

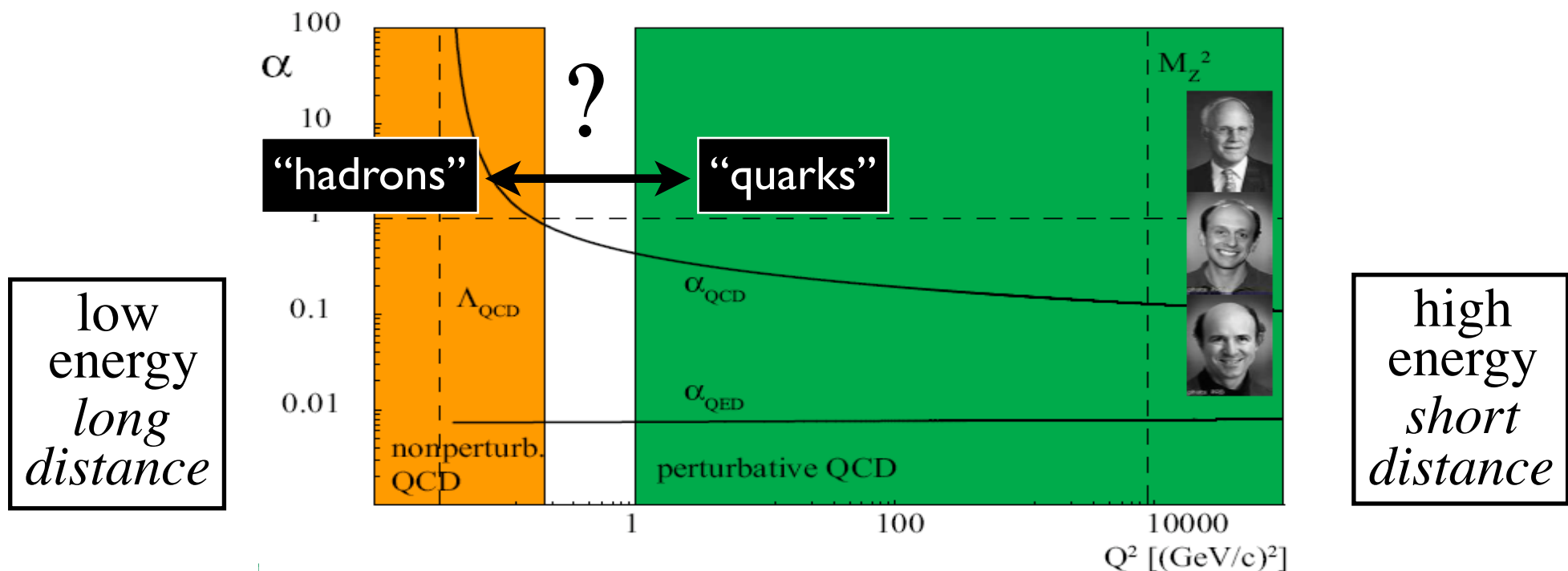
Connecting resonance and deep-inelastic phenomena through quark-hadron duality

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Outline

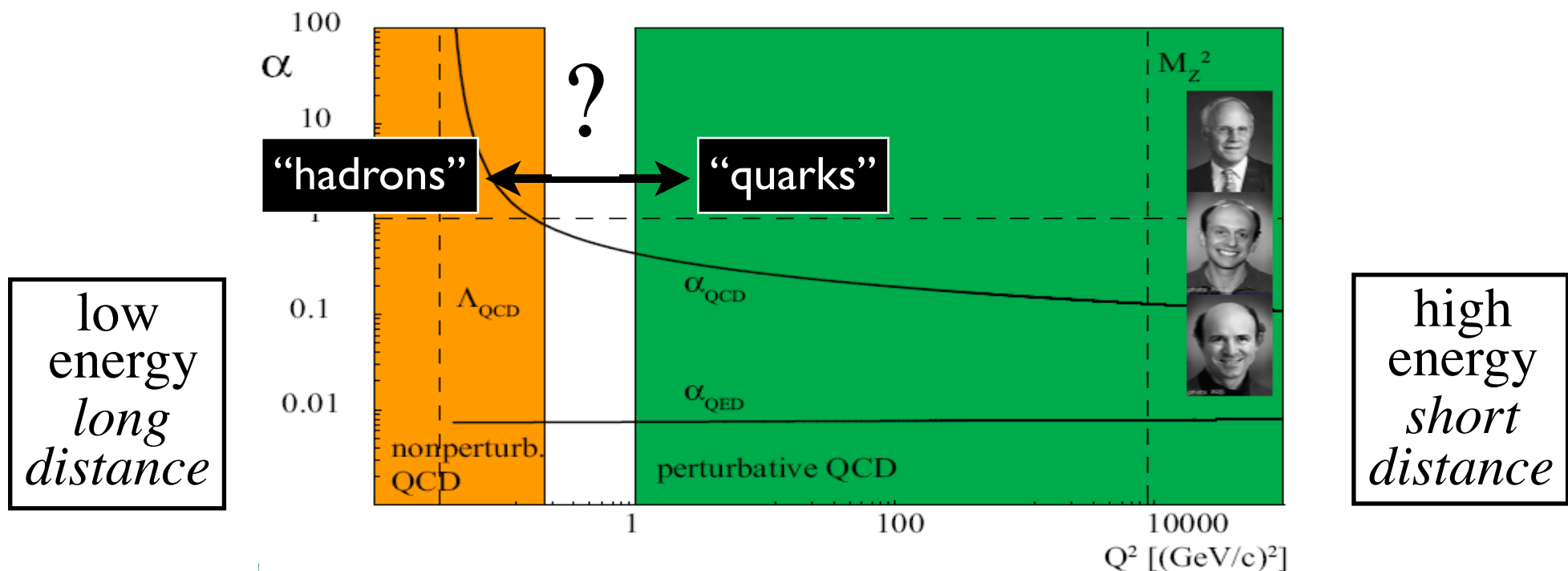
- Historical perspective
- Duality in QCD
 - resonances & higher twists
- Local duality
 - truncated moments
 - insights from models
- Applications of duality
 - global PDF analysis
 - single-hadron production
- Outlook



- Duality hypothesis: complementarity between *quark* and *hadron* descriptions of observables

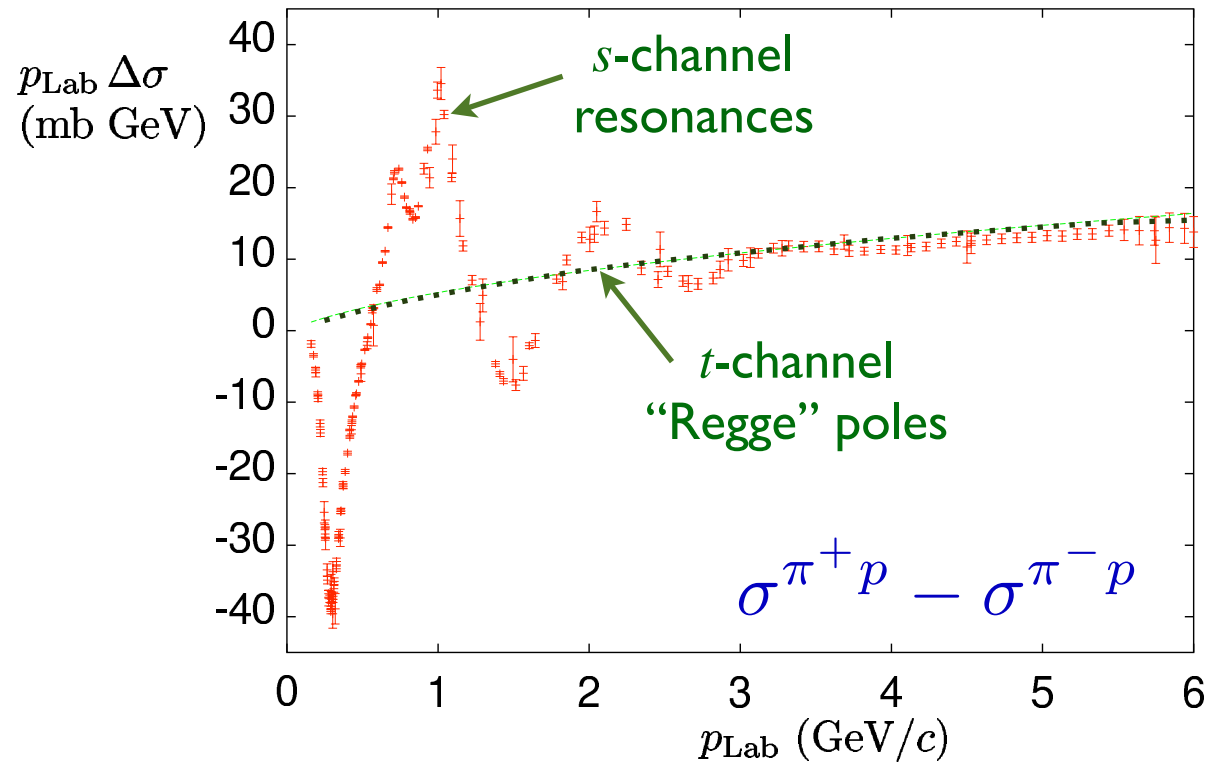
$$\sum_{\text{hadrons}} = \sum_{\text{quarks}}$$

