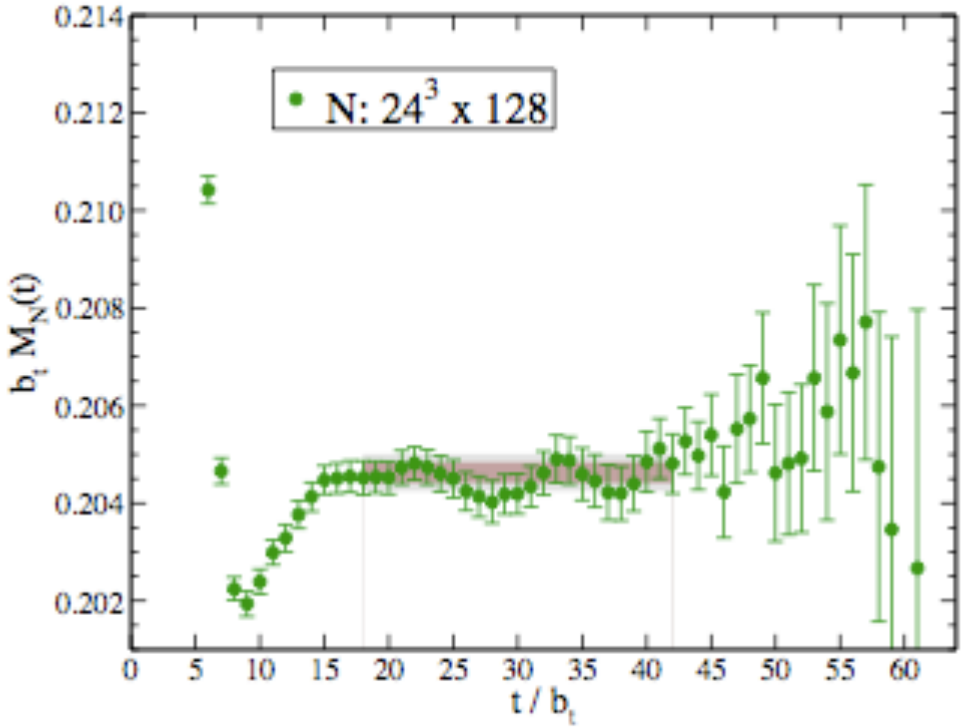
nucleons:

$$\frac{\sigma}{\langle C \rangle} \sim \frac{1}{\sqrt{N}} \times \exp\left(M_N - \frac{3m_\pi}{2}\right)t$$

for baryons, the noise grows exponentially with time

NPLQCD, Phys.Rev. D84 (2011) 014507

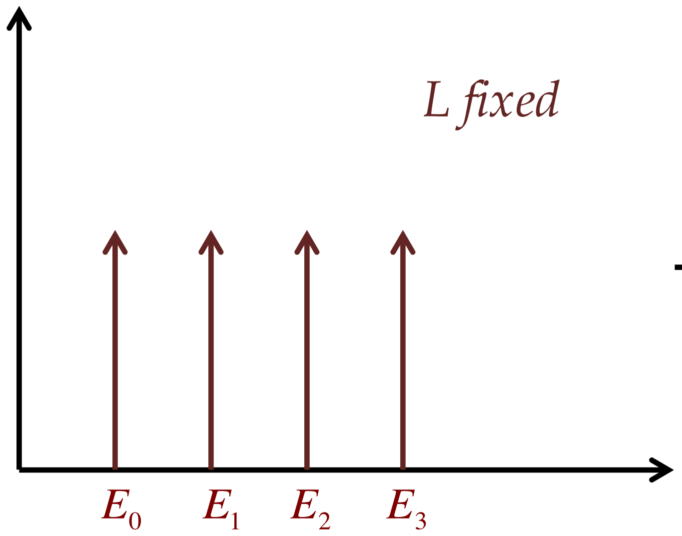




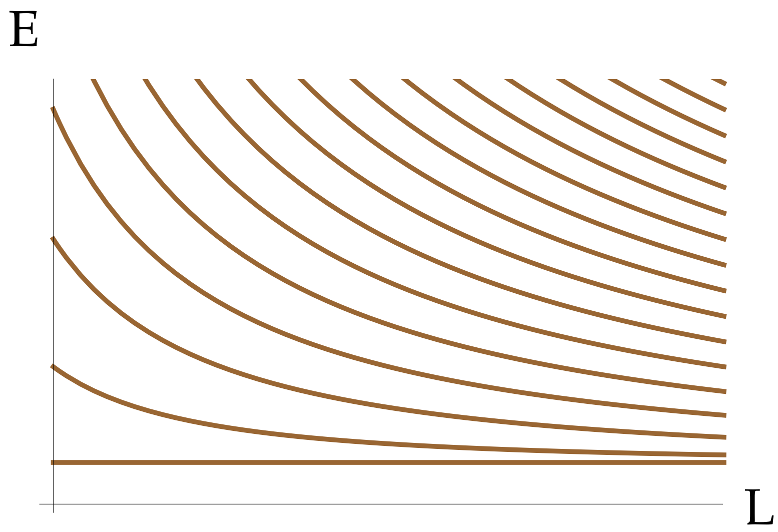
hadron-hadron scattering?

For two **non-interacting** particles of mass m in a volume L^3 and zero CoM momentum :

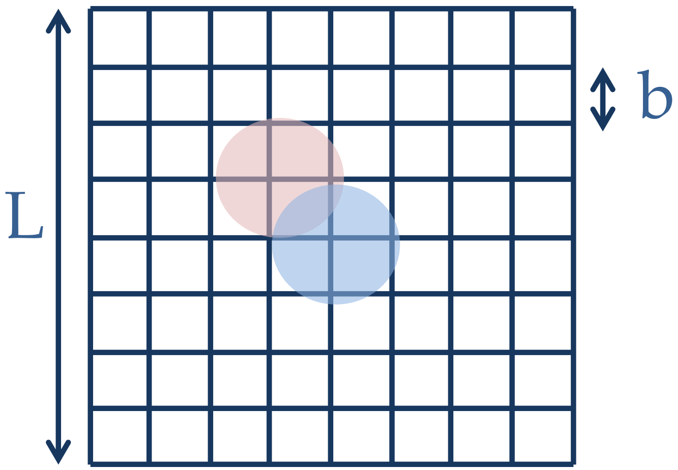
$$E_n = 2\sqrt{m^2 + |\vec{p}|^2} = 2\sqrt{m^2 + \left(\frac{2\pi}{L}|n|\right)^2}\,,\; n_i \in Z$$



spectral function

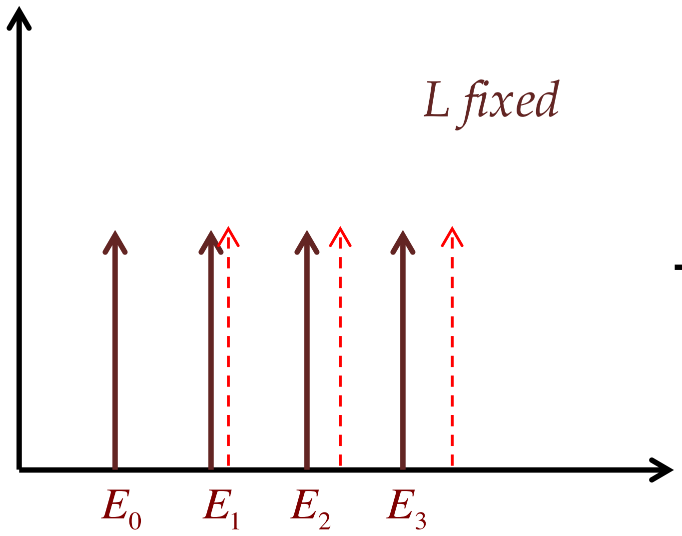


energy spectrum

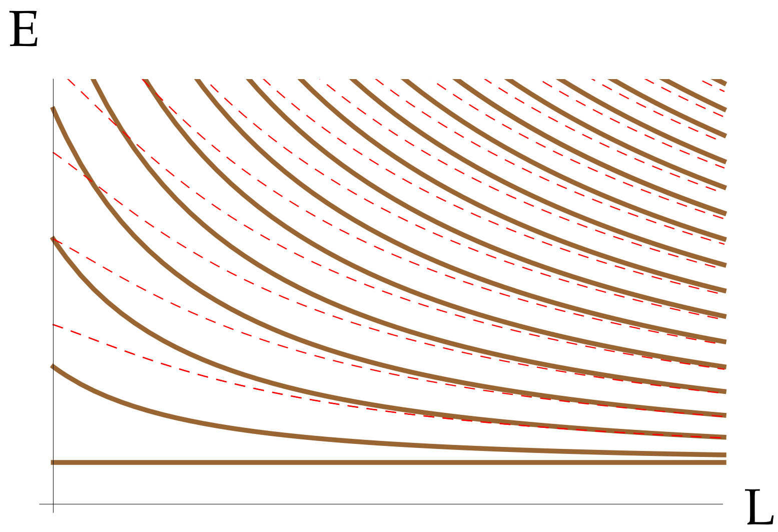


two **interacting** particles of mass m in a volume L^3 and zero CoM momentum :

