Few-body systems in QCD

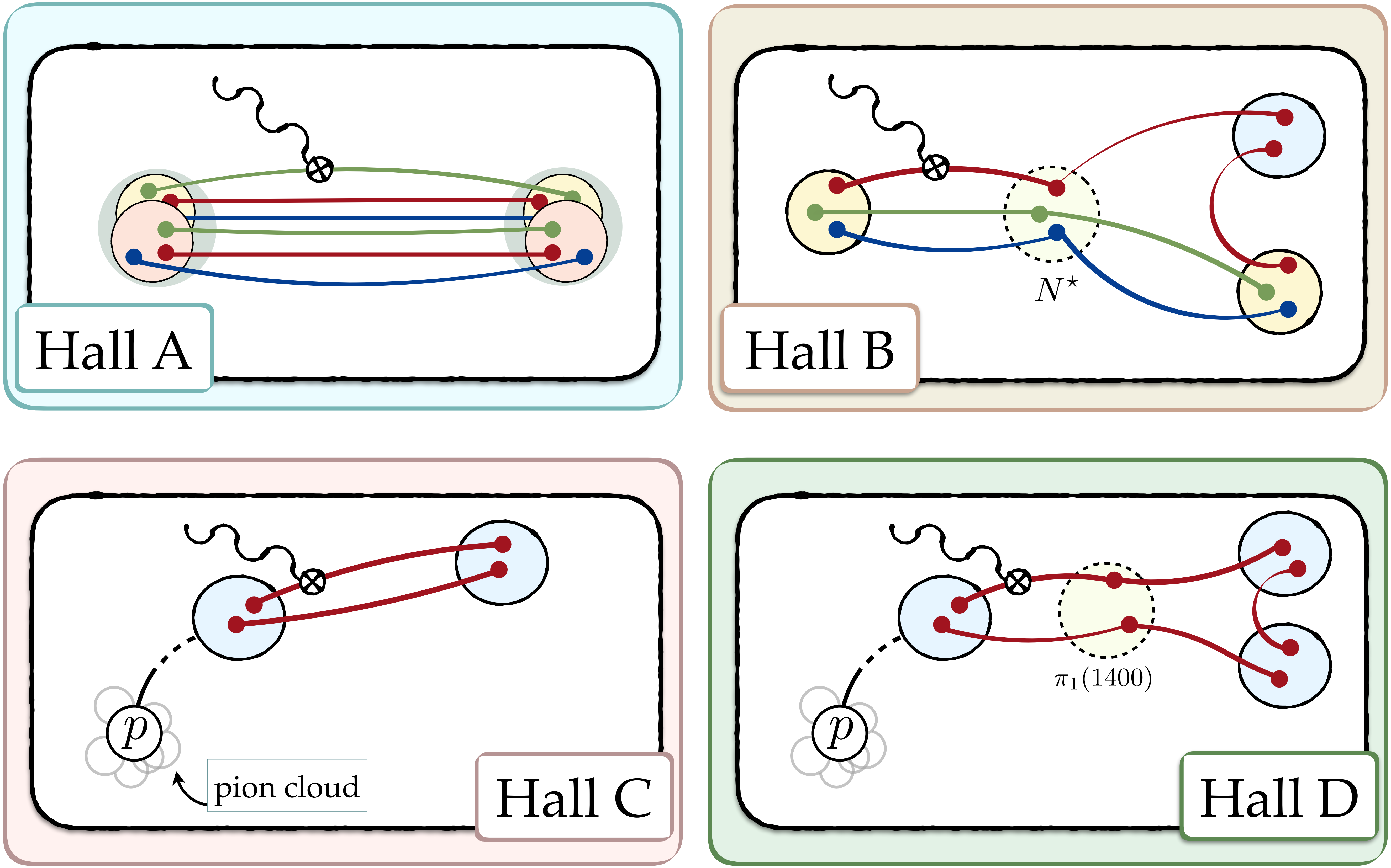
from spectroscopy to structure

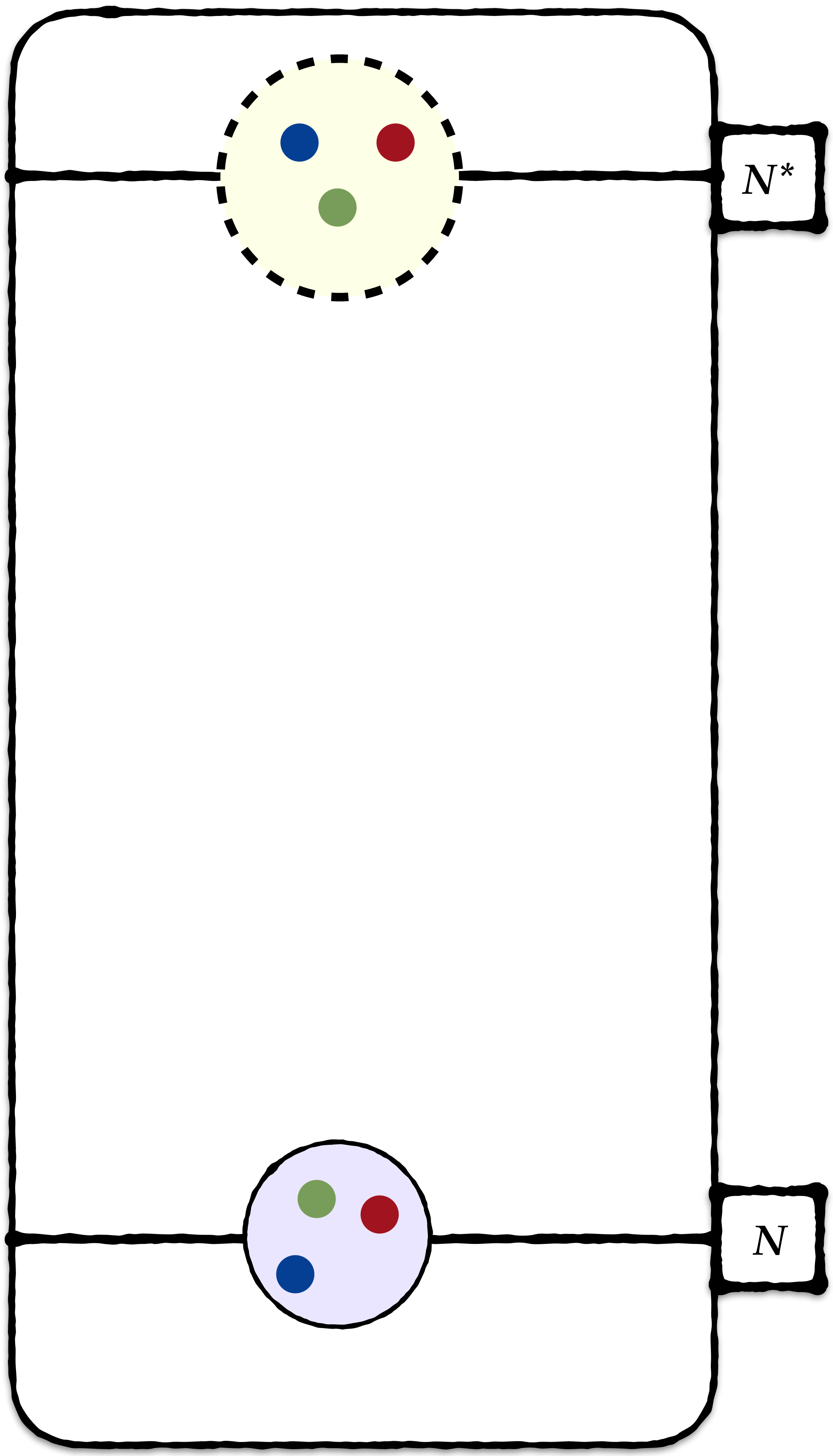
Raúl Briceño

Jefferson Lab

JLab Feb, 2017

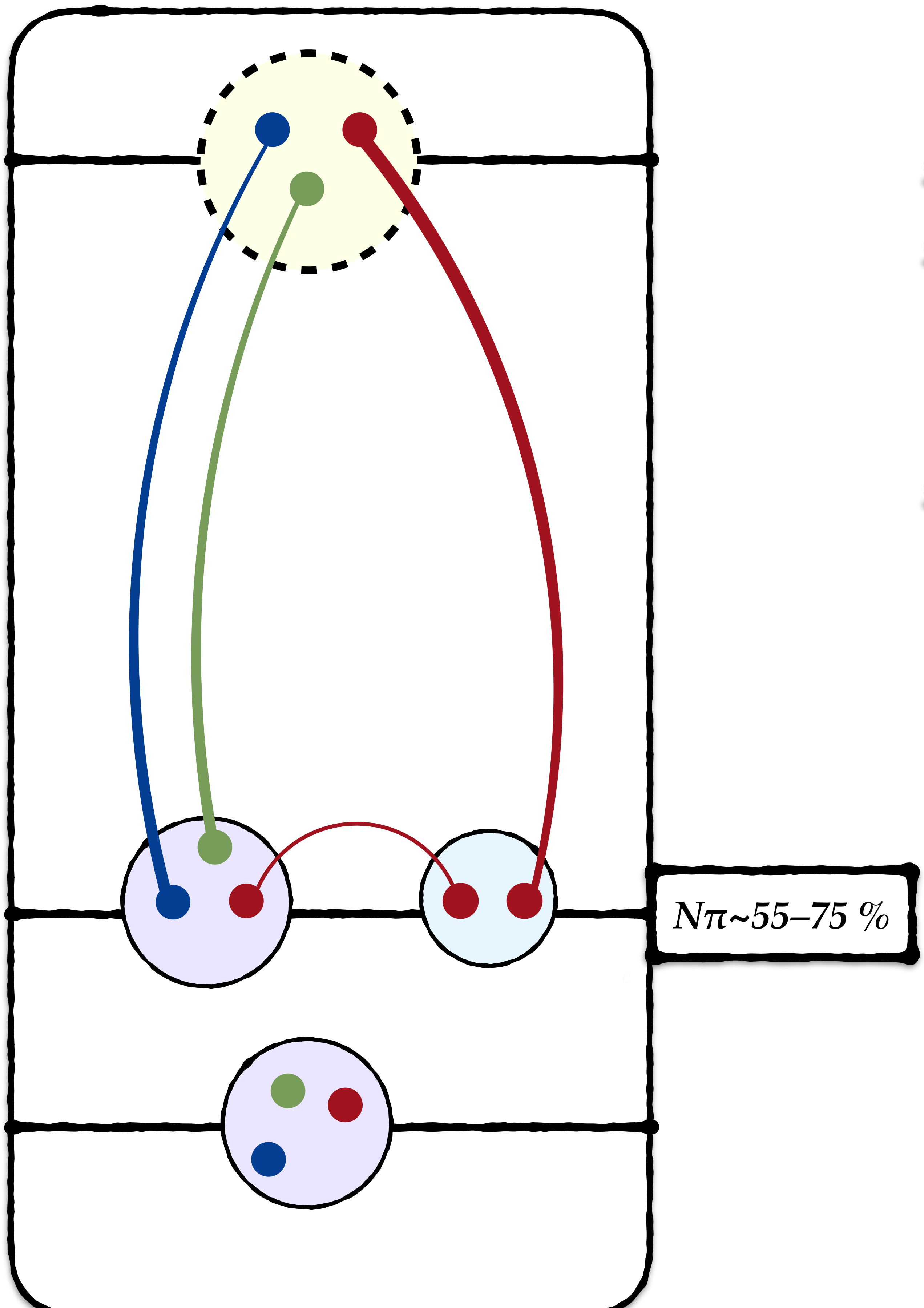
12GeV is now!





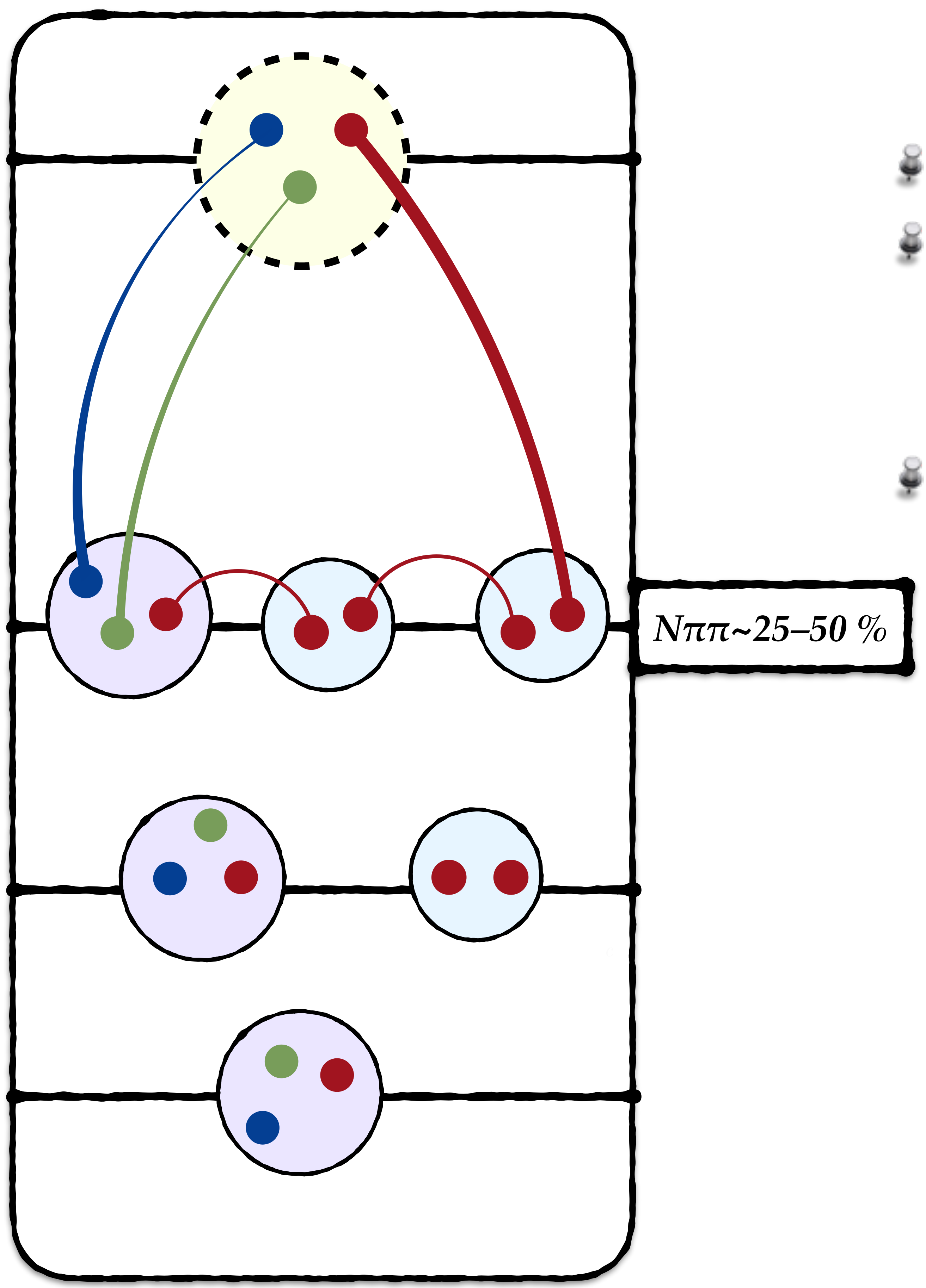
the Roper

- Excited state of the nucleon



the Roper

- Excited state of the nucleon
- Dynamical enhancement in amps.
  - Complex pole
  - Fairly broad
- Strongly coupled to:
  - $N\pi$



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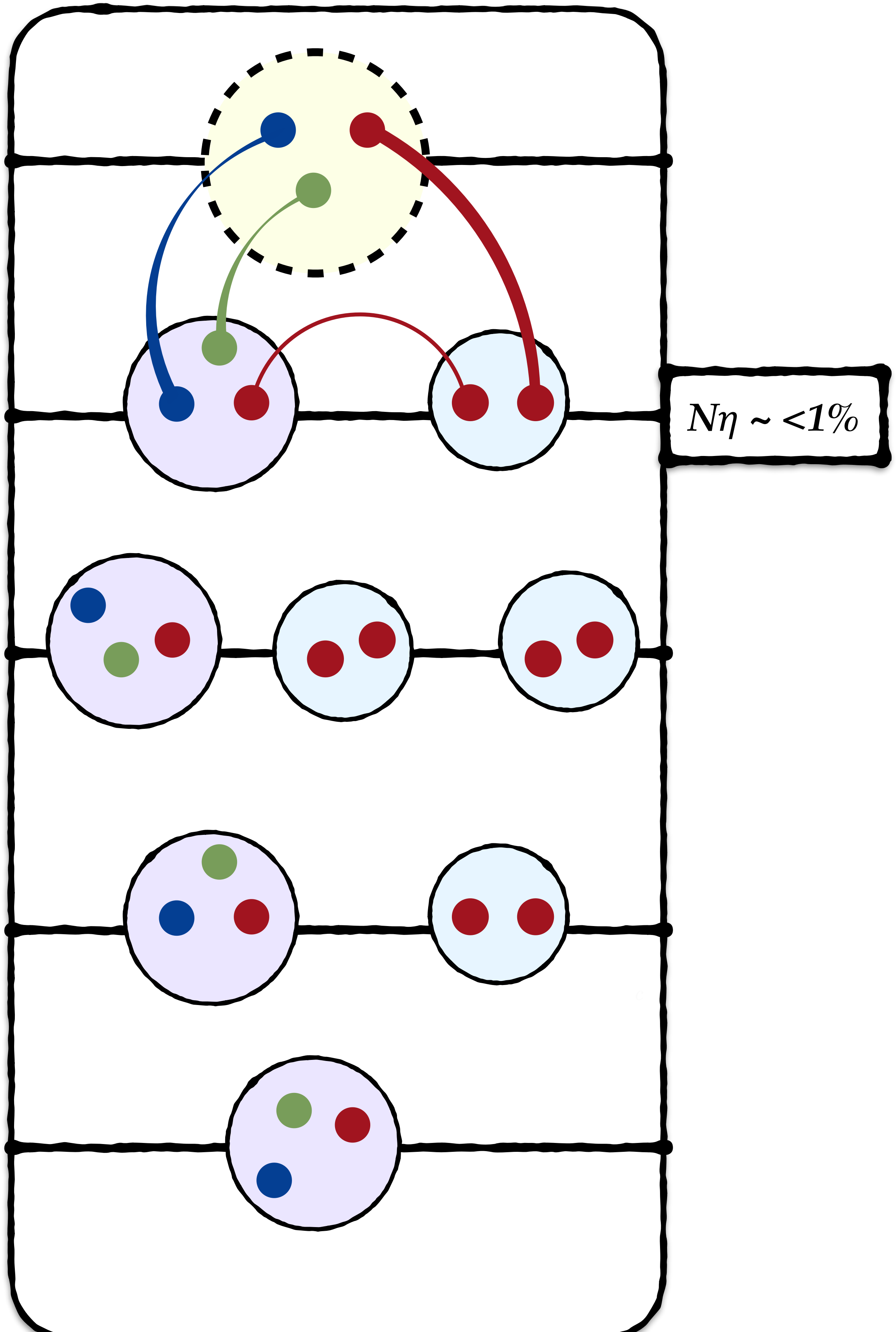

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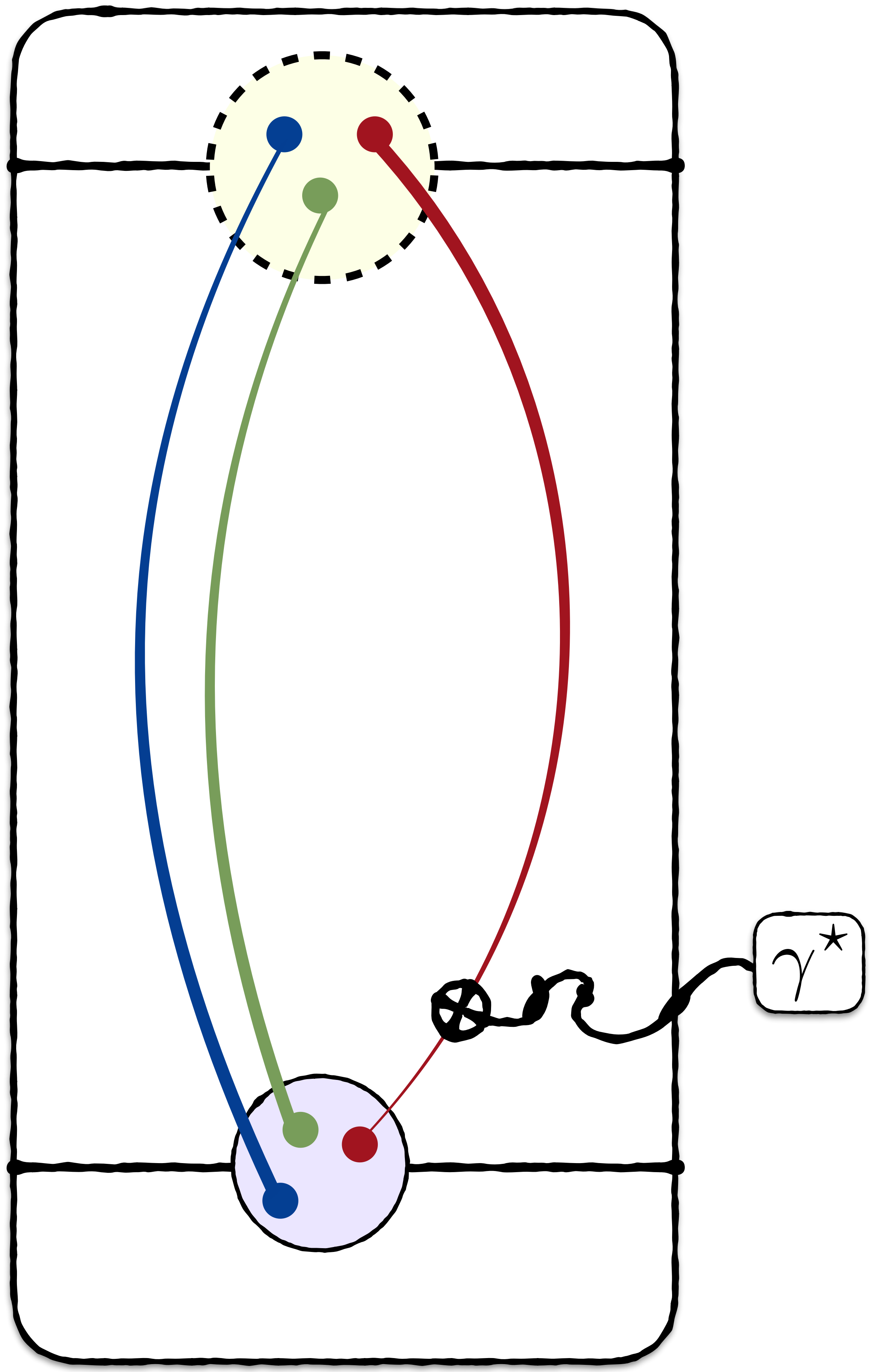

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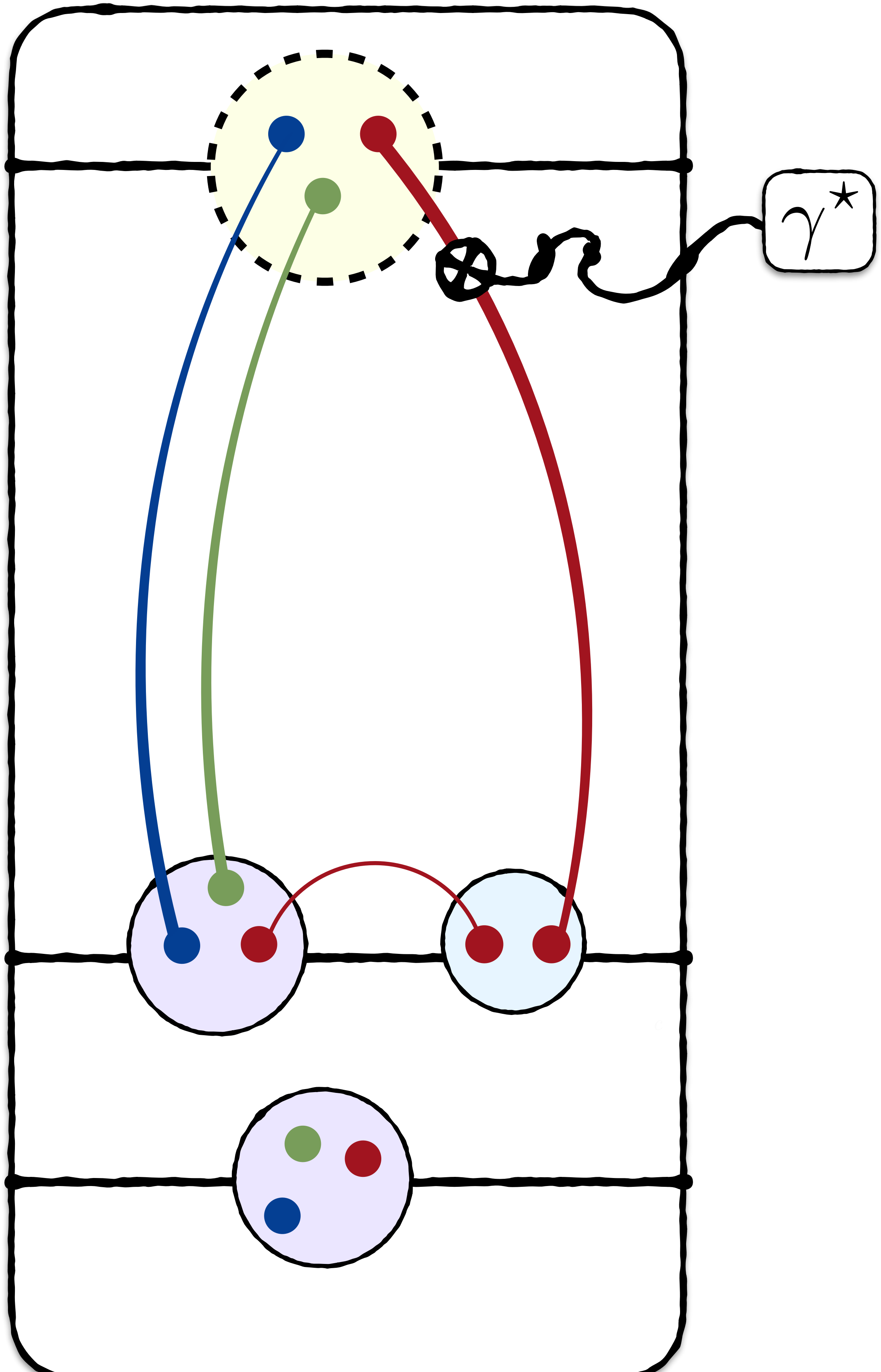

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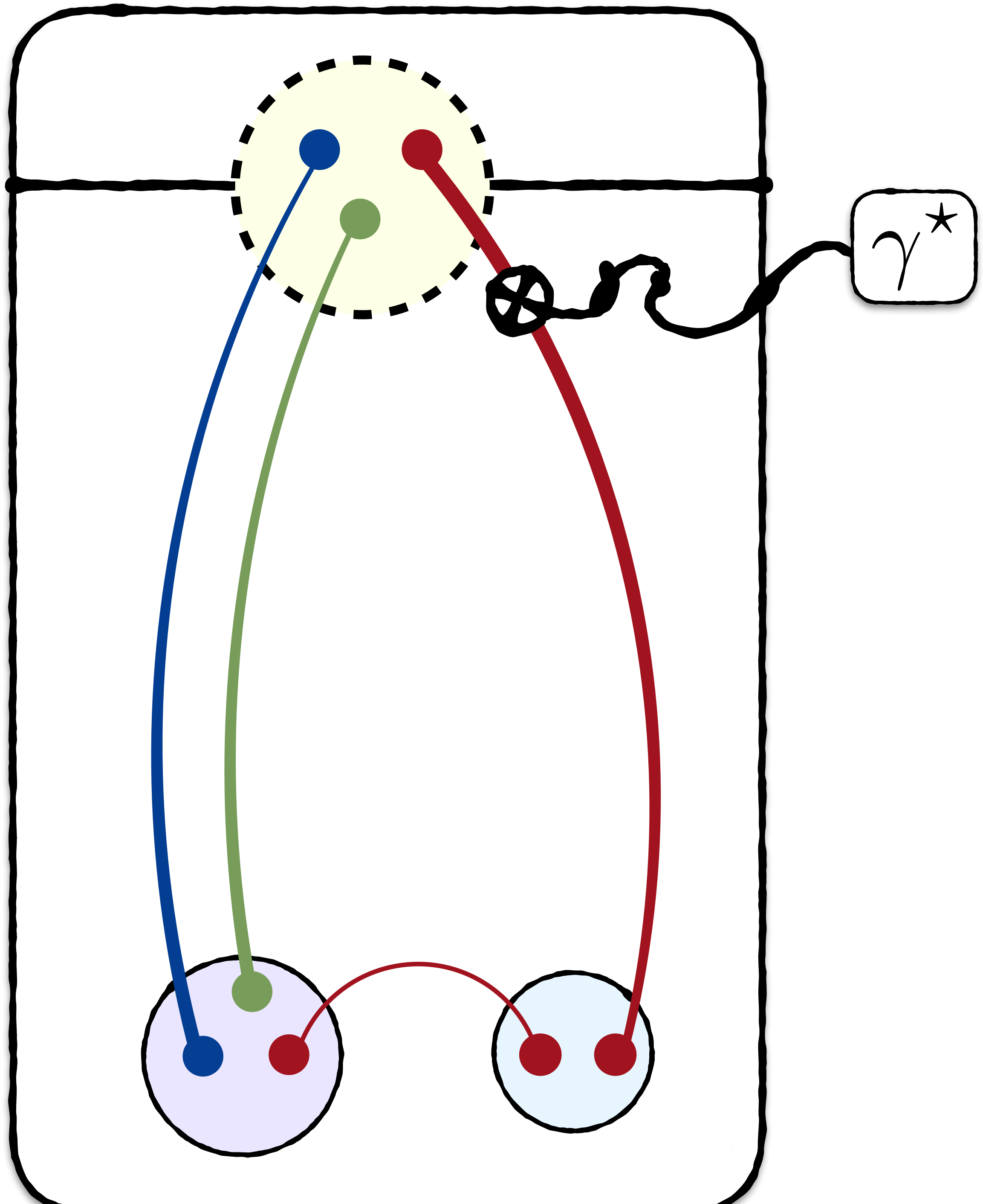
- Elastic form factors?



