Nuclear physics and stellar physics: a continuous synergy

Scilla Degl'Innocenti

(Physics Department, Pisa University, INFN Pisa)

The described works have been performed in collaboration with:

Stellar physicists

E. Tognelli (Physics Department, Pisa University, INFN Pisa)

P. G. Prada Moroni (Physics Department, Pisa University, INFN Pisa)

G. Valle (Physics Department, Pisa University, INFN Pisa)

M. Dell'Omodarme (Physics Department, Pisa University)



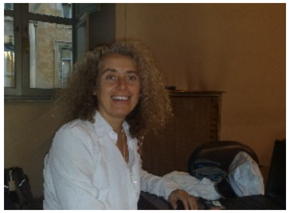
Nuclear physicists

L. Marcucci (Physics Department, Pisa University, INFN Pisa)

B. Ricci (Physics Department, Ferrara University, INFN Ferrara)

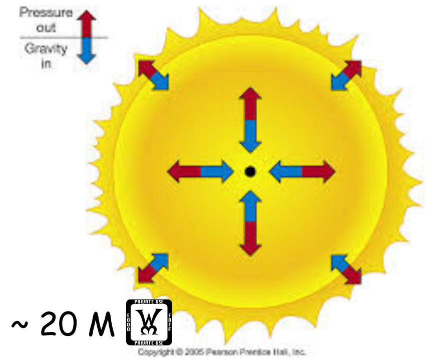
The Trojan Horse Method group (Physics Department, Catania University, INFN Catania), in particular with A Tumino, G. Pizzone, L. Lamia, C. Spitaleri

G. Fiorentini (Physics Department, Ferrara University, INFN Ferrara)




The stars in few words



- **Hydrostatic equilibrium**: stars are gaseous systems in equilibrium between the pressure (gas+radiation pressure) and the gravitational force → the equilibrium configuration is a sphere

M ↑ T ↑ L ↑

$$\frac{L}{L_\odot} \propto \left(\frac{M}{M_\odot}\right)^{\alpha}$$ $\alpha \approx 2 \div 4$ for $M > \sim 20\ M_\odot$

- **Thermodynamic equilibrium** between matter and radiation → an **equation of state (EOS)** is needed

⟶ relation among luminosity (L), Radius (R) and surface (effective) temperature (T_e): $L=4\pi R^2 \sigma T_e^4$

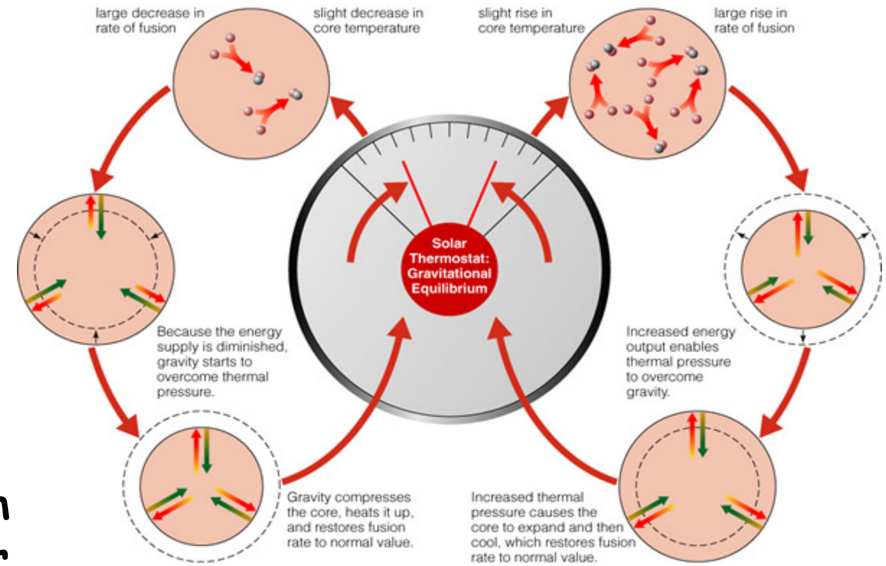
- **Thermal equilibrium:** the amount of energy per unit time which exits from a given spherical region of infinitesimal thickness (shell) direct outward is equal to the amount of energy which enters in the shell plus the energy possibly produced in the shell itself

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$ ε = energy production per unit mass and time = $\varepsilon_{nucl} + \varepsilon_{grav} - \varepsilon_{\nu}$

- **For the most of their life stars are powered by nuclear fusions**

Solar luminosity: $L_\odot \approx 3.83 \times 10^{33}$ erg/s   Solar mass: $M_\odot \approx 1.989 \times 10^{33}$ g

- Stars are systems with feedback: nuclear reaction efficiency exactly compensates radiation losses from the surface



- Energy transport mechanisms:

✓ radiation transport → opacity, k : the sum of all the mechanisms of photon-matter interaction which remove energy from the outgoing flux, averaged over the photon frequency distribution.

The photon mean free path λ_{γ} depends on the stellar opacity and density ϱ: $\lambda_{\gamma} = k\,\varrho$

✓ convection or electronic conduction, can be present under specific conditions

⬇

Integration of stellar equilibrium equations through a numerical code

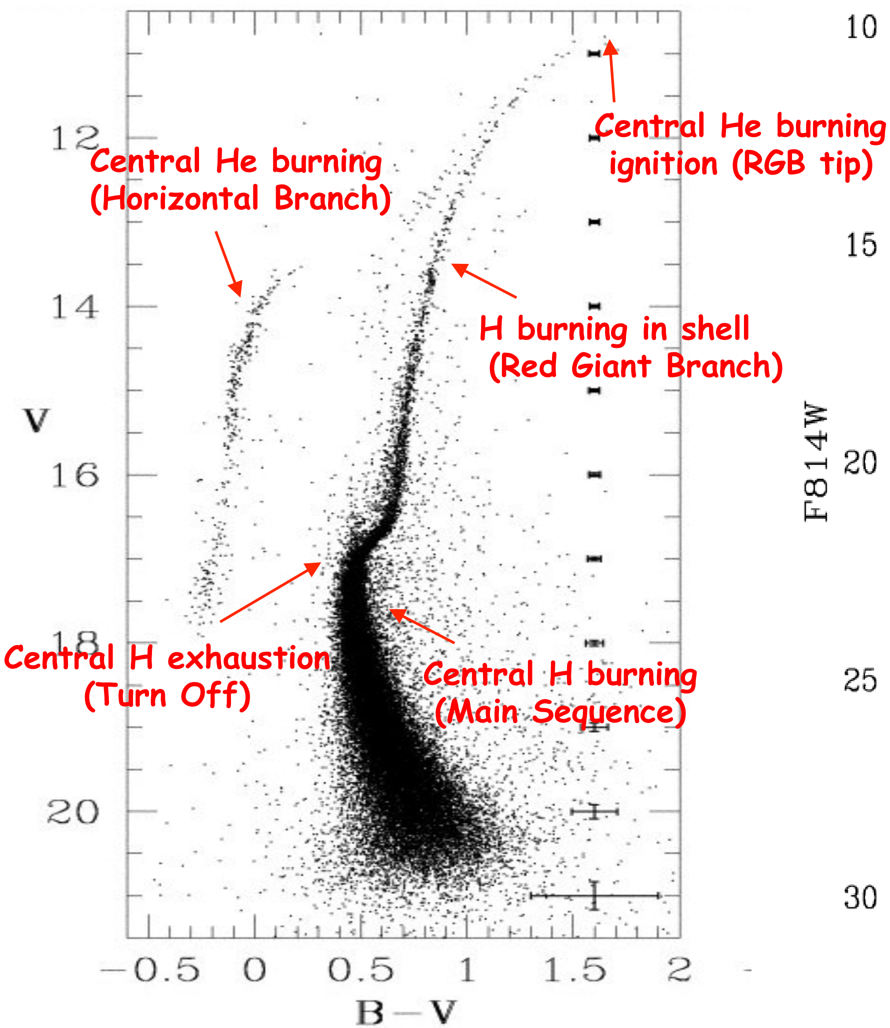
⬇

Internal structure and observables quantities during the stellar lifetime

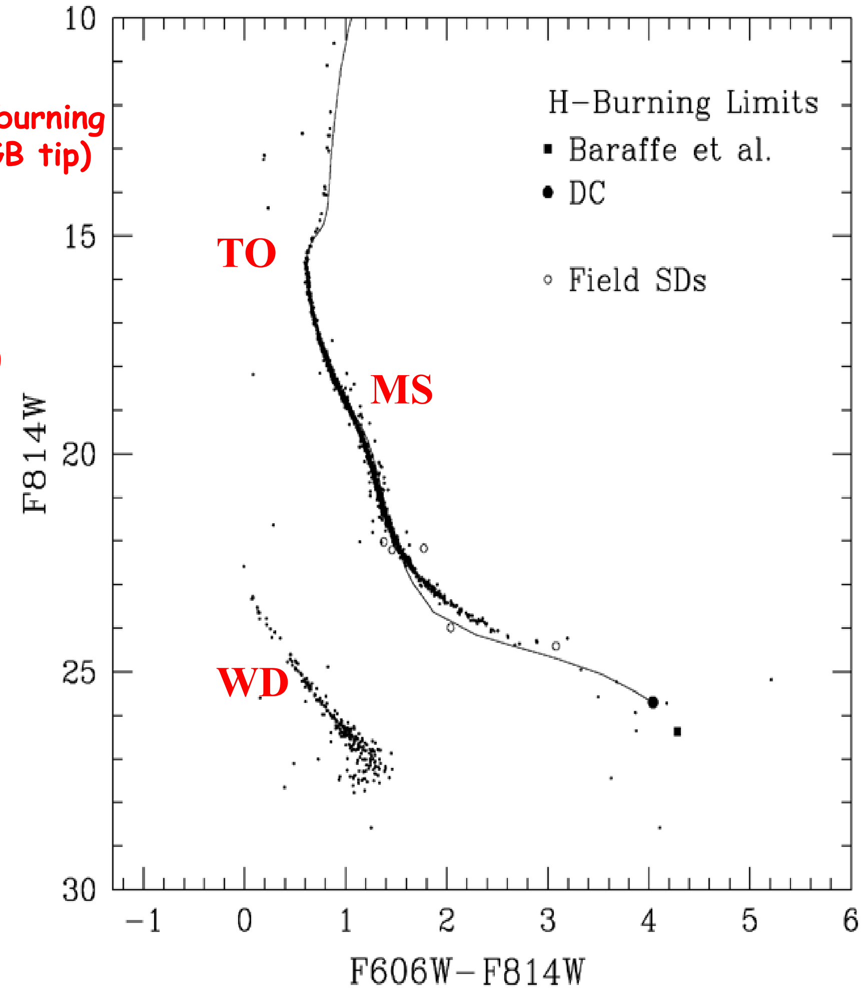
Main observables: electromagnetic energy flux and surface color ➡ luminosity and (surface) effective temperature

The most of stars born in clusters

Hertzsprung-Russell (HR) diagram of old (globular) galactic clusters

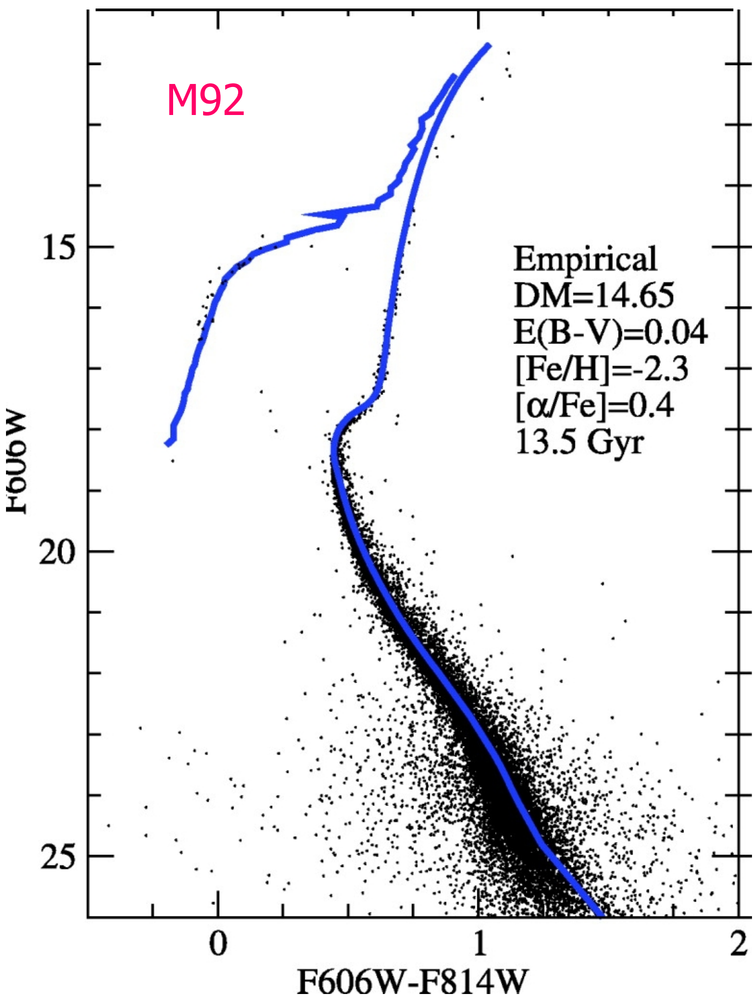


Momany et al. (2002)



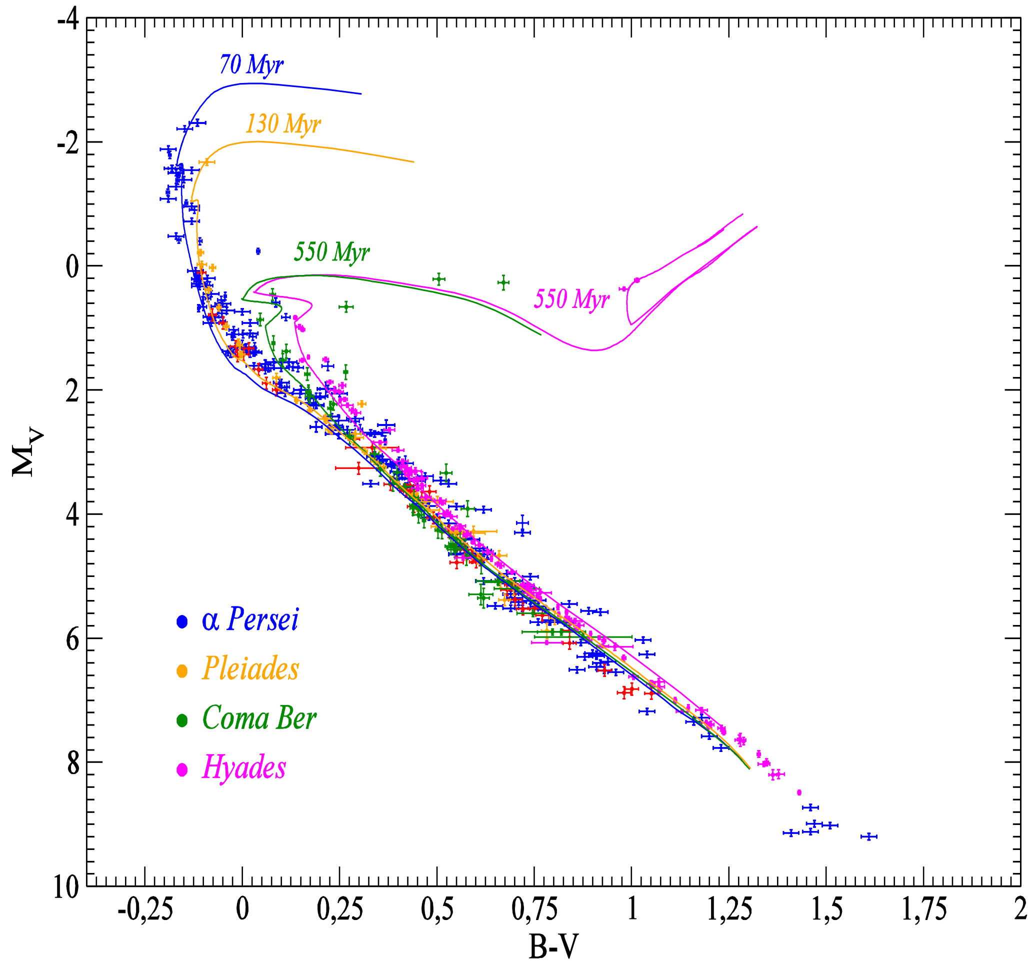
Richer et al. (2008)

General good agreement between theory and observation



Dotter et al. 2007

Local open clusters observed by the Hipparcos satellite



Models from the «Pisa Evolutionary Library»:
http://astro.df.unipi.it/stellar-models

A very precise evaluation of errors in stellar models is now mandatory

GAIA Satellite

About 1 billion of stars up to 50 Kpc

distances to 1% for ~10 million stars to 2.5 kpc

distances to 10% for ~100 million stars to 25 kpc

proper motions and radial velocities

Photometry, low resolution spectroscopy

+

The Gaia ESO Spectroscopic survey

high quality spectroscopy of ≈ 1 million of stars in the galaxy

radial velocities

Kepler Satellite

Asteroseismic data

Stellar mass and radius estimates based on average seismic properties combined with non seismic observables

See e.g Boruki et al. 2010, Gilliland et al. 2010, Gilmore et al. 2012,Turon et al. 2012

Stellar models are the results of complex calculations relying on:

- **Input physics:** EOS, radiative and conductive opacities, nuclear reaction cross sections, neutrino emission rates, element diffusion efficiency etc.