→ can use either set of *complete* basis states to describe physical phenomena

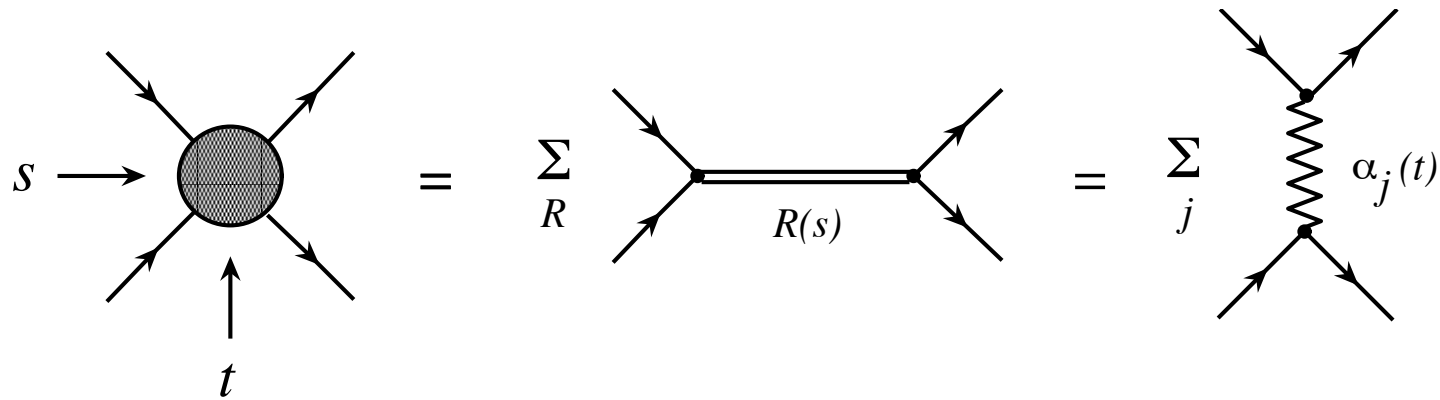


- In practice, at *finite energy* typically access only *limited* set of basis states
- Question is not “*why* duality exists”, but
 - *how* it arises?
 - how can we make *use* of it (in a controlled way)?

Duality in hadron-hadron scattering



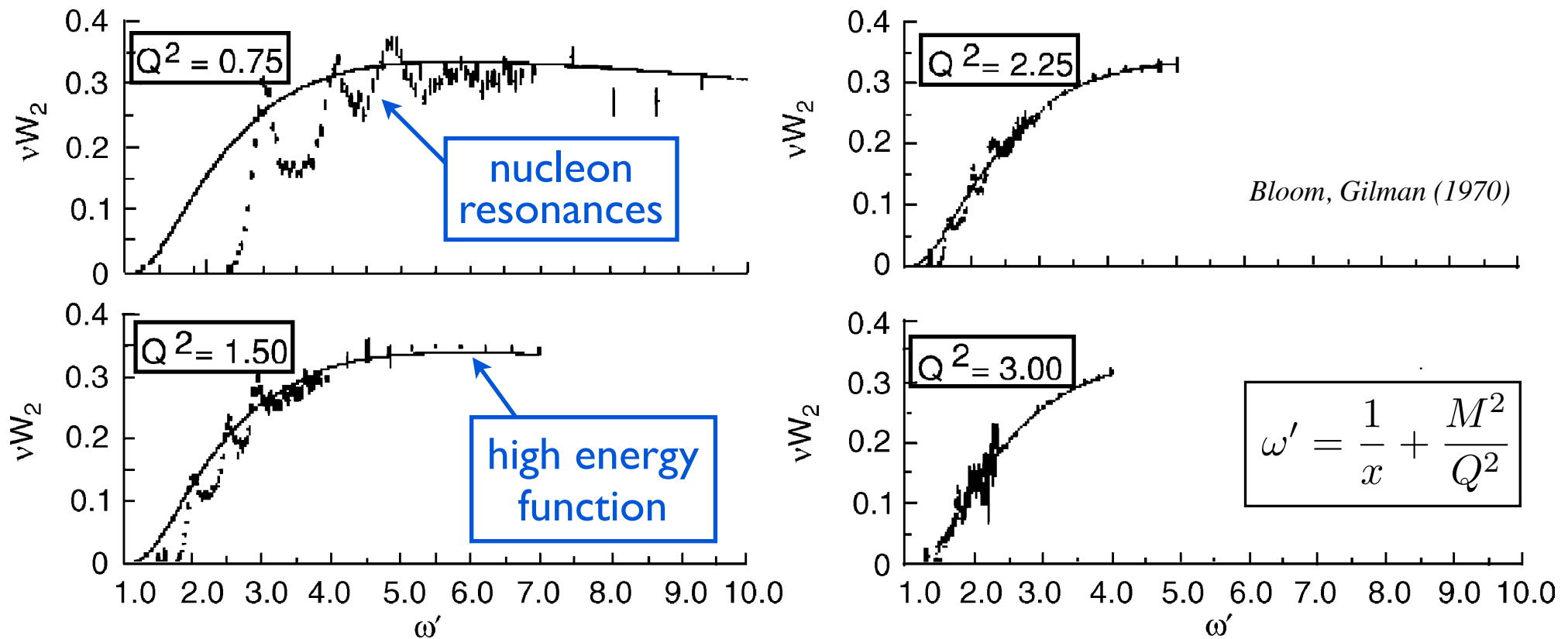
Igi (1962)
Dolen, Horn, Schmidt (1968)



$s - t$ channel duality

Duality in electron-proton scattering

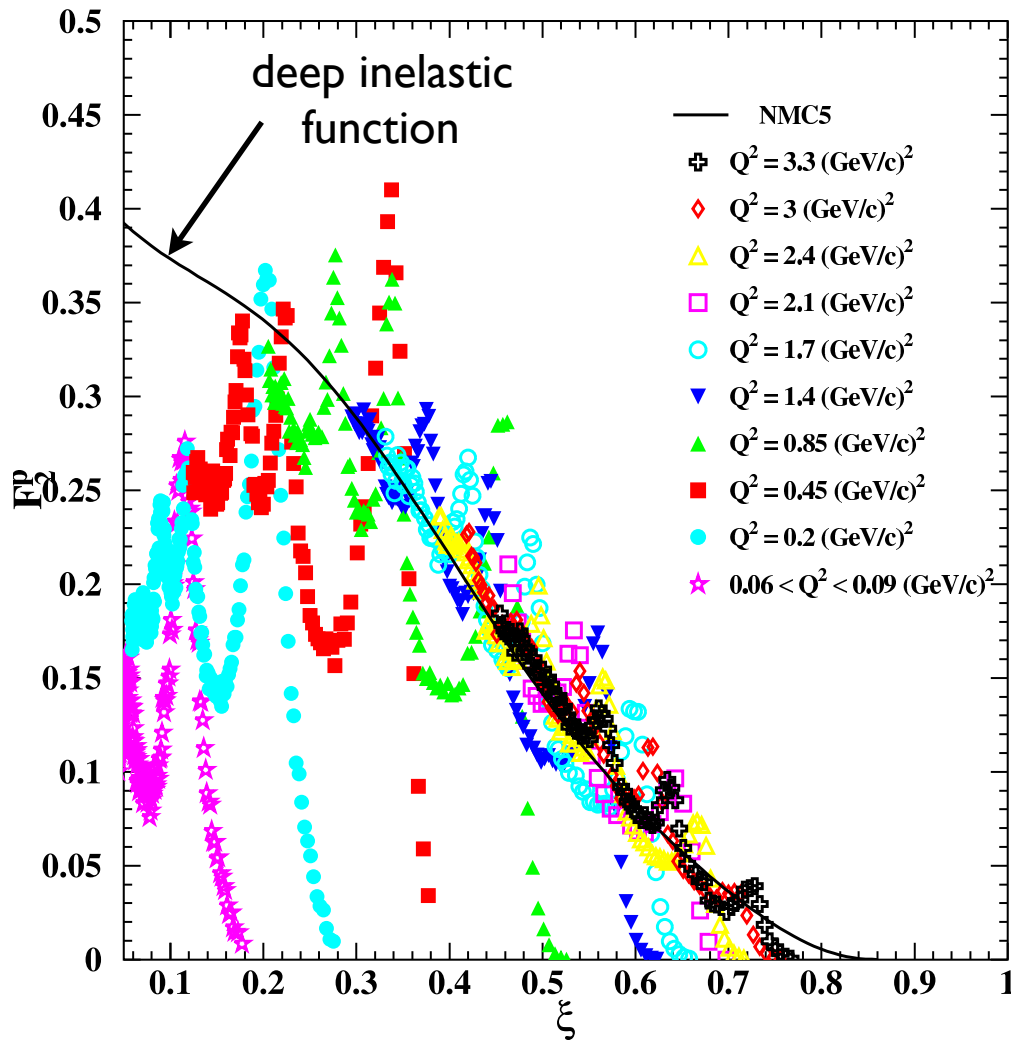
“Bloom-Gilman duality”