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demand for lattice:

- Stable states generated *"exactly"*
- Resonant/non-resonant amplitudes are generated *"exactly"*
- QED/weak can be introduced perturb. or non-perturb.

Broad goals

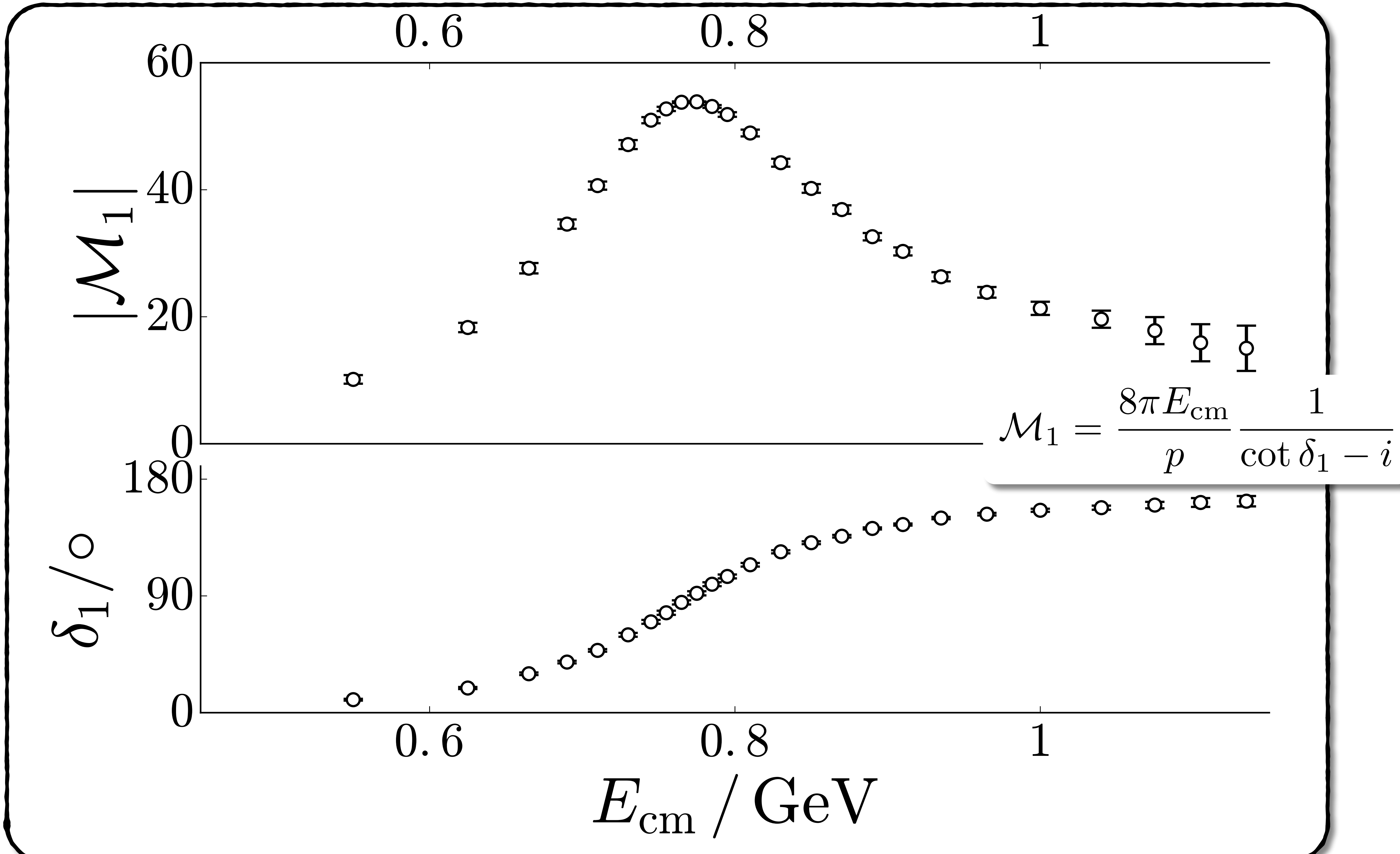
- Strongly coupled **2**-body
- Strongly coupled **2, 3**-body
- Spin-dependent amps.
- Narrow resonances
- Broad resonances
- Photo-, electro-production
- Transition form factors
- Elastic form factors

Broad goals

	formalism	numerical
Strongly coupled **2**-body	☐	☐
Strongly coupled **2, 3**-body	☐	☐
Spin-dependent amps.	☐	☐
Narrow resonances	☐	☐
Broad resonances	☐	☐
Photo-, electro-production	☐	☐
Transition form factors	☐	☐
Elastic form factors	☐	☐

A pseudo-quantitative definition

(bump in cross sections/amplitude - e.g., ππ scattering in ϱ-channel)



Protopopescu et al. (1972)

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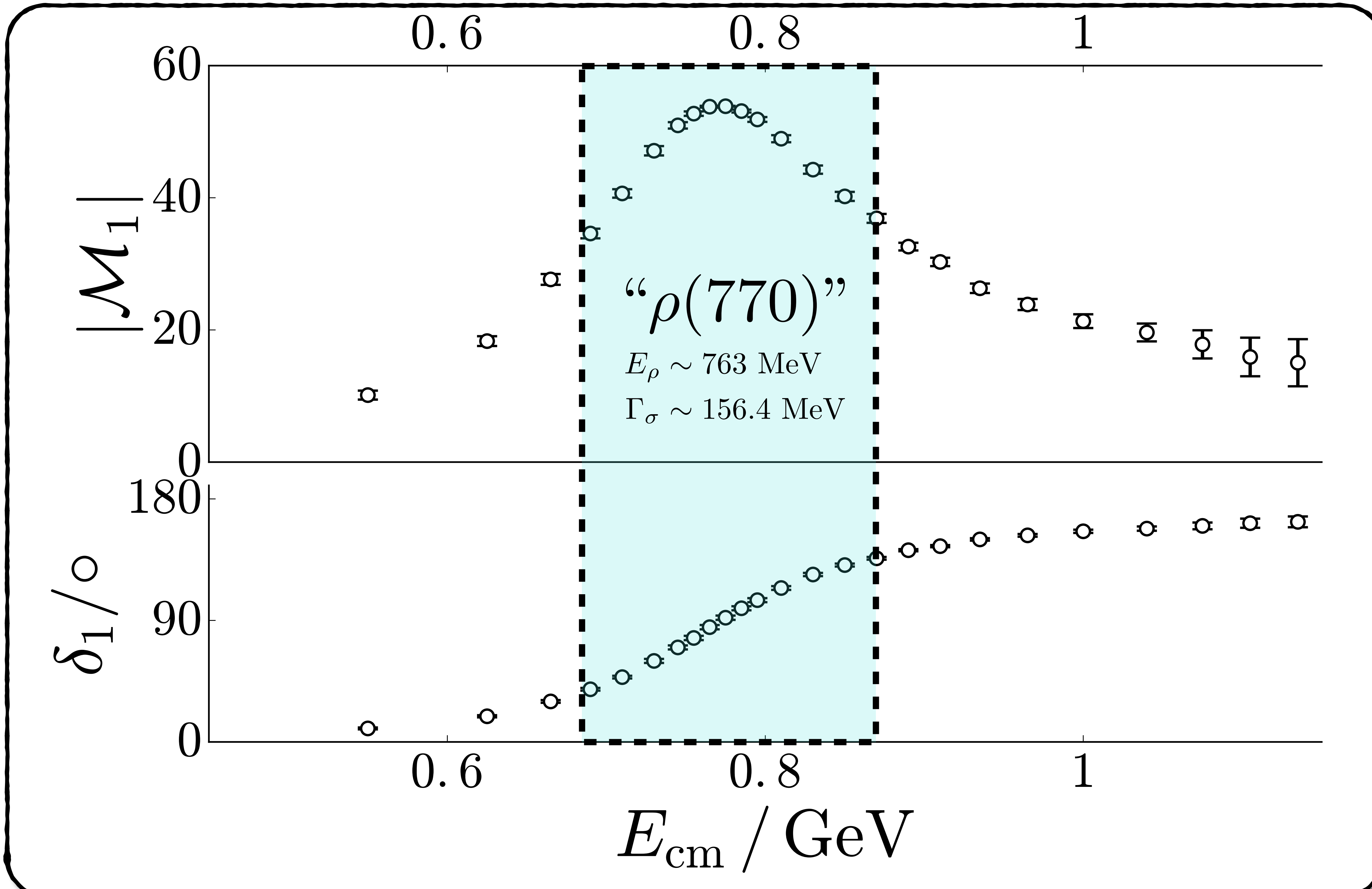
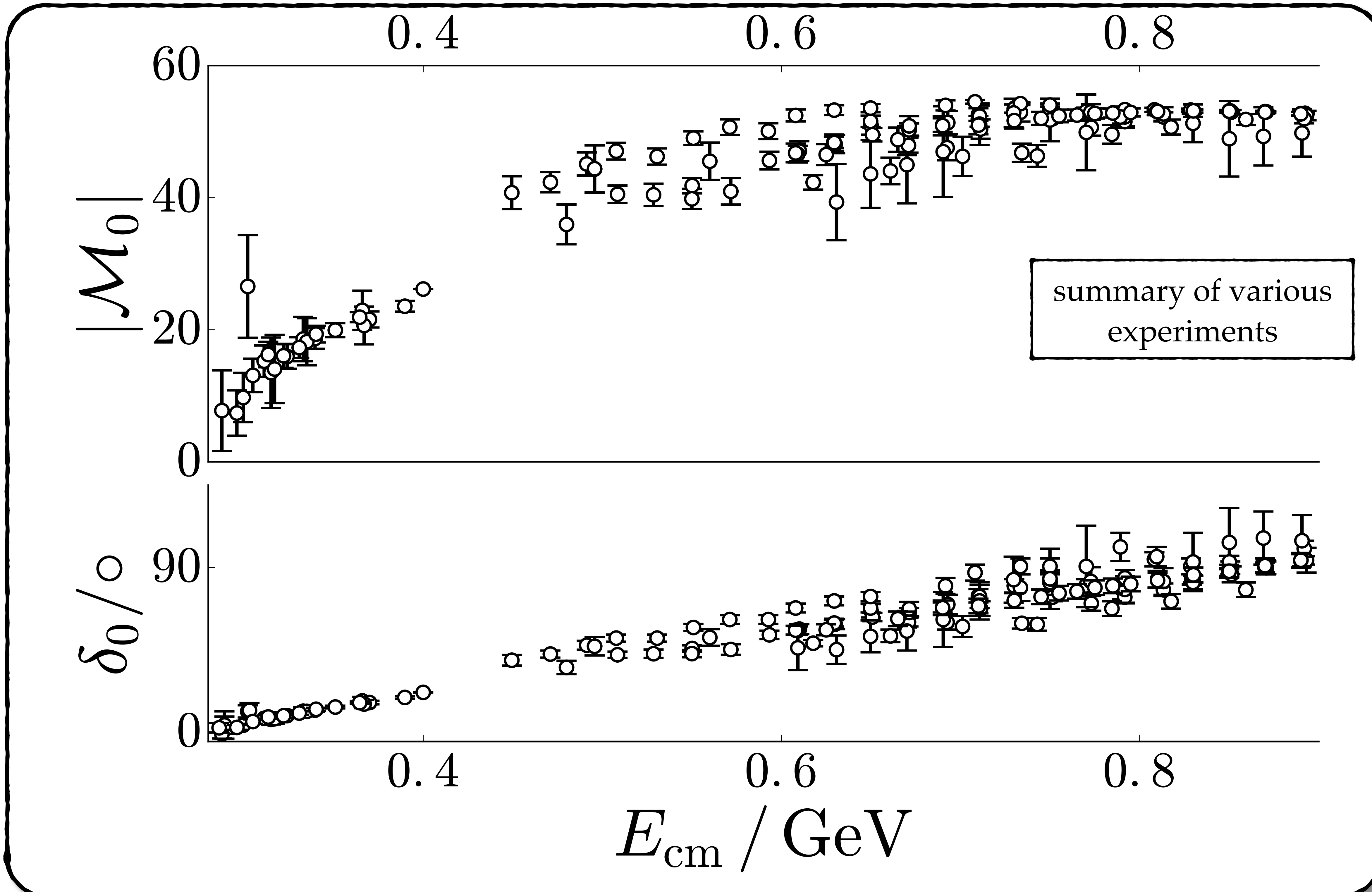


Protopopescu et al. (1972)

A counter example

(Isoscalar, scalar $\pi\pi$ scattering)



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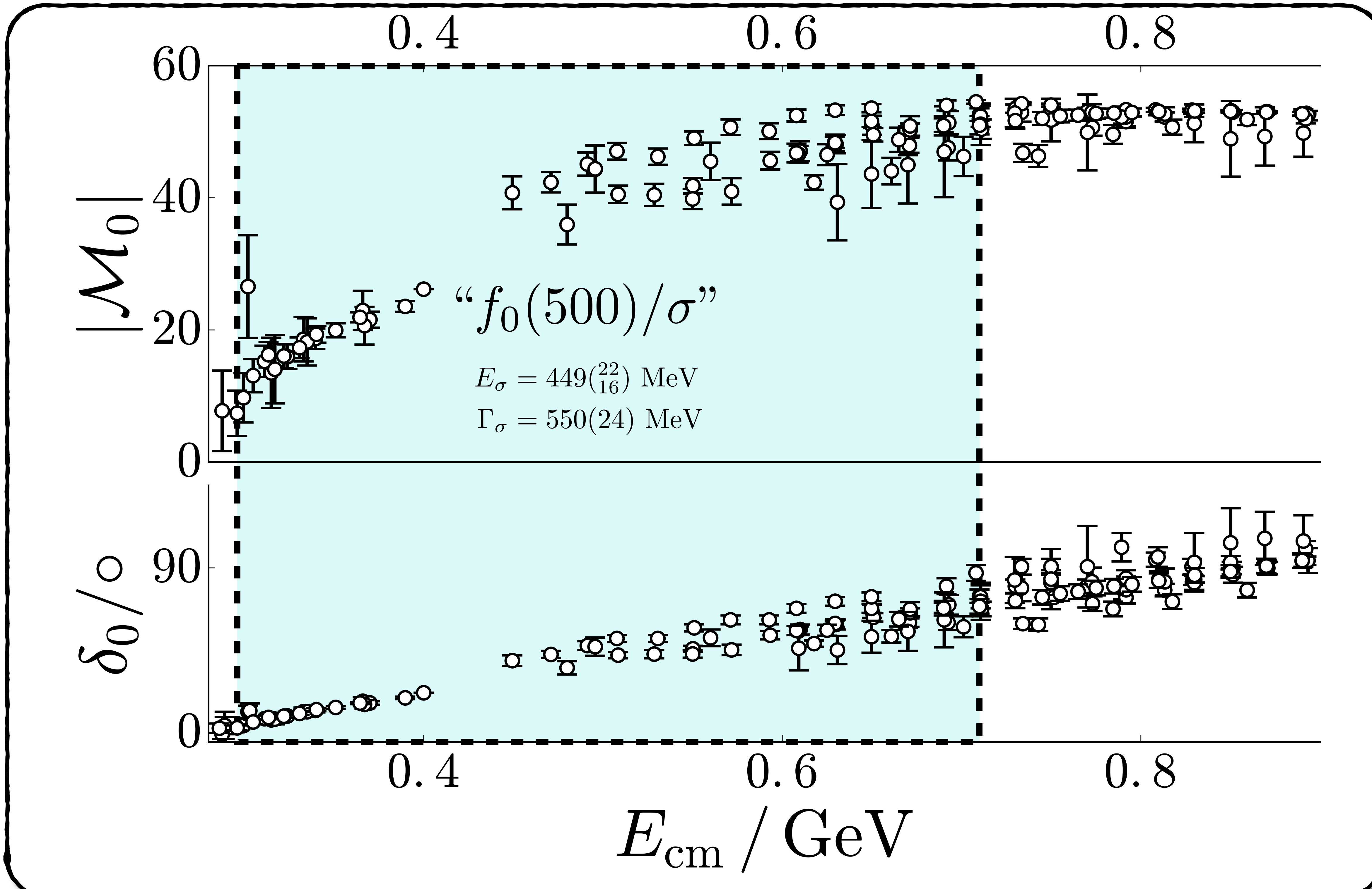
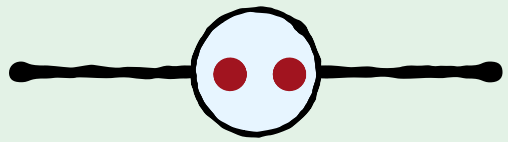


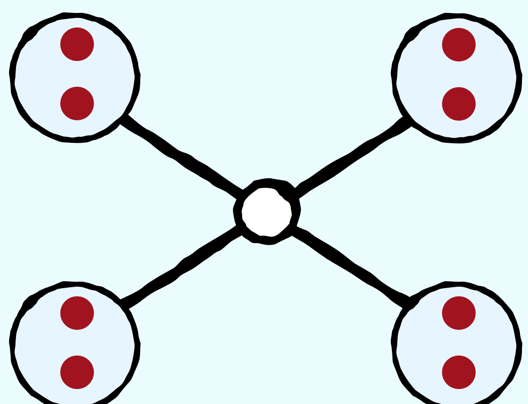
Quantitative definition

propagator:



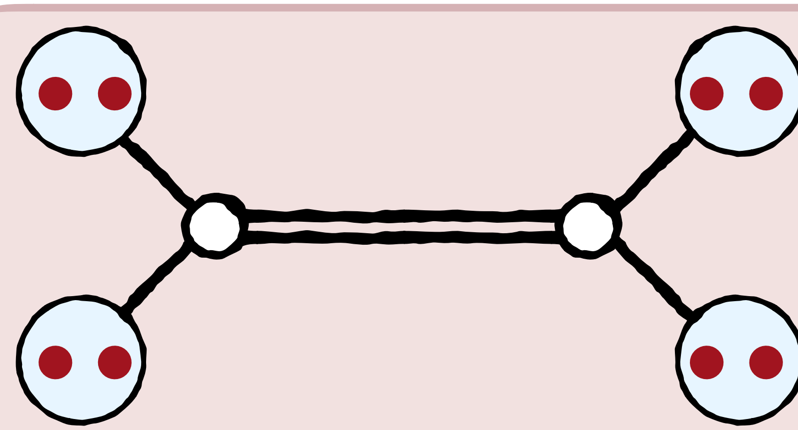
$$\sim \frac{iZ}{p^2 - m^2} = \frac{iZ}{s - m^2}$$

scattering amplitude:



$$= i\mathcal{M}$$

…near bound state or resonance:

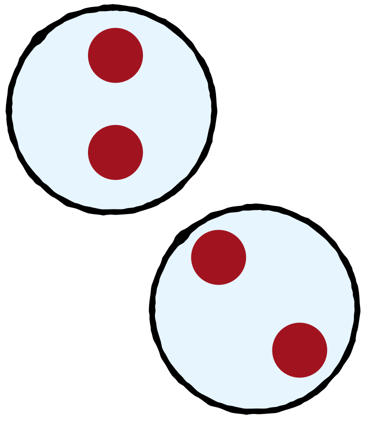


$$\sim \frac{ig^2}{s - s_0}, \quad s_0 = (E_0 - \tfrac{i}{2}\Gamma)^2$$

Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monter Carlo sampling

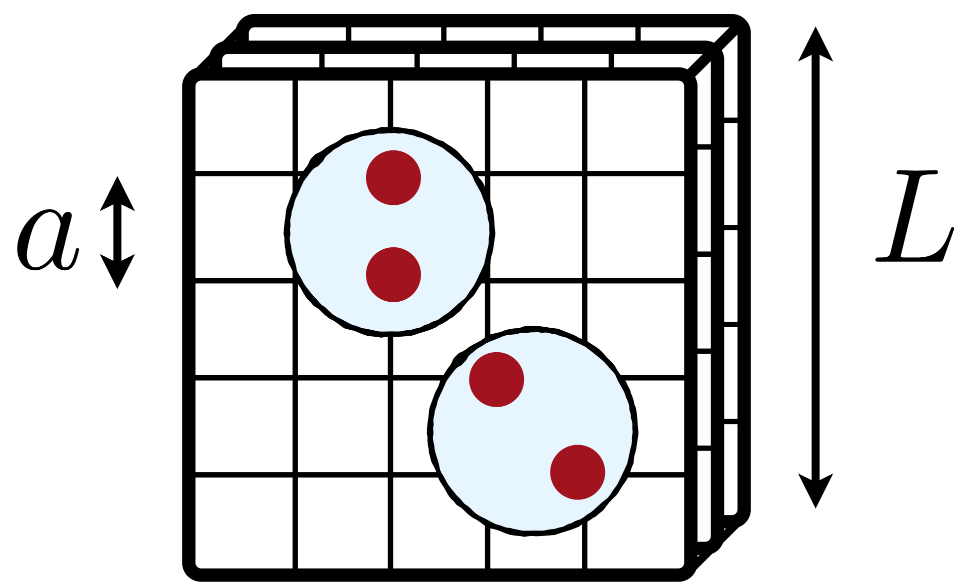
$$\int \mathcal{D}U\,\mathcal{D}q\,\mathcal{D}\overline{q}\;\; e^{iS_M} \to \int \mathcal{D}U\,\mathcal{D}q\,\mathcal{D}\overline{q}\;\; e^{-S_E}$$



Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monter Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume

$$D_\mu = \left(\qquad \right) \updownarrow (L/a)^3 \times (T/a)$$



Never free!

No asymptotic states!

No scattering!

Lattice QCD

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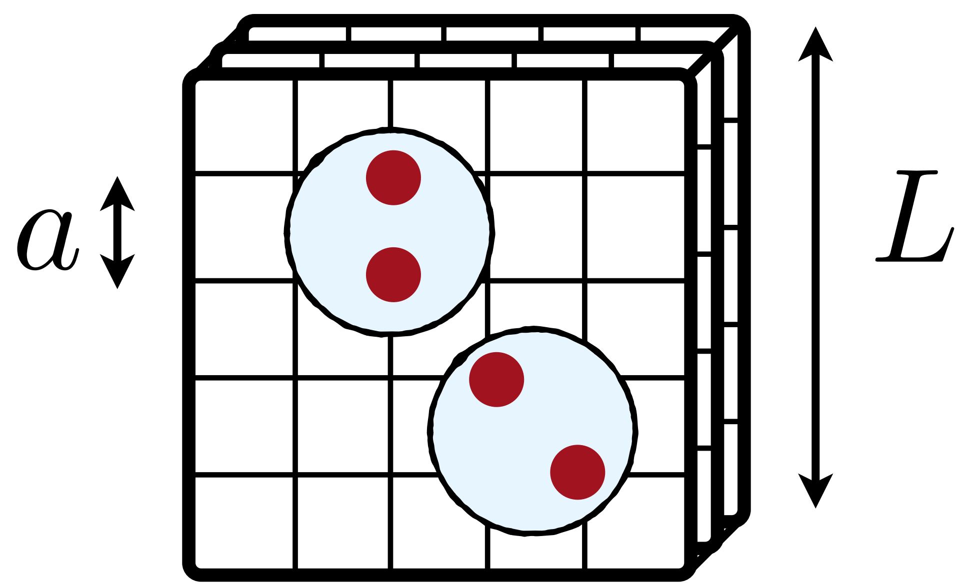
- quark masses: $m_q \to m_q^{\text{phys.}}$



Advantage over experiment!

Lattice QCD

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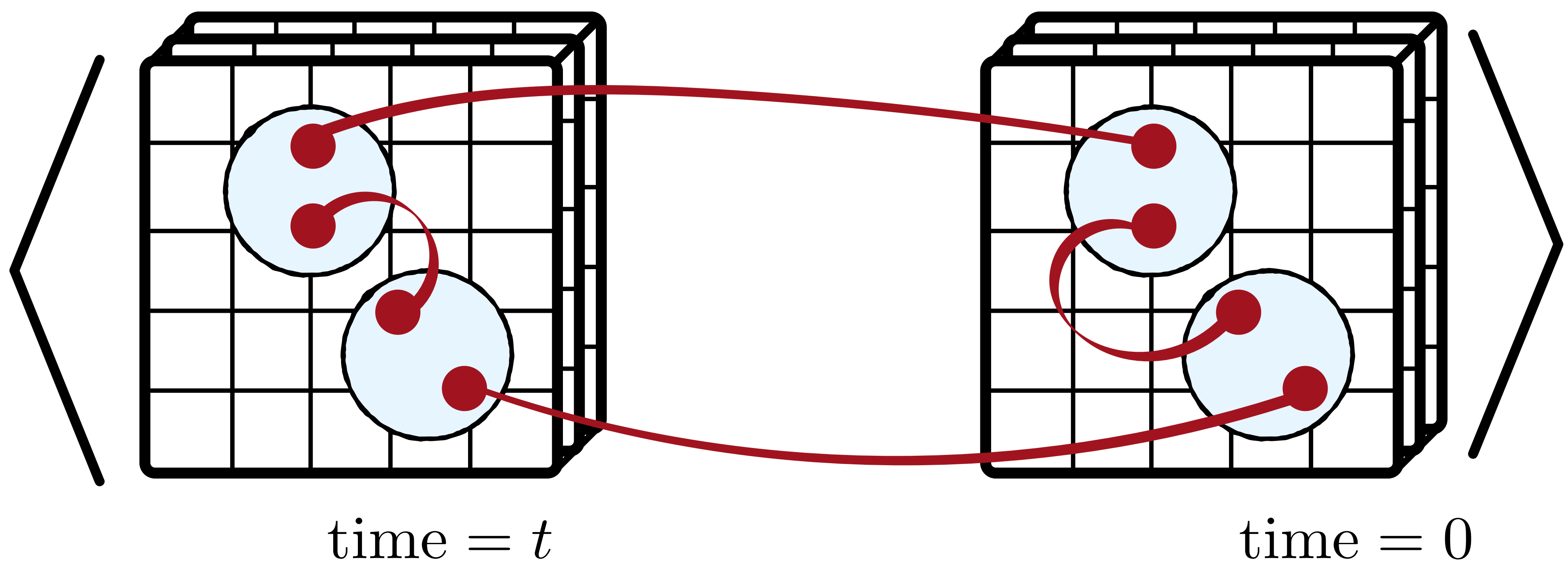
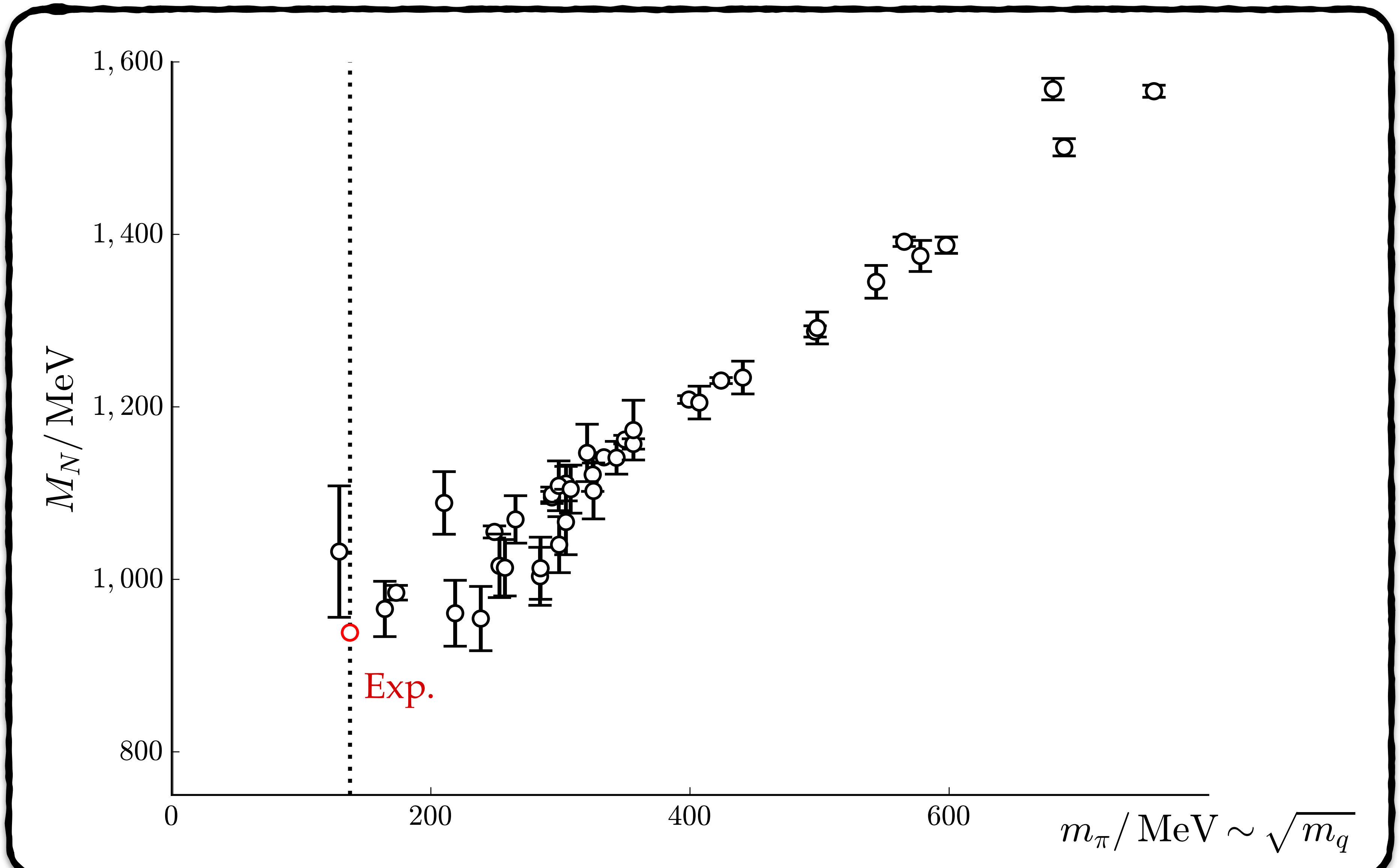
- Monter Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume
- quark masses: $m_q \to m_q^{\text{phys.}}$
- Correlation functions: spectrum, matrix elements