- **Chemical composition:** initial helium, Y, and metal*, Z, fractional abundance in mass, heavy – elements mixture, etc.

Each of these ingredients is affected by not negligible **uncertainties**

↓

Stellar models are still affected by significant **uncertainties**

I will focus **only** on the effects of the **input physics** uncertainties from the **central H Burning** (Main Sequence, MS) to **the central He burning** (Horizontal Branch, HB) phases of **low mass stars** → **old stellar clusters**

---

* In astrophysics all the elements heavier than helium are called "metals"

Nuclear fusions

Energetic — **Element nucleosynthesis**

The fusion reactions for the evolutionary phases of interest happen among thermalized charged nuclei of the stellar plasma screened by the plasma electrons through tunnel effect

$$\sigma(E) = \frac{1}{E}\, S(E)\, \exp\left(-\frac{2\pi Z_1 Z_2 e^2}{\hbar v}\right)$$

Penetration probability through the Coulomb barrier
(strong energy dependence)

The "astrophysical factor", a function smoothly varying with E, which includes the nuclear information and the normalization of the cross section; $E = \frac{1}{2}\mu v^2$

Reaction rate

$$r = f\, \frac{1}{1+\delta_{12}}\, n_1 n_2 \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(KT)^{3/2}} \int_0^\infty \sigma(E)\, E\, \exp\left(-\frac{E}{KT}\right) dE$$

f=the plasma electron screening factor; $\mu = \frac{m_1 m_2}{m_1 + m_2}$; n=number density

Uncertainties in low mass stellar models due to the adopted input physics

- **Evaluation of the global uncertainty in stellar models due to input physics**
- **The complexity of stellar evolution calculations hampers an analytical evaluation of the impact of the variation of the chosen inputs on stellar models calculation**
- **The problem must be addressed by direct computation of perturbed stellar models**

↓

Simplest approach: change a given input physics at a time keeping all the other inputs and parameters fixed (see e.g. Chaboyer et al. (1995), Cassisi et al. (1998), Brocato et al. (1998), Castellani & Degl'Innocenti (1999), Castellani et al. 2000; Imbriani et al. 2001; Salaris et al. 2002; Imbriani et al. 2004; Weiss et al. 2005, Valle et al. 2009; Tognelli et al. 2011)

↓

The method does not allow to quantify the possible interactions among the different input physics

Systematic and simultaneous variation on a fixed grid* of the main input physics within their current range of uncertainty, in a way to obtain a full crossing of the perturbed input values.

Main advantage: no a priori independence among input physics is assumed

Main disadvantage: very computationally expensive

* Similar works have been performed varying simultaneously the input physics adopting a Monte Carlo technique (see e.g. Chaboyer et al. 1996, 1998; Chaboyer & Krauss 2002; Krauss & Chaboyer 2003; Bjork & Chaboyer 2006)

Main physical inputs perturbed in the calculations and their assumed uncertainty

PARAMETER (p_i)	DESCRIPTION	UNCERTAINTY (Δp_i)
p_1	$p(p, e^+\nu_e)^2H$ reaction rate	1% (1)
p_2	$^{14}N(p,\gamma)^{15}O$ reaction rate	10% (2)
p_3	Radiative opacity, k_r	5% (3)
p_4	Microscopic diffusion velocities	15% (4)
p_5	Triple-α reaction rate	20% (5)
p_6	neutrino emission rate	4% (6)
p_7	Conductive opacity k_c	5% (7)

(1) Adelberger et al. (2011)

(2) error on the reaction rate for the $^{14}N(p,\gamma)^{15}O$ in the range $10^6 \div 10^8$ K, as presented in Imbriani et al. (2005)

(3) see e.g. Rose (2001), Seaton & Badnell (2004), Badnell et al. (2005), Valle et al. (2012)

(4) Thoul et al. (1994)

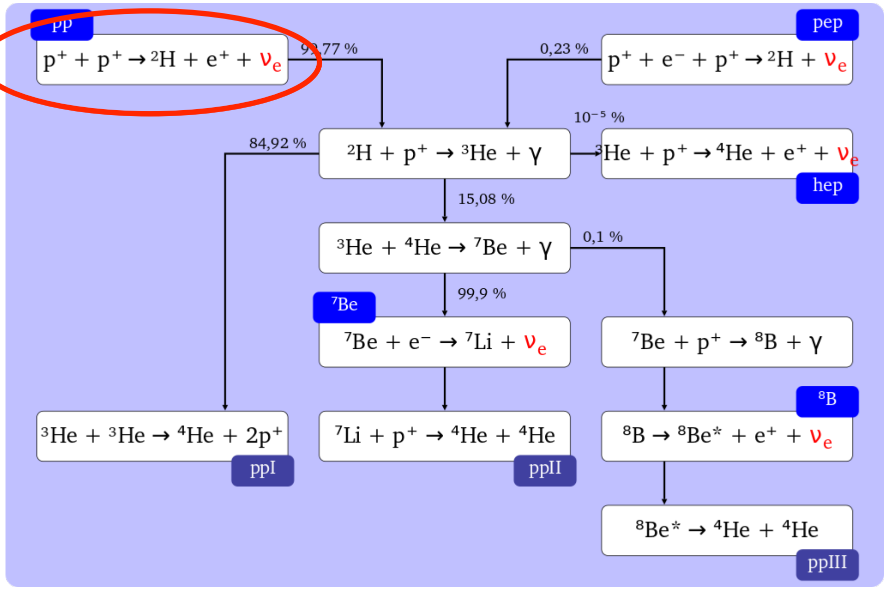
(5) Fynbo & Diget (2014)

(6) Haft et al. (1994)

(7) Potekhin (2011)

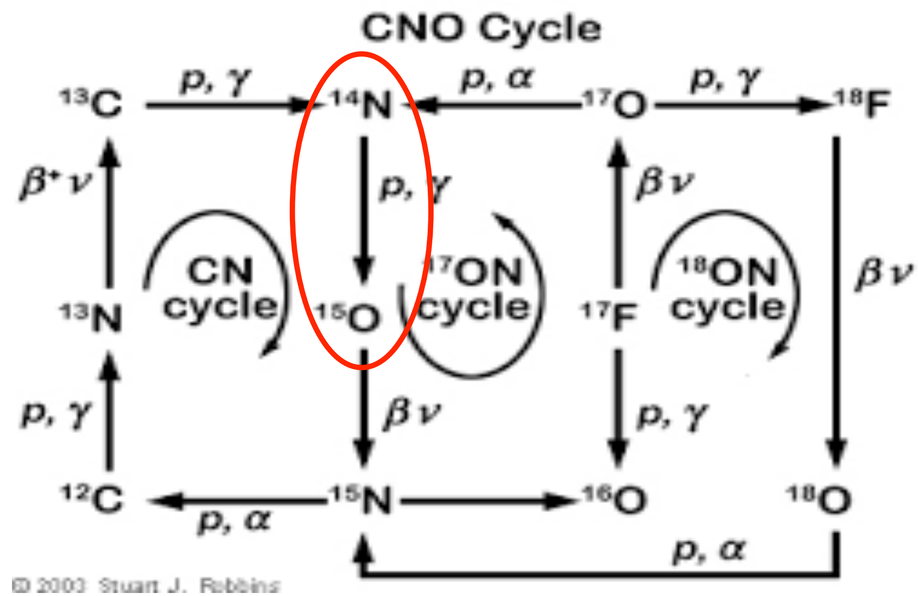
Hydrogen burning reactions

Proton proton chain



By Dorottya Szam https://commons.wikimedia.org/

CN-NO bicycle



In each case: $4\,^1H \rightarrow {}^4He + 2e^+ + 2\nu_e$ **(Q≈26.7 MeV)**

Triple α

$$
\begin{aligned}
{}^4He + {}^4He &\rightleftharpoons {}^8Be \\
{}^8Be + {}^4He &\rightarrow {}^{12}C + \gamma
\end{aligned}
$$

Physical uncertainty for a typical low mass star

Fixed mass and chemical composition ($M=0.9\ M_\odot$, $Z=0.006$, $Y=0.26$)

Stellar models covering all the possible combinations of simultaneously perturbed input physics → exploration of the edges of the variability region

$3^7 = 2187$ models from central H burning to the central He burning, with the same M, Z, Y

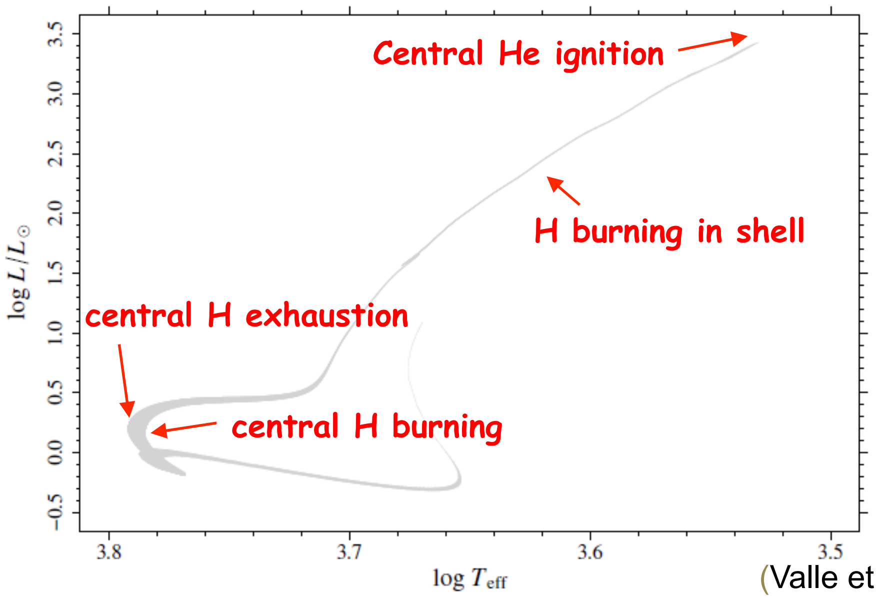
↓

Global physical uncertainty in stellar models

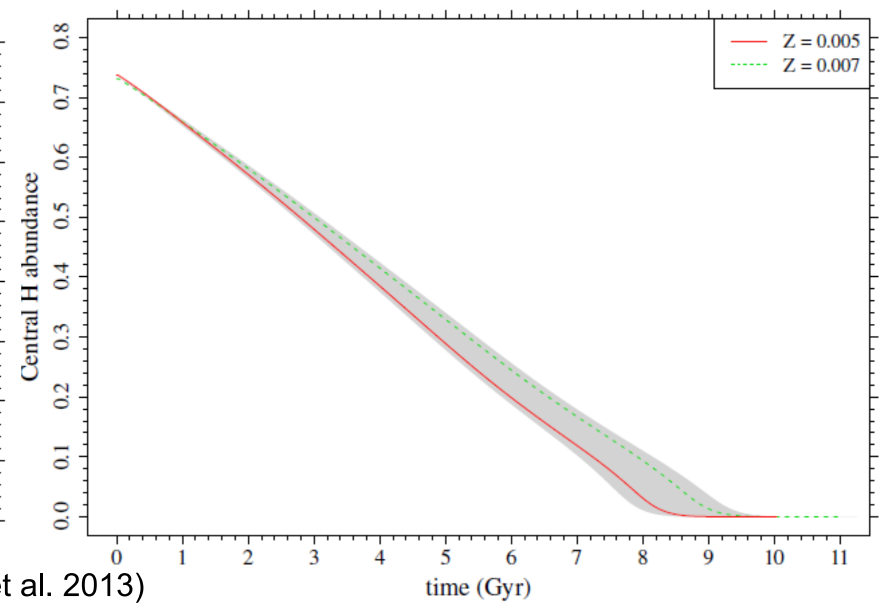
Global physical uncertainty

First error stripes associated to theoretical stellar models

HR diagram of the 0.9 $M_{\odot}$ model



Time evolution of central H abundance for the 0.9 $M_{\odot}$ model



(Valle et al. 2013)

M=0.9 M , Z=0.006, Y=0.26

Total physical uncertainty for the selected stellar evolutionary parameters

Table 2. Total range of variation in theoretical predictions of selected quantities for our reference case, i.e. $M = 0.90\ M_{\odot}$ with $Z = 0.006$ and $Y = 0.26$, due to input physics uncertainties.

	quantity	variation range	range extension	
Luminosity at the central H exhaustion	$\log L_{\mathrm{BTO}}$	[0.334 - 0.376] dex	0.042 dex	± 6%
Central H exhaustion time	t_{H}	[9.83 - 11.26] Gyr	1.43 Gyr	± 6.5 %
Luminosity and core He mass at the central He ignition	$\log L_{\mathrm{RGBtip}}$	[3.38 - 3.44] dex	0.06 dex	± 1 %
	$M_{\mathrm{c}}^{\mathrm{He}}$	[0.4796 - 0.4879] $M_{\odot}$	0.0083 $M_{\odot}$	± 3 %
Central He burning luminosity	$\log L_{\mathrm{ZAHB}}$	[1.52 - 1.61] dex	0.09 dex	± 0.85 %

For such small perturbations no significant interactions occur among the physical inputs

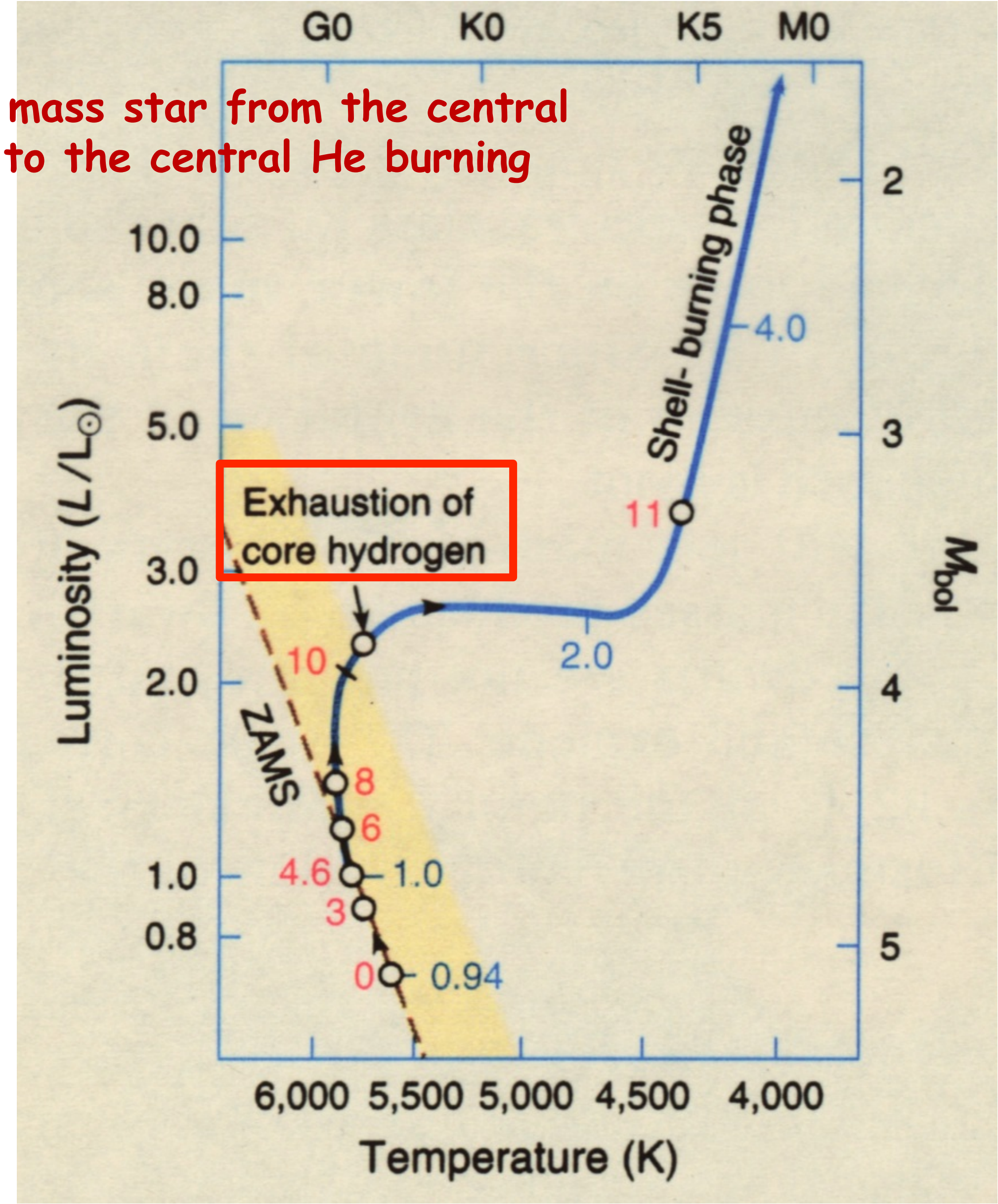
It is possible to disentangle the effects of the different physical inputs on the above selected stellar quantities

The dependence of the evolutionary quantities on physical inputs was explored by means of linear regression models

Main results on selected evolutionary characteristics

The luminosity of central H exhaustion

Evolution of a low mass star from the central H burning ignition to the central He burning ignition



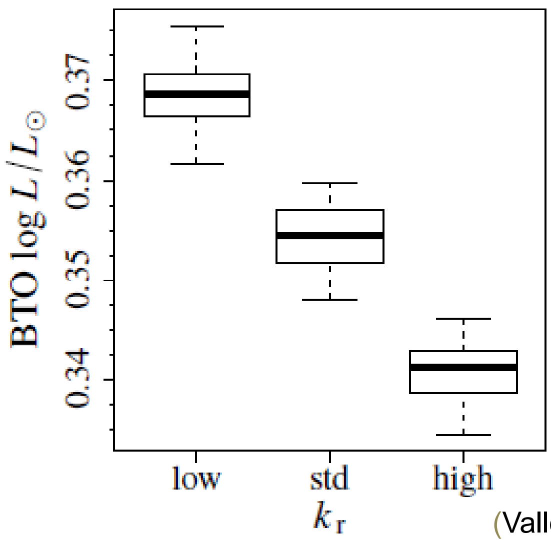
(Figure from http://crab0.astr.nthu.edu.tw/hchang/ga1/)

Impact of uncertainty of individual physical inputs

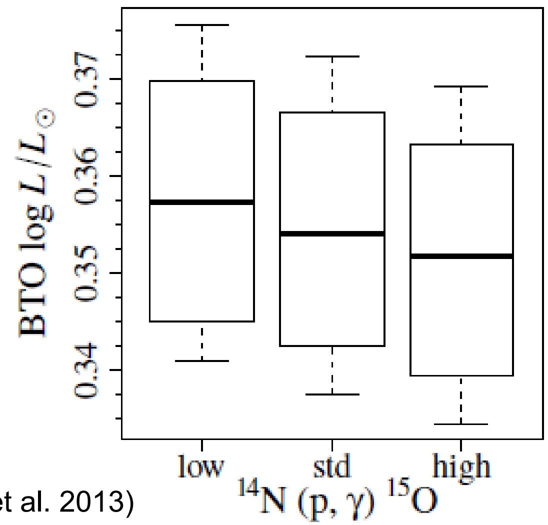
Luminosity at the central H exhaustion (BTO)

The black lines show the medians of the dataset

Opacity variation



$^{14}N(p,\gamma)^{15}O$ cross section variation



(Valle et al. 2013)

the positions of the medians are related to the effect of the parameter while the extension of the whiskers is due to the variation of all the other parameters.