“hadrons” $\frac{2M}{Q^2} \int_0^{\nu_m} d\nu \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega' \nu W_2(\omega')$ “quarks”

finite-energy sum rules

Duality in electron-proton scattering



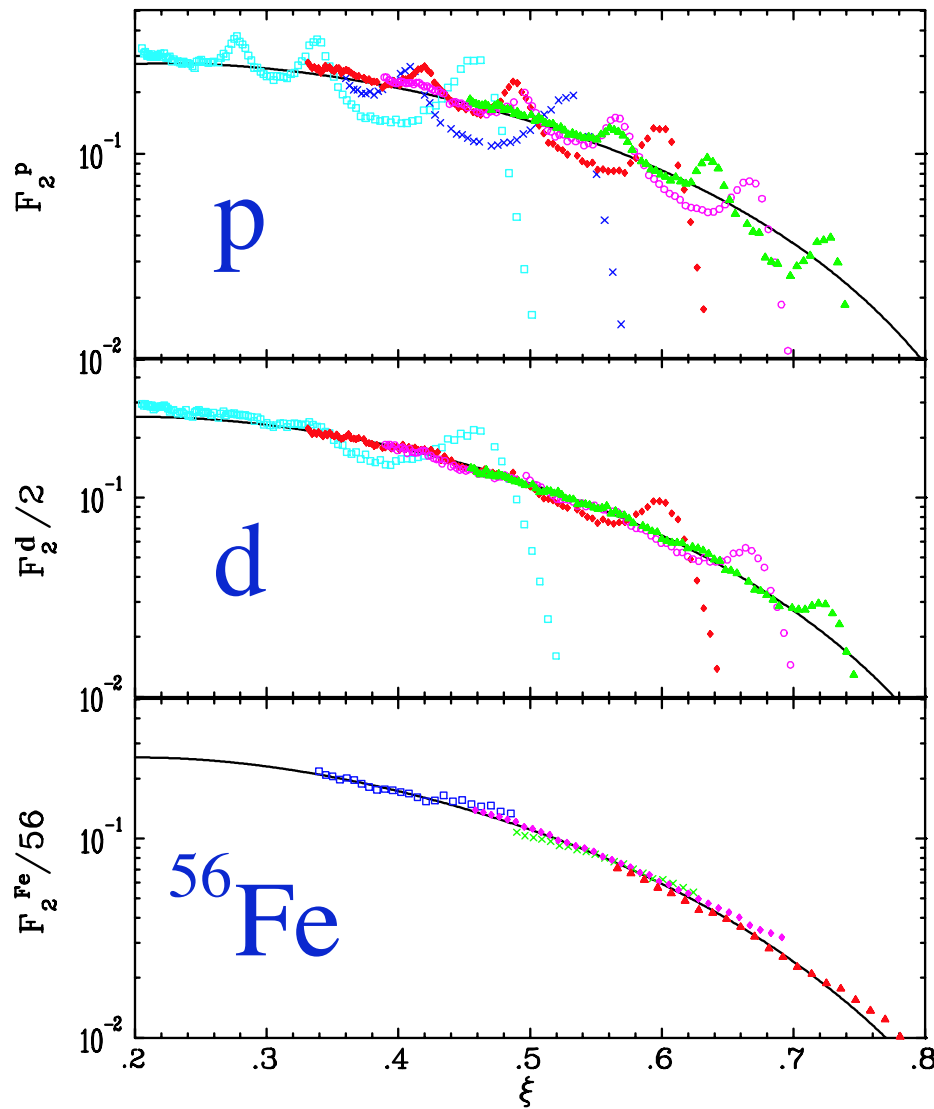
Niculescu et al. (2000)

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2 x^2 / Q^2}}$$

■ average over resonances
(strongly Q^2 dependent)

$\approx Q^2$ independent
scaling function

Duality in electron-nucleus scattering

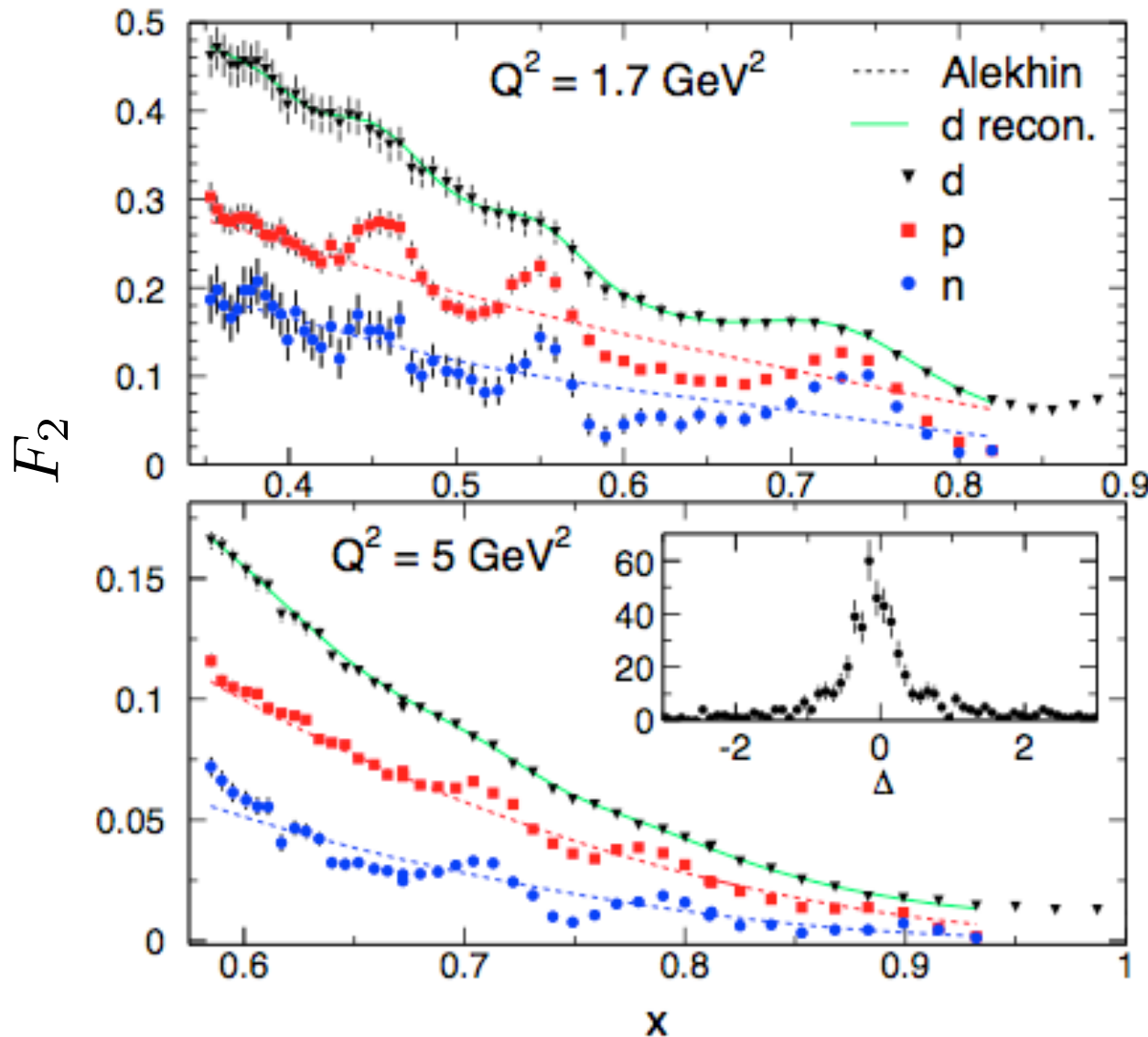


WM, Ent, Keppel (2005)

- further resonance averaging from Fermi smearing in *nuclear* structure functions

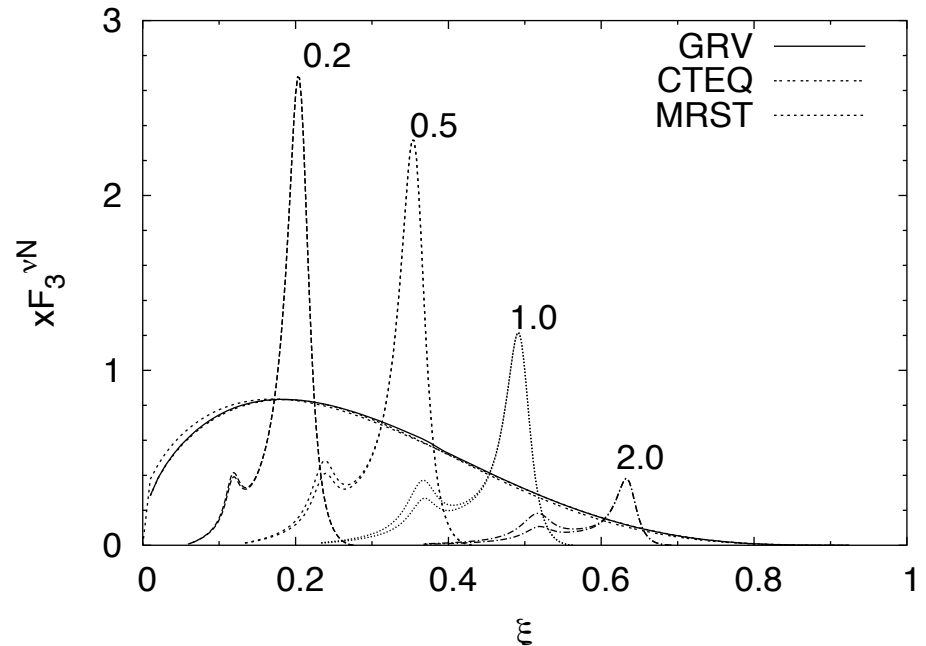
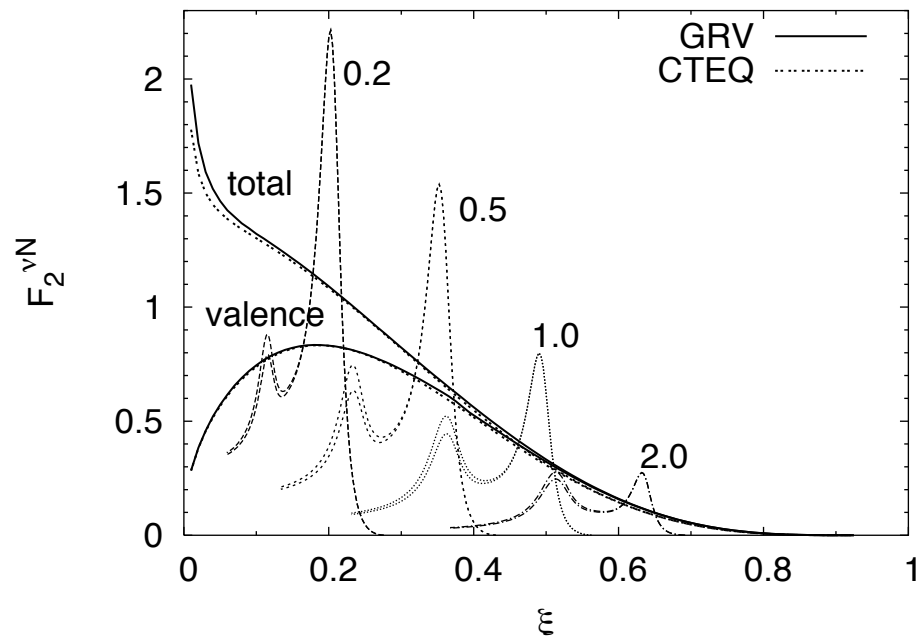
Duality in electron-neutron scattering

No free neutron targets, but (new) iterative method allows neutron resonance structure function to be extracted



■ evidence for duality also in neutron!