$C_L(t, \mathbf{P}) =$



Status of LQCD

- *Simple* properties of QCD stable states [non-composite states]
  - physical or lighter quark masses [down to m_π~120 MeV] ✓
  - non-degenerate light-quark masses: N_f=1+1+1+1 ✓
  - dynamical QED ✓



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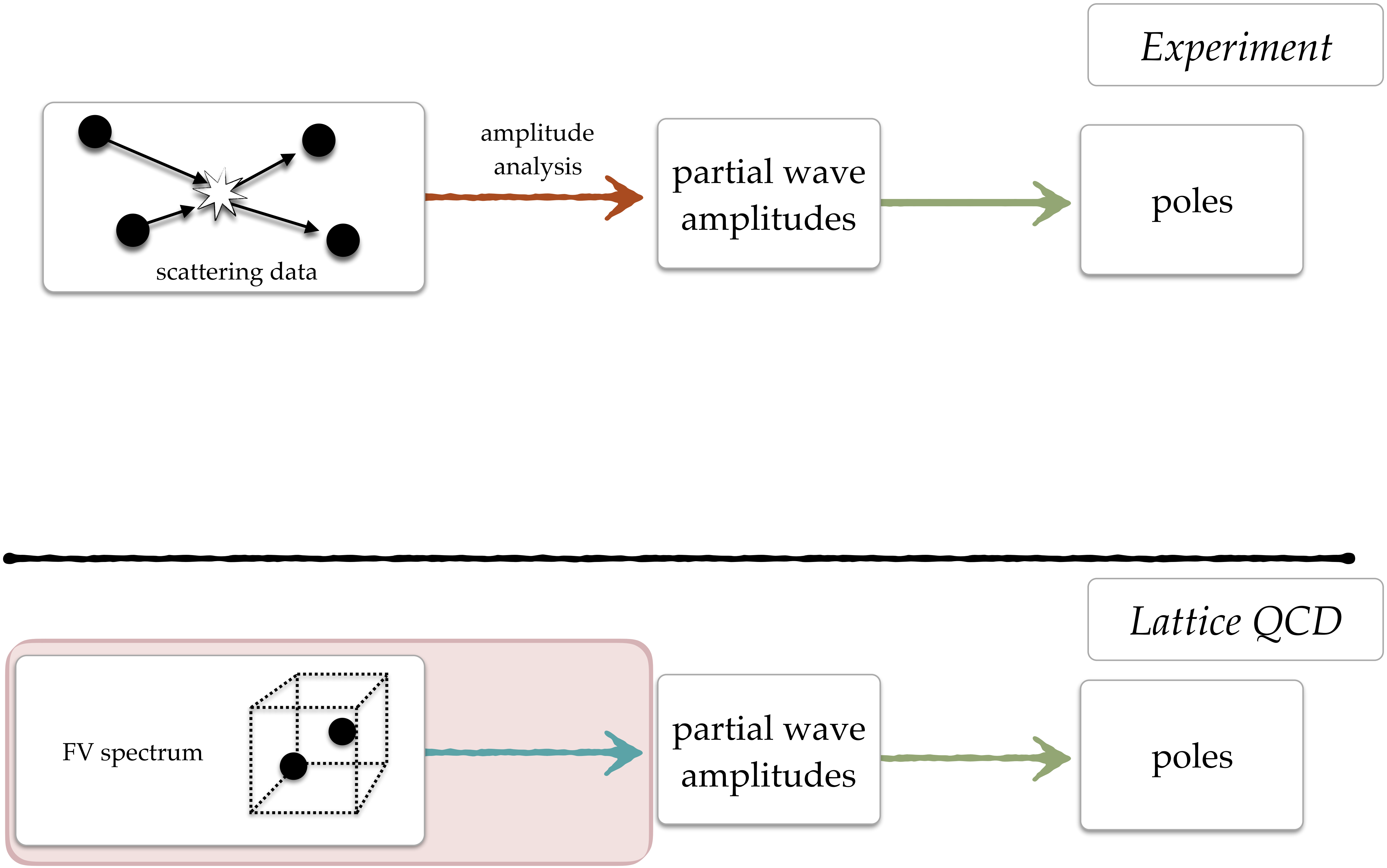
  - physical or lighter quark masses [down to m_π~120 MeV] ✓
  - non-degenerate light-quark masses: N_f=1+1+1+1 ✓
  - dynamical QED ✓
- Frontier of lattice: multi-particle physics
  - scattering / reactions
  - composite states
    - bound states
    - hadronic resonances

Formal development:
- under way
- more needed

Benchmark calculations:
- exploratory
- proof of principle
- unphysical quark masses [m_π=236, 391 MeV]
- …



RB, Hansen, Sharpe - arXiv:1701.07465 [hep-lat] (2017)
RB - Phys.Rev. D89 (2014) no.7, 074507.
RB, Davoudi - Phys.Rev. D87 (2013) no.9, 094507.
RB, Davoudi - Phys.Rev. D88 (2013) no.9, 094507.

Two-point functions

$$C_L(t,\mathbf{P}) \equiv \int_L d\mathbf{x} \int_L d\mathbf{y}\ e^{-i\mathbf{P}\cdot(\mathbf{x}-\mathbf{y})} \langle 0|T\mathcal{A}(t,\mathbf{x})\mathcal{B}^\dagger(0,\mathbf{y})|0\rangle$$

Dispersive representation:

$$C_L(t,\mathbf{P}) = \int_L d\mathbf{x} \int_L d\mathbf{y}\ e^{-i\mathbf{P}\cdot(\mathbf{x}-\mathbf{y})} \sum_n \langle 0|\mathcal{A}(t,\mathbf{x})|n,L\rangle\langle n,L|\mathcal{B}^\dagger(0,\mathbf{y})|0\rangle$$
$$= L^6 \sum_n e^{-E_n t}\langle 0|\mathcal{A}(0)|n,L\rangle\langle n,L|\mathcal{B}^\dagger(0)|0\rangle$$

Diagrammatic representation:

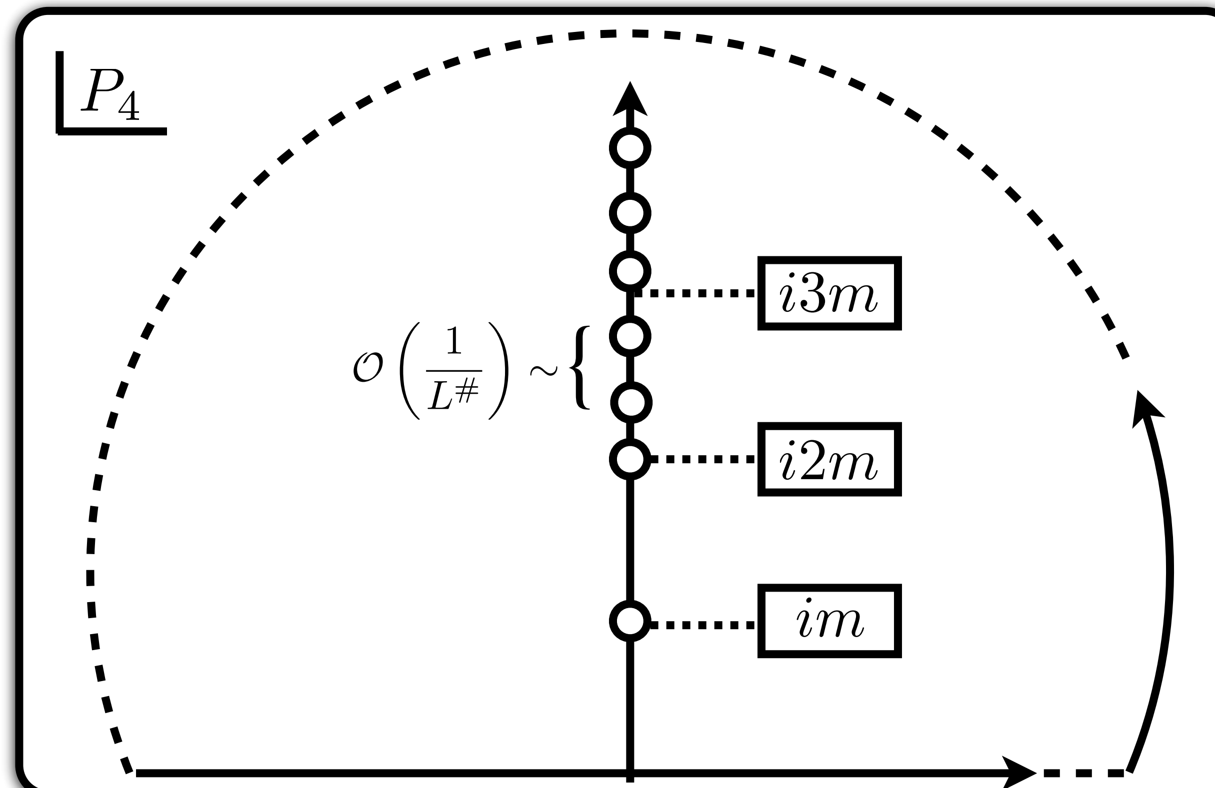
$$C_L(t,\mathbf{P}) \equiv L^3 \int \frac{dP_4}{2\pi} e^{iP_4 t}\, C_L(P)$$

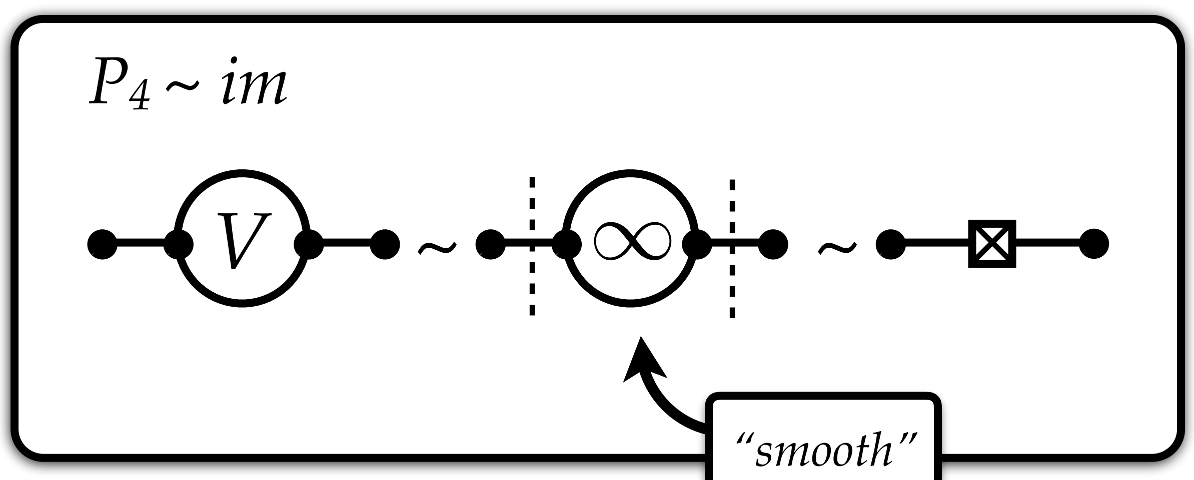
$C_L(P) =$ sum over all finite volume, momentum space Feynman diagram

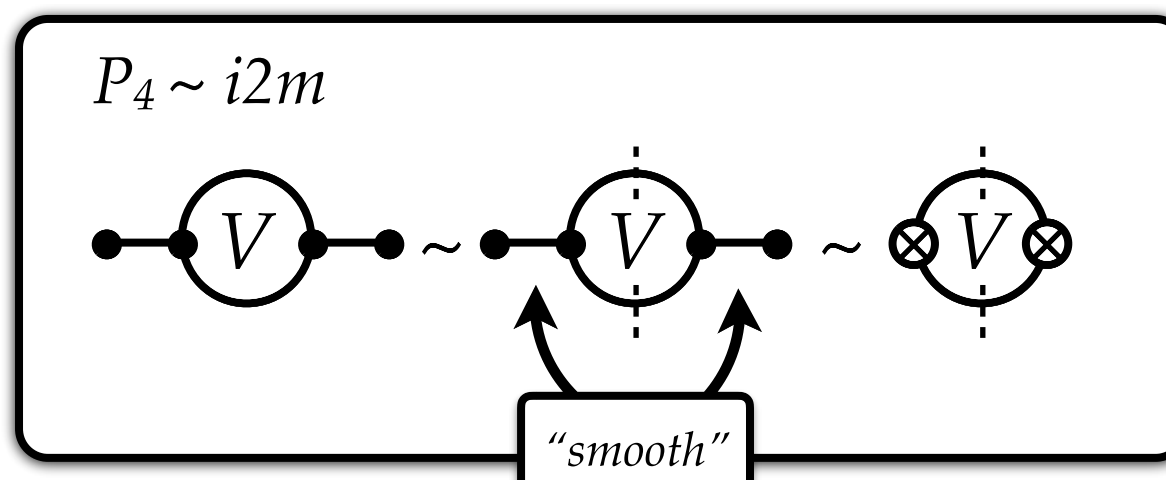
Diagrammatic representation

$$C_L(t,\mathbf{P}) \equiv L^3 \int \frac{dP_4}{2\pi} e^{iP_4 t}\, C_L(P)$$

1. Euclidean
2. Tower of poles
3. Importance of diagram depends on P_4
4. On-shell states:
   - Propagate
   - Infinite-volume: imaginary contribution
   - Finite-volume: power-law effects



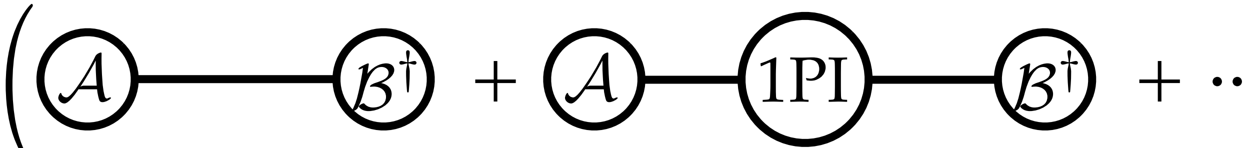




One-particle systems

Consider $P_4 \sim im$:

$$C_L(t, \mathbf{P}) \equiv L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t} \Big(\quad \Big)$$



$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t} \frac{A \ z(P) \ B^\dagger}{p^2 + m^2} + \cdots$$

$$= \frac{L^3 A \ B^\dagger}{2\sqrt{\mathbf{P}^2 + m^2}} e^{-t\sqrt{\mathbf{P}^2 + m^2}} + \cdots$$

Equating this to the dispersive representation:

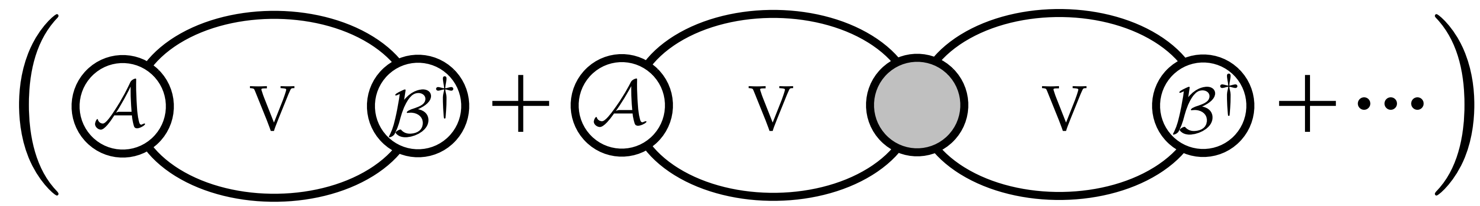
$$E_0 = \sqrt{\mathbf{P}^2 + m^2}$$

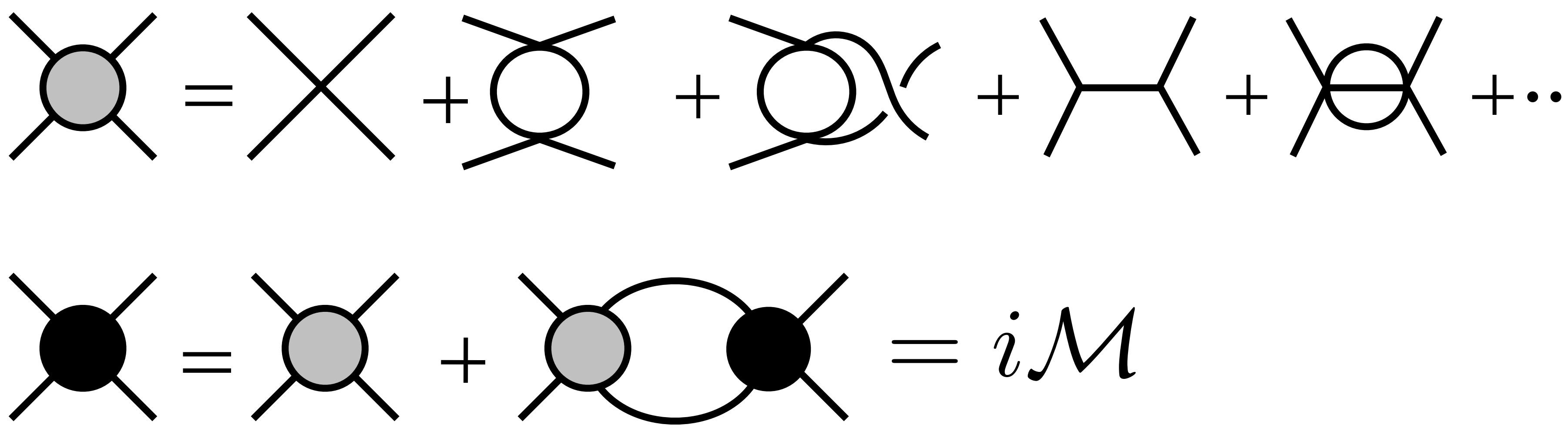
$$\langle 0 | \mathcal{A}(0) | E_0, L \rangle \langle E_0, L | \mathcal{B}^\dagger(0) | 0 \rangle = \frac{A \ B^\dagger}{2 L^3 E_0}$$

Conclusion: masses and decay constants of stable states can be reliably extracted!

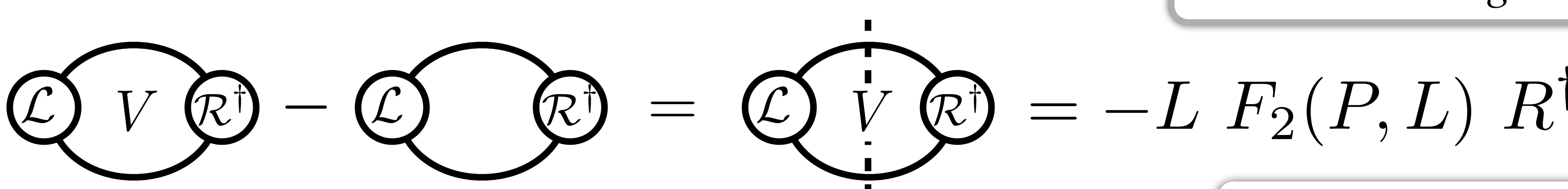
Two-particle systems

Consider $P_4 \sim i2m$:

$L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t}$ 



Matrices in *angular momentum*



Kim, Sachrajda, & Sharpe (2005)
RB (2014)

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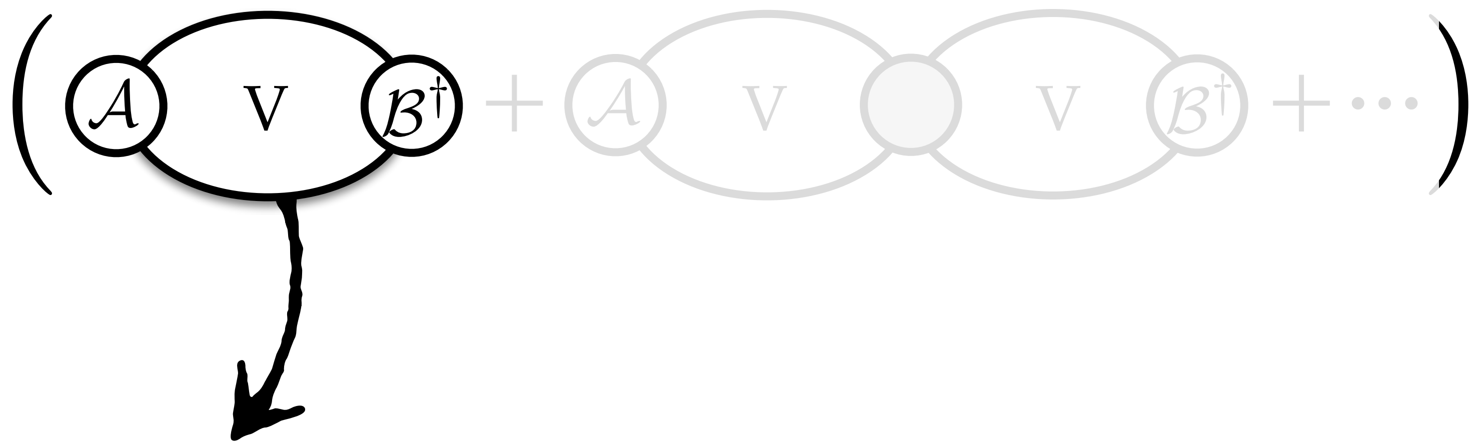

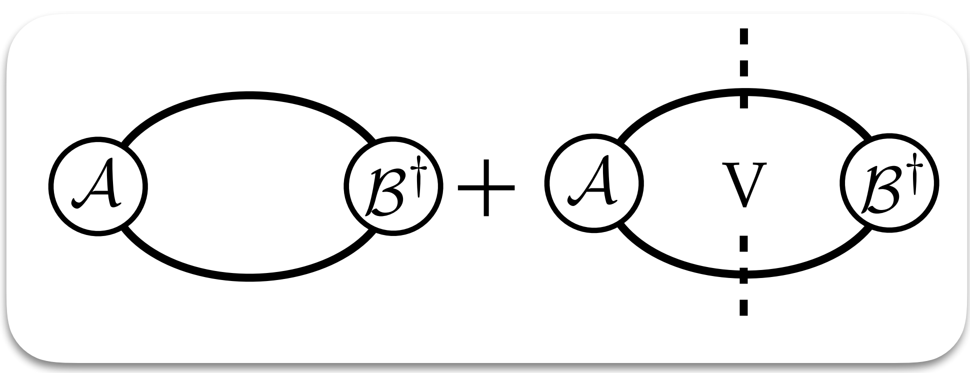

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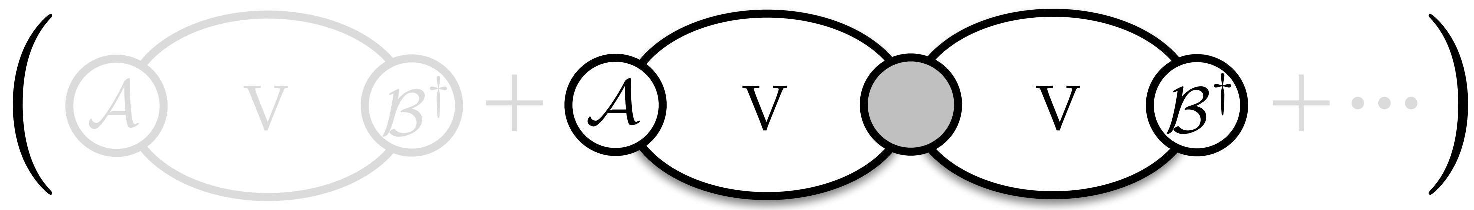

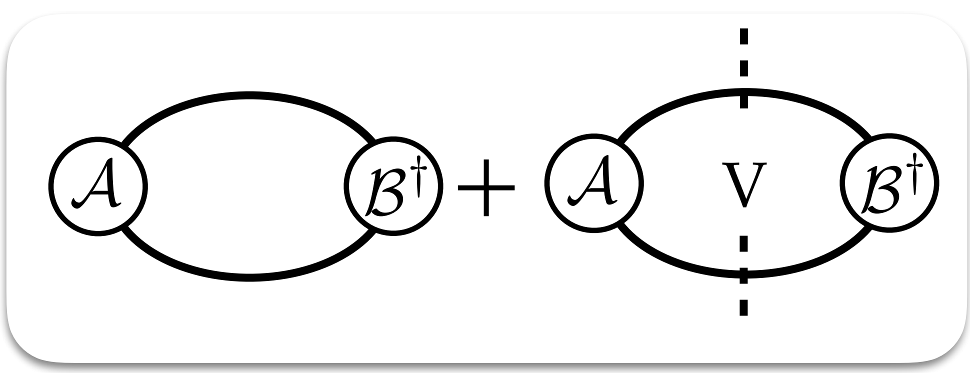

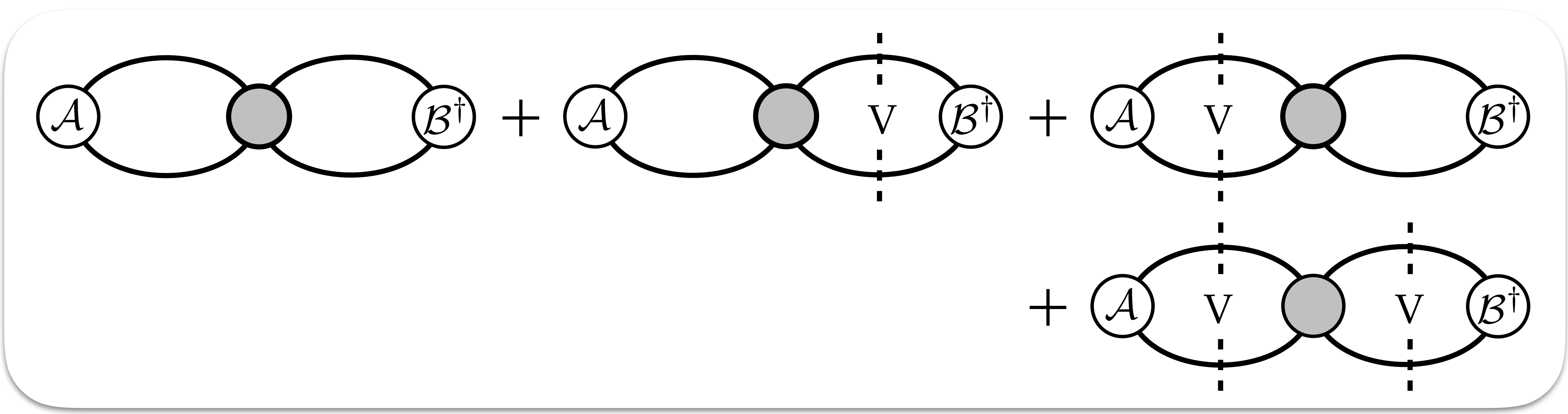

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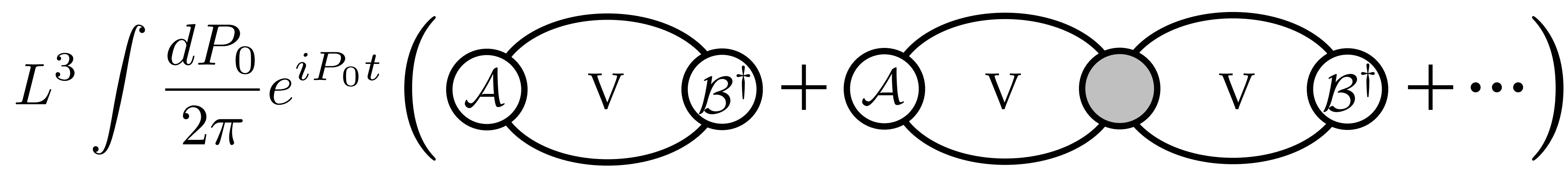
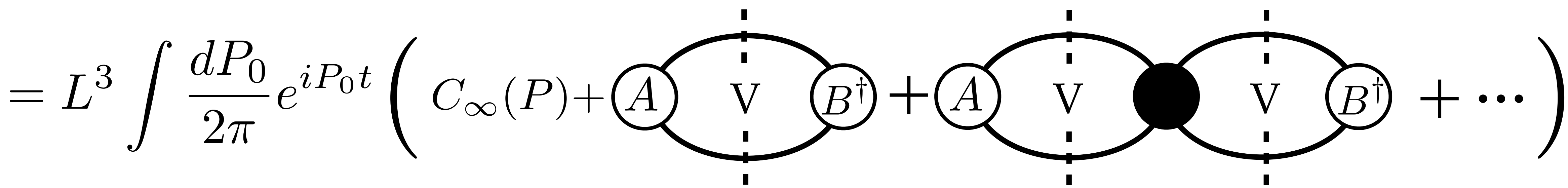
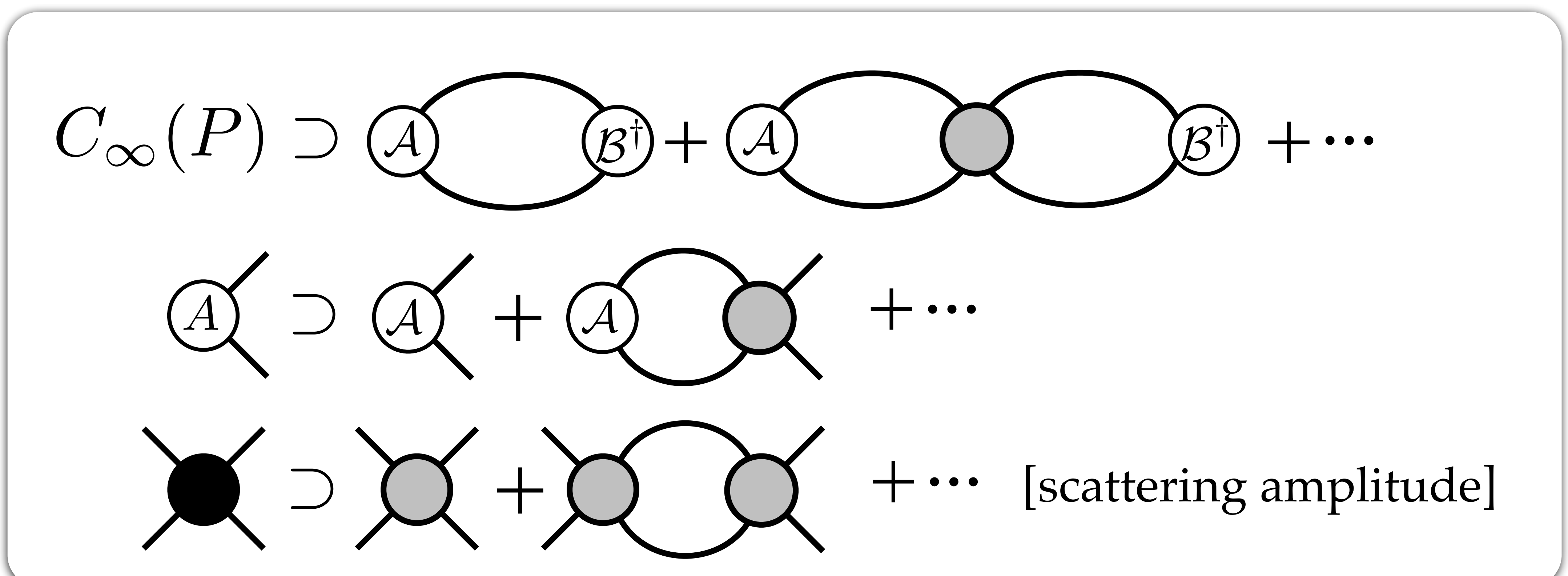