The larger the separation of the medians with respect to the dimension of the boxes and the greater the importance of a given parameter

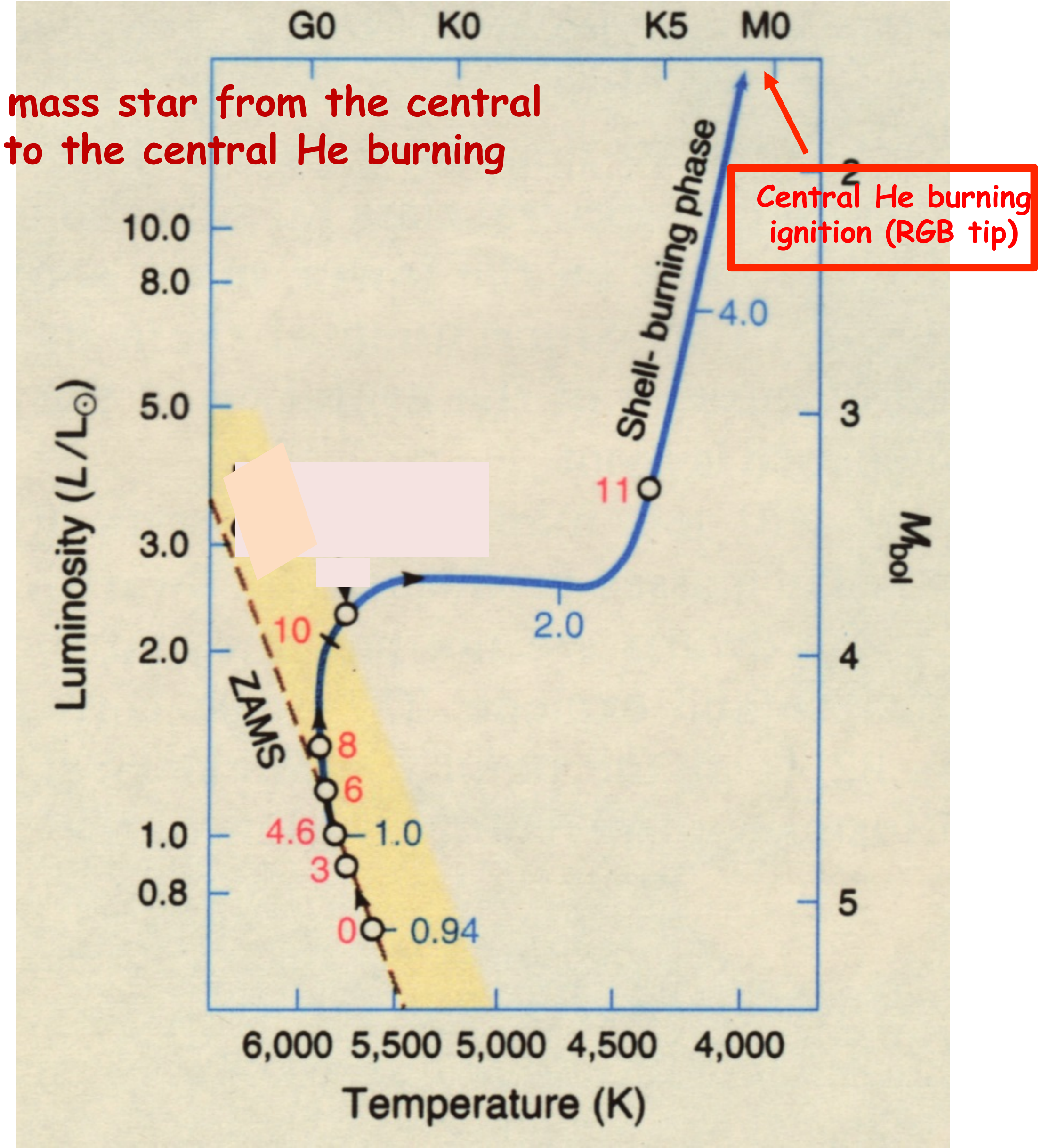
- The radiative opacity uncertainty largely dominates $\Delta\log(L_{BTO}/L_{\odot})$ ~ ± 0.014 dex [Δk_r ~5%, $\Delta\log(L_{BTO}/L_{\odot})$~-0.28 Δk_r]

- The second most important input is $^{14}N(p,\gamma)^{15}O$ cross section $\Delta\log(L_{BTO}/L_{\odot})$~±0.0028 dex [$\Delta\sigma(^{14}N(p,\gamma)^{15}O)$ ~10%, $\Delta\log(L_{BTO}/L_{\odot})$~-0.028 $\Delta\sigma(^{14}N(p,\gamma)^{15}O)$]

For each parameter pi the boxplots are a convenient way to summarize the variability of the data according to the three values of pi (1.00−Δpi, 1.00, and 1.00 + Δpi, labeled as low, std, and high in the plots). The black thick lines show the median of the data set, while the box marks the interquartile range, i.e. it extends form the 25th to the 75th percentile of data. The whiskers extend from the box until the extreme data, the bottom whisker ranges from the sample minimum to the first quartile while the top whisker from the third quartile to the sample maximum. While the position of the medians are related to the effect of the parameter in study in each plot, the extension of the box and whiskers are due to the variation of all the other parameters.

The luminosity of central He burning ignition

Evolution of a low mass star from the central H burning ignition to the central He burning ignition

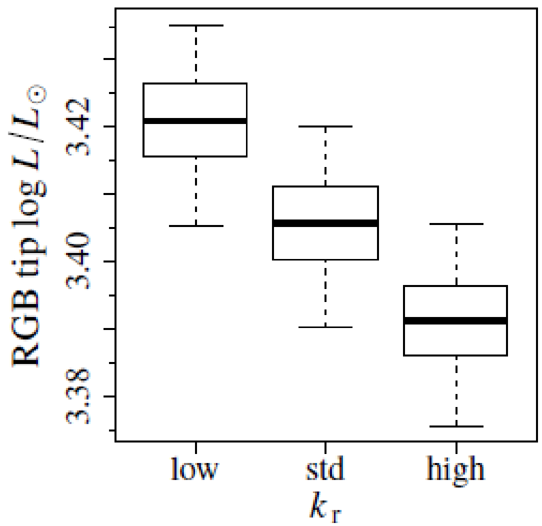


(Figure from http://crab0.astr.nthu.edu.tw/hchang/ga1/)

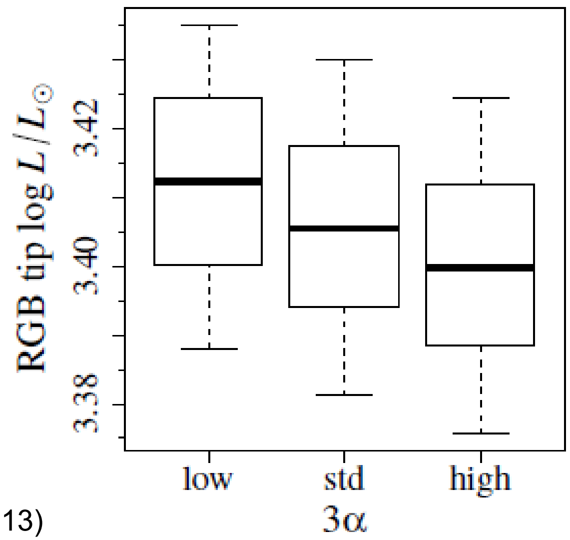
Impact of uncertainty of individual physical inputs

Luminosity at the central He ignition (RGB tip)

Opacity variation



Triple-α cross section variation



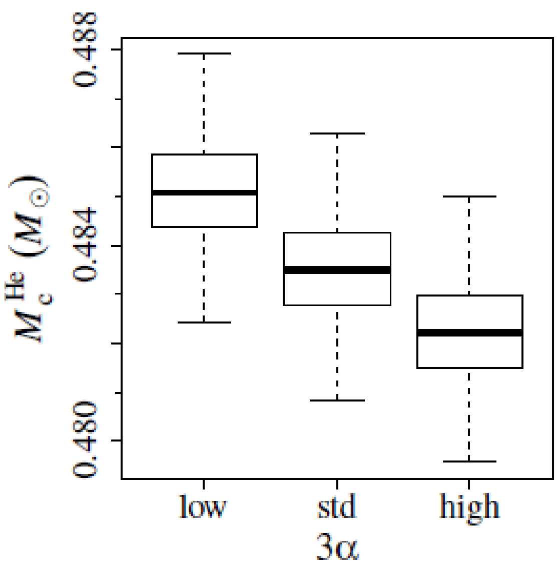
(Valle et al. 2013)

- The most important factor is again the radiative opacity $\Delta \log L_{tip}/L_{\odot}$ ~ ± 0.015 dex [Δk_r ~5%, $\Delta \log L_{tip}/L_{\odot}$ ~-0.3 Δk_r]
- The second most important input is the Triple-α cross section $\Delta \log L_{tip}/L_{\odot}$~ ± 0.006 dex [Δσ(Triple-α)~20%, $\Delta \log L_{tip}/L_{\odot}$ ~-0.03Δσ(Triple-α)]

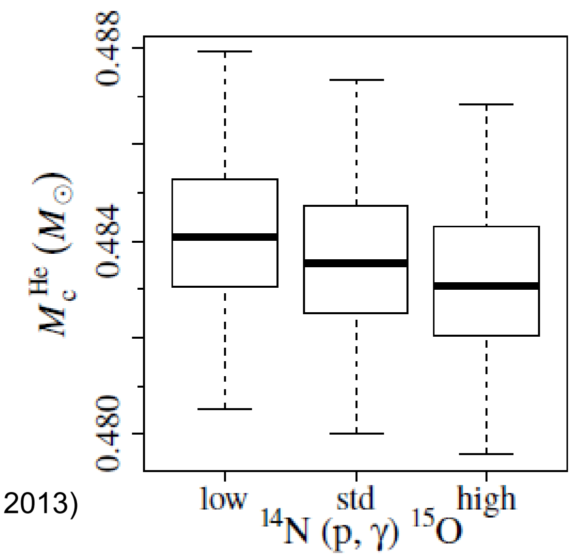
Impact of uncertainty of individual input physics

He core mass at the central He burning ignition

Triple-α cross section variation



$^{14}N(p,\gamma)^{15}O$ cross section variation



(Valle et al. 2013)

- The larger uncertainty arises from the Triple-α cross section $\Delta M_c^{He}/M_\odot \sim \pm 0.0014$ [$\Delta\sigma$(Triple-α) ~20%, $\Delta M_c^{He}/M_\odot \sim -0.007\ \Delta\sigma$(Triple-α)]

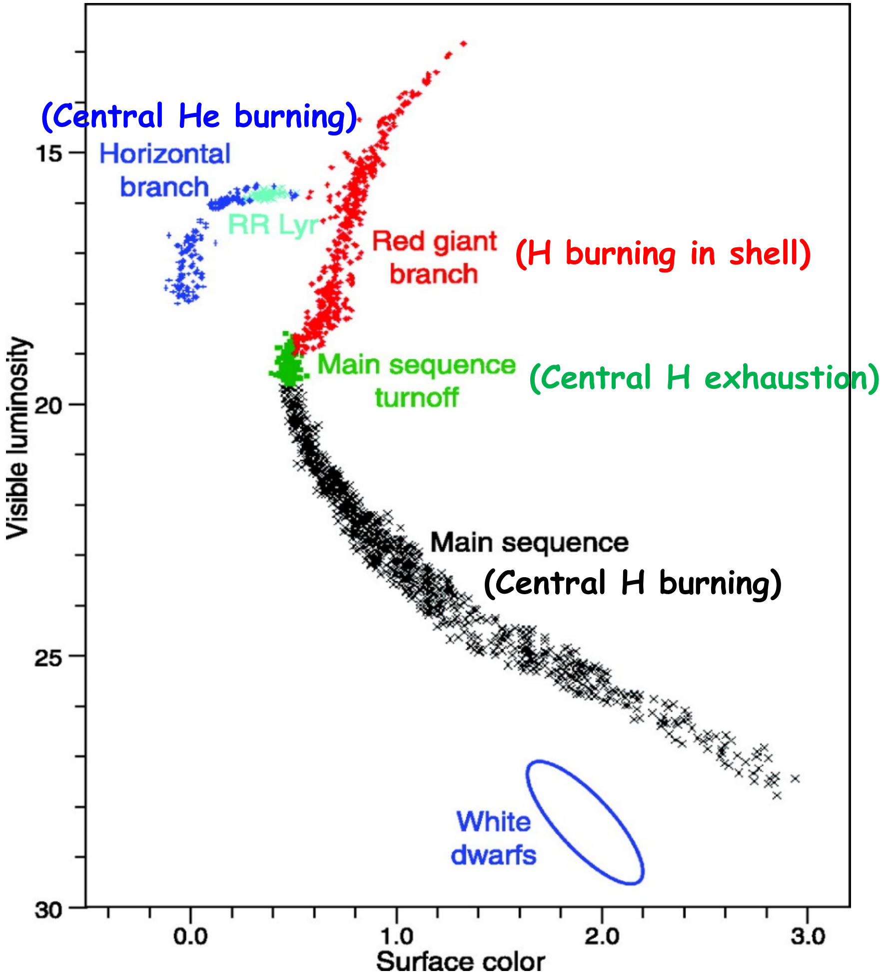
- However the larger dependencies are the ones from the thermic neutrinos emission rate, the radiative and conductive opacities:
  $\Delta M_c^{He}/M_\odot \sim -0.013\ \Delta k_r$, $\Delta M_c^{He}/M_\odot \sim -0.012\ \Delta k_c$, $\Delta M_c^{He}/M_\odot \sim 0.016\ \Delta\nu$
  [each of them has an effect of the same order of ~ 45% of the Triple-α one]

- $^{14}N(p,\gamma)^{15}O$ cross section effect ~ 35% of the Triple-α one: $\Delta M_c^{He}/M_\odot \sim \pm 0.0005$
  [error~10%, $\Delta M_c^{He}/M_\odot \sim -0.005\Delta\sigma(^{14}N(p,\gamma)^{15}O)$]

Stellar cluster populations

Stars that have (at least on first approximation) the same original chemical composition, age and distance from us but different masses

Isochrones/synthetic stellar clusters

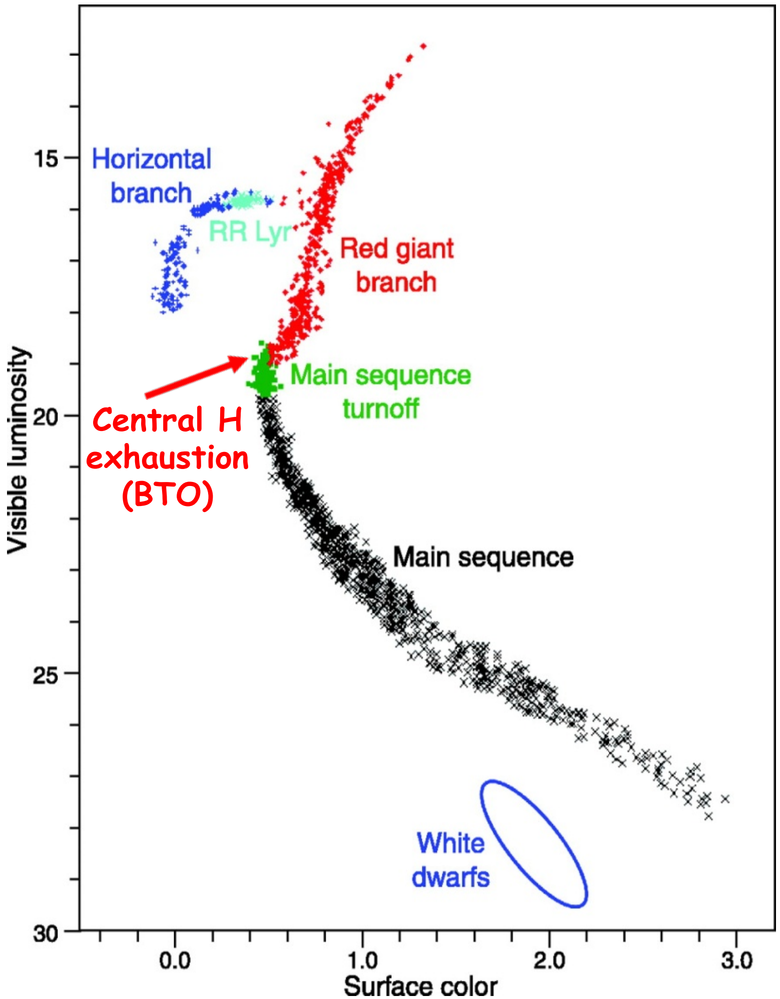


Krauss et al. (2003)