Duality in neutrino-nucleon scattering



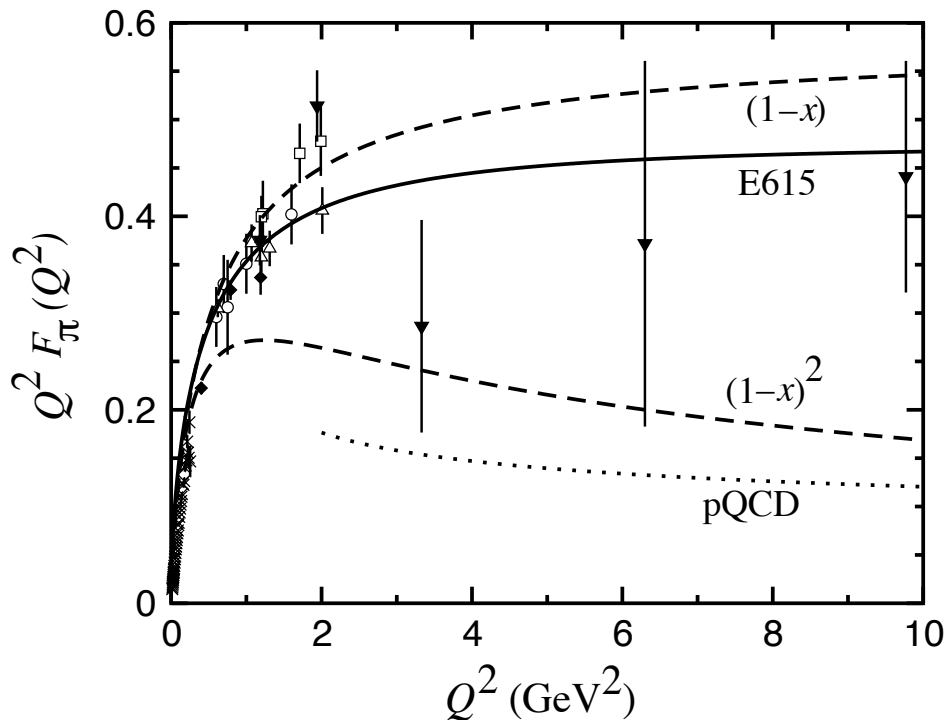
Lalakulich, WM, Paschos (2007)

- indications of duality in neutrino structure functions
from models of weak transition matrix elements
from resonance neutrino-production data (FNAL, ANL)

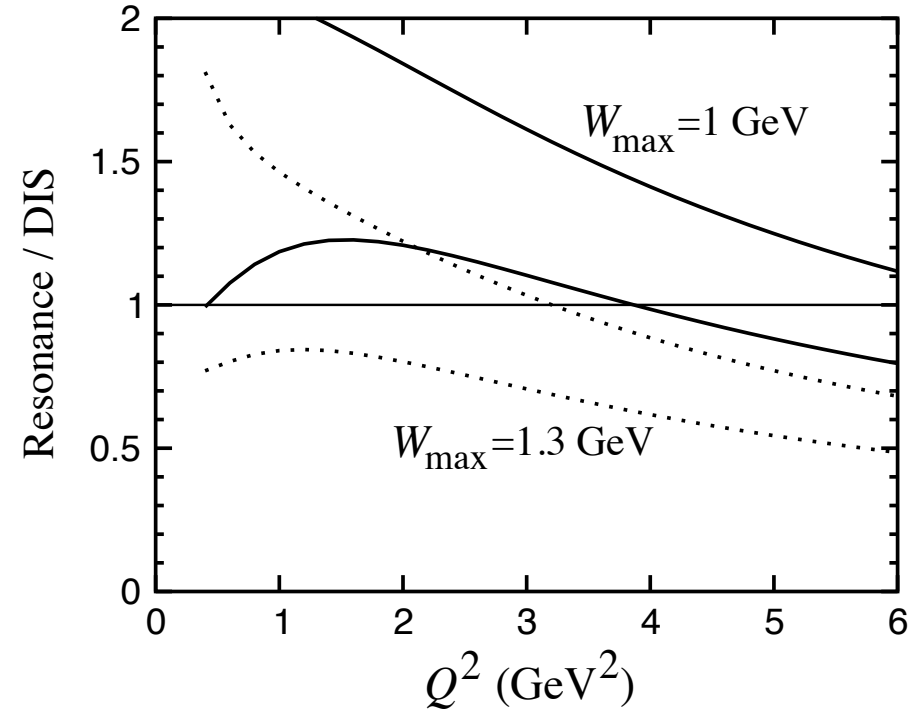
Duality in electron-pion scattering (?)

Extend finite-energy sum rule to threshold region

$$[F_\pi(Q^2)]^2 \approx \int_1^{\omega_{\max}} d\omega \, \nu W_2^\pi(\omega) \quad \omega = 1/x, \quad \omega_{\max} = 1 + (W_{\max}^2 - m_\pi^2)/Q^2$$



→ Q^2 dependence of FF correlated with $x \rightarrow 1$ behavior of SF



→ nontrivial relation between L and T cross sections?

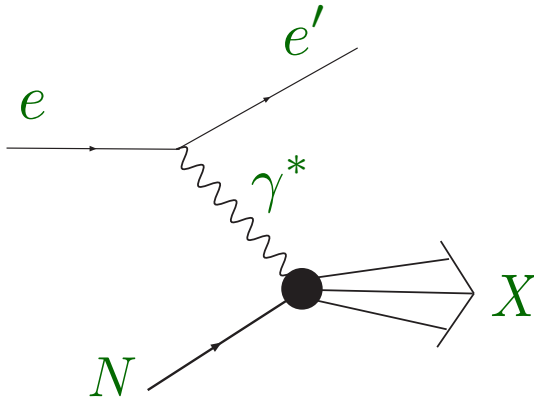
$$F_\pi^2 + \left(1 + \frac{m_\rho^2 - m_\pi^2}{Q^2}\right) F_{\pi\rho}^2$$

Duality in QCD

— *global duality* —

Duality and QCD

■ Kinematics of inclusive deep-inelastic scattering (DIS)



$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2 \frac{\theta}{2}}{Q^4} \left(2 \tan^2 \frac{\theta}{2} \frac{F_1}{M} + \frac{F_2}{\nu} \right)$$

$$\nu = E - E'$$

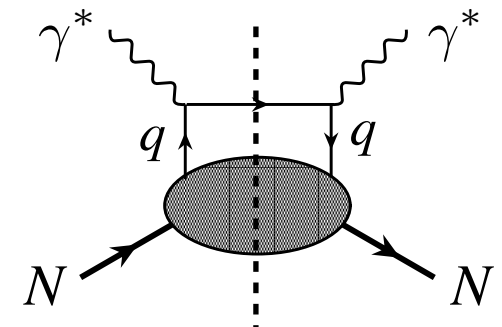
$$Q^2 = \vec{q}^2 - \nu^2$$

$$W^2 = M^2 + Q^2 \frac{(1-x)}{x}$$

$$x = \frac{Q^2}{2M\nu}$$

■ In *deep-inelastic* region ($W \gtrsim 2 \text{ GeV}$, $Q^2 \gtrsim 1 \text{ GeV}^2$) structure functions given by parton distributions

$$F_2(x, Q^2) \stackrel{\text{LO}}{=} x \sum_q e_q^2 q(x, Q^2)$$



Duality and QCD

■ Operator product expansion in QCD

→ expand *moments* of structure functions in powers of $1/Q^2$

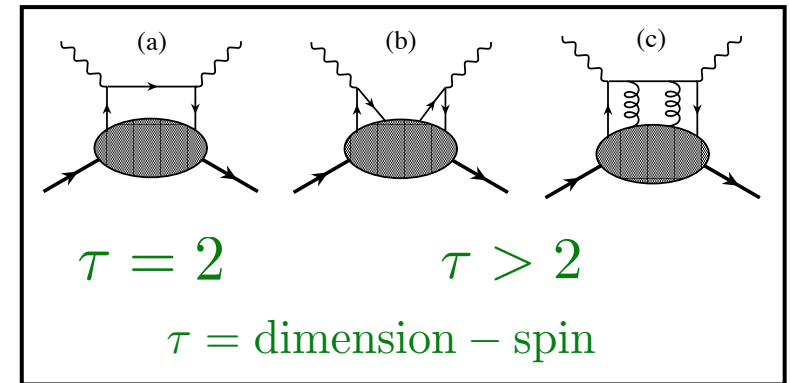
$$M_n(Q^2) = \int_0^1 dx \, x^{n-2} F_2(x, Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \dots$$

matrix elements of operators
with specific “twist” τ

$$e.g. \langle N | \bar{\psi} \gamma^+ \psi | N \rangle$$

$$\langle N | \bar{\psi} \tilde{G}^{+\nu} \gamma_\nu \psi | N \rangle$$

etc.