$$E_n \neq 2\sqrt{m^2 + \left(\frac{2\pi}{L}\right)^2 |n|^2}, n_i \in Z$$



spectral function



shift in the energy spectrum

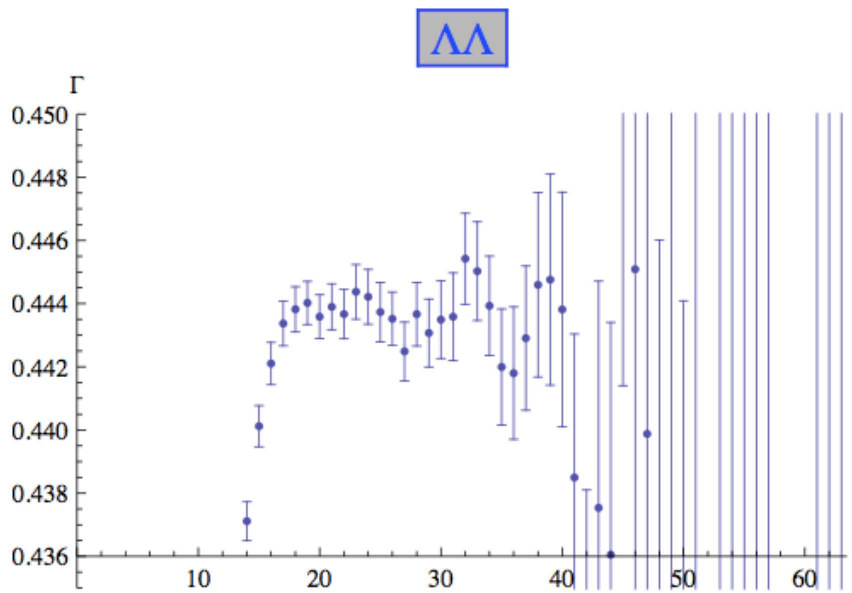
Lüscher, Hamber, Marinari, Parisi , Rebbi (QFT);
Uhlenbeck 1930's; Bogoliubov 1940's; Lee, Huang, Yang 1950's

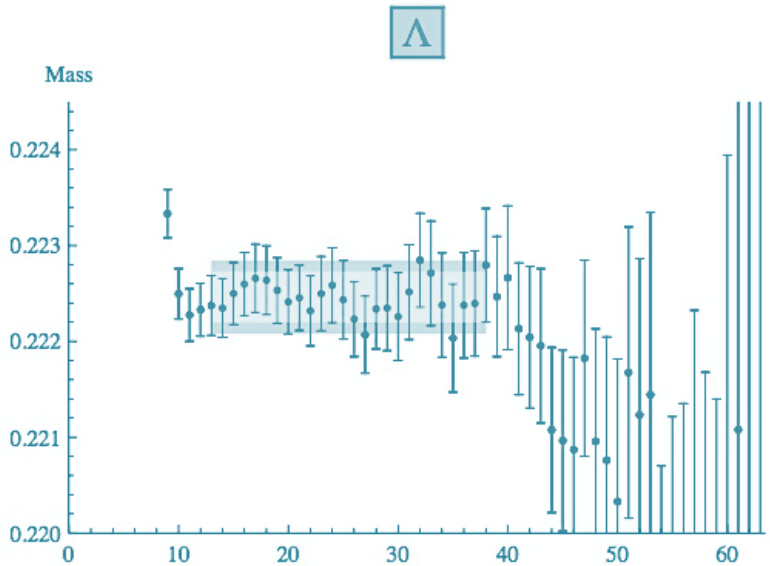
Two particles placed in finite volume suffer from energy shifts, ΔE, which depend on their interactions

We extract the energy of the interacting system of hadrons for a given {m_π , L, b}

$$G_{\Lambda\Lambda}(t) = \frac{C_{\Lambda\Lambda}(t)}{C_\Lambda(t)\, C_\Lambda(t)} \rightarrow A_0\, e^{-\Delta E_{\Lambda\Lambda} t} \quad \rightarrow \quad \frac{1}{t_J} \log \frac{G(t)}{G(t+t_J)} \rightarrow \text{extract} \quad \Delta E$$

$$\Delta E_{\Lambda\Lambda} = E_{\Lambda\Lambda} - E_\Lambda - E_\Lambda$$

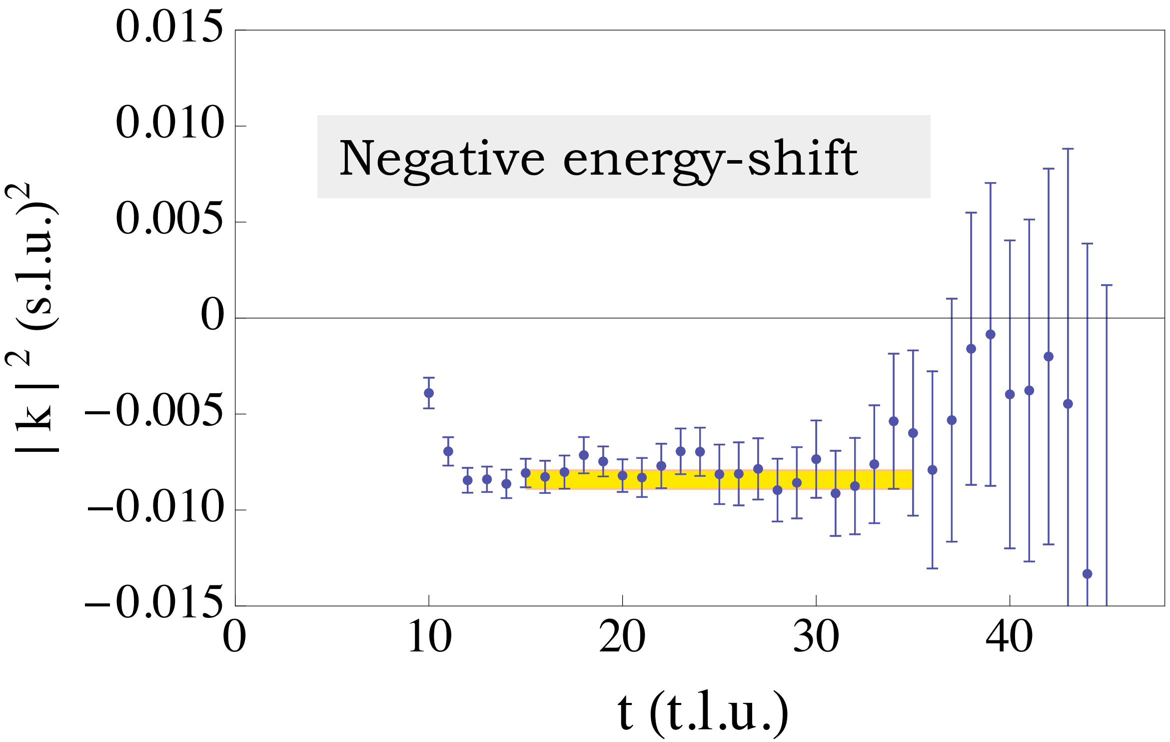




$$\left(J^\pi = \left(\frac{1}{2}\right)^+ ; \qquad \Lambda_\alpha(\vec{x},t) = \varepsilon^{ijk} s^i_\alpha(\vec{x},t) \left(u^{j^T}(\vec{x},t) C \gamma_5 d^k(\vec{x},t) \right) \right)$$

We can also extract the energy of the interacting system for a given {m_π, L, b} set

$$G_{\Lambda\Lambda}(t) = \frac{C_{\Lambda\Lambda}(t)}{C_\Lambda(t)\, C_\Lambda(t)} \longrightarrow A_0\, e^{-\Delta E_{\Lambda\Lambda} t} \qquad \rightarrow \qquad \frac{1}{t_J} \log \frac{G(t)}{G(t+t_J)} \longrightarrow \text{extract} \quad \Delta E$$



ΛΛ channel

$$\left(J^\pi = \left(\frac{1}{2}\right)^+ ; \qquad \Lambda_\alpha(\vec{x},t) = \varepsilon^{ijk} s^i_\alpha(\vec{x},t) \left(u^{j^T}(\vec{x},t)\, C\gamma_5\, d^k(\vec{x},t) \right) \right)$$

Scattering states:

Beane, Bedaque, Parreño, Savage, Phys. Lett. B585 (2004) 106-114

$$\Delta E_0 = \frac{p^2}{M} = \frac{4\pi a}{ML^3}\left[1 - c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2 + \ldots\right]$$

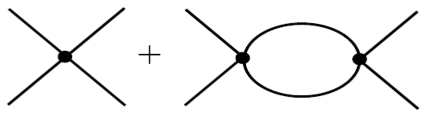
Ground state energy shift

Recovering M. Lüscher, Commun. Math. Phys. 105, 153 (1986) (L >> a)

extract the scattering lentgh

Bound states?