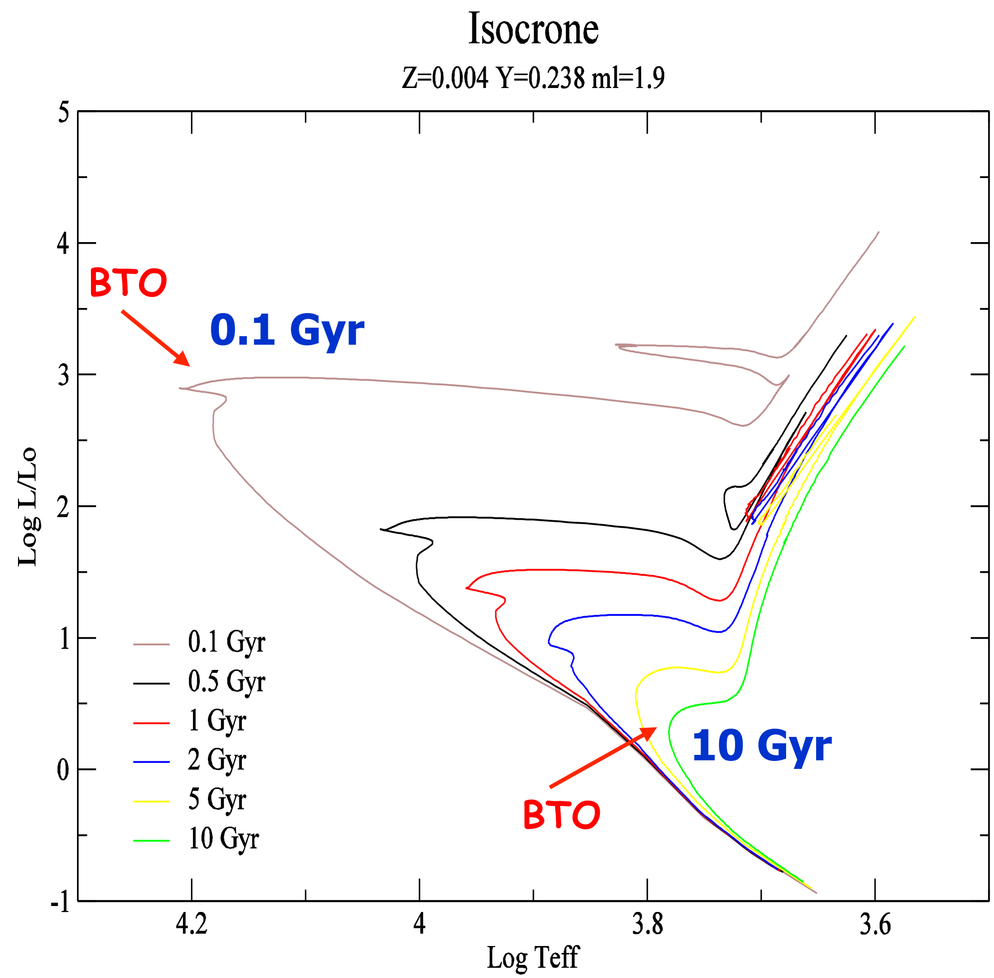
(The theoretical counterpart of an observed cluster HR diagram)

Absolute age determination

The luminosity of the central H exhaustion region (BTO) is a powerful age indicator



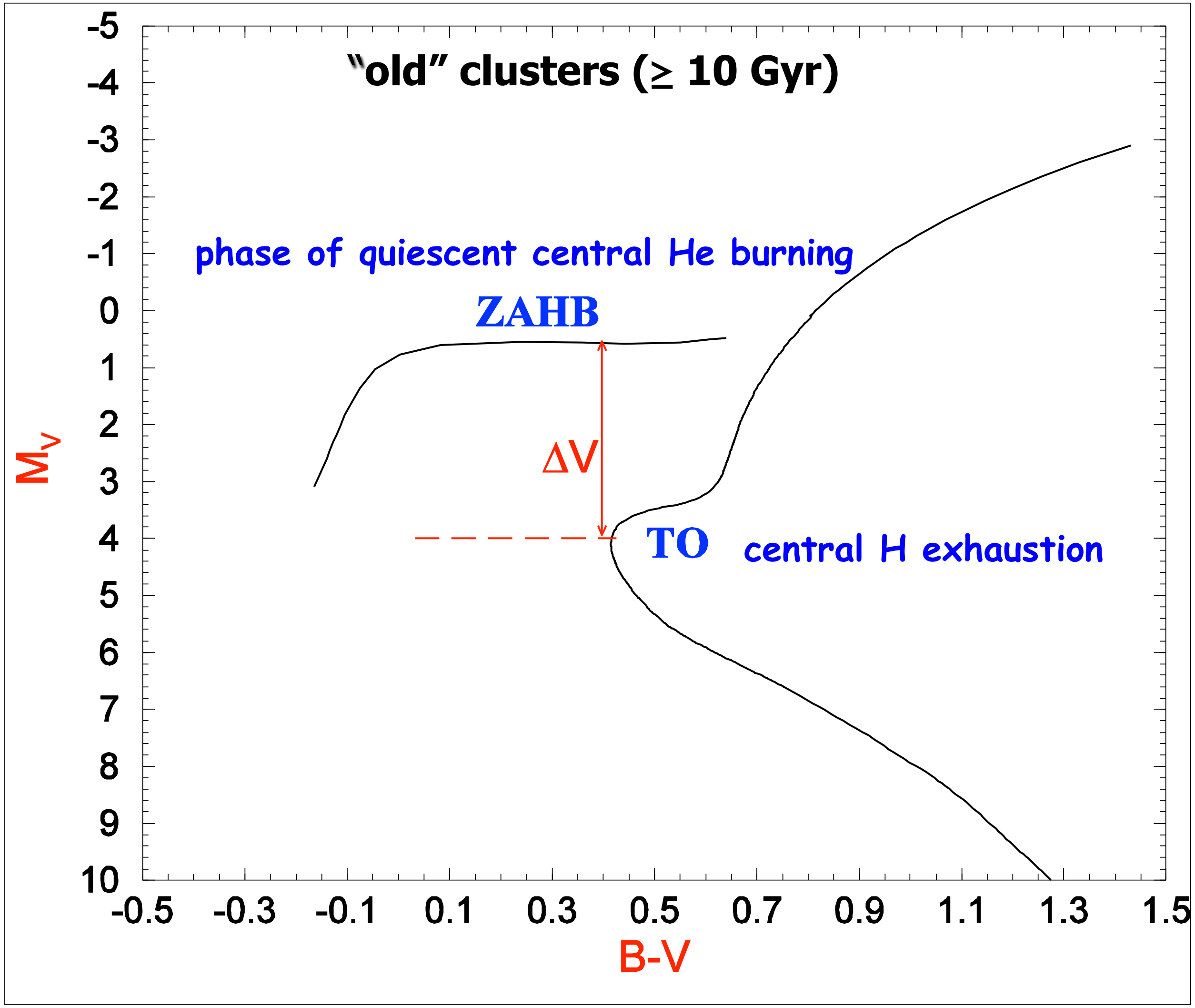
Krauss et al. (2003)



Isochrones from the Pisa evolutionary library. Stellar models and isochrones available at the link: `http://astro.df.unipi.it/stellar-models`

The "vertical method" for cluster age determination

(Iben & Faulkner 1968)



For ages around 10 Gyr the $\Delta V_{TO\text{-}HB}$ parameter scales approximately as:

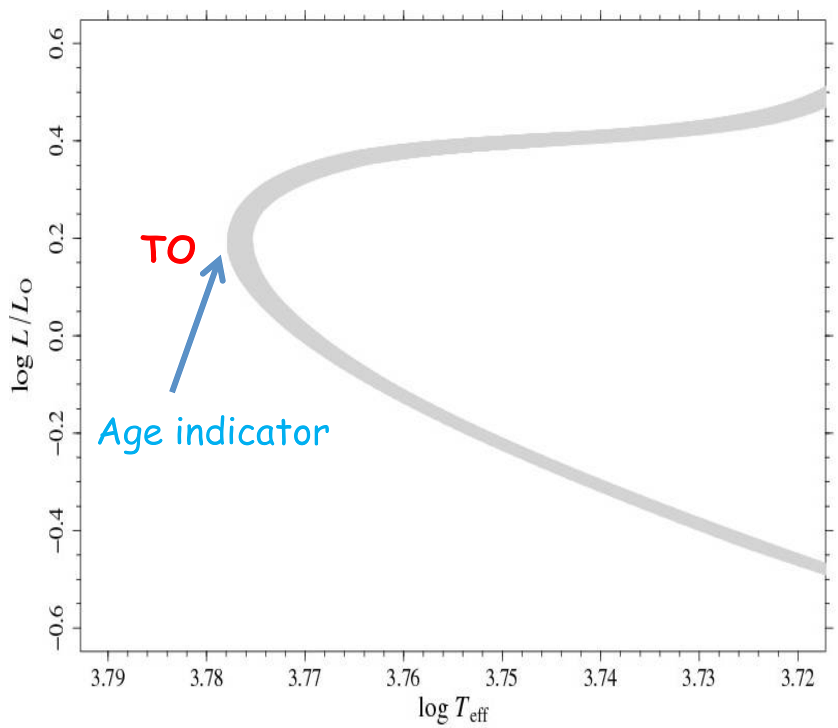
$\delta \Delta V_{TO\text{-}ZAHB} / \delta t \sim 0.1$ mag Gyr^{-1}

Global physical uncertainty in stellar isochrones

Isochrones in the range 8-14 Gyr at a fixed chemical composition (Z=0.006 Y=0.26):

For **each** set of all the possible combinations of **perturbed input physics**, we computed: **12** stellar tracks with **$M = 0.4 - 1.1\ M_{\odot}$ → $3^4 = 81$ grids** of 12 stellar tracks for a total of **972 tracks** and **567 isochrones**

Error stripe associated to theoretical stellar isochrones (12 Gyr)



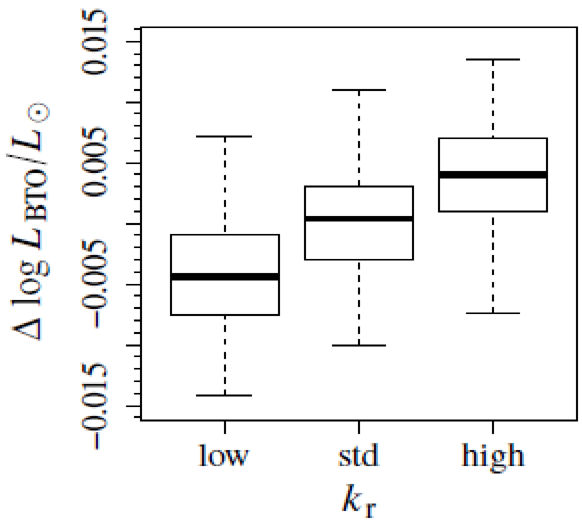
(Valle et al. 2013)

- $\Delta \log L_{BTO}^{ISO}/L_{\odot} \sim \pm\ 0.013$ dex

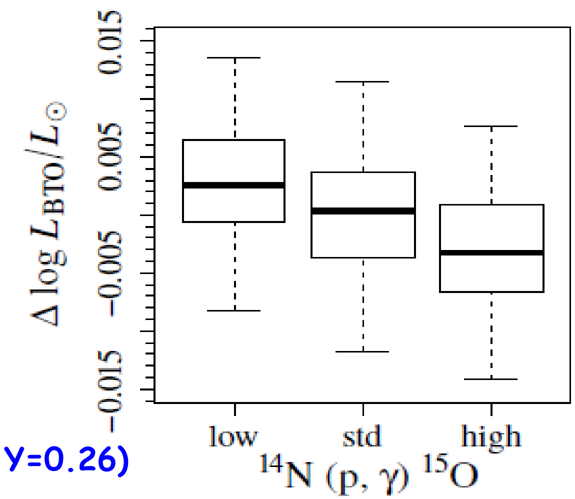
Impact of uncertainty of individual input physics

Isochrone H exhaustion luminosity

Opacity variation



$^{14}N(p,\gamma)^{15}O$ cross section variation



(Z=0.006 Y=0.26)

(Valle et al. 2013)

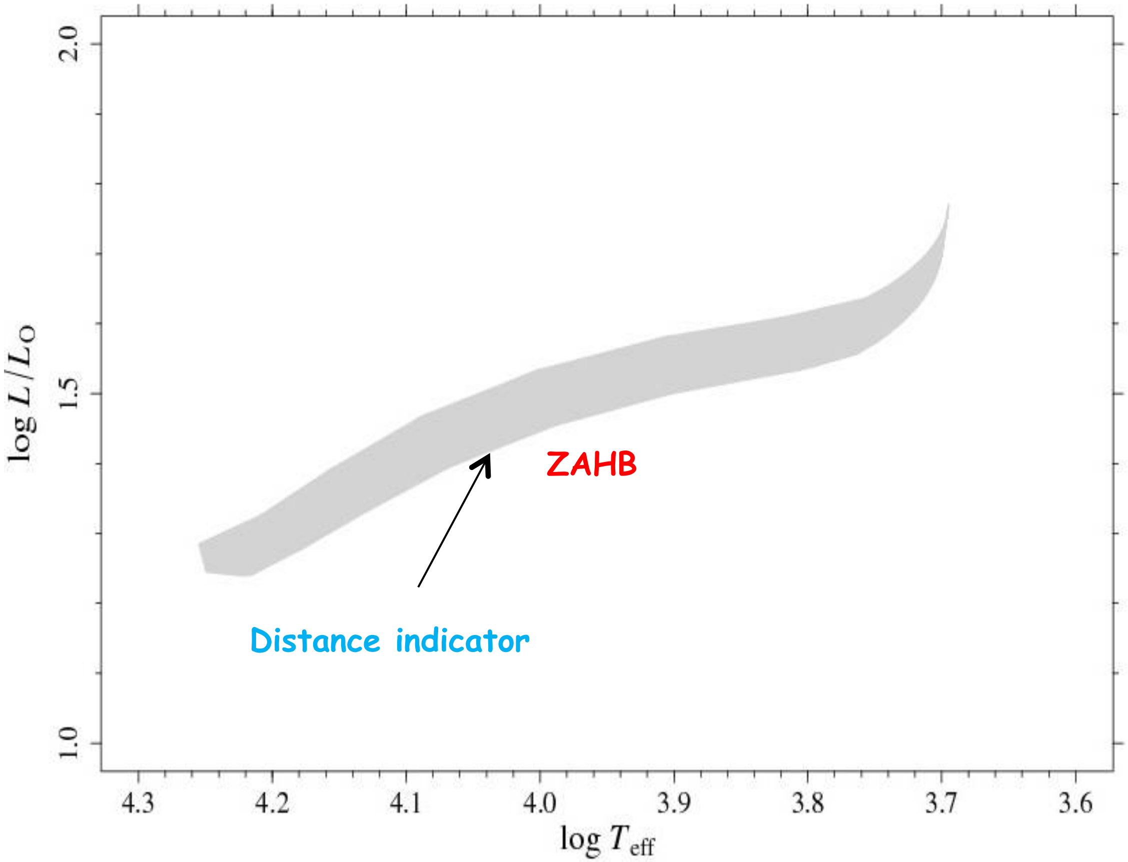
- No physical input definitely dominates

- The most important factor is the radiative opacity $\Delta \log L_{BTO}^{ISO}/L_\odot \sim \pm 0.004$ dex
[error ~5%, $\Delta \log L_{BTO}^{ISO}/L_\odot \sim 0.08\, \Delta k_r$]

- The second most important input is the $^{14}N(p,\gamma)^{15}O$ cross section:
$\Delta \log L_{BTO}^{ISO}/L_\odot \sim \pm 0.003$ dex
[error~10%, $\Delta \log L_{BTO}^{ISO}/L_\odot \sim -0.03 \Delta\sigma(^{14}N(p,\gamma)^{15}O)$]

- The highest dependence is the one from the $p(p,e^+\nu_e)^2H$ cross section ($\Delta \log L_{BTO}^{ISO}/L_\odot \sim 0.10 \Delta\sigma(p(p,e^+\nu_e)^2H)$) but, due to the low error (~1%), the effect is low: $\Delta \log L_{BTO}^{ISO}/L_\odot \sim \pm 0.001$ dex