After some massaging:



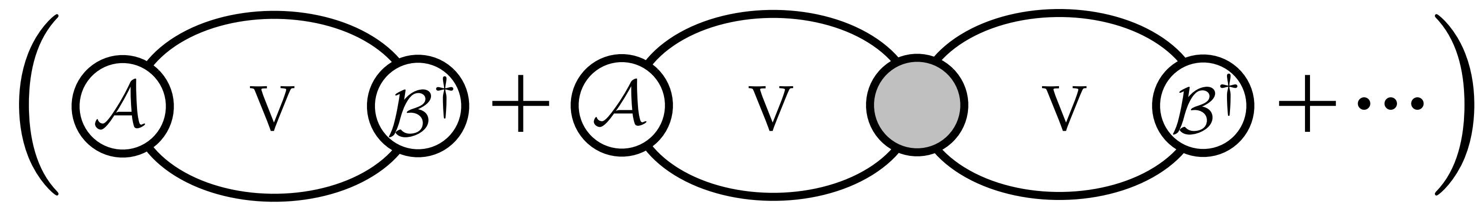
Where,



Two-particle systems

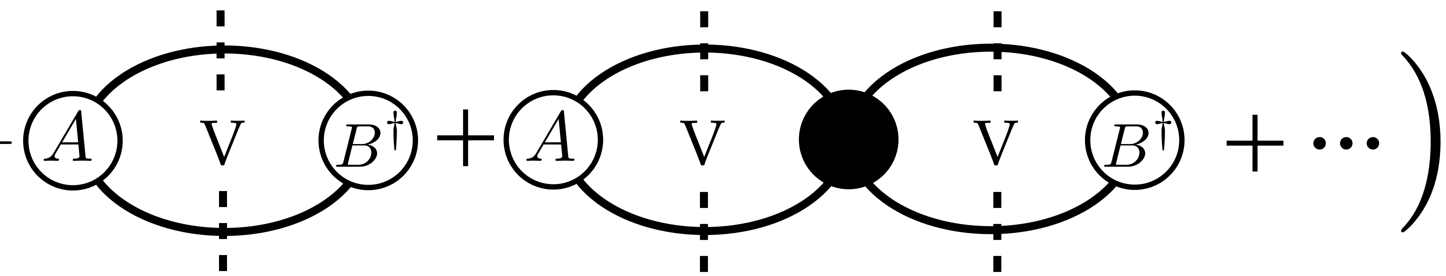
Consider $P_4 \sim i2m$:

$$L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t}$$



After some massaging:

$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t} \Big(C_\infty(P) + \cdots$$



$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t} \left(C_\infty(P) - A(P) \frac{1}{F_2^{-1}(P,L) + \mathcal{M}(P)} B^\dagger(P) \right)$$

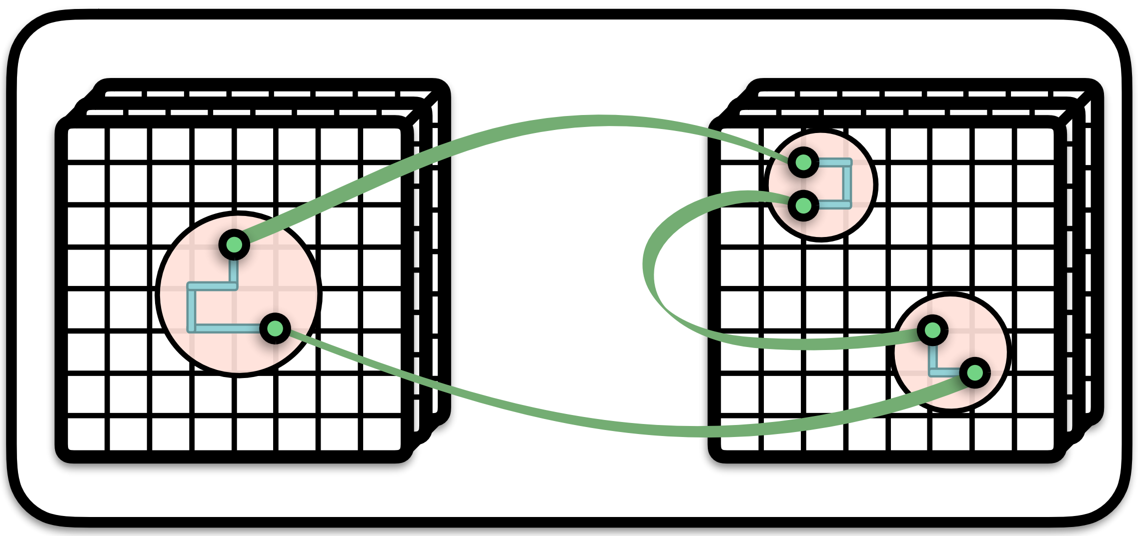
$$= L^3 \sum_n e^{-E_n t} A_n \, \mathcal{R}_n \, B_n^\dagger$$

poles satisfy: $\det[F_2^{-1}(P,L) + \mathcal{M}(P)] = 0$

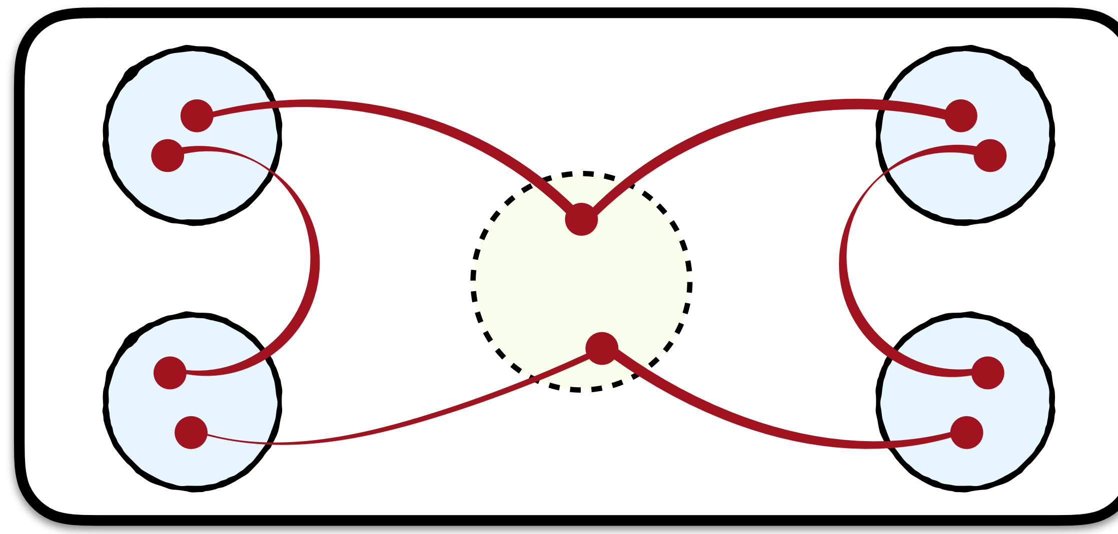
We will come back to the residues later…

Lüscher formalism

$$\det[F_2^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$



finite volume spectrum



scattering amplitude

E_L = finite volume spectrum

L = finite volume

F_2 = known function

$\mathcal{M}$ = scattering amplitude

Lüscher formalism

$$\det[F_2^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Bernard, Lage, Meißner & Rusetsky (2008) [Nπ systems]
- Gockeler, Horsley, et al. (2012) [Nπ systems]
- RB, Davoudi, Luu & Savage (2013) [generic spinning systems]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- **RB (2014) [Most general 2-body result: inelastic, spinning particles]**

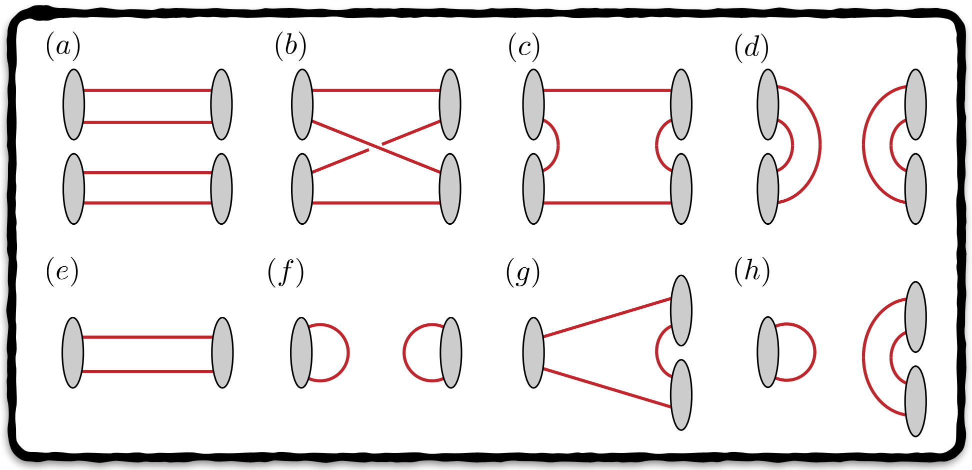
Extracting the spectrum

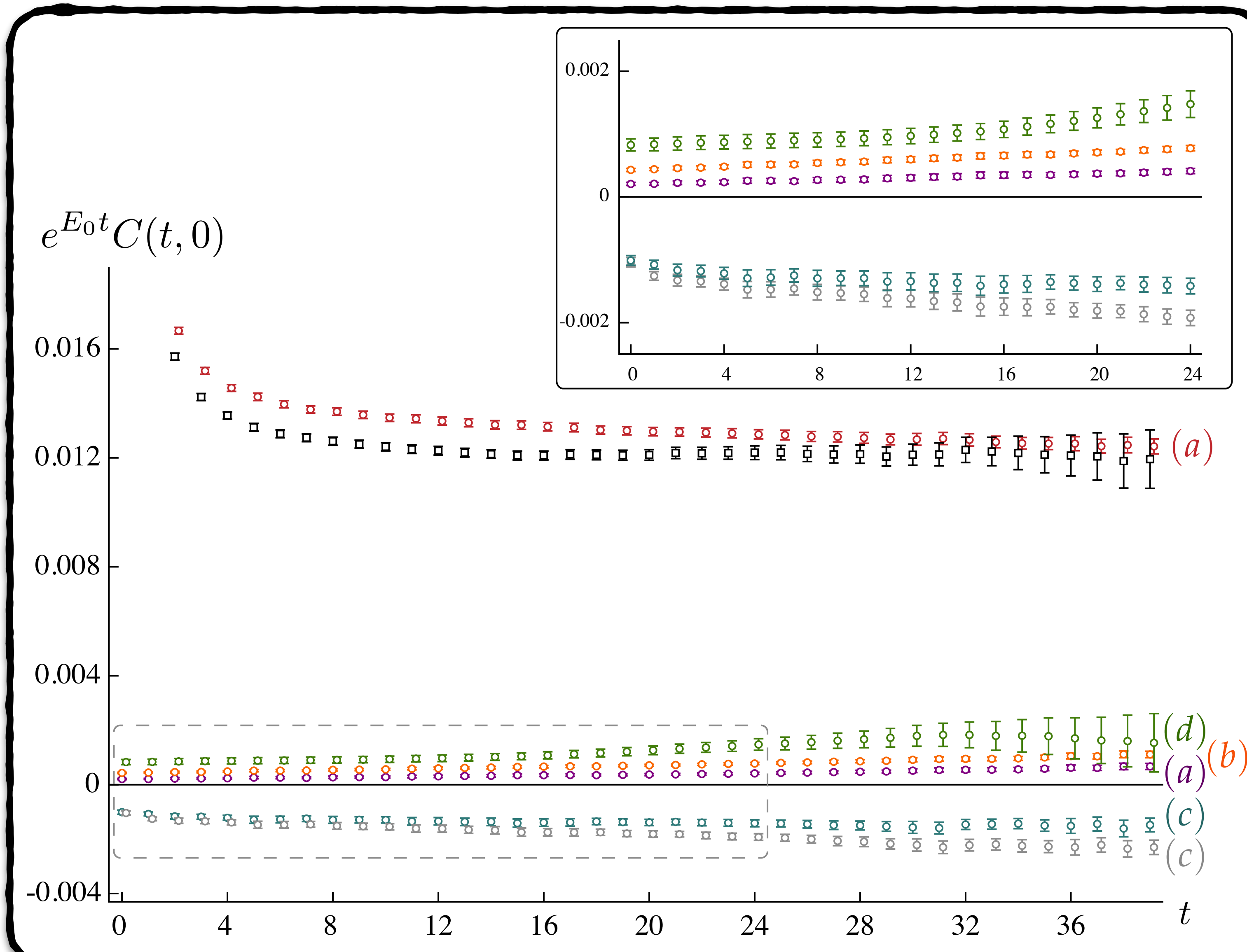
Two-point correlation functions:

$$C^{2pt.}_{ab}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}^\dagger_a(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z^\dagger_{a,n} e^{-E_n t}$$

- Evaluate **all** Wick contraction

e.g. isoscalar: $\pi_{[000]}\pi_{[110]}$, $m_\pi = 236$ MeV





RB, Dudek, Edwards, Wilson - PRL (2017)

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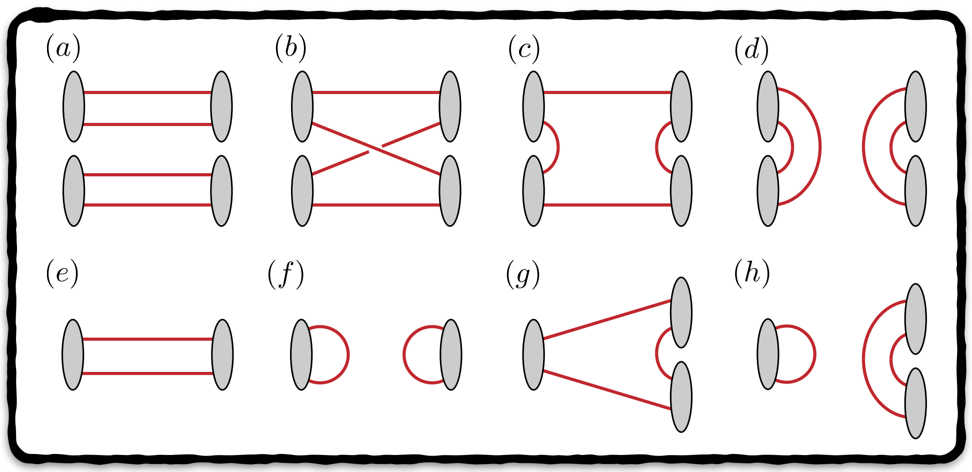

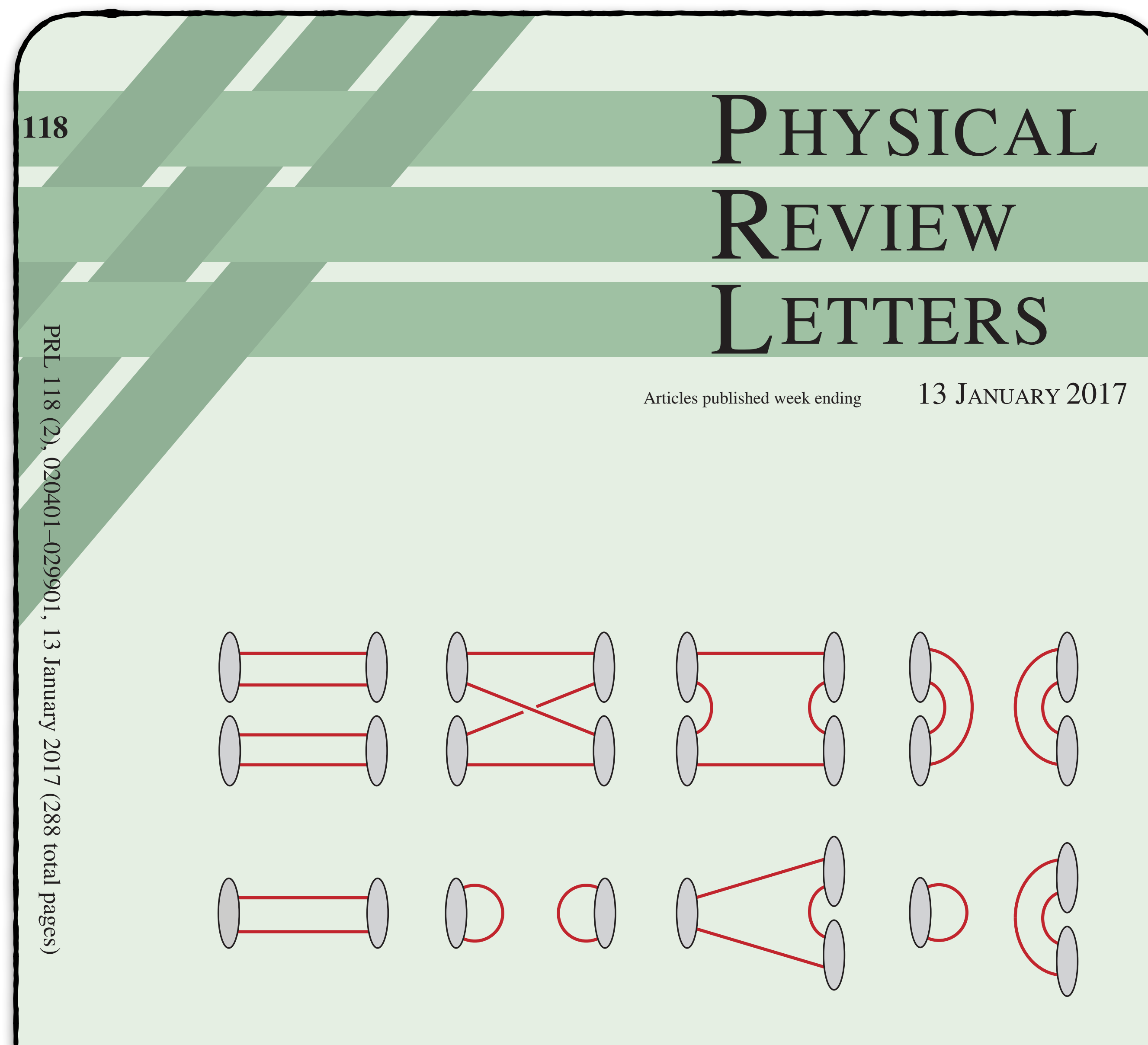

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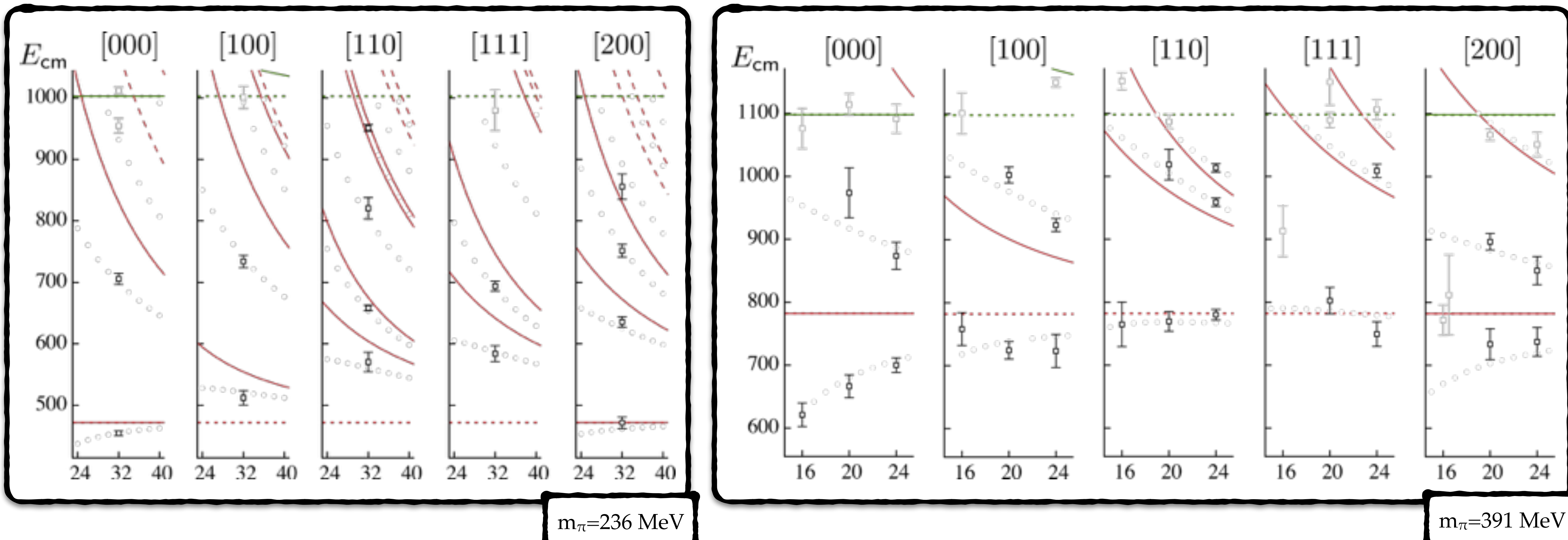
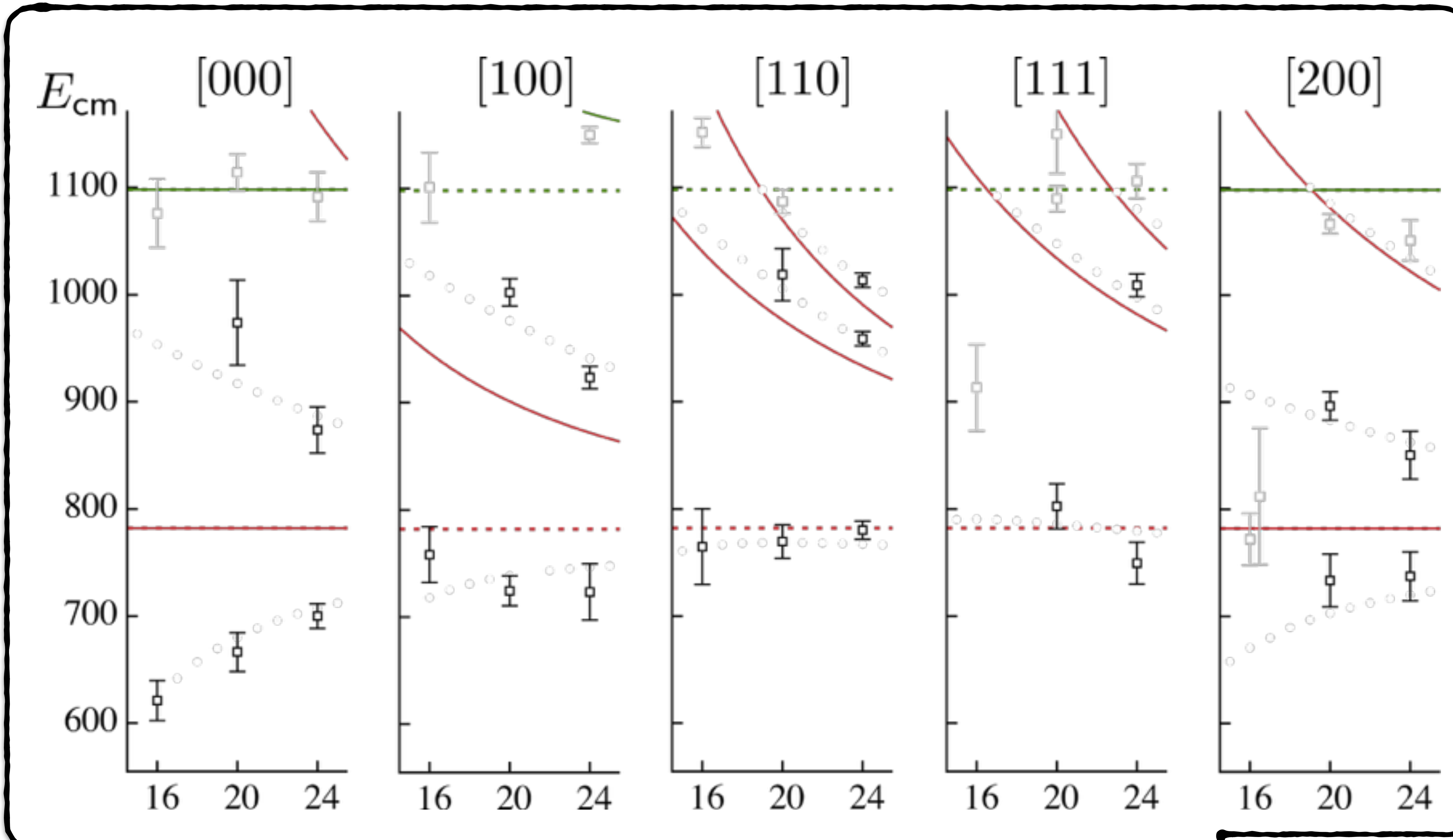
$$C^{2pt.}_{ab}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z^\dagger_{a,n} e^{-E_n t}$$

- Evaluate **all** Wick contraction
- Use a large basis of operators…



RB, Dudek, Edwards, Wilson - PRL (2017)