$\mathcal{A} \sim$



$+ \; \ldots \; = \dfrac{4\pi}{M}\dfrac{1}{p\cot\delta(p) - ip}$

infinite volume

b.s. $p^2 = -\gamma^2$

$$\cot\delta(i\gamma) = i$$

finite volume:

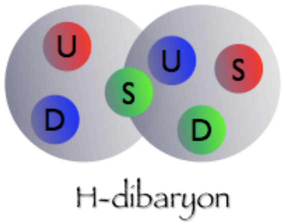
$$\cot\delta(i\gamma)\Big|_{k=i\gamma} = i - i\sum_{\vec{m}\neq\vec{0}}\frac{e^{-|\vec{m}|\gamma L}}{|\vec{m}|\gamma L}$$

$k^2 < 0, \quad k = i\kappa$

$$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L} + \ldots\right)$$

$\kappa \to \gamma$ for large L

$$B_\infty = \frac{\gamma^2}{M}$$



PREDICTION (Bag model)

$m_H - 2\,m_\Lambda \sim -81$ MeV

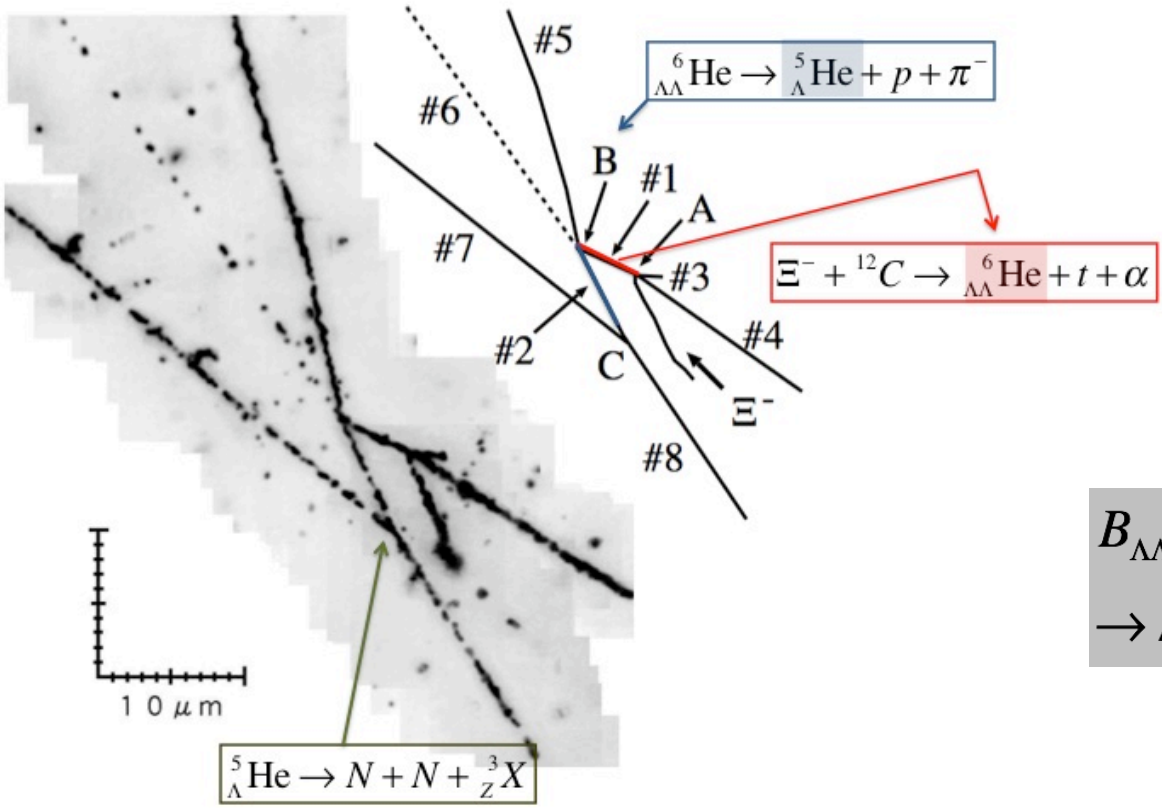
R.L. Jaffe, Phys. Rev. Lett. 38, 195 (1977); 38, 617 (1977) (E)

$$A=2,\ s=-2,\ J=0,\ I=0$$

$$\Lambda\Lambda - \Xi N - \Sigma\Sigma$$

$$SU(3)_f \rightarrow \Psi_H = \frac{1}{\sqrt{8}}\left(\Lambda\Lambda + \sqrt{3}\Sigma\Sigma + 2\Xi N\right)$$

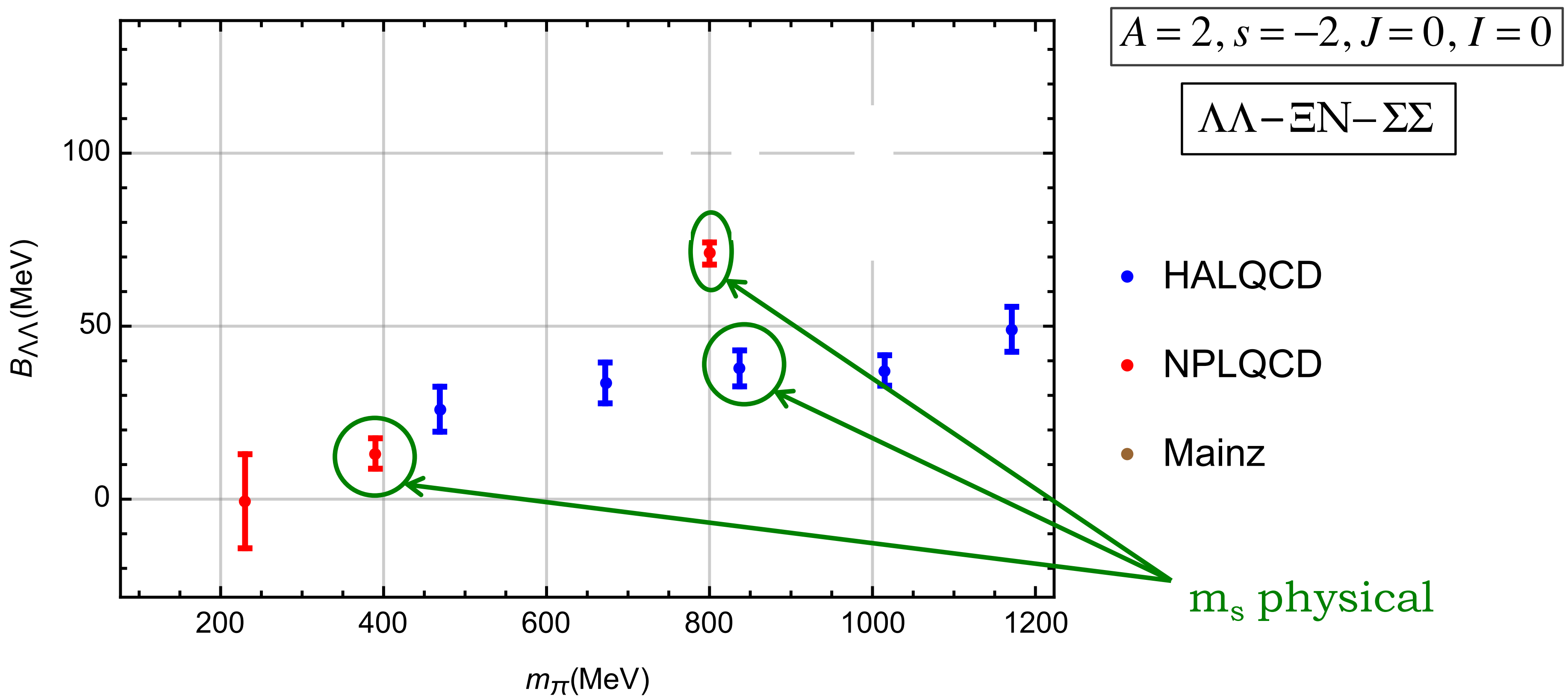
NAGARA EVENT, , KEK-E373, *Takahashi et al. PRL 87 (2001) 212502*



Experimental constraints

$$B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}_{g.s.}) = 6.91 \pm 0.16 \text{ MeV}$$

$$\rightarrow \Delta B_{\Lambda\Lambda} - 2B_\Lambda = 1.01 \pm 0.20^{+0.18}_{-0.11} \text{MeV}$$



NPLQCD, PRL 106, 162001 (2011) n_f=2+1, b_s = 0.12 fm, L: 2, 2.5, 3, 3.9 fm, m_π = 390 MeV
NPLQCD, Mod.Phys.Lett. A26 (2011) n_f=2+1, b_s = 0.12 fm, L: 4fm, m_π = 230 MeV
NPLQCD, PRD87 (2013) n_f=3, b_s = 0.145 fm, L: 3.4, 4.5, 6.7 fm, m_π = 807 MeV

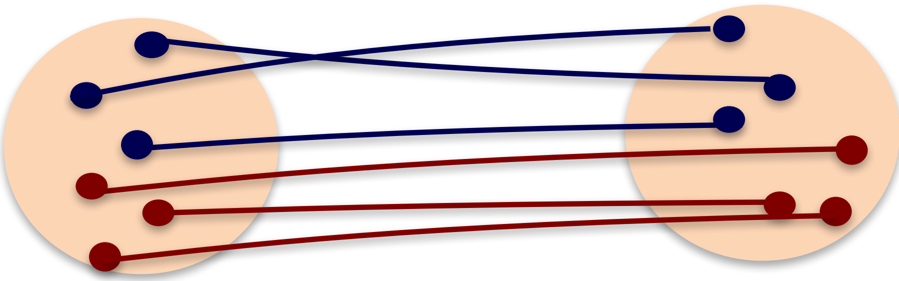
HALQCD, PRL 106, 162002 (2011), NPA 881 (2012)
n_f=3, b_s = 0.12 fm, L: 4 fm m_π = 469, 670, 830, 1015, 1171 MeV

Going beyond A=2

✓ Larger complexity as compared to calculations for single hadrons

$$\left((A+Z)!\,(2A-Z)!\right)$$

$$^{3}H \rightarrow 2880 \qquad {}^{4}He \rightarrow 518\,400$$



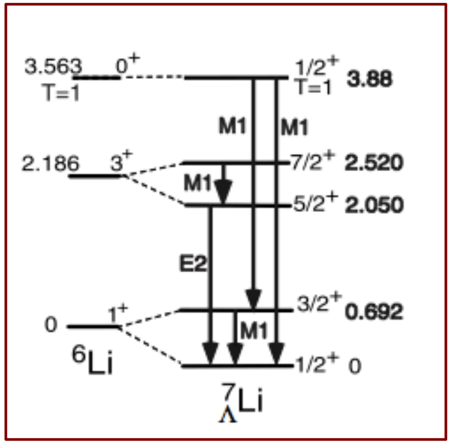
Wick contractions ~ $N_u!\,N_d!\,N_s!$

Detmold & Savage, PRD82 (2010) 014511
Detmold & Orginos, PRD87 (2013) 11, 114512
Doi & Endres, Comput.Phys.Commun. 184 (2013) 117