Error stripe of the initial central He burning luminosity



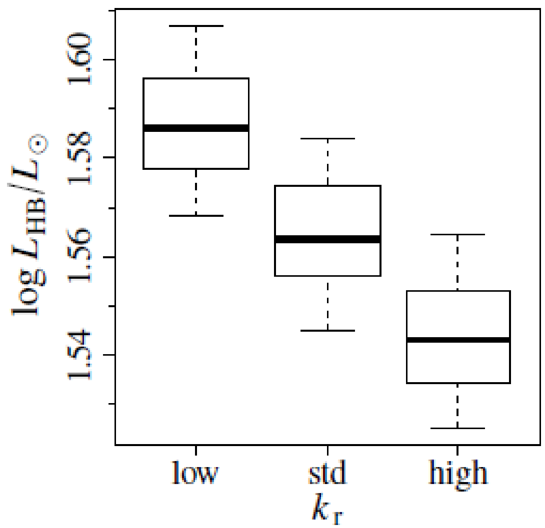
(Valle et al. 2013)

- $\Delta \log L_{ZAHB}/L_{\odot} \sim \pm\ 0.045$ dex

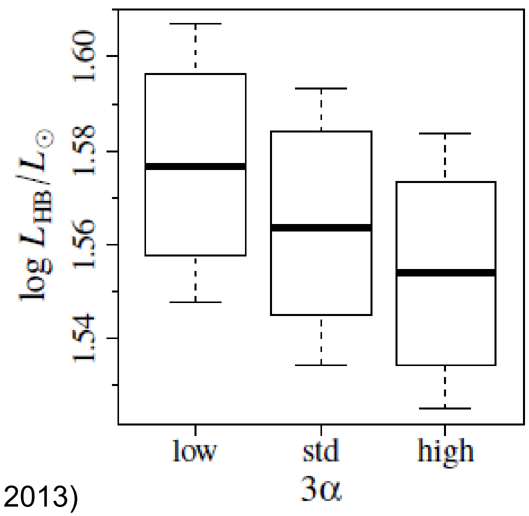
Impact of uncertainty of individual physical inputs

Luminosity at the initial central He burning (ZAHB)

Opacity variation



Triple-α cross section variation

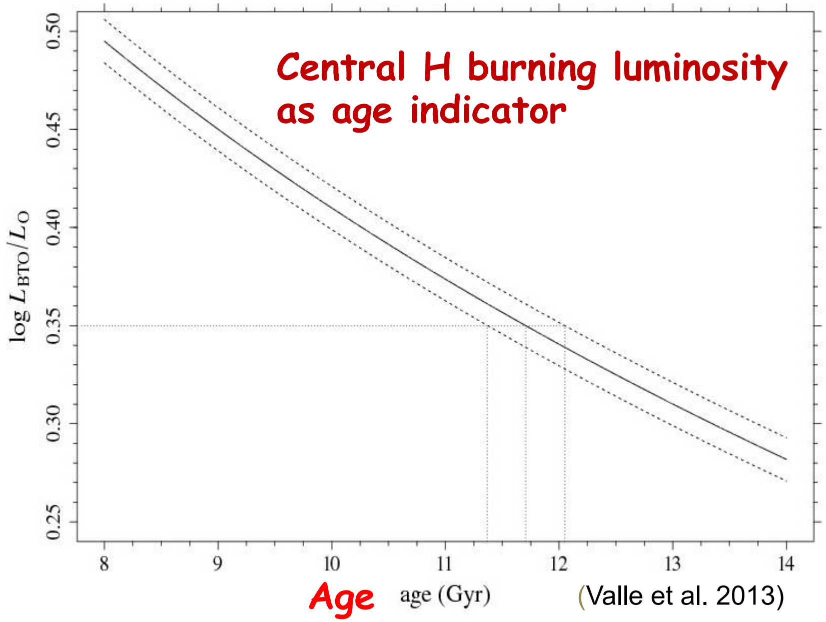


(Valle et al. 2013)

- The most important factor is again the radiative opacity $\Delta \log L_{ZAHB}/L_{\odot}$ ~ ± 0.022 dex [error ~5%, $\Delta \log L_{ZAHB}/L_{\odot}$ ~-0.43 Δk_r]
- The second most important input is the Triple-α cross section $\Delta \log L_{ZAHB}/L_{\odot}$~±0.011 dex [error~20%, $\Delta \log L_{ZAHB}/L_{\odot}$ ~-0.06Δσ(Triple-α)]
- The dependence on the thermic neutrino production is more or less the same as the one on the Triple-α rate but the associated uncertainty (and thus also the effect) is lower: error~4%, $\Delta \log L_{ZAHB}/L_{\odot}$~-0.002 dex

Global uncertainty on age determination of old stellar clusters

By adopting the vertical method the total uncertainty on age determination due to physical inputs is ± 1.25 Gyr



(Valle et al. 2013)

The TO luminosity is weakly sensitive to variations of the physical inputs (see also e.g. Chaboyer 1995, Brocato et al. 1998), whereas the ZAHB luminosity, due to the more advanced evolutionary phase, is more dependent on the uncertainties on the adopted input physics

- For a given TO luminosity the age variation is ± 0.37 Gyr
- In most cases the estimated uncertainties of the evolutionary parameters and their dependence on the physical parameters depend only in a mild way on the assumed chemical composition (Valle et al. 2013b)

See also uncertainty evaluations by Chaboyer et al. 1996,1998, 2002,Krauss & Chaboyer 2003

To improve theoretical evolutionary predictions for low mass stars up to the central He burning

- Huge effort to increase the precision of radiative opacity calculations

- Reduction of the uncertainty on the Triple alpha reaction rate

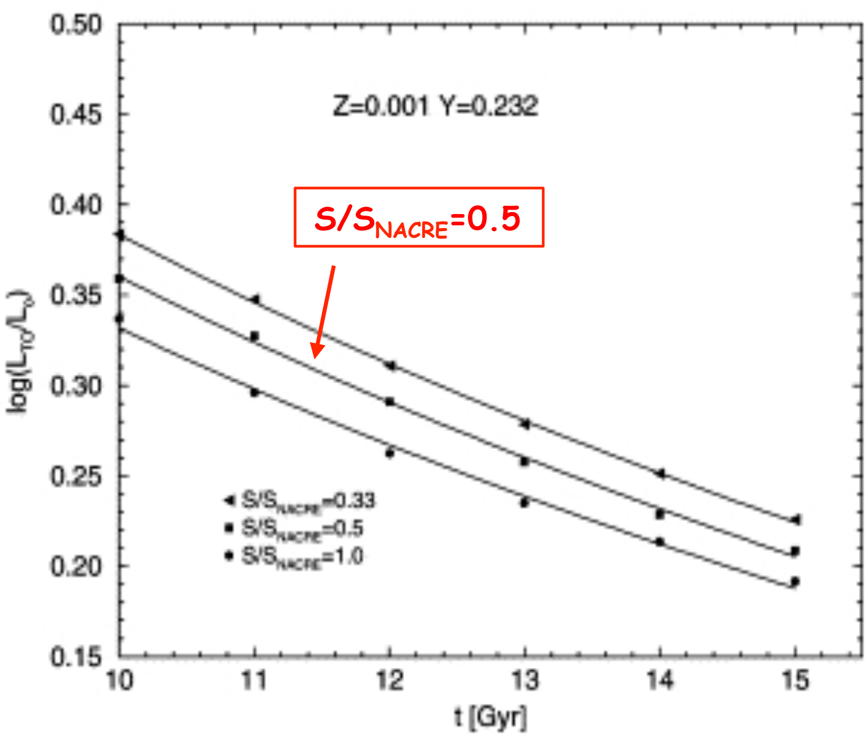
Surprises are always possible!

In 2004 the LUNA collaboration (Formicola et al. 2004, Imbriani et al. 2005) found a value for the $^{14}N(p,\gamma)^{15}O$ astrophysical factor S(0) which was half of the previous quoted estimates → this value was confirmed by more recent measures of the same group (see e.g. Marta et al. 2008 and references therein) $S(0)=1.57 \pm 0.13$ KeV-b

The cross section update influences the age determination only for old clusters

(see Straniero et al. 2002, Imbriani et al. 2004, Degl'Innocenti et al. 2004, Weiss et al. 2005)

$\sigma(^{14}N(p,\gamma)^{15}O) \downarrow \quad L_{TO} \uparrow \quad M_c^{He} \uparrow \quad L_{CNO}^{HB} \downarrow \quad L_{3\alpha}^{HB} \uparrow$



(Degl'Innocenti et al. 2004)

$\Delta \mathrm{Log}L_{HB} \sim 0.01$ (Z=0.0002)

$\Delta \mathrm{Log}L_{HB} \sim -0.01$ (Z=0.001)

↓

Maximum age variation by adopting the vertical method ~ 1 Gyr

Stellar physics effects of updates of the $p(p, e^+\nu_e)^2H$ cross section

- Recently (Marcucci, Schiavilla, Viviani 2013, MSV13) the pp astrophysical S-factor S(E) has been calculated taking into account also the contribution of the P-partial wave in the initial pp state
- The estimated theoretical uncertainty on S(E) is of few ‰ (about a factor 7 lower than previous evaluations)
- S(E) is often expressed as the first three terms of a Maclaurin series in E:

$$S(E) = S(0) + S'(0)E + \frac{1}{2}S''(0)E^2 + \ldots$$

Marcucci et al. 2013, MSV13 : $S(0) = (4.030 \pm 0.006) X\ 10^{-25}$ MeV b

Adelberger et al. 2011, AD11: $S(0) = (4.01 \pm 0.04) X\ 10^{-25}$ MeV b

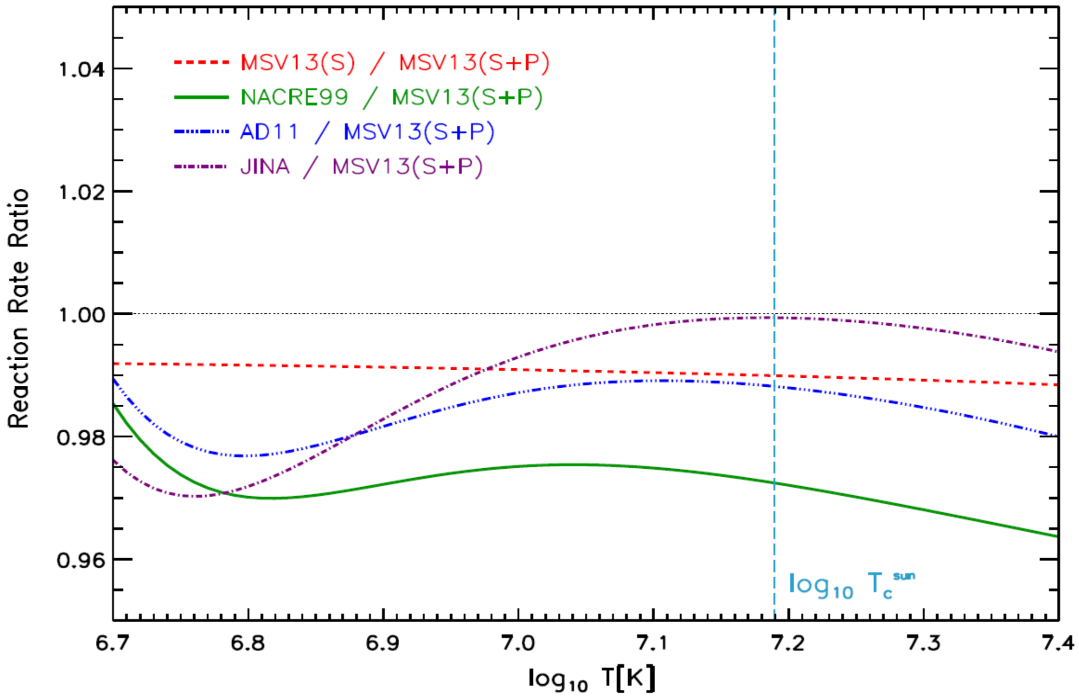
- We calculated the pp rate adopting the S(E) evaluation by MSV13 with an without the P-partial wave contribution: MSV13(S+P), MSV13(S)

$$R[\mathrm{cm^3\,mol^{-1}\,s^{-1}}] = \frac{3.73 \times 10^{10}}{\sqrt{\hat{\mu}\,T_9{}^3}} \int_0^{\infty} S(E) \exp\left(-2\pi\eta - 11.605\frac{E}{T_9}\right) \mathrm{d}E$$

where $\hat{\mu} = 0.504$ is the p-p reduced mass in atomic mass units, T_9 is the temperature in units of 10^9 K, and η is the Sommerfeld parameter expressed as $\eta = 0.1575(\frac{\hat{\mu}}{E})^{1/2}$.

The routine for the pp rate calculation is available at the link: http://astro.df.unipi.it/stellar-models/pprate/

Comparison of pp reaction rates



(Tognelli et al. 2013)

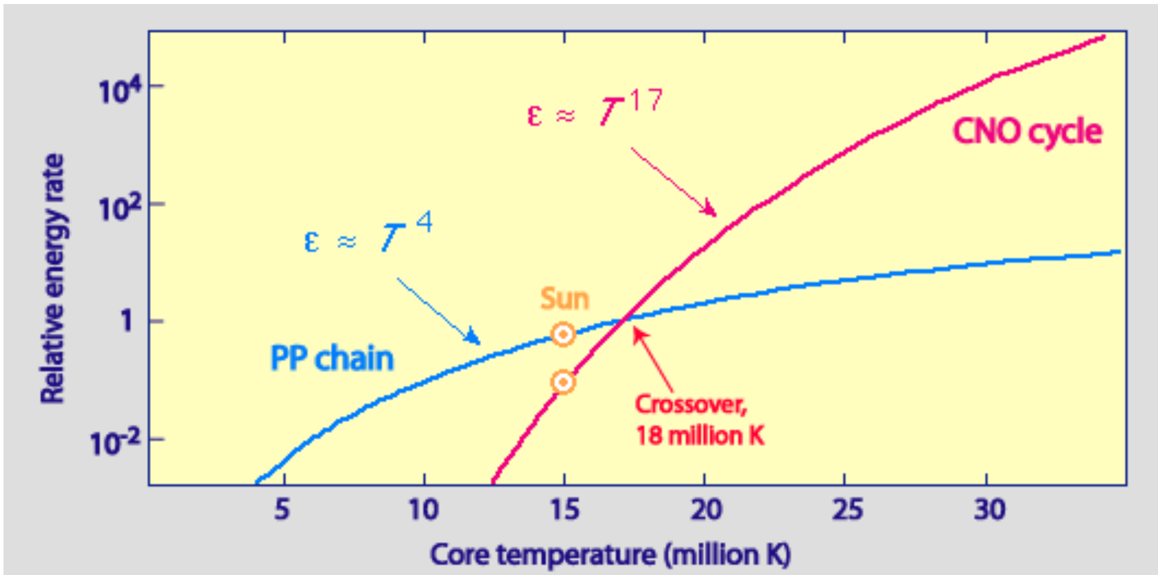
MSV13= Marcucci, Schiavilla, Viviani 2013
AD11=Adelberger et al. 2011
NACRE99 = Angulo et al. 1999
JINA= Cyburt et al. 2011

Results shown only in the temperature range of astrophysical interest for central H burning stars

- The error on the rate, derived from the one of S(E), is of the order of few ‰
- The effect on the rate of the inclusion of the P-partial wave contribution is ~1%
- The largest relative variation (~3÷4%) is found for the NACRE compilation which adopts S(0) different by ~2% from the present one
- The temperature behaviour of the JINA and AD11 rates is different from the present one, but the relative rate differences remain within ~2% (AD11) or ~2%÷3% (JINA)

Central H burning in stellar clusters

The pp chain and the CN-NO bicycle reactions experience different coulombian barriers and thus have different ignition temperatures and temperature dependence



(Figure from http://iasf.milano.inaf.it)

M_{star} ↑ T_c ↑

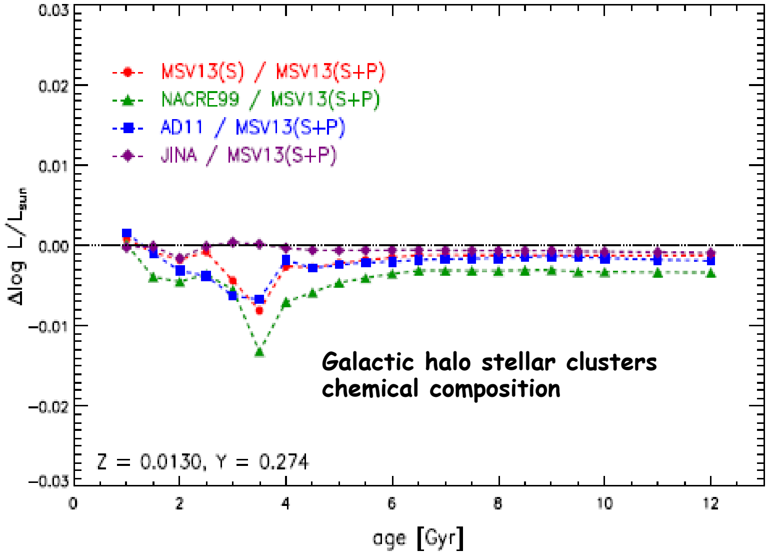
- stars with masses lower than about 1.2 $M_{\odot}$ burn H in the center mainly through the pp chain

- stars with masses higher than about 1.2 $M_{\odot}$ burn H in the center mainly through the CN-NO bicycle

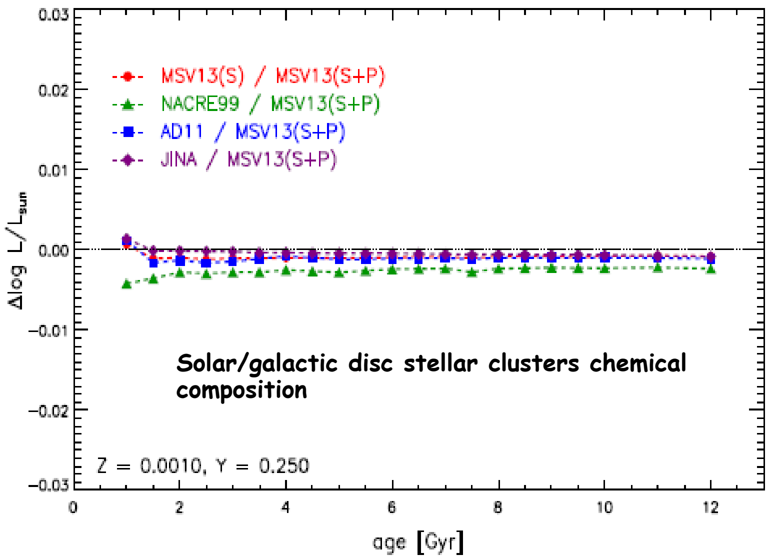
The pp contribution to the central H burning is not negligible for cluster ages older than about 1 Gyr

Difference in the central H exhaustion luminosity-age relation

Analysis for cluster ages from 1 to 12 Gyr and two different chemical compositions



The rate obtained from the Marcucci et al. 2013 S(E) with the contribution of the P-partial wave is taken as reference



Differences in central H exhaustion luminosity are generally lower than ~5 ‰

The $p(p, e^+\nu_e)^2H$ cross section is now known with such an high precision that it does not constitute anymore a significant uncertainty source in the age evaluation of stellar clusters

Standard Solar Models

Bahcall (1995): "A SSM is one which reproduces, within uncertainties, the observed properties of the Sun, by adopting a set of physical and chemical inputs chosen within the range of their uncertainties".