Duality and QCD

■ Operator product expansion in QCD

→ expand *moments* of structure functions in powers of $1/Q^2$

$$\begin{aligned} M_n(Q^2) &= \int_0^1 dx \, x^{n-2} F_2(x, Q^2) \\ &= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \dots \end{aligned}$$

■ If moment $\approx$ independent of Q^2

→ “higher twist” terms $A_n^{(\tau>2)}$ small

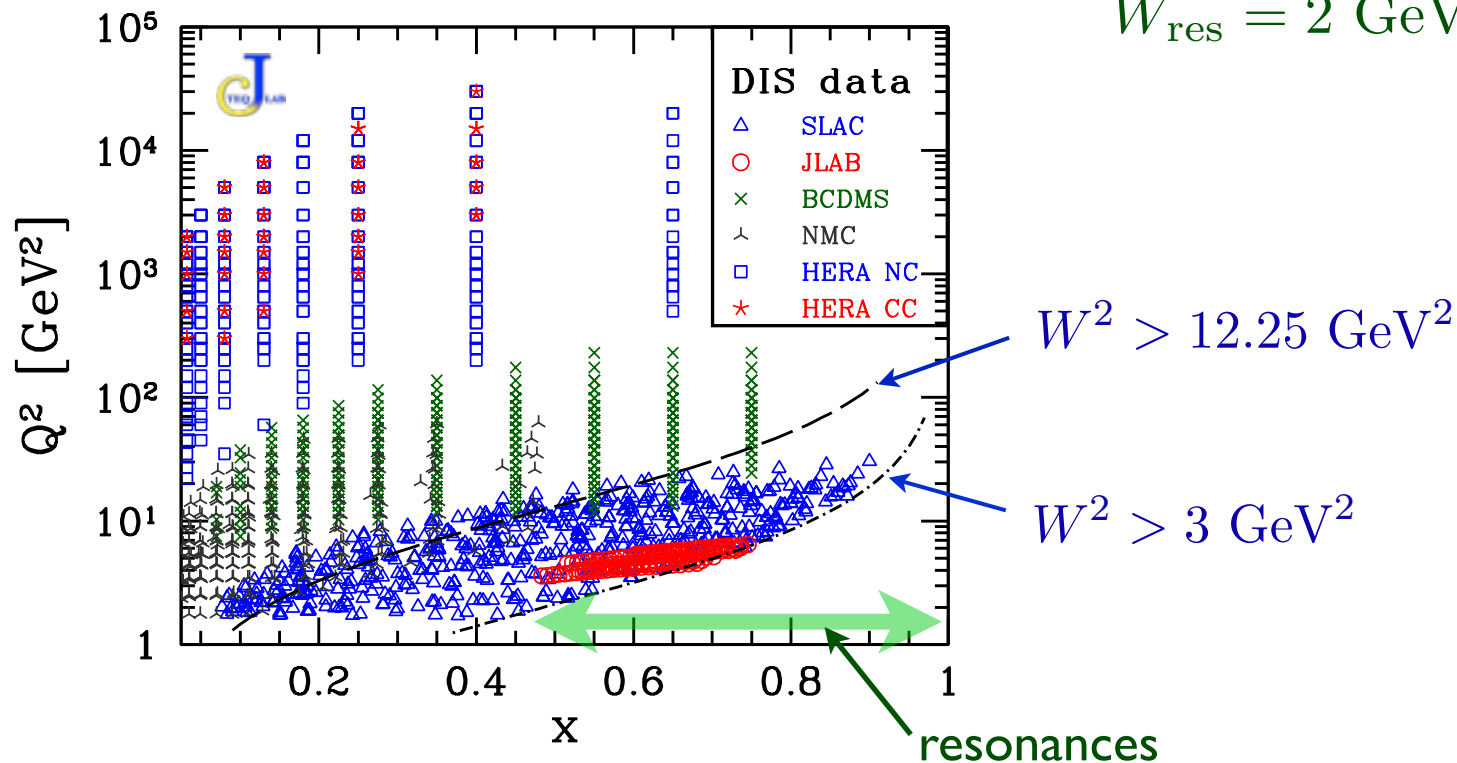
■ Duality $\longleftrightarrow$ suppression of higher twists

Duality and QCD

- Note: at finite Q^2 , from kinematics *any* moment of *any* structure function (of *any* twist) must, by definition, include the resonance region

$$W^2 = M^2 + Q^2 \frac{(1-x)}{x} \quad \longrightarrow \quad x_{\text{res}} = \frac{Q^2}{W_{\text{res}}^2 - M^2 + Q^2}$$

$$W_{\text{res}} = 2 \text{ GeV} \quad \Longrightarrow \quad x_{\text{res}} \approx 0.24 \quad \text{at } Q^2 = 1 \text{ GeV}^2$$



Duality and QCD

- Note: at finite Q^2 , from kinematics *any* moment of *any* structure function (of *any* twist) must, by definition, include the resonance region
- Resonance and DIS regions intimately connected
 - resonances an *integral* part of scaling structure function
 - e.g.* in large- N_c limit, spectrum of zero-width resonances is “maximally dual” to quark-level (smooth) structure function

Local Duality

— *truncated moments* —

Truncated moments

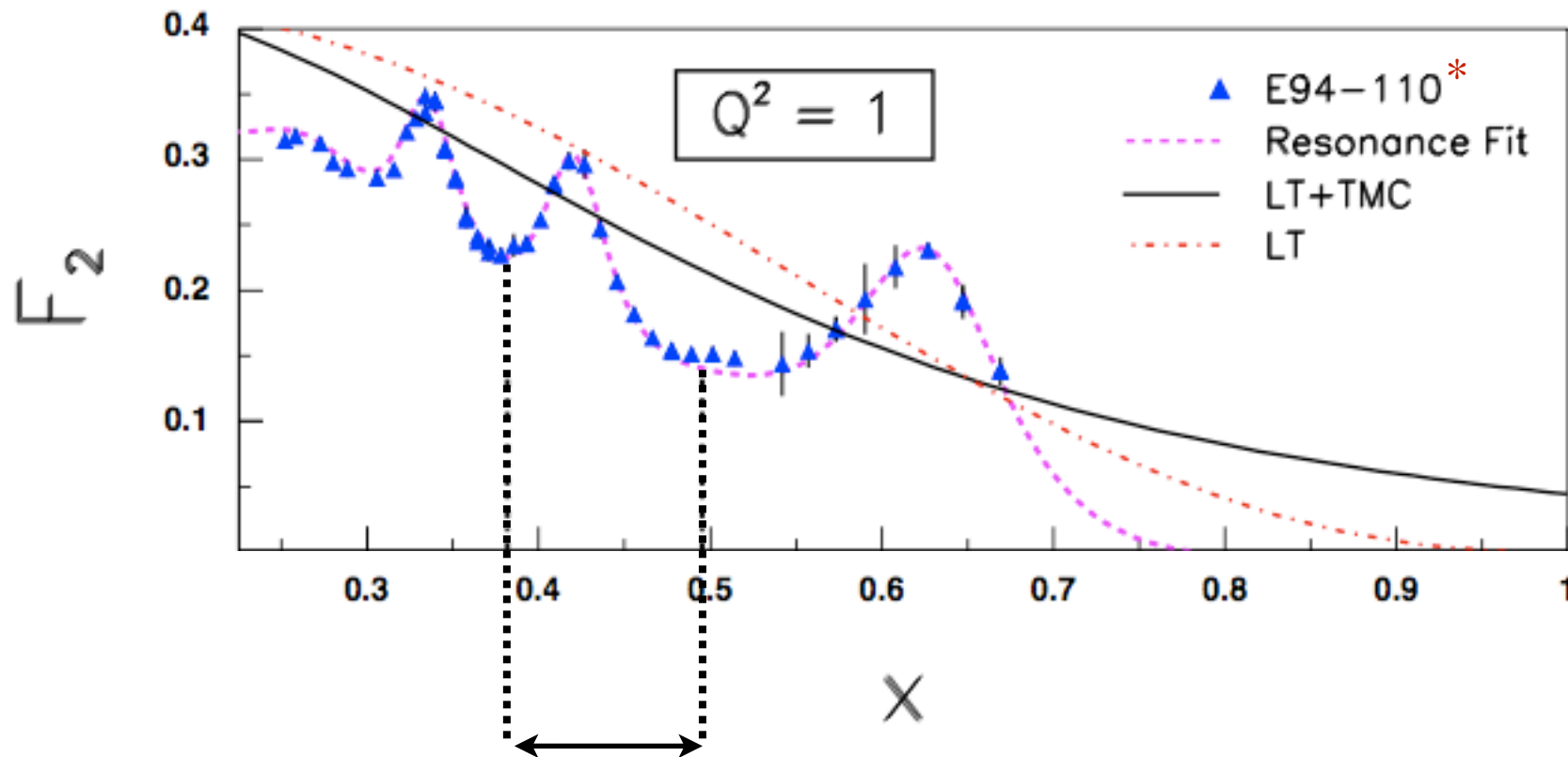
- Complete moments can be studied via twist expansion
 - Bloom–Gilman duality has a precise meaning
(*i.e.*, duality violation = higher twists)
- Rigorous connection between local duality & QCD difficult
 - need prescription for how to average over resonances
- *Truncated* moments allow study of restricted regions in x (or W) within pQCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx \, x^{n-2} F_2(x, Q^2)$$