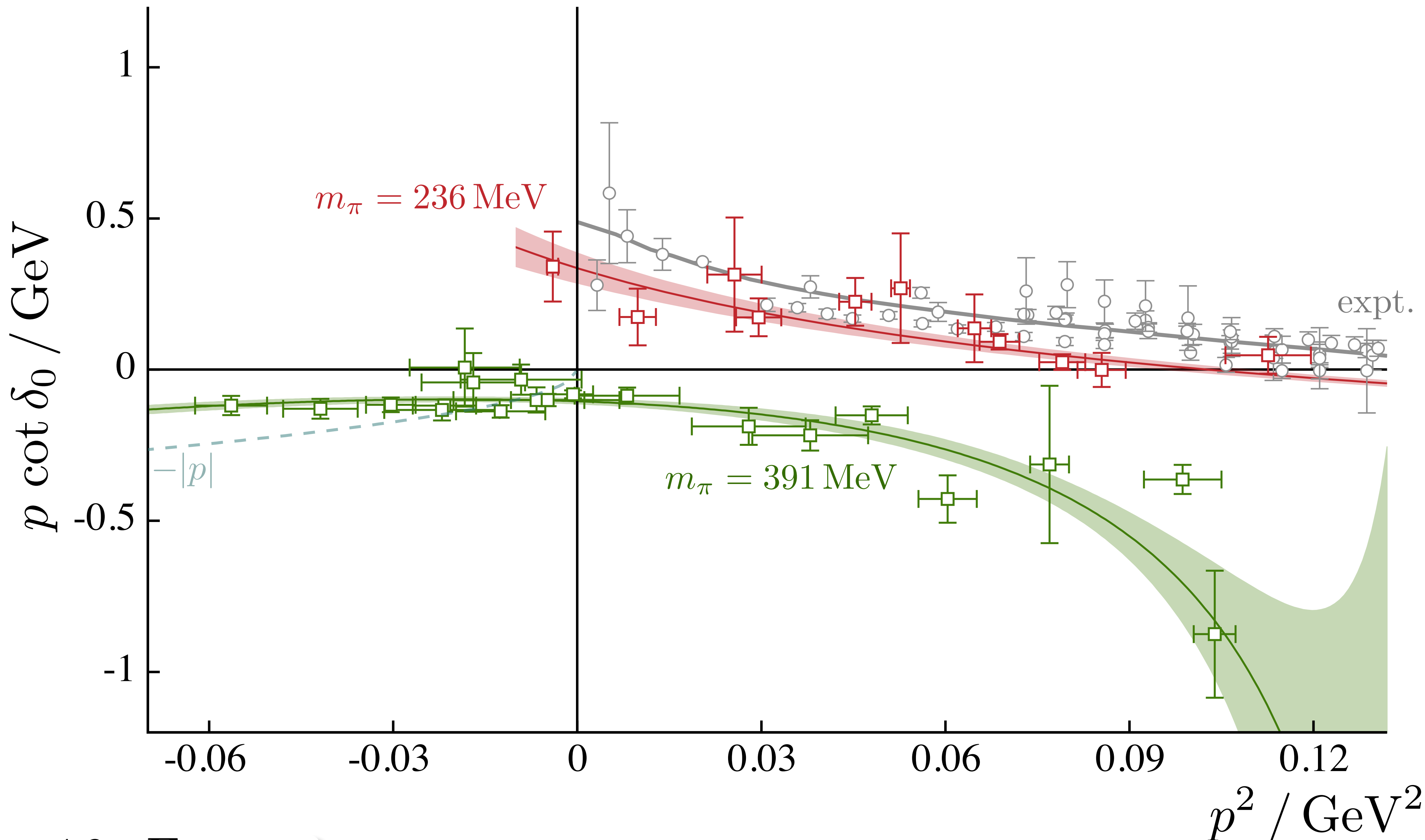
Extracting the spectrum



m_π=391 MeV

RB, Dudek, Edwards, Wilson - PRL (2017)

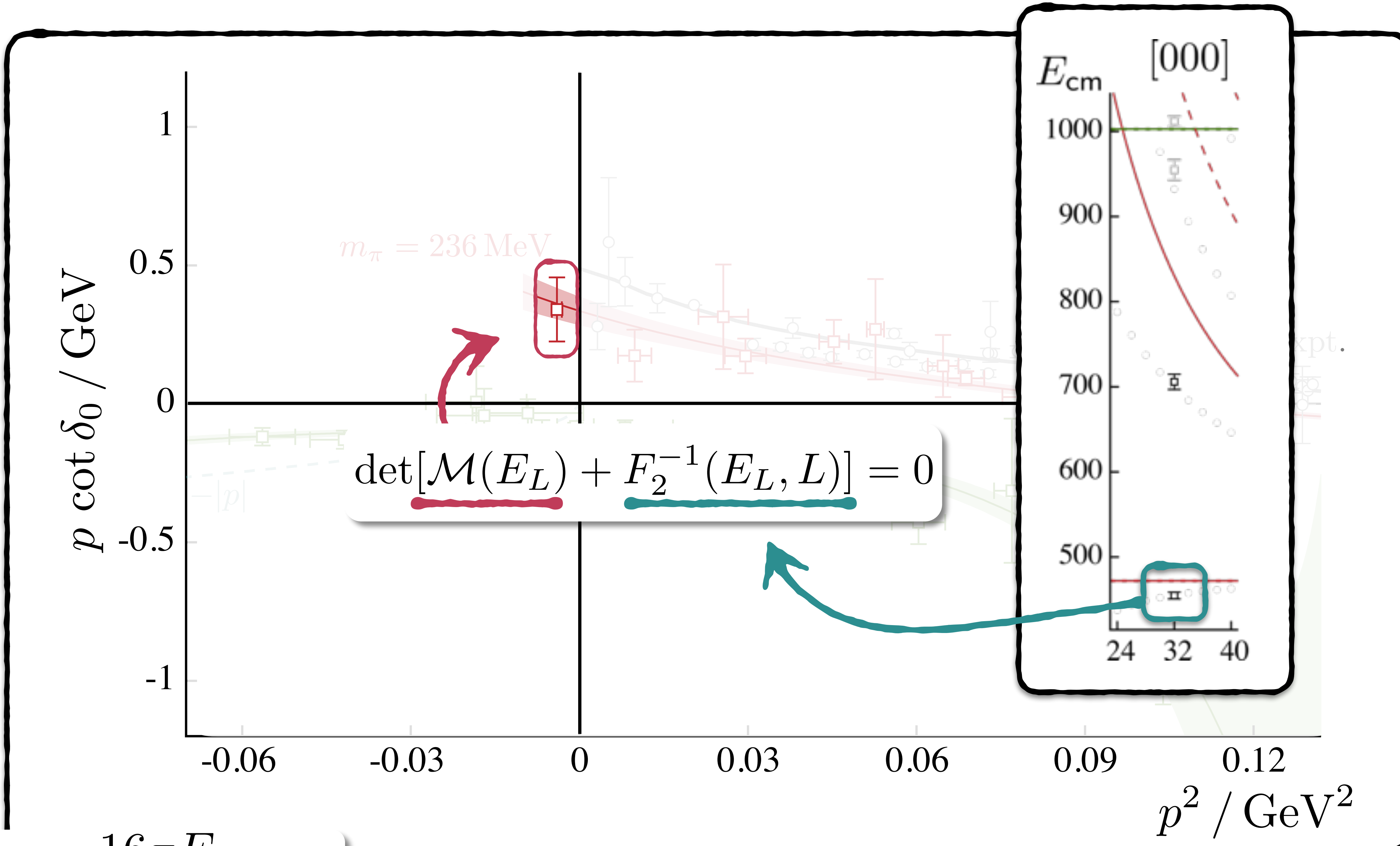
Isoscalar $\pi\pi$ scattering



$$\mathcal{M}_0 = \frac{16\pi E_{\text{cm}}}{p \cot \delta_0 - ip}$$

RB, Dudek, Edwards, Wilson - PRL (2017)

Isoscalar $\pi\pi$ scattering

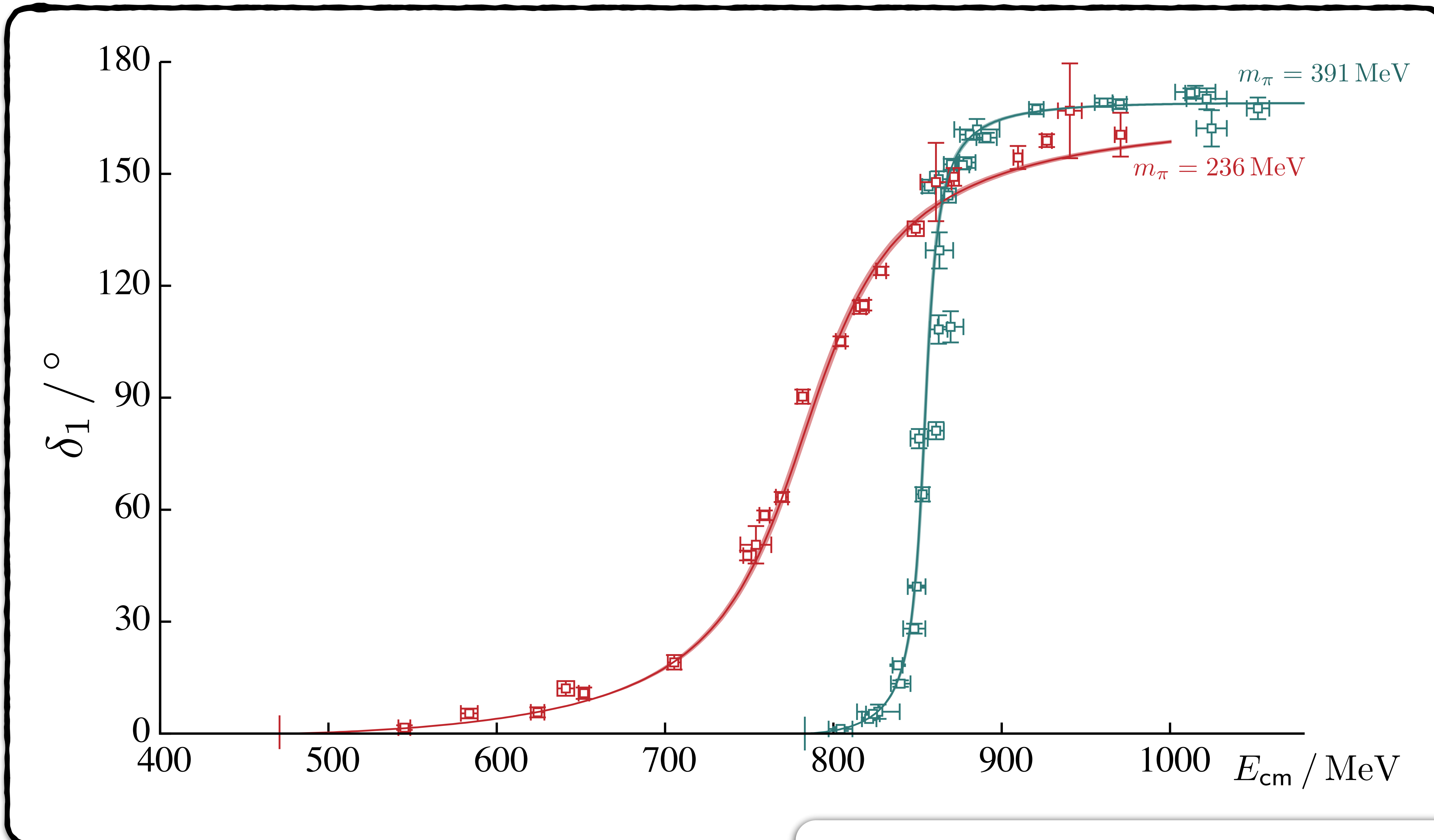


$$\det[\mathcal{M}(E_L) + F_2^{-1}(E_L, L)] = 0$$

$$\mathcal{M}_0 = \frac{16\pi E_{\rm cm}}{p\cot\delta_0 - ip}$$

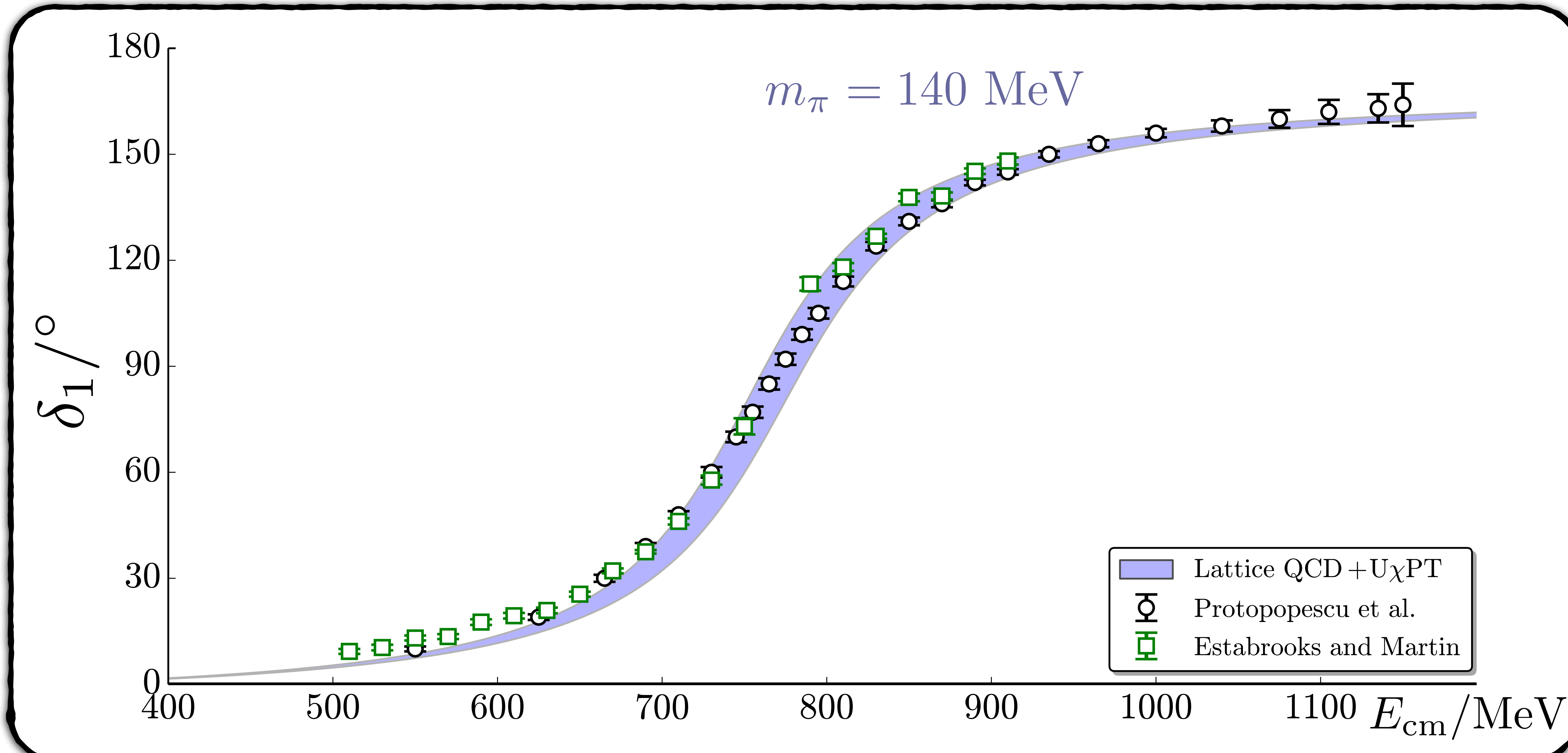
RB, Dudek, Edwards, Wilson - PRL (2017)

Isovector $\pi\pi$ scattering



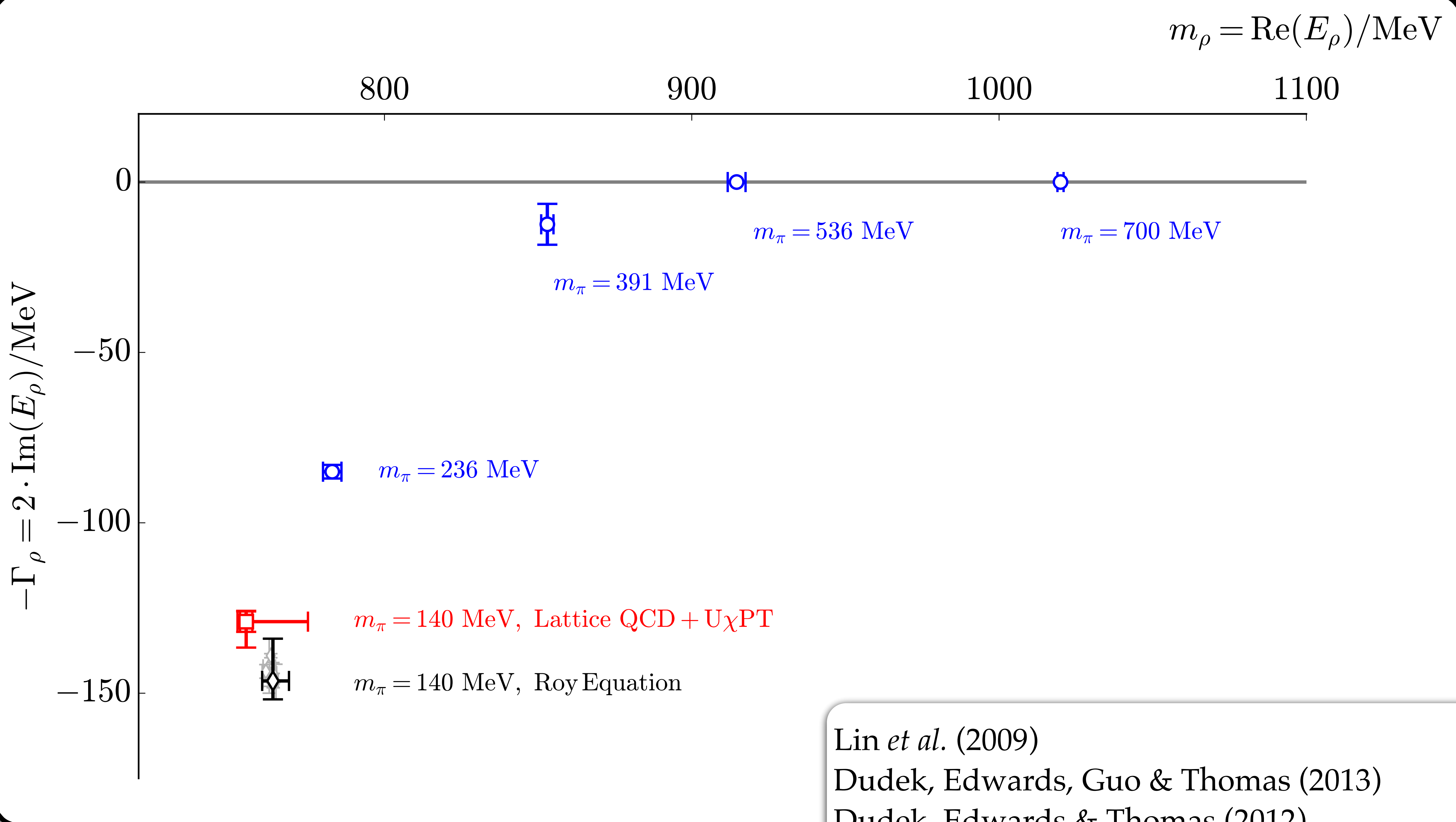
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

Comparison with experiment



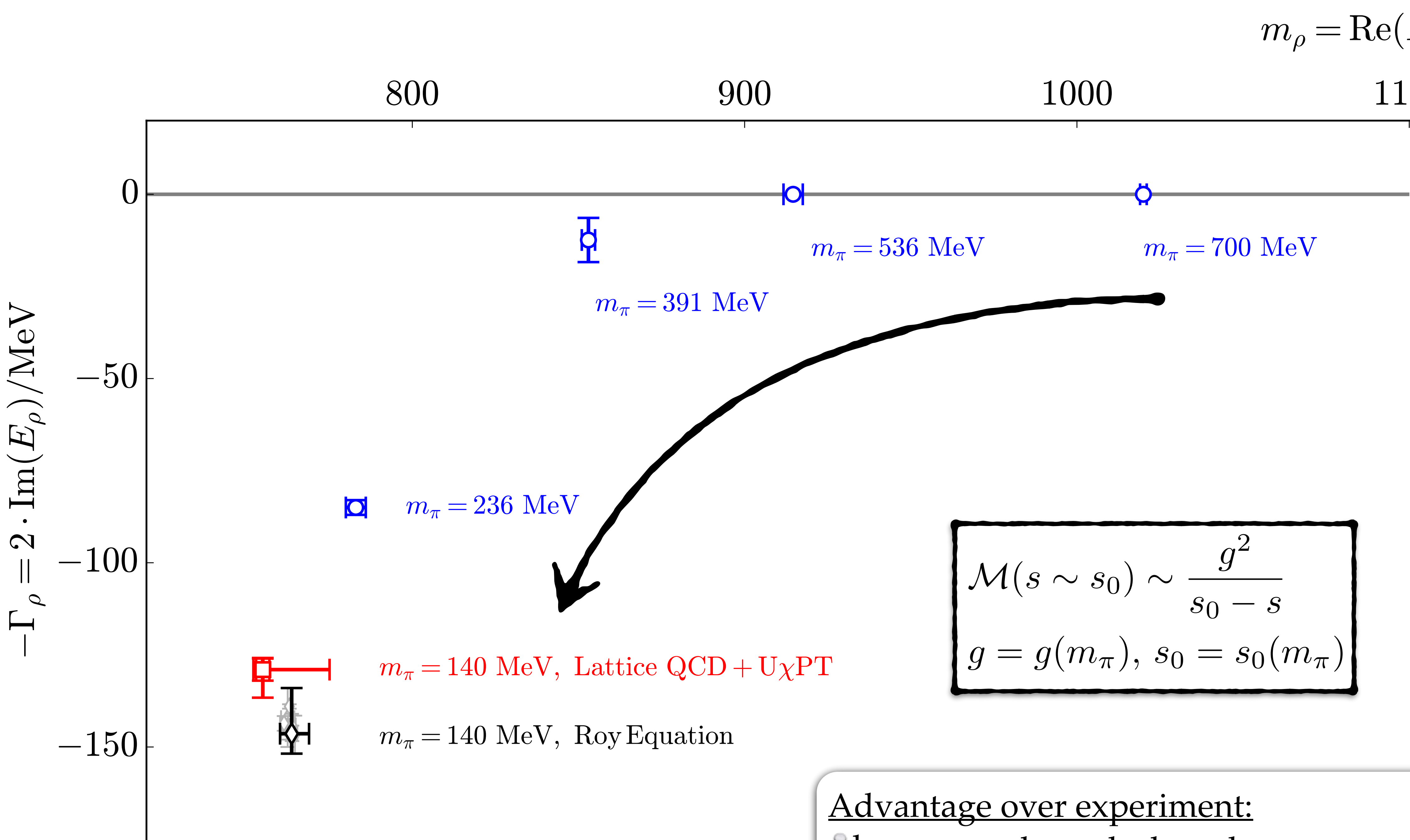
Bolton, RB & Wilson (2016)

The ϱ vs m_π



Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

The ϱ vs m_π



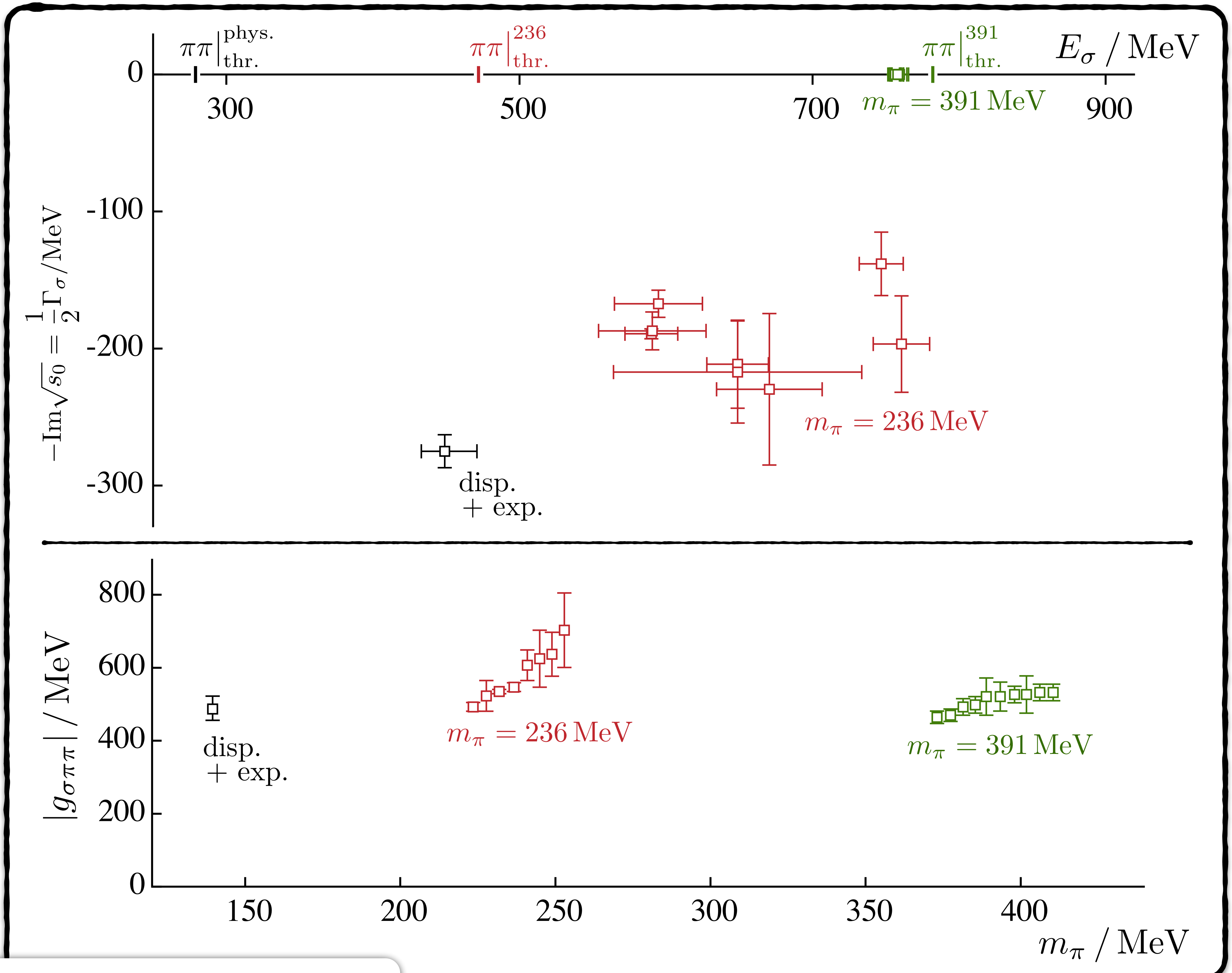
$$\mathcal{M}(s \sim s_0) \sim \frac{g^2}{s_0 - s}$$

$$g = g(m_\pi),\ s_0 = s_0(m_\pi)$$

<u>Advantage over experiment:</u>

- heavy quarks make broad resonances bound
- **unambiguously track poles in complex plane**

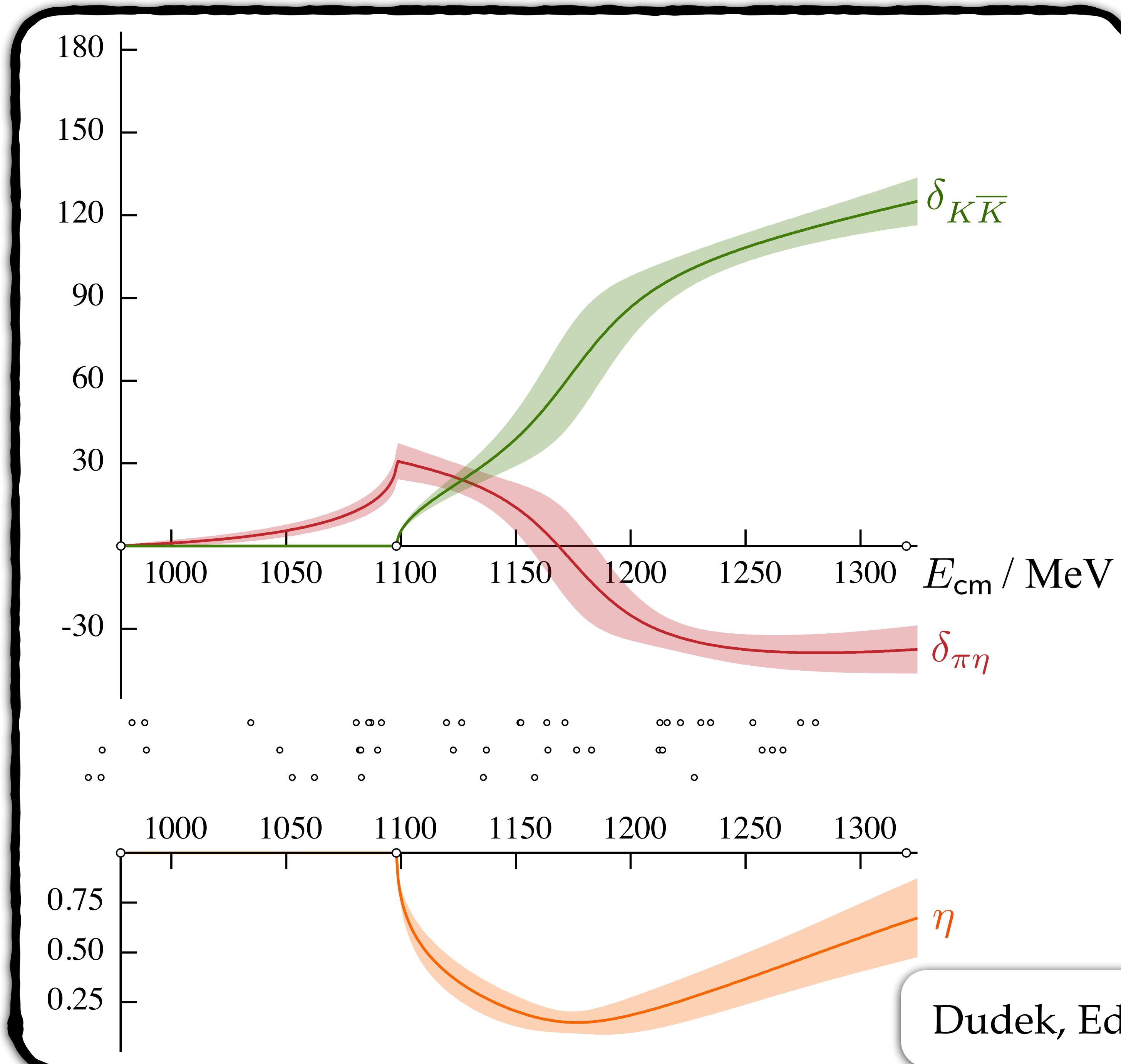
The σ / f$_0$(500) vs m$_\pi$



RB, Dudek, Edwards, Wilson - PRL (2017)

Going higher in energy

- *Coupled channels:* e.g., $\pi\eta$, $K\overline{K}$



m_π=391 MeV

Dudek, Edwards & Wilson (2016)