Fixed quantities		
Solar mass	$M_\odot=1.989\times10^{33}$g 0.01%	Kepler's 3rd law
Solar age	$t_\odot=4.57\times10^{9}$yrs 0.1%	Meteorites

Quantities to match		
Solar luminosity	$L_\odot=3.827\times10^{33}$erg s^{-1} 0.04%	Solar constant
Solar radius	$R_\odot=6.9566\times10^{10}$ cm 0.01%	Angular diameter
Solar photospheric metals/hydrogen ratio	$(Z/X)_\odot= 0.0183$* ~10%	Photosphere and meteorites

+ inferred helioseismic observables:

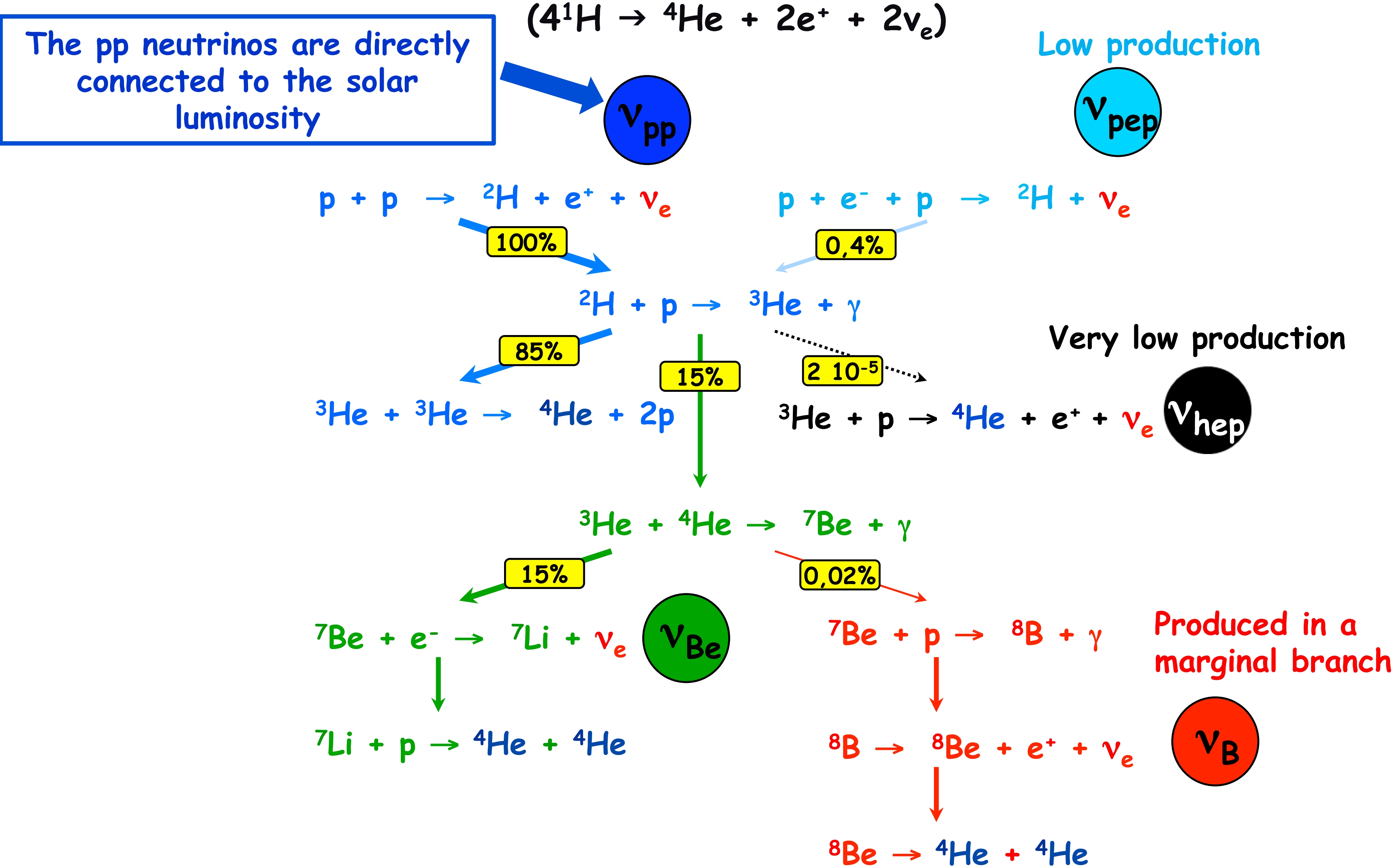
- **Extension of the convective envelope**
  $R_{cz}/R_\odot = 0.713 \pm 0.001$
  (Basu & Antia 1997)
- **The present surface He abundance**
  $Y_s = 0.2485 \pm 0.0034$
  (Basu & Antia 2004)
- **The sound speed profile**

*The precise value for the solar chemical composition is still under debate

Data from the 2015 IAU resolutions B2 and B3

Solar neutrino production: the proton proton chain
(highly dominant ~ 88.5% of the energy production)

$(4\,^{1}H \rightarrow\ ^{4}He + 2e^{+} + 2\nu_e)$



(Adapted from D. Vignaud at Neutrinos at the forefront of particle physics and astrophysics, 2012)

CN-NO by-cycle

(disfavoured ~ 1.5% of the solar energy production)

$(4\,^1H \rightarrow \,^4He + 2e^+ + 2\nu_e)$

CN-cycle ~ 1.49%

$$\begin{cases} ^{12}C + p \rightarrow \,^{13}N + \gamma \\ ^{13}N \rightarrow \,^{13}C + e^+ + \nu_e + \gamma \quad (\nu_N) \\ ^{13}C + p \rightarrow \,^{14}N + \gamma \\ ^{14}N + p \rightarrow \,^{15}O + \gamma \\ ^{15}O \rightarrow \,^{15}N + e^+ + \nu_e + \gamma \quad (\nu_O) \\ ^{15}N + p \rightarrow \,^{12}C + \,^4He + \gamma \end{cases}$$

NO-cycle ~ 0.01%

$$\begin{cases} ^{15}N + p \rightarrow \,^{16}O + \gamma \\ ^{16}O + p \rightarrow \,^{17}F + \gamma \\ ^{17}F \rightarrow \,^{17}O + e^+ + \nu_e + \gamma \quad (\nu_F) \\ ^{17}O + p \rightarrow \,^{14}N + \,^4He + \gamma \end{cases}$$

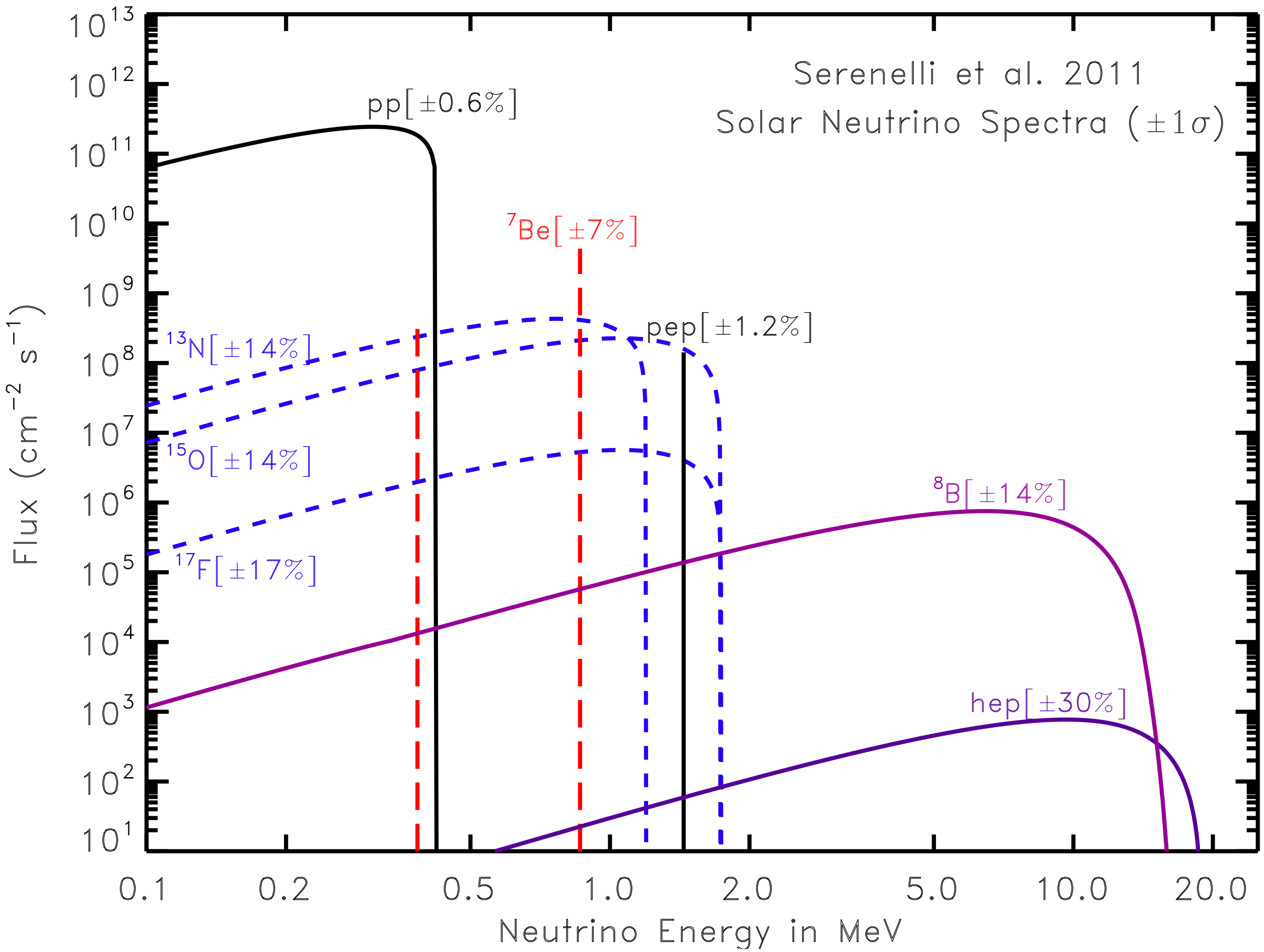
A rough estimate of the total neutrino flux can be obtained from the solar constant $K_\odot = L_\odot / \pi \langle D \rangle^2 \sim 1360.8 \pm 0.5\ W/m^2$ and the mean Q value for H burning Q~ 26.7 MeV ; $\langle D \rangle$ = 1 A.U.~ 150 x 10^6 Km

$$\Phi_{tot} = \frac{2K_\odot}{Q} \approx 6.4 \cdot 10^{10} \frac{\nu}{cm^2 s}$$

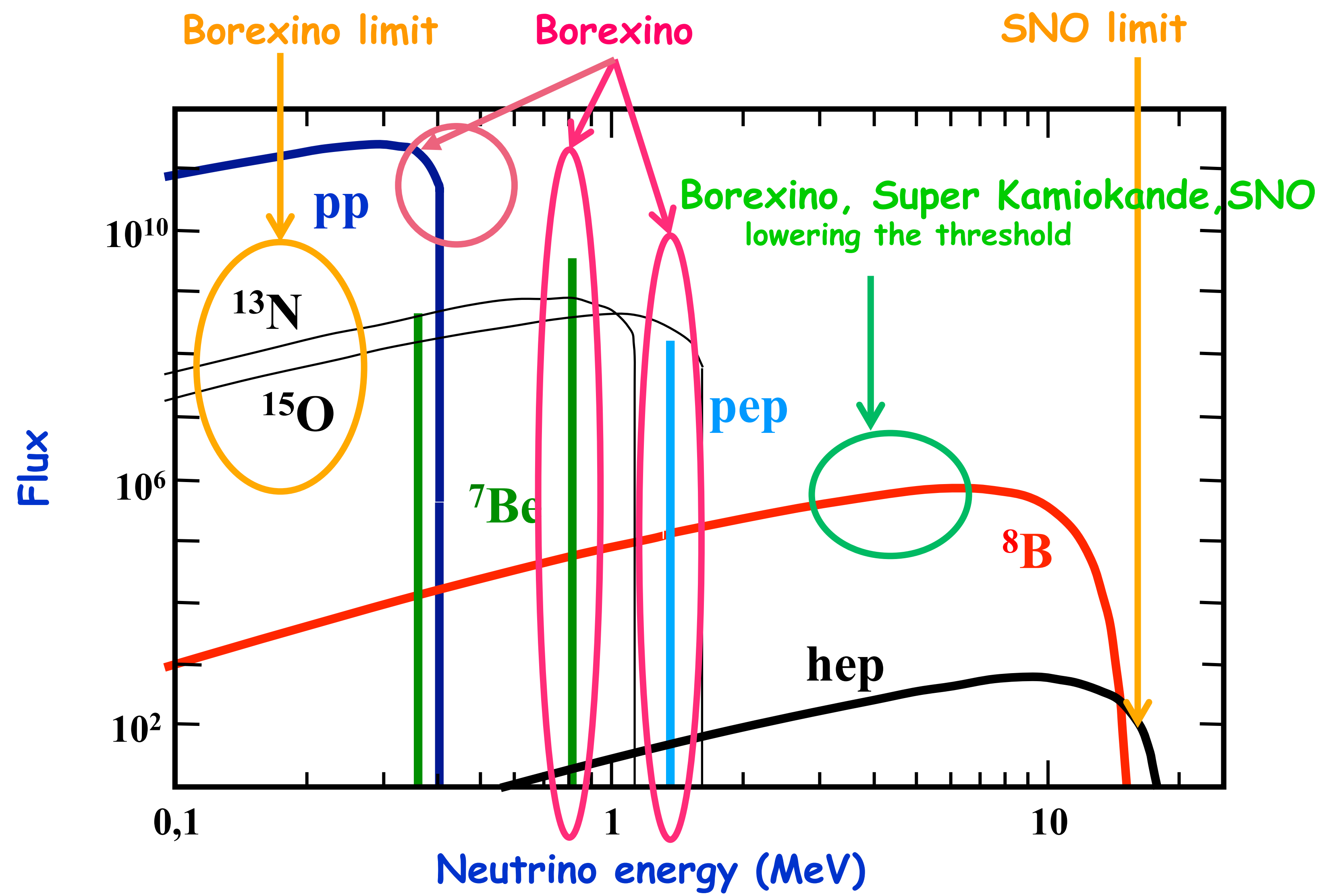
The total solar neutrino luminosity can be roughly estimated as:

$$L_\nu \approx 2 \cdot 10^{38} \nu/s$$

Energy spectrum of solar neutrinos



Towards spectroscopy of solar neutrinos



Experimental results for solar neutrinos

Solar neutrinos already individually detected

pp [Borexino, radiochemical Ga experiments]

^{7}Be [Borexino, radiochemical Cl experiment]

pep [Borexino]

^{8}B [SNO, Super-Kamiokande, Borexino]

Solar neutrinos still to be individually detected

CNO Borexino upper limit < 7.7 10^{8} $cm^{-2}s^{-1}$

hep SNO upper limit < 2.3 × 10^{4} cm^{-2} s^{-1}
SK upper limit < 1.5 10^{5} $cm^{-2}s^{-1}$

For experimental results see e.g. : Aharmin B. et al. 2013 (SNO collaboration), Cravens et al. 2008, Abe et al. 2001 (Super Kamiokande collaboration), Bellini et al. 2011, 2012, Bellini et al. , 2014, Physical Review, Bellini et al., 2014, Nature (Borexino Collaboration)

Solar neutrino fluxes inferred from global fits to solar neutrino data

($cm^{-2}s^{-1}$ units)

Flux	Solar value	error
pp	6.05×10^{10}	0.6%
pep	1.46×10^{8}	1.2%
hep	18×10^{3}	45%
^{7}Be	4.82×10^{9}	4.5%
^{8}B	5.00×10^{6}	3%
^{13}N	$\le 6.7 \times 10^{8}$	
^{15}O	$\le 3.2 \times 10^{8}$	
^{17}F	$\le 59 \times 10^{6}$	