Forte, Magnea (1999)

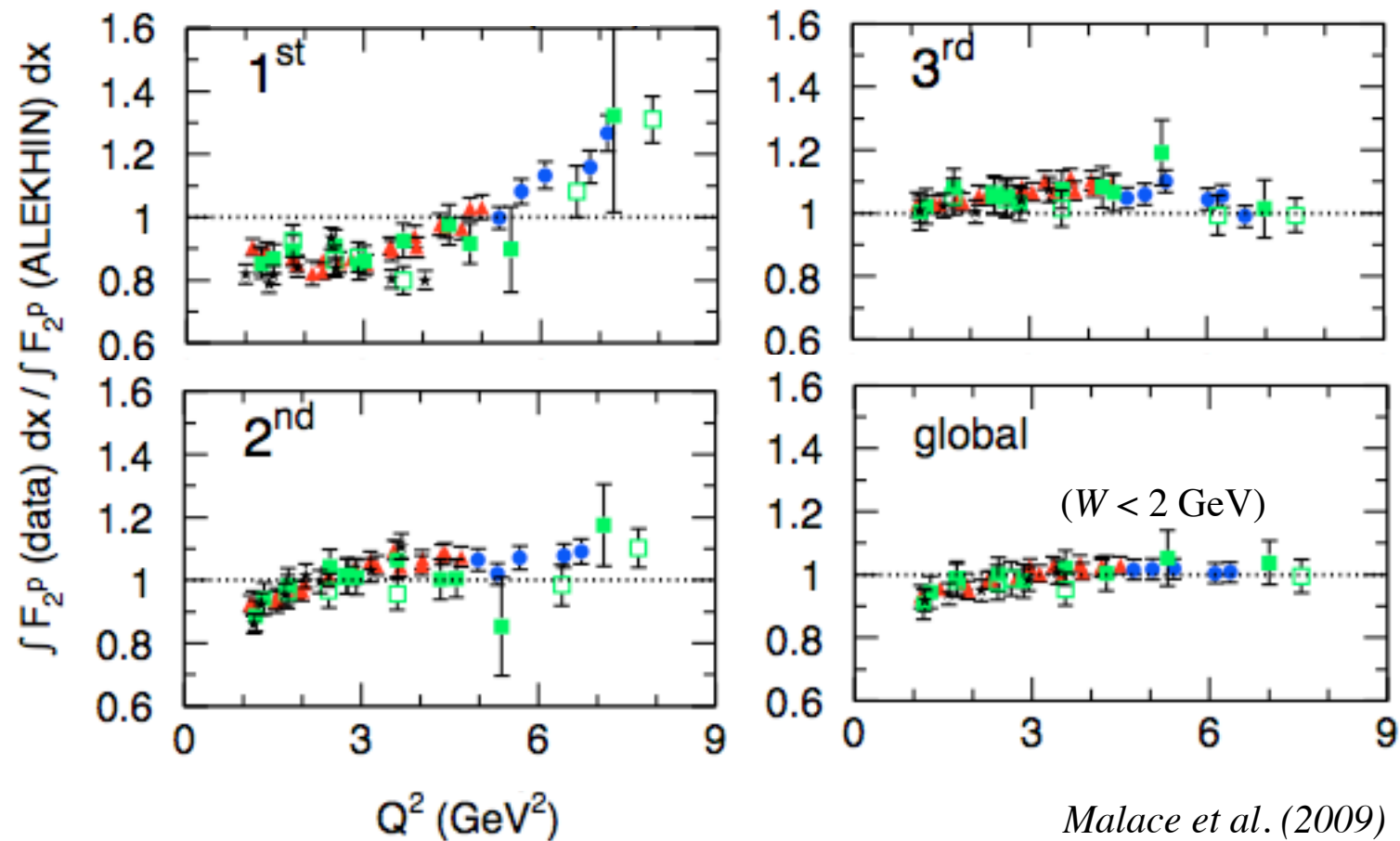
Psaker, Malace, Keppel, WM (2008)

Truncated moments



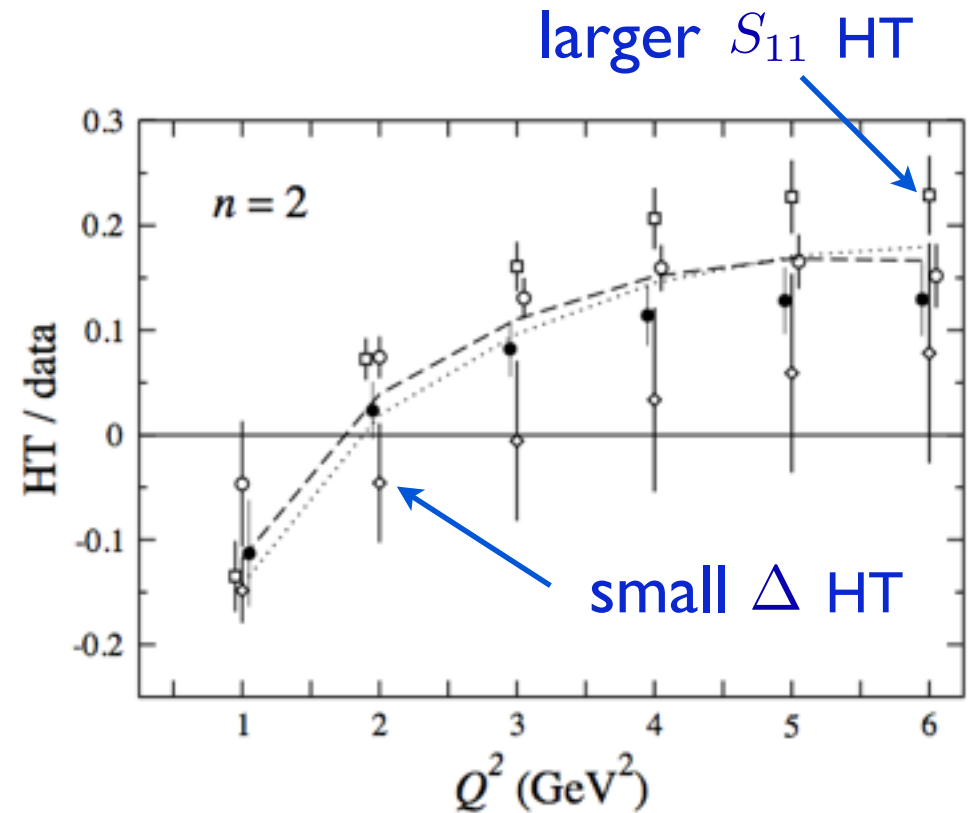
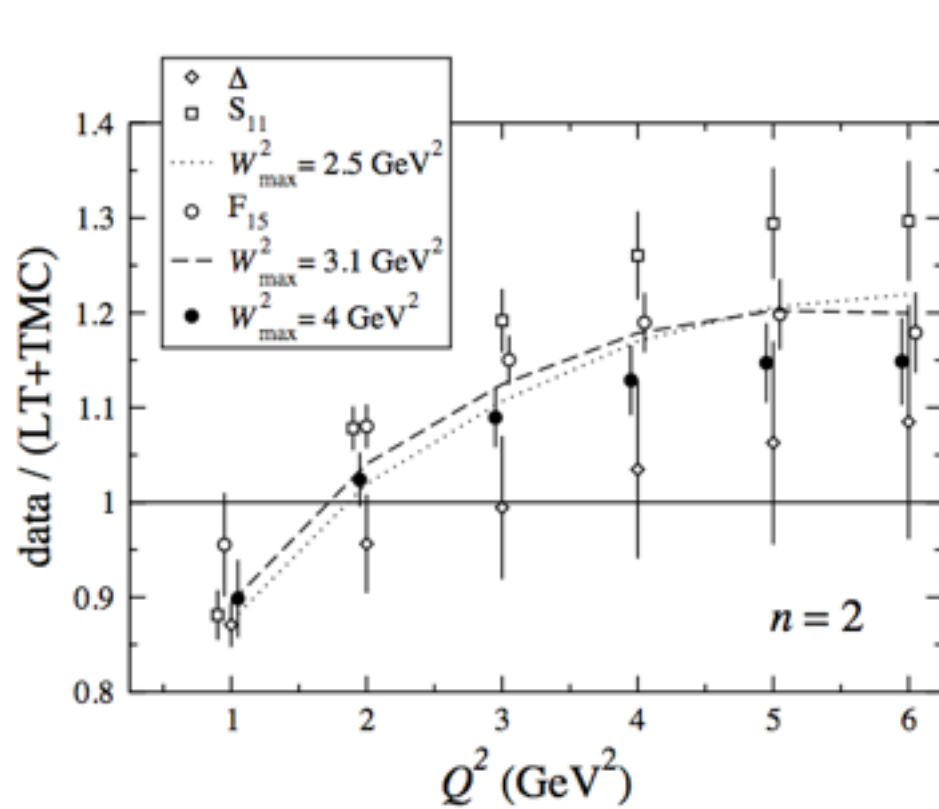
how much of this region is leading twist ?