✓ Demand larger lattice volumes

✓ Demand better accuracy

$$\frac{\sigma}{\langle C\rangle} \sim \frac{1}{\sqrt{N}}\exp\left[A\left(M_N - \frac{3m_\pi}{2}\right)t\right]$$



✓ Small energy splittings in nuclear physics

↓

Need of high statistics calculations

Going beyond A=2

Perform calculations at heavier light-quark masses:

✓ signal-to-noise ratio improves

✓ reduced computational resources to generate LQCD configurations, *i.e.*, larger statistics

$SU(3)_f$

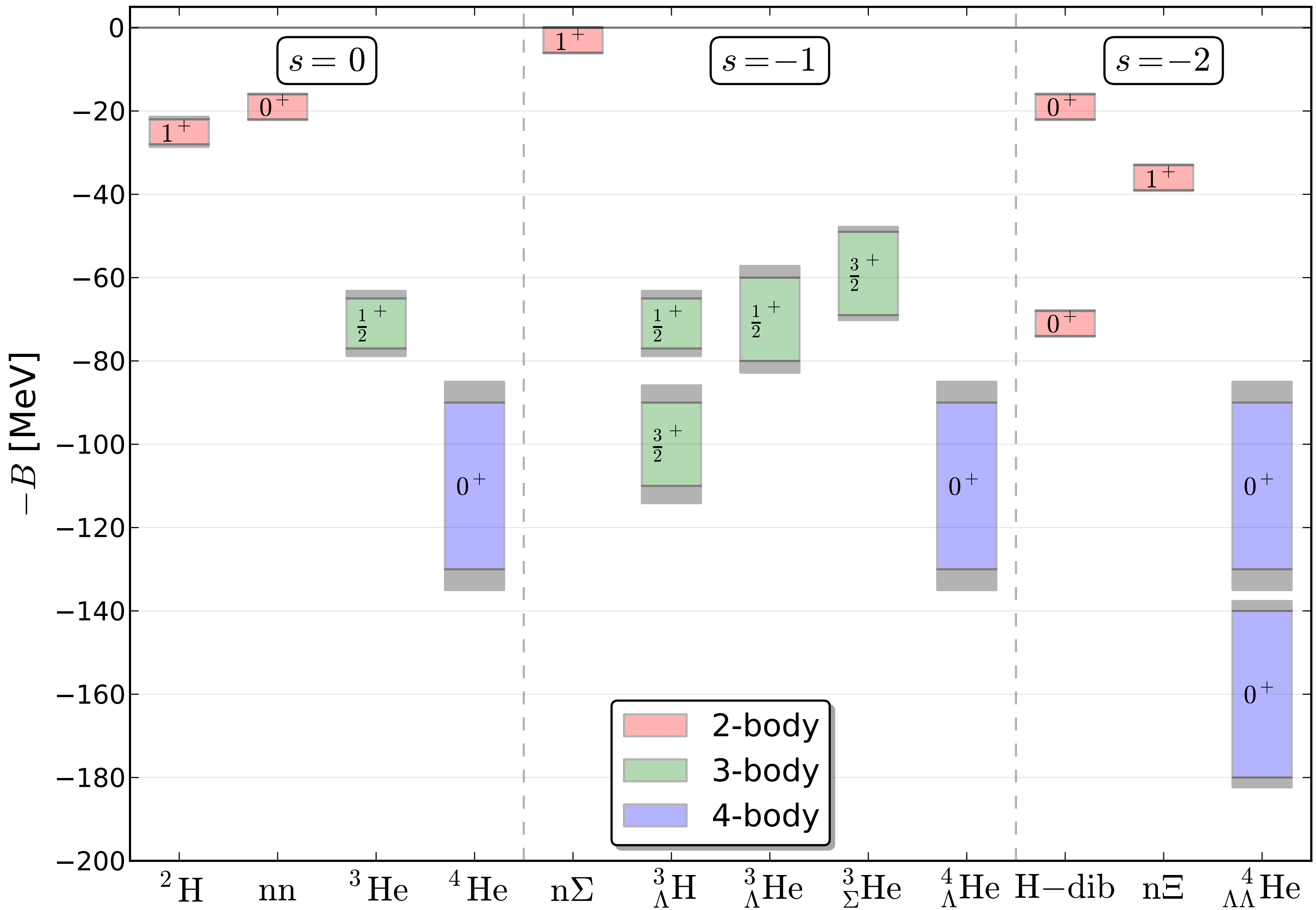
no continuum extrapolation

L/b	T/b	β	$b\,m_q$	b (fm)	L (fm)	T (fm)	m_π (MeV)	$m_\pi L$	$m_\pi T$	$N_{\rm cfg}$	$N_{\rm src}$
24	48	6.1	−0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	96
32	48	6.1	−0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	72
48	64	6.1	−0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1905	54

infinite volume extrapolation

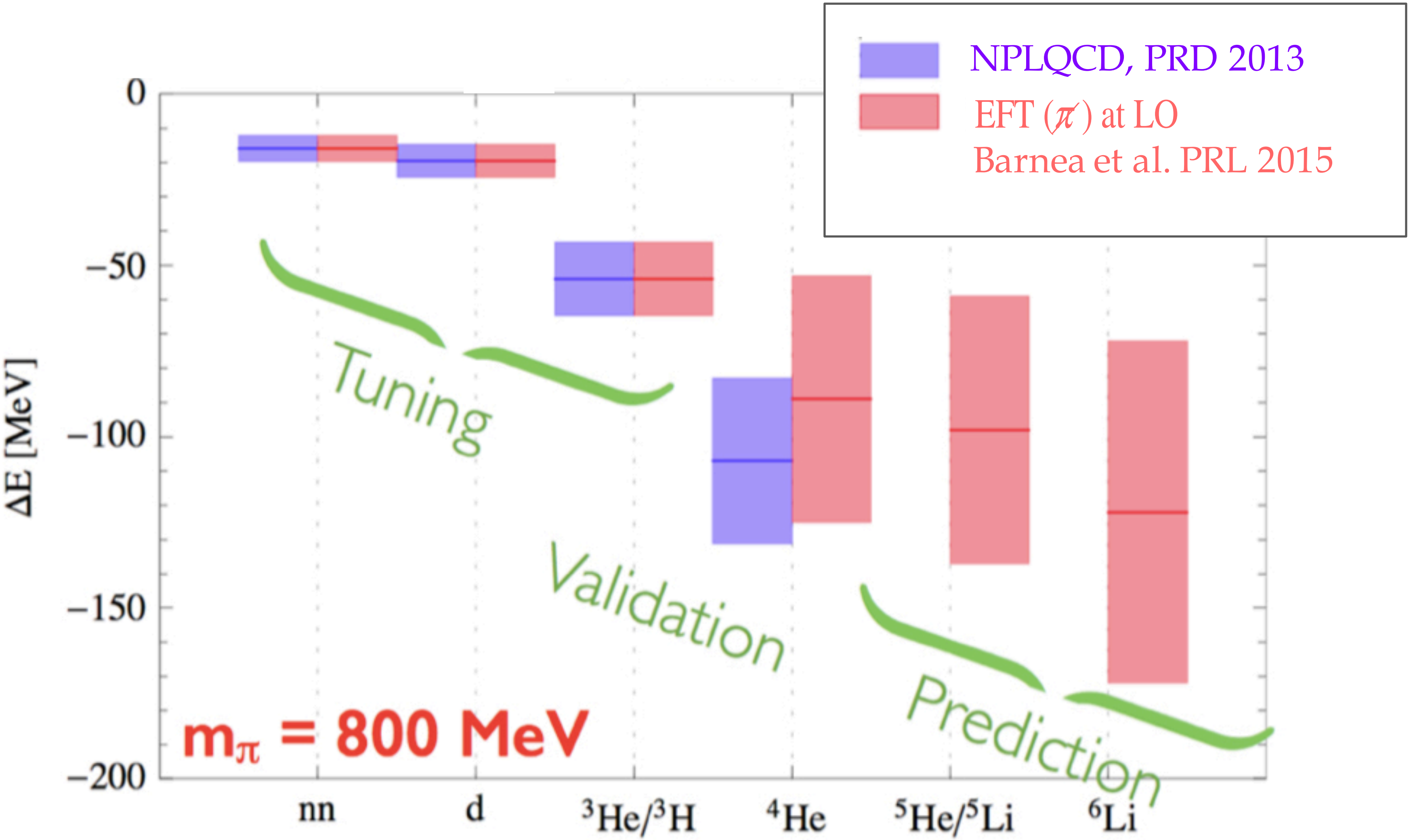
NPLQCD Phys. Rev. D87 (2013) 3, 034506

$SU(3)_f$



no e.m. interactions

(hadronic labels for (J^π, I, s, A) states) many body: EFT + EIHH/AFDMC

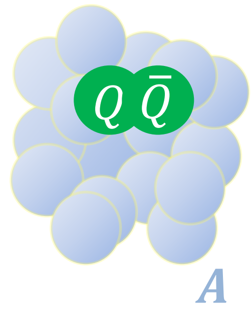


Reusing the nuclear correlation functions
Quarkonium-nucleus

PRD 91, 114503 (2015)

Brodsky, Schmidt, de Téramond, PRL 64, 1011 (1990)
(spin-spin correlations in pp scattering)

$\bar{Q}\Gamma Q$



no Pauli blocking
no quark-exchange

$\mathcal{A}$

$m_\pi = m_K \sim 807$ MeV
b = 0.145 fm
L ~ 3.4, 4.5 and 6.7 fm

EXPERIMENTAL SEARCH PROGRAM
ATHENNA (Jlab), PANDA@FAIR, JPAC

For the compound system:

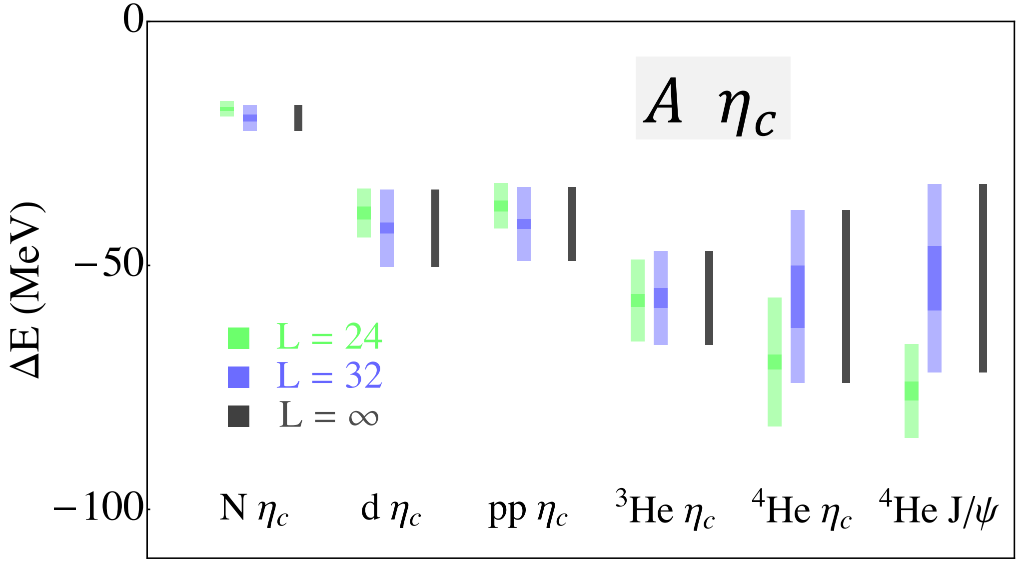
$$C_{\mathcal{AB}}(t) = \left\langle 0 \middle| \chi_{\mathcal{A}}(t) \chi^\dagger_{\mathcal{B}}(0) \middle| 0 \right\rangle$$

$\chi_{\mathcal{A}} = \chi_A \, \chi_{\bar{Q}\Gamma Q}$

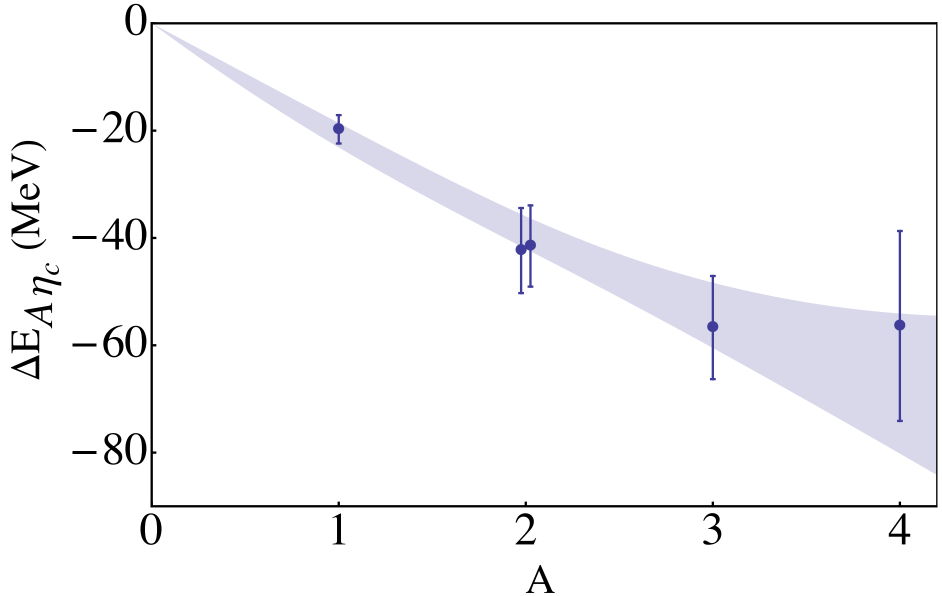
(Rel. Heavy Quark action for the charmonium)

Combination of analysis methods (included in the systematics)

→ one-state, two-states and $\mathcal{R}(t) = \frac{C_{\mathcal{AB}}(t)}{C_{AB}(t)C_{\overline{Q}\Gamma Q}(t)}$



$$B_{AQ\bar{Q}} \sim \alpha_V^{Q\bar{Q}} - \alpha_S^{Q\bar{Q}} A^{-1/3}$$

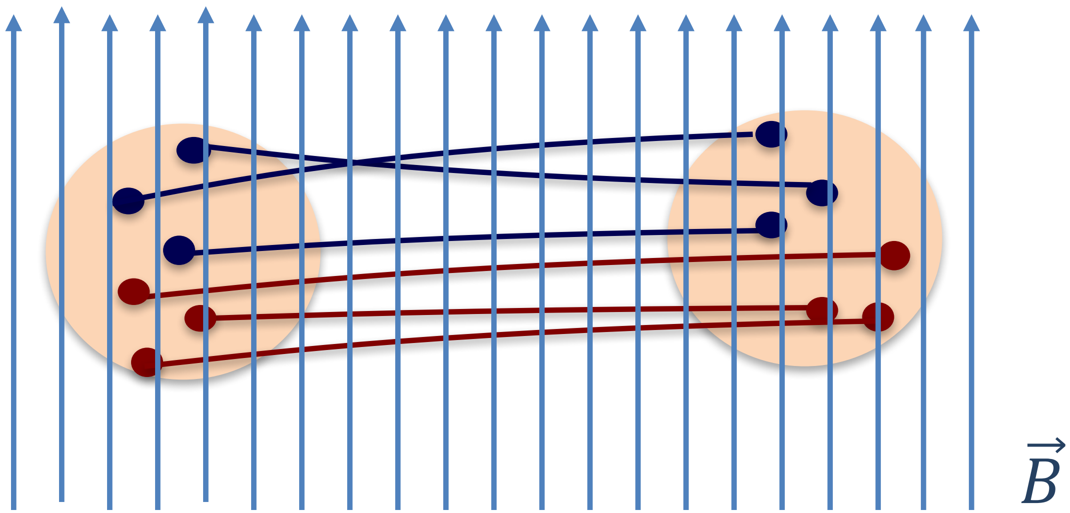


Reusing the nuclear correlation functions
Magnetic moments of nuclei

NPLQCD, Phys. Rev .Lett. 113 (2014) 25, 252001

G. t'Hooft, 1979

Background field method: Uniform, time-independent background magnetic field



$$e|B| = \frac{6\pi}{L^2}\tilde{n}, \quad \boxed{\vec{B} = \hat{z}\cdot B}$$

$$U_\mu^b(x) \to U_\mu^b(x) U_\mu^{\text{ext}}(x)$$

$$U_0^{\text{ext}}(x) = U_3^{\text{ext}}(x) = 1 \qquad U_1^{\text{ext}}(x) = \begin{cases} 1 & x_1 \neq L-b \\ e^{-i\beta x_2} & x_1 = L-b \end{cases} \qquad U_2^{\text{ext}}(x) = e^{i\beta x_1}$$

$$\beta \sim q_q B b^2$$

Reusing the nuclear correlation functions
Magnetic moments of nuclei

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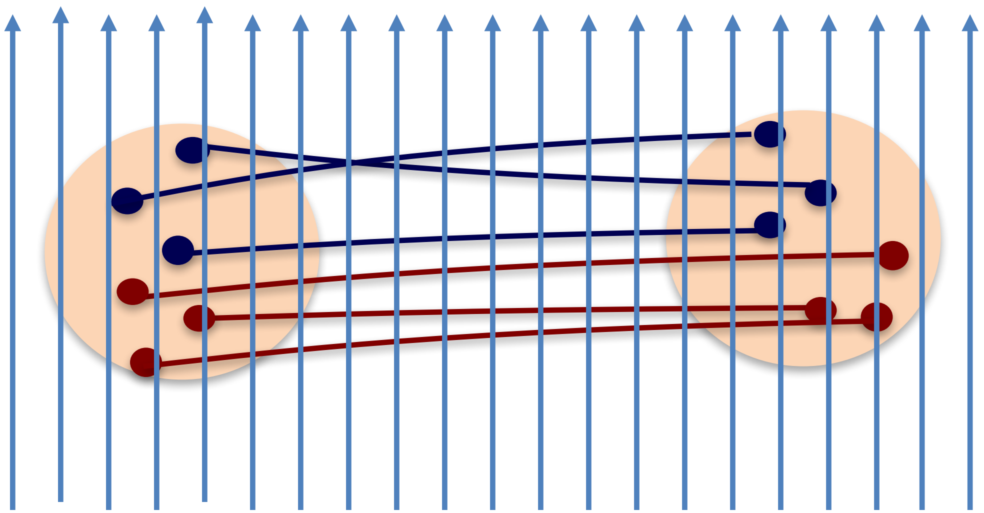


$$e|B| = \frac{6\pi}{L^2}\tilde{n}, \quad \boxed{\vec{B} = \hat{z}\cdot B}$$

$$E(B) = M + \underbrace{\frac{|QeB|}{2M}} - \mu\cdot B - 2\pi\beta|B|^2 + \dots$$