(Serenelli 2016)

These results have been obtained from the neutrino signal in the various experiments, in the neutrino flavour oscillation framework, with the constraints that the sum of the thermal energy generation rates associated with each of the solar neutrino fluxes concides with the solar luminosity.This last constraint strongly bound pp and pep fluxes

Experimental results are in agreement with solar models predictions in the neutrino flavor oscillation framework and within theoretical and experimental uncertainties

Dependence of solar neutrino fluxes on different $p(p, e^+\nu_e)^2H$ reaction rates

- An increase of the pp rate leads to a decrease of the solar temperature*
- The dependence of the solar neutrino fluxes on central temperature variation can be approximated as a power law (Bahcall 1989, Bahcall & Ulmer 1996):

$$\Phi \propto T^{\alpha}$$

Flux	α
pp	-0.7
pep	-1.2
^{7}Be	10
^{8}B	20
NO	20
^{17}F	23

Update of Castellani et al. 1997 for solar models with diffusion and Grevesse& Sauval 1998 chemical composition

- Be neutrinos strongly depends on T_c, due to Gamow factor in ^{3}He+^{4}He
- B neutrinos has a stronger dependence due both to ^{3}He+^{4}He and (mainly) to ^{7}Be+p
- NO strongly depends on T_c, due to Gamow factor in ^{14}N+p
- For the conservation of total flux, pp neutrinos decrease with increasing T_c
- The pep rate goes approximatly as R_{pp} $T^{-0.5}$

*See e.g. Bahcall & Ulrich 1988, Bahcall 1989, Castellani et al. (1993), Degl'Innocenti et al. 1998, Antia & Chitre 1999, Serenelli et al. 2013

Effect on the solar neutrino fluxes of the adoption of different $p(p, e^+\nu_e)^2H$ reaction rates

	MSV13(S+P) reference	MSV13(S)	NACRE99	AD11	JINA
		relative differences			
T_c [10^7 K]	1.56214	−1‰	−3‰	< 1‰	−1‰
Φ^{ν}_{pp} [10^{10}]	6.009	1‰	2‰	< 1‰	1‰
Φ^{ν}_{pep} [10^8]	1.434	−2‰	−6‰	1‰	−1‰
Φ^{ν}_{hep} [10^3]	8.405	−1‰	−3‰	2‰	2‰
Φ^{ν}_{Be-7} [10^9]	4.996	**−1%**	**−3%**	< 1‰	−9‰
Φ^{ν}_{B-8} [10^6]	4.525	**−3%**	**−7%**	4‰	**−2%**
Φ^{ν}_{N-13} [10^8]	3.368	**−2%**	**−6%**	6‰	**−1%**
Φ^{ν}_{O-15} [10^8]	2.621	**−3%**	**−8%**	7‰	**−2%**
Φ^{ν}_{F-17} [10^6]	5.513	**−3%**	**−8%**	7‰	**−2%**

(Neutrino fluxes are in $cm^{-2}s^{-1}$ units)

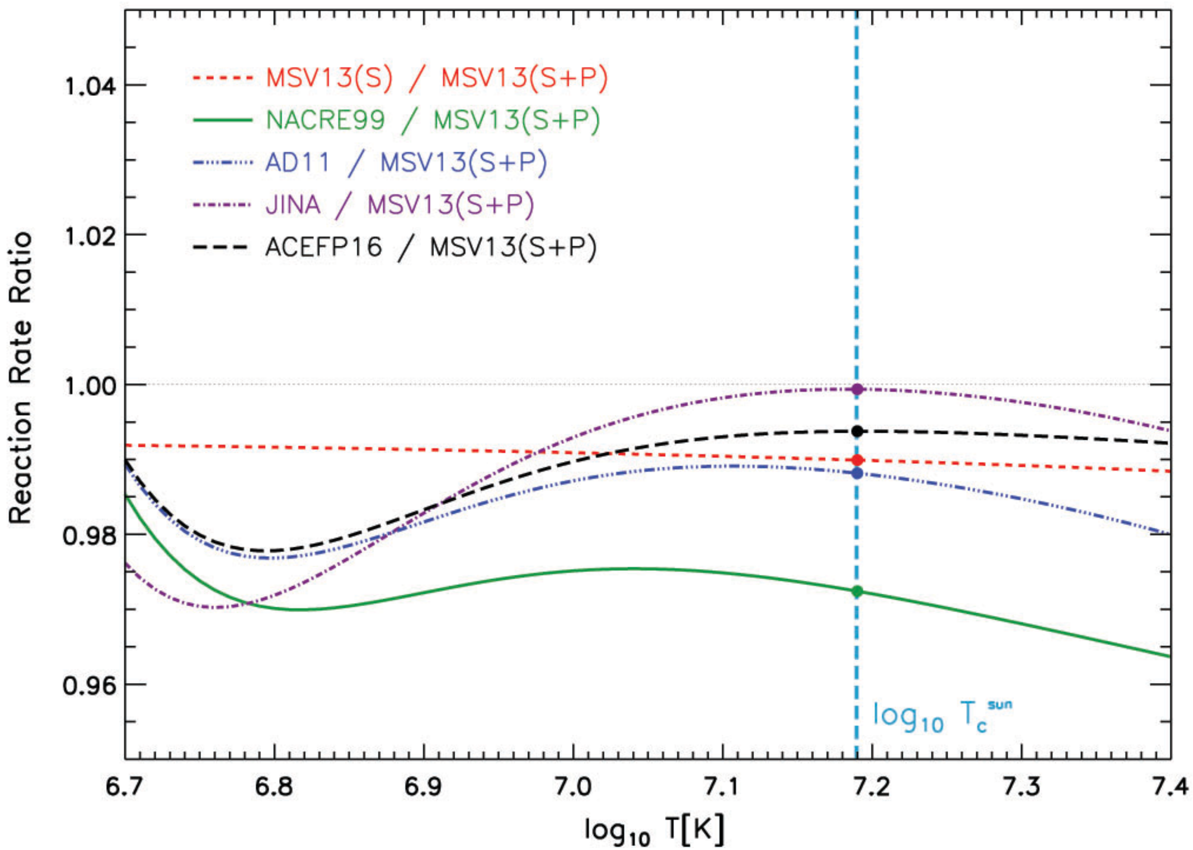
- Among all the models the differences in the original helium and metallicity abundances required to obtain a SSM are negligible
- The largest difference in the central temperature (for the NACRE rate) is of the order of 3‰ which leads to a difference of 7-8% in the ^8B/CNO neutrino fluxes
- The inclusion of the P-partial waves in the pp rate leads to a maximum of 3% effect on the ^8B and CNO neutrino fluxes

Due to the very small error, the $p(p, e^+\nu_e)^2H$ rate does not affect in a significant way standard solar models and neutrino fluxes calculations.

Very recently Acharya et al. 2016 made a careful quantitative analysis of the theoretical uncertainty for the proton proton reaction rate

Marcucci et al. 2013, MSV13 : $S(0) = (4.030 \pm 0.006) X\ 10^{-25}$ MeV b

Acharya et al. 2016, MSV13 : $S(0) = (4.047^{+0.024}_{-0.032}) X 10^{-25} MeV\ b$



The corresponding reaction rate is intermediate between MSV13 with and without the P-partial wave contribution -> the effects of the rate change with respect to MSV13 are negligible

Solar neutrino dependence on various physical and chemical inputs

The production efficiency of neutrinos from the different reactions depends on:

- Environmental inputs (Lum.,opacity, age,Z/X…)
  which affect physical conditions of the medium where they are produced, mainly the temperature

- Nuclear inputs (cross sections for the pp chain and CNO cycle reactions)

Dependence of solar neutrino fluxes on physical and chemical inputs

The sensitivity of the ν fluxes (and of the solar central temperature) to (small) changes of physics and chemical inputs can be expressed in terms of power laws.

Y \ X	S_{pp}	S_{33}	S_{34}	S_{17}	$S_{1,14}$	L	Z/X	opa	age	dif
pp	0.114	0.029	-0.062	0	-0.019	0.73	-0.076	-0.12	-0.088	-0.02
Be	-1.03	-0.45	0.87	0	-0.027	3.5	0.60	1.18	0.78	0.17
B	-2.73	-0.43	0.84	1	-0.02	7.2	1.36	2.64	1.41	0.34
N	-2.59	0.019	-0.047	0	0.83	5.3	1.09	1.82	1.15	0.25
O	-3.06	0.013	-0.038	0	0.99	6.3	2.12	2.17	1.41	0.34
T_c	**-0.14**	**-.0024**	**0.0045**	**0**	**0.0033**	**0.34**	**0.078**	**0.14**	**0.083**	**0.016**

(Update of Castellani et al. 1997 for solar models with diffusion and Grevesse& Sauval 1998 chemical composition)

$$\frac{Y}{Y_{SSM}} \propto \left(\frac{X}{X_{SSM}}\right)^{\alpha}$$

Values of $d\ln Y/ d\ln X$ computed by using models including element diffusion and Grevesse & Sauval 1998 chemical composition

See also Bahcall 1989, Bahcall & Ulmer 1996, Haxton & Serenelli 2008, Serenelli et al. 2013

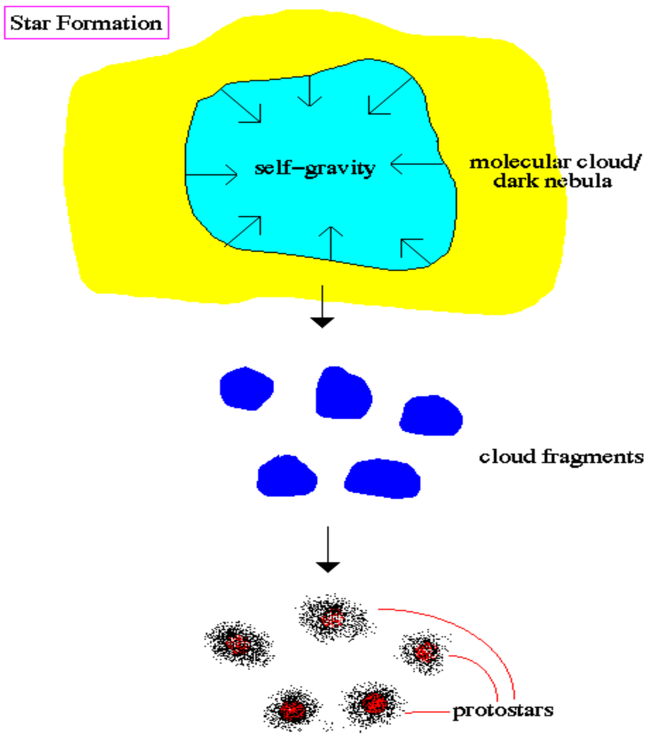
S(0) for $^{7}Be(p,\gamma)^{8}B$ and $^{14}N(p,\gamma)^{15}O$ cross sections

$$S(0)_{17} = 20.8 \pm 0.7(expt) \pm 1.4(theor) \text{ eV-b}$$

$$S(0)_{1,14} = 1.66 \pm 0.12 \text{ KeV-b}$$

Adelberger et al. 2011

The still present uncertainty on the $^{7}Be(p,\gamma)^{8}B$ and $^{14}N(p,\gamma)^{15}O$ cross sections leads to an indetermination on ^{8}B and ^{13}N, ^{15}O neutrino fluxes of the order of 6÷7.5%



- Stars form from molecular clouds of gas and dust
  - Star formation is a very complex process not yet fully understood
    - An hydrostatic core forms and accretes gas and dust while the protostar is heavily obscured by the cloud

The Pre-Main Sequence Phase begins when:

- the main accretion phase is completed
- The star has already sweeped out the residual gas from the original cloud and it is optically visibile as a new star
- The star is a bright and large object

Pre-Main Sequence evolution

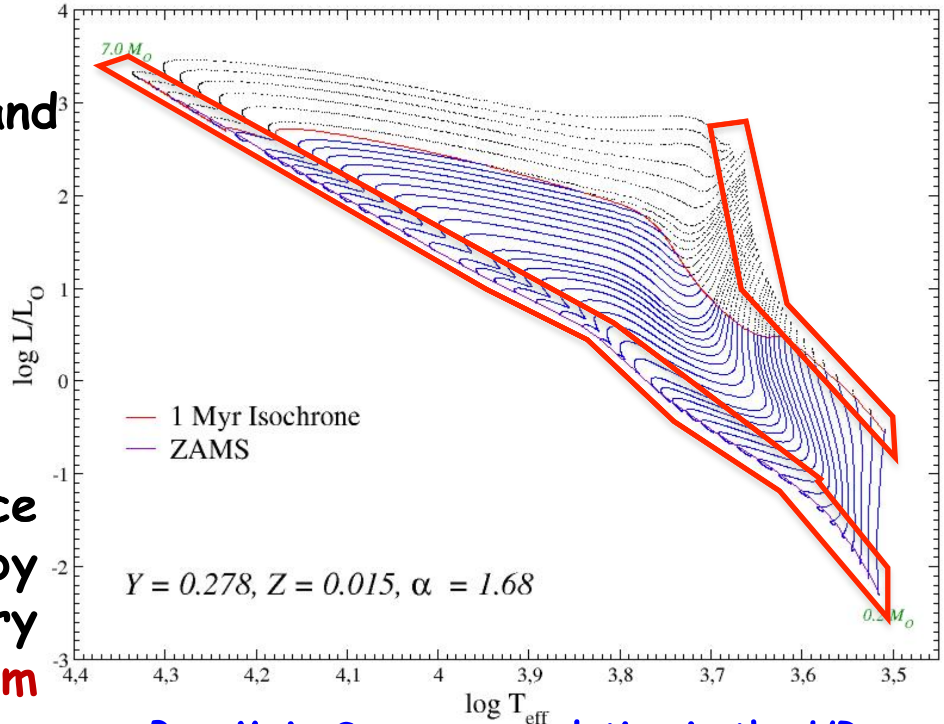
The PMS phase begins …stars are bright, cold and large objects

No nuclear reactions → but they are bright → large energy losses

Stars contract on thermal timescales increasing their internal temperature and decreasing their luminosity

Until in the central regions the temperature for H burning is reached

The PMS phase ends …
…stars enter in the Main Sequence with the first model fully supported by H-burning in which the secondary elements are at their equilibrium configuration



Pre-Main Sequence evolution in the HR diagram of stars of different masses

(Tognelli, Prada Moroni & Degl'Innocenti 2011)

Pre-Main Sequence evolution

At the beginning of the PMS evolution stars are fully convective

As soon as the internal temperature increases a radiative core develops (for masses lower than about) and grows with time

In convective regions matter is fully mixed -> homogeneous chemical composition

Unluckily the efficiency of the external convection cannot be calculated with precision

Mixing Length Theory (Bohm-Vitense 1968)

The convection efficiency depends on the free parameter:

α ($l_c = \alpha H_P$) $$H_p = -\frac{dr}{d\ln P}$$

Calibration of α: Sun

There is **no a priori** reason that guarantees that the solar calibration holds for stars of **different masses** and/or in different **evolutionary stages** (see e.g. Canuto & Mazzitelli 1992, D'Antona & Mazzitelli 1994, 1998, Montalban et al. 2004, Siess & Livio 1997, Ludwig et al. 1999, Trampedach et al. 1999, 2007)

The pratical consequence is that we are not yet able to firmly predict the extension and the temperature gradient of the convective envelopes and thus surface temperature, T_{eff}, and the radius, R of stars with an outer convective envelope

Light elements burning

During the PMS evolution temperatures for light elements burning are reached in the stellar convective envelope too

If the burning temperatures are reached at least at the bottom of the convective envelope the surface abundances change too

T ~ 10^6 K $\quad$ $$^2\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma$$

T ~ 2×10^6 K $\quad$ $$^6\mathrm{Li} +^1\mathrm{H} \rightarrow^3\mathrm{He} +^4\mathrm{He}$$

T~ 2.5×10^6 K $\quad$ $$^7\mathrm{Li} + {}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$$

T ~ 3.5×10^6 K $\quad$ $$^9\mathrm{Be} +^1\mathrm{H} \rightarrow^4\mathrm{He} +^6\mathrm{Li}$$

T ~ 4.2×10^6 K $\quad$ $$^{11}\mathrm{B} +^1\mathrm{H} \rightarrow^4\mathrm{He} +^8\mathrm{Be}$$

- **The light elements burning (except deuterium burning) does not influence the energy production, due to their low original abundance.** Fractional abundances in mass: $X_D \approx 2 \times 10^{-5}$, $X_{6Li} \approx 7.5 \times 10^{-10}$, $X_{7Li} \approx 1.1 \times 10^{-8}$, $X_{Be} \approx 1.8 \times 10^{-10}$, $X_{11B} \approx 5.3 \times 10^{-9}$ for disc stars*

- **The surface abundances strongly depend on the extension of the convective envelope and thus on the stellar mass and metallicity**

*see e.g. Geiss & Gloeckler 1998, Linsky et al. 2006, Steigmann et al. 2007, Jeffries 2006, Lodders et al. 2010, 2009, Cunha2010

Light elements burning cross sections

We adopted the **cross sections** for light elements burning measured by the **Trojan Horse Method, THM, group** (see e.g. Lamia et al. 2015, 2013, 2012, Lattuada et al. 2001, Tumino et al. 2014, Pizzone et al. 2005, Pizzone et al. 2003).