RB

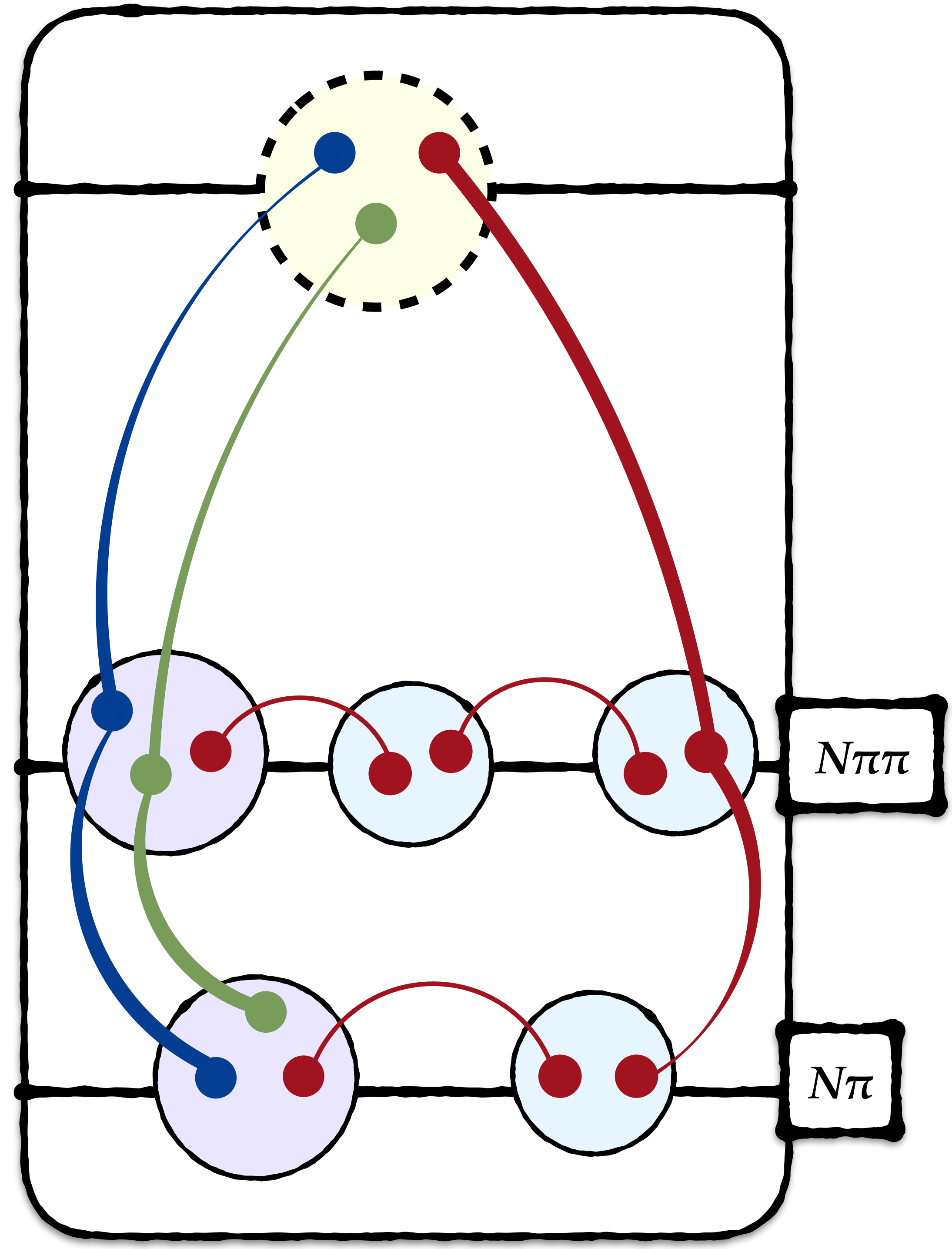
Going higher in energy

- *Coupled channels*
- *Beyond two particles:*

$$\det\left[1+\begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix}\begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\mathrm{df},3} \end{pmatrix}\right]=0$$

RB, Hansen & Sharpe (2017)

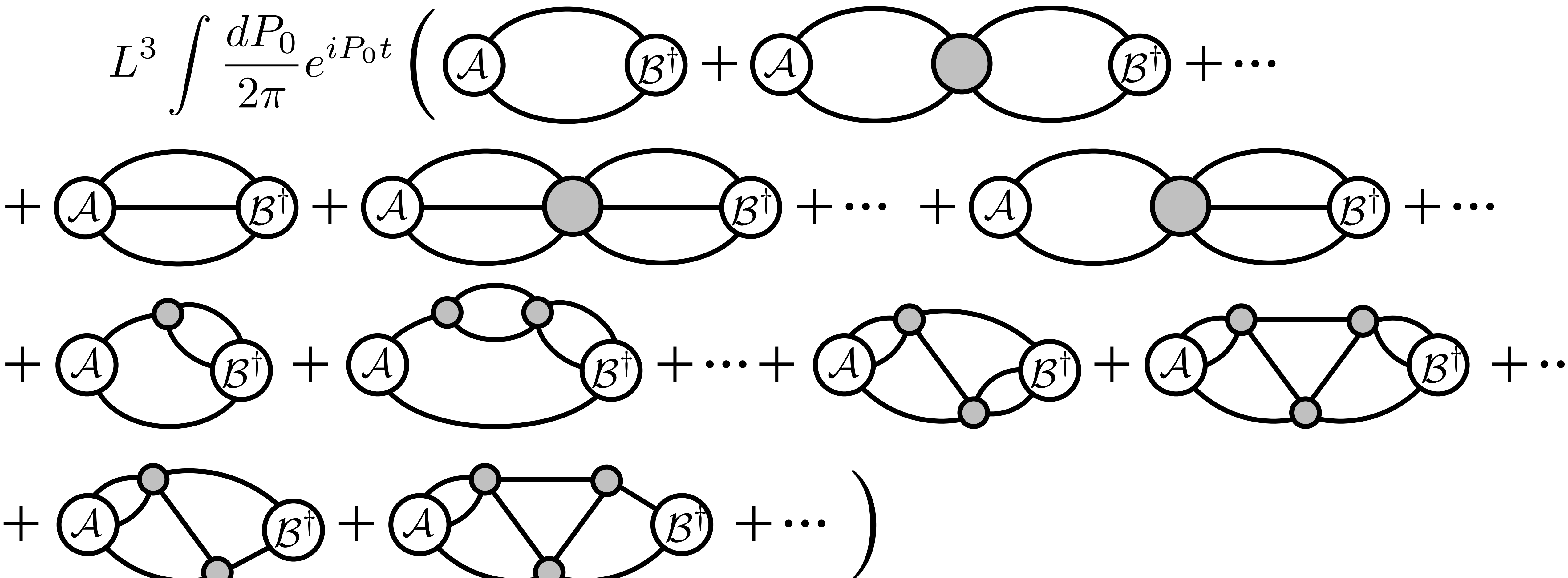
"could be used to extract 3N forces, the Roper and much more"



Two,three-particle systems

Consider $P_4 \sim i3m$:

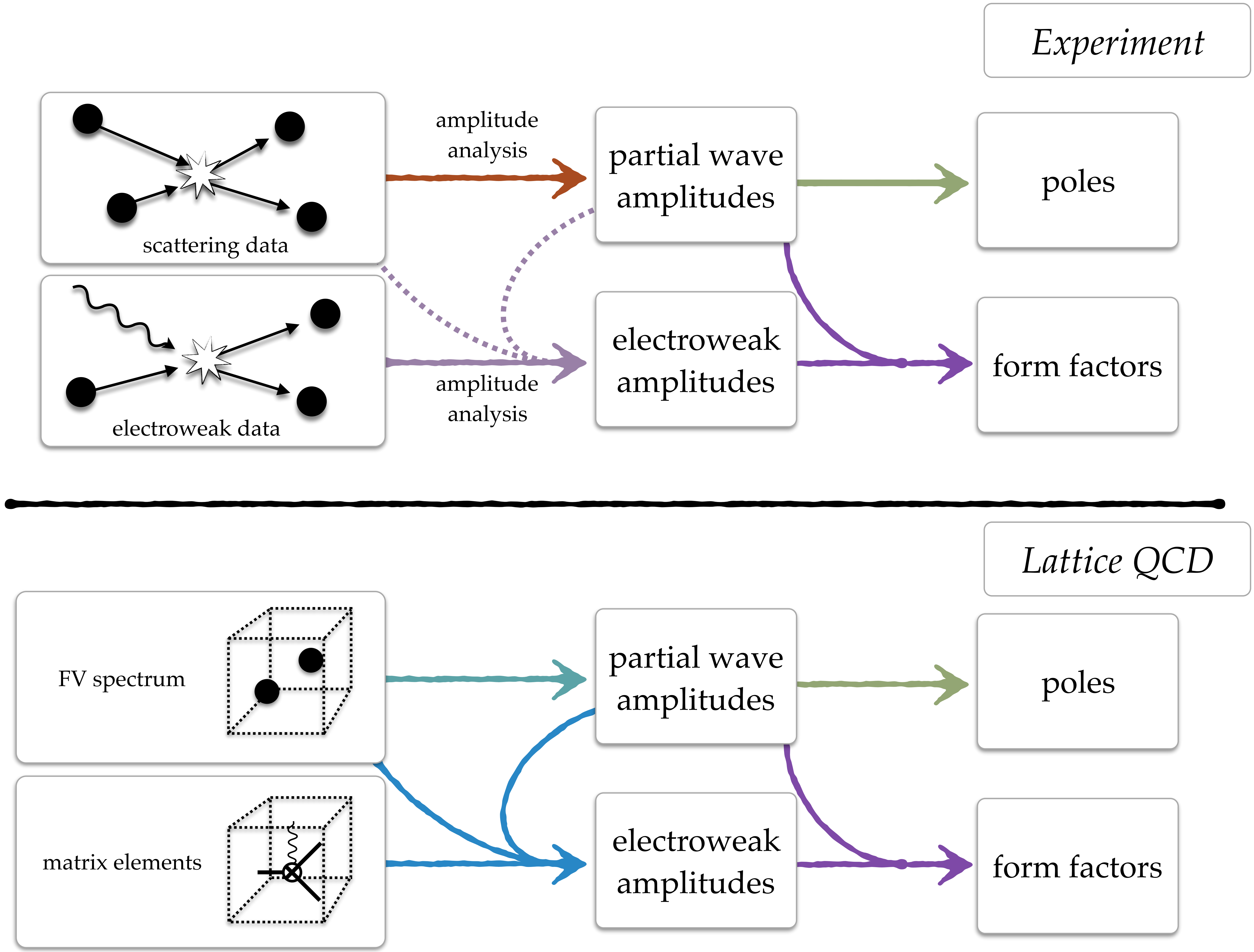
$$L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t} \Big(\cdots \Big)$$



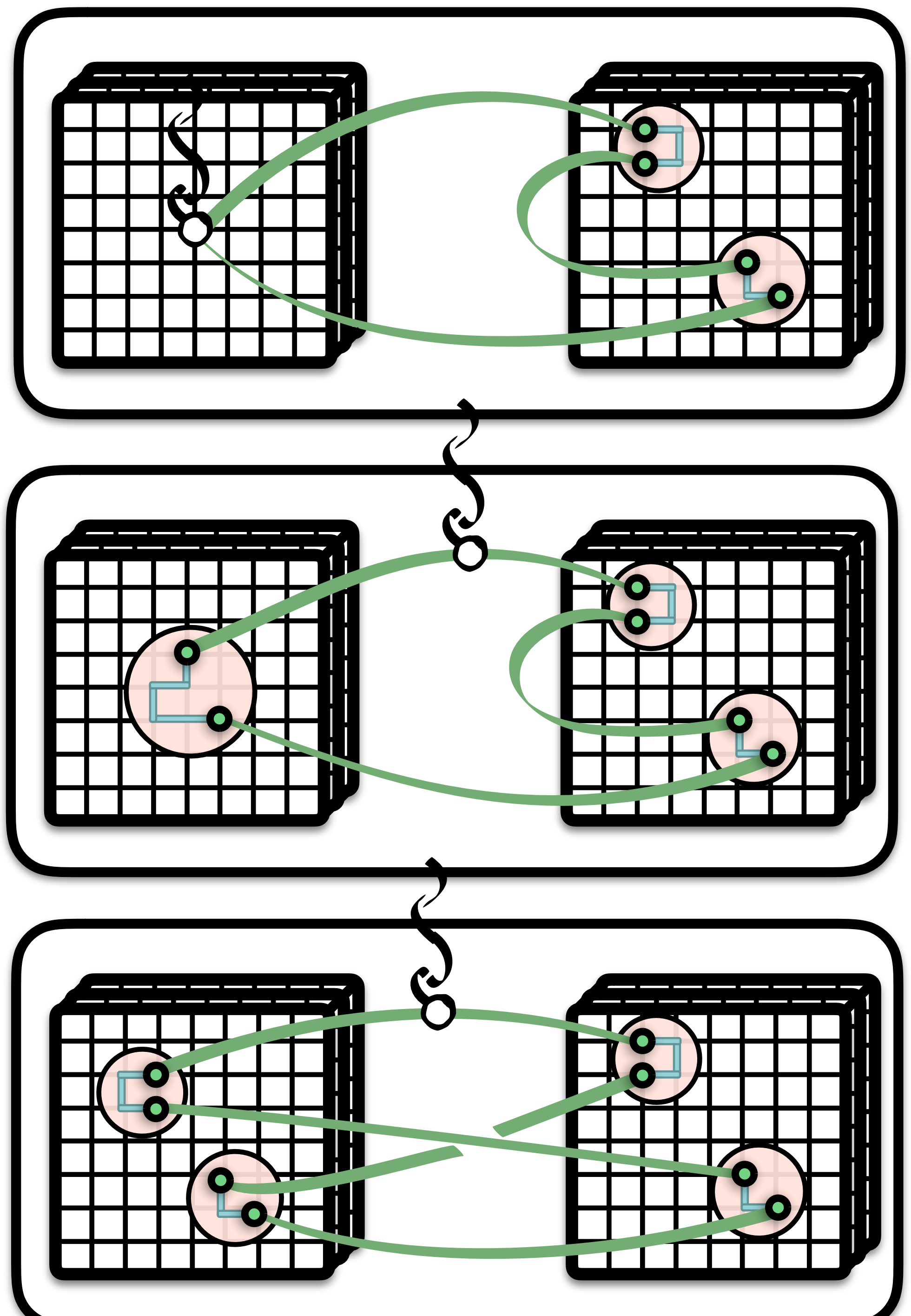
After *substantial* massaging:

$$\text{poles satisfy:} \quad \det\left[1 + \begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix} \begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\mathrm{df},3} \end{pmatrix}\right] = 0$$

Not the final result! Does not accommodate for resonant processes…underway!


Experiment
scattering data
amplitude analysis
partial wave amplitudes
poles
electroweak data
amplitude analysis
electroweak amplitudes
form factors
Lattice QCD
FV spectrum
partial wave amplitudes
poles
matrix elements
electroweak amplitudes
form factors

Beyond spectroscopy



$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{0}\rangle_L\right| = \sqrt{L^3}\sqrt{\mathcal{V}\ \mathcal{R}\ \mathcal{V}}$$

$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right| = \sqrt{\mathcal{H}\ \mathcal{R}\ \mathcal{H}}$$

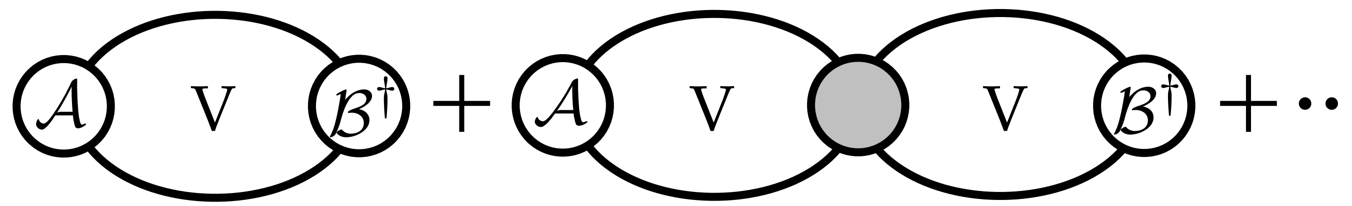
$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{2}\rangle_L\right| = \frac{1}{\sqrt{L^3}}\sqrt{\mathrm{Tr}\left[\mathcal{R}\ \mathcal{W}_{L,\mathrm{df}}\ \mathcal{R}\ \mathcal{W}_{L,\mathrm{df}}\right]}$$

RB, Hansen (2016)
RB, Hansen (2015)
RB, Hansen, Walker-Loud (2014)

Two-particle systems

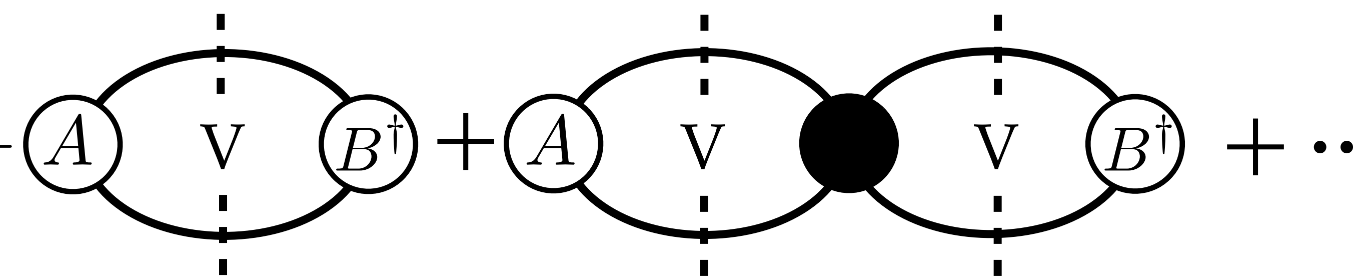
Consider $P_4 \sim i2m$:

$$L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t} \Big(\cdots \Big)$$



After some massaging:

$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t} \Big(C_\infty(P) + \cdots \Big)$$



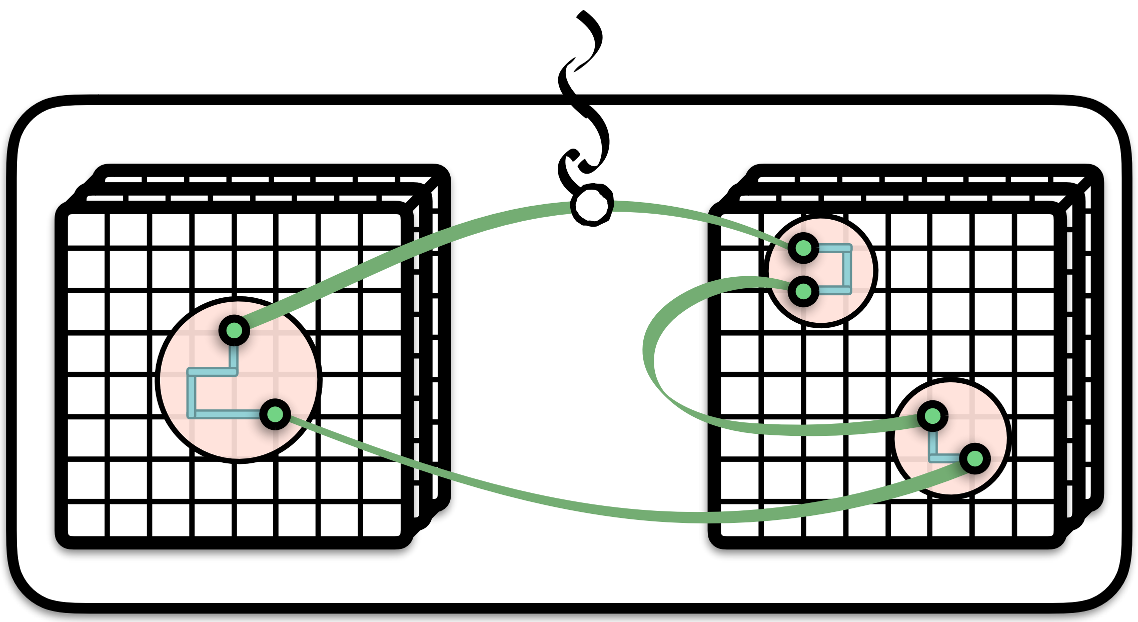
$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t} \left(C_\infty(P) - A(P) \frac{1}{F_2^{-1}(P,L) + \mathcal{M}(P)} B^\dagger(P) \right)$$

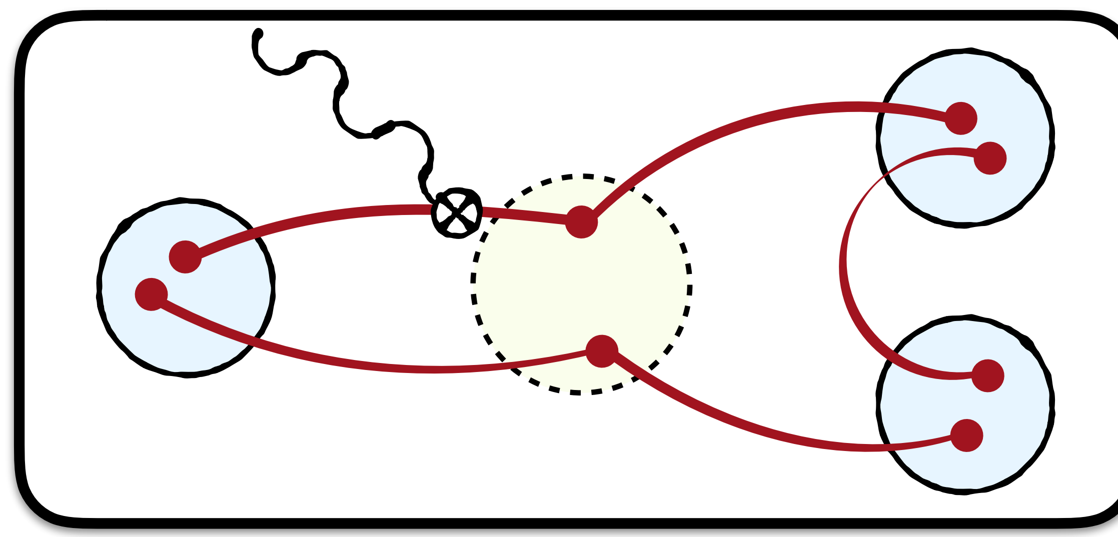
$$= L^3 \sum_n e^{-E_n t} \, A_n \, \mathcal{R}_n \, B_n^\dagger$$

$\mathcal{R}_n$: F.V. residue for 2-particle states. Explains how infinite-volume and F.V. states are mapped onto each other.

One-to-two transition

$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right| = \sqrt{\mathcal{A}\,\mathcal{R}\,\mathcal{A}}$$





finite volume matrix element

$\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L =$ finite matrix element

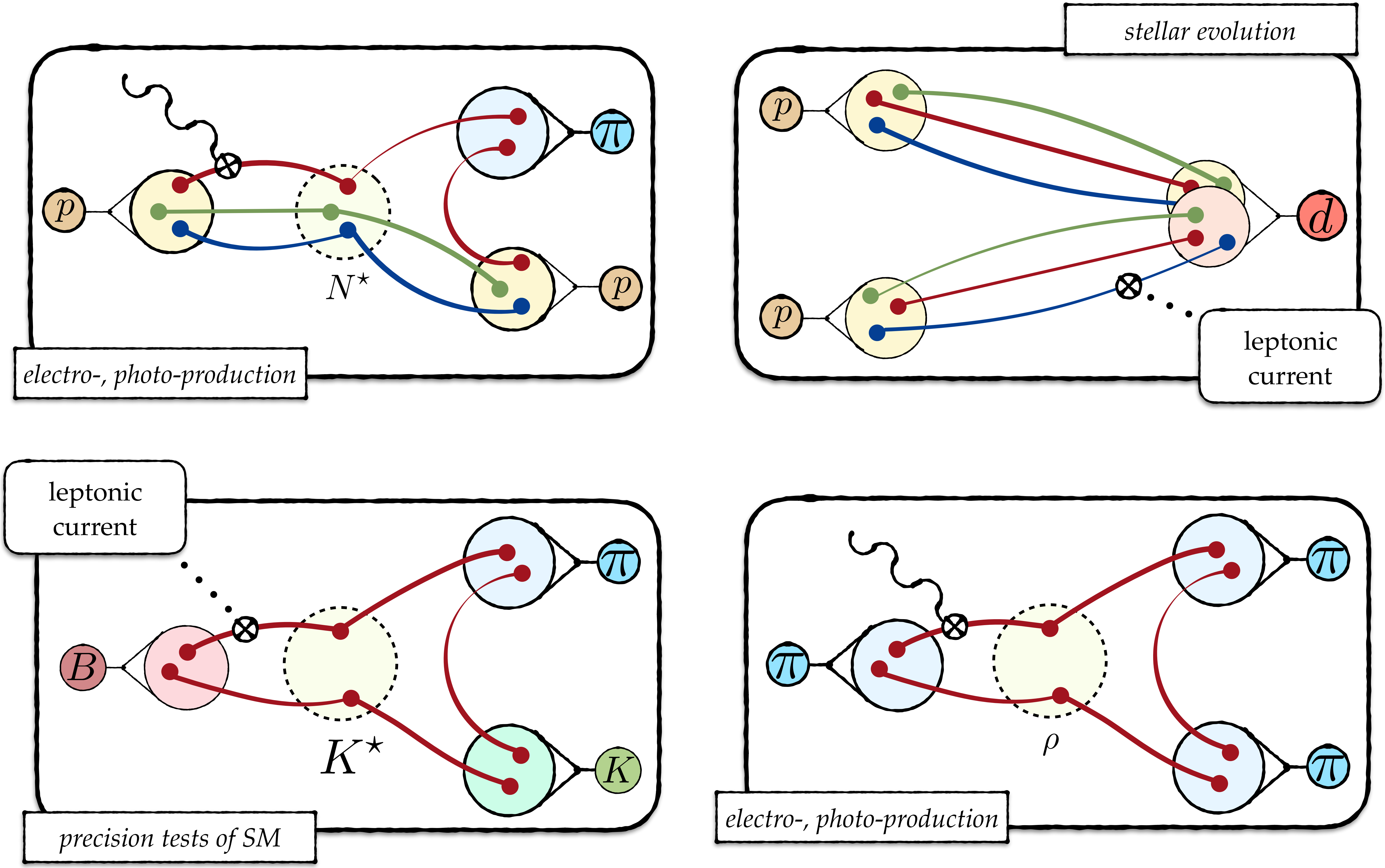
$\mathcal{R} =$ known function

electroweak amplitude

$\mathcal{A} =$ electroweak amplitude

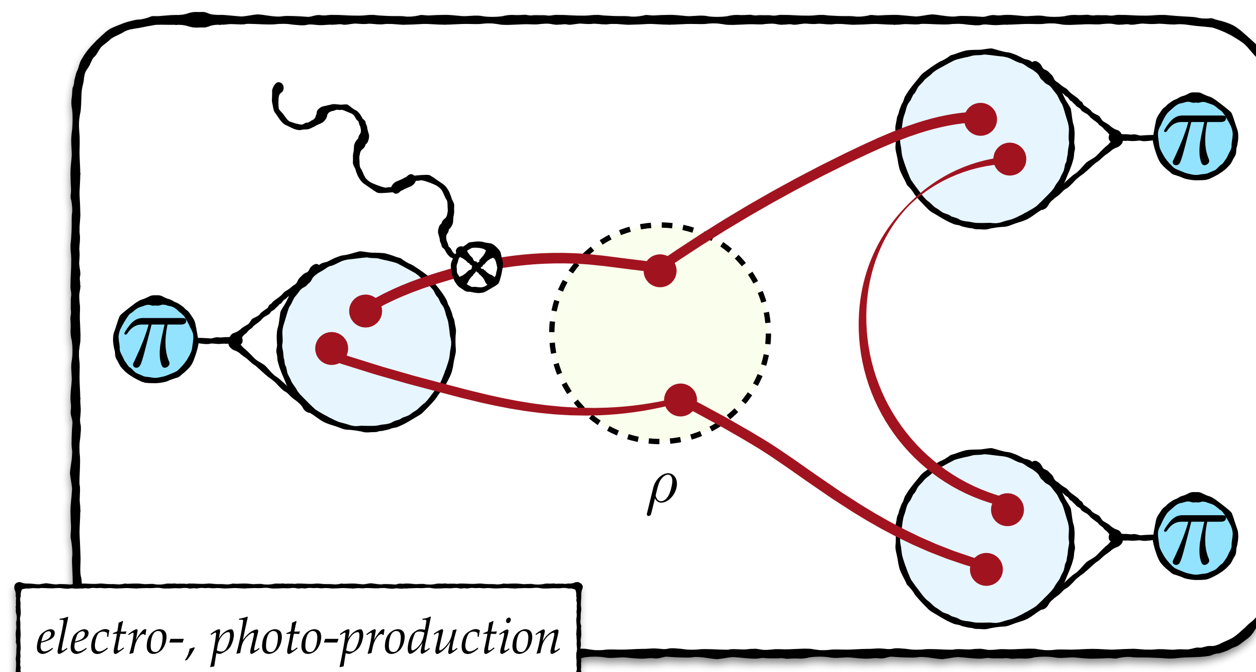
RB, Hansen (2015)
RB, Hansen, Walker-Loud (2014)