Truncated moments



→ duality appears in various resonance regions

Truncated moments



→ higher twists < 10–15% for $Q^2 > 1 \text{ GeV}^2$

Resonances & twists

- Total “higher twist” is *small* at scales $Q^2 \sim \mathcal{O}(1 \text{ GeV}^2)$
- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)
 - nontrivial interference between resonances
- Can we understand this dynamically, at quark level?
- Can we use resonance region data to learn about *leading twist* structure functions (and *vice versa*)?
 - expanded data set has potentially significant implications for global quark distribution studies

Local Duality

— *insights from models* —

Scaling functions from resonances

■ Earliest attempts predate QCD

→ *e.g.* harmonic oscillator spectrum $M_n^2 = (n + 1)\Lambda^2$
including states with spin = $1/2, \dots, n+1/2$

(n even: $I = 1/2$, n odd: $I = 3/2$)

Domokos et al. (1971)

→ at large Q^2 magnetic coupling dominates

$$G_n(Q^2) = \frac{\mu_n}{(1 + Q^2 r^2 / M_n^2)^2} \quad r^2 \approx 1.41$$

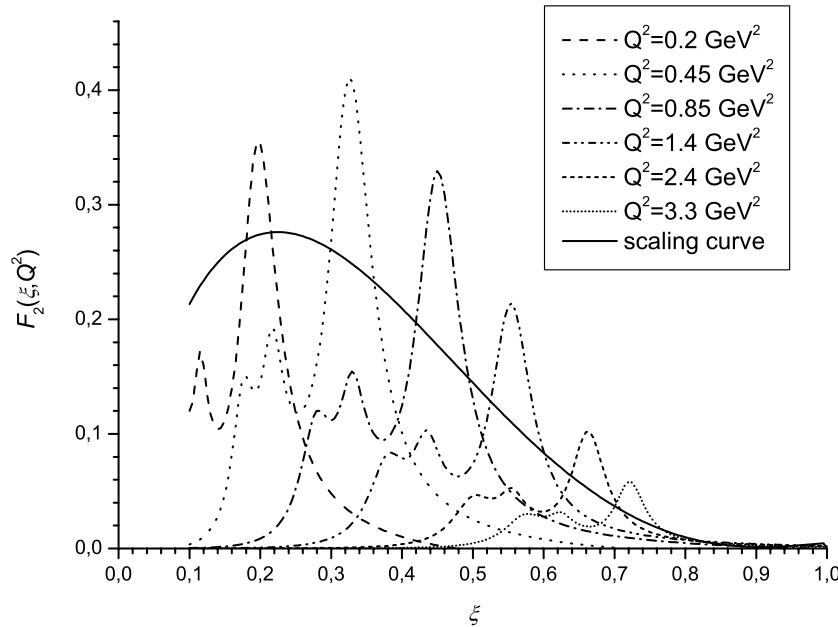
→ in Bjorken limit, $\sum_n \longrightarrow \int dz$, $z \equiv M_n^2 / Q^2$

$$F_2 \sim (\omega' - 1)^{1/2} (\mu_{1/2}^2 + \mu_{3/2}^2) \int_0^\infty dz \frac{z^{3/2} (1 + r^2/z)^{-4}}{z + 1 - \omega' + \Gamma_0^2 z^2}$$

→ scaling function of $\omega' = \omega + M^2 / Q^2$ ($\omega = 1/x$)

Scaling functions from resonances

■ Phenomenological analyses at finite Q^2



21 isospin-1/2 & 3/2
resonances (mass < 2 GeV)

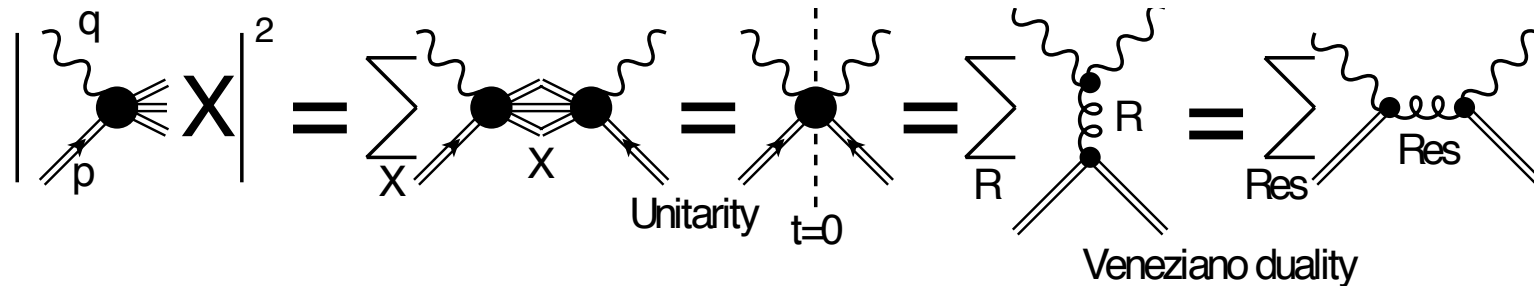
Davidovsky, Struminsky (2003)

→ valence-like structure of dual function suggests
“two-component duality”:

- valence (Reggeon exchange) dual to resonances $F_2^{(\text{val})} \sim x^{0.5}$
- sea (Pomeron exchange) dual to background $F_2^{(\text{sea})} \sim x^{-0.08}$

Scaling functions from resonances

■ Explicit realization of Veneziano & Bloom-Gilman duality



$$V(s, t) = \frac{\Gamma(1 - \alpha(s))\Gamma(1 - \alpha(t))}{\Gamma(2 - \alpha(s) - \alpha(t))}$$

$$\rightarrow s^{\alpha(t)} \text{ high } s, \text{ low } |t|$$

→ Veneziano model not unitary,
has no imaginary parts

→ generalization of narrow-resonance approximation,
with nonlinear, complex Regge trajectories

$$D(s, t) = \int_0^1 dz \left(\frac{z}{g} \right)^{-\alpha_s(s(1-z))-1} \left(\frac{1-z}{g} \right)^{-\alpha_t(tz)-1}$$

“dual amplitude with Mandelstam analyticity” (DAMA) model

Jenkovszky et al.

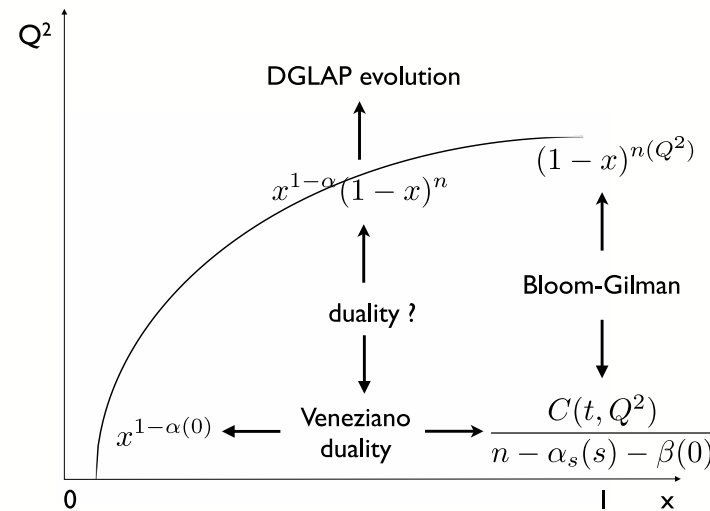
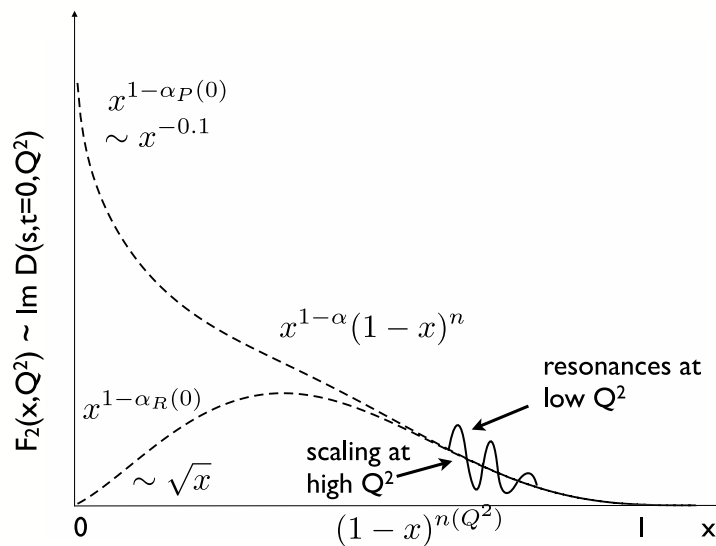
Scaling functions from resonances

■ Explicit realization of Veneziano & Bloom-Gilman duality

→ for large x and Q^2 , have power-law behavior

$$F_2 \sim (1-x)^{2\alpha_t(0) \ln 2g / \ln g}$$

where parameter g can be Q^2 dependent



Jenkowszky, Magas, Londergan, Szczepaniak (2012)