Landau levels
(charged particles)

$$\delta E^B = E^B_{+j} - E^B_{-j}$$

$$\delta E^B = -2\mu|B| + \gamma_3|B|^3$$

LQCD calculations of magnetic moments of light nuclei $SU(3)_f$

Use background magnetic fields

$m_\pi \sim 800$ MeV

$$e|B| = \frac{6\pi}{L^2}\tilde{n}, \quad \vec{B} = \hat{z}\cdot B \quad (e|B| \sim 0.046\,\tilde{n}\ \mathrm{GeV}^2)$$

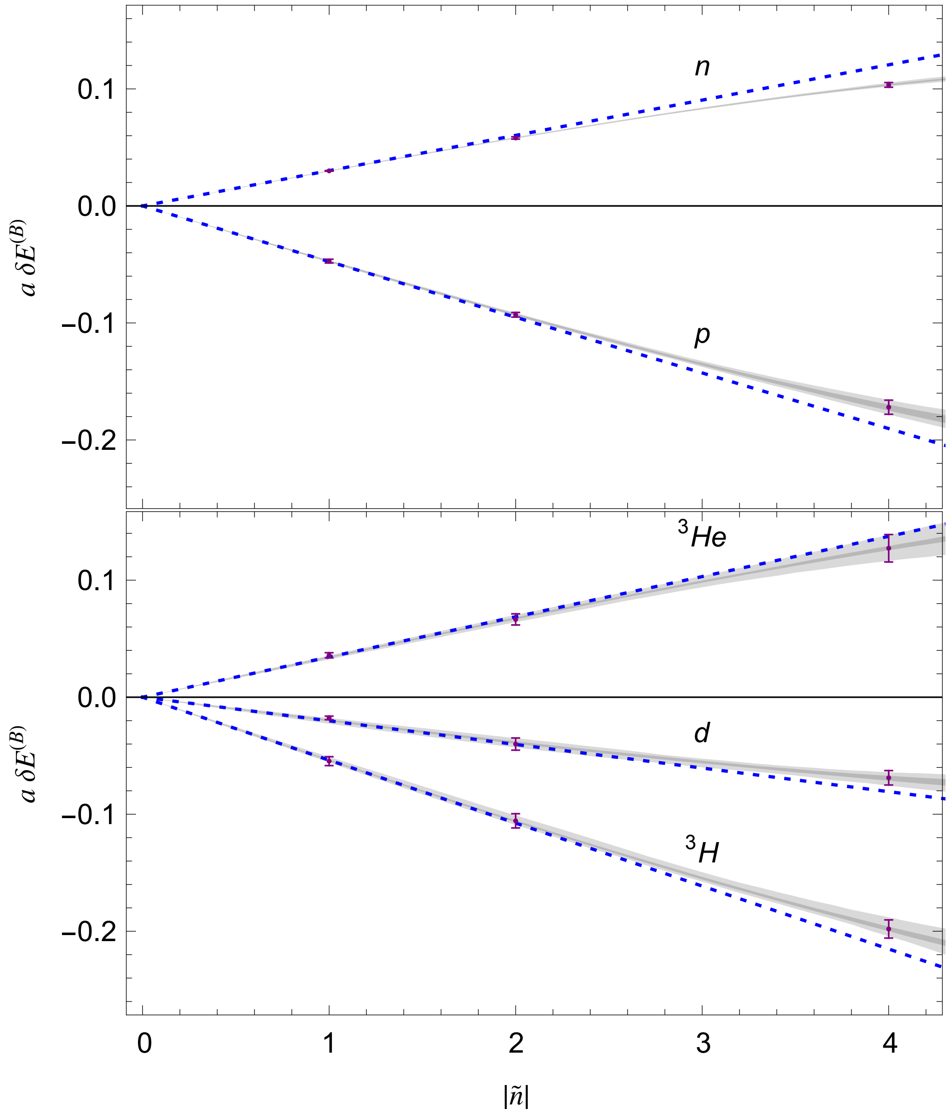
From our calculated correlation functions:

$$R(B) = \frac{C^B_{j_z}(t)\,C^0_{-j_z}(t)}{C^B_{-j_z}(t)\,C^0_{j_z}(t)} \xrightarrow{t\to\infty} Ze^{-\delta E^B t}$$

$$\delta E^B = E^B_{+j} - E^B_{-j}$$

$$E(B) = M + \frac{|QeB|}{2M} - \mu\cdot B - 2\pi\beta|B|^2 + \ldots$$

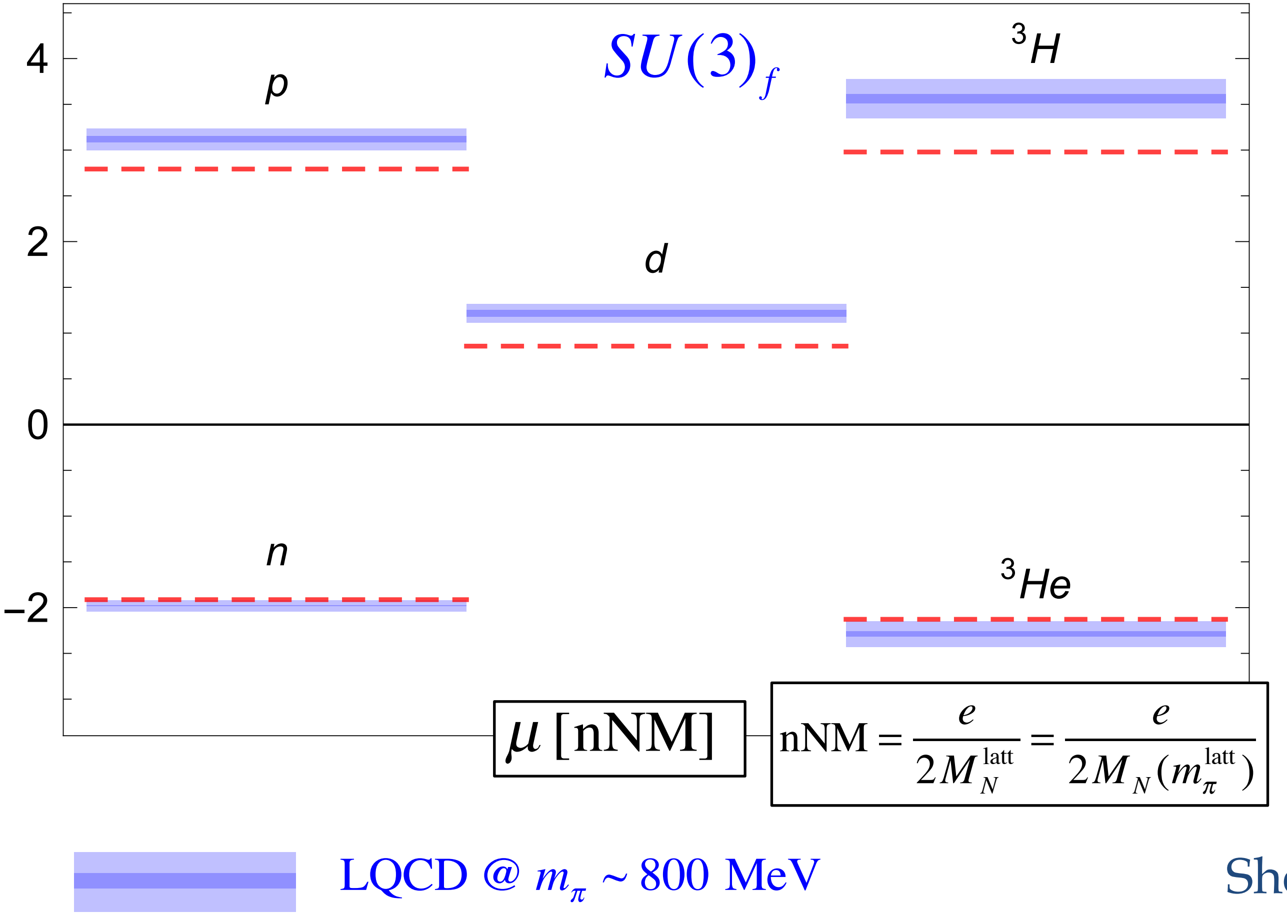
$$\delta E^B = -2\mu|B| + \gamma_3|B|^3$$

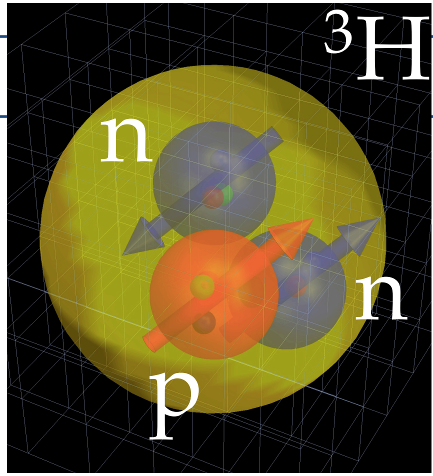


NPLQCD, Phys. Rev .Lett. 113 (2014) 25, 252001

LQCD calculations of magnetic moments of light nuclei

NPLQCD, Phys. Rev .Lett. 113 (2014) 25, 252001





$j_{nn} = 0$

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Artist's impression of a triton. Neutrons in blue and protons in red, with quarks inside; the arrows indicate the alignments of the spins. Image: William Detmold, MIT

Shell-model predictions

$$\mu(^3H) = \mu_p$$

$$\mu(^3He) = \mu_n$$

$$\mu_d \sim \mu_n + \mu_p$$

Nowadays we have calculations of :