One-to-two transition

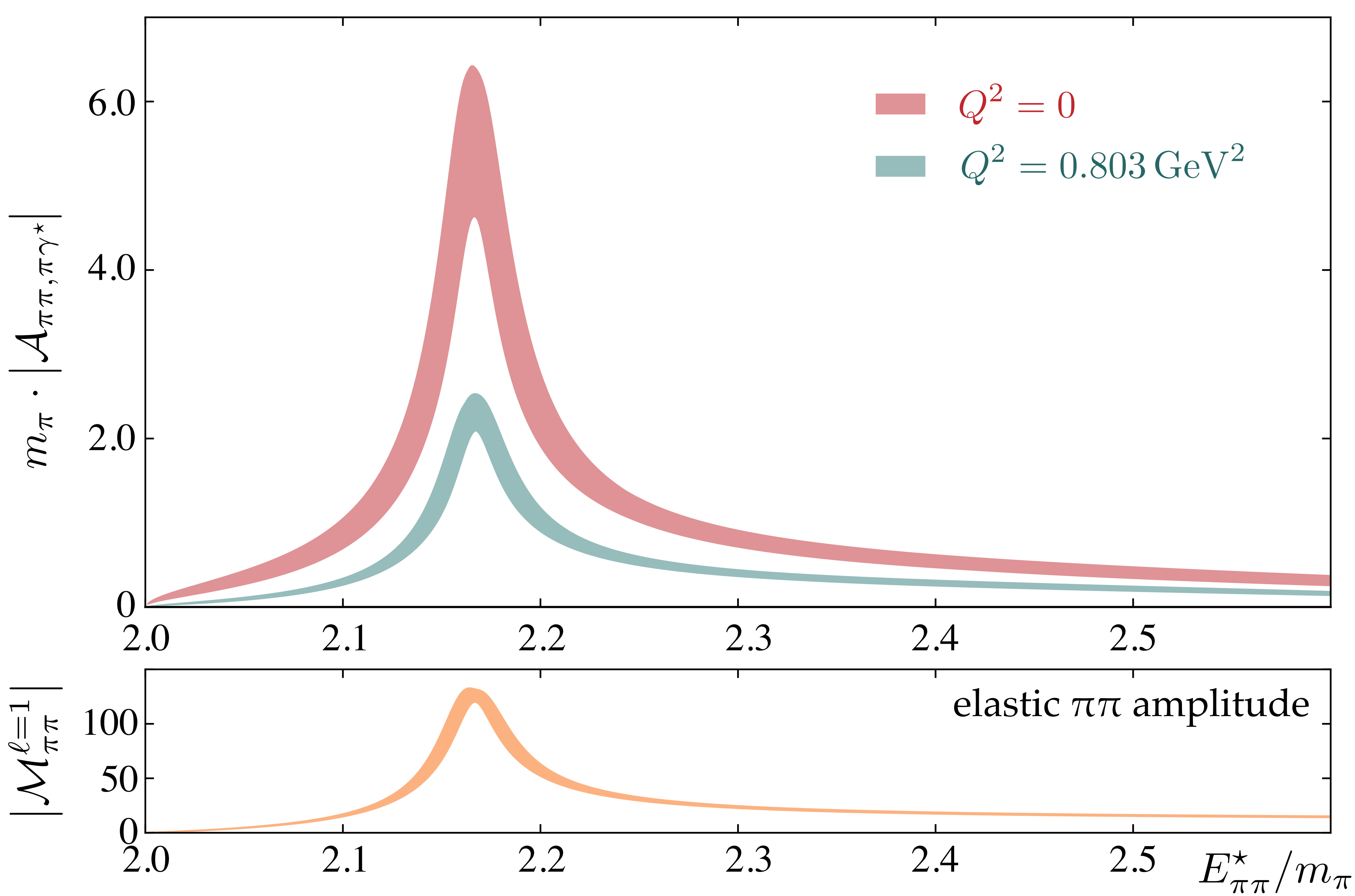


One-to-two transition

- ρ-to-π form factor
- chiral anomaly
- anomalous magnetic moment of the muon [g-2]
- first resonant 1-to-2 process
- proof of principle



$\pi\gamma^*$-to-$\pi\pi$ amplitude

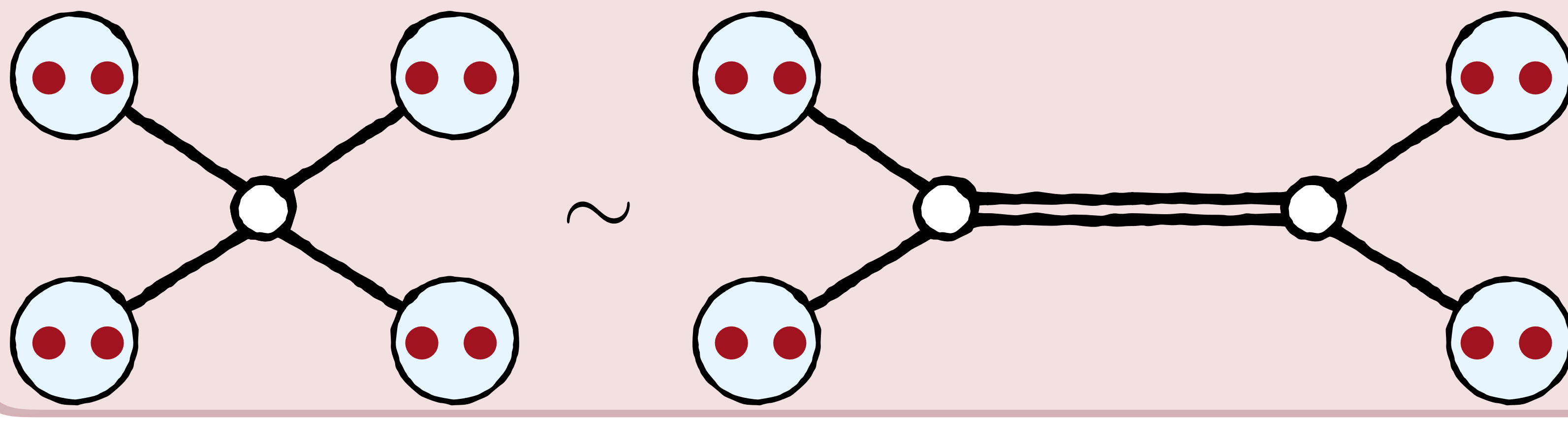


m_π=391 MeV

RB, Dudek, Edwards, Thomas, Shultz, Wilson - PRL (2015)

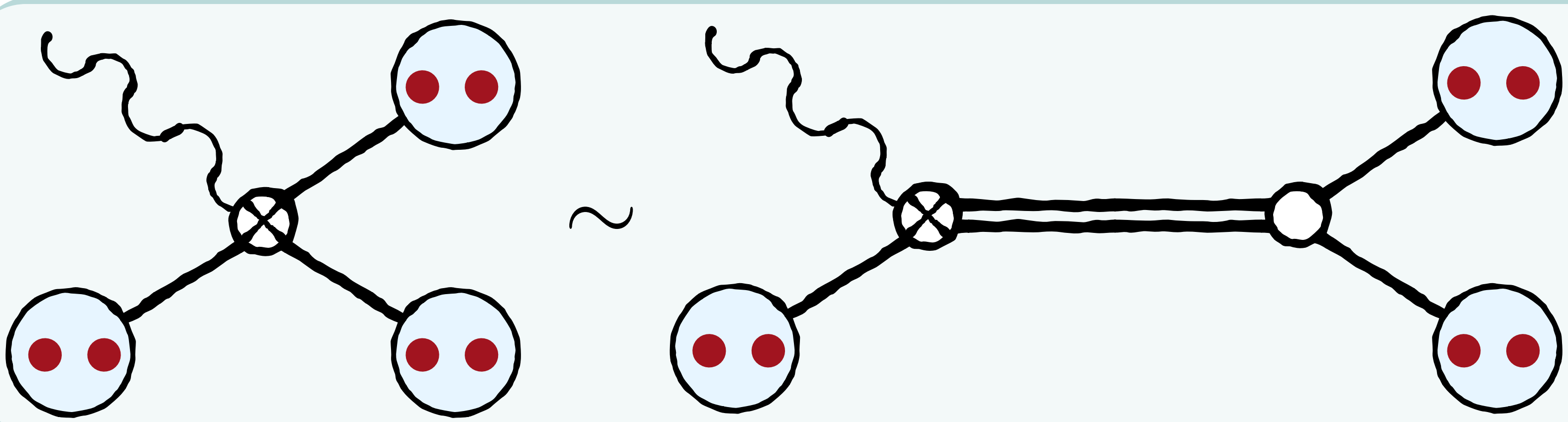
Explanation

- *ππ-to-ππ amplitude:*



$$\sim \frac{i|g_{\rho,\pi\pi}|^2}{s - s_0}$$

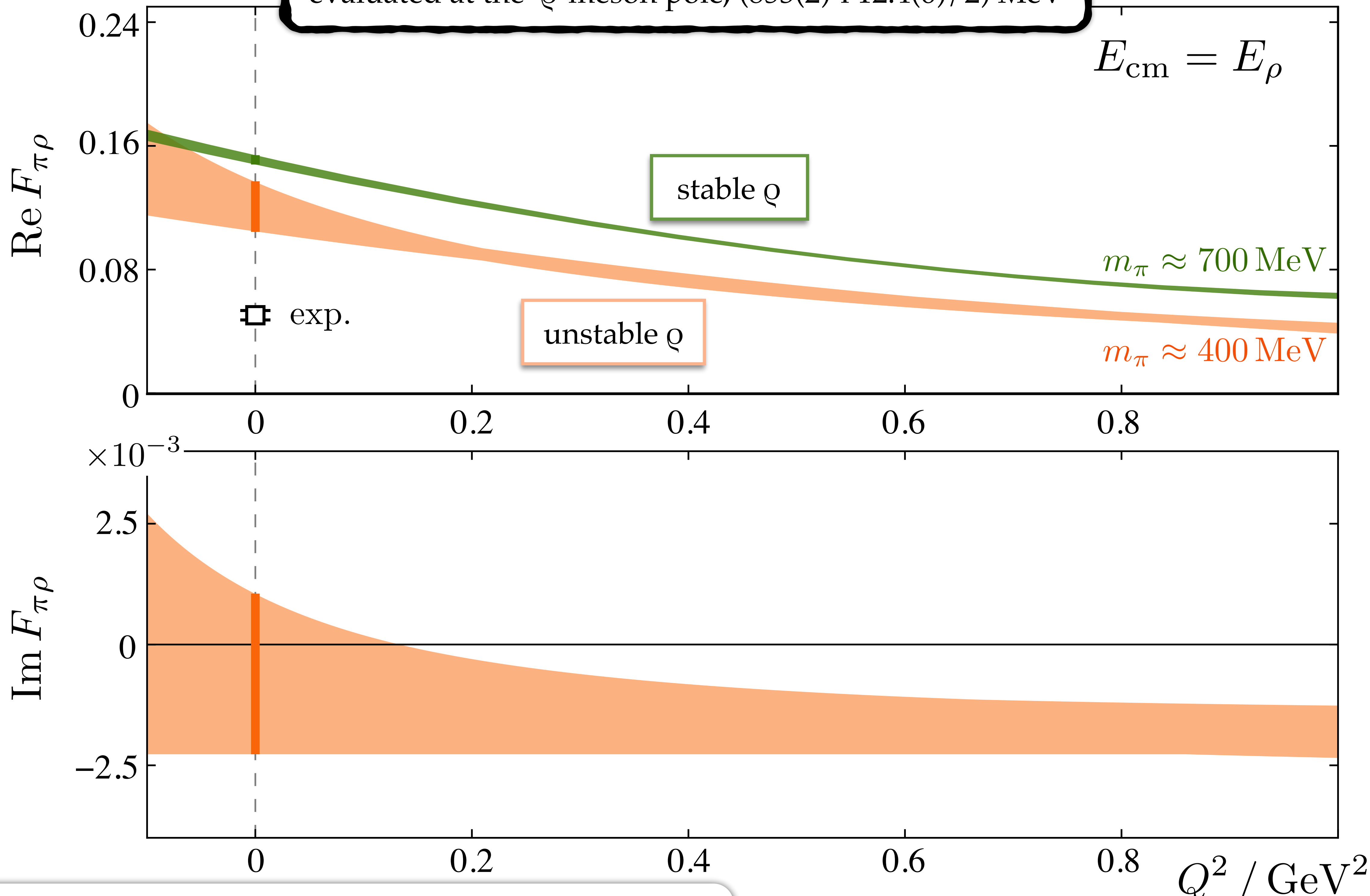
- *πγ*-to-ππ amplitude:*



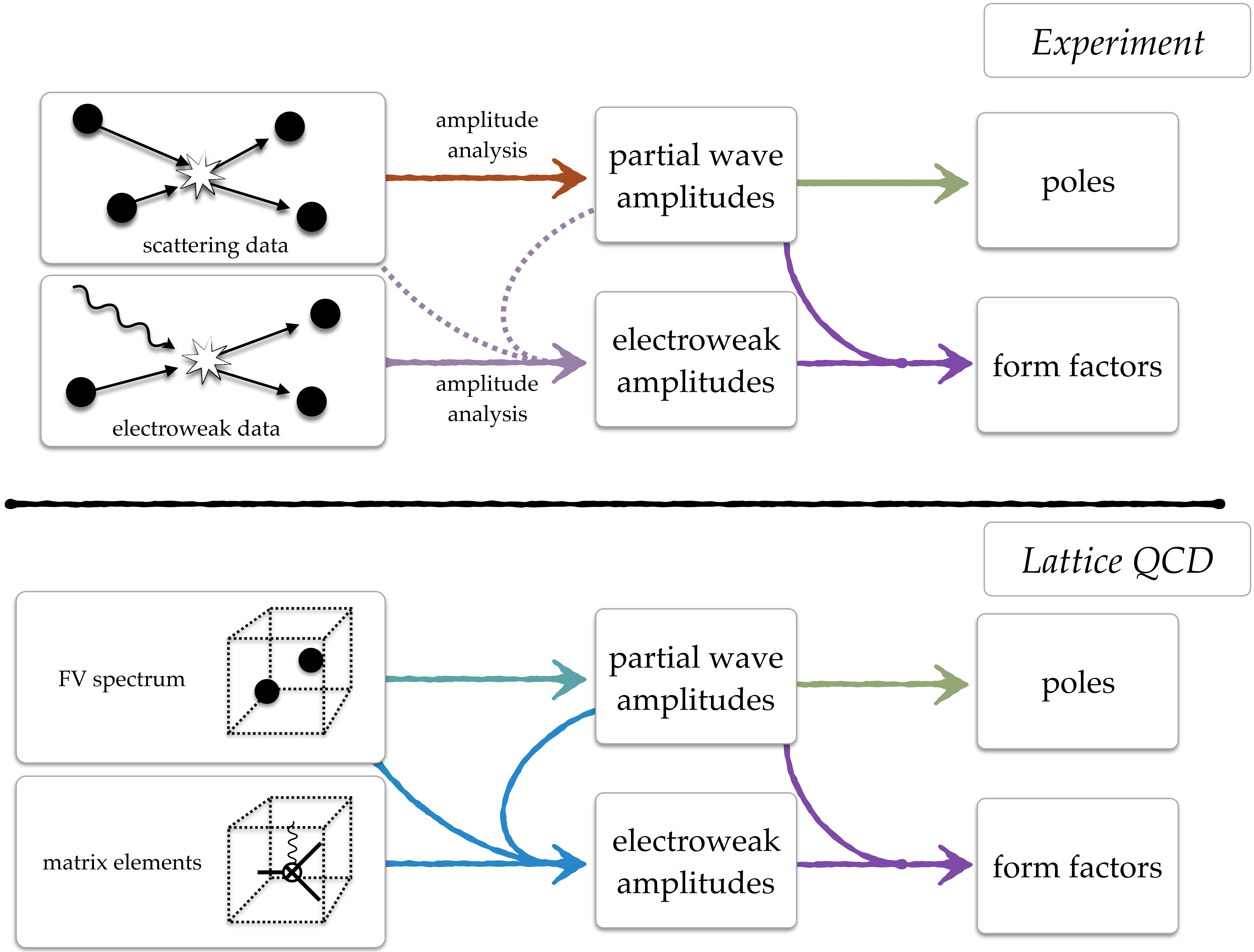
$$\sim \frac{iF_{\pi\rho}\ g_{\rho,\pi\pi}}{s - s_0}$$

Form factor at ϱ pole

evaluated at the ϱ-meson pole, (853(2)-i 12.4(6)/2) MeV



Shultz, Dudek, & Edwards (2014)
RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)


Experiment
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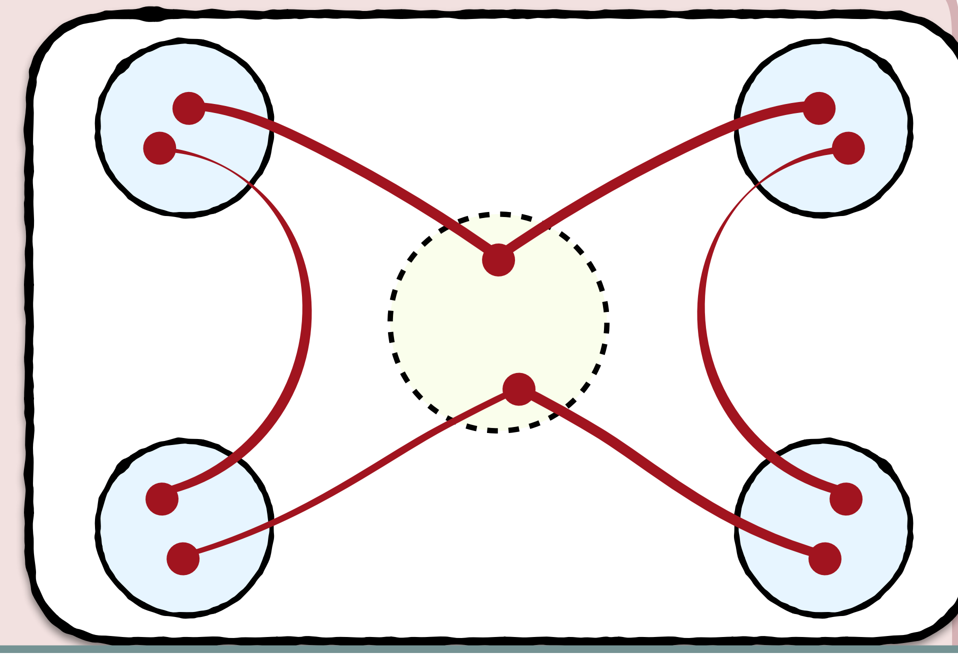
poles
form factors
Lattice QCD
FV spectrum
matrix elements
partial wave amplitudes
electroweak amplitudes
poles
form factors

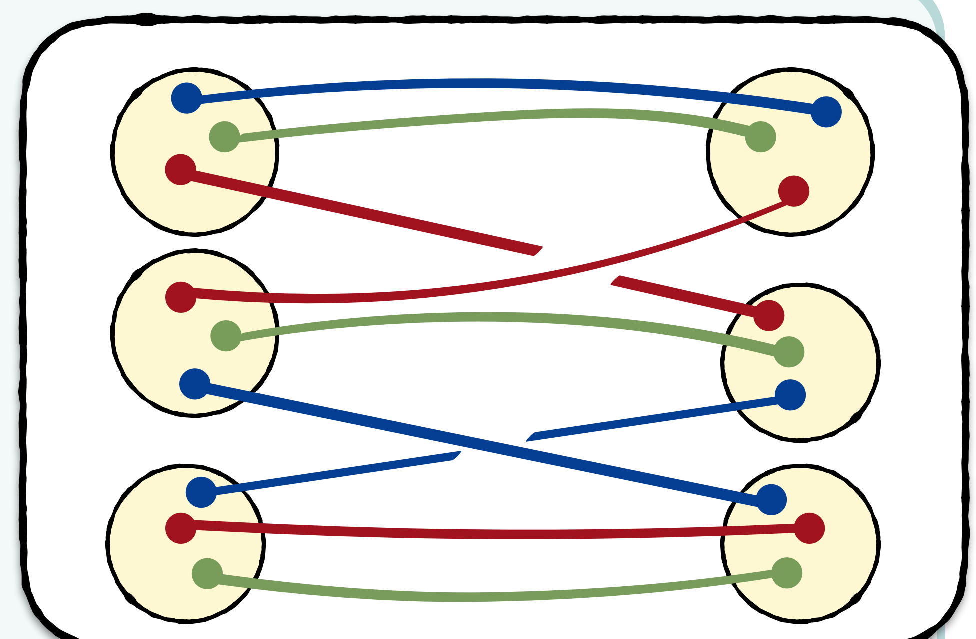
The future of spectroscopy

- Formalism: complete and tested
- Only a handful of channels considered
- Much more underway
- No systems with intrinsic spin to date



RB (2014) [inelastic, spinning 2-particles]

- Formalism: incomplete [1-2yrs]
  - 2-body resonances
  - Multichannel, asymmetric masses, spin
- Untested [3-5yrs]

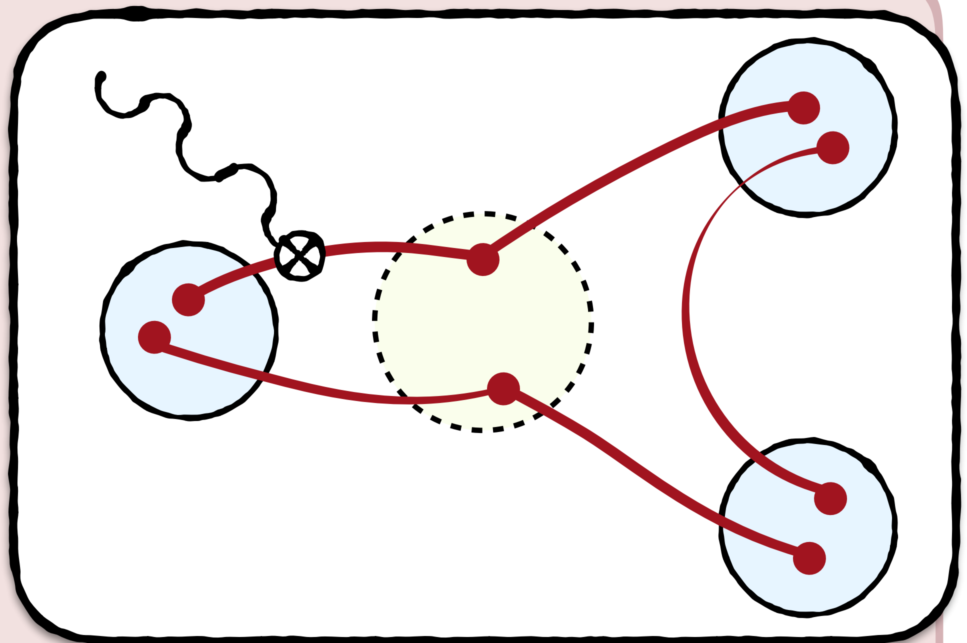


Complimentary to experiment!

RB, Hansen & Sharpe (2017)

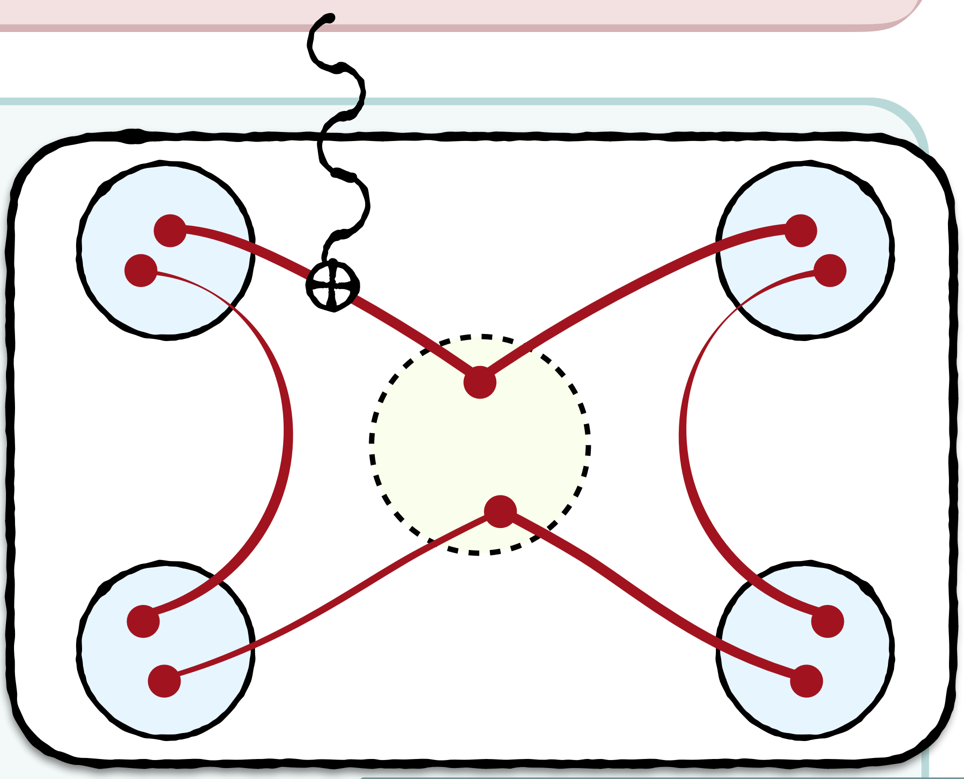
The future of structure

- Formalism: complete and tested
- Only one calculation to date
- Quark-mass exploration [~1-3yrs]
- Baryons to come…



RB & Hansen (2015)

- Formalism: incomplete [~1-2yrs]
- Untested
- First calculation:
  - $\pi\pi\gamma^*$-to-$\pi\pi$ [~2-4yrs]
  - First elastic f.f. of a composite state



RB & Hansen (2016)

Complimentary to experiment!