Applications of Duality

CTEQ-JLab (CJ) global PDF analysis

- Global QCD analysis of high-energy scattering data, including large- x , low- Q^2 region
- Systematically study effects of Q^2 & W cuts

cut0: $Q^2 > 4 \text{ GeV}^2$, $W^2 > 12.25 \text{ GeV}^2$

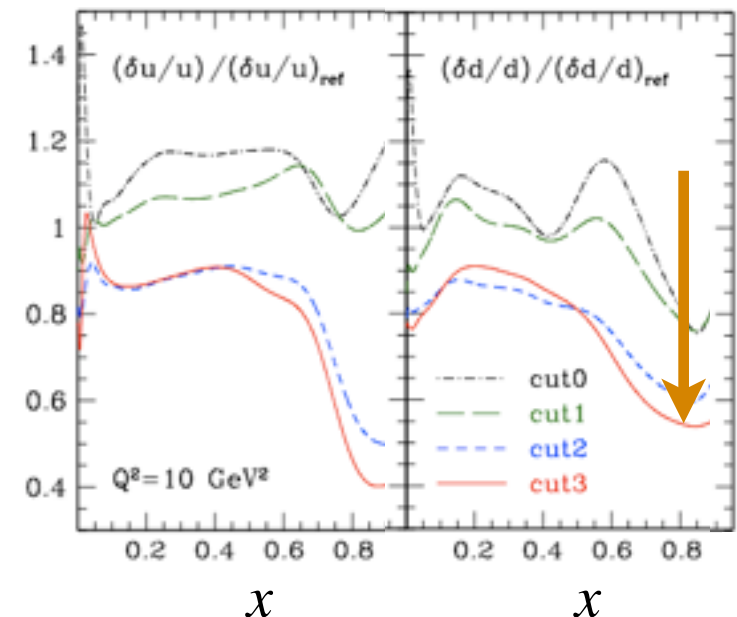
cut1: $Q^2 > 3 \text{ GeV}^2$, $W^2 > 8 \text{ GeV}^2$

cut2: $Q^2 > 2 \text{ GeV}^2$, $W^2 > 4 \text{ GeV}^2$

cut3: $Q^2 > m_c^2$, $W^2 > 3 \text{ GeV}^2$

factor 2 increase
in DIS data from
cut0 $\rightarrow$ cut3

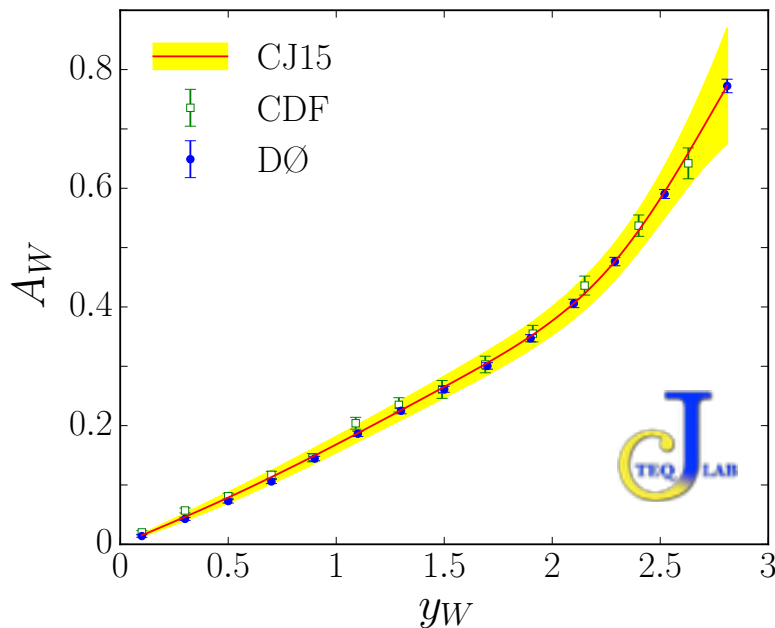
- $\rightarrow$ larger database with weaker cuts significantly reduced errors, especially at large x
- $\rightarrow$ up to $\sim 40\text{--}60\%$ error reduction when cuts extended into near-resonance region



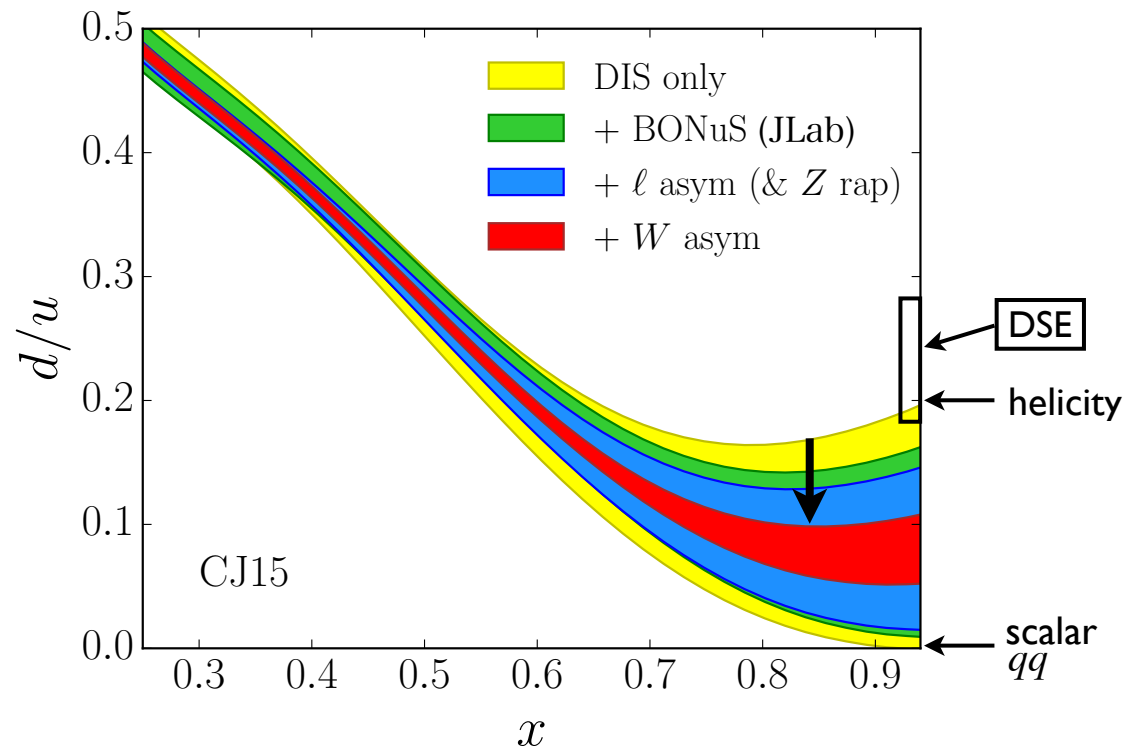
CTEQ-JLab (CJ) global PDF analysis

Valence d/u ratio at high x

→ significant reduction of PDF errors with new JLab tagged neutron & FNAL W -asymmetry data



Accardi, WM, Owens (2016)



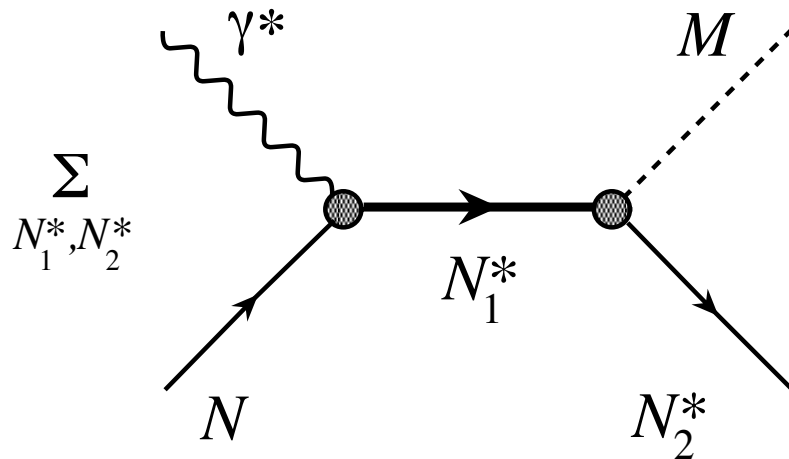
→ extrapolated ratio at $x = 1$

$$d/u \rightarrow 0.09 \pm 0.03$$

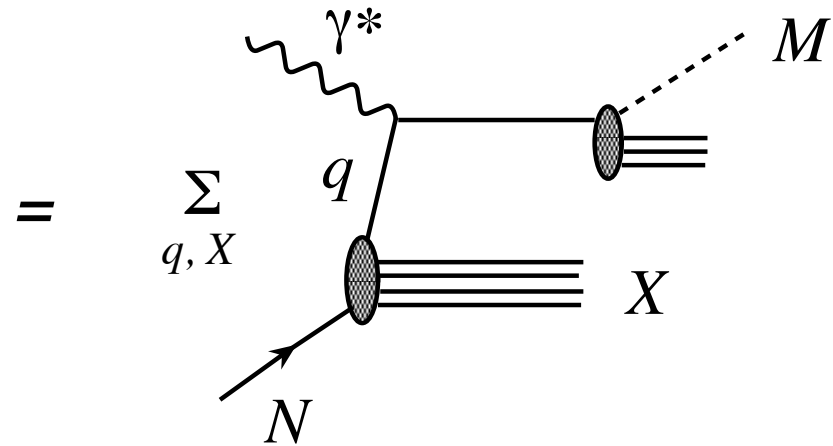
→ upcoming experiments at JLab (MARATHON, BONuS, SoLID) will determine d/u up to $x \sim 0.85$

Duality in (semi-inclusive) meson production

- Extend duality to less inclusive processes, such as meson electroproduction



s-channel resonance
excitation and decay



parton level scattering
and fragmentation

$$\sum_{N_2^*} \left| \sum_{N_1^*} F_{\gamma N \rightarrow N_1^*}(Q^2, M_1^*) \mathcal{D}_{N_1^* \rightarrow N_2^* M}(M_1^*, M_2^*) \right|^2 = \sum_q e_q^2 q(x, Q^2) D_q^M(z, Q^2)$$

Afanasev, Carlson, Wahlquist (2000)
Hoyer (2002)
Close, WM (2009)

Outlook

- Confirmation of duality (experimentally & theoretically) suggests origin in dynamical cancelations between resonances
 - explore more realistic descriptions based on phenomenological $\gamma^* NN^*$ form factors (→ CLAS N^* program)
 - incorporate nonresonant background in same framework

- Practical application of duality
 - use resonance region data to constrain PDFs at high x
 - application to other processes, *e.g.*, semi-inclusive DIS, DVCS / GPDs, ...