$^7Li(p,\alpha)^4He$ cross section

S(0)=53 ± 5 KeV b

(Lamia et al. 2012)

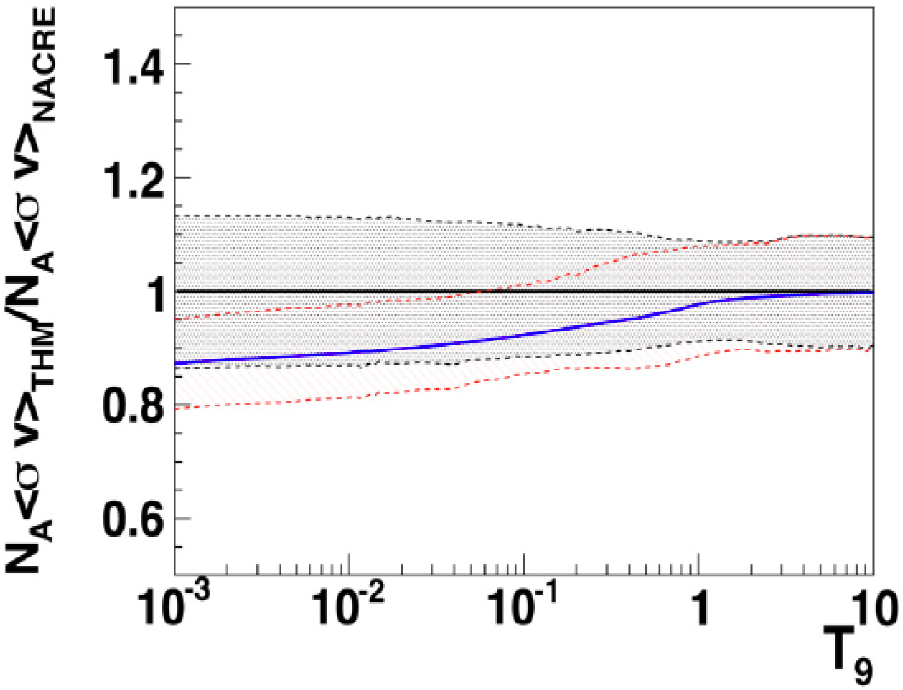


Fig. 4. Ratio of the adopted THM $^7Li(p,\alpha)^4He$ reaction rate to that evaluated by the NACRE compilation (Angulo et al. 1999) (full blue line), together with THM upper and lower limit (red dashed lines). This is compared with the upper and lower values recommended in NACRE (black dashed lines).

General features of Lithium burning in pre-MS

1 - Onset of lithium burning:

Fully convective star

increasing **M** -> increases $T_c = T_{ce}$ ->Li burning onsets **earlier**

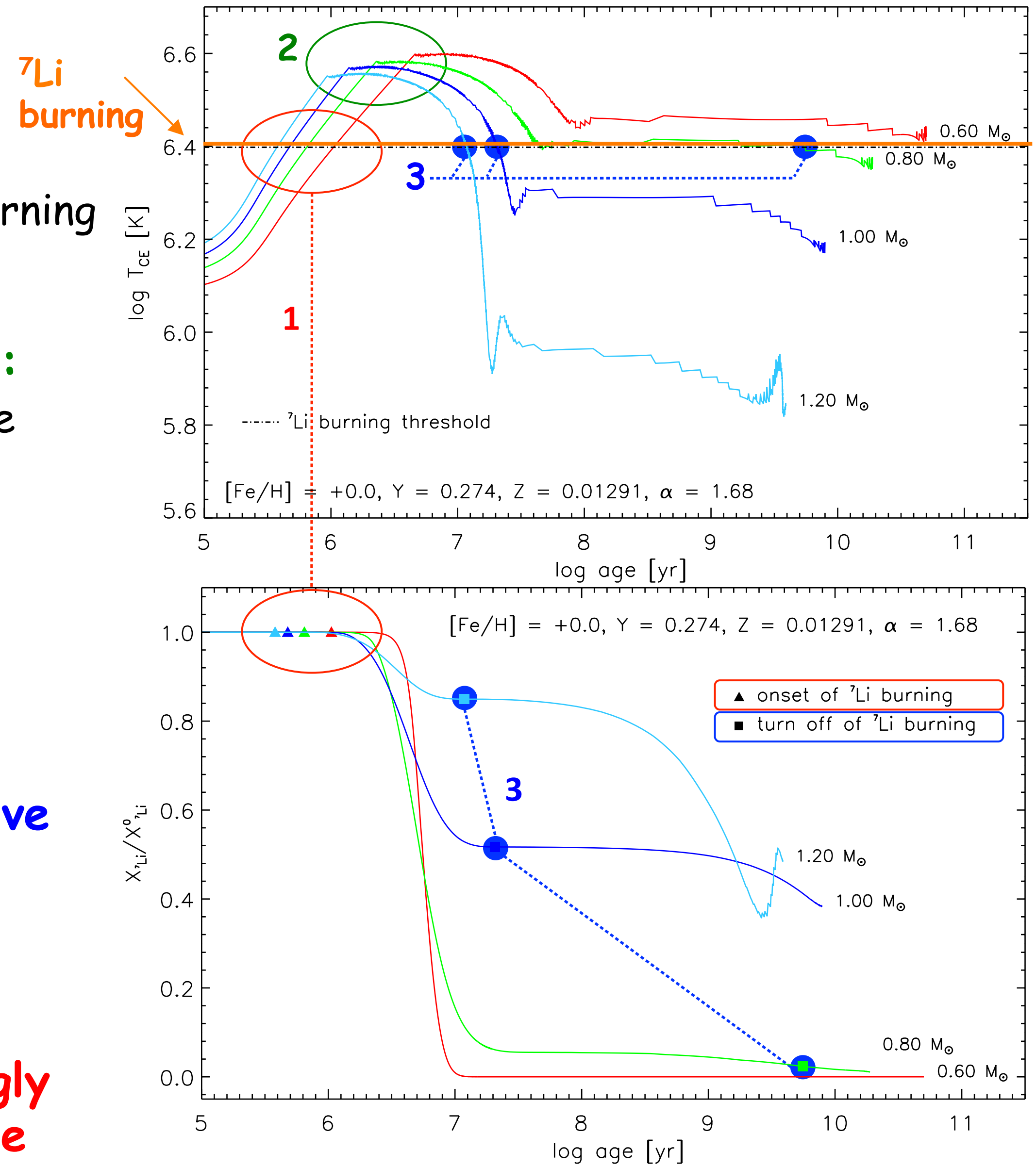
2 - Formation of a radiative core:

Lithium burning still possible at the base of the convective envelope if $T_{CE} \geq T_{Li}$.

increasing **M** -> decreases **convective envelope** extension -> decreases T_{ce} -> Li-burning stops **earlier**

3 - The extension of the convective envelope reduces:

T_{CE} decreases, lithium burning is halted.

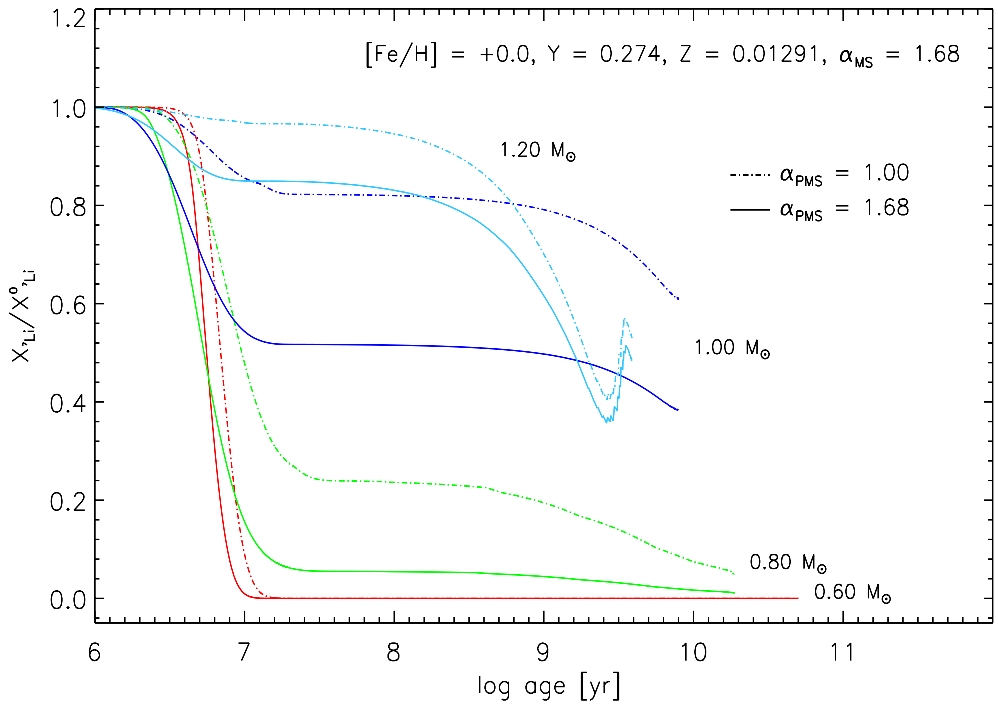


PMS ^{7}Li surface depletion strongly depends on the mass and on the metallicity of the star

General features of Lithium burning in pre-MS

Lithium pre-MS depletion is **extremely sensitive** to the convection efficiency

-



(Tognelli et al. 2012)

The lower is the convection efficiency, the lower the temperature at the bottom of the convective envelope the less efficient is the lithium depletion

Standard models fail in reproducing the observed ^{7}Li abundances in young clusters if a solar/MS convection efficiency is assumed in PMS ➡ hints for a reduced convection efficiency

(see e.g. D'Antona & Mazzitelli 1994, 1997, Ventura et al. 1998, Schlatt & Weiss 1999, Piau & Turck-Chièze 2002, D'Antona & Montalban 2003, Landin et al. 2006, Eggenberger et al. 2012, Tognelli et al. 2012, Sommers & Pinsonneualt 2014 and references therein)

Uncertainties on chemical and physics inputs

- Chemical composition

Uncertainties on Y_P, $(Z/X)_\odot$, [Fe/H], $\Delta Y/\Delta Z$, solar mixture

- Δ[Fe/H] ≈ ±0.01, ±0.1 (we adopts ±0.05);
- ΔY_P ≈ ±0.001 (Cyburt 2004, Steigman 2006, Peimbert et al. 2007);
- $\Delta Y/\Delta Z$ ≈ 2 ± 1 (Casagrande et al. 2007);
- $\Delta(Z/X)_{sun}$ ≈ +25/-10 % (see discussion in Tognelli et al. 2012)

Y_P : primordial helium abundance.
$(Z/X)_\odot$: photospheric metal-to-hydrogen ratio abundance in the Sun.
$\Delta Y/\Delta Z$: helium-to-metal enrichment ratio.

$$[Fe/H] = log\frac{N_Fe/N_i}{(N_Fe/N_H)_\odot}$$

- Physics Input

Opacity coefficients (radiative opacity): uncertainty of about ± 5% from differences between OPAL 2005 (see e.g. Iglesias & Rogers 1996) and OP (Seaton et al. 1994, Badnell et al. 2005), see also Neuforge-Verheecke et al. 2001, Bahcall et al. 2005, Badnell et al. 2005, Valle et al. 2012).

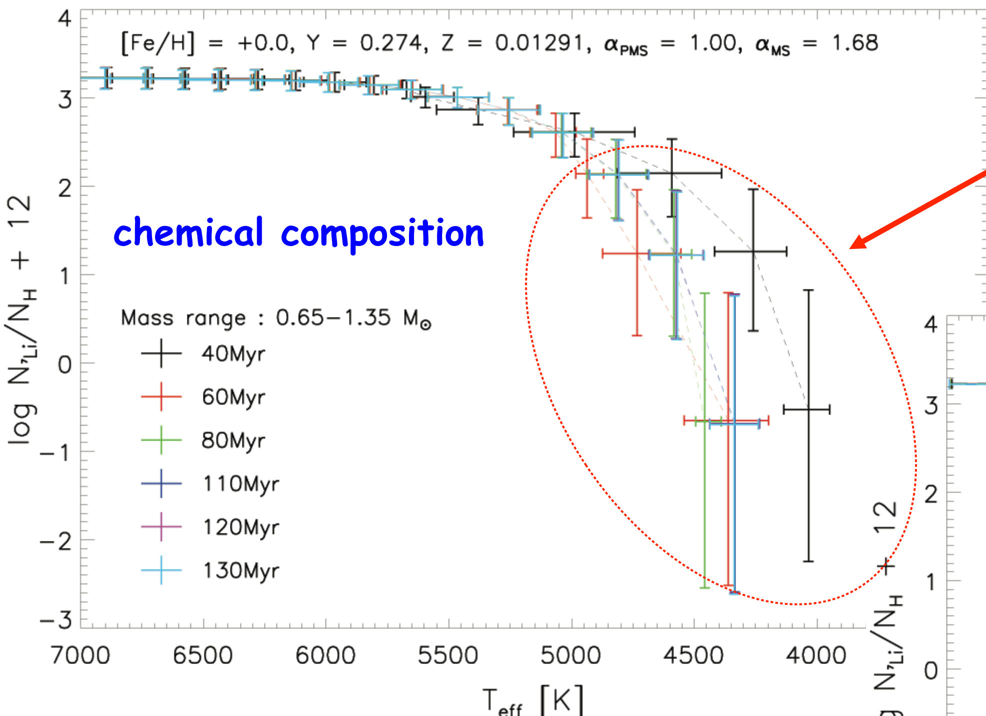
$^7Li(p,\alpha)^4He$ reaction Rates: uncertainty about ± 10% (Lattuada et al. 2001; Pizzone et al. 2003, Adelberger et al. 2011, Lamia et al. 2012)

EOS: uncertainties evaluated by comparing the results obtained from several EOS widely used in the literature (OPAL2001 , Rogers & Nayfonov 2002, SCVH95 Saumonn, Chabrier & VanHorn 1995, PTEH, Pols et al. 1995)

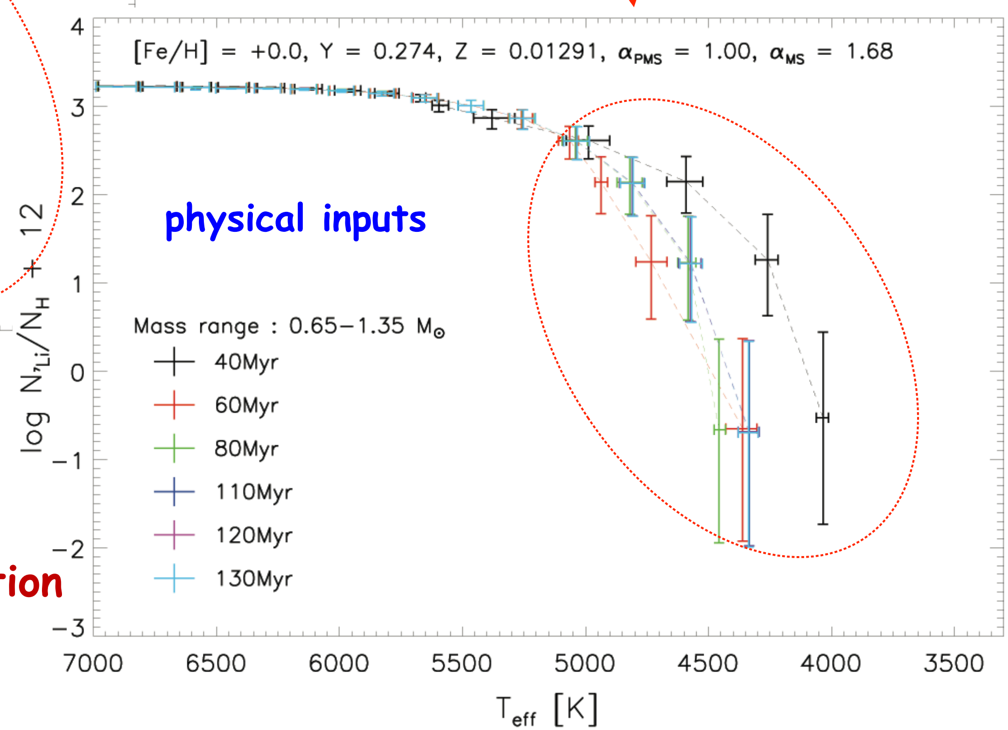
Initial D and ^{7}Li values : X_D= 2 · 10^{-5} (see e.g. Geiss & Gloeckler 1998, Linsky et al. 2006, Steigmann et al. 2007)
A(Li) = log N_{Li}/N_H + 12 = 3.2 ± 0.2 -> X_{7Li} ≈ 7 x 10^{-9} ÷ 1x10^{-8} in dependence on the stellar metallicity (see e.g. Jeffries 2006, Lodders et al. 2009)

Uncertainties on predicted surface lithium abundance

Error bars: we **quadratically added** the differences in surface lithium abundances and effective temperature obtained taking into account the quoted error sources



The **uncertainties** on the **chemical composition, physical inputs** have **a strong effect** on the predictions of surface lithium abundance in particular for **low-mass stars**



Tognelli, Degl'Innocenti, Prada Moroni 2012

The uncertainty on the ^{7}Li (p,α)^{4}He reaction rate has a minor effect

Conclusions

(for low mass stars from the PMS to the initial central He burning)

- Cross section values for light elements and H burning reached a so high precision (in particular the pp reaction!) that they are no more major sources of uncertainties → nuclear physics was kind to us!

- A further reduction of the uncertainties for $^{7}Be(p,\gamma)^{8}B$ and $^{14}N(p,\gamma)^{15}O$ reaction rates could improve even more the precision of the predictions for ^{8}B and CNO solar neutrino fluxes

- The present uncertainty of the triple α reaction rate has not negligible effects on stellar evolutionary calculations