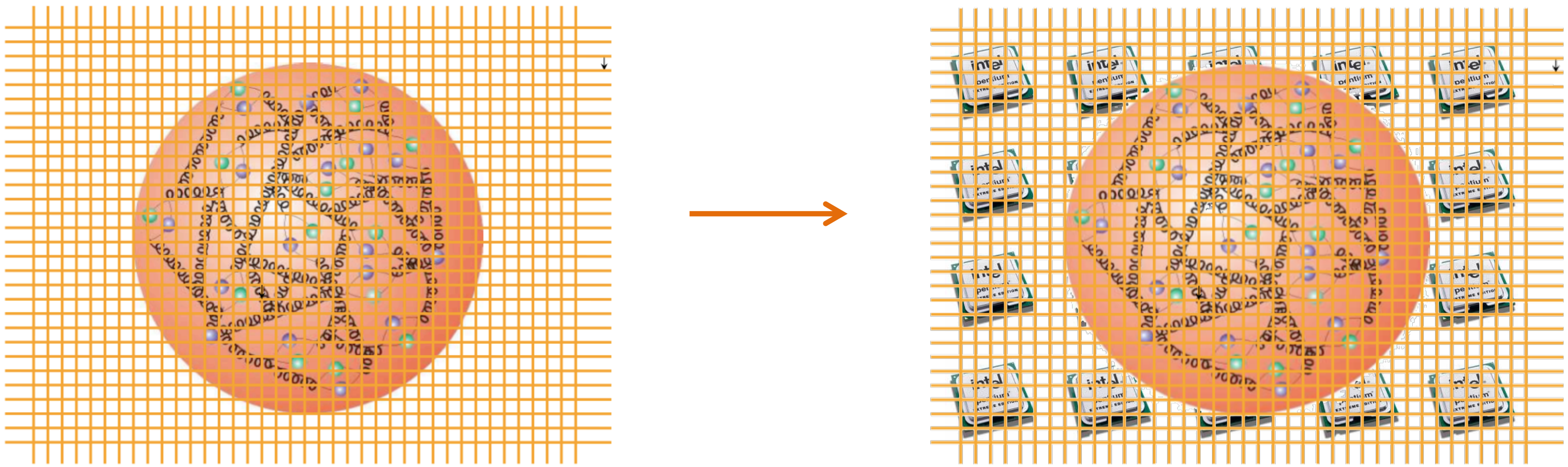
Summary & Prospects

- ✓ nucleon-nucleon interactions
- ✓ hyperon-nucleon interactions
- ✓ hyperon-hyperon interactions
- ✓ first exploration of s-shell (hyper) nuclei at the SU(3) symmetric point (m_π ~ 800 MeV)
- ✓ magnetic properties of light nuclei have been addressed at unphysical values of m_π
- ✓ the $np \to d\gamma$ cross section has been studied (not shown here)

We continue our program including:

- analysis of m_π ~ 450 MeV lattice data
- present run at m_π ~ 300 MeV

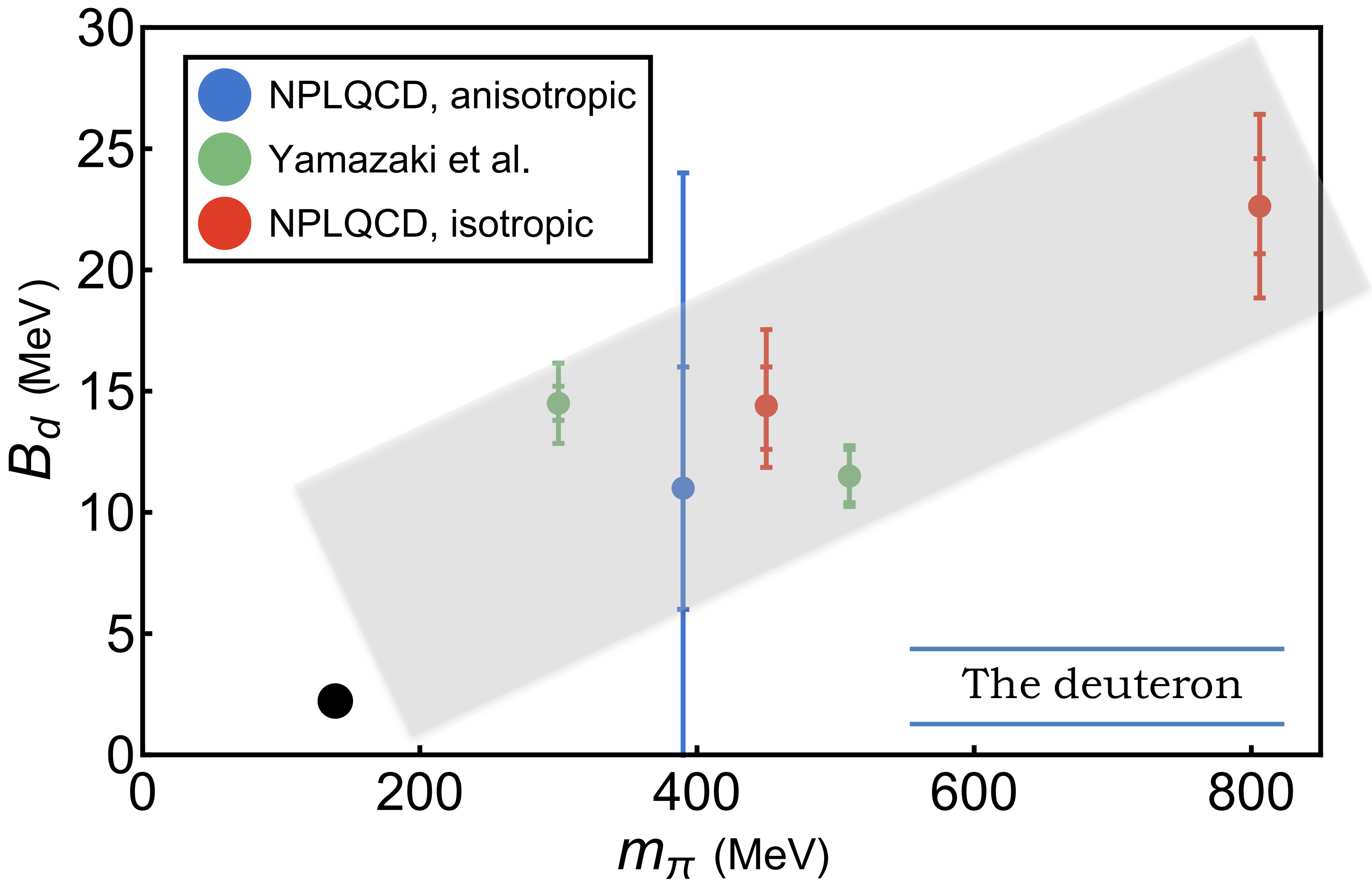
Agenda: light nuclear matrix elements of the axial current, $\nu d \to npe$ …



The nucleon-nucleon system

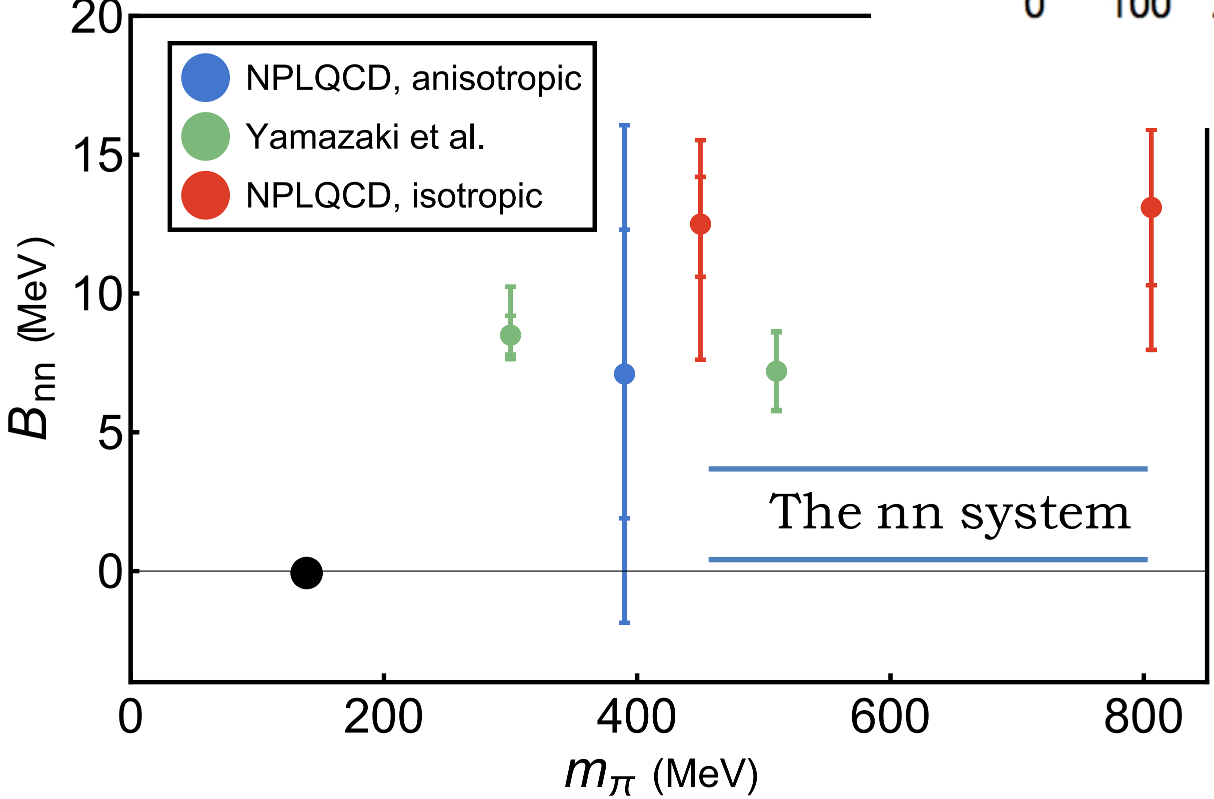
Major challenge in LQCD calculations for Nuclear Physics:

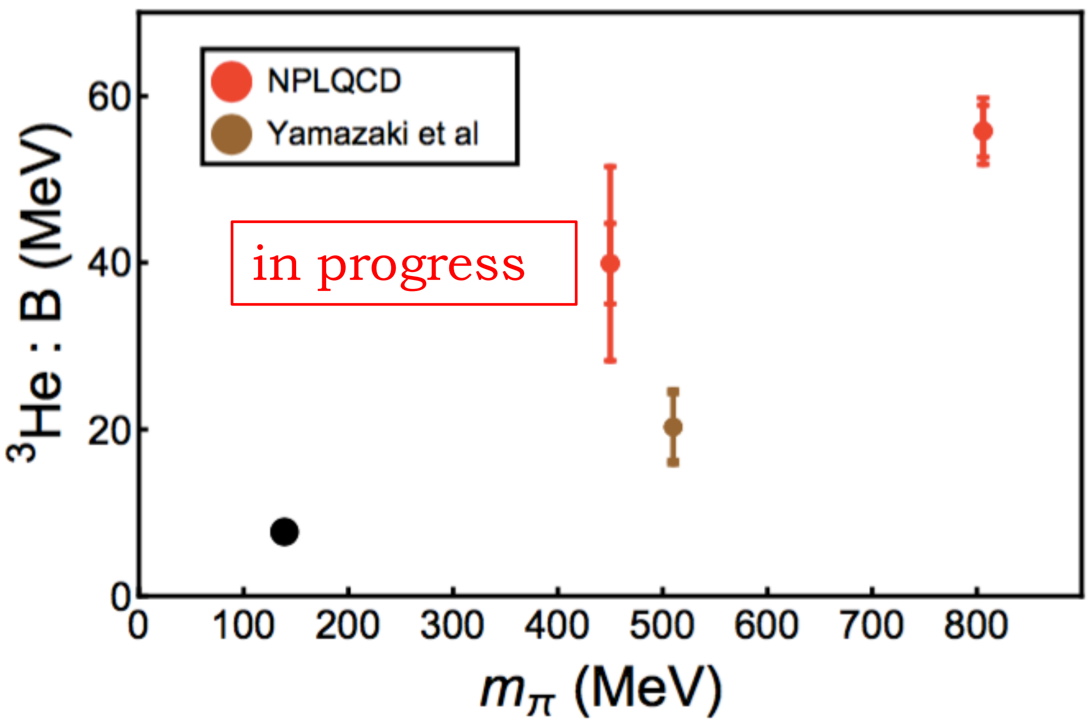
recover the experimentally known properties of the two-nucleon systems

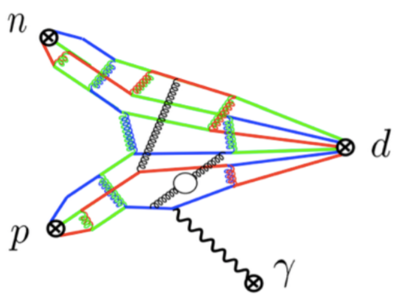


.

no clear pattern for the nn binc







$np \to d\gamma$

Phys. Rev. Lett. 115 (2015) 13, 132001

Illustration of how the long-distance and short-distance contributions can be isolated with lattice QCD.

E1, M2, …

$$\sigma(np \to d\gamma) = \frac{e^2\left(\gamma_0^2 + |\vec{p}^2|\right)^3}{M^4\gamma_0^3|\vec{p}|}\left|\tilde{X}_{M1}\right|^2 + \ldots$$

Bethe, Longmire (1950)
Noyes (1965)

$$\tilde{X}_{M1} = \frac{Z_d}{-\frac{1}{a_1} + \frac{1}{2}r_1|\vec{p}|^2 - \mathrm{i}|\vec{p}|} \times \left[\frac{\kappa_1\gamma_0^2}{\gamma_0^2 + |\vec{p}|^2}\left(\gamma_0 - \frac{1}{a_1} + \frac{1}{2}r_1|\vec{p}|^2\right) + \frac{\gamma_0^2}{2}l_1\right]$$

$$l_1 = \tilde{l}_1 - \sqrt{r_1 r_3}\kappa_1$$

with $\kappa_1 = \frac{\kappa_p - \kappa_n}{2}$ isovector nucleon magnetic moment

$$Z_d = \frac{1}{\sqrt{1-\gamma_0 r_3}}$$

EFT ($\not\pi$) *Kaplan (1997); Kaplan, Savage, Wise (1998, 1999) van Kolck (1999); Beane, Savage (2001)*

$np \to d\gamma$ cross section

Phys.Rev.Lett. 115 (2015) no.13, 132001

Detmold, Savage, NPA743 (2004) 170-193

Illustration of how the long-distance and short-distance contributions can be isolated with lattice QCD.

$$\left[p\cot\delta_1 - \frac{S_+ + S_-}{2\pi L}\right]\left[p\cot\delta_3 - \frac{S_+ + S_-}{2\pi L}\right] = \left[\frac{|eB|\, l_1}{2} + \frac{S_+ - S_-}{2\pi L}\right]^2$$

with $\quad S_\pm \equiv S\left(\frac{L^2}{4\pi^2}\left(p^2 \pm |eB|\kappa_1\right)\right)$

$$\Delta E_{{}^3S_1,{}^1S_0}(\vec{B}) = 2\left(\kappa_1 + \gamma_0 Z_d^2\, \tilde{l}_1\right)\frac{e}{M}\left|\vec{B}\right| + O\left(\left|\vec{B}\right|^2\right)$$

$$\delta E_{{}^3S_1,{}^1S_0}(\vec{B}) \equiv \Delta E_{{}^3S_1,{}^1S_0}(\vec{B}) - \left[E_{p,\uparrow} - E_{p,\downarrow}\right] + \left[E_{n,\uparrow} - E_{n,\downarrow}\right] \to 2\bar{L}_1\frac{\left|e\vec{B}\right|}{M} + O\left(\left|\vec{B}\right|^2\right)$$

$np \to d\gamma$ cross section

Phys.Rev.Lett. 115 (2015) no.13, 132001

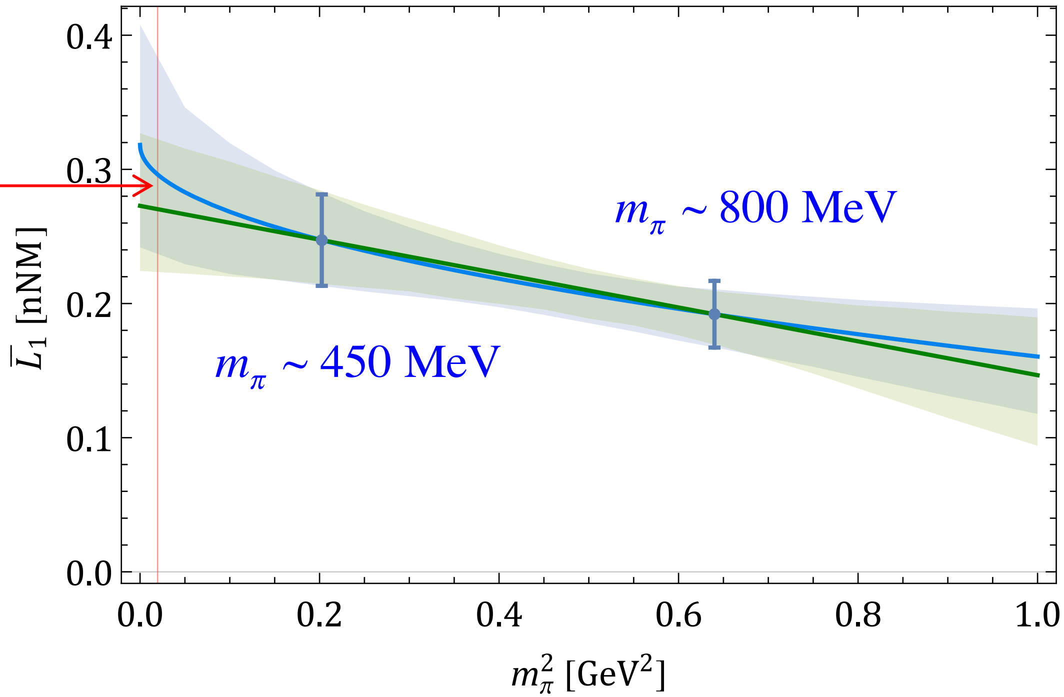
$\bar{L}_1^{lqcd} = 0.285^{+63}_{-60}$ nNM

$l_1^{lqcd} = -4.48^{+16}_{-15}$ fm

$\sigma^{lqcd} = 332.4^{+5.4}_{-4.7}$ mb

($\sigma^{exp} = 334.2 \pm 0.5$ mb)

($v_{neutron} = 2200$ m/s)



$V = 32^3 \times 96$, $b \sim 0.12$ fm

$$\Delta E_{^3S_1, {}^1S_0}(\vec{B}) = 2\left(\kappa_1 + \gamma_0 Z_d^2 \tilde{l}_1\right)\frac{e}{M}|\vec{B}| + O\left(|\vec{B}|^2\right)$$

A magnetic field mixes the $I_z = j_z = 0$ np states in the ${}^1S_0$ and ${}^3S_1 - {}^3D_1$ channels

$$\delta E_{^3S_1, {}^1S_0}(\vec{B}) \equiv \Delta E_{^3S_1, {}^1S_0}(\vec{B}) - \left[E_{p,\uparrow} - E_{p,\downarrow}\right] + \left[E_{n,\uparrow} - E_{n,\downarrow}\right] \to 2\bar{L}_1 \frac{|e\vec{B}|}{M} + O\left(|\vec{B}|^2\right)$$