Broad goals

	formalism	numerical
Strongly coupled **2**-body	☐	☐
Strongly coupled **2, 3**-body	☐	☐
Spin-dependent amps.	☐	☐
Narrow resonances	☐	☐
Broad resonances	☐	☐
Photo-, electro-production	☐	☐
Transition form factors	☐	☐
Elastic form factors	☐	☐

Broad goals

	formalism	numerical
Strongly coupled **2**-body	✓	✓
Strongly coupled **2, 3**-body	!	✗
Spin-dependent amps.	✓	✗
Narrow resonances	✓	✓
Broad resonances	!	!
Photo-, electro-production	✓	✓
Transition form factors	✓	✓
Elastic form factors	✓	✗

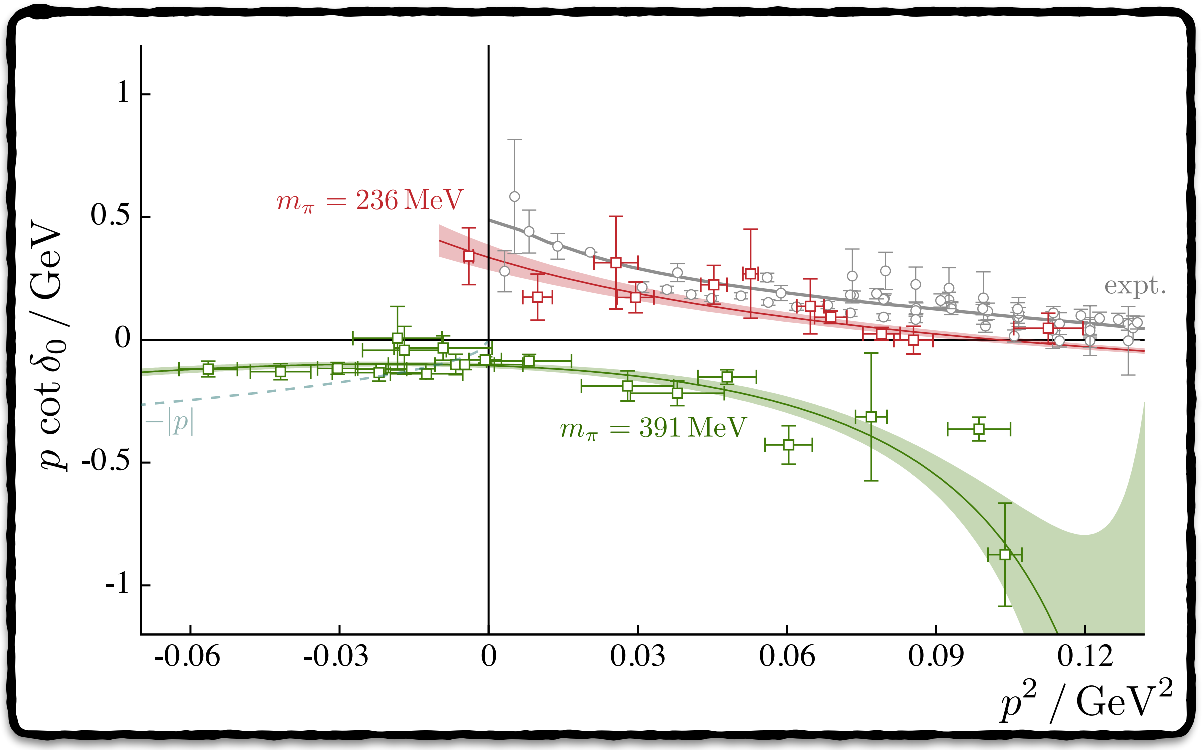
Broad goals

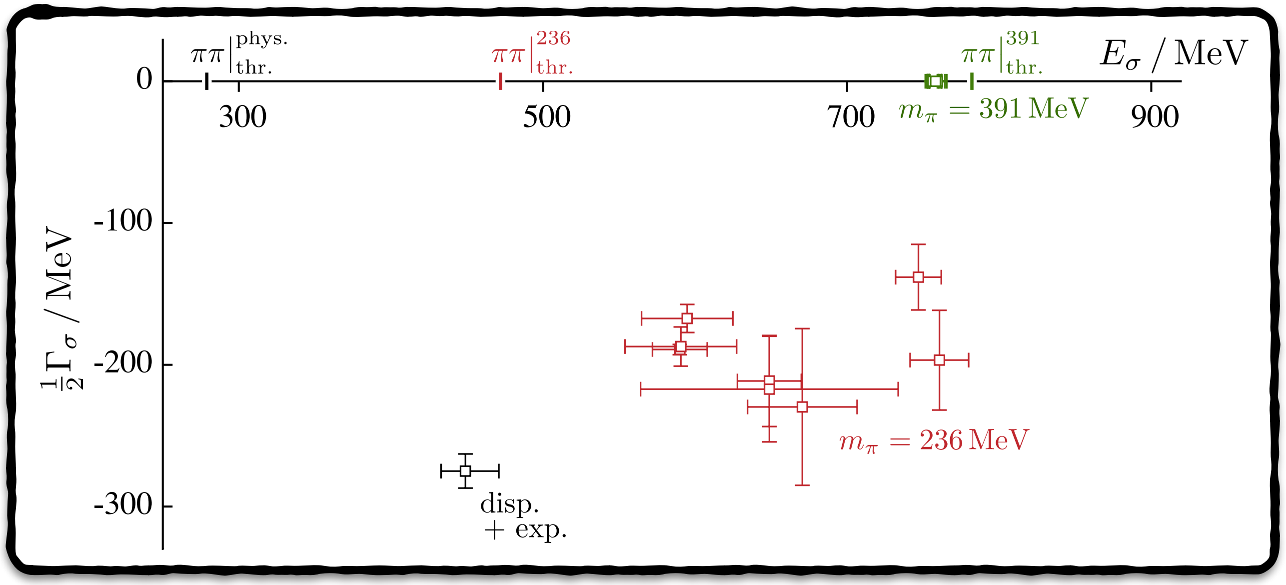
	5 yrs. ago	formalism	numerical
Strongly coupled **2**-body	!	✓	✓
Strongly coupled **2, 3**-body	✗	!	✗
Spin-dependent amps.	✗	✓	✗
Narrow resonances	!	✓	✓
Broad resonances	✗	!	!
Photo-, electro-production	✗	✓	✓
Transition form factors	✗	✓	✓
Elastic form factors	✗	✓	✗

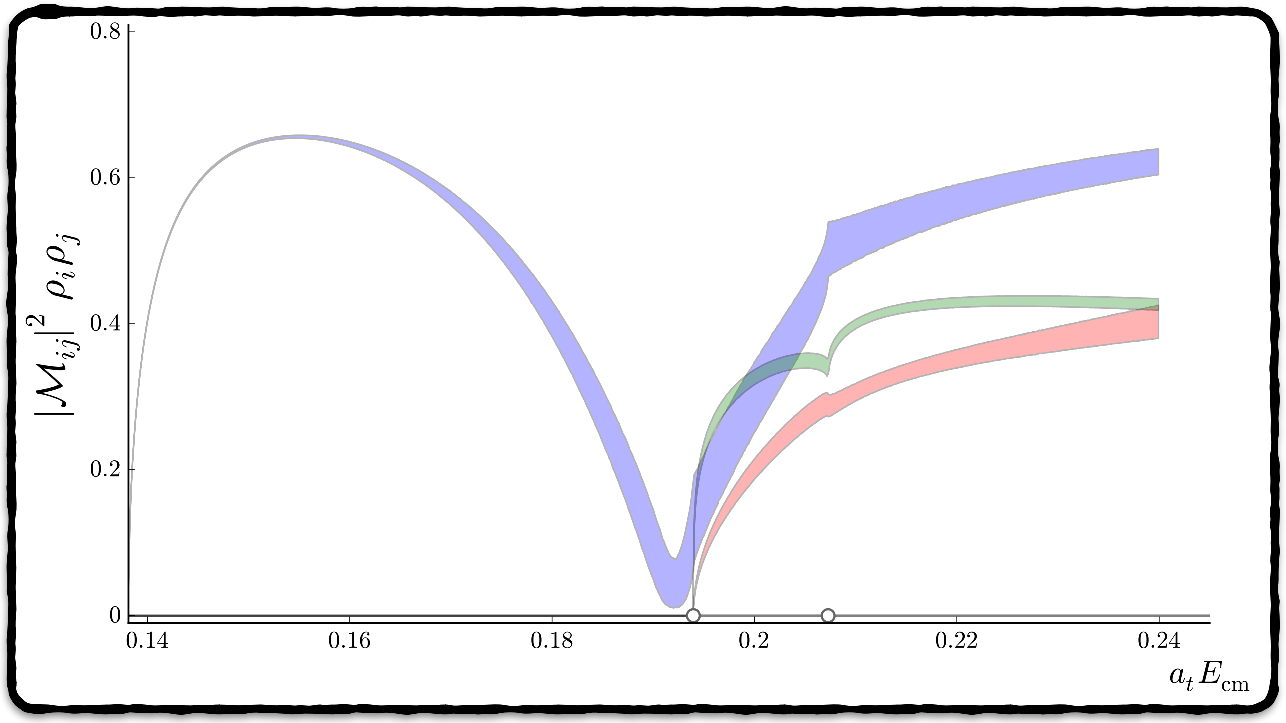
Broad goals

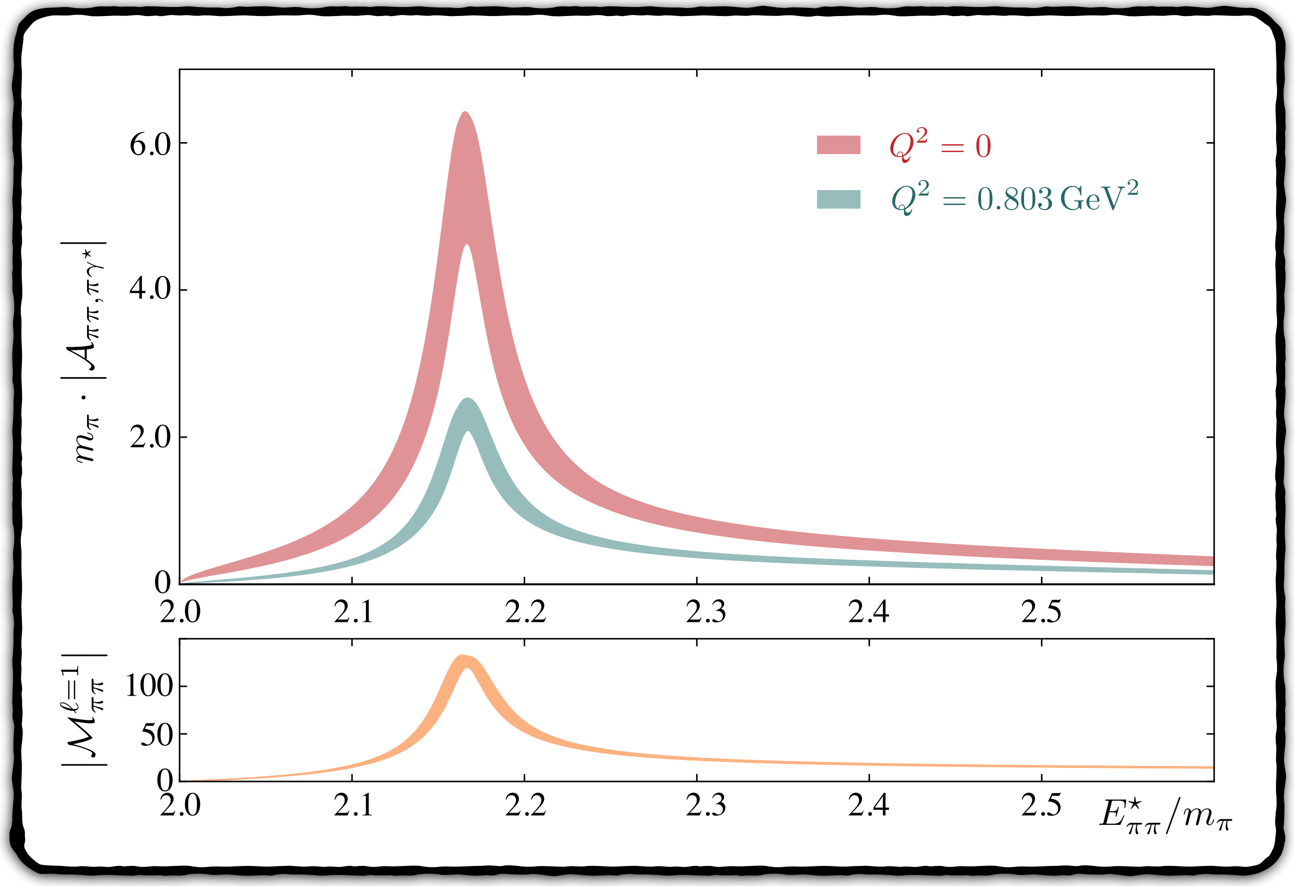
- Strongly coupled **2**-body
- Strongly coupled **2, 3**-body
- Spin-dependent amps.
- Narrow resonances
- Broad resonances
- Photo-, electro-production
- Transition form factors
- Elastic form factors

The big picture!









Collaborators & references

formalism


Hansen

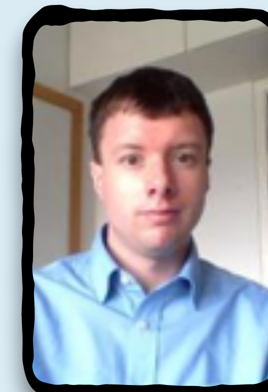

Walker-Loud


Sharpe

numerical


Wilson


Shultz


Thomas


Bolton


Dudek


Edwards

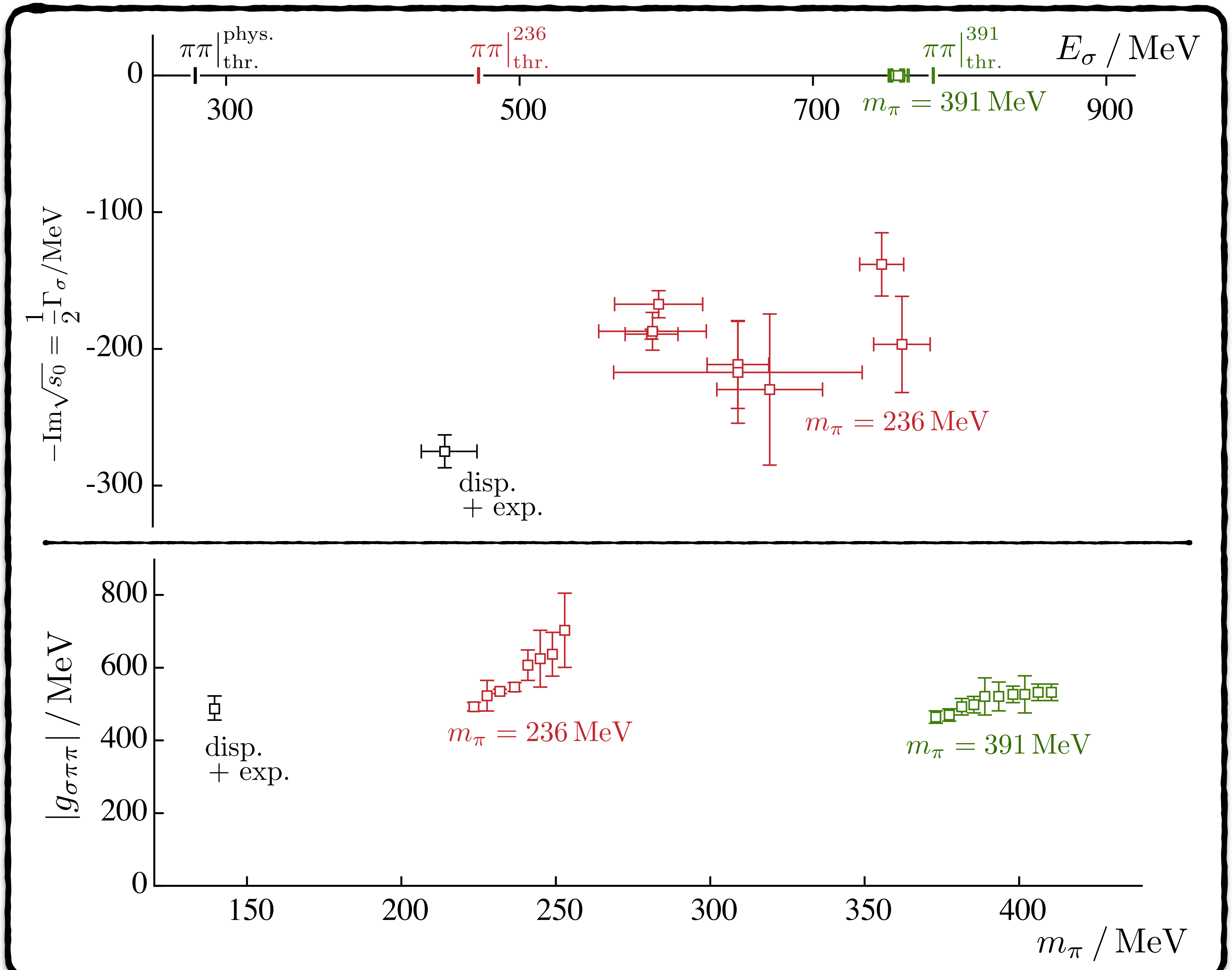
HadSpec Collaboration

RB, Hansen, Sharpe - arXiv:1609.09805 [hep-lat] (2016)
RB, Hansen - Phys.Rev. D94 (2016) no.1, 013008 .
RB, Hansen - Phys.Rev. D92 (2015) no.7, 074509.
RB, Hansen, Walker-Loud - Phys.Rev. D91 (2015) no.3, 034501.
RB - Phys.Rev. D89 (2014) no.7, 074507.

RB, Dudek, Edwards, Wilson - Phys.Rev.Lett. 118 (2017) no.2, 022002.
RB, Dudek, Edwards, Thomas, Shultz, Wilson - Phys.Rev. D93 (2016) 114508.
RB, Dudek, Edwards, Thomas, Shultz, Wilson - Phys.Rev.Lett. 115 (2015) 242001
Wilson, RB, Dudek, Edwards, Thomas - Phys.Rev. D92 (2015) no.9, 094502

The σ / f₀(500) vs m_π

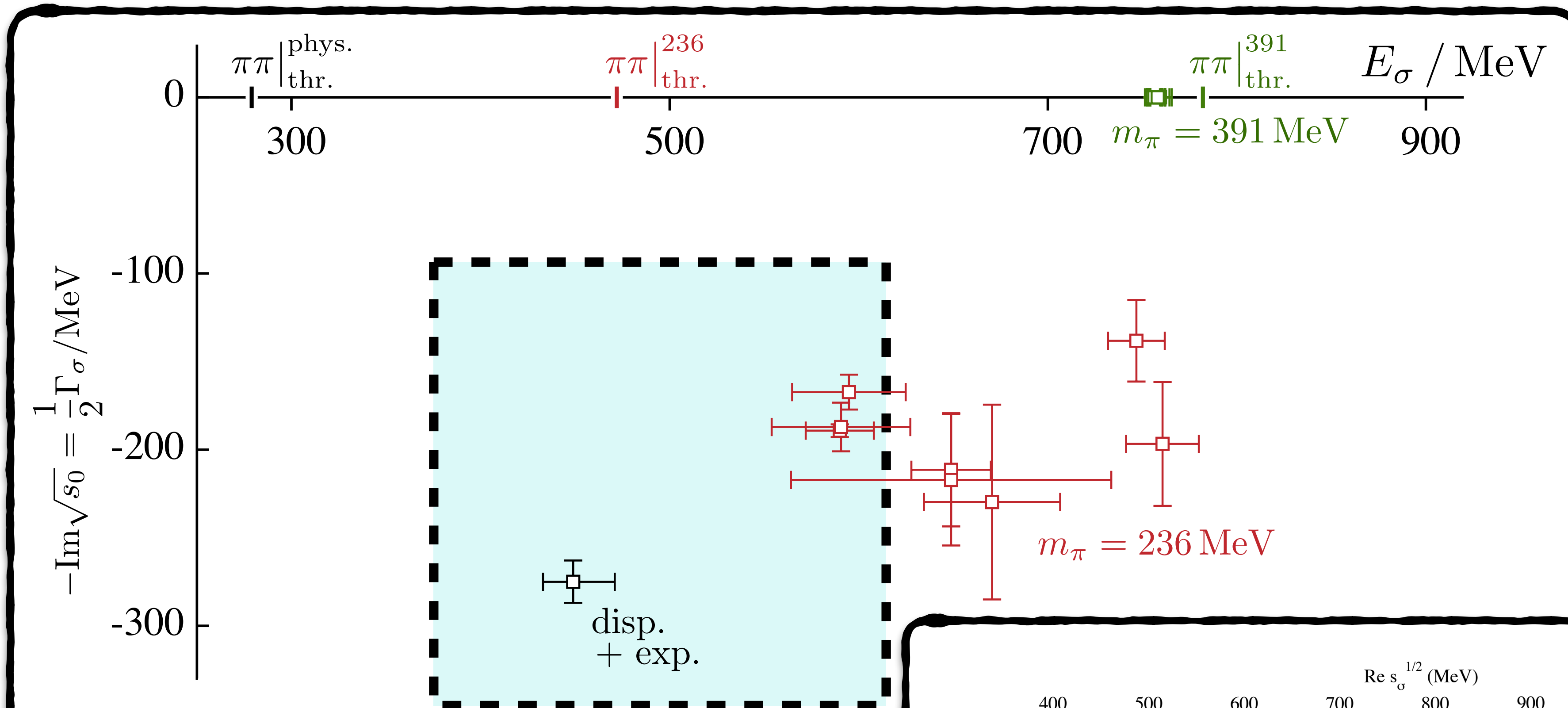
$$s_0 = (E_\sigma - \tfrac{i}{2}\Gamma_\sigma)^2, \qquad g^2_{\sigma\pi\pi} = \lim_{s\to s_0} (s_0 - s)\, t(s)$$



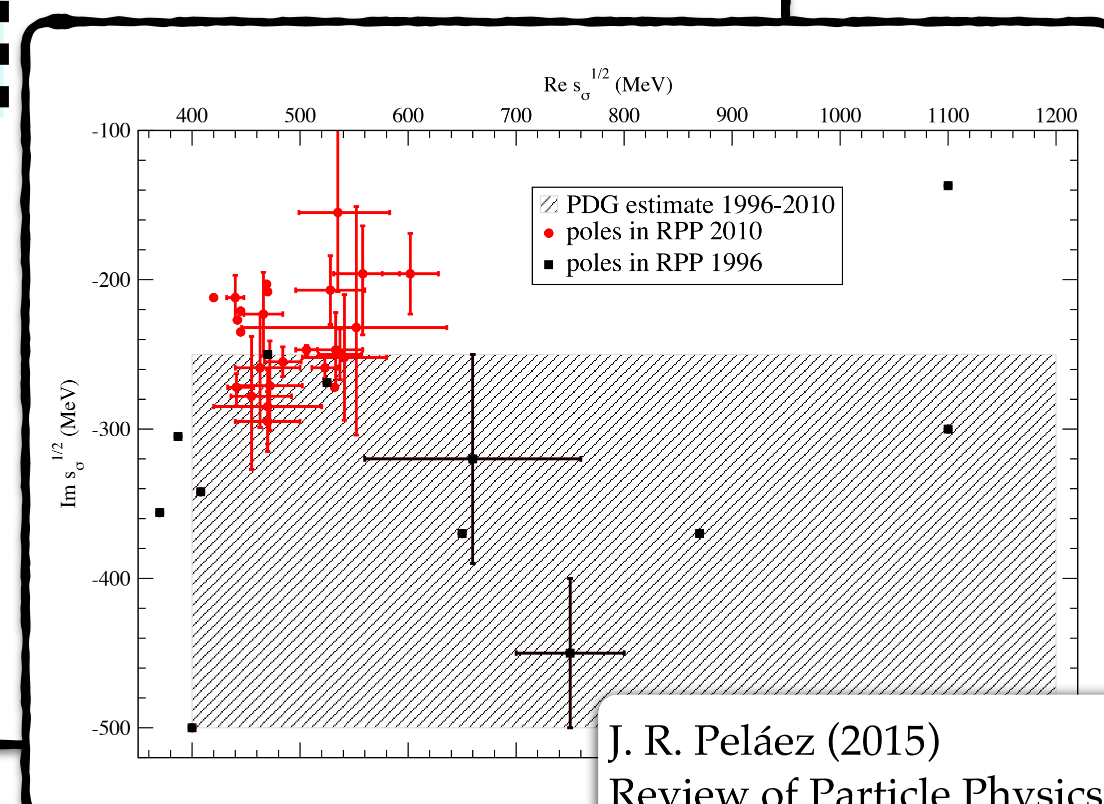
disp. +exp. = Peláez (2015), Caprini, et al. (2006), & Garcia-Martin et al. (2011)

The σ / f₀(500) vs m_π

$$s_0 = (E_\sigma - \tfrac{i}{2}\Gamma_\sigma)^2, \qquad g^2_{\sigma\pi\pi} = \lim_{s\to s_0}(s_0 - s)\,t(s)$$



Historical perspective



J. R. Peláez (2015)
Review of Particle Physics (RPP)

Unitarized χPT

$$\mathcal{M}_{\mathrm{U}\chi\mathrm{PT}} = \mathcal{M}_{\mathrm{LO}} \frac{1}{\mathcal{M}_{\mathrm{LO}} - \mathcal{M}_{\mathrm{NLO}}} \mathcal{M}_{\mathrm{LO}}$$

$$\begin{aligned}
S &= 1 + 2i\sigma\mathcal{M} \\
\mathcal{M} &= (\mathrm{Re}(\mathcal{M}^{-1}) - i\sigma)^{-1} \\
\mathcal{M}^{-1} &= \mathcal{M}_{\mathrm{LO}}^{-1} \frac{1}{1 + \mathcal{M}_{\mathrm{LO}}^{-1}\,\mathcal{M}_{\mathrm{NLO}} + \ldots} = \mathcal{M}_{\mathrm{LO}}^{-1}\left(1 - \mathcal{M}_{\mathrm{LO}}^{-1}\,\mathcal{M}_{\mathrm{NLO}} + \ldots\right) \\
\mathrm{Re}(\mathcal{M}^{-1}) &= \mathcal{M}_{\mathrm{LO}}^{-1}\left(1 - \mathcal{M}_{\mathrm{LO}}^{-1}\,\mathrm{Re}(\mathcal{M}_{\mathrm{NLO}}) + \ldots\right)
\end{aligned}$$

Dobado and Pelaez (1997)

Oller, Oset, and Pelaez (1998)

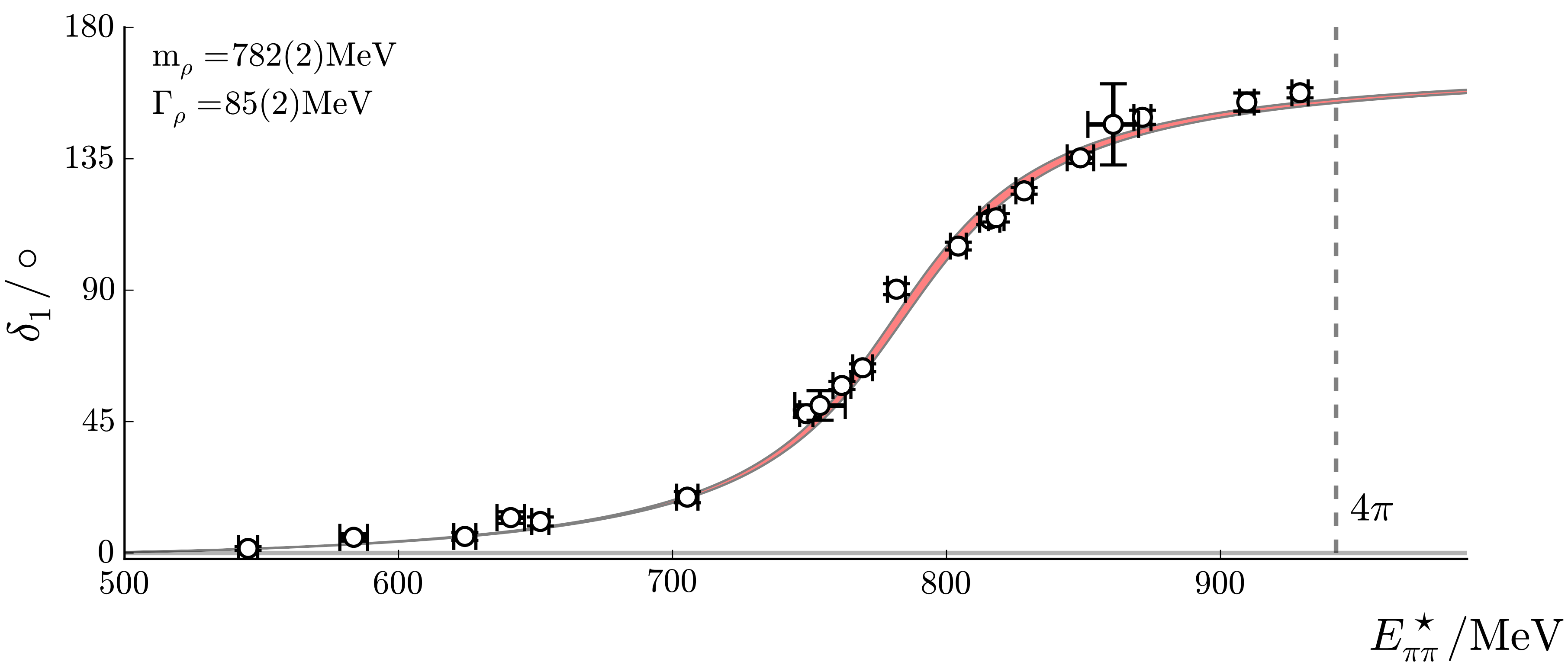
Oller, Oset, and Pelaez (1999)

Chiral fit

$$\alpha_1 \equiv -2\ell_1^r + \ell_2^r,\ \alpha_2 \equiv \ell_4^r$$

$$\alpha_1(770\text{ MeV}) = 14.7(4)(2)(1) \times 10^{-3}$$

$$\alpha_2(770\text{ MeV}) = -28(6)(3)\binom{01}{11} \times 10^{-3}$$

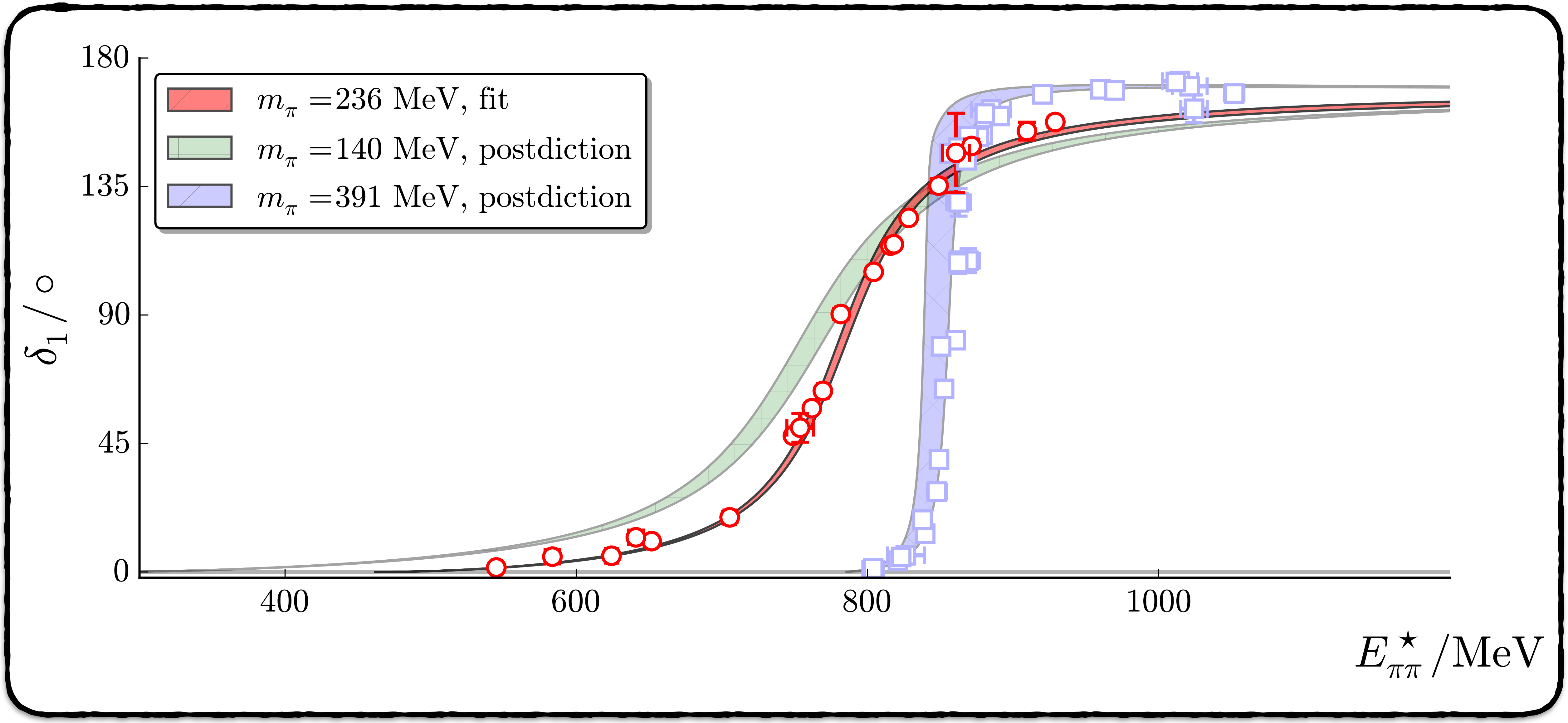


previos results:

$$\alpha_1(770\text{ MeV}) \in [9, 13] \times 10^{-3}$$

$$\alpha_2(770\text{ MeV}) \in [1, 12] \times 10^{-3}$$

m_π dependence



σ / $f_0(500)$ vs